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Editorial by William Baker

This issue contains teaching research articles from a variety of countries in the global community of mathematics teacher-researchers investigating the use of internet-based technology, flexible teaching methods, ethnomathematics, realistic mathematics problems within problem-solving, hypothetical learning trajectories, the use of geometry to assist visualization, collaborative group-work within problem-solving with realistic and non-routine problems. The different papers in this article attempt to show improvement in student learning, motivation, conceptual understanding, and problem-solving ability.

Flexible Teaching-Learning Modality in Mathematics Education of a State University in West Philippines

Authors: Ronalyn M. Bautista, Dominga C. Valtoribio (p.5)

The paper explores the implementation of flexible teaching and learning modalities in mathematics education in response to the COVID-19 pandemic. It focuses on assessing the modalities used by Mathematics Teacher Educators (MTEs), learners' perceptions of flexible teaching and learning, opportunities for improvement, and developing an action plan to enhance these modalities. The teaching research study emphasizes the implementation of flexible instructional approaches. The research involves a mixed-methods research design to gather data and analyze results in a state university in West Philippines. The possible audience for this paper includes mathematics educators, MTEs, educational researchers, curriculum developers, and policymakers interested in enhancing teaching and learning modalities in mathematics education, especially in response to challenges like the COVID-19 pandemic.

Development of Discovery-Based Etnobra (Ethnomathematics Geogebra) Geometry Learning Model to Improve Geometric Skills in Terms of Student Learning Styles and Domicile.

Authors: Hamidah, Jaka Wijaya Kusuma, Sigit Auliana (p.25)

The paper presents a research study on improving students' geometry skills by developing a culture-based Geometry learning design based on Geogebra to implement an Ethnomathematics model. The study, which takes place in Indonesia, proposes a new approach to teaching geometry based on students' learning styles and domicile. This model was found practical and effective in enhancing students' geometry skills. The possible audience for this paper includes educators, researchers in the field of education, and those interested in the integration of culture and technology in teaching and learning

The Use of Variation Theory of Learning in Teaching Solving Right Triangles
Authors: Joel B. Mendoza, Minie Rose C. Lapinid (p.58)

This research examines how Variation Theory is employed as a pedagogical tool to enhance students' problem-solving abilities in trigonometry, in the Philippines. This topic is important as disseminating the principles of Variation Theory among instructors can substantially improve students' learning outcomes. The article describes learning trigonometry under the usage of Variation Theory. It Focuses on the variation patterns of *contrast*, *separation*, and *generalization* to help students discern that not all triangles are solvable, especially when there is a lack of given information. The primary audience for this study is Mathematics instructors of trigonometry or branches of math that use trigonometry.

How Students Understand the Area under a Curve: A Hypothetical Learning Trajectory
Authors: Aniswita, Ahmad Fauzan, Armiati (p.80)

This paper focuses on how students understand the area under the curve in contextual problems and how the Hypothetical Learning Trajectory (HLT) can help students discover the concept. This paper studies how students understand the concept of the area under a curve over a closed interval in Integral Calculus, in a college-level class in Indonesia. This paper presents teaching research on how students can improve their understanding of the area under a curve, using a Hypothetical Learning Trajectory. This article is appropriate for Calculus instructors interested in using HLTs to assist student learning.

The Role of Ethnomathematics in South-East Asian Learning: A Perspective of Indonesian and Thailand Educators
Authors: Gusti Ayu Putu Arya Wulandari, I Putu Ade Andre Payadnya, Kadek Rahayu Puspadewi, Sompob Saelee (p.101)

The paper explores ethnomathematics in Indonesia and Thailand, to understand its importance in teaching mathematics. Participants included educators and pre-service teachers, who provided positive responses through questionnaires and interviews.
The study emphasizes the need to integrate ethnomathematics into education, highlighting its close ties to cultural development. More specifically, this paper addresses the role of ethnomathematics in Southeast Asian learning, focusing on educators' perspectives in Indonesia and Thailand. This proposal seeks to understand the challenges, and opportunities educators face in implementing ethnomathematics. The research methodology is a descriptive qualitative approach, involving teaching activities, questionnaires, interviews, and documentation. Teaching activities showcase the effectiveness of ethnomathematics instruction in enhancing students'

understanding of mathematical concepts through real-world and cultural contexts. Questionnaires and interviews provide insights into educators' perspectives on ethnomathematics, revealing positive recognition in Indonesia and Thailand, with nuanced differences.

Effectiveness of the CORE Learning Model: A Case Study of Learning the Method of Coordinates in a Plane in Vietnam

Authors: Duong Huu Tong, Pham Sy Nam, Nguyen Thi Nga, Le Thai Bao Thien Trung,Tang Minh Dung, Bui Phuong Uyen, Nguyen Nguyen Chuong (p 120)

The authors studied the CORE (connecting, organizing, reflecting, and extending) method for teaching geometry. This study is a quasi-experimental design with a treatment group of 47 tenth-grade students and 49 students in the control group. The results found that students in the treatment group performed better academically than the control group, taught using a traditional teaching method. The problems used in the experimental group were interesting real-life problems that connected to student life.

Visualizing Math Word Problems: Impact on First-Grade Students' Problem-Solving Performance

Authors: Nihan Şahinkaya, Zeynep Çiğdem Özcan , Selda Obalar (p.146)

This study contributes to the literature on problem-solving. The authors explore the use of visualization to support student problem-solving. The methodology was both quantitative and qualitative. Participants consisted of 41 first-grade students and interviews were conducted with eight of the students. Findings reveal an improvement in problem-solving through visualization.

Geometric Reasoning to Reinventing Quadratic Formula: Designing the Learning Trajectory

Authors: Sani Sahara, Dadang Juandi, Turmudi Turmudi, Agus Hendriyanto, Lukman Hakim Muhaimin, Matawal D. Bulus (p.164)

This paper explores geometry as a valuable tool for teaching students how to solve quadratic equations. By incorporating geometric concepts, this study aims to establish a strong foundation of knowledge to support students' comprehension of the quadratic formula. This research strives to provide insights and strategies for enhancing students' understanding of mathematical concepts.

The research methodology was a qualitative reflection on student answers and teacher comments that are well documented. The results suggest that improvement is achieved through using interactive tools developed by the authors. By employing these tools, students were empowered to explore and discover solutions to quadratic equations.

Unlocking the Future: Mathematics Teachers' Insight into Combination of M-learning with Problem-Based Learning Teaching Activities

Authors: Mohamad Ikram Zakaria, Nik Abdul Hadi Noor Nasran, Abdul Halim Abdullah, Najua Syuhada Ahmad Alhassora, Rasidi Pairan, Wanda Nugroho Yanuarto. (p.196)

This paper focuses on using technology to support mathematics education within problem-solving. It reviews the benefits and shortcomings of internet-based technology in Mathematics education. The authors' research methodology was qualitative interviews on the impact of integrating mobile technology within problem-solving in mathematics education. Despite the limited number of participants (three), the authors concluded that the overall effect was positive.

Mathematical Problem Design to Explore Students' Critical Thinking Skills in Collaborative Problem Solving

Authors: Arif Hidayatul Khusna, Tatag Yuli Eko Siswono, Pradnyo Wijayanti (p.217)

The authors explored using student social interactions, and non-routine problems to improve problem-solving and critical thinking in the classroom. The authors contextualized their study within Vygotsky's sociocultural theory

The Situation of Mathematical Problem Solving and Higher Order Thinking Skills in Traditional Teaching Method and Lesson Study Program

Author: Hosseinali Gholami (p.241)

This paper employs qualitative and quantitative research to study students' problem-solving and higher-order thinking skills. The study employs a traditional teaching program (control group), and compares this to a "Lesson Study" program (experimental group). The Lesson Study program used collaborative learning and problem-solving techniques. The results revealed better problem-solving and higher-order thinking in the Lesson Study group.

Development of Student Self-Efficacy for Mathematics Learning in Indonesia

Authors: Destiniar, Ali Syahbana, Tika Dwi Nopriyanti (p.265)

These authors developed a self-efficacy instrument for mathematics learning. Self-efficacy is an important area of study in mathematics education, given its impact on successful math learning. The authors use this instrument with middle and high school students.

The Problem Corner by Ivan Retamoso (p.290)

Flexible Teaching-Learning Modality in Mathematics Education of a State University in West Philippines

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Abstract: In response to paradigm shifts in education, teacher education institutions have increasingly adopted flexible learning modalities such as online courses, blended learning approaches, and personalized instruction to meet the diverse needs of students and prepare educators for the demands of modern classrooms. This study used a mixed-method research design to gain a comprehensive understanding of the flexible teaching and learning modalities it brought to mathematics teacher educators (MTEs) and learners in a state university in West Philippines. Since ethical considerations preclude random sampling, the study comprised nonrandom participants using purposive sampling, where 189 learners were surveyed, and six MTEs and 16 learners were interviewed. A researcher-made interview guide was prepared to describe the flexible teaching and learning modalities and opportunities of the MTEs. Also, a researcher-made questionnaire was used to describe the learners' general assessment of the flexible teaching and learning modalities in mathematics education. Quantitative data were gathered online, while qualitative data were gathered through interviews. Results showed that flexible teaching-learning approaches have implications for education, overcoming barriers, and promoting engagement. The findings highlight the importance of technology integration, inclusive assessment modalities, infrastructure support, and professional development opportunities in enhancing the quality and accessibility of flexible teaching and learning, ultimately leading to inclusive educational environments. Therefore, the educational institution must adopt flexible instructional modalities, emphasize diverse assessment methods, and personalize the learning experience by integrating online platforms and various assessments for effective mathematics instruction.

Keywords: COVID-19 pandemic, flexible instruction, new normal, mathematics education

INTRODUCTION

The coronavirus disease (COVID-19) has prompted global challenges and adjustments in the education sector in that it halted the operation of educational institutions (OECD, 2020) and caused a crucial disruption in the educational system (Dayagbil et al., 2021; Fuchs & Tsaganea, 2020), where no one knows when it will end (Tria, 2020). In the Philippines, particularly in higher education institutions (HEIs), the COVID-19 pandemic replaced the traditional face-to-face classroom with flexible teaching and learning. These paradigm shifts in the educational system posed challenges for both students and teachers (Lapitan et al., 2021; Zheng et al., 2020), including parents and school administrators. Nevertheless, changes in the educational system do not hamper quality education. To continue the delivery of instruction, the Commission on Higher Education

(CHED, 2020) promulgated the guidelines on flexible teaching and learning implemented by public and private HEIs.

HEIs offering teacher education programs faced the new norm and adopted flexible teaching and learning approaches in preparing future teachers. Cassidy et al. (2016) defined flexible teaching and learning as a pedagogical approach that allows place, time, and audience flexibility but does not focus on technology utilization. Flexibility in time is considered the most crucial aspect of flexible teaching and learning for students' education (Dimarucot et al., 2021). Flexible teaching and learning have always been the most appropriate approach to broadening access to education. Accordingly, HEIs are prompted to redesign the educational system using information and communications technology (ICT) and the available alternative delivery modes of instruction (Pawilen, 2021). Malipot (2021) echoed the Commission on Higher Education's (CHED) view that traditional face-to-face learning may no longer be appropriate in HEIs, as it advocated for implementing flexible teaching and learning in the school years that followed. Like traditional face-to-face, flexible teaching and learning have merits and limitations for teacher education institutions (TEIs).

TEIs were confronted with the challenges of flexible teaching and learning. Teacher educators (Gayon & Tan, 2021; Jones & Kessler, 2020) and learners (Gocotano et al., 2021; Ozudogru, 2021) experienced difficulties during the pandemic period, especially during the virtual classroom (Konuk, 2021). Nevertheless, flexible teaching and learning could be an agent in promoting a learner-centered environment as TEIs deviate from traditional to innovative pedagogical approaches; with the uncertainty that TEIs experience, teacher educators and preservice teachers continue embracing flexible teaching and learning. This study is necessary to determine the modalities derived from teacher educators and learners in these challenging times. Besides, this study agreed and argued with several studies for flexible teaching and learning in TEIs. Furthermore, the study proposed an action plan to enhance teacher educators' flexible teaching and learning modalities that can be used by the TEI involved, which can be extended to other TEIs. The unanticipated transition to flexible teaching and learning has brought challenges and opportunities (Carrillo & Flores, 2020). While there is an extensive study on flexible teaching and learning among HEIs, how TEIs engaged in the sudden shift to flexible teaching and learning is limited. With the COVID-19 pandemic, this study sought to establish the modalities the MTEs and learners employed in crafting a more relevant, flexible teaching and learning environment.

Flexible teaching and learning options may include digital and non-digital technologies, face-to-face or in-person learning, out-of-classroom learning modes, or a combination of these delivery modes (CHED, 2020). Factors such as internet access, availability of devices, and strengthening online learning platforms are crucial considerations when designing a flexible teaching and learning environment. Teacher educators and learners must be technologically literate and innovative (Gocotano et al., 2021), as technological education is necessary for meeting emerging standards and challenges (Levinson, 2020). Flexible teaching and learning in TEIs address the gaps between teaching and learning in the new normal (Jigyasu et al., 2021). It refers to educational techniques and systems that offer learners enhanced choice, convenience, and personalization (Khan, 2019). Various learning modes are employed, such as full-time online, blended learning, flipped classroom, and distance learning (Bates, 2019). Flexible teaching and learning helped learners manage their activities according to their needs and interests (Urgel, 2020). TEIs and MTEs must respond promptly to the shift to flexible teaching and learning by creating an atmosphere that supports preservice teachers (Gayon & Tan, 2021). They must regularly revise their faculty development plans to enhance performance and promote social issue participation and sensitivity (Pawilen, 2021). ICT literacy is, thus, critical for MTEs and learners. MTEs can provide course packages for preservice teachers, whether online or offline (Gayon & Tan, 2021). Accordingly, learners must develop self-directed and self-regulated learning skills in a flexible teaching and learning environment.

Theoretical Framework

The study's theoretical framework is built on the Framework for Capacity Building of Teacher Education Institutions, the Asynchronous Course Delivery (ACCORD) Framework, and the Guidelines on Flexible Learning Implementation by the Commission on Higher Education. The Framework for Capacity Building of Teacher Education Institutions provides guides for improving teacher preparation courses through six strategic dimensions. These dimensions focus on vision and philosophy, program development, professional learning, ICT integration, partnerships, and research and evaluation (Lim et al., 2011). The ACCORD Framework outlines a systematic approach to delivering high-quality online courses that allow for asynchronous learning (Abisado et al., 2020). It includes course design, management, assessment, student support, instructional technology, and instructor support. Meanwhile, the Guidelines on Flexible Learning Implementation guide HEIs to implement flexible teaching and learning programs (Commission on Higher Education, 2020). These guidelines comprise course design, faculty development, assessment, infrastructure and technical support, and student services. Considering these frameworks, the study explored flexible teaching and learning modalities for educators and learners. It focused on curriculum, assessment, professional development, ICT integration, instructional technology, instructor support, and student services. The study sought to contribute to improving teacher education programs and effectively implementing flexible teaching and learning in response to evolving educational needs and challenges.

Conceptual and Analytical Framework

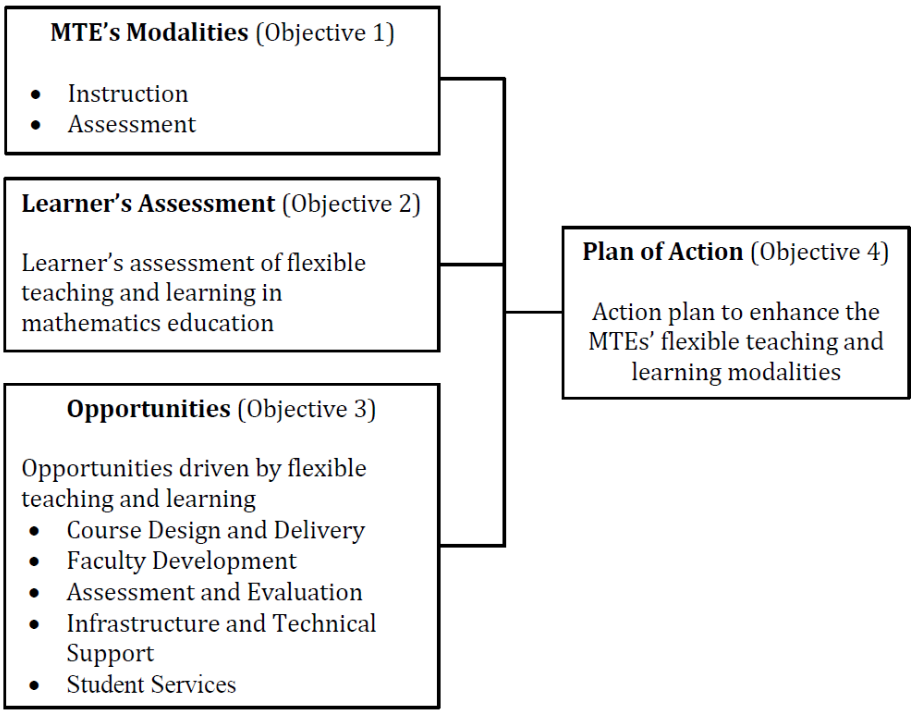
Figure 1 (see next page) illustrates the research paradigm focused on flexible teaching and learning in mathematics education. As portrayed in the framework, learners' general evaluation of flexible teaching and learning modalities in mathematics education was established through the quantitative phase (Objective 2). Meanwhile, the qualitative phase described the MTEs' modalities along instruction and assessment (Objective 1) and opportunities driven by flexible teaching and learning for MTEs and learners along course design and delivery, faculty development, assessment and evaluation, infrastructure and technical support, and student services (Objective 3). The findings from Objectives 1, 2, and 3 formed the foundation for creating an action plan to improve the flexible teaching and learning modalities of MTEs (Objective 4).

Research Objectives

This study determined the flexible teaching and learning modalities among MTEs and learners in a state university in West Philippines. Specifically, it aimed to:

1. determine the flexible teaching and learning modalities employed by the MTEs in terms of instruction and assessment;

2. determine the learners' general assessment of the flexible teaching and learning modalities in Math Education;
3. determine the opportunities provided when implementing flexible teaching and learning in terms of course design and delivery, faculty development, assessment and evaluation, infrastructure and technical support, and student services; and
4. prepare a plan of action to enhance the flexible teaching and learning modalities of MTEs.



METHODOLOGY

Research Design

A mixed-method research design, compounding quantitative and qualitative research methods (Creswell et al., 2003), was employed to understand the flexible teaching and learning implementation in mathematics education. This research design helped to provide insights into the effectiveness of flexible teaching and learning in mathematics education and guide future educational policy and practice. The quantitative phase determined the learners' general assessment of flexible teaching and learning modalities in mathematics education. On the other hand, the qualitative phase described the MTEs' modalities and the MTEs' and learners' opportunities driven by flexible teaching and learning. Quantitative and qualitative results served as a basis for crafting an action plan to enhance the flexible teaching and learning modalities of MTEs.

Research Participants

The study's respondents were mathematics teacher educators (MTEs), core faculty handling math subjects, and BSEd-Math and BEEd third-year and fourth-year students (learners) in a state university in West Philippines. These participants experienced flexible teaching and learning implemented from the first semester of SY 2020-2021 to SY 2022-2023. An invitation letter indicating informed consent and voluntary participation was sent to the participants. The quantitative study utilized nonrandom sampling. This sampling refers to selecting individuals from a population in a nonrandom manner. A sample of 189 learners participated in the quantitative study to determine their general assessment of the flexible teaching and learning modalities in mathematics education, along with online and modular teaching and learning. For the qualitative study, purposive sampling was employed since these participants were held to share their rich experiences in flexible teaching and learning challenges posed by the COVID-19 pandemic, where modalities and opportunities were derived. Four MTEs were interviewed about their flexible teaching and learning modalities regarding instruction and assessment. Meanwhile, 16 learners and six MTEs participated in describing the opportunities provided to MTEs and learners during the implementation of flexible teaching and learning.

Research Instrument

A researcher-made interview guide was prepared to describe the flexible teaching and learning modalities employed by the MTEs regarding instruction and assessment. The interview questions on instruction modalities asked about teaching methods used in flexible learning, the development of instructional materials, flexible course policies, consultation hours, and other instructional modalities implemented. On the other hand, the interview questions on assessment modalities looked into assessment methods, including written works and performance tasks, scoring rubrics, and major exams. It also asked about the validity of the assessment tools.

A researcher-made questionnaire based on a four-point Likert scale was used to describe the learners' general assessment of the flexible teaching and learning modalities in mathematics education, along with online and modular teaching and learning. The statements for online teaching and learning described the benefits of flexible teaching and learning through online options, including convenience, access to course materials and resources, interaction with professors and peers, and better work-life balance. Meanwhile, the statements for modular teaching and learning described the benefits of flexible teaching and learning through printed and electronic modules, including flexibility, well-designed materials with assessments, independent study, and a better understanding of course material. The following qualitative descriptions were employed: 4 = Strongly Agree; 3 = Agree; 2 = Disagree; and 1 = Strongly Disagree. The instrument was pilot-tested on non-participants and tested with validity and reliability. The instrument used in the study demonstrated high internal consistency with Cronbach's alpha coefficients of 0.879 (online) and 0.929 (modular), indicating that the instrument reliably measures the learners' assessment.

Finally, self-made semi-structured interview questions guided the researcher in describing the opportunities provided to MTEs and learners during the implementation of flexible teaching and learning in terms of course design and delivery, student services, faculty development, assessment and evaluation, and infrastructure and technical support, based on CHED Memorandum Order no. 4, series of 2020. This instrument was submitted to three experts, a mathematics professor, a teacher educator, and a qualitative researcher, for validation to ensure its relevance for the study.

Data Gathering Procedure

The Saint Mary's University Research Ethics Board approved the protocol at Saint Mary's University, Bayombong, Nueva Vizcaya, Philippines (Code: 2Sem 203 309). Ethical factors such as conflict of interest, privacy confidentiality and data protection, risk/benefit ratio, informed consent, and terms of reference were considered during the evaluation process. Other than the approval from the University President through the College Dean of the research locale, informed consent from the participants was secured before gathering the data. The participants completed an informed consent form. A Google Form was utilized where the participants read the content and indicated their consent by ticking the box at the end of the form. After their consent, they were directed to another link for data gathering. Participants who declined had to indicate their reasons before submitting the form.

The researcher gathered data about the flexible teaching and learning modalities employed by the MTEs regarding instruction and assessment through face-to-face focus group discussions (FGD) to answer the first objective. For the second objective, a questionnaire collected data on the learners' general assessment of the flexible teaching and learning modalities in mathematics education. The questionnaire was translated into a Google Form to reach the target participants easily. Finally, for the third objective, virtual and face-to-face individual interviews were used to collect data describing the opportunities provided to MTEs and learners while implementing flexible teaching and learning.

The virtual interview was conducted through Google Meet, ensuring participants had the necessary devices and reliable access to the internet to participate effectively. Participants who did not attend face-to-face interviews or FGDs were provided free load cards, as those who chose virtual interviews or FGDs. The conversations from the interviews or FGD were audio-recorded with the participant's permission. Participants were encouraged to tell their stories and answer the questions in English or Filipino. A mobile phone recorded and replayed the conversations, aiding in data transcription.

Treatment of Data

For the first and third objectives, the qualitative data collected were evaluated using an inductive approach. The inductive analysis followed the three-step guide: Encoding verbatim, coding and deriving codes, and general code development. Transcripts from the interview and FGD were encoded verbatim. Responses in Tagalog were translated into English. Then, codes were derived from the data encoded, reflecting the patterns from the participant's narratives. Finally, general codes emerging from the codes/patterns were obtained and reported, answering the flexible teaching and learning modalities. Concerning the second objective, the quantitative data about the participants' flexible teaching and learning assessment were analyzed with mean and standard deviation descriptives. The following qualitative descriptions were employed: 3.50-4.00 = Strongly Agree; 2.50-3.49 = Agree; 1.50-2.49 = Disagree; and 1.00-1.49 = Strong Disagree.

RESULTS

Flexible Teaching and Learning Modalities Employed by the MTEs

The findings from the study highlight the flexible teaching and learning modalities employed by mathematics teacher educators (MTEs).

Instruction. MTEs utilized a combination of modular instruction and online learning. [*We utilize a combination of modular instruction and online learning, sometimes conducted in synchronous and asynchronous formats. It can be challenging since not everyone has access to the internet. (Teacher Bravo)*]. Modular instruction benefited learners without internet access, allowing them to continue learning offline [*Modular instruction only because some students do not have internet connectivity. Even though they have an online presence, we use group chat to send their modules and reminders, and I address their questions through group chat or private messages. The module itself is detailed, with lectures and exercises. I prefer not to conduct online classes with one or two students missing. Their reason is valid since they do not have internet connectivity, so I cannot assure the quality of teaching. (Teacher Alpha)*]. Online components such as synchronous discussions and asynchronous tasks were also integrated to enhance engagement and accessibility [*I conducted my classes online for Assessment and Evaluation in Mathematics and Research in Mathematics. I used Google Meet synchronously for discussions, while tasks were assigned asynchronously through Google Classroom. I provided feedback on students' outputs directly on Google Classroom. Online consultations were conducted through the group chat. (Teacher Delta)*]. The MTEs emphasized consultation, support, monitoring, and feedback, and flexible classroom policies to accommodate learners' needs and promote effective learning [*I do not have designated consultation hours, but I told them they could contact me anytime they have questions, and I will respond to them through private messaging. I encourage them to post their questions in the group chat because others may share them. (Teacher Alpha)*]. Besides, modular learning was found to promote personalized instruction and self-learning [*But the great thing about modular learning is that it promotes self-learning, which enables personalized instruction tailored to each student's learning requirements and preferences. (Teacher Foxtrot)*].

Assessment. The MTEs employed both formative and summative assessment methods. Formative assessments included exercises, pre-tests, post-tests, activity sheets, and problem sets [*We provide activity sheets and problem sets. They have weekly tasks to assess their readiness for the next lesson. (Teacher Bravo)*]. In contrast, summative assessments comprised major exams and performance tasks [*We have major exams like midterm and final exams and performance tasks like real-world problem-solving tasks and group projects to enhance their critical thinking, creativity, and collaboration. (Teacher Echo)*]. The MTEs ensured the validity and fairness of assessments by aligning them with the course syllabus and using rubrics and a table of specifications [*We have a rating scale for each problem-solving activity. We also have a Table of Specifications for major* exams (Teacher Alpha)]. They also provided timely feedback to learners, monitored their progress, and engaged in open forums to address their concerns [*I provide feedback and timely checking of their school tasks in specific subjects. I also encourage them regarding academic integrity. (Teacher Charlie)*].

Learners' General Assessment of the Flexible Teaching and Learning Modalities in Math Education

The findings from the learners' assessment of flexible teaching and learning modalities in math education denote that learners agree overall with implementing these modalities.

Online Instruction. In the case of online instruction (Table 1), learners appreciated the freedom to learn from anywhere and at any time through synchronous and asynchronous options

(Mean = 3.34, SD = 0.63). They found the integration of online learning into the course syllabus and the orientation provided to be beneficial.

Indicators	Mean	SD	QD
1. Flexible teaching and learning allowed me to have the freedom to learn from anywhere and at any time, as it offered both online synchronous and asynchronous options.	3.46	0.56	Agree
2. This learning mode was integrated into the course syllabus, and we were thoroughly oriented about its usage.	3.42	0.57	Agree
3. The online classes were conducted using popular teleconferencing tools, and I could participate in them regardless of location or device.	3.32	0.69	Agree
4. This mode provided me with the convenience and flexibility to join classes from home.	3.38	0.62	Agree
5. My professors were available for online academic advising, feedback, and consultation, which helped me to stay on track with my studies.	3.22	0.67	Agree
6. Assessments were assigned or administered using tools like Google Classroom and Google Forms, making monitoring my performance and progress easy.	3.46	0.58	Agree
7. I could connect with my professors and classmates online, allowing me to interact and collaborate with my peers, further enhancing my learning experience.	3.33	0.65	Agree
8. Online platforms allow access to course materials and resources, making it easy for me to review and revisit course content.	3.36	0.63	Agree
9. This learning mode provided me with a better work-life balance, as I could attend classes and complete assignments at a pace that suited my lifestyle.	3.25	0.65	Agree
10. Overall, flexible teaching and learning via online options proved to be an effective and convenient mode of education for me.	3.17	0.70	Agree
Mean	**3.34**	**0.63**	**Agree**

Table 1. Learners' general assessment of the flexible teaching and learning modalities in math education and online instruction. (Note: 3.50-4.00 = Strongly Agree, 2.50-3.49 = Agree, 1.50-2.49 = Disagree, 1.00-1.49 = Strong Disagree; SD = Standard Deviation, QD = Qualitative Description)

Modular Instruction. Similarly, in the case of modular instruction (Table 2), learners generally agreed with the flexible modalities employed (Mean = 3.37, SD = 0.59). They acknowledged the inclusion of printed and electronic modules in the course syllabus and were well-oriented about this learning mode.

Indicators	Mean	SD	QD
1. Flexible teaching and learning included printed and electronic modules in the course syllabus.	3.44	0.56	Agree
2. We were thoroughly oriented about this learning mode and its implementation.	3.37	0.59	Agree
3. The printed modules were accessible at the college or designated distribution areas, while the electronic modules were easily accessible through platforms such as Google Classroom, Messenger, or email.	3.41	0.60	Agree
4. Instructional modules were provided at the start of the semester and were constantly updated throughout the course.	3.35	0.59	Agree
5. Modular learning allowed me to pace and schedule my personal and academic tasks, as I could study at my own pace and schedule.	3.39	0.59	Agree
6. The modules were well-designed, and the discussions, illustrations, and examples helped me enhance my understanding of the course material.	3.32	0.61	Agree

7. The modules indicated assessments, including pre-and post-tests, exercises, and activities, which allowed me to assess my learning and track my progress.	3.48	0.54	Agree
8. The modules were designed for independent study, allowing me to take ownership of my learning and become an active participant in my education.	3.33	0.58	Agree
9. Modular learning allowed me to better understand the course material, as I could revisit the modules at any time and focus on the areas I needed to improve.	3.29	0.61	Agree
10. Overall, flexible teaching and learning through printed and electronic modules proved to be an effective and convenient mode of education, providing me with the necessary resources and support to succeed in my studies.	3.32	0.62	Agree
Overall	**3.37**	**0.59**	**Agree**

Table 2. Learners generally assess the flexible teaching and learning modalities in math education along with modular instruction. (Note: 3.50-4.00 = Strongly Agree, 2.50-3.49 = Agree, 1.50-2.49 = Disagree, 1.00-1.49 = Strong Disagree; SD = Standard Deviation; QD = Qualitative Description)

Opportunities Provided during the Implementation of Flexible Teaching and Learning
Implementing flexible teaching and learning has provided various opportunities for teachers and learners, as revealed through interviews and focus group discussions.

Course Design and Delivery. The MTEs had the opportunity to integrate technology into their teaching, using software applications and digital platforms to enhance engagement and cater to different learning styles [*"I used software applications in my lessons, such as GeoGebra and Desmos, for graphing purposes." (Teacher Charlie)*]. They also emphasized responsiveness to learners' needs, providing detailed modules and offering various learning options such as online and modular learning, e.g., *"I make sure that the module is detailed with enough examples, step by step, especially since some cannot access the internet." (Teacher Alpha).* Additionally, teachers promoted self-learning and independent exploration by providing comprehensive materials and resources, allowing students to learn independently, e.g., *"They have learned through their independent learning and self-learning, which has made the process engaging and not boring." (Teacher Bravo).*

Faculty Development. The MTEs had the opportunity to expand their technological and digital literacy through training programs and webinars [*"Because of webinars, my digital skills and computer literacy have improved." (Teacher Charlie)*]. They could learn from experts worldwide through virtual conferences using platforms like Zoom and Google Meet [*"Through Zoom, numerous webinars became popular. Unlike before, when the university would select whom to send to seminars during face-to-face sessions, now anyone willing can attend through Zoom or Google Meet." (Teacher Alpha)*]. Online courses and advanced studies were also accessible [*"There are many opportunities. I was able to enroll in graduate school. I also took free courses through Coursera and Coursebank." (Teacher Delta)*], allowing teachers to pursue further education and enhance their qualifications.

Assessment and Evaluation. During flexible teaching and learning, the MTEs have provided flexibility in assessment, allowing students to access exams and evaluations from anywhere with an internet connection [*"I became flexible because the students and their learning were affected. Exams and performance tasks were still required, but I always made sure that no one would be left behind in the class." (Teacher Delta)*]. Authentic assessment methods, such as real-world problem-solving activities and alternative evaluation forms, were incorporated to promote creativity and critical thinking skills [*"I included other forms of assessment, such as projects, presentations, case studies, portfolios, group work, and online conversations, rather than depending simply on conventional tests or quizzes." (Teacher Echo)*]. Technology played a role in efficient evaluation, with online platforms like Google Classroom enabling automated assessment and streamlined feedback processes [*"I have used various online platforms for my assessments. These digital platforms provide individualized feedback, like Google Forms, which has helped us promptly address our learners' needs and provide personalized guidance for improvement." (Teacher Foxtrot)*].

Infrastructure and Technical Support. The MTEs benefited from technological support, including free Wi-Fi, laptops, and other resources provided by their institutions [*"The university provided free Wi-Fi, load card, and laptops to connect with our learners." (Teacher Bravo)*]. Access to digital resources was also facilitated, eliminating the need for teachers to invest in their equipment [*"The university installed free Wi-Fi in each college and provided laptops. As teachers, we did not need to invest as the university provided the necessary resources for flexible learning and teaching. Unlike in a traditional setting, where we had to use our laptops." (Teacher Alpha)*]. Teachers could organize course materials, track student progress, and communicate effectively through digital platforms, ensuring efficient learning process management [*"I can effectively organize course materials, track student progress, give timely comments, and encourage involvement using digital platforms and tools. Using these skills, I can spot learning gaps and fill them, communicate meaningfully with students, and modify lessons to fit each student's requirements." (Teacher Foxtrot)*].

Student Services. During flexible teaching and learning, students were given various opportunities for student services, as reflected by the CommunicaToAL: Communication Tools and Applications on Learning and SuppoSe: Support to Students. Communication tools and applications play a crucial role in facilitating learning and connectivity. Students expressed their appreciation for platforms like Google Classroom, Zoom, Google Meet, Messenger, and Gmail, which allowed them to continue their studies and gain knowledge even during the pandemic [*"Flexible teaching and learning made me access support services, mostly any time, as it gives me freedom or free time to do activities, ask queries to my instructors online, and attend consultation meetings through Zoom and Google Meet without any hassle in preparation." (BEEd Learners - PPC Campus)*]. These tools enabled them to stay connected with their teachers and classmates, regardless of location. Additionally, students utilized math-related tools such as Mathway, Photomath, Gauthmath, Symbolab, Geogebra, Desmos, and scientific calculator apps to enhance their understanding of mathematical concepts [*"We have become aware of various applications such as Symbolab, Gauthmath, Mathway, Photomath, and Desmos, which assist us in checking our answers and meeting our learning needs in mathematics. These software tools have been helpful in our studies. Since we do not have access to scientific calculators due to their high cost, we have downloaded scientific calculator apps on our mobile phones as an alternative. This has helped us save on expenses." (BSEd Math Learners - PPC Campus)*]. Online tutorials, resources, and platforms like YouTube and educational websites further supported their learning [*"We have become familiar with using various tools that can aid us in the learning process. We often rely on YouTube tutorials to understand how to use different tools effectively. In addition to that, we* *frequently utilize Google for research and finding additional resources." (BSEd Math Learners - Main Campus)*].

Student support services were also extended through bridging connections and increasing engagement. Online events, webinars, and extracurricular activities organized by student organizations and the university allowed students to participate in activities of interest [*"We can participate in extracurricular activities and pursue our professional interests online. The school offers online training and seminars that align with our academic and professional interests. They have also organized activities that do not require us to leave our homes, such as online events organized by the student council." (BEEd Learners - Main Campus)*]. Learning materials were accessible through flash drives, online libraries, and module bags [*"The university provided bags and flash drives to store our files and learning materials." (BSEd Math Learners - PPC Campus)*].

Plan of Action to Enhance the Flexible Teaching and Learning Modalities of MTEs

The proposed action plan (see Appendix) aims to enhance the flexible teaching and learning modalities of MTEs. The plan emphasizes revising teaching methods, incorporating technology, revising assessment modalities, and promoting timely feedback to create a flexible and inclusive learning environment. The research findings support the action plan's objectives, aligning with the need for increased flexibility, inclusivity, and technology-driven education.

DISCUSSION

Flexible Teaching and Learning Modalities Employed by the MTEs

The MTEs successfully implemented flexible and inclusive instruction combining modular and online learning. They employed various assessment methods to evaluate learners' progress and provided timely feedback. These findings align with previous research and contribute to learners' long-term educational development. Online resources, communication channels, and flexible classroom policies facilitated effective teaching and learning experiences. Generally, Flexiclusive: Flexible and Inclusive Instruction emerged among MTEs, who aimed to create a flexible and inclusive learning environment by combining modular instruction with online learning opportunities. Multiple resources were utilized to accommodate different learners, allowing them to engage with the content in diverse ways, fostering a supportive learning environment by addressing questions and concerns, and emphasizing flexibility and open communication channels to accommodate individual circumstances and needs. This finding is favorable since modular and online learning fostered autonomy and self-directed learning (Bacomo et al., 2022), benefitting learners' long-term educational development. Meanwhile, the general code emerging was MTEs' ComPass Tick: Comprehensive Assessment and Timely Feedback, which highlights their commitment to using various assessment modalities aligned with learning objectives. In connection with Harris and Jones (2021) and Van Nuland et al. (2020), using the best modalities to create assessments, grading complex tasks using rubrics, and aligning test questions and performance tasks to learning objectives are all techniques instructional designers can use to create high-quality assessments.

Learners' General Assessment of the Flexible Teaching and Learning Modalities in Math Education

The accessibility of printed modules at designated areas and electronic modules through platforms like Google Classroom was recognized. Learners appreciated the continuous updating of instructional modules and the flexibility to pace and schedule their tasks. Assessments indicated in the modules allowed them to assess their learning and track progress. However, the perceived effectiveness of modular learning for improving understanding was slightly lower. Although Sanchez et al. (2022) reported challenges among learners during modular instruction, this study showed that printed modules were accessible. This finding suggests that learners found the materials available through different mediums, catering to their preferences or circumstances. However, the effectiveness of modular learning for improving understanding was perceived as slightly lower, although still generally positive. Hamora et al. (2022) indicated that there might be room for further enhancements in its implementation.

The learners acknowledged the convenience of attending classes from home, accessing course materials and resources online, and the availability of online academic advising. However, there were varying opinions regarding online education’s effectiveness and convenience. Like Muthuprasad et al. (2021), the results imply that not all learners viewed online education as equally beneficial or convenient. The result suggests that some learners may have found online learning effective and convenient, similar to Almahasees et al. (2021), while others may have had reservations or encountered difficulties with this mode of education. Individual learning preferences, technological factors, and personal situations could influence these differing views.

These findings suggest that while learners generally appreciate the flexible modalities used in online and modular instruction, there are differing perspectives on the effectiveness and convenience of these modes of education. Individual learning preferences, technological considerations, and personal circumstances may contribute to these opinions. Educators need to consider these factors when designing and implementing flexible teaching and learning modalities in math education, aiming to enhance the overall learning experience for all learners.

Opportunities Provided during the Implementation of Flexible Teaching and Learning

During the implementation of flexible teaching and learning, mathematics educators (MTEs) have embraced CuRing: Course Reengineering as a pivotal opportunity. Similar to the perspectives proposed by Cassidy et al. (2016) and Hamora et al. (2022), this underscores the importance of redesigning and adapting courses to suit the needs of flexible teaching and learning. MTEs are leveraging digital tools and innovative instructional methods to ensure the delivery of high-quality mathematics education. In addition, faculty development emerges as a crucial opportunity during flexible teaching and learning, encapsulated by CoPeD: Continuing Professional Development. This ongoing learning process enables MTEs to grow as educators, empowering them to effectively navigate the challenges of flexible teaching and learning while delivering top-notch education to their mathematics students (Al-Thani et al., 2021). These opportunities strengthen the need to continually review and revise mathematics instruction with the emerging trends while equipping each mathematics educator with the needed training and preparation towards resiliency in classroom instruction.

Assessment and evaluation also play a vital role in implementing flexible teaching and learning, with the emergence of IPasA: Inclusive Practices in Assessment. MTEs are embracing assessment flexibility to address the challenges posed by the pandemic and ensure equitable opportunities for all learners. They are incorporating authentic assessment methods to foster creativity, collaboration, and critical thinking skills through real-world problem-solving activities. These tasks are necessary to develop metacognitive awareness and advance mathematical performance among learners (Oficiar et al., 2024; Pentang et al., 2023). In mathematics teaching, the integration of technology, as emphasized by Tech EmpowerEd: Empowering Education Through Technology, presents a significant opportunity for enhancing pedagogical practices. Similar to the findings of Dayagbil et al. (2021), providing technological resources can empower educators to deliver flexible teaching approaches tailored to diverse student needs. As Asio et al. (2021) highlighted, leveraging technology can substantially augment the effectiveness of online instruction, fostering an environment conducive to learning in the digital space. Flexible situations have created opportunities for inclusive assessment and technology integration in mathematics teaching. The inclusive assessment ensures fairness for all learners, while technology integration enhances engagement and personalized learning experiences. Educators adapt traditional approaches, embrace innovative assessment methods, and leverage digital tools to support diverse student needs in changing educational contexts, particularly mathematics.

Technology also provided opportunities to support flexible teaching-learning in mathematics regarding communication. Mariano-Dolesh et al. (2022) and Santiago et al. (2021) underscored that communication tools and applications are vital in facilitating mathematical learning and connectivity among students and educators. By engaging with these digital resources, learners can deepen their comprehension, verify solutions, and explore various mathematical concepts, enriching their educational experiences. Implementing flexible teaching and learning methodologies enables students to access essential services and resources and sustains their engagement in mathematical education amidst the challenges posed by the pandemic. Aligning with the OECD (2020), such support ensures that learners' well-being and fundamental needs are addressed, cultivating a nurturing environment conducive to their academic endeavors. Thus, by harnessing technology and adopting flexible instructional approaches, educators can effectively empower students to thrive in their mathematical learning journey, irrespective of external circumstances.

Plan of Action to Enhance the Flexible Teaching and Learning Modalities of MTEs

The proposed action plan offers a comprehensive strategy for enhancing the flexible teaching and learning modalities within mathematics classrooms, explicitly focusing on MTEs. By prioritizing the revision of teaching methods, integration of technology, adaptation of assessment modalities, and facilitation of timely feedback, the plan aims to foster a more adaptable and inclusive learning environment. Through adopting diverse teaching techniques, such as active learning and collaborative problem-solving, MTEs can engage students with varied learning styles more effectively. Moreover, incorporating digital tools and resources enables MTEs to provide interactive learning experiences and facilitate remote learning when necessary. By revising assessment methods to include a broader range of formative and summative approaches, MTEs can better assess students' understanding and skills in mathematics. Additionally, promoting timely feedback through peer assessment or digital platforms ensures that students receive personalized guidance and support to enhance their learning outcomes. Overall, the action plan emphasizes the importance of flexibility, inclusivity, and technology integration in mathematics education, aligning with research findings that underscore the need for innovative approaches to teaching and learning in this field.

CONCLUSION AND RECOMMENDATION

Conclusion

The MTEs have successfully implemented a combination of modular instruction and online learning to address learners' challenges, providing personalized instruction, flexibility, and offline access to educational materials. Integration of online components has enhanced engagement and accessibility, supported by communication channels for seeking support and addressing concerns. The MTEs have effectively monitored student progress, provided timely feedback, and employed diverse assessment methods aligned with learning objectives. These flexible teaching and learning modalities have catered to learners' needs, fostering a supportive and inclusive environment. Learners appreciated the flexibility of online and modular instruction, allowing them to learn anytime and anywhere. Online assessments helped monitor progress, but there was less agreement on the effectiveness and convenience of online teaching. Similarly, learners recognized the benefits of modular learning, but slightly less agreement was observed in this aspect. Flexibility is valued, but improvements can be made to enhance online instruction and optimize the benefits of modular learning.

The study findings highlight flexible teaching and learning implementation opportunities for both MTEs and learners. MTEs demonstrated adaptability and creativity in course design, utilizing digital platforms, multimedia elements, and online discussions to cater to learners' needs. They actively engaged in faculty development, enhancing technological proficiency and teaching experience. Assessment methods have become more flexible and technology-driven. Infrastructure and technical support facilitated the implementation by providing necessary resources. These opportunities empowered MTEs to create personalized learning experiences, fostered student engagement, and improved learning outcomes. Faculty development initiatives kept MTEs updated, while inclusive assessment modalities promoted creativity and critical thinking. The provision of infrastructure and technical support eliminated access barriers for seamless implementation. Enhancing flexible teaching and learning modalities is crucial for inclusive education. The proposed action plan includes a comprehensive review of teaching methods and materials, faculty professional development, and integration technology. Alternative assessment methods and transparent criteria are emphasized. Execution of the plan fosters continuous improvement and collaboration among faculty. Success is measured by increased participation in professional development, feedback from faculty and students, and technology integration. Prioritizing flexibility creates engaging and inclusive learning environments that meet diverse learner needs. The alignment between findings and the action plan enhances the validity and relevance of proposed strategies for effective teaching and learning.

Recommendations

Educational institutions and teachers should embrace flexible teaching and learning modalities by integrating modular and online learning, providing personalized instruction, and accommodating learners' circumstances. Institutions should support teachers in creating high-quality modules and using online platforms effectively. Communication channels, such as group chats and online consultations, should be established to address learners' questions. Teachers should monitor student progress, provide timely feedback, and adjust teaching strategies accordingly. Assessment methods should combine formative and summative approaches, ensuring validity and fairness through rubrics and precise alignment with objectives. Timely feedback should be given to support learners' progress and encourage improvement.

To improve flexible teaching and learning in online and modular instruction, educators should focus on enhancing effectiveness through instructional design and pedagogical strategies. They should also conveniently streamline the online learning experience with user-friendly interfaces, clear instructions, and technical support. They must also provide additional support and guidance for learners in modular learning through clear instructions, communication channels, and scaffolding resources. These measures optimize learner engagement and outcomes. This study recommends several ways to enhance the implementation of flexible teaching and learning in mathematics education. These include prioritizing teacher professional development, integrating technology into instruction, promoting educator collaboration, and using inclusive assessment methods. These measures aim to improve instructional strategies, support teacher growth, and enhance students' learning experiences in mathematics. To enhance flexible teaching and learning, an educational institution should execute a well-developed action plan, including reviewing teaching methods, providing professional development, integrating technology, and implementing flexible assessments. They should monitor success indicators, gather feedback, and continuously refine the plan based on evolving needs. Fostering a culture of collaboration among faculty members is essential for sustaining and expanding flexible teaching and learning modalities.

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APPENDIX

Action Plan to Enhance the Flexible Teaching and Learning Modalities

Area	Tasks/ Objectives	Activities	Strategy of Execution	Performance Indicator
Instruction	Conduct a comprehensive review of current teaching methods and materials to identify areas for improvement.	Form a review committee to assess current teaching methods and materials. Gather feedback from faculty members through surveys or interviews. Analyze data to identify strengths and weaknesses. Document findings and recommendations for improvement.	Allocate budget for committee meetings and consultations. Invest in research materials and resources. Conduct surveys or focus groups with faculty and students	Number of teaching methods and materials reviewed. Identification of specific areas for improvement. Development of a comprehensive report outlining the findings and recommendations.
	Develop a professional development program for faculty to enhance their skills in flexible teaching strategies.	Assess faculty members' needs and interests through surveys. Design workshops and webinars on flexible teaching strategies. Create user-friendly learning materials for the program. Offer ongoing support and mentoring during implementation.	Design and develop training materials and resources Conduct workshops or training sessions. Provide incentives or honoraria for guest speakers or experts	Number of faculty members participating in the professional development program. Completion of training sessions or workshops on flexible teaching strategies. Evaluation of faculty members' skills and knowledge enhancement through pre-and post-training assessments or surveys.
	Provide resources and support for the implementation of technology-enhanced teaching methods.	Identify and curate a list of user-friendly technology tools. Develop user guides and tutorials for faculty members. Provide hands-on training sessions for using technology tools. Establish a helpdesk or support system for technical assistance.	Invest in technology infrastructure and equipment. Purchase educational software or platforms. Provide training and technical support for faculty	Availability and accessibility of technology resources provided to faculty members. Number of faculty members utilizing technology-enhanced teaching methods. Feedback from faculty members regarding the effectiveness and usefulness of the provided resources and support.
	Encourage collaboration and sharing of best modalities among faculty members.	Create an online platform for faculty members to connect and share ideas. Organize faculty meetings or workshops for knowledge exchange. Facilitate peer mentoring and observation programs. Recognize and reward contributions to best modality sharing.	Organize faculty forums or conferences. Develop an online platform or community for sharing the best modalities. Provide incentives or rewards for faculty members who contribute or present their best modalities.	Number of collaborative activities or initiatives among faculty members. Participation and engagement in sharing best modalities through workshops, presentations, or online platforms. Feedback from faculty members indicating increased collaboration and knowledge sharing.
Assessment	Review and revise existing assessment modalities to align with flexible teaching methods.	Evaluate current assessment methods to identify areas for improvement. Modify assessments to accommodate flexibility in learning environments.	Allocate budget for committee meetings, consultations, and coordination efforts. Invest in assessment analysis tools or software. Conduct faculty and student surveys or focus groups.	Number of assessment modalities reviewed and revised. Alignment of assessment modalities with flexible teaching methods.

Area	Tasks/ Objectives	Activities	Strategy of Execution	Performance Indicator
		Ensure assessments align with course objectives and learning outcomes.	Design and distribute communication materials for revised assessment modalities	Feedback from faculty and students on the effectiveness of revised assessment modalities.
	Explore alternative assessment methods, such as project-based assessments and formative assessments.	Research and consider alternative assessment approaches suitable for flexible teaching. Introduce project-based assessments to assess the practical application of knowledge. Incorporate formative assessments for continuous feedback and progress monitoring.	Research and acquire resources on alternative assessment methods. Develop training materials and resources. Conduct workshops or training sessions. Share success stories through online platforms or events.	Identification and introduction of alternative assessment methods. Adoption and implementation of project-based assessments and formative assessments. Feedback from faculty and students on the suitability and impact of alternative assessment methods.
	Provide training and support for faculty to design and implement flexible assessments.	Offer training sessions on designing flexible assessments. Provide resources and examples of effective, flexible assessment strategies. Support faculty in implementing flexible assessments in their courses.	Develop a professional development program. Provide resources and materials for faculty training. Allocate budget for mentoring or coaching sessions. Establish an online platform or community.	Number of faculty members receiving training on flexible assessment design. Completion of training sessions or workshops on flexible assessments. Faculty members' feedback indicates increased confidence and competence in designing and implementing flexible assessments.
	Establish clear and transparent assessment criteria and rubrics.	Develop clear assessment criteria aligned with learning objectives. Create transparent rubrics to guide evaluation and grading. Communicate assessment criteria and rubrics to students for clarity.	Develop assessment criteria and rubrics. Design and distribute communication materials. Conduct training sessions on assessment criteria and rubrics. Regularly review and update assessment criteria and rubrics	Development of clear assessment criteria and rubrics aligned with learning outcomes. Communication of assessment criteria and rubrics to faculty and students. Feedback from faculty and students on the clarity and usefulness of assessment criteria and rubrics.

Development of Discovery-Based Etnobra (Ethnomathematics Geogebra) Geometry Learning Model to Improve Geometric Skills in Terms of Student Learning Styles and Domicile.

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Abstract: The low knowledge and love of students for culture, plus based on preliminary analysis, it is known that geometry skills are relatively low. The a need for harmony between student and teacher learning styles to maximize student engagement and focus in the classroom. So this study aims to create a culture-based Geometry learning design that effectively improves students' geometry skills in terms of students' learning styles and domicile. Furthermore, specifically examines geometry skills in terms of learning styles and student domicile. The subjects of the study were students of SMP Negeri 1 Ciruas and SMP Negeri 1 Tanara, Serang Regency. The research used the R&D method of prototype development model and quality research method. The results of the study found that the invention-based Ethnobra (Ethnomathematical Geogebra) design was valid, practical, and effective in improving students' geometry skills. The results of the subsequent analysis found that urban students had better geometry skills than rural students. It's also known that students with a tendency towards kinesthetic learning styles have the highest geometry skills. This study recommends that teachers use the Ethnobra model by integrating Geogebra applications in Geometry learning, especially for students who go to urban schools and with kinesthetic learning styles.

Keywords: ethnomathematics, geogebra, geometric skills, learning styles, domicile

INTRODUCTION

One of the scopes of junior high school mathematics is Geometry. Geometry is one of the objects of mathematical study that is quite abstract and difficult to understand (Al Afgoni et al., 2020; Karapınar &; Alp İlhan, 2018). In learning geometry, geometry skills are needed, namely the ability of learners to observe objects, build definitions based on the inherent characteristics of things, recognise relationships between objects, and apply them in solving geometry problems (VanHiele, 1959). However, the results of previous studies showed a low level of students' geometry skills (Hamidah et al., 2022; Hamidah &; Kusuma, 2020; Haviger & Vojkůvková, 2014, 2014, 2015; Verner et al., 2019). Van Hiele divided geometry skills into five levels: level 0 (visualization), level 1 (analysis), level 2 (abstraction), level 3 (deduction), and level 4 (rigor). The results of previous it is show that the level of geometry thinking of Van Hiele students is mostly only up to the level of visualization studies (ALTUN, 2018; Asemani et al., 2017; Karapınar &; Alp İlhan, 2018; Şefik et al., 2018).

The initial analysis results found that 48.3% at level 0, 34.5% of level 1 students, and 17.2% at level 2. Based on the interview results, students stated that their difficulties in dealing with geometry problems were drawing objects in geometric shapes from story problems, imagining abstract figures, proof problems, and complications due to forgetting formulas. The results of the initial research on student errors, in general, students make many mistakes in reading and understanding the meaning of the problem. Observations and teacher interviews show that learning habits in the classroom refer to school books without development. The book is very rigid and outdated, that is, it provides definitions and formulas, examples of problems with their solutions, after which practice questions. In the era of Society 5.0, this method is outdated and less optimal for developing student abilities. So in this study, an effective geometry learning design in the era of Society 5.0 will be designed to improve students' geometry skills.

The era of society 5.0, puts humans as the main component and utilizes technology to solve problems. To realize this, learning by utilizing the Geogebra application can optimize students' geometry skills. Geogebra is one of the *tools* that can clarify and facilitate students' understanding of abstract objects (Chivai et al., 2022; Dockendorff &; Solar, 2018; Korkmaz, 2021). Geogebra provides visual experience, develops experimental processes, and draws geometric objects easily and precisely (Alkhateeb &; Al-Duwairi, 2019; Celen, 2020; Yorganci, 2018; Zengin, 2018). So Geogebra was chosen as a tool for learning Geometry to investigate Geometry problems. Technology development is very rapid, accompanied by the demands of technology-based education. Geogebra is one of the learning media that can demonstrate and invite students to be actively involved in constructing Geogebra objects to make them easier to understand and more meaningful. Geogebra is designed to help students develop experimental, problem-oriented, and discover Geometry concepts (Çolakoğlu, 2018). So that the learning design is designed to be problem-oriented and invites students to discover the concept of Geometry.

Furthermore, ethnomathematics is interpreted as the study of mathematics and its relationship with culture in the context of social life in society (Prahmana &; D'Ambrosio, 2020; Verner et al., 2019). Similarly, ethnomathematics is a study that examines mathematical ideas in various cultures that show reciprocal relationships (Pradhan, 2017; Rosa & Gavarrete, 2017). Learning with an Ethnomathematical approach will make learning richer and more meaningful and students' love for their area will emerge (Rosa et al., 2016; Rosa & Gavarrete, 2017). Furthermore, learning with an Ethnomathematical approach that develops in culture in an area will strengthen students with their customs and environment (Prieto et al., 2015). In other words, mathematics and culture will be an interesting scientific context, because students will learn mathematics based on the culture they understand and are relevant to their daily lives.

The design then chose an Ethnomathematical approach by including Geogebra as an application that would invite students to be oriented to Cultural problems in finding Geometry concepts, to improve students' geometry skills. Ethnomathematical studies began to be introduced by (D'Ambrosio, 1985), and continue to be researched and developed (D'Ambrosio, 1999). (D'Ambrosio, 1999) Learning with an Ethnomathematical approach that develops in culture in an area will strengthen students with their customs and environment (Knijnik, 2002; Mosimege, 2012; Prahmana &; D'Ambrosio, 2020; Prieto et al., 2015). So that the Ethnomathematics approach is used to create meaningful learning and increase students' love of their culture.

Previous research has discussed Geogebra to overcome students' difficulties understanding Geometry material. However, no one has used Geogebra with Ethnomathematics as a foundation for designing their learning designs. Therefore, this study intends to find problem-oriented learning designs using Ethnomathematics as a cultural approach and then utilizing Geogebra to manifest the era of society 5.0. The design is called Etnobra, one of this study's novelties. Furthermore, this study uses an Ethnomathematical approach closely related to Culture in the daily student environment. However, it is a well-known tendency that students who live in cities and villages have different understandings of culture and different technological facilities. So the Ethnomathematical approach and the use of applications may have a negative or positive effect on students' understanding of concepts when viewed from their domicile. For this reason, this study will examine whether the domicile of students attending school (cities and villages) affects students' geometry skills using learning with the Ethnobra model (Geogebra Ethnomathematics).

Every child is born with different abilities, especially in terms of absorbing, processing, understanding and conveying information, this ability is called a learning style (Nugraha &; Rahman, 2021; Tatminingsih, 2022). Learning style is called collaboration on how a person absorbs and processes all the information obtained (Chetty et al., 2019; Costa et al., 2020; Leyton-Román et al., 2020; Wang et al., 2019). So the a need for harmony between student and teacher learning styles to maximize student engagement and focus in class to get satisfactory results. There are three learning styles, namely visual, auditorial and kinesthetic learning styles (Buşan, 2014; Chetty et al., 2019; Leasa et al., 2017; Saga et al., 2015).

This study discusses three independent variables, namely Ethnobra learning design, learning style, and domicile, and one dependent variable is geometry skills. The framework of thinking about the relationship between variables, namely, the learning style and domicile of students attending school affects students' geometry skills after being given learning with the Ethnobra model. The following is summarized in the form of a flow chart.

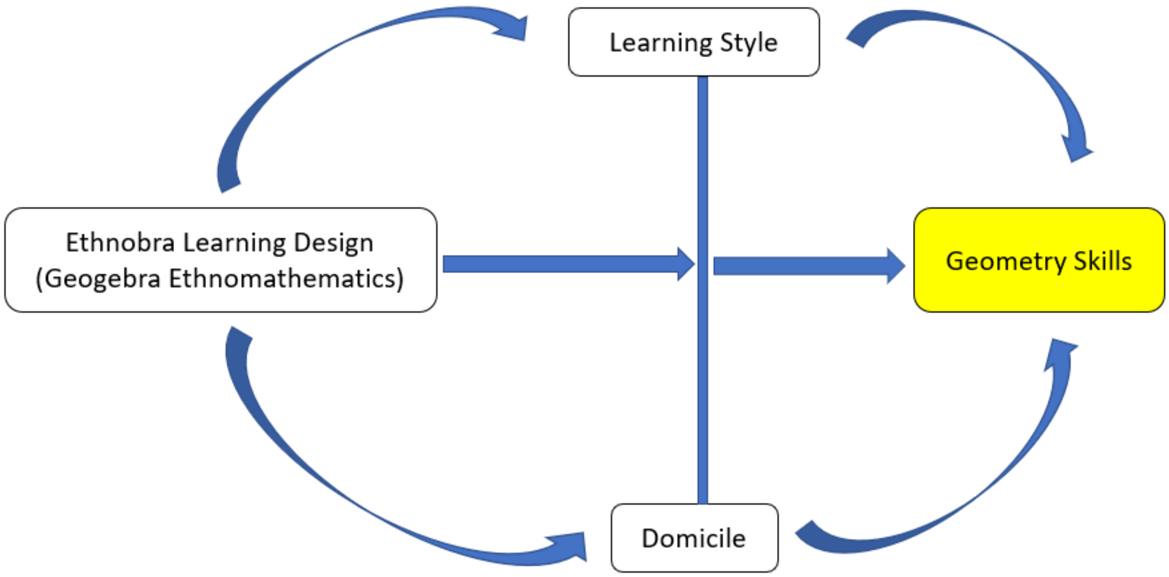


Figure 1: Research Model

Based on the reasons above, it is necessary to develop a culture-based Geometry learning design that effectively improves students' geometry skills regarding learning styles and domicile. The learning design is designed using technology that makes it easier for students to understand geometry lessons, namely the Geogebra application, so the learning design developed is an Ethnobra learning model (Geogebra Ethnomathematics). This research has two objectives, the first is to develop a quality Ethnobra (Ethnomathematical Geogebra) learning model with valid, practical, and effective categories and the second goal is to analyze the geometry ability of students taught with the Ethnobra (Ethnomathematical Geogebra) learning model in terms of student learning styles and domicile.

METHODS

The research began in the odd semester of 2023-2024 in July 2023, the subjects of the study were students of SMP Negeri 1 Ciruas and SMP Negeri 01 Tanara class VIII. The subjects were selected by purposive sampling, considering that SMP Negeri 1 Ciruas represented urban areas and SMP Negeri 01 Tanara represented rural areas. Class VII was selected based on line and flat build materials. Based on the research objectives, this study uses R&D methods to develop learning models and uses a qualitative approach to analyze students' geometry skills regarding learning styles and domicile of students attending school.

Development Research

In the R&D method, the model development procedure used is a prototype development model, with stages namely (1) preliminary research, (2) prototyping stage, and (3) assessment stage (

Plomp, 2013; Van den Akker et al., 2006). Systematically the stages of his research are described as follows.

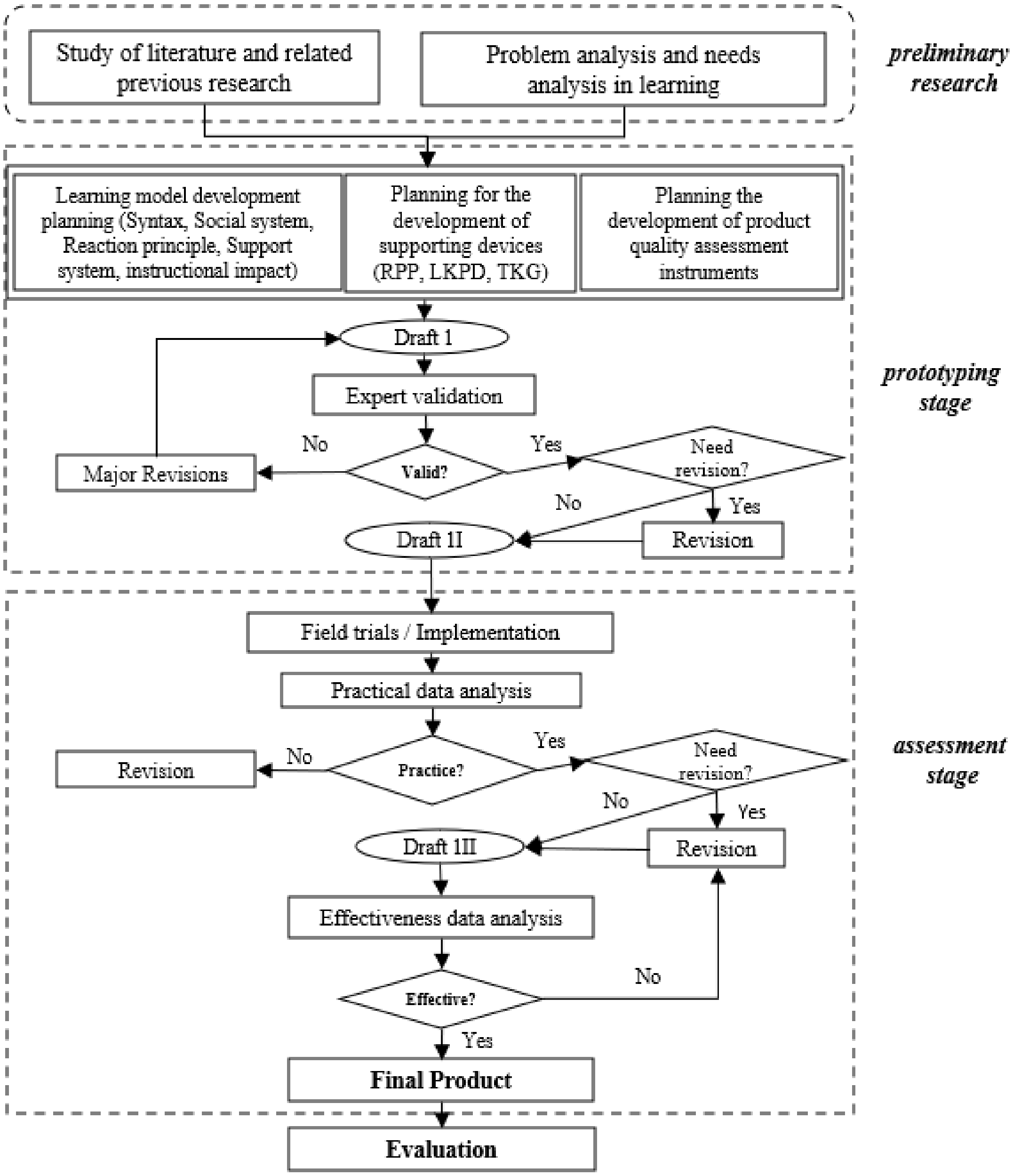


Figure 2: Research Development Model

The first stage, preliminary research, includes the literature and field studies used as a needs analysis on the development of learning designs and comparisons with relevant research. Field studies are conducted to analyze the needs of learning design to measure the initial ability to think the geometry of students and the culture in the student environment so that learning designs are tailored to the needs of students and teachers. The prototyping stage is carried out to produce a draft of learning design in the form of a draft of learning model development planning, a draft of planning for the development of supporting devices (RPP, LKPD, and geometry skill tests), and a draft of product quality assessment instrument development. The product development phase produces draft grids, instruments, and Geometry skill assessment rubrics. The activity continued with testing, namely expert validation and revision.

Furthermore, the assessment stage is carried out field trials, and an analysis of the practicality and effectiveness of the learning design developed. The learning model was implemented on grade VIII junior high school students at SMP Negeri 1 Tanara and SMP Negeri 1 Ciruas. After receiving feedback, the learning design is evaluated and improved to obtain a valid, practical, and effective final product to improve students' geometry skills.

The instruments used in this study include instruments to assess product quality, including three aspects: validity, practicality and effectiveness. Validators use product validity assessment instruments to assess the quality of validity of the product developed. The validator assessment sheet consists of a validity assessment sheet for the learning model and its tools (RPP, LKPD, and geometry skill test). Product practicality assessment instruments are assessed from two criteria, namely based on the results of practicality questionnaires from experts/observers and teachers who state that the learning model developed can be applied and (2) practical questionnaires from students, namely in real-time in the field, students as users state that the developed model is easy to apply. The assessment instrument for the effectiveness of the learning model developed is determined by the criteria for achieving students' geometry skills that are better than students with conventional learning.

Qualitative Research

After conducting development research, this study used qualitative research methods to describe students' geometry skills with Ethnobra learning based on visual, auditorial, and kinesthetic learning styles. The instruments used were geometry skill tests and learning style questionnaires. In this study, data were obtained from two schools with urban area school backgrounds and schools with rural backgrounds. The two schools will be given learning with the Ethnobra model, geometry skill tests, and learning style questionnaires. Based on learning style questionnaire data in each school, two students were selected to be interviewed regarding geometry skills and Ethnobra learning models.

The learning style questionnaire consists of 36 statement items prepared based on a grid of visual, auditorial, and kinesthetic learning styles validated and declared valid by experts for use. The preparation of the questionnaire instrument uses the Likert scale with 5 answer choices, namely always, often, sometimes, rarely, and never. The geometry skill test is made based on five indicators of geometry skills, namely those that have been declared valid by validators, namely visual, verbal, drawing, logic, and applied (Connolly, 2010; Jebur, 2020; Molina &; Mason, 2021; Trimurtini *et al.*, 2022; Tzagkourni *et al.*, 2021). Furthermore, unstructured interviews were conducted to obtain in-depth information. The interview guidelines used were an outline of questions about the Ethnobra learning model and students' geometry skills asked of the research subjects.

The validity of the data in this study includes the degree of trust (credibility), transferability criteria, dependability criteria (dependability), and certainty criteria (confirmability). Triangulation in research compares data on written test results of geometry skills, observation data of students' geometry skills in class, and data on geometry skills interviews (triangulation method). It also compares data reductions from subjects within the same group of students' geometry skill levels (triangulation of data sources).

RESULTS AND DISCUSSION

Based on the objectives of this study, the first is to develop a quality Ethnobra (Ethnomathematical Geogebra) learning model with valid, practical, and effective categories and the second goal is to analyze the geometry skills of students taught with the Ethnobra (Ethnomathematical Geogebra) learning model in terms of student learning styles and domicile.

Development Research Results

The first result is the development of learning models and their tools, which are based on prototyping model procedures including the stages of (1) preliminary research, (2) prototyping phase, and (3) assessment phase.

Preliminary Research

At the preliminary research stage, namely to find out the problems that occur in the implementation of learning geometry subjects in class. Collect information about the needs in geometry learning, the learning model used, the use of culture in classroom learning, and the learning tools teachers use in the learning process. Pre-field survey and preliminary analysis activities are carried out to achieve the objectives mentioned above.

The results of the pre-survey of classroom learning: show that mathematics teachers in schools carry out classroom learning by applying learning models less relevant to student characteristics and mathematics learning objectives. The current condition of mathematics learning implementation is currently running in schools, has not implemented student-centred learning, and has not paid attention to cultural aspects. Learning also has not utilized social interaction patterns in organizing students to learn to be actively involved in reconstructing geometry knowledge through problem-solving sourced from facts and cultural environments.

Results of curriculum analysis: The study of mathematics should align with the goals and content of the mathematics curriculum, which encompasses: 1) comprehending mathematical principles, involves the capacity to elucidate the connections between these principles and to employ them flexibly, accurately, efficiently, and with precision.; 2) utilizing patterns as hypotheses during problem-solving, and having the capacity to formulate generalizations from existing occurrences or data; 3) applying logical thinking to natural phenomena, conducting mathematical operations for simplification, and scrutinizing constituent elements in problem-solving within mathematical and non-mathematical contexts. This encompasses the capability to comprehend issues, construct mathematical models, resolve these models, and interpret the resulting solutions, all of which are valuable for addressing real-world problems in daily life; 4) effectively conveying concepts, logical reasoning, and crafting mathematical proofs through the use of complete sentences, symbols, tables, diagrams, or other means to elucidate situations or issues; 5) possessing an attitude that values the practicality of mathematics in daily life, which includes curiosity, attentiveness, and enthusiasm for learning mathematics, along with a determined and self-assured approach to problem-solving; 6) exhibit attitudes and conduct that align with the principles of mathematics and learning, including adhering to rules, maintaining consistency, upholding agreements, showing tolerance, respecting diverse viewpoints, displaying politeness, promoting democracy, demonstrating resilience, fostering creativity, valuing the context and environment, fostering cooperation, ensuring fairness, practicing honesty, being meticulous, exercising flexibility and openness, and having a willingness to share emotions with others; 7) engage in physical activities that apply mathematical understanding; 8) utilize basic educational tools and technological outcomes for conducting mathematical tasks. Although there is no firm separation for the compatibility between the goals to be achieved and the domain of competence, it can be identified that the focus of the subject objectives is to be achieved when students learn the basic competencies of a particular domain. By taking into account the description of the competencies learned by junior high school students and the objectives of junior high school mathematics subjects, the following are the focus of subject objectives to be achieved when students learn the basic competencies of junior high school mathematics in certain areas, namely attitudes, knowledge, and skills. Based on curriculum analysis, a learning model is designed that covers these three domains. The material chosen is the subject of Geometry sub-material triangles and quadrilaterals because the development of learning models focuses on students' geometry skills.

Analysis of student characteristics: The outcomes of the examination of student traits are known to students of SMP Negeri 1 Ciruas who come from urban school backgrounds. Students are provided with wifi, a strong network, and infocus facilities in some classes. Meanwhile, students at SMP Negeri 1 Tanara come from rural school backgrounds far from urban areas. There is no wifi in the school, and no infocus can be used to support technology-based learning. However, when conducting direct interviews with class teachers about culture, it was known that most students at SMP Negeri 1 Ciruas preferred foreign cultures due to globalization.

Meanwhile, students at SMP Negeri 1 Tanara are still familiar with regional culture and there are still few who are influenced by foreign culture. One of the reasons is technology that students in rural areas cannot easily access. In other words, technology can have both a positive impact and a negative impact on students.

Preliminary analysis of students' geometry skills: It is known that 48.3% at level 0, 34.5% at level 1 students, and 17.2% are at level 2. Based on the interview results, students stated that their difficulties in dealing with geometry problems were drawing objects in geometric shapes from story problems, imagining abstract shapes, proof problems, and difficulties due to forgetting formulas. The results of the initial research on student errors in solving geometry problems are that students generally make many mistakes in reading and understanding the meaning of the problem. Observations and teacher interviews show that learning habits in the classroom refer to school books without development.

Results of theoretical studies: Based on the analysis above, theories that support overcoming existing problems are studied. One of them is designing an effective learning model for geometry learning. The design of the designed learning model is called Ethnobra. The Ethnobra learning model stands for Ethnomathematics and Geogebra. Ethnomathematics is used to create meaningful learning and increase students' love for their culture and Geogebra is one of the learning media that can demonstrate and invite students to be actively involved in constructing Geogebra objects so that they are easier to understand and more meaningful. In learning, it is designed so that the student centre is discovery-based. The information obtained is used for designing development products, namely learning models, supporting devices, and instruments to assess product quality.

Prototyping Phase

At this stage, produce a draft of learning design in the form of a draft of learning model development planning, a draft of planning for the development of supporting devices (RPP, LKPD, and TKG/geometry ability tests), and a draft of product quality assessment instrument development. The product development phase produces learning design drafts, grids, instruments, and Geometry skill assessment rubrics.

The learning model is designed to invite students to make discoveries with the help of the Geogebra application by covering the components of the learning model, which include (1) syntax, (2) social systems, (3) reaction principles, and (4) support systems (5) learning impacts and companion impacts. The design of the Learning model is then presented in the form of Draft I of the Ethnobra learning model. In addition to the components of the learning model, supporting tools are included, namely RPP, Student Worksheets (LKPD), and Geometry Skills Test (TKG).

The design of the learning model is as follows.

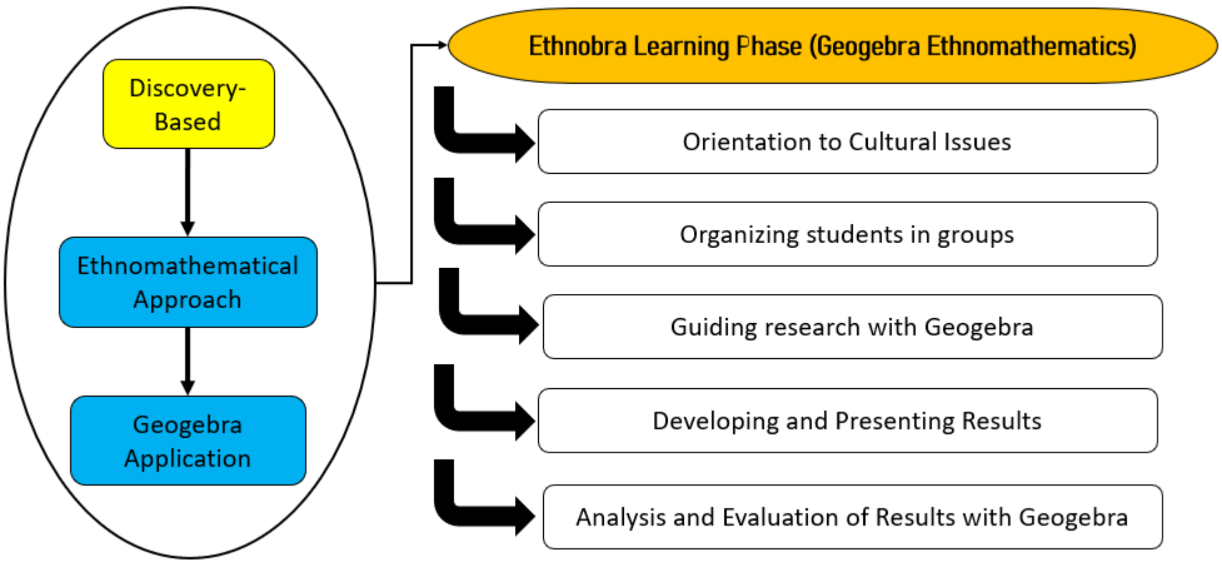


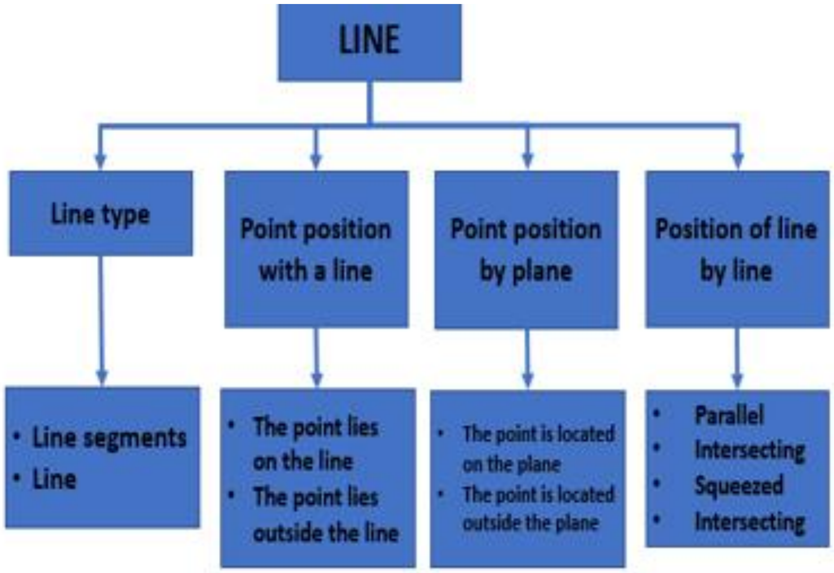
Figure 3: Ethnobra Learning Design (Geogebra Ethnomathematics)

Figure 3 is an Ethnobra learning design design accompanied by discovery-based learning steps. Furthermore, LKPD is prepared as a worksheet containing activities and independent practice questions for students related to geometry material done manually and with the help of the Geogebra application. Student worksheets (LKPD) are arranged into four learning activities, namely learning activity 1 (point, line, field, line ray, line segment, and angle), learning activity 2 (relationship between point, line, and plane), learning activity 3 (congruence and awakening properties), learning activity 4 (flat building). Each learning activity presents problems related to ethnomathematics which then presents the steps of investigation with the Geogebra application. For the learning process, each meeting is carried out based on five stages of Ethnobra learning (Figure 3). The following is an example of LKPD designed in learning activity 1 line material.

Ayo Kita Mengenal Budaya Banten

Batik Banten memiliki pola pengulangan cap dengan tujuan untuk efisiensi, mempercepat, dan memperbanyak hasil produksi. Motif batik Banten memiliki garis yang tebal, isen-isen kasar, ukuran motif yang cukup besar. Awal mula kemunculan batik Banten adalah keinginan pemerintah Provinsi untuk menginventarisasi kekayaan budaya setempat. Bidaya wilayah setempat menjadi ciri khas batik Banten. Proses pengkajian batik Banten telah dilakukan pada tahun 2002, kemudian diperkuat dengan surat keputusan Gubernur pada tahun 2003 untuk membentuk panitia peneliti. Penelitian dilakukan dengan mengambil sumber data arkeologis untuk menemukan kondisi Banten di masa lalu. Sumber motif berasal dari bangunan arkeologis pada pemerintahan Sultan Maulana Hasanuddin, pendiri kesultanan Banten. Hasil penelitian pada tahun 2004 berhasil mengumpulkan 75 motif batik Banten. Beberapa diantaranya telah diproduksi yaitu Surosowan, Pasulaman, Pasepen, sabakingking, Srimanganti, dan masih banyak lagi.

Motif Srimanganti misalnya, diambil dari nama bangunan istana yang digunakan oleh raja bertatap muka dengan rakyatnya. Warna motif ini adalah merah tua, coklat tua, dan hitam. Makna motif Srimanganti adalah sifat raja yang arif, teguh hati, dan pemberani.



Let's Get to Know Banten Culture

Banten Batik has a stamp repetition pattern with the aim of efficiency, speeding up, and multiplying production results. Banten batik motifs have thick lines, rough isen-isen, the size of the motif is quite large. The beginning of the emergence of Banten batik was the desire of the provincial government to inventory the local cultural wealth. Bidaya local area is the hallmark of Banten batik. The Banten batik assessment process was carried out in 2002, then strengthened by the Governor's decree in 2003 to form a research committee. The research was conducted by taking archaeological data sources to find the condition of Banten in the past. The source of the motif comes from archaeological buildings during the reign of Sultan Maulana Hasanuddin, the founder of the Banten sultanate. The results of research in 2004 managed to collect 75 Banten batik motifs. Some of them have been produced, namely Surosowan, Pasulaman, Pasepen, sabakingking, Srimanganti, and many more.

The Srimanganti motif, for example, is taken from the name of the palace building used by the king face to face with his people. The colors of this motif are dark red, dark brown, and black. The meaning of Srimanganti's motif is the nature of the king who is wise, determined, and brave.

Setelah kamu amati, pola batik di atas terdiri dari titik, garis, dan bangun-bangun datar. Gambarkan bentuk apa saja yang ada pada batik di atas!

After you observe, the batik pattern above consists of points, lines, and flat shapes. Describe what shapes are on the batik above!

Sebutkan bagaimana kedudukan dari titik dan garis, kedudukan titik dan bidang, serta kedudukan garis dan garis yang dapat kamu temukan dari gambar batik di atas? Gambar kan!

Mention how the position of points and lines, the position of points and planes, and the position of lines and lines that you can find from the batik picture above? Draw it right!

Figure 4: Example 1 Student Worksheet

Figure 4 is an example of LKPD, a worksheet given to students. The worksheets are packed by introducing Banten culture, namely batik, to attract and provide cultural understanding to students. Furthermore, students are asked to observe and analyze the shape of batik patterns and look for their relation to the teaching material. This activity is directed to be completed in groups to create interesting and memorable discussions and questions and answers.

Kegiatan 1

Part 1
1. Buatlah segmen garis AB
2. Buatlah titik C dan titik E yang berada diantara segmen garis AB
3. Buatlah titik D da n titik F yang tidak berada di antara segmen garis AB

4. Kedudukan titik C adalah .... segmen garis AB
5. Kedudukan titik D adalah .... segmen garis AB
6. Kedudukan titik E adalah .... segmen garis AB
7. Kedudukan titik F adalah .... segmen garis AB

Part 2
1. Buatlah persegi panjang ABCD
2. Buatlah titik E dan titik F yang berada ditengahpersegi panjang ABCD
3. Buatlah titik G da n titik H yang tidak berada di tengahpersegi panjang ABCD

Tentukan!
a. Kedudukan titik E adalah .... persegi panjang ABCD
b. Kedudukan titik F adalah .... persegi panjang ABCD
c. Kedudukan titik G adalah .... persegi panjang ABCD
d. Kedudukan titik H adalah .... persegi panjang ABCD

Activities 1

Part 1
1. Create an AB line segment
2. Make point C and point E between the AB line segments
3. Make a point D and a point F that is not between the segments of line AB

4. The position of point C is.......................... against line segment AB
5. The position of point D is .... against line segment AB
6. The position of point E is .... against line segment AB
7. The position of point F is .... against line segment AB

Part 2
1. Create an ABCD rectangle
2. Make point E and point F in the center of the ABCD rectangle
3. Make a point G and a point H that is not in the center of the ABCD rectangle

Specify!
a. The position of point E is.......................... against rectangle ABCD
b. The position of point F is.......................... against rectangle ABCD
c. The position of point G is.......................... against rectangle ABCD
d. The position of point H is.......................... against rectangle ABCD

Kegiatan 3

Kerjakan menggunakan aplikasi GeoGebra! Kemudian isilah titik-titik yang disediakan!

1. Buatlah kubus ABCD.EFGH !
2. Berapakah panjang rusuk kubus ABCD.EFGH yang kamu buat?
3. Buatlah titik R yang merupakan titik tengah segmen garis AB!
4. Tentukan jarak antara titik F dengan titik R!
5. Tentukan jarak antara titik R dan segmen garis GH dengan aplikasi Geogebra!
6. Buatlah bidang CDEF, dengan menghubungkan titik C, titik D, titik E, dan titik F!
7. Hitunglah berapa jarak antara titik R dengan bidang CDEF!
8. Buatlah titik S, yang merupakan titik tengah segmen garis GH!
9. Hitunglah berapa jarak antara garis AS dan garis RG!
10. Buatlah segmen garis AC, dengan menghubungkan titik A dan titik C!
11. Hitunglah jarak antara garis AC dan garis DH!

Buat kesimpulan materi dari rangkaian kegiatan yang dikerjakan!

Activities 3

Do it using the GeoGebra app! Then fill in the dots provided!

1. Create an ABCD cube. EFGH!
2. What is the length of the ribs of the cube ABCD. EFGH you created?
3. Make the point R which is the midpoint of the line segment AB!
4. Determine the distance between point F and point R!
5. Determine the distance between the R point and the GH line segment with the Geogebra app!
6. Create a CDEF field, by connecting point C, point D, point E, and point F!
7. Calculate the distance between the point R and the CDEF field!
8. Make point S, which is the midpoint of the GH line segment!
9. Calculate what is the distance between the US line and the RG line!
10. Make a segment of the AC line, connecting point A and point C!
11. Calculate the distance between the AC line and the DH line!

Make material conclusions from the series of activities carried out!

Figure 5: Example 2 Student Worksheet

Figure 5 is the next activity sheet on the designed LKPD. In this activity, students were asked to discuss the first activity manually and then do the second activity using the Geogebra application.

In the final activity, students are asked to make material conclusions based on the activities carried out. LKPD functions to guide students in the activity of constructing new knowledge in each meeting. The next learning tool is TKG, which is a geometry skill test made based on indicators of geometry skills, namely visual, verbal, drawing, logic, and applied (Connolly, 2010; Jebur, 2020; Molina &; Mason, 2021; Trimurtini *et al*., 2022; Tzagkourni *et al.*, 2021).

Assessment Phase

The activity continued by assessing the quality of the learning model and its tools. Assessing the quality of learning requires instruments of validity, practicality and effectiveness. For the validity of the model and supporting devices for the implementation of the model, validation was requested by three teams of experts.

Validity Assessment Results

Validation is carried out on the learning model and its devices. The formula used is (Arikunto, 2012):

$$P = \frac{\sum R}{N} \times 100\%$$

Information:
P : Percentage of score searched
$\sum R$: number of answers given by validators / selected choices
N : Maximum or ideal number of scores
The criteria or level of achievement of the instruments used in model development are as follows (Arikunto, 2012).

Achievement Level (P)	Qualification	Information
81% – 100%	Excellent	Very well deserved. No revision required
61% – 80%	Good	Feasible, no revision required
41% – 60%	Good enough	Less worthy needs revision
21% – 40%	Not good	Not worth it, needs revision
< 20%	Very unfavourable	Very unfeasible, needs revision

Table 1: Instrument Criteria.

The summary of the validation calculation results is presented in the table.

Object	Aspects	Attainment Rate (P)	Information
Ethnobra Learning Model	Content validation	82%	Very decent
	Construct validation	89%	Very decent
RPP (Learning implementation plan)	Indicator Formulation	78%	Proper
	Purpose	85%	Very decent
	Time Allocation	99%	Proper
	Material	82%	Very decent
	Learning activities (conformity to Ethnobra syntax)	89%	Very decent
LKPD (Student worksheet)	Content suitability	78%	Proper
	Conformity of construction requirements	75%	Proper
	Technical compliance	78%	Proper
TKG (geometry skill test)	Question items	88%	Very decent
	Time allocation suitability	85%	Very decent
	General validity	87%	Very decent

Table 2: Summary of Validity Assessment Results

Practicality Assessment Results

The final product's practicality is determined by whether it fulfills the specified criteria of (1) experts/observers and teachers stating that the learning model developed can be applied and (2) in real-time in the field, students as users state that the developed model is easy to apply. The findings from the evaluation of practicality are displayed within the table.

Assessment	Aspects	Amount That Gives a Minimal Good/Practical Rating (P)	Conclusion
Teacher	Easy to implement	83 %	According to teachers, the model developed is said to be practical because the average teacher assesses more than 80% with a minimal practical category
	Accuracy of time allocation estimates with implementation	83%	
	Possible achievement of learning objectives	85%	

Student	Easy of understanding	89%	According to students, the model developed is said to be practical because the average student assesses more than 80% with a minimal practical category

Table 3: Results of the Practicality Assessment

Effectiveness Assessment Results

The effectiveness of the learning model developed is determined by the criteria for achieving competence by individual students, as seen from geometry skills results. The results of the assessment of students' geometry skills after being given learning with the Ethnobra learning model compared to conventional learning are presented in the table.

Aspects	Learning Model	
	Ethnobra	Conventional
Average Geometry skills	83	62
Normality Test Results	Sig=0.10>0.05 Normal distributed data	Sig=0.55>0.05 Normal distributed data
Homogeneity Test Results	Sig=0.88>0.05 Score variance of both homogeneous classes	
Average Difference Test Results (T-Test)	Sig=0.000<0.05 Hypothesis rejected	
Conclusion	The geometry skills of students provided learning with the Ethnobra model are better than with conventional learning	

Table 4: Results of the Effectiveness Assessment

The significance test results show that the geometry skills of students given learning with the Ethnobra model outperforms conventional learning methods. This shows that the Ethnobra learning model is more effective for students' geometry skills than conventional learning.

Qualitative Research Results

Filling out the learning style questionnaire was carried out to identify each student's learning style. The questionnaire was given to urban area students at SMP Negeri 1 Ciruas totaling 27 students and rural area students, namely at SMP Negeri 01 Tanara totaling 34 students. The results are presented in the following table.

Learning Style	Number of Students Based on School Domicile			
	City		Village	
Visual	14	51.85%	22	64.71%
Auditorial	9	33.33%	12	35.29%
Kinesthetic	4	14.81%	0	0%
Sum	27	100%	34	100%

Table 5: Results of Student Learning Style Questionnaire

Based on the results of the questionnaire data calculation, it is known that the trend of student learning styles varies in each school. In general, it is known that students in urban and rural areas tend to have visual learning styles, however, none of them have kinesthetic learning style tendencies. Furthermore, the geometry skill test results are presented to students and reviewed based on their learning style.

Learning Style	Average Geometry Skills Test	
	City	Village
Visual	74	70.32
Auditorial	82.11	70.42
Kinesthetic	88.5	0
Sum	81.54	70.37

Table 6: The results of students' geometry skill tests are reviewed based on style

From the table above, it is known that the average geometry skill test results of students in urban areas are higher than the average geometry skill test results of students in rural areas. Moreover, considering different learning styles, it's evident that students with a kinesthetic learning preference exhibit superior geometry skills compared to students with other learning styles. However, in rural areas, there are no students displaying a preference for kinesthetic learning style.

A statistical test was carried out to determine the significance of the difference in the average geometry skill test of students in urban and rural areas after learning with the Ethnobra model, with the calculation results presented in the following table.

Normality Test Results	Homogeneity Test Results	Average Difference Test Results (T-Test)	Conclusion
Urban School Class Sig=0.65>0.05 Normal distributed data Rural School Classes Sig=0.25>0.05 Normal distributed data	Score variance of both homogeneous classes	Sig=0.000<0.05 Hipotesis ditolak	There are significant differences in geometry skills of urban and rural students after learning with the Ethnobra model

Table 7: T-Test Geometry Skills Test Students In Urban and Rural Areas

From the table above, it can be concluded that the geometry skills of urban area students are better than those of rural area students after being given learning with the Ethnobra model. Furthermore, in-depth interviews were conducted with several selected subjects to get more in-depth information. The selection of interview subjects is selected based on the following table.

Learning Style	School Domicile	Student Code
Visual	City	VK1, VK2, and VK3
	Village	VD1, VD2, and VD3
Auditorial	City	AK1, AK2, and AK3
	Village	AD1, AD2, and AD3
Kinestetik	City	KK1, KK2, and KK3
	Village	-

Table 8: Research Subjects Interviews

The subjects in this study were 27 students who went to school in urban areas and 34 students who went to school in rural areas. The results of the descriptive analysis in Table 5 are known to be the distribution of student learning style trends. Furthermore, 15 students were selected to be interviewed, namely three students each representing the student's learning style and domicile. The results of interviews with the 15 students and based on observations concluded that when viewed from a school domicile, rural students stated that it was still difficult to follow the learning process delivered (Ethnobra learning model) because they were not familiar with technology-based learning. However, students claimed to be very interested in learning related to culture, students stated that they became more familiar with their culture and felt that learning became more interesting because the culture they mentioned was recognized. Facts in the field also support this, it is known that to conduct technology-based research, the school must move students to classes with electricity. In the school, almost all classes have no electricity and have never done learning using PowerPoint. Students claimed to be interested in the Geogebra application but stated that they could not use it at home because they did not have a laptop, computer, or mobile phone to install it. So, the teacher must vary learning, one of which is by utilizing the Geogebra application.

From these findings, several appropriate steps are needed to overcome how to control these factors so that students' geometry skills can be developed optimally. The steps are to familiarize students who go to school in rural areas with technology-based learning. The school should be given an understanding of the importance of involving technology in the learning process in this growing era, so they will try to facilitate. At least every classroom has electricity, and schools have at least one infocus that teachers can use to implement technology-based learning. Based on input from teachers when interviewed, that teachers want to develop their abilities in technology-based learning but school facilities are still limited. In other words, students will get used to using technology in understanding the material if the teacher familiarizes technology-based learning. Then, teachers will familiarize technology-based learning if the school facilitates supporting facilities and infrastructure. Furthermore, in addition to supporting facilities and infrastructure, teachers also need knowledge in the form of training on how to use technology in learning, one of which is using the Geogebra application in explaining Geometry material. Geometry is an abstract lesson, so it will be more optimal if the learning process is technology-based.

Another thing that catches the eye is the colloquial language in which students in rural areas speak most regional languages. Students are very comfortable and relaxed with friends and teachers in regional languages during the learning process. This certainly needs to be a concern because there are good and bad sides. The good side is to preserve regional languages, and support for learning with the Ethnobra model (Ethnomathematical Geogebra). The student's understanding becomes more profound because the use of regional languages makes the student comfortable and confident thus helping him understand concepts in a more familiar and familiar context. However, the downside is that there are limited learning materials, learning materials in regional languages may be more limited than in national or international languages. This can limit students' access to a wider and more diverse range of information. Further, the downside is the possibility of an inaccurate understanding of the material. Students who are very thick in regional languages sometimes do not understand Indonesian, so it can sometimes cause inaccurate understanding or defects in learning materials. Some specific terms or concepts in certain subjects may not have direct equivalents in regional languages, as a result, students encounter challenges comprehending the content presented by the teacher. In other words, a solution is needed to overcome it, namely as a teacher needs to understand very well how the characteristics of the students he teaches before deciding to use regional languages or Indonesian when teaching. Because the use of regional languages when teaching is not always bad nor always good, so choosing the right time in the use of regional languages or not has a major influence on optimal student understanding.

Furthermore, another interesting fact is known otherwise in students of urban areas. Students stated that they follow the technology-based learning process because they are accustomed to using technology-based learning and are often taught with mathematics applications. Students claim to be interested in the Geogebra application. This is supported by the student stating that they will install the Geogebra application after school. This student statement also proves that at home students are facilitated with technology-based learning, and few students have mobile phones with wifi networks at home. Nevertheless, students expressed less interest in material associated with culture. Students state that the culture conveyed is unknown, so having to understand is twice as difficult as understanding the culture and the material. According to students, this is considered ineffective for understanding the material. The solution, Culture-based modules/textbooks that will be used is designed as well as possible so that the development is focused on attracting students' attention and introducing Culture. In other words, there is no presentation in the module that asks students to solve questions about Culture.

Another found that students in urban areas tended toward kinesthetic learning styles, and had a much better average score on geometry skills than other learning styles. This can contribute to the high average geometry skills of students in urban areas compared to those in rural areas. The learning process with the Ethnobra model directs students to understand the material by practising the Geogebra application. So that students with kinesthetic learning styles understand the material presented more than others. This finding is important information for teachers, in particular, that it is very important to know the tendency of the learning styles of students taught in order to choose the right learning model. Teachers can also combine several different learning methods to embrace all varied student learning styles so as to help increase the understanding of the material to the maximum.

Furthermore, it is analyzed based on the results of students' work solving geometry skill problems. Based on the student's answers to question number 5, the indicator of geometry skills is applied.

The problem is: Pak Ali has a rectangular coffee plantation with a length of 35 m and a width of 25 m. Around the garden, the road will be widened, with a widening of 1.5 m.

a. Sketch a flat wake based on the information above!
b. If Mr Ali wants to sell his coffee plantation at a price of Rp.3,000,000 per m2, then how much money should Mr Ali receive after widening the road?

The results of students' answers are reviewed from the following learning styles and domicile.

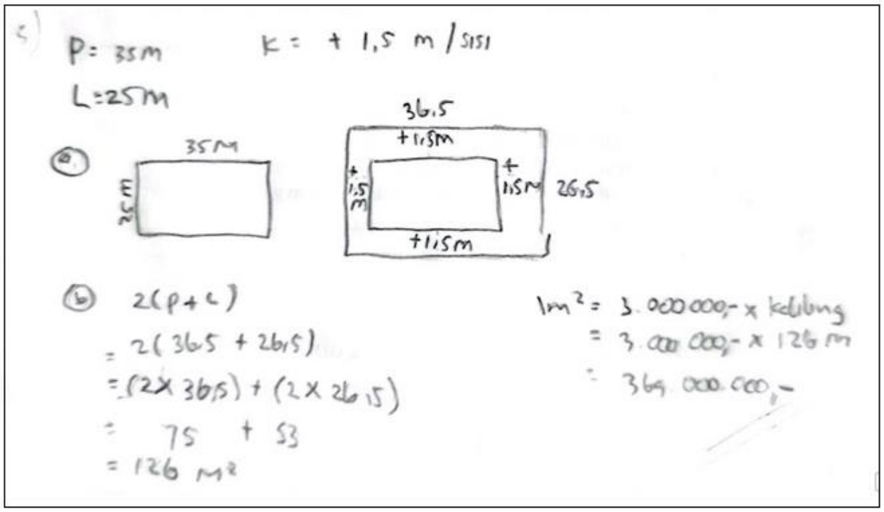


Figure 6: Student Work Results from KK1 Code

Figure 6. Is the result of the work of students with kinesthetic learning style tendencies and schooling in urban areas. Of all the students' answers, only a small part answered correctly and close to correct, the rest answered incorrectly question number 5. The results of this work show that students with kinesthetic learning styles have good understanding skills in solving problems. As Hernandez et al. (2020) mentioned kinesthetic learners have a broader understanding. This student is called KK1, according to him to solve question number 5 the first step that comes to mind is to draw it. From the sketches given, it shows that students understand the problem and know what to do to solve it.

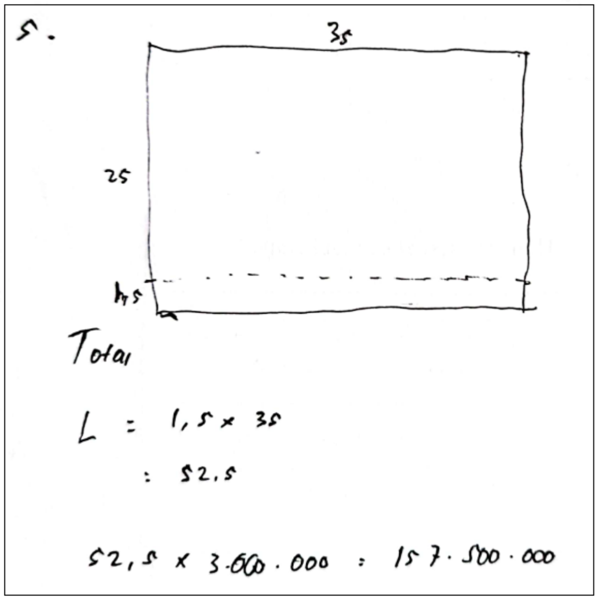


Figure 7: Student Work Results from VD1 Code

Figure 7 results from the work of students with visual learning style tendencies and rural schools. The results of students' answers show that students do not properly understand the instructions on the questions. It is stated that "Around the garden will be widened the road, with a widening of 1.5 m". However, students describe the dilation on only one side. Furthermore, determining the square area is still wrong so the final result written is also wrong. When asked, the student stated that "determining the area of a rectangle is length times width, and in the problem, it is known that the length is 35m and the width after widening is 1.5m". This student's statement informs that students have not been able to think realistically, because the width of the 25m road that has widened has shrunk to 1.5m. This is an expert opinion that one of the characteristics of someone with a visual learning style is to understand images better than written instructions (Chetty et al., 2019).

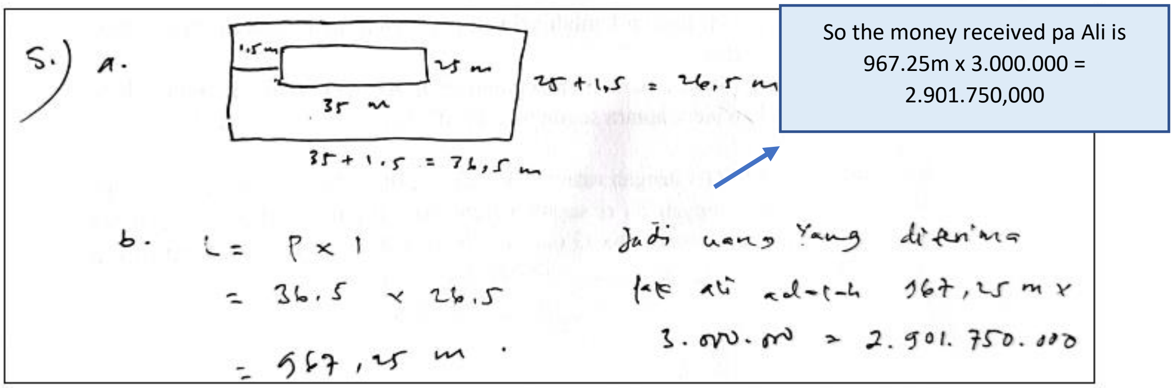
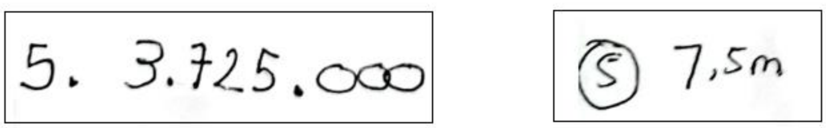


Figure 8: Student Work Results Code VK1

Figure 8 shows students' work with trends in visual learning styles and urban schools. The sketch of the picture made is not much different from the student in figure 7 which shows that students do not properly understand the instructions on the problem. The difference is that urban students are more realistic in the square area written by increasing the width on two sides, but the answer is still wrong. So it is very important to understand the problem to provide the right solution correctly.



a) b)

Figure 9: Student Work Results Code AK1 (a) and AD1 (b)

Figure 9 shows the answers of students with auditorial learning style tendencies who go to school in urban and rural areas. Interestingly, almost all answers of students with auditorial learning styles give answers that are direct to the results and wrong. When asked how to get these results, both students gave a long presentation even though it was still incorrect. However, from this fact, it is known that students with auditorial learning style tendencies can provide much exposure but it is not easy to write it down. This is by the opinion of Kusumawarti &; Subiyantoro (2020), that one of the characteristics of auditorial people is that they like to talk, discuss, and explain things at length.

Another interesting thing was that students who go to rural schools have shy characteristics when asked and use the regional language (Javanese) more often in class. Meanwhile, students who go to urban schools have the characteristics of talking a lot and do not hesitate to joke when discussing, and no students are found who use regional languages when learning. This also needs attention for future research to consider the good and bad sides of using culture (regional languages) in classroom learning, both for students who go to school in urban and rural areas.

Ethnobra Learning Implementation Results

Learning with the Ethnobra model was carried out in both schools, namely SMP Negeri 1 Ciruas which represented urban area schools and SMP Negeri 01 Tanara which represented rural area schools. Here is the implementation of Ethnobra in schools.

Figure 10: Material Delivery with Ethnobra Model in Urban Classroom

Figure 10 shows how the material is delivered with the Ethnobra model (Ethnomathematical Geogebra) to students who go to school in urban areas.

Figure 11: Material Delivery with Ethnobra Model in Rural Classroom

Figure 11 shows how the material is delivered with the Ethnobra model (Ethnomathematical Geogebra) to students who go to school in rural areas. In urban areas, learning is carried out in ordinary classes with facilities that support technology-based learning, namely with electricity and infocus already available in class. However, unlike classes in rural areas, learning was moved to the hall because none of the classes had electricity. This is an obstacle that requires improvement because technology-based learning requires electricity.

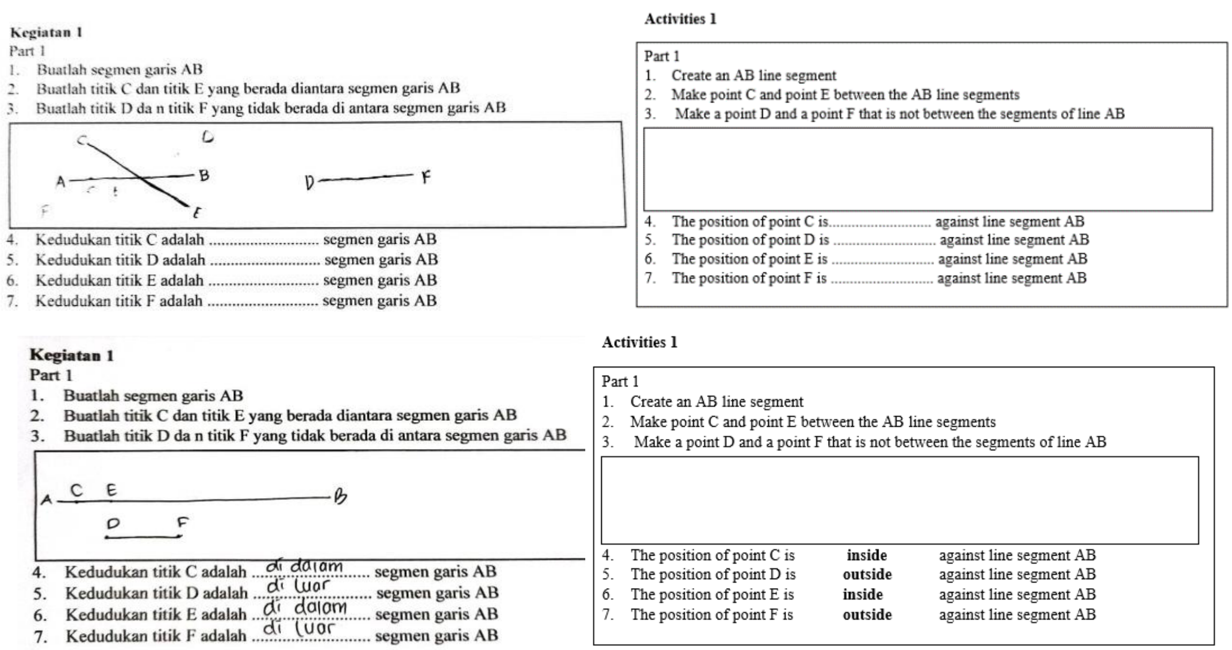
The series of learning activities with the Ethnobra model is in the LKPD (student worksheet) which is designed to invite students to find material from the activities provided as shown in Figure 4 and Figure 5. Activities are made in the form of two steps to complete, namely manually and with the Geogebra application done in discussion. The activity sheet is also designed based on culture by relating the material to the surrounding culture, one of which is Banten batik.

The results of interviews with teachers are known several important points about the effectiveness of learning Geometry with the Ethnobra model (Geogebra Ethnomathematics). The teacher stated that the Ethnobra model is learning that combines Ethnomathematics-based learning with the help of the Geogebra application. Teachers admitted that they were very enthusiastic about the new approach because they could see the value in integrating culture, traditions, and social contexts in mathematics learning then teachers were given the convenience of visualizing the material with the help of the Geogebra application. According to him, students seem more motivated and happy when learning takes place because students associate mathematical material with their daily experiences and realities, and students can be interactively involved in exploring their skills solving geometry problems. However, teachers also stated several obstacles and challenges faced when applying learning with the Ethnobra model, including a feeling of lack of experience so that sometimes there is a sense of uncertainty about how to integrate cultural aspects into mathematics learning effectively. Another challenge is a technical challenge, according to the teacher the use of technology in the classroom is a challenge, because they have no previous experience with the GeoGebra application. So the teacher stated that it was necessary to spend additional time to understand and master this tool before delivering the material to students.

Figure 12: Students Discuss LKPD Ethnobra (Geogebra Ethnomathematics)

In rural and urban schools, students are enthusiastic about discussing the worksheets distributed. The discussion lasts 10-15 minutes, finds the results of the answers and concludes the expected material. The worksheet design is made culturally, making it more interesting and familiar to students. However, there are facts in the field that state that the cultural aspect helps it understand geometry material and there are also those who feel disturbed and complicated by the cultural aspect. In this case, it is known that the use of Culture provides two arguments, namely students feel helped and feel difficult. Thus, in implementing the Ethnobra model, teachers should know the characteristics of the students taught first. If students find it helpful, then the use of Culture in understanding teaching material can continue to be explored more. However, if students find it difficult then the teacher still uses Culture as a variation of learning and attracts students' interest and love for Culture, but it can be reduced exploration and balanced by exploring more Geogebra applications.

The following picture presents one of the results of the student's answers to completing the activity sheet.


Kegiatan 1
Part 1
1. Buatlah segmen garis AB
2. Buatlah titik C dan titik E yang berada diantara segmen garis AB
3. Buatlah titik D da n titik F yang tidak berada di antara segmen garis AB

4. Kedudukan titik C adalah .... segmen garis AB
5. Kedudukan titik D adalah .... segmen garis AB
6. Kedudukan titik E adalah .... segmen garis AB
7. Kedudukan titik F adalah .... segmen garis AB

Activities 1
Part 1
1. Create an AB line segment
2. Make point C and point E between the AB line segments
3. Make a point D and a point F that is not between the segments of line AB

4. The position of point C is.......................... against line segment AB
5. The position of point D is .... against line segment AB
6. The position of point E is .... against line segment AB
7. The position of point F is .... against line segment AB

Kegiatan 1
Part 1
1. Buatlah segmen garis AB
2. Buatlah titik C dan titik E yang berada diantara segmen garis AB
3. Buatlah titik D da n titik F yang tidak berada di antara segmen garis AB

4. Kedudukan titik C adalah ...di dalam... segmen garis AB
5. Kedudukan titik D adalah ...di luar... segmen garis AB
6. Kedudukan titik E adalah ...di dalam... segmen garis AB
7. Kedudukan titik F adalah ...di luar... segmen garis AB

Activities 1
Part 1
1. Create an AB line segment
2. Make point C and point E between the AB line segments
3. Make a point D and a point F that is not between the segments of line AB

4. The position of point C is **inside** against line segment AB
5. The position of point D is **outside** against line segment AB
6. The position of point E is **inside** against line segment AB
7. The position of point F is **outside** against line segment AB

Figure 13: Results 1 of Student Discussion to solve one of the LKPD Questions

Figure 13 shows that students do not understand the basic concepts of geometry before being explained using Geogebra. One of the commands in the worksheet is to create point D and point F. Almost all students draw the DF line. This clearly shows that students have been unable to follow the worksheet's instructions. So that the learning process becomes longer than planned. However, almost all students become more understanding and interested when practising with the Geogebra application using a laptop or mobile phone as shown in Figure 14 and Figure 15.

Figure 14: Students Practice Geogebra Applications to Solve Geometry Problems

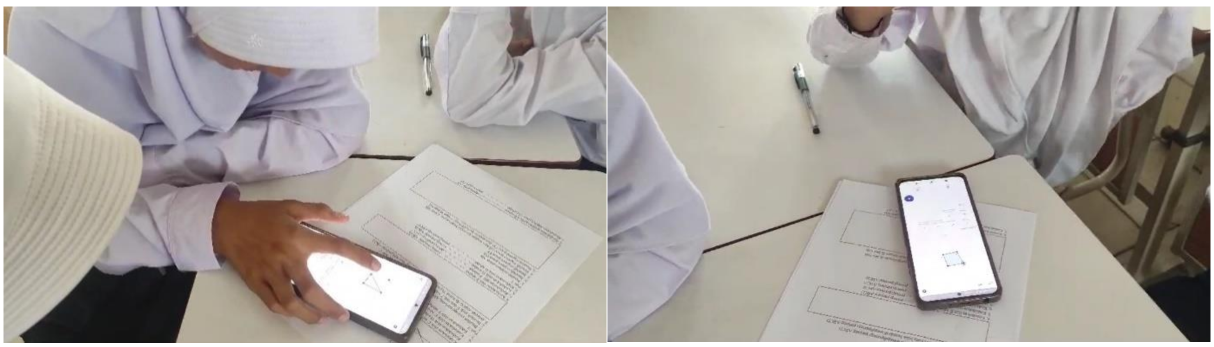
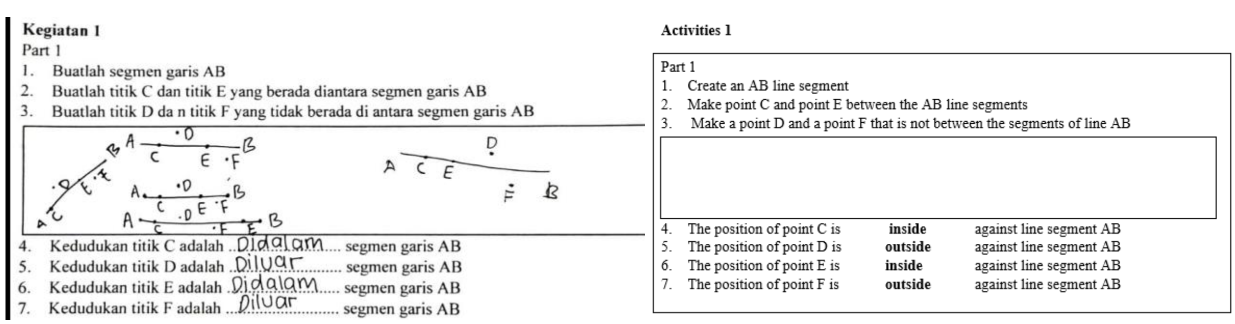


Figure 15: Students Try to Use Geogebra to Complete LKPD

Figure 15 is a student activity discussing completing activities in LKPD using the Geogebra application on a mobile phone. Before using a cellphone, students are explained how to use the Geogebra application to solve Geometry problems and then practice directly as shown in Figure 14. At the beginning of the activity, there were still many obstacles because students were not used to it, so they spent much time explaining how to use the Geogebra application instead of explaining the material. However, after students get used to it, they become faster at solving questions and LKPD activities. Furthermore, students claimed it was easier to solve geometry problems and the visualization was clearer with the Geogebra application. That is, at the beginning of learning to apply the Ethnobra model requires additional time to explain how to use the Geogebra application so that at the next meeting learning becomes smooth. As revealed by previous researchers Geogebra can make it easier for students to solve geometry problems and improve their abilities significantly (Adelabu et al., 2022; Yimer, 2022).



Kegiatan 1

Part 1

1. Buatlah segmen garis AB
2. Buatlah titik C dan titik E yang berada diantara segmen garis AB
3. Buatlah titik D da n titik F yang tidak berada di antara segmen garis AB
4. Kedudukan titik C adalah ..Didalam.... segmen garis AB
5. Kedudukan titik D adalah .Diluar......... segmen garis AB
6. Kedudukan titik E adalah .Didalam..... segmen garis AB
7. Kedudukan titik F adalah ...Diluar........... segmen garis AB

Activities 1

Part 1

1. Create an AB line segment
2. Make point C and point E between the AB line segments
3. Make a point D and a point F that is not between the segments of line AB
4. The position of point C is **inside** against line segment AB
5. The position of point D is **outside** against line segment AB
6. The position of point E is **inside** against line segment AB
7. The position of point F is **outside** against line segment AB

Figure 16: Results 2 of Student Discussion to solve one of the LKPD Questions

Figure 16 is one of the results of student discussion after being explained about the position of points on lines with the Geogebra application. Students give correct answers and show understanding of the commands on the questions given.

Figure 17: Students actively ask questions during learning

The picture above shows the activeness of students responding to the material being delivered in both rural and urban classes. Not a few students ask questions and give answers when asked. This shows that the purpose of developing a learning model has succeeded in inviting students to be actively involved during the learning process.

CONCLUSION

This study concludes that a valid, practical, and effective Etnobra (Ethnomathematical Geogebra) learning model was developed to improve students' geometry skills. The validity of the learning design and the tools developed is indicated by the average score of 3 material and media expert validators, each of which falls into the very valid category. The practicality of the learning model is evidenced by the average score of teacher and student assessments, each of which is included in the very practical category. At the same time, its effectiveness is demonstrated by improving students' geometry skills after learning with the Ethnobra model. The subsequent analysis concluded that urban students have better geometry skills than rural students. Urban and rural students have the most visual learning style, then rural students do not have a kinesthetic learning style. Students with kinesthetic learning styles have the highest geometry skills than students with other learning styles. Students with auditorial learning styles who go to school in rural and urban areas have the characteristic of talking a lot but have difficulty writing. So the development of the Ethnobra model is very effective for improving students' geometry skills in urban areas, especially for students with kinesthetic learning styles.

The involvement of Ethnomathematics in learning makes learning more memorable and increases students' knowledge about culture. Furthermore, learning using the Geogebra application is more interesting and trains students to learn with discovery. In addition, the use of this application invites students and teachers to be technologically literate which is a significant advantage. Teachers must be familiar with using technology, especially the application of Geogebra in geometry teaching and learning. In addition to geometry learning, Geogebra applications can also be applied in learning algebra material.

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The Use of Variation Theory of Learning in Teaching Solving Right Triangles

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Abstract: Teachers are often perplexed realizing students ending up with different understandings of the same lesson after attending the same class. This study investigates the use of Variation Theory as a pedagogical design tool in improving students' problem-solving skills in trigonometry. This action research utilizing the 'Learning Study' approach was conducted in a Filipino-Chinese private school in a highly urbanized city in the Philippines. Two Grade 10 intact classes consisting of a total of 41 students and three mathematics teachers participated in the learning study. Selected students were interviewed to validate students' intended and lived objects of learning. The video-recorded lessons were examined to determine the alignment of the intended and enacted objects of learning. The analysis of the pretest and posttest showed the use of different patterns of variation and invariance in teaching was able to address students' misconceptions and difficulties in solving right triangles and helped the students understand problems better. Thus, the Variation Theory of learning as a pedagogical design tool is deemed effective in improving the students' knowledge, procedural, and problem-solving skills, and in bridging the gap between the intended objects of learning and the lived objects of learning.

Keywords: Variation Theory, Learning Study, Object of learning, Problem-solving, Trigonometric ratios

INTRODUCTION

Over the years, different studies have been conducted in order to determine how students learn (Felder & Brent, 2005; Garfield, 1995). Findings from these studies have suggested different approaches on how teaching can be conducted and how learning can be facilitated (Prince, 2004; Dunlosky et al., 2013). Despite these efforts to make teaching and learning more effective, "educators, researchers, and policy-makers worldwide continue to struggle to understand the needed changes to improve educational outcomes and educational attainment for students, particularly in the content areas such as science and mathematics" (O'Dwyer et al., 2015, p. 1). There still seems to have no consensus reached as to which kind of teaching is the best that would ensure effective learning (Lo, 2012). Educators are still puzzled as to how two students sitting in the same class, given the same instruction and using the same materials, end up with a different understanding of the concept taught (Bussey et al., 2013). The Philippines is no exception to this problem. The performance of the Filipino students in international large-scale assessments such as the Programme for International Student Assessment (PISA) is a glaring indicator the country has consistently performed poorly in mathematics (OECD, 2019).

One interesting thing to ponder is how our neighboring East Asian countries such as Singapore, Hong Kong SAR, South Korea, Chinese Taipei, and Japan performed very well in the said study. The results published by the International Association of Educational Assessment (IAEA) showed that these countries continue to dominate the rankings and outperform other participating countries notwithstanding a pronounced gap of 48 points is observed between these top performing countries and the next highest performers (Mullis et al., 2016). Several studies have been conducted as to how these East Asian countries outperformed other countries in international benchmarking tests despite the unfavorable classroom image such as large classes, teacher-dominated classroom, among others (Mok, 2006; Wong, 2013; Lim, 2007). The East Asian paradox intrigued many educators which led to conducting studies among Asian classrooms. Findings of the studies showed that contrary to the notion of the outsiders that Asian classrooms are teacher-dominated and students seemingly practice rote memory learning, the teachers presented the lessons with variations (Lim, 2007; Mok, 2006). These findings support Ference Marton’s study and his Theory of Variation. The Variation Theory (VT) provides a framework that the learners must experience variation in the critical feature of a concept, within limited space and time, in order for the concept to be learnable. VT started to gain popularity in Hong Kong, Mainland China, and Sweden. Studies were also conducted in Brunei, Japan, and Malaysia with results revealing positive effects of using VT as a pedagogical tool in designing lessons.

According to Marton and Booth (1997), one possible reason for students’ difference in understanding of the concept is the difference in their perspective of the lesson taught. One common mistake that teachers make is to assume that their students comprehend the lesson the way they expect them to. This assumption becomes a barrier to facilitating learning. As pointed out by Pang and Lo (2012), students experience a phenomenon differently; therefore, teachers must craft and deploy a pedagogy that suits the varying learning styles and other differences that impact comprehension.

From the theoretical point of view of phenomenography, every individual experience a certain phenomenon in a unique way. Hence, variation occurs among individuals who experienced the same phenomenon which leads to difference in conceptions (Samuelsson & Pramling, 2016). In order for understanding of the concept to happen, students’ perspective should be drawn to the intended similar aspect. Discernment of this perspective allows students to learn what the teachers ought them to know. “The aspects of the phenomenon and the relationships between them that are discerned and simultaneously present in the individual’s focal awareness define the individual’s way of experiencing the phenomenon” (Marton & Booth, 1997, p. 101).

The first researcher has been teaching at the secondary school level for more than ten years when the study was conducted. Based on his teaching experience, a common problem among students is difficulty in solving problems. It is in this context that the researcher conducted this study using VT in the hope that it can improve the problem-solving skills of the students in Trigonometry. As

Lo (2012) pointed out, improvement in teaching can be done by changing the mindset through determining students' views on the concepts being taught since these are primarily the reasons for the variation in the attainment of the learning outcomes.

Variation Theory

Variation theory is a theory of learning and experience which explains "how a learner might come to see, understand, or experience a given phenomenon in a certain way" (Orgill, 2012, p. 3391). Moreover, this theory claims that an object may be interpreted by people differently, which results in different understanding (Lo, 2012). In view of this, the theory states that for learning to happen, discernment of the critical aspects of learning must take place. Discernment only happens when the students are directed at the object of learning.

According to Marton and Pang (2006), there are four ways wherein discernment of variation can happen: contrast, separation, generalization, and fusion.

> a. "The principle of contrast. To discern quality X, a mutually exclusive quality ~X needs to be experienced simultaneously. For instance, to understand what a fraction is, students need to be presented with non-examples, such as a whole number or a decimal.
> b. The principle of separation. To discern a dimension of variation that can take on different values, the other dimensions of variation need to be kept invariant or varying at a different rate. For instance, if teachers want students to understand the relationship of a numerator to the value of a fraction, then they may keep the denominator invariant but vary the numerator. In this way, students' attention will be drawn to the numerator, which has been separated from the other critical aspects that affect the value of the fraction.
> c. The principle of generalization. To discern a certain value, X_1, in one of the dimensions of variation X from other values in other dimensions of the variation, X_1 needs to remain invariant while the other dimensions vary. For instance, to help students to generalize the concept of 1/2, teachers may give all kinds of examples that involve 1/2, say half of a pizza, half of an apple, half of an hour, etc.
> d. The principle of fusion. To experience the simultaneity of two dimensions of variation, these two dimensions need to vary simultaneously and be experienced by the learner. For instance, to enable students to understand the two critical aspects of numerator and denominator in determining the value of a fraction, teachers may vary both the numerator and the denominator at the same time, systematically, such as 1/2, 2/3, 3/4, 4/5, etc." (Pang, 2008, pp. 5-6)

Several research studies have already been carried out to explore the use of VT as a guiding principle for pedagogical design in teaching. The findings of the study conducted by Lam (2012) in examining the effectiveness of VT as a pedagogical tool to help students learn chemical reaction rates by using the group-based scientific investigation methodology proved VT to have helped academically-challenged students understanding their lessons.

According to Lo (2012), the tenets of VT complement the other teaching principles that are deemed effective by the academic community. Specifically, the VT principles guide the teachers in sharpening the "focus of the object of learning, which resulted in the students acquiring a better understanding of the role of characteristics and interaction with a storyline" (Tong, 2012). The theory also serves as a reference for educators in designing proper pedagogical approaches to assist students in determining the object of learning. However, a particular pattern of variation and invariance must be in place to cater to different objects of learning (Marton & Pang, 2006). Given this context, VT can be considered as a "theoretical grounding to understand some of the necessary conditions of learning so that wise pedagogical decisions can be made. The principles of VT imply what features of the object of learning have to be invariant and what should vary in the students' experience" (Lo & Marton, 2012, p. 7).

According to Kullberg et al. (2017), studies on the use of systematic variation in teaching mathematics within a VT framework prove that the theory, as a design principle, can help students notice specific contents of the lesson, which eventually result in better understanding and learning. Hence, it is essential to carefully select the factors that are critical to learning and to keep the unimportant invariant.

The findings of Cheng (2016) are in parallel with the conclusions of other studies about the use of VT in teaching mathematics (e.g., Al-Murani, 2007; Mhlolo, 2013). The latter studies assert that using the variation framework as a pedagogical design helps students discern certain aspects of the lessons and therefore, increases their ability to comprehend the concepts being discussed. Lo (2012) stressed further that using VT as a guiding principle for pedagogical design ensures that teachers employ effective teaching strategy and learning activities that are focused on the object of learning and critical aspects. This prevents the lesson from deviating from its objective and wasting valuable teaching time, and avoids students discerning other objects of learning that are inappropriate and not worth learning.

In a study conducted by Pang & Lo (2012), teachers who intentionally used the tools of VT in teaching were better compared to those who unknowingly used variation in teaching. The former group was able to effectively manage the class discussion and to inject changes in the lesson plan which resulted in enhanced student learning. This highlights the need for teachers and course designers to have a full grasp of the critical aspects of learning and the patterns of variation and invariance that help students notice them. Moreover, teachers should have a clear plan on how they can draw their students toward these patterns of variation and invariance. As Marton & Pang (2013) noted, "if an aspect that we want our students to notice is varied against an invariant background, it is more likely that students will discern it."

Object of Learning

An *object of learning* refers to the "specific insight, skill, or capability that the students are expected to develop during a lesson or during a limited sequence of lessons" (Marton & Pang, 2006). It is used to denote "the 'what' aspect of teaching and learning" (Häggström, 2008). In the context of the classroom, the object of learning encompasses everything that students are supposed to learn from what the teachers are teaching.

There are three perspectives that are used in VT to study the object of learning, namely the (1) lived object of learning, (2) intended object of learning, and (3) enacted object of learning. In these three perspectives, VT observes and evaluates the object of learning. The *lived object of learning* is the object of learning from the students' perspective. It describes what students actually learn which is influenced by what they perceive as important or valuable (Marton et al., 2004 as cited in Häaggström, 2008). Their experience within the learning environment provides the basis for how they will make sense of the object of learning presented to them. Hence, it requires a close observation of how a particular object of learning is being developed and implemented vis-à-vis the direct and indirect aspects of learning during class discussions. On the other hand, when the lens by which classroom scenarios are viewed come from the perspective of the teachers, it is referred to as the *intended object of learning*. It denotes the intention of the teacher to help students acquire specific skills and capabilities by using his or her sphere of knowledge and experience. This is manifested in the end-to-end process of creating instructional materials (Marton & Tsui, 2004).

Lastly, the *enacted object of learning* deals with the perspective of a researcher with regard to the kind of learning experience that transpires in a particular situation. This is "co-constituted in the interaction between learners and the teacher or between the learners themselves. It is described by the researcher from the point of view of what was afforded to the learners." (Runesson, 2005, p. 7). The researcher examines the direct and indirect objects of learning that both impact learning in positive and negative manners (Marton & Pang, 2006). According to Runesson (2005), there are factors other than the teachers' intentions that dictate the possibility of learning; the classroom, books, and other instructional materials as well as the interactions among the teachers and students comprising the learning environment that influence the enacted object of learning.

The interrelationship of the three perspectives on the objects of learning is illustrated in Figure 1 adapted from Häaggström (2008). A Venn diagram is used to present the interplay of the different perspectives in particular instances. The partial overlap between the intended and enacted objects of learning indicate that not all intended objects of learning may be enacted in the classroom and/or not all objects enacted in the classroom are part of the intended object of learning. The same is true with the partial overlap between the enacted and lived objects, and the partial overlap between the intended and the lived objects. The central part which is common to all three circles indicates the intended object of learning that was enacted by the teacher and lived by the students. In an ideal scenario, the intended, enacted and lived objects of learning must totally overlap.

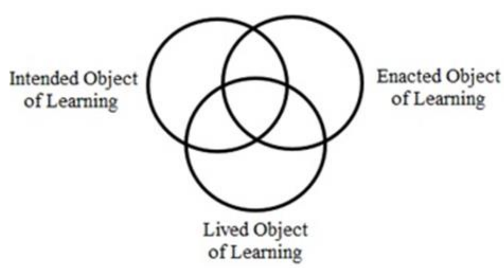

Figure 1: The object of learning.
Source: Häggström (2008)

Learning Study

The Learning Study is an approach where a group of teachers, ranging from two to six members, collaborate to determine effective ways to help students absorb a particular object of learning

(Cheng & Lo, 2013). According to Marton and Pang (2006), Learning Study is inspired by the *design experiment* (Brown, 1992; Collins, 1992) where a lesson plan is taught to a control group and an experimental group, with the latter's lesson subjected to the VT (Kirkman, 2014) and the *Japanese lesson study* (Elipane, 2012; Stigler & Hiebert, 1999) where the lesson plan is subjected to the teach-review-teach-review cycle (Kirkman, 2014). To be specific, the Learning Study approach follows a systematic process, beginning with identifying the specific object of learning that students have difficulty comprehending. Once the need has been determined, the group proceeds with planning the lessons. The main goal of the research lesson plan is to help students absorb the object of learning. To achieve this, the teachers share their knowledge and experiences related to the particular subject. Kirkman (2014) added that it is also essential that assessment of the depth and breadth of knowledge of the students is taken into consideration when planning the lesson. From the insights gained in this step, the group then identifies which method helps them facilitate learning effectively. One of the teachers from the group then uses the lesson plan to teach the students while the rest observe so that they would know what needs to be enhanced, if necessary. Should there be revisions, another teacher can facilitate the discussion to a different group of students; this step illustrates the use of VT which Stigler and Hiebert (1999, as cited by Kirkman, 2014) noted as the border line between the Learning Study and the Lesson Study. Aside from this, Cheng and Lo (2013) added that the Learning Study focuses on the object of learning and how it can be taught to students in the easiest way possible while the Lesson Study looks at the different factors that affect the lesson such as management style and methodology of teaching. There are also two main features that distinguishes the Learning Study from its origins (Marton & Pang, 2006):

1. Focus. The Learning Study has a narrower focus compared to the design experiment and is only concerned about identifying how a particular object of learning can be effectively taught to the students.

2. Teacher's Role. In the Learning Study, the teachers are mainly responsible in determining how the framework can be utilized in the lesson plan design and implementation.

The Learning Study has two aspects as described by Marton and Pang (2006). Aside from pooling teachers' valuable experiences to improve teaching and learning, it aims to build cutting-edge learning environments for theoretically grounded research studies.

Problem Statement

Since Trigonometry has been identified as one of the difficult areas in mathematics due to its abstract nature (Dhungana et al., 2023), this study has chosen to apply VT in one of the lessons in Trigonometry. The purpose of this study is to improve the problem-solving skills of students in solving right triangles in Trigonometry using the VT. Specifically, it sought to answer the following questions:

1. What are the intended objects of learning based on students' pretest results?

2. What are the patterns of variations and invariance in the learning study plan and the enacted objects of learning?
3. What are the lived objects of learning?

METHOD

Research Design

The research follows an action research design which uses the Learning Study approach. Using this approach, the researcher focused on the object of learning and sought strategies that aim to help improve the facilitation of learning problem-solving skills. The study adapted the learning study procedure by Lo (2012) which uses the VT as a guiding principle throughout its entire process following a "systematic process of inquiry which involves planning, implementation and evaluating a research lesson" (Cheng & Lo, 2013, p. 5).

Research Participants

The first researcher who was also the lesson implementer, was joined by two teacher-observers in designing the research lesson plan. The teacher-observers were given a briefing on how to conduct a learning study. The teacher-observers had at least 10 years of teaching experience.

Two Grade 10 intact classes from a private school in a highly urbanized metropolitan area in the Philippines were chosen as the research lesson participants for the two cycles of the learning study. The age of the students ranges from 14 to 16. The classes were composed of Filipino and foreign students, mostly of Chinese descent. The students already had a background on right triangles which was discussed when they were in Grade 9. The first class where the first cycle of the learning study was conducted had 17 students and this cycle is referred to as the pilot study. The second class where the second cycle of the learning study was conducted had 24 students and this cycle is referred to as the main study.

Purposive sampling was used in selecting participants to be interviewed in order to have representatives for different levels of students' performance. In this connection, the researcher selected six (6) students from each learning study cycle. The sample was composed of two (2) above average, two (2) average and two (2) below average performers in the written tests.

Research Instruments

The pre- and posttest were written tasks composed of eight short response questions on trigonometric ratios and its application in real life, particularly the angle of elevation and angle of depression. These were constructed by the researcher and validated by three seasoned Mathematics teachers. The posttest utilized the same questions given to the students during the pretest. The reason for using the same set of questions is to help chart the progress of their understanding. The tests were written in English since this is the primary medium of instruction in the school.

The interviews enabled the researcher to dig deeper on the thoughts of the students and validate students' responses in the pre- and posttests. General and open-ended questions were used to probe and uncover students' thought processes with regard to understanding the object of learning. These served as reference for mapping out students' cognitive processes when solving problems.

The research lessons for the two (2) classes were video-recorded and transcribed verbatim. The researcher also took note of nonverbal cues such as facial expressions used by the students during the research lesson implementation. In addition, the researcher used the video-recording to examine whether the intended object of learning is aligned with the enacted object of learning. Data gathered from these research instruments were triangulated to validate or to cross-check the results of each instrument.

Data Gathering Procedure

Briefing sessions were conducted to provide participants an overview of the study and informed consents to participate in the study were secured from them. The pretest was administered for 60 minutes to determine the objects of learning and the patterns of variations and invariance for the research lesson. By administering the written tasks, the researcher was able to have an initial assessment of the capability of the students to understand the concept, as well as determine different methods of solving. Moreover, the tasks enabled the researcher to see the critical aspects of learning that aid in solving these types of problems. The results of the pretest written tasks were then used in the next step: the preparatory meetings. This step aimed to produce essential materials for the study such as research lesson plan, activity worksheets, presentation slides, and other teaching aids. The researcher set preparatory meetings with the teacher-observers to design the research lesson and to plan how to teach students the trigonometric ratios and its application in solving real-life problems. The following guide questions from Marton and Pang (2006) helped the teachers in this step:

1. What are the important points of teaching this topic?
2. What common errors and confusions do students have when learning this topic?
3. How do students make sense of the topic?

Moreover, the teachers were also asked how they facilitated learning in terms of the same object of learning in their previous teaching engagement. They were guided by the following questions as suggested by Marton and Pang (2006):

1. How did you handle the same object of learning in the past?
2. What do you think are the critical aspects of understanding this topic?
3. What were the difficult points of teaching this topic in the past?
4. How could we help students from the phenomenon?

The outputs from the preparatory meetings were used in preparing the research lesson. The research lesson implementation lasted for four days with 80-minute time allotment for each day. The lessons were videotaped and were used to analyze the enacted object of learning in terms of variance and invariance in the actual classroom contexts as the researcher implemented the lesson plan in the classes in line with his own personal style and with any modifications that he considered necessary. The two other teachers involved observed the research lessons and gave comments in the post-lesson meeting for evaluation and modification of the lessons. For example, the number of review questions on Day 1 was reduced and the illustrations used for angle of elevation and depression were improved.

The posttest was conducted to determine how much of the intended object of learning was experienced (lived) by the students. Just like the pretest, students were given 60 minutes to answer the questions in the posttest. Two sets of semi-structured interviews were conducted for each cycle of the research lesson. The first set of interviews were conducted after the pretest and the second set of interviews after the learning study. These were recorded and transcribed verbatim. Students were asked to elaborate their answers in the pre- and posttest and to expound on what they have learned about the lessons, the misconceptions that have been cleared, and what they still need to understand. During the posttest interview, the students were asked to give feedback on the teaching approach, activities, and examples given. The research lesson plan was again slightly modified based on students' suggestions and comments from the interviews.

Data Analysis

A scoring rubric was used to reliably evaluate the students' performance in the written tasks. A set of descriptors was developed to help the researcher categorize students' answers into different levels of understanding. The following points are assigned for each correct response corresponding to knowledge of concept questions: 1 point for a correct answer and another 1 point for correct explanation. As for the problem-solving questions, the following points are given for each correct response: illustration (1 point), equation (1 point), solution (2 points), final answer (1 point).

To check whether there is a significant difference between the pretest and posttest results related to students' knowledge in the two cycles of the research lesson, paired t-test was used. To check whether there is a significant difference between the posttest results related to students' knowledge in the two cycles of the research lesson, Welch t-test for unequal sample size and unequal variance was conducted. Descriptive summaries of the mean scores for each question in the posttest of the two learning study cycles were computed to assess whether the students have acquired (lived) the intended object of learning and to determine whether statistically significant differences exist between the posttest of the two cycles by means of independent samples t-tests. Statistical tests of the hypothesis using the differences in gain scores for each question and total gain scores posttest in the two cycles were also analyzed. These descriptive summaries provided an understanding of how students in the first and second cycles differed in relation to the lived object of learning.

This study used the framework of variation to analyze the research lessons in a qualitative way, that is, how the teacher handled the object of learning and implemented the lesson plan in the two cycles. The analysis does not describe the details of the teaching processes. Rather, it focuses on the patterns of variation and invariance in the lessons. In this study, the teacher used the initial research lesson plan, teaching and learning materials designed in the teachers' preparation meeting in the first cycle and used a modified lesson plan in the second cycle based on feedback from the learning study group.

RESULTS

Intended Objects of Learning

The analysis of the pretest revealed the following: (1) the students have limited knowledge in solving right triangles, (2) the students have no prior knowledge of angle of elevation and angle of depression, (3) the students have no prior knowledge of trigonometric ratios, and (4) the students have no idea on how to solve problems involving right triangles.

Some of the students were able to determine which of the triangles is solvable. However, the reasons they provided showed they have limited understanding on how to determine whether the given right triangle is solvable or not. Nonetheless, they tried to recall their previous lessons on right triangles as reflected in some of the reasons they gave: (a) the measure of the length of the three sides can be solved, (b) the measures of the three angles can be solved, (c) Pythagorean Theorem can be used, (d) 30°-60°-90° Triangle Theorem can be used to find the measure of the missing side, (e) The sum of the angles in a triangle is 180°, and (c) “I don’t know/I have no idea”. Some students provided no reason at all.

On questions that require students to determine which trigonometric ratio can be used to solve the unknown part of the triangle, to identify the angle of elevation and angle of depression, and solving real-life problems on right triangles, no one among the students were able to answer any of the questions correctly. However, some attempts were made to answer the given questions. This includes (a) drawing a figure based on the given problem, (b) using Triangle Angle Sum Theorem to find the measure of the angles, (c) using Pythagorean Theorem to find the measure of the sides despite having only one given side measurement, and (d) putting random numbers. Some students got correct answers in the multiple-choice type questions by guessing as they were not able to provide a valid explanation to their answers. The reasons provided by the students include the following: (a) “It looks elevated”, (b) “I don’t have any idea yet”, (c) “I just guessed”, (d) “The line of sight is going up”, and (e) no answer at all. The results showed that the students lack the necessary knowledge of the concepts involved in the questions such as trigonometric ratios, and angle of elevation and angle of depression.

Based on the above results, the learning study group decided to have the following intended objects of learning, that is, to develop students’ capability to discern that: (OL1) a right triangle can be solved if either of the following are given: (a) two sides, or (b) an acute angle and a side; (OL2) For OL1-b, the choice of trigonometric ratio to use will vary depending on which pair of side and angle measures are given; and (OL3) trigonometric ratios can be used to model and solve real-life problems that involve right triangles.

Patterns of Variations in the Lesson and the Enacted Objects of Learning

In order to help the students 'live' the intended objects of learning, several patterns of variations were created by the researcher through the inputs of the teacher-observers. The principles of *contrast*, *separation*, and *generalization* were used.

Activity 1: Finding the Unknown Measures of a Right Triangle

Materials: ruler, protractor, calculator

A. Which of the following sets of given would make it possible to solve ΔABC, where $\angle C = 90°$? Draw and label the triangle based on each given. Solve the triangle. Show your solution.

1. $\angle BAC = 34°, \angle ABC = 56°$
2. $\angle BAC = 34°, AC = 4\ cm$
3. $\angle ABC = 56°, AC = 4\ cm$
4. $AC = 4\ cm, AB = 4.8\ cm$
5. $AC = 4.8\ cm$

B. Based on part A, what are the conditions (given) needed to solve a right triangle?

Figure 2: Activity for OL1

For OL1, *separation* was used. The right triangle remained invariant while the given conditions were variant. In this activity, students were given a right triangle with five different combinations of measurements given. These are as follows: (1) two acute angles, (2) an acute angle and its adjacent leg, (3) an acute angle and its opposite leg, (4) two sides, and (5) only one side. Please see Figure 2. Here is an excerpt of the class discussion:

T: So, based on the activity, when do we say that a right triangle is solvable?
S: When you can form a unique right triangle. That is when the given is at least the measurement of an angle and a measurement of a side.
T: What kind of angle?
S: Acute angle.
T: Aside from that, are there any other conditions that would allow us to solve a right triangle?
S: If measurements of two sides are given.

For OL2, *separation* and *generalization* were used. The students were shown a right triangle and were then asked to give the trigonometric ratio that can be used to find the value of the labeled part. The teacher kept the triangle, the measure of the acute angle, and the missing side invariant while making the side with known measure variant. The purpose of the activity is for the students to discern that the trigonometric ratio to be used is largely dependent on which parts of the triangle the measurements are given albeit they can use either the primary or its secondary trigonometric ratio counterpart (e.g. cosine and secant in solving for b given hypotenuse equals 17 units and acute angle equals 28°).

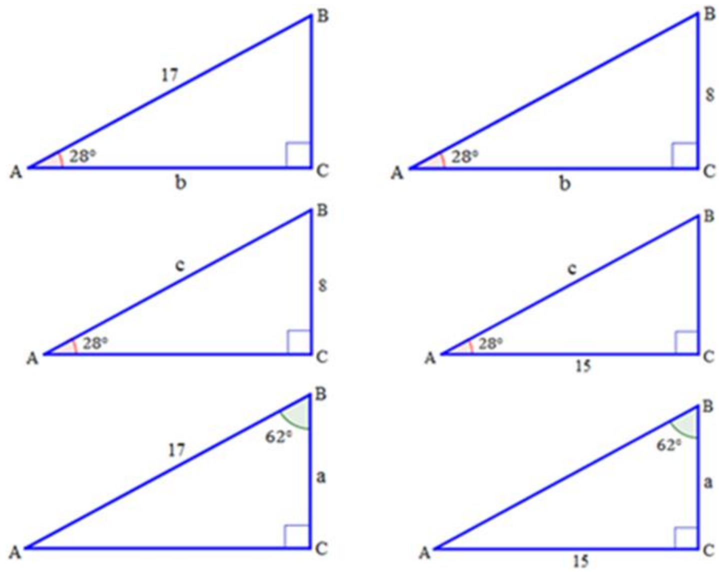

Figure 3: In each pair of triangles, the measure of the acute angle and the unknown were kept invariant while the given measure of the side varied

T: What can you say about the examples?
S: We can use two different trigonometric ratios to find the unknown measure.
T: Is that true to all the examples that we had?
S: Yes, sir!
T: Do you have any generalization or conclusion?
S: In certain given measurements, for example, when an angle and an adjacent leg are given, we can either find the hypotenuse using cosine or secant.
T: Correct! Anything else?
S: If an acute angle is given, you can find the measure of the other acute angle and find the unknown using that angle and the given sides.
T: Correct! Any other answers?
S: More than one trigonometric ratio can be used to find the measure of the unknown sides.

Activity 3: Solving Right Triangles

A. Solve ΔABC where $\angle C = 90°$. Round your final answer to the nearest hundredths. Show your solution.

1. $\angle B = 40°, AC = 5\ cm$

2. $\angle A = 50°, AB = 7.8\ cm$

3. $\angle A = 25°, BC = 10\ cm$

Figure 4: Solving right triangles using different trigonometric ratios

Furthermore, in order to reinforce this object of learning, the students solve right triangles presented in Figure 4. Variations were given thereby the use of trigonometric ratios in the initial step also vary. This pattern of variation helps the students discern that the choice of trigonometric ratios depends largely on the pair of given measures. This is validated by the student's response:

T: What can you conclude based on the activity?
S: The activity showed that the choice of trigonometric ratio to use will be based on the given.

Understanding the concept of angle of elevation and angle of depression is paramount in solving problems involving trigonometric ratios. In order for the students to discern the concept of angle of elevation and angle of depression, the teacher used *contrast* as a pattern of variation. See Figure 5 for the examples and non-examples of angle of elevation.

Through *contrast*, the students' attention was focused on the critical aspects of the angle of elevation and the angle of depression – that is the location of both the observer and the object, the line of sight, and the horizontal line. The students were able to discern the definition of both the angle of elevation and angle of depression by identifying the similarities among examples and contrasting these with the non-examples. The following is an excerpt of the class discourse.

T: The figures on the board show examples and non-examples of angle of elevation. What do you observe in the figures shown?
S: In the examples, the observer is somewhat below the object.
T: That's right. What else?
S: The line of sight is above the observer.
T: Okay. What else?
S: The observer is always looking above.
T: Any other observations?
S: The angle is formed by the horizontal line and the line of sight.
T: That's correct! So, what separates the examples and the non-examples of the angle of elevation?
S: In the non-example, the object is above the observer and so is the line of sight. But if we look at the angle, the angle is formed by the vertical line and the line sight. In the non-example, the angle is formed by the horizontal line and the line of sight but the object is below the horizontal line.
T: So based on your observations, what is an angle of elevation?
S: The angle of elevation is an angle formed by the horizontal line and the line of sight.
T: Hmmm… Do you think the definition you gave is complete? Look at this non-example (pointing at the red-boxed figure at the right). The angle is also formed by the horizontal line and the line of sight. But why is that figure a non-example?
S: Ahhh… The object should be above the observer.
T: So what is an angle of elevation?
S: The angle of elevation is an angle formed by the horizontal line and the line of sight where the object is above the horizontal line.
T: Very good! Now take a look at the next set of figures. We have examples and non-examples of angles of depression. What are your observations?
S: The observer is always looking downward.
T: What else?

S: Just like in angle of elevation, the angle is formed by the horizontal line and the line of sight.
T: So, based on the examples and non-examples, what is an angle of depression?
S: The angle of depression is an angle formed by the horizontal line and the line of sight where the object is below the horizontal line.

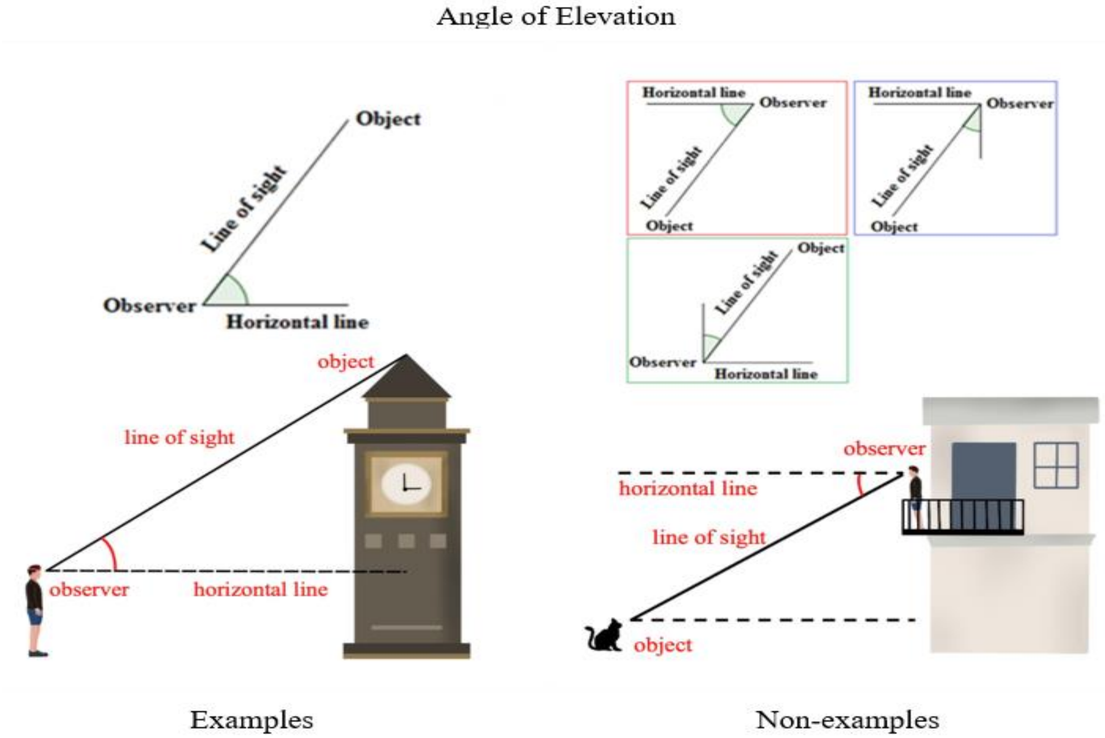

Figure 5: Examples and non-examples of angle of elevation

The designed plan was carried out and the teacher-observers noted that the teacher was able to enact the intended object of learning. Some of the remarks from the observers during the post-lesson discussion are the following:

> *Observer 1: The teacher is commended for directing the students' attention to the objects of learning… The teacher has facilitated well in directing the students' attention to the important things to consider/focus in order to come up with the definition."*
> *Observer 2: The activities are well-thought out. It has enacted the intended object of learning.*

Lived Objects of Learning in the Two Cycles of Research Lesson

The analysis of students' answers in the posttest in the two cycles of the research lesson in problem-solving revealed the usual mistakes some of the students still make. These are: (a) incorrect manipulation of the equation (Figure 6), (b) carelessness (Figures 7), (c) use of incorrect trigonometric ratio, and (d) failure to understand the problem presented (Figures 9). Nonetheless, the majority of the students were able to correctly draw the correct illustration to represent the given problem, write the correct equation and solve the given measure in question.

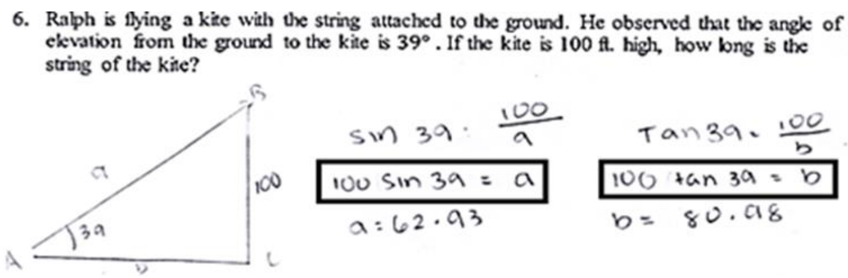

Figure 6: Incorrect manipulation of the working equation

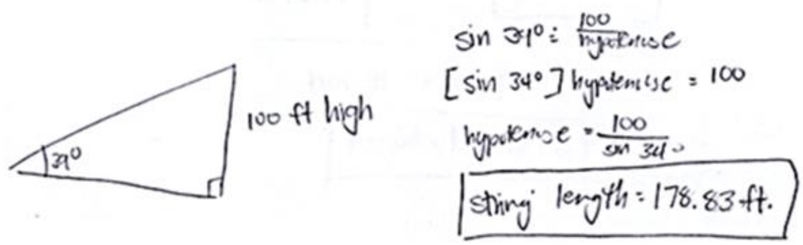

Figure 7: Carelessness in copying the angle measure

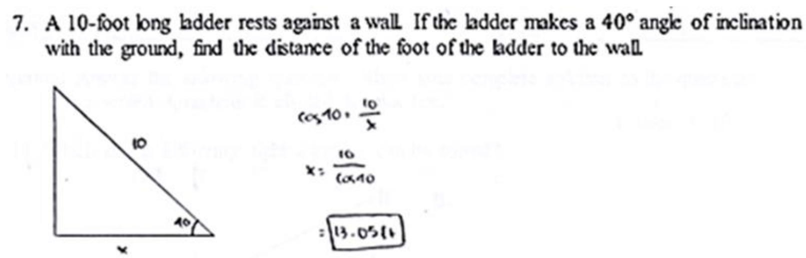

Figure 8: The use of the incorrect ratio

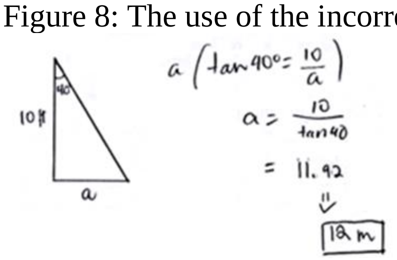

Figure 9: Failure to draw a correct figure by a student

Pilot Study

Table 1 shows the summary of the pre- and post-test results in the pilot study. The computed p-values are less than 0.05 significance level for OL1, OL2, and OL3 show that there is a statistically significant difference in the pre- and post-test mean scores. It can be concluded that the change in scores in each object of learning is due to the intervention done by the teacher – the use of VT in teaching the lesson. The posttest results further show a significant improvement among the students in terms of the three objects of learning. Thus, the pronounced gap between the intended object of learning and the lived object of learning before the research lesson was also addressed.

Table 1: Summary of the Pre-Test and Post-Test Results of the Pilot Study

Object of Learning	Test Items	Full Score	Pretest		Posttest		Pretest vs. Posttest	Level of Significance
			Mean	SD	Mean	SD	*t*	(p = 0.05)
OL1	1a to 1f	12	4.647	1.281	11.235	1.059	22.125	0.000
OL2	2a to 2c & 3	11	1.294	0.824	9.118	2.928	13.137	0.000
OL3	4 to 8	20	1.059	0.998	16.941	3.638	19.015	0.000
Overall	1a to 8	43	7	2.169	37.294	6.711	23.256	0.000

Main Study

Table 2 shows the summary of the pre- and posttest results of the main study. The table shows the computed p-values for each object of learning. Since all p-values < 0.05, it implies that there is a significant difference between the performance of the students before and after the research lesson. The post-test mean score for each object of learning revealed a significant improvement in the students' lived object of learning. The overall mean score of 39.21 which is 91.19% of the total score 43 points is considered high indicating students' lived objects of learning are very close to the intended objects of learning.

Table 2: Comparison of Pre-Test and Post-Test Results of the Main Study

Object of Learning	Test Items	Full Score	Pretest		Posttest		Pretest vs. Posttest	Level of Significance
			Mean	SD	Mean	SD	*t*	(p = 0.05)
OL1	1a to 1f	12	4.083	1.412	10.833	1.374	15.087	0.000
OL2	7 to 10	11	0.542	0.498	10.167	1.700	27.915	0.000
OL3	11 to 15	20	1.000	1.555	18.167	2.734	32.361	0.000
Overall	1 to 15	43	5.625	2.563	39.208	4.153	38.364	0.000

The average score of the students in the pilot and main study are 37.294 and 39.208 out of a possible score of 43, translating to 86.73% and 91.18%, respectively. To check whether there is a significant difference between the performance of the students in terms of understanding and ability to solve problems involving trigonometric ratios, the posttest results in the two cycles of the research lesson were subjected to Welch t-test for unequal sample size and unequal variance was conducted. The 4.45% difference between the post-test results in the two cycles of the research lesson is not significant (p-value > 0.05). This implies that the minimal changes in the lesson are not significant to greatly affect the manner in which the lessons were taught between the two classes leading to small and non-significant differences in their lived objects of learning.

The use of different patterns of variations enabled the students to see the critical aspects of the lesson. Thus, discernment of the object of learning occurred. This is likewise validated by students' responses in the interview when asked how the use of patterns of variations helped them. The answers are summarized below.

a. It enabled the students to see deeper meaning of the topics.

"The different examples and explanations used showed me the deeper meaning by showing critical aspects in trigonometry … and how useful trigonometry is…"

b. It helped the students in relating one problem with another problem.

"It helped me in solving different problems, such as figures where I have to solve different measurements and relating problems to each other so I can solve the problem in a similar way and at the same time, knowing that I can use solutions for specific types of problems."

c. It guided the students on how to approach the lesson.

"The teachers' varying examples give out a distinct guide on how to tackle a problem… especially the examples that use the same unknown but have different parts that were given measures—it just really makes my mind work a little more to process the said problem…"

DISCUSSION AND CONCLUSIONS

Marton et al. (2004) underscored the objects of learning to be clearly stated and identified and Kullberg et al. (2017) advised administering a test in order to identify students' prior knowledge. Particularly, we noted that VT has also been conducted in solving right triangles by Peng et al. (2017) but the focus had been in the use of VT in the six typical phases of the national pattern of teaching of problem solving in China. In this study, the learning study group teachers, based on the context of the test results, deemed OL1 to be essential in teaching solving right triangles using Trigonometric ratios to Grade 10 students as teaching through progressive variation problems and core connections (Gu et al., 2017). Students previously learned solving right triangles using Pythagorean theorem and solving the missing angle measure using the Triangle Angle Sum Theorem. This is being connected to the current lesson on solving right triangles using Trigonometric ratios. Moreover, we included the concept of angle of elevation and angle of depression as most of the word problems would make use of these terminologies.

In this study, we illustrated the use of variation patterns of *contrast*, *separation* and *generalization* to help students discern that not all triangles are solvable especially when there is lack of given information. When a triangle is solvable, students were made to discern when to use the Pythagorean Theorem, the Triangle Angle Sum Theorem and the Trigonometric Ratios, all of which are dependent on which part (side and/or angle) the measures are given and their relation to the unknown measure one wish to solve. Furthermore, students were taught to correctly solve a problem through proper illustration of the model representation of the problem by discerning between examples and non-examples of angle of elevation and angle of depression, neither of these angles are formed by a vertical line with the observer's line of sight. Watson (2017) argues that the use of variation in mathematics teaching should draw out students' attention to "dependency relationships" that are invariant in mathematics and how such careful use of variation can lead to abstraction of new ideas. Likewise, categorization of the different parts of a triangle as hypotenuse, a given acute angle, its opposite side and its adjacent side had played an important role as Gu et al. (2017) states that categorization is an important mathematical thinking method in VT.

The intended objects of learning were drawn out from students' pretest results and discussed by the teachers in the learning study. Students' lived objects of learning as reflected in their posttests were very close to the intended objects of learning by way of the patterns of variations in the enacted objects of learning. While there are students who still exhibit mistakes, these are minimal as most students were able to draw a correct illustration based on the given problem, write a correct equation and answer the given question. The students who do not usually perform well in class have made significant improvements after the research lesson. Some students who have below average academic ability before the research lesson were able to answer all the problem-solving questions correctly and were able to get a perfect score in the posttest despite performing poorly in the pretest. The consistent high scores of the students in the two cycles of the research lesson are indications that the use of VT as a pedagogical design tool is effective in bridging the gap between the intended object of learning and the lived object of learning of the students. Careful selection and proper use of different variation patterns such as *contrast*, *separation*, and *generalization* for the enactment of objects of learning, proved to be effective in helping the students keep their focus and discern the critical aspects of the lesson to achieve the intended objects of learning, thereby narrowing the gap between the intended and lived objects of learning. The results of the study confirmed the productive applicability of VT (Clarke, 2017). Peng et al. (2017) explains that VT allows learners to develop their capability to discern which aspects must be considered in the process of achieving one's goal even in an unfamiliar situation. Exposing students to discerning important aspects of mathematical concepts and procedures help them assimilate the skills even beyond the classroom lessons.

The following pedagogical implications based on the findings of the study are thus recommended. The researchers posit that in order to ensure the intended object of learning will be lived by the students, the teacher should plan teaching and structure learning activities that will keep the students' focus on the critical aspects of the lesson which are the intended objects of learning for which VT has been of general utility. For the students to live the intended object of learning, the enacted object of learning has to be as close to the intended object of learning. As with any skill, problem solving demands a considerable level of motivation, effort and devotion from students to imbibe a good understanding of the underlying concepts involved in the problems and mastery of the procedures. A learning study as a form of professional learning community, is considered to be a good opportunity for teachers to collaborate to better their teaching practice as there is more than 1 pair of critical eyes looking at the lesson and at least two heads in sharing their expertise, content and pedagogical knowledge, and experiences.

Future research may consider a long-term study to further examine the long-term effects of the use of VT as a pedagogical design tool in the problem-solving skills of the students most especially those of low academic ability. Likewise, a study on using variation theory in online and blended learning modes of delivery may be conducted.

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How Students Understand the Area under a Curve: A Hypothetical Learning Trajectory

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Abstract: The area under the curve is a fundamental concept for students to build their understanding of the Definite Integral. This research reveals how students comprehend the area under the curve in given contextual problems and how the Hypothetical Learning Trajectory (HLT) can help students find the concept. This research follows the development research model of Gravemeijer and Cobb, which consists of three stages: Preparing for the experiment, Experimenting, and Retrospective analysis. This research involved three students with varying abilities: low, medium, and high. Data were collected through the analysis of student work documents and in-depth interviews. The research findings indicate that students often struggle with determining the area under the curve. Students approach the area by using various polygons. With the guidance of their lecturers, students discovered that the area under the curve can be approached by partitioning the area into polygons. The more polygons used, the closer the approximation becomes. In other words, the HLT designed in this study facilitates students in understanding the concept of the area under the curve, formulated as $A = \lim_{n\to\infty} \sum_{i=1}^{n} A_i$.

Keywords: Area under the Curve, Partition, Polygon, Hypothetical Learning Trajectory, Realistic Mathematic Education.

INTRODUCTION

The definite integral is one of the important concepts in Calculus (Greefrath et al., 2021; Hashemi et al., 2020; Jones, 2015; Stevens, 2021; Ural, 2020) studied at the college level. Apart from its support for other subjects, the definite integral has broad applications, including the calculation of areas, volumes of objects, lengths of curves, work, fluid pressure, moments, the center of mass, and more (Varberg et al., 2016). This concept should be well mastered by students.

Several studies have revealed that students have difficulty understanding the concept of definite integrals. It was found that students often fail to understand the definition of a definite integral (Grundmeier et al., 2006; Rasslan & Tall, 2002). This is in line with Aniswita's research which found that only 7% of students could answer Integral Of course questions using the definition correctly (Aniswita et al., 2023). The same thing was also found by Hidayat et al., (2021) only 10% of students were able to construct a definite integral definition. According to Purnomo et al (2022), the students' difficulties were caused by weak mathematical literacy and an incomplete mathematization process.

The concept of definite Integral is built from Riemann sums (Sealey, 2006, 2014). which begins with the concept of the area under the curve (Grundmeier et al., 2006; Orton, 1983; Yost, 2008). In other words, to understand the concept of a definite integral, students must first grasp the concept of the area under a curve.

To provide a strong understanding of concepts, learning needs to be well-designed so that it can facilitate students discovering these concepts (Clement & Sarama, 2004; Gravemeijer, 2004). Learning design or learning trajectory is a potential step to improve the learning process to achieve goals (Daro et al., 2011; Wilson et al., 2013; Clement & Sarama, 2014; Simon, 1995; Simon et al., 2018). Learning trajectory in mathematics learning was first introduced by Simon (1995) with the term Hypothetical Learning Trajectory (HLT). HLT consists of three components, namely: 1) learning objectives; 2) learning activities; and 3) the learning process hypothesis.

According to Gravemeijer (1998) and Larsen (2013), HLT in mathematics learning is connected to principles of Realistic Mathematics Education (RME), namely: 1) guided reinvention; 2) didactical phenomenology; and 3) emergent models (Gravemeijer, 1999). RME is an approach to learning mathematics based on the Freudental view that mathematics is a human activity and learning mathematics is essentially 'doing mathematics’ or 'mathematizing' (Barnes, 2005; Dickinson & Hough, 2012; Drijvers, 2018; Gravemeijer & Terwel, 2000; Kwon, 2002; Marja Van Den Heuvel-Panhuizen, 2003; Van den Heuvel-Panhuizen, M; Drijvers, 2014; Webb et al., 2011). RME was first developed in the Netherlands in 1968. This approach is inspired by the philosophy of constructivism, especially social constructivism (Gravemeijer, 2020b, 2020a) which views that knowledge is constructed by humans through their interactions with the environment (Ansyar, 2015; Schunk, 2012). So learning must start from contextual problems that are close and meaningful to students (Aziza, 2020). Students solve problems by using the knowledge they have (model of) to find concepts (model for). During the discovery process, with the help of lecturers, their knowledge develops along with the process of horizontal mathematization and vertical mathematization (Barnes, 2005; Fauzan, 2002; Gravemeijer, 2020b; Gravemeijer & Doorman, 1999; Guler, 2018; Kwon, 2002; Marja Van Den Heuvel-Panhuizen, 2003; Rasmussen, 2014; Yvain-Prébiski & Chesnais, 2019).

Research on HLT with the RME approach has been carried out quite a lot and is growing rapidly. This development inspired the Mathematics Teaching Research Journal to publish a special issue related to this research MTRJ, vol 13 no 4, winter 2021 edition (http et al., n.d.). This research was conducted at all levels, both from elementary school to university level. At the university level, multiple studies were undertaken, including investigations by Zandiah & Rasmussen (2010) on geometry, Larsen (2013) on algebra, Cárcamo et al. (2019) on algebra, Andrews-Larson et al. (2017) on linear transformation, Syafriandi et al. (2020) on statistics, and Yarman et al. (2020) on differential equations. Based on this, it is necessary to design HLT which can facilitate students to find the concept of the area under the curve. This article will reveal how students understand the concept of the area under the curve and how the designed HLT can facilitate students in finding the formal concept of the area under the curve.

METHOD

This research is Gravemeijer and Cobb's design research model, which consists of three phases, namely: 1) Preparing for the experiment, (2) Experimenting in the classroom, and (3) Retrospective analysis (Gravemeijer & Cobb, 2013). To fulfill the research objectives of investigating students' understanding of the area under the curve and the role of HLT in facilitating their comprehension, a qualitative descriptive approach was used. In the initial phase of preparing for the experiment, the final learning goal and starting point are identified to develop an anticipated learning process and corresponding HLT. The subsequent phase of experimenting in the classroom focuses on implementing and testing the formulated learning process. Lastly, the retrospective analysis phase evaluates and potentially revises the HLT that was designed. Here is the description:

Preparing for The Experiment

The first step is to formulate the objective of studying the area under the curve. From the literature review, the learning objective of the area under the curve emphasizes the procedural ability to calculate the area under the curve using polygons. According to researchers, students need to understand that the area of a curved plane can be determined by partitioning the area into polygons. Then, the researcher interviewed ten students to determine the starting point in learning. Students were selected to represent low, medium, and high levels of academic ability. In addition, gender and communication skills were thoughtfully taken into consideration. This investigation entailed specific inquiries regarding the students' preferred learning modalities and the essential educational resources for facilitating a profound understanding of the subject matter. Finally, the author formulated an HLT for the area under the curve based on these two things.

Classroom Experiment and Retrospective Analysis

The focus of these two phases is on testing and improving the effectiveness of the HLT (Hypothetical Learning Trajectory) area under the curve that has been designed. At this stage, the HLT can transform into a localized instructional theory specifically aimed at teaching the area under the curve. Gravemeijer and Cobb (2006) emphasize that this phase involves a cyclical process where thinking and experimentation in instructional development are intertwined, as shown in Figure 1.

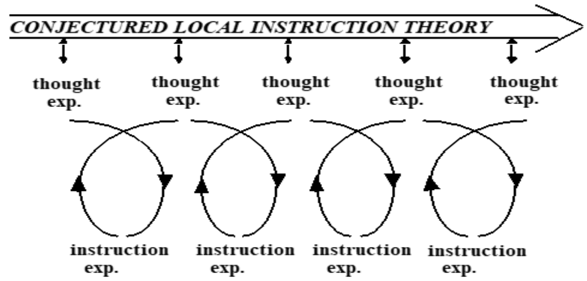


Figure 1: The reflective relation between the theory and the gap

Before conducting the experiment, the validation of HLT area under the curve was validated by five experts, including three mathematicians, one educational technology expert, one educational evaluation expert, and one language expert. The experiment involved three students who possessed high, medium, and low academic abilities, which were determined by their performance in the Differential Calculus course. These students were selected randomly. Data was collected through the analysis of student work documents and in-depth interviews conducted during the learning process. The purpose of these interviews was to understand how students approached the given activities and to determine whether their anticipation could guide them toward the desired trajectory.

RESULTS

Preparing For The Experiment

Based on the literature review and student characteristics, the HLT for the area under the curve was formulated. Iceberg from HLT is given in Figure 2.

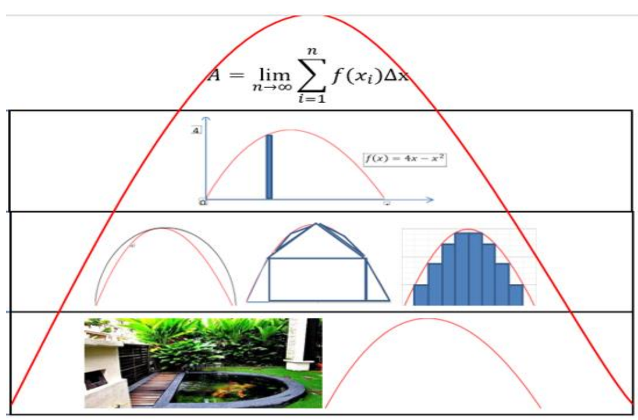


Figure 2: "Iceberg" Area under a Curve

In accordance with the "iceberg" HLT the area under the curve consists of two activities, namely:

Activity 1

The purpose of this activity in Activity 1 is for students to be able to predict/approach and determine the area using polygons. Contextual problems are given in Figure 3

Mr. Syaiful has a koi fish pond. How do you calculate the surface area of Mr. Syaiful's koi fish pond? Explain the answer!

Figure 3: Contextual problem Koi Fish Pond Surface Area

Hypothesis and anticipation of Activity 1 as in Table 1

The Hypothesis of Student Activities	*Anticipation*
Students are confused about determining the surface area of a fish pond	The lecturer asks probing questions. Take a look at the surface shape of Mr Syaiful's koi fish pond, how do you get to its area?
Students approach the surface area of a fish pond with one flat plane such as a circle, triangle, square, rectangle, trapezoid	The lecturer asks probing questions. Is the approximation you found the best approximation? What about the area that does not include the surface area of the pool?
Students approach the surface area of a fish pond by dividing the area into several polygons of different types	The lecturer asks probing questions. Is the approximation you found the best approximation? Is it easy to calculate the area of these polygons?
Students approach the surface area of a fish pond by dividing the area into several similar polygons	The lecturer asks probing questions. Is the approximation you found the best approximation? How do you make the area that does not include the area of the fish pond getting smaller and closer to the real thing?
Students approximate the surface area of a fish pond by dividing the area into more polygons	The lecturer asks probing questions. What can you conclude from the activities you did?

Students are able to find that the more the surface of the fish pond is partitioned, the area of all polygons will approach the surface area of the actual pond	The lecturer asks probing questions. When do you think the area of the entire polygon equals the actual area of the fish pond?
Students are able to find that the actual surface area of a pool can be determined by dividing the area into infinity and then adding up the areas of all the polygons.	The lecturer emphasizes the student's findings that to determine the area of a curved plane by partitioning the area into $n \to \infty$

Table 1: *Hypothetical Learning Process Koi Fish Pond Surface Area*

Activity 2

The purpose of the activity in activity 2 is for students to be able to determine the formula for the area under a curve using polygons. Contextual issues are given in Figure 4

If the curved line on the surface plan of Mr. Syaiful's koi pond is a function curve y=f(x), then determine:

1. Approximate surface area of the pool if partitioned by 8 polygons!
2. The approximate surface area of the pool if it is partitioned by n polygons!
3. What is the actual surface area of the fish pond? explain the answer!
4. Based on answer 2) construction of the formula for the area under the curve f(x) from a to b!

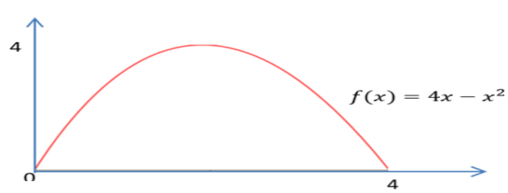


Figure 4: Contextual problem the area under the curve

Hypothesis and anticipation of Activity 2 as in Table 2

The Hypothesis of Student Activities	Anticipation

Question 1 Students partition the area into eight parts but there are still errors	The lecturer asks probing questions. Look at the partition area under the curve! Where are the polygons formed?
The student partitioned the area into eight parts but still got the size of the polygon wrong	The lecturer asks probing questions. Look at polygon 1, polygon 2, and so on, what's the width? Can it also be determined
Students are able to approximate the surface area of a fish pond by dividing the area into eight polygons	The lecturer emphasizes the students' findings, that the approximate area under the curve is the sum of the areas of eight polygons
Question 2 Students have not been able to determine the size of a polygon if it is partitioned by n	The lecturer asks probing questions. Pay attention to polygons 1, 2, …, and polygon n. Is it possible to specify the width of the polygon? Then how to determine the length of each polygon?
Students are able to determine the size of polygons but are still wrong about the surface area of a fish pond with n polygons	The lecturer asks probing questions. Notice the size of the polygons? How to determine the approximate surface area of a fish pond?
Students are able to find the approximate surface area of a fish pond with n polygons	The lecturer emphasizes the students' findings, that the approximate area under the curve is the total area of n the polygons, namely $L \approx \sum_{i=1}^{n} L_i$
Question 3 Students have not been able to find the actual surface area of the pond.	The lecturer asks probing questions. How do you determine the area under the curve (surface area of the fish pond) in the form of a curved plane?
Students are able to find the surface area of the actual fish pond	The lecturer emphasized the students' findings, that the surface area of the fish pond namely $L = \lim_{n\to\infty} \sum_{i=1}^{n} \left(4\left(\frac{4i}{n}\right) - \left(\frac{4i}{n}\right)^2\right)\frac{4}{n}$
Question 4 Students have not been able to find the area under the curve for any function f(x) from a to b	The lecturer asks probing questions. Based on the previous activity, what can you conclude to determine the area under the curve with a boundary from a to b?

Students are able to find the area under the curve for any function f(x) from a to b namely $\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$	The lecturer emphasized the students' findings, that the surface area under the curve for any function f(x) from a to b namely $\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$ with $\Delta x = \frac{b-a}{n}$, and $x_i = a + i\Delta x$

Table 2: *Hypothetical Learning Process Area Under the Curve*

Experimenting in The Classroom

Activity 1

Students are asked to determine a method or strategy for calculating the surface area of Pak Syaiful's koi fish pond. Before starting Activity 1, the lecturer conveys the aim of the activity and reminds the term polygon and the area of several flat planes such as rectangles, triangles, trapeziums, and others. The three students with low (MR), medium (MS), and high (MT) abilities had difficulty determining the surface area of the pool. The lecturer, using the HLT that has been designed, guides students to find ways or strategies to calculate the surface area of the fish pond. The following is the description.

Students with Low Ability (MR)

Students with low ability (MR) are not able to understand the questions well. This can be seen from the empty student answer sheets and from interviews. Students need quite a lot of time to find a way to determine the surface area of Mr. Syaiful's fish pond. The following are the results of interviews and student worksheets during the discovery process.

Lecturer: *Do you understand what is meant by the question?*

Student (MR): *Understand Ma'am, but I cannot determine the surface area of the fish pond because the size of the pond is unknown.*

Lecturer: *Take a look at the surface shape of Mr Syaiful's koi fish pond, how do you get to its area?*

Student (MR): *hmmm (student thinks for a long time). I approximated the surface area of the fish pond using triangles, rectangles, and circles Ma'am.*

Lecturer: *What about areas that are outside the flat surface but include the surface area of the fish pond?*

Students return to the surface area of the fish pond using a trapezium. The answers as in Figure 5

Figure 5: Students MR Approach Using Circles, Triangles, Rectangles And Trapezoids

Lecturer: *Is this the best approximation?*
Student (MR): *Yes Ma'am, when compared with approaches using rectangles and triangles, the trapezoidal approach is better*
Lecturer: *OK. What about the area outside the trapezoid including the surface area of the fish pond? In your opinion, when is this approximation the best approximation?*
Student (MR): *The best approximation is when this area (pointing) gets smaller*
Lecturer: *What is the strategy to make the area even smaller?*
Students re-approximate the surface area of the pool by dividing the surface of the pool into several dissimilar polygons as in Figure 6

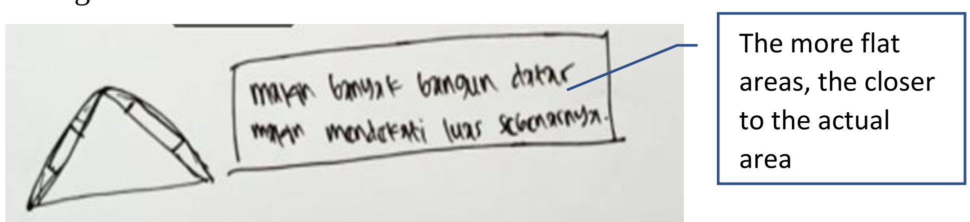


Figure 6: Students MR Approach By Dividing The Area Into Several Different Polygons

Lecturer: *Which is easier to calculate area by dividing into similar or different polygon?*
Student (MR): *It's easier if the polygon is similar*
Then, the student approximates the surface area of a pool with similar polygons as in Figure 7

Figure 7: Students MR Approach By Dividing The Area Into Several Similar Polygons

Lecturer: *When do you think the area of all polygons is the same as the surface area of the pool?*
Student (MR): *I can't do it, Ma'am, because there will always be some leftovers.*
Lecturer: *Just imagine, what if the surface of the fish pond was divided into 100 flat areas, 1 million flat areas, and so on.*
Student (MR): *The remaining areas are getting smaller, Ma'am, almost approaching 0*
Lecturer*: When do you think the remainder of the area is equal to 0?*
Student (MR): *hmm (thinks for a long time, then hesitantly says), when divided by infinity*

Lecturer: *OK, so what can you conclude about how to determine the surface area of Mr. Syaiful's koi fish pond?*
Student (MR): *Divide the surface of the fish pond into infinite polygons.*

The answers of students with low abilities (MR) with the help of HLT can be seen from the development of models used by students to solve problems until they find formal concepts as in Figure 8.

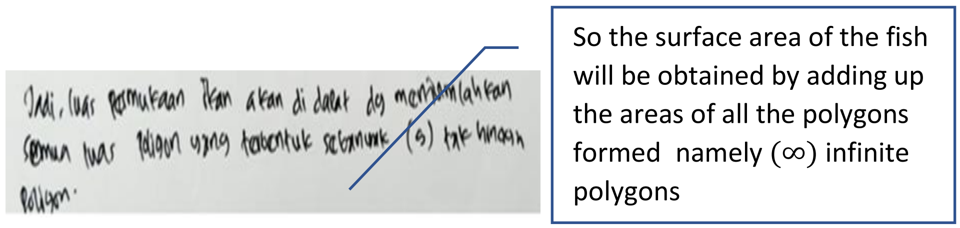


Figure 8: Students MR Can Conclude How To Calculate A Curved Plane By Partitioning n $\rightarrow \infty$

Students with Medium Ability (MS)

Students with medium abilities (MS) can understand the questions well, students immediately approach the surface area of the fish pond with various flat areas (polygons). Students need quite a long time to try out various polygon shapes.

Lecturer: *How do you determine the surface area of a fish pond?*
MS Student: *I tried to approach with triangles, rectangles, and trapezoids*
Lecturer: *Is the approximation that you found the best approximation?*
MS Student: *Not yet Ma'am, because there are still many remaining areas that are not included in the triangle, rectangle, and trapezium.*
Lecturer: *What is the strategy to make the area smaller?*
MS Student: *I divided it into several different polygons, and this is better Ma'am than just approaching it with one polygon.*
The answer as in Figure 9

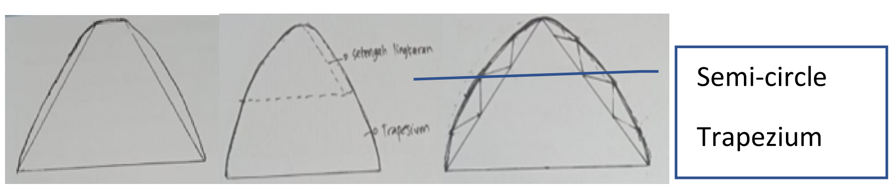


Figure 9: Students MS Approach By Dividing The Area Into Several Different Polygons

Lecturer: *In your opinion, which is easier to determine the area of similar or dissimilar polygons?*
MS Student: *It's easier to use a similar flat surface*
Lecturer: *In your opinion, when is the area of all polygons the same as the actual area? Try to imagine, what if the surface of the pool was divided into 100 polygons, 1 million polygons, and so on. What about the remaining areas?*
MS student: *the less it is, the more it goes to 0*

Lecturer: *So what can you conclude?*
MS Student: *The surface area of a pool can be determined by dividing the area into an infinite number of similar polygons.*

The answer as in Figure 10 and Figure 11

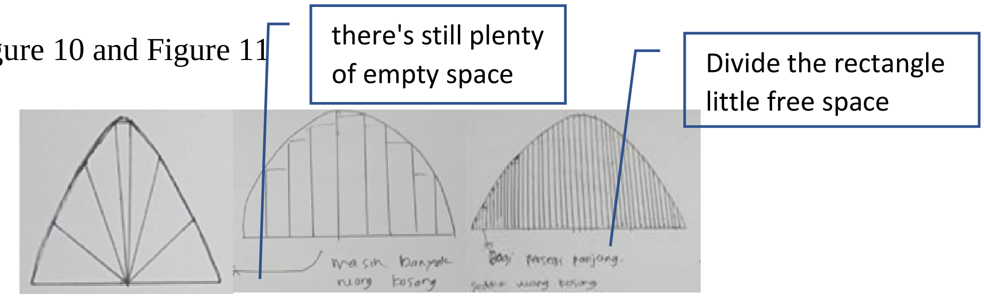


Figure 10: Students MS Approach By Dividing The Area Into Several Similar Polygons

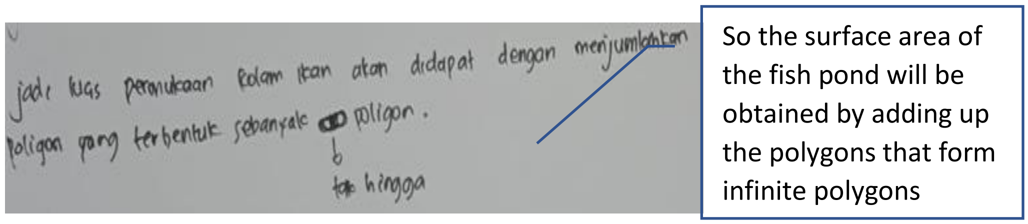


Figure 11: Students MS Conclude How To Calculate A Curved Plane By Partitioning N →∞

The answers of students with medium abilities (MS) with the help of HLT can be seen from the development of models used by students to solve problems until they find formal concepts as in Figure 9- Figure 11.

Students with High Ability (MT)

Students with high abilities (MT) can understand the questions well, students immediately approach the surface area of the fish pond with various polygons. Students need a relatively short time to try out various polygon shapes.

Lecturer: *How to determine the surface area of a fish pond?*
Student (MT): *Partition it into several polygons, Ma'am, then calculate the area of the area of all polygons.*
Lecturer: *Is the area of the entire polygon the surface area of the fish pond?*
Student (MT): *No Ma'am, because there are areas that are not included in the surface area of the pool.*
Lecturer: *So what can you conclude?*
Student (MT): *If the surface of the pool is divided more and more, the area of the entire area will be closer to the actual area.*
The answer as in Figure 12

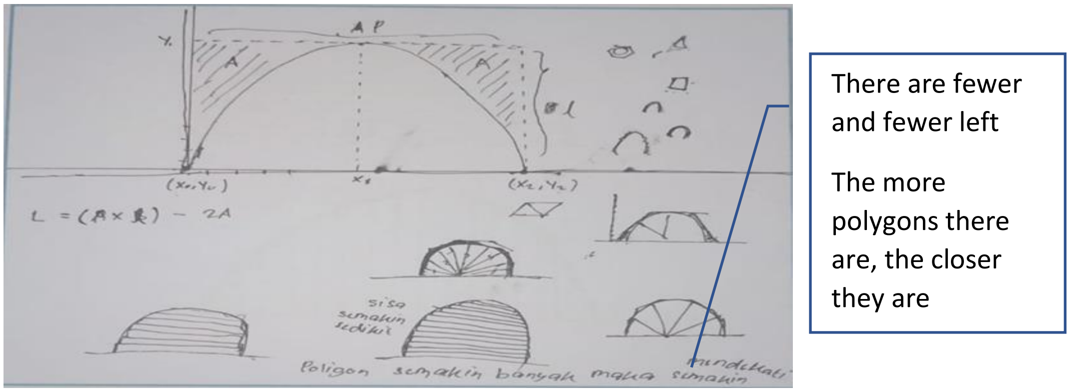


Figure 12: Students MT Approach By Dividing The Area Into Several Polygons

Lecturer: *When do you think the area of a polygon is the same as the actual area?*
Student (MT): *hmm (thinks for a moment) When divided by n it goes to infinity.*

The answers of students with high abilities (MT) with the help of HLT can be seen from the development of models used by students to solve problems until they find formal concepts as in Figure 13

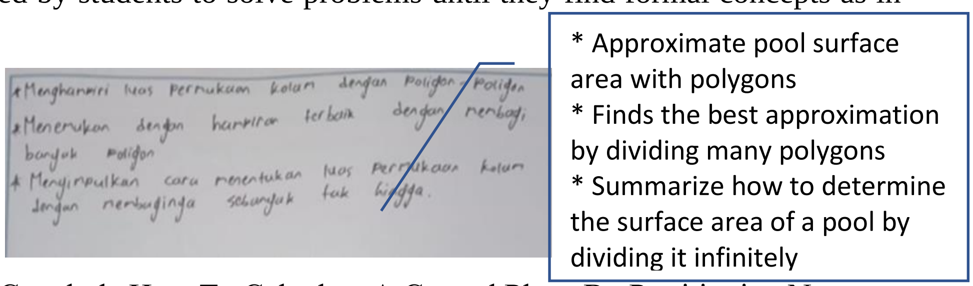


Figure 13: Students MT Conclude How To Calculate A Curved Plane By Partitioning N →∞

Activity 2

Students are asked to find a formula for determining the area under the curve from the problem of the surface area of a fish pond whose curved side is a graph of the function f(x)=4x-x^2. Before starting the activity, the lecturer conveys the purpose of the activity and reminds you about the sigma function and notation. The three students with low (MR), medium (MS) and high (MT) abilities still have difficulty determining this. The lecturer, using the HLT that has been designed, guides students to find the formula for determining the area under the function curve f(x) from a to b. Here's the description:

Students with low ability (MR)

Students with low ability (MR) need quite a long time to determine the surface area of Mr. Syaiful's fish pond, especially when the area is divided into n polygons. The following are the results of interviews and student worksheets during the discovery process. Students (MR) divide the area into 8 parts but it is not correct.
Lecturer: *Look at the partition, is it divided into eight?*
Student (MR): *hmm (thinking and paying attention to the picture), not yet ma'am (smiling)*
Lecturer*: Can you determine the area of each polygon?*

Student (MR): *No, I can't, Ma'am, because the length of the polygon is unknown.*
Lecturer: *Look at the curved line (while pointing at the graph), if the x value is known, can this value (while pointing at the graph) be determined?*
Student (MR): *(smiling) Yes, Ma'am, by substituting the value of x into the function f(x)*
Then students (MR) determine the size of the polygon. Calculate the area of the polygons and add up their areas as an approximation of the surface area of a fish pond with partitions of eight as in Figure 14

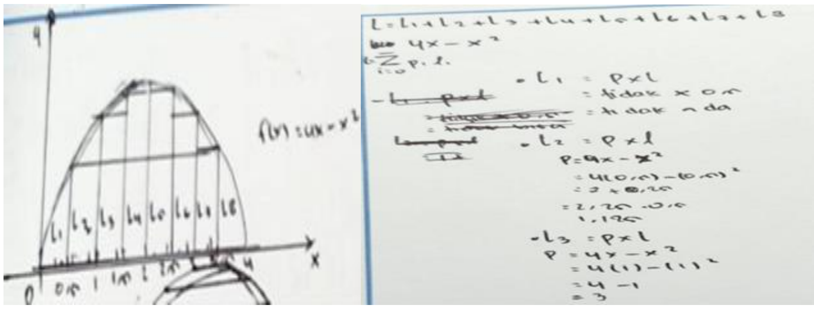


Figure 14: Students Determine The Approximate Surface Area Of A Fish Pond With Eight Partitions

To answer the second question, students (MR) are confused about how to divide it into n parts, this can be seen from the students' answer sheets which are still empty.

Lecturer: *Can you calculate the area of all polygons?*
Student (MR): *(while thinking and hesitating to answer) I can't, Ma'am, because the value of n is unknown.*
Lecturer: *Haven't you studied sigma notation? Try to pay attention to the area of polygons 1, 2, 3, and so on. Can you see the connection?*
Student (MR): *hmm (thinks for a long time then smiles) yes Ma'am*
Then students determine the surface area of the fish pond with partition n as ini Figure 15

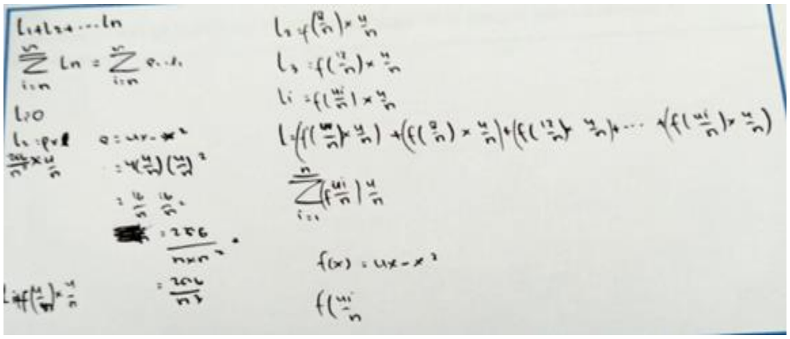


Figure 15: Students Determine The Approximate Surface Area Of The Mikan Pool With Partition n

To answer question 3 to determine the surface area of a fish pond, MR students can actually answer in words but are confused about how to formulate it.

To answer question 4, determine the formula for the area under the curve of the function f from a to b, students (MR) have difficulty. This can be seen from the empty Student (MR) answer sheets.

Lecturer: *Pay attention to your answer to question number 3, if you substitute for any function f(x) and the limit is not from 0 to 4 but from a to b, can you find the formula?*

Student (MR): *(hmm), yes Ma'am*

The answers of students with low abilities (MR) with the help of HLT can be seen from the development of models used by students to solve problems until they find formal concepts as in Figure 16:

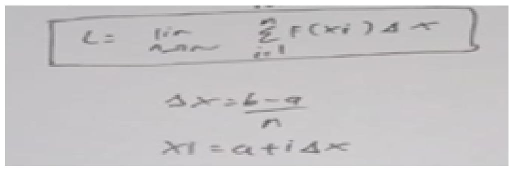


Figure 16: Students Generalize The Area Formula For The Area Under The Curve

Students with Medium Ability (MS)

Students with a medium ability (MS) are almost the same as students with low ability for questions number 1, 3, and 4. However, students (MS) tend to solve them more quickly. For question number 2, MS students were able to determine the size and area of the polygon but there were still errors. The following are interviews and student work results:

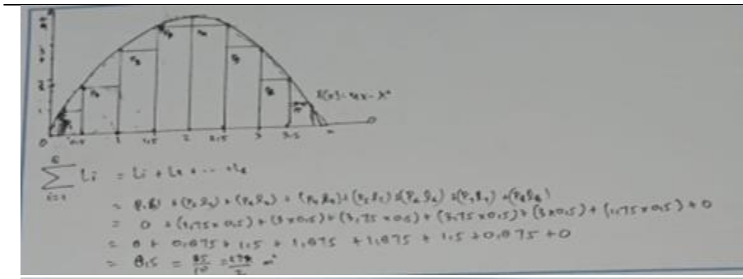
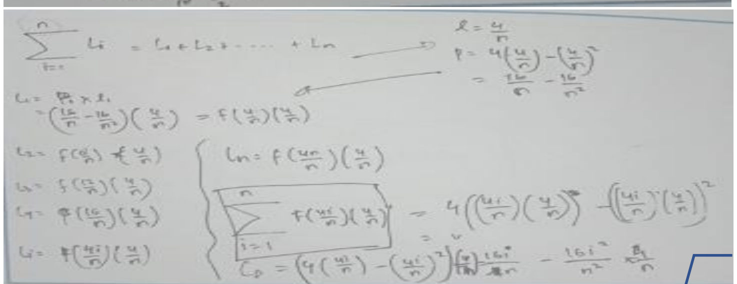
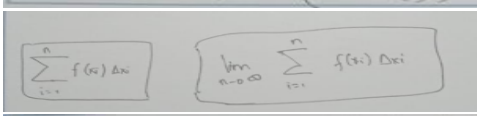
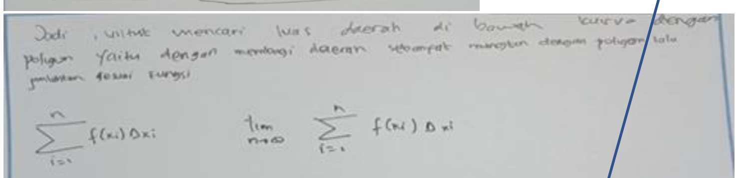
Lecturer: *How to determine the surface area of a pool if divided by n?*

Student (MS): *Calculating the area of all polygons then adding them? But I'm confused about how to determine the total size and area because n is unknown*

Lecturer: *Is there a relationship between the areas of polygon 1, 2, 3, and so on?*

Student (MS): *(smiling) yes Ma'am.*

The answers of students with medium abilities with the help of HLT can be seen from the development of models used by students to solve problems until they find formal concepts as in Table 3:

	Students determine the approximate area of the pool using 8 polygons
	Students determine the surface area of the pool using n polygons
	Students formulate the surface area of the fish pond
	Students generalize the formula for the area under the curve

So to find the area under a curve using polygons, divide the area into as many polygons as possible and then add them up.

Table 3: *Student Answers (MS) During the Discovery Process in Activity 2*

Students with High Ability (MT)

Students with high abilities (MT) were able to solve question number 1, question 3, and question 4. Students (MT) had a little difficulty solving question number 2 to determine the surface area of a fish pond if divided into n polygons.

Lecturer: *How to determine the surface area of a fish pond if it is approached by n polygons?*

Student (MT): *Determine the area of each polygon then add them up, ma'am. But I am constrained because there are n polygons.*

Lecturer: *Have you studied sigma form? Try to pay attention to the area of polygon 1, polygon 2, polygon 3, and so on. Do you see the connection?*

Student (MT): *hmmm (student thinks then smiles), yes Ma'am*

The answers of students with high abilities (MT) with the help of HLT can be seen from the development of models used by students to solve problems until they find formal concepts as in Table 4:

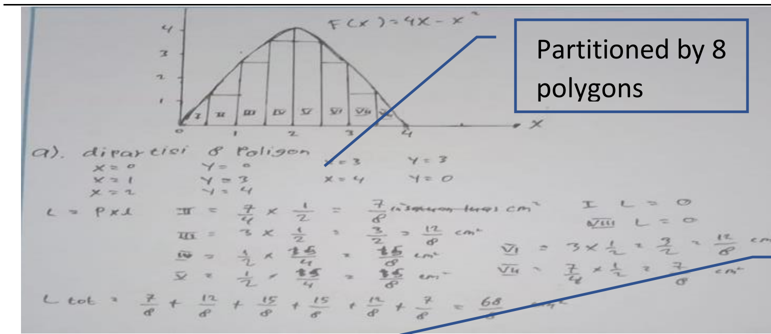
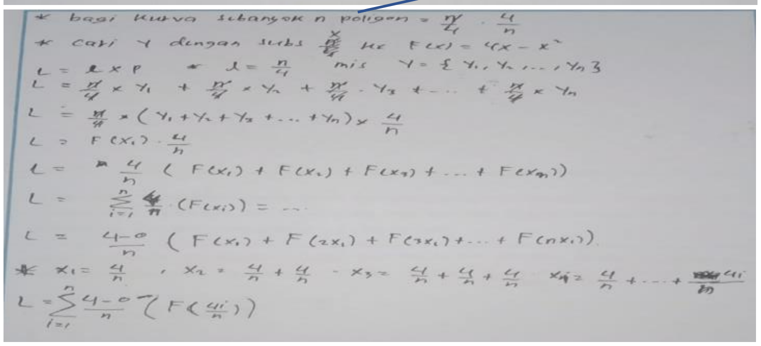
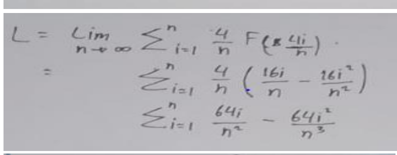
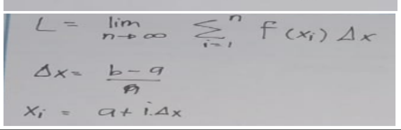
	Students determine the approximate surface area of the fish pond if it is divided into 8 polygons
	Students are able to approximate the surface area of a fish pond by dividing the area into n polygons
	Students determine the formula for the surface area of a fish pond
	Students generalize the formula for the area under the curve

Table 4: *Student Answers (MT) During the Discovery Process in Activity 2*

Retrospective Analysis

Based on the experimental results, for activity 1 there are several changes to the HLT that have been designed, namely adding two hypotheses and anticipation, namely 1) when students cannot see the relationship between many partitions with the best approximation. The anticipation that is carried out is a probing question, how can the area outside the flat area which is still the surface area of the fish pond become smaller? 2) When students are not able to see that the area of all flat areas (polygons) is the same as the surface area of a fish pond when the area in the partition is infinite. The anticipation that is carried out is a probing question, what if the surface of the fish pond is divided into 100 flat areas, 1 million flat areas, and so on? When is the area of the entire plane equal to the surface area of the fish pond? For activity 2, there are several changes to the HLT that have been designed, namely anticipating the first question, namely 1) when students are not yet able to determine the length of the polygon. The anticipation is a probing question, pay attention to the graph of the function and one of the polygons, if the area of origin is known, can the length of the polygon be determined? For question 2, you need to add hypothesis and anticipation, namely when students are confused about dividing an area into n polygons. The anticipation is a probing question, can you describe some of the partitions and determine their sizes? Anticipation of the second hypothesis was changed to a more specific probing question. Haven't you learned sigma notation? Take a look, is there a relationship between the areas of polygon 1, polygon 2, and so on? For question 4, anticipation is changed to be more specific, pay attention to the answer to question 3, what if the function is arbitrary f(x) and the limit is from a to b?

DISCUSSION AND CONCLUSIONS

This research describes how students understand the area under the curve and how the designed HLT can facilitate students to discover the concept of the area under the curve. The area under the curve is the basis for understanding Definite Integrals or Riemann Integrals. This integral is built from the Riemann sum which is the algebraic sum of the area under the curve (Varberg et al., 2016). Based on the research results, it can be seen that students have difficulty determining the area under the curve. Students are only able to approximate the surface area of the pool by using a flat plane or a polygon. This is because students are accustomed to calculating area using formulas without knowing how the concept of area was discovered.

The existence of intervention in the form of RME-based HLT can help students discover the concept of area under the curve. This is in accordance with the opinion of Clement and Sarama (2004) that learning flow can increase students' understanding of concepts. Through contextual problems given, students develop their knowledge by gradually discovering formal mathematical concepts. In accordance with the opinion of Gravemeijer (2020), during the discovery process, student knowledge is formed and developed. According to Vygotsky, a social constructivist figure (Santrock, 2008), children actually already have rich concepts but are not yet systematic and organized, so external intervention is needed, one of which is RME-based HLT. In accordance with the opinion of Cárcamo et al (2019), RME-based HLT has a great opportunity to improve students' reasoning abilities and make Integral Learning more meaningful. This is also supported by research by Aziza (2020) which recommends that Calculus learning must be connected to students' real lives, which is one of the characteristics of RME.

There were several changes to the draft HLT that was designed, this is in accordance with the opinion of Simon & Tzur (2004) that the HLT must be modified regularly and through an iterative process. HLT must be adapted to student characteristics and be dynamic. In line with Gravemeijer and Cobb (2006), there is a reflective relationship between learning theory and its implementation and the development of HLT. In other words, HLT can change based on the experiments carried out. Of course, this is a challenge for lecturers in designing HLT that suits student characteristics.

Based on the research results, it can be concluded that students have difficulty determining the area of an area whose sides are curved lines. The existence of HLT helps students discover the concept of the area under the curve. There are several revisions to the HLT, namely the addition of two hypotheses and their anticipation in activity 1. For activity 2 there is a change in anticipation when students are not able to determine the size of the polygon in the first question, namely "Pay attention to the graph of the function and one of the polygons, if the area of origin is known, what is the length of the polygon can it be determined?" Subsequent revisions to the anticipation of questions 2, questions 3, and questions 4 were changed to be more specific. Students with low abilities tend to need quite a long time and quite a lot of experiments to come to this conclusion and tend to use words. Medium and high students tend to be relatively fast and draw conclusions using symbols. Likewise, when formulating the area under the curve, high and medium ability students with the help of HLT tend to formulate it more quickly.

A limitation of this research is that the research subjects only involved three students with heterogeneous abilities. For further research, it is recommended to expand the research subject and also designs HLT on other Definite Integral topics such as Riemann sums, definitions, and properties of definite integrals including the Fundamental Theorem of Calculus (FTC). The hope is that Integral learning will certainly become more meaningful and effective.

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The Role of Ethnomathematics in South-East Asian Learning: A Perspective of Indonesian and Thailand Educators

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The ethnomathematics field plays a crucial role in understanding how students can comprehend, express, manipulate, and apply mathematical concepts. However, it is acknowledged as a complex concept, particularly in Asian countries like Indonesia and Thailand, requiring a comprehensive understanding. This study aims to bridge this gap by exploring the cross-cultural perspectives of mathematics educators in Indonesia and Thailand and examining the differences in ethnomathematics application between the two countries. Participants included lecturers, teachers, and pre-service teachers, and data was collected through questionnaires and interviews. Before the questionnaire is distributed, comprehensive learning is carried out in the involved schools and universities to provide a direct experience of ethnomathematics education for educators and students. The analytical process involved data reduction, presentation, drawing conclusions or verification, and ensuring data validity. Positive responses from mathematics educators from the teaching and learning activity, supported with the questionnaire average scores in Indonesia at 4.77 and Thailand at 4.57. This study emphasizes the significance of integrating ethnomathematics into education, highlighting its connection to cultural growth. It notes differences in ethnomathematics application between Indonesia and Thailand, shaped by cultural, educational, and institutional contexts. The authors recommend math educators consider integrating cultural aspects into teaching for enhanced inclusivity and longevity.

Keywords: Ethnomathematics, Perspectives, Difference, Educators, Teaching and Learning Activity

INTRODUCTION

In addition to its connection with the progress of science, mathematics holds a vital role in cultural advancement. The emergence of ethnomathematics, a form of culture-based mathematics, has gained prominence in contemporary mathematics education. Ethnomathematics involves studying mathematical practices within diverse cultural groups, encompassing urban and rural communities, labor groups, specific age groups of children, and indigenous communities (Rodríguez-Nieto & Alsina, 2022).

Indonesia and Thailand, recognized for their cultural richness in Southeast Asia, have embraced ethnomathematics within their educational landscapes. Prahmana and D'Ambrosio (2020) found that in Indonesia, ethnomathematics is integrated into the formal mathematics education curriculum, going beyond mere exploration. Examples include the study of modulo in the Balinese naming system (Suryanatha & Apsari, 2022) and the examination of ethnomathematics in Balinese weaving crafts (Puspadewi & Putra, 2014), showcasing how mathematical concepts can be derived from cultural practices.

Similarly, in Thailand, the exploration of ethnomathematics has extended into school education. Suryana et al. (2022) explored ethnomathematics in the five tones of the Thai language, revealing its applicability to topics such as curves and functions in mathematics learning. Despite the evident potential, the implementation of ethnomathematics in education faces various challenges. Concerns expressed by Sarı and Yüce (2020) highlight issues such as the lack of inclusion in classroom materials, inadequate teacher training, and the perceived conflict with traditional curriculum expectations, affecting its successful integration into mainstream education.

The challenges in implementing ethnomathematics have led to suboptimal learning experiences for students, impacting academic achievement. However, ethnomathematics remains integral to enhancing students' mathematical abilities. Ricardo's research (2016) emphasizes that integrating ethnomathematics into mathematics education expands learning beyond the classroom, incorporating real-world experiences and cultural interactions. In the context of the fourth industrial revolution, ethnomathematics serves as a vital bridge between technological advancements and the preservation of cultural heritage (Setiana, 2020).

Recognizing the importance of ethnomathematics in education and the challenges in its implementation, researchers argue for an intercultural perspective on its role, particularly in culturally rich countries like Indonesia and Thailand (Fouze & Amit, 2023). This research aims to fill the gap by conducting a comprehensive study on the perspectives of educators from both nations regarding the significance and implementation of ethnomathematics. While previous studies have acknowledged the presence and potential of ethnomathematics in these countries, this research uniquely focuses on the viewpoints of educators.

Through an exploration of the intercultural perspective on the importance of ethnomathematics learning (Meaney et al., 2021). The study aims to understand the cultural nuances shaping the reception and application of ethnomathematics in educational settings. The novelty of this research lies in its explicit focus on educators' perspectives, offering insights into challenges, opportunities, and potential strategies for integrating ethnomathematics into the curriculum, thus promoting more effective and culturally responsive practices within mathematics education.

The subsequent sections delve into the methodology employed in this research, outlining the research location and participants, research design and procedure, data collection techniques, and data analysis techniques. Subsequently, the outcomes of the teaching and learning activity, questionnaire score analysis, and comprehensive interviews with representative respondents are discussed in the third section, revealing their responses to the role of ethnomathematics in learning, along with diverse opinions and inputs. Finally, the conclusion summarizes the research findings, acknowledges limitations, and suggests avenues for future studies.

LITERATURE REVIEW

Ethnomathematics Learning

The term ethnomathematics was introduced by D'Ambrosio, a Brazilian mathematician, in 1997. Linguistically, ethnomathematics originates from the prefix "ethno," which broadly refers to socio-cultural aspects, including language, jargon, ethical codes, myths, and symbols. The base word "mathema" means to know, explain, understand, and engage in activities such as measuring, classifying, modeling, and drawing conclusions. The suffix "tics" comes from the word techne, meaning technique. D'Ambrosio (1985) define ethnomathematics as mathematics that practiced within cultural groups, such as national communities, tribes, labor groups, children from specific age groups, and professional classes.

The field of ethnomathematics is relatively new but holds the potential to revolutionize how students learn mathematics by introducing them to different cultures. By interacting with local cultures outside the classroom and using mathematics as a medium, ethnomathematics can create a new environment where mathematics learning is not confined to the classroom. Consequently, the learning process, teaching strategies, and learning materials can all center around ethnomathematics. Aprilianingsih & Rusdiana (2019) stated that the topics studied in ethnomathematics are diverse, including: a) Symbols, concepts, principles, and mathematical abilities present in a community; b) Mathematical differences or similarities within a community and the factors contributing to those differences or similarities; c) Specific and interesting aspects within a particular group, such as language use, attitudes, thinking patterns, etc., relevant to mathematics; and d) Various aspects of societal life related to mathematics, such as social, economic, cultural, and political conditions.

Realistic Mathematics Education

In the context of realistic mathematics education, the learning process starts with something tangible or related to students' real-life experiences. "Realistic" here doesn't just mean something concrete or physically present but can also refer to something that can be imagined by students (Afriansyah, 2016). In the teaching and learning activities, things that students can imagine related to the learned concepts can be utilized to enhance the learning process (Febrian & Perdana, 2017).

Therefore, students can meaningfully engage in learning and attribute significance to what they are studying.

Soedjadi (2014) explains that Realistic Mathematics Education (RME) is an innovation in mathematics education aligned with constructivist theory. RME focuses more on the potential that students possess, which should be developed. The teacher's belief in this potential will influence how they manage mathematics teaching. This, in turn, will impact how students cultivate habits aligned with their abilities. Both of these factors will influence the teaching culture of educators and how the learning culture of students should be shaped. Thus, this mathematics education innovation will not only change the understanding of mathematical concepts and their connections but will also transform the learning culture into a more dynamic yet ethically grounded framework.

METHOD

Research Location and Participants

The research was conducted at Mahasaraswati University Denpasar and Bansomdejchaopraya Rajabhat University, Thailand, along with secondary schools affiliated with both institutions. The research took place during the Even Semester of the Academic Year 2022/2023. The participants in this research were mathematics educators consist of: lecturer, high school teachers, secondary school teachers, and pre-service teachers from Indonesia (138 participants) Thailand (145 participants).

Research Design and Procedure

1) Research Design

This study employed a descriptive qualitative research method aiming to provide a more detailed perspective on the intercultural aspects of mathematics educators concerning the significance of ethnomathematics in mathematics education. Qualitative research is intended to understand phenomena such as the experiences of research subjects, including behaviors, perceptions, motivations, actions, and more (Semiawan, 2010). The qualitative approach was chosen to reveal a deeper understanding of the intercultural perspective of mathematics educators regarding the significance of ethnomathematics in mathematics education. The study will employ a qualitative descriptive research approach, gathering data directly from questionnaire responses and interviews.

2) Research Procedure

The research procedure encompasses the activities designed by the researchers to be implemented during the research. The steps involved in this research procedure are as follows: a) Determining the research steps through a focus group discussion with partner institutions, b) Selecting research participants from both institutions, b) Establishing the analysis procedures for examining the participants' perspectives, d) Preparing research instruments such as questionnaires and interview guidelines, e) Consulting research instruments with the research team and experts, f) Conducting instrument pilot tests to ensure their validity and reliability, g) Implementing the research by distributing instruments and conducting interviews, h) Examining and analyzing the acquired data, and i) Drawing conclusions from the data analysis.

Data Collection Techniques

Several data collection methods were utilized in this research, including the following:

1) Teaching Acitivity

Learning takes place prior to the collection of other data with the aim of introducing ethnomathematics to both teachers and students. This introductory phase is essential in laying the foundation for understanding and engaging with ethnomathematics concepts within the educational setting. Through these early learning sessions, the intention is to familiarize educators and students with the principles, applications, and cultural contexts associated with ethnomathematics. This initial exposure creates a conducive environment for subsequent data collection, ensuring that participants have a basic understanding of the subject matter and are better equipped to provide informed insights during the research process. The emphasis is on establishing a solid groundwork that facilitates a more meaningful and effective exploration of ethnomathematics within the educational context.

2) Questionnaire Technique

Sugiyono (2014) stated that a questionnaire is a data collection technique where the researcher provides a list of written questions or statements to be answered by respondents. In this study, a questionnaire was used to obtain intercultural perspectives on the role of ethnomathematics learning among educators from Indonesia and Thailand. The questionnaire, consisting of 20 questions with score 1-5 from Strongly Disagree to Strongly Agree, specifically designed to assess educators' perspectives, was created using Google Forms and distributed through WhatsApp and Line. The points of questionnaire were: 1) The ethnomathematics approach can help students understand mathematical concepts in a more real and relevant way, 2) The use of ethnomathematics in learning mathematics can increase students' interest and motivation towards mathematics subject, 3) Ethnomathematics helps students understand that mathematics can be found in various cultures and contexts of everyday life, 4) Ethnomathematics integration in learning can help increase inclusion and respect for cultural diversity in the classroom, 5) Using ethnomathematics in learning mathematics can help students develop broader and creative problem-solving skills, 6) Ethnomathematics helps broaden students' views of mathematics and see it as something more than formulas and calculations, 7) I believe that the use of ethnomathematics in learning can help reduce cultural disparities and increase equity in mathematics education, 8) The integration of ethnomathematics in mathematics learning provides opportunities to enrich students' learning experiences and make them more interesting and relevant, 9) Ethnomathematics helps students understand and appreciate the various ways of thinking and approaches to mathematics that exist in various cultures, 10) The integration of ethnomathematics in learning mathematics can help students see the relevance of mathematics in life, 11) The use of ethnomathematics in learning can motivate students to develop critical and analytical thinking skills, 12) Ethnomathematics helps to dispel negative stereotypes related to mathematics and opens opportunities for all students to achieve success in this subject, 13) The integration of ethnomathematics in learning mathematics can help students build a deeper and abstract understanding of mathematical concepts, 14) I believe that the use of ethnomathematics in learning mathematics can help students develop an appreciation of the cultural diversity around them, 15) Ethnomathematics helps students relate mathematical concepts to real-world situations and reinforces their understanding of the relevance of mathematics in everyday life, 16) The integration of ethnomathematics in learning mathematics can help increase students' confidence in facing the challenges of mathematics, 17) I believe that ethnomathematics can help overcome cultural and language barriers experienced by students in learning mathematics, 18) The use of ethnomathematics in learning mathematics helps students gain a deeper understanding of cultural values related to mathematics, 19) The integration of ethnomathematics in learning mathematics can help students gain a global perspective and understand how mathematics is applied in various cultural contexts, and 20) I believe that ethnomathematics can help develop social awareness and teach students about fairness, equality, and respect for differences in mathematical contexts.

3) Interview Technique

Interviews involve obtaining information or data for research purposes through a question-and-answer process, conducted face-to-face between the interviewer and the respondent using an interview guide (Taherdoost, 2022). Interviews, conducted as part of this research, involved engaging 8 participants, including middle school teachers, high school teachers, mathematics education lecturers, and pre-service teachers from Indonesia and Thailand. The selection criteria for participants ensured representation, focusing on individuals with a strong understanding of ethnomathematics and culture-based mathematics learning, as evidenced by their responses to preliminary questionnaires. Employing an unstructured interview format, the discussions aimed to explore deeply the perspectives of mathematics educators regarding the significance of integrating cultural elements into mathematics education. The interview point of questions was: 1) 1What do you know about ethnomathematics? 2) Name and explain one of the ethnomathematics concepts that you know according to your culture and your environmental situation! 3) In your opinion, how can the use of ethnomathematics help students to develop their understanding of abstract mathematical concepts? 4) In your opinion, can the ethnomathematics approach help connect mathematics with students' daily lives? and 5) Do you consider it significant to include ethnomathematics that based on local culture and local wisdom in learning mathematics? Why? Throughout the interview, detailed notes were taken to capture the nuances of the discussions, providing valuable insights into the perceptions and beliefs of educators. These insights contributed significantly to the research objectives, shedding light on the role of culture in mathematics learning.

4) Documentation

Documentation encompasses the gathering of data on variables from diverse sources, including records, transcripts, books, student grade lists, attendance records, and more (Lambert & Lambert, 2013). Specifically, in this study, the documentation method was employed to collect data on the implementation of ethnomathematics teaching in the participating institutions. This involved gathering the results of students' worksheets and documentation on how students reacted to ethnomathematics. These documents provide valuable insights into the effectiveness of ethnomathematics teaching, students' comprehension of the concepts, and their engagement with the material, thereby contributing significantly to the research objectives.

Data Analysis Technique

The research employed a qualitative descriptive data analysis technique, with the following steps:

1) Data Reduction

Data reduction involves sharpening, categorizing, directing, discarding unnecessary data, and organizing the remaining data to draw and verify final conclusions. This process includes selecting, focusing, simplifying, and abstracting the raw data written in field notes. The data reduction steps in this research were as follows: a) Examining the questionnaire results and grouping them based on participant responses, b) Transforming the questionnaire data obtained from the participants into notes for use in the interviews, c) Simplifying the conducted interview results into a well-structured and organized form, and then transforming them into notes. In grouping the questionnaire results, positive responses were obtained from an average of agree and strongly agree responses with a score of 4-5.

2) Data Presentation

Data presentation involves organizing a set of structured information that allows for drawing conclusions and taking action. In this stage, the questionnaire results were organized according to the participants.

3) Drawing Conclusions or Verification

Verification is part of a comprehensive configuration activity that can answer research questions and objectives. Comparing the questionnaire results with the interview outcomes can lead to conclusions about the educators' perspectives.

4) Data Validity Check

After analyzing the available data to find answers to the research problem, the next step involves examining the validity of the findings. To determine the validity of the findings, a verification technique is required. In this research, data validity checking employed the triangulation technique. The type of triangulation used in this research was source triangulation, comparing and cross-checking the degrees of belief of information obtained through different times and methods in the qualitative approach. The source triangulation stage conducted in this research involved comparing the results of the questionnaire with the interview outcomes.

RESULTS AND DISCUSSION

Results

The research results describe how the researcher analyzed the outcomes obtained from teaching activities, questionnaires, and interviews. From various presentations, specific perspectives of educators regarding ethnomathematics education were obtained.

1) Teaching Activity Results

Before the questionnaire providing insights from mathematics educators is distributed, two learning sessions are conducted at each level using ethnomathematics-based instruction. The content utilized in the lessons varies depending on the distribution of topics at each level, but generally revolves around geometry. The following are explanations for each learning session conducted. Learning is conducted in a limited manner due to various constraints such as permissions, effectiveness, time, and other factors. Education in Indonesia is focused on the Middle School (SMP) and Senior High School (SMA) levels, while in Thailand, education is concentrated at the university level. This choice of educational focus may be influenced by educational policies, national curricula, and local conditions and needs in each country. Despite the limited scope of learning, efforts to understand and apply ethnomathematics principles at different educational levels remain a positive step in enhancing the understanding of mathematical concepts through cultural contexts and local environments.

The measurement and validation of the effectiveness of teaching activities in ethnomathematics education involve employing various assessment methods to ensure the robustness of the instructional approach. For instance, before distributing questionnaires to gather insights from mathematics educators, two learning sessions are conducted at each level - middle school and high school in Indonesia, and university-level in Thailand - utilizing ethnomathematics-based instruction. The content of these sessions varies but generally focuses on geometry, such as calculating the area and perimeter of plane figures or analyzing architectural elements in traditional Balinese houses. During these sessions, students engage in Problems Based Learning (PBL), where they analyze problems, construct solutions, present their answers, and discuss results with peers. The effectiveness of these activities is measured through the students' ability to comprehend and apply mathematical concepts within cultural contexts, as demonstrated by their learning outcomes and participation in discussions (Fouze & Amit, 2023).

Although the teaching activity was conducted at the college level in Thailand, it still provides valuable background information for participants in the research who are high school, middle school teachers, and pre-service teachers. The college-level teaching in Thailand offers insights into advanced math concepts and teaching methods. High school and middle school teachers can learn from these approaches to better prepare their students. Pre-service teachers can also gain ideas for culturally responsive teaching. Understanding how ethnomathematics is used at college level can help all teachers promote cultural inclusivity in math education (Meaney et al., 2021).

Moreover, the learning activities provide valuable background information for participants who are not students (Tremblay et al., 2012). For example, the exploration of ethnomathematics at the high school and middle school levels in Indonesia offers insights into how cultural elements are integrated into mathematics education at different educational stages. Similarly, the university-level learning activities in Thailand demonstrate the adaptability of mathematical concepts to diverse linguistic structures, highlighting the importance of integrating cultural and linguistic elements into the learning process. By elucidating these learning activities and their outcomes, participants gain a deeper understanding of how ethnomathematics is implemented in educational settings and its impact on students' mathematical learning experiences.

Learning at the Middle School Level in Indonesia

At the middle school level, ethnomathematics instruction focuses on the concepts of area and perimeter of plane figures. During this stage, students engage in Problems Based Learning (PBL) where the general steps they follow are: 1) Analyzing the problems provided by the teacher, 2) Constructing solutions to the problems, 3) Presenting the obtained answers, 4) Discussing the results with their friends.

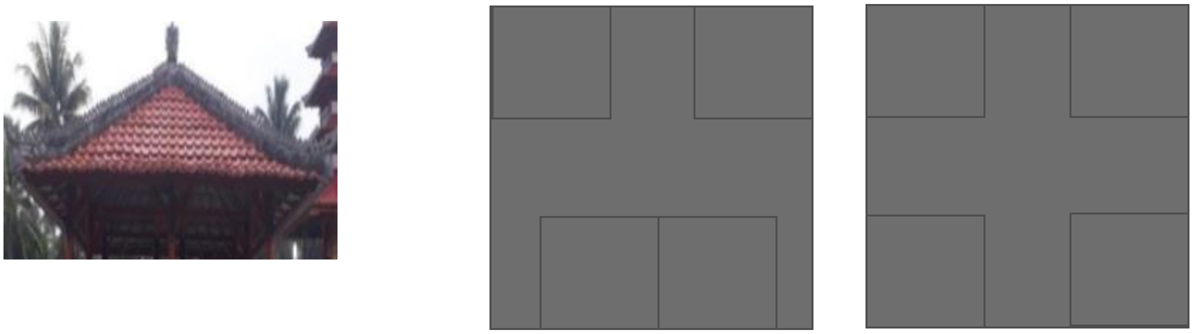
The problem presented by the teacher involves calculating the area of a square in Taledan, which is one of the places of worship for Balinese Hindus. In practice, students are tasked with measuring the area of the Taledan to determine how many Janur (coconut's leaves) are needed to create the Taledan. Following this, students are asked to express their opinions and share what they have learned in discovering the area of a square using the Taledan.

Figure 1. Taledan

The learning outcomes indicate that students not only can calculate the area of the square region in Taledan but also comprehend the structure of Taledan, formed by several rectangles arranged in a meaningful manner. As a result, students are able to calculate the square's area based on the total area of the arrangement of rectangles. Without complex calculations, students can determine the length of Janur needed to create one Taledan. This demonstrates that ethnomathematics instruction enables students to connect various mathematical concepts, leading to a deeper understanding (Rodríguez-Nieto & Alsina, 2022).

Learning at High School Level in Indonesia

At the high school level, the curriculum emphasizes mathematical modeling and geometric positioning, specifically in relation to Sikut Satak, an architectural element found in traditional Balinese houses. In this context, teachers utilize Problems Based Learning (PBL) and introduce challenges related to organizing traditional Balinese Sanggah buildings, sacred places for Balinese Hindus, to adhere to the guidelines of Sikut Satak. In this scenario, students are tasked with placing the designated buildings and developing their mathematical models.



Fugure 2. Ground Plan in Sikut Satak

The results of this learning experience indicate that students are proficient in creating the desired mathematical models, displaying enthusiasm for the lessons as they are highly relevant to their own homes. However, students from urban areas, accustomed to living with limited space, may face some challenges in constructing mathematical models. This is because urban buildings typically employ simplified Balinese designs.

The teaching process highlights the close connection between ethnomathematics instruction and Realistic Mathematics Education (RME). Ethnomathematics proves effective in helping students understand mathematical concepts through their surrounding environment. This is because ethnomathematics emphasizes the understanding of mathematics through real-world contexts, specifically cultural elements that are deeply rooted in the students' own experiences. (Nasir, 2021)

Learning at University Level in Thailand

In Thailand, the learning process takes place at the university level, where students are presented with challenges to analyze the concepts of lines and curves within the framework of the Thai Tonal Language. In this academic setting, students exhibit the capability to discern various mathematical concepts such as upward and downward functions, quadratic curves, gradients, and more, all expressed through the intricacies of the Thailand Tonal Language. The exercises provided to students engage them in identifying and applying mathematical principles within the unique linguistic context, highlighting the adaptability of mathematical concepts to diverse linguistic structures.

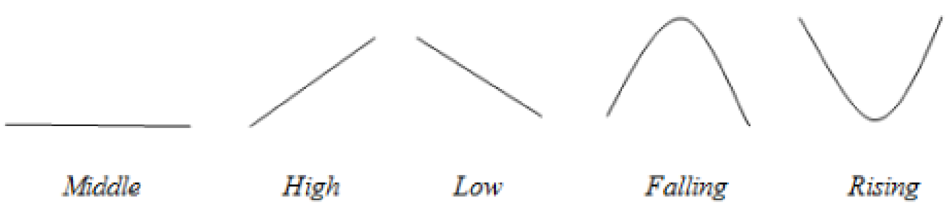


Figure 3. Concept of Line and Curve in Thailand Tonal Language

This approach not only enhances students' mathematical skills but also emphasizes the importance of integrating cultural and linguistic elements into the learning process. By exploring mathematical concepts within the specific linguistic nuances of the Thai Tonal Language, this university-level education contributes to a more holistic understanding of mathematics, fostering a deeper connection between language and mathematical reasoning. The results showcase the effectiveness of this pedagogical approach in bridging mathematical concepts with the linguistic intricacies of the Thai context (Suryana et al., 2022).

The Difference of Ethnomathematics Application Between Indonesia and Thailand

The observed differences in the application of ethnomathematics between Indonesia and Thailand can be attributed to several factors, including cultural, educational, and institutional differences. One possible reason for these variations is the unique cultural contexts and historical backgrounds of each country (Cimen, 2014). Indonesia, with its diverse cultural heritage and rich traditions, may approach ethnomathematics with a focus on traditional Balinese culture and Hindu influences, as evidenced by the teaching activities involving Taledan and Sikut Satak. On the other hand, Thailand's ethnomathematics application may be influenced by its distinct cultural practices and linguistic nuances, as seen in the exercises related to the Thailand Tonal Language.

Additionally, differences in educational policies, curricula, and teaching methodologies between Indonesia and Thailand can also shape the application of ethnomathematics. For instance, Indonesia's emphasis on Problems Based Learning (PBL) at middle school and high school levels reflects a pedagogical approach aimed at fostering critical thinking and problem-solving skills among students within the context of ethnomathematics. In contrast, Thailand's university-level education may prioritize theoretical concepts and mathematical modeling, incorporating ethnomathematics through linguistic analysis and cultural interpretation. Moreover, institutional support and resources dedicated to ethnomathematics education may vary between Indonesia and Thailand, influencing the depth and breadth of its integration into the curriculum. Differences in teacher training programs and professional development opportunities related to ethnomathematics could also contribute to disparities in its implementation across educational settings (Machaba & Dhlamini, 2021).

The implications of these variations for the broader field of ethnomathematics are significant (Setiana, 2020). Firstly, they highlight the need for culturally responsive teaching practices that are sensitive to the unique cultural and linguistic backgrounds of students. Educators must recognize and embrace cultural diversity in their instructional approaches to ensure equitable learning experiences for all students. Secondly, the observed differences underscore the importance of cross-cultural collaboration and exchange in ethnomathematics research and practice. By sharing insights, experiences, and best practices across borders, scholars and educators can enrich the field and promote global understanding and appreciation of ethnomathematics (Ogunkunle et al., 2015). Lastly, these variations emphasize the dynamic nature of ethnomathematics as it evolves and adapts to different cultural, social, and educational contexts. Continued research and dialogue are essential for advancing ethnomathematics as a transformative force in mathematics education worldwide.

2) Questionnaire Results

The questionnaire was distributed among a diverse group of respondents, including university lecturers, high school teachers, middle school teachers, and pre-service teachers across educational institutions in Indonesia and Thailand. Upon collecting and analyzing the questionnaire responses, a comprehensive score recapitulation was compiled. This recapitulation highlights the perspectives of educators from both countries concerning the significance of ethnomathematics in the learning process.

Table 1. Recapitulation of Average Scores of Indonesian Respondents

No	Respondent	Indonesia			Thailand		
		Number of Respondent	**Score**	**Criteria**	**Number of Respondent**	**Score**	**Criteria**
1	Lecturer	30	4,90	Positive	10	4,75	Positive
2	High School Teacher	55	4,53	Positive	50	4,62	Positive
3	Middle School Teacher	40	4,78	Positive	28	4,10	Positive
4	Pre-Service Teacher	20	4,85	Positive	50	4,80	Positive
	Total	**145**	**Average = 4,77**		**138**	**Average = 4,57**	

Table 1 provides a comprehensive summary of average scores derived from Indonesian respondents, including 30 lecturers, 55 high school teachers, 40 middle school teachers, and 20 pre-service teachers. The data indicates a notably high average score of 4.77, signifying a robust recognition of the importance of ethnomathematics within the Indonesian educational framework. These scores suggest a positive outlook and a clear acknowledgment of the value of incorporating cultural aspects into mathematics education. This trend is consistent across all respondent categories, emphasizing a strong endorsement of the integration of cultural elements in fostering a deeper understanding of mathematical concepts. These findings align with contemporary research advocating for the advantages of incorporating ethnomathematics into diverse educational contexts, promoting a richer comprehension of mathematical concepts through a culturally relevant lens (Machaba & Dhlamini, 2021).

In a similar vein, Table 1 presents the average scores from Thai respondents, encompassing 10 lecturers, 50 high school teachers, 28 middle school teachers, and 50 pre-service teachers. The table reveals a slightly lower overall average score of 4.57, indicating a comparable yet marginally more reserved stance towards the incorporation of ethnomathematics within the Thai educational setting. Although the scores still reflect a high level of acknowledgment, subtle variations in the averages suggest differing perspectives on the degree to which cultural elements should be integrated into mathematics education. Recent research underscores the importance of considering cultural context in mathematics instruction, emphasizing the potential for enhanced student engagement, and understanding through culturally relevant pedagogical practices (Owan, 2019). These nuances highlight the need for ongoing research and collaborative initiatives to bridge the gap between cultural diversity and mathematical education, fostering inclusive learning environments that cater to the diverse needs of students within varied sociocultural contexts.

3) Interview Results

To delve deeper into these perspectives, interviews were conducted with one representative from each respondent group. The outcomes of these interviews with Indonesian and Thai educators reveal strikingly similar perceptions regarding the significance of ethnomathematics in the learning process. The following excerpt is from an interview with a representative middle school teacher.

The points of interview show positive attitude of the teacher towards ethnomathematics. the interview section provided valuable insights into the significance of ethnomathematics in contemporary education. The exchange between the researcher and the teacher highlighted the acknowledgment of ethnomathematics as a crucial component in today's learning environment, facilitating a deeper understanding of diverse cultures and fostering inclusivity among students. The discussion also underscored the pivotal role of ethnomathematics in connecting theoretical mathematical concepts with real-world applications, thus enriching students' learning experiences. Moreover, the teacher's reflections on the transformative potential of ethnomathematics emphasized its capacity to shape the future of mathematics education by revolutionizing teaching practices and engaging students on a meaningful level. Overall, the interview concluded on a positive note, with the researcher expressing gratitude for the teacher's valuable insights and expertise in the field of ethnomathematics.

The insights shared by the lecturer resource person provide a compelling perspective on the enduring significance of ethnomathematics in contemporary learning environments. The interviewee emphasizes that the dynamic nature of culture necessitates the integration of ethnomathematics as a crucial component of the educational process. This viewpoint underscores the evolving nature of cultural contexts, highlighting the importance for educators to incorporate diverse cultural perspectives into mathematical instruction. These insights align with recent scholarly literature, which emphasizes the crucial role of culturally responsive pedagogy in enhancing students' engagement and understanding of mathematical concepts within diverse societal frameworks (Caingcoy, 2023). The lecturer's assertion further underscores the role of ethnomathematics in fostering a deeper appreciation of cultural diversity, thereby paving the way for inclusive educational practices that resonate with students from various cultural backgrounds.

Discussion

The perspectives shared by high school educators illuminate the intricate challenges associated with incorporating ethnomathematics into complex curricula. While they recognize its significance in enhancing problem-solving skills, concerns arise about seamlessly integrating ethnomathematics into advanced material. This echoes recent research emphasizing the delicate balance needed to harmonize cultural perspectives with complex mathematical concepts in the high school context (d'Entremont, 2015; Yılmaz, 2020). Educators stress the need for comprehensive strategies to integrate ethnomathematics effectively, aligning it with the demanding requirements of high school mathematics education.

Middle-school educators highlight the interconnected nature of ethnomathematics and realistic mathematics learning. They emphasize how ethnomathematics aids students in understanding mathematical concepts within real-world contexts. This echoes contemporary research emphasizing the importance of real-life applications in fostering conceptual understanding and problem-solving skills in mathematics (Kaitera & Harmoinen, 2022). Feedback from middle school educators emphasizes the potential of ethnomathematics as a catalyst for promoting authentic learning experiences, nurturing students' ability to recognize the practical relevance of mathematical concepts in their surroundings. This underscores the transformative potential of ethnomathematics in cultivating students' holistic mathematical understanding within the middle school curriculum.

The viewpoint of pre-service teachers sheds light on the promising role of ethnomathematics as a catalyst for meaningful mathematics learning. Their endorsement underscores the transformative potential of culturally responsive pedagogical practices, aligning with recent research emphasizing the role of culturally relevant teaching approaches in fostering students' intrinsic motivation and interest in mathematics (Kenan, 2018). The optimistic outlook on ethnomathematics among pre-service teachers highlights its capacity to stimulate students' curiosity and engagement, fostering a positive learning environment that nurtures their appreciation for the cultural diversity embedded within mathematical concepts. This underscores the need for educators to adopt innovative pedagogical strategies, integrating ethnomathematics and fostering a dynamic learning ecosystem that aligns with students' evolving educational needs and aspirations.

Interview findings also reveal differences in the application of ethnomathematics in the educational context between Indonesia and Thailand. While both countries recognize its significant role in fostering cultural inclusivity and enhancing students' understanding of mathematical concepts, interviews with respondents from Thailand suggest a limited integration of ethnomathematics within the learning process. The Thai educational landscape appears to demonstrate a relatively lower prevalence of ethnomathematics in instructional practices, consistent with recent research emphasizing the challenges of implementing ethnomathematics in diverse educational settings, particularly where traditional pedagogical approaches are prevalent (Jun-on & Suparatulatorn, 2023). In contrast, interviews with Indonesian respondents reflect a more proactive approach to integrating ethnomathematics, emphasizing its relevance and practical application within the Indonesian educational framework (Mania & Alam, 2021). This discrepancy underscores the need for tailored educational policies and collaborative efforts to promote the effective integration of ethnomathematics in diverse cultural contexts, fostering inclusive learning environments that cater to the multifaceted needs of students within the Southeast Asian region.

This research also points out the differences in how ethnomathematics is applied in Indonesia and Thailand are influenced by cultural, educational, and institutional factors. For example, Indonesia focuses on its rich cultural heritage, while Thailand emphasizes linguistic nuances. Educational policies, teaching methods, and support also play a role. These differences highlight the need for teaching practices that respect cultural diversity and encourage collaboration between countries. Overall, ethnomathematics is evolving to fit various contexts, emphasizing the importance of ongoing research and discussion in shaping mathematics education globally (Nur et al., 2020).

In conclusion, discussions with various educational stakeholders underscore the potential of integrating ethnomathematics within the educational framework to foster a culturally inclusive and engaging learning environment. While underscoring the importance of embracing diverse cultural perspectives in mathematics education (Hill & Hunter, 2023), the discussions also highlight the challenges associated with seamlessly integrating ethnomathematics into advanced high school curricula (Freire & McCray, 2020). Furthermore, insights underscore the transformative potential of ethnomathematics in promoting authentic learning experiences, fostering a deeper connection between theoretical knowledge and practical applications within the middle school curriculum (Ogunkunle et al., 2015). By adopting culturally relevant teaching practices, ethnomathematics has the potential to stimulate students' curiosity, engagement, and appreciation for the cultural diversity embedded within mathematical concepts, envisioning a more inclusive and dynamic educational landscape (Nasir, 2021).

CONCLUSIONS

The findings of this research highlight positive responses from mathematics educators regarding the significance of ethnomathematics in education. The average scores of respondents in Indonesia and Thailand, at 4.77 and 4.57 respectively, indicate a strong acknowledgment of the importance of ethnomathematics, particularly in the Indonesian educational context. Despite challenges in material integration, especially at higher levels, the study concludes that ethnomathematics, in incorporating culture into the teaching of mathematics in Indonesia and Thailand, holds great potential to enhance students' understanding of mathematical concepts in everyday life. These positive outcomes can lay the groundwork for the development of a culturally-based ethnomathematics curriculum that is suitable and applicable across all levels of education. Additionally, this research also found that he differences in the application of ethnomathematics between Indonesia and Thailand stem from unique cultural, educational, and institutional contexts, shaping teaching methods and priorities in mathematics education. This will materialize meaningful mathematics teaching and learning activity and enhance students' abilities in understanding concepts, mathematical thinking, and problem-solving.

However, it is important to acknowledge the primary limitation of this research, which is the restricted number of respondents and participating institutions. The study focused on educational institutions in Bali, its surrounding regions, and Thailand, with a notable emphasis on affiliated or collaboratively associated institutions with the research entity. This limitation may impact the generalizability and broader applicability of the study's findings. Future research endeavors are encouraged to involve a more extensive and diverse pool of respondents from various cultural backgrounds. This expansion is expected to enhance the understanding of different cultural contexts related to mathematics education, allowing for the identification of diverse cultural patterns in the comprehension and application of mathematical concepts. Consequently, such research initiatives will contribute significantly to underscoring the importance of cultural integration in an inclusive and sustainable approach to mathematics education.

Considering these conclusions, the authors encourage mathematics educators to consider about integrating their teaching methods with cultural aspects in math education, making it more inclusive and sustainable. By adding diverse cultural views into math lessons, teachers can make learning more interesting and relevant for students from different backgrounds. This can help students understand math concepts better and see how they relate to the real world. It also encourages critical thinking and problem-solving skills by letting students explore math through different cultural perspectives. Plus, when teachers respect and appreciate students' cultures, it creates a supportive and welcoming learning environment where everyone feels valued. Overall, including cultural elements in math education can make learning more enjoyable and meaningful for students, helping them succeed academically and personally.

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Effectiveness of the CORE Learning Model: A Case Study of Learning the Method of Coordinates in a Plane in Vietnam

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Abstract: Coordinate geometry is an important part of mathematics. It helps students develop thinking, logic, and problem-solving skills. This study was conducted to test the effectiveness of the CORE learning model in promoting students' mathematical problem-solving skills when they learn the method of coordinates in a plane. Consequently, this study used mixed methods as a quasi-experiment with a non-equivalent control group design, with assessment tools including pre-test, post-test, classroom observation, and attitude survey. The data collected were quantitatively analyzed with JASP and qualitatively analyzed. The analysis findings demonstrate that the students in the experimental group performed better academically in terms of knowledge and problem-solving skills and had more optimistic learning attitudes. In particular, a correlation test was performed on the pre-and post-test scores of the experimental group. It showed that with a correlation level of 0.810, according to the Hopkins reference table, the scores of the students in the experimental group were higher than those of the control group due to the effectiveness of the CORE learning model in promoting students' problem-solving abilities. In addition, the study identified certain limitations and proposed new research directions for the future.

Keywords: CORE learning model, Mathematical problem-solving skills, Mathematics learning outcome, Method of coordinates in a plane

INTRODUCTION

CORE stands for four words with unifying functions in the learning process, including connecting, organizing, reflecting, and extending. These phases connect old and new information, organize diverse material, reflect on everything students learn, and develop a learning environment. Yaniawati et al. (2019) argue that CORE is one of the learning models based on constructivist theory, which states that students can construct their knowledge by interacting with their environment.

According to Calfee (2010), the CORE learning model involves discussion techniques that can impact students' knowledge acquisition and ability to think critically by keeping them interested. The CORE model expects students to be able to construct their knowledge by connecting and organizing new knowledge with old knowledge, then rethinking the concept being learned, and students are expected to expand knowledge in the learning process. Many studies show the diverse application of this model in many domains of mathematical knowledge, such as conics (Salinas & Pulido, 2016), computational methods course (Khor et al., 2020), and trigonometric material (Yaniawati et al., 2019). From this, this learning model contributes to increasing aspects such as problem-solving (Arizal et al., 2018; Irawan & Iasha, 2021; Son et al., 2020), mathematical communication and connection (Yaniawati et al., 2019), mathematical reasoning ability (Atiyah & Priatna, 2023), and creative thinking (Ardiyanto et al., 2022; Saregar et al., 2021).
The ability to solve mathematical problems plays an important role in mathematics education and is studied by many educators (Alabdulaziz, 2022; Arizal et al., 2018; Gunawan et al., 2023; Jacinto & Carreira, 2023; Putri et al., 2022; Rocha et al., 2024). At the same time, students' mathematical problem-solving skills could be enhanced by applying instructional approaches. Still, little research has been done on applying the CORE learning model in math instruction to improve students' problem-solving skills in Vietnam. For these reasons, the study investigated the effectiveness of the CORE learning model in teaching the method of coordinates in a plane to promote students' problem-solving skills.

LITERATURE REVIEW

CORE learning model

Many researchers use the CORE learning model as an instructional approach in mathematics education. Irawan and Iasha (2021) aimed to improve the mathematical problem-solving abilities of elementary school students using this model. Wiharso and Susilawati's (2020) study as a quasi-experiment aimed to compare the results of students taught with the CORE model and students taught in a traditional learning style. Meanwhile, Saregar et al. (2021) conducted a study on 60 eighth-grade students in a high school using a purpose-sampling technique. The results of this study have proven that the CORE learning model effectively enriches students' creative thinking skills. So, what phases does this model include? What role does each phase play?

The CORE model includes four cyclical phases: connecting, organizing, reflecting, and extending. At each phase, students are directly involved in thinking and acting and are trained in listening, speaking, reading, writing, teamwork, and skills such as purposive observation, thinking, comparison, analysis, synthesis, practical skills, evaluation, and self-assessment.

During the "Connecting" phase, teachers can introduce issues related to the new lesson to attract students' attention to the content, making students realize the need and desire to research and explore new content. Teachers can ask questions or have students discuss in groups to help students recall or activate knowledge that students previously knew related to new content. When asking students to discuss what they already know, teachers can find out how much each student knows and identify any misconceptions they may have about mathematics that need to be cleared up. In the "Organizing" phase, students arrange and organize the ideas they had in the previous phase in their way, such as mind maps, charts, and tables. Therefore, learners must be active, proactive, and creative. If the learners are not active, proactive, and creative, no teacher can help them master the lesson content. The above activities will help students appropriately use the available knowledge to create discovery ideas based on guiding questions and adjusting teacher actions for students instead of answers. With this activity, students will synthesize the knowledge they have learned through problem-solving and critical thinking. In addition, in this phase, students are in the center, and teachers play a consulting role, guiding students in arranging and organizing their ideas to solve problems.

In the "Reflecting" phase, the students contemplate and reflect on the products they made in Phase 2. The teacher has the role of concluding and correcting scientific knowledge. The aim is to improve knowledge about possible misunderstandings and consolidate knowledge. In the "Extending" phase, students apply the knowledge they have just acquired with the existing knowledge base to expand and condense their understanding through new experiences to deepen their knowledge, become more skillful, and know how to apply it to different situations and circumstances, especially practical situations. Teachers act as advisors to help students summarize key content, deepen lessons, and create opportunities for students to expand their knowledge.

Regarding the advantages of the CORE model, the "Connecting" phase helps students focus and pay more attention to the lesson because they feel interested and excited compared to approaching the lesson with traditional teaching methods. The "Organizing" phase helps students have many opportunities to exchange and discuss with each other so that they can express their thoughts and approach the problem through many different perspectives from the opinions of other students. In the group, students summarize the whole problem. Mastering all the activities during this phase helps keep the classroom atmosphere exciting and not boring and increases the student's ability to acquire knowledge. Teachers' lesson preparation becomes simpler and more systematic, helping to create diverse activities for students to experience. This process helps teachers reduce the time spent teaching theory and instead create discovery and practice activities to form new knowledge. This is in line with the current educational trend, which is student-centered.

Regarding the limitations when applying the CORE model in teaching, the "Organizing" and "Reflecting" phases require students to have certain learning abilities and efforts. Arranging and organizing their ideas in the "Connecting" phase or giving feedback on the products they made in Phase 2 is difficult for all students. If the student does not pass that, the results of these phases are limited, or the student does not complete the learning task. Many students can use group activities to work individually and influence their environment. When teachers spend too much time on each phase, it will more or less cause boredom for students, and the CORE model will no longer be effective.

Problem-solving skills

Problem-solving skills are essential in mathematics and everyday life. One can easily solve any problem by having various problem-solving skills. When studying mathematics, students learn abstract concepts and make real-world connections between those concepts and their applications in everyday life. Through this learning, students can understand how to apply mathematics in real-life contexts and develop problem-solving skills. These skills are one of the aspects taught in mathematics.

Polya explains the four main phases of problem-solving: understanding the problem, planning the solution, executing the plan, and checking the results (as cited in Daulay & Ruhaimah, 2019). On the other hand, Polya's approach describes general problem-solving steps and is not limited to mathematical problems. Students' ability to solve mathematical problems includes readiness, creativity, knowledge, skills, and application in everyday life. These skills also have a close relationship with other factors such as written feedback (Santos & Barbosa, 2023), creative thinking (Saregar et al., 2021), ability to mathematical connections (Sari & Karyati, 2020), students' problem-solving beliefs in mathematics (Sintema & Jita, 2022), and student cognitive styles (Son et al., 2020). Many educational approaches have been used to enhance students' mathematical problem-solving skills, such as learning devices with CORE models (Arizal et al., 2018) and digital subtraction games (Erbilgin & Macur, 2022), the use of effective learning media (Gunawan et al., 2023), the CORE learning model (Irawan & Iasha, 2021; Son et al., 2020), technology (Jacinto & Carreira, 2023), realistic mathematics education (Putri et al. al. 2022), and GeoGebra (Suratno & Waliyanti, 2023).

Teaching the method of coordinates in a plane

In the research work "Teaching math solutions on the topic of the method of coordinates in a plane for high school students", the author Hoa (2017) provided a theoretical basis for the history of the formation of the method of coordinates, mathematical ability, factors affecting students' math solving skills, and pedagogical measures to foster math-solving ability in teaching math problem-solving on the method of coordinates in a plane for high school students. The illustrative examples refer only to two objects: a straight line and a circle. In the research work "Teaching the topic of three conic sections in the high school program towards competency development", the author Bang (2019) has provided a theoretical basis for mathematical competencies; the competencies are formed through specialized teaching about three conics, historical development of three conics in mathematics, teaching theorems, properties, solving exercises about three conics in the direction of capacity development; Develop specific lesson plans on teaching three conics.

In the research "Developing problem-solving skills for students in teaching the content of the method of coordinates in a plane", the author Cuong (2018) has provided a theoretical basis and discusses the relationship between problem-solving skills in mathematics and the mathematical competencies of high school students and some pedagogical measures to develop problem-solving competencies for students in teaching math subjects, such as straight lines, circles, and ellipses. In his research, the author applied information technology to teaching three conics. The authors have created a digital environment to help students interact and understand each quadratic curve's nature, shape, and equation, such as circle, ellipse, hyperbola, and parabola (Salinas, 2017).

However, there is no research on applying the CORE learning model to teaching the method of coordinates in a plane to promote mathematical problem-solving skills for 10th-grade students.

Research Objectives and Questions

The purpose of the study was to evaluate the effectiveness of employing the CORE learning model in the context of teaching the method of coordinates in a plane. Therefore, this research was conducted to answer the following questions:

(1) Is there a significant difference in learning outcomes between students instructed by the CORE learning model (experimental group) and students taught using conventional methods (control group)?

(2) Are the students' learning outcomes in the experimental group significantly different before and after the intervention?

(3) Is there any improvement in students' math problem-solving skills with the CORE learning model?

(4) What is the attitude of the students in the experimental group toward learning with the CORE learning model?

The Study's Context

The method of coordinates in a plane was the research subject for grade 10 students in the Vietnam General Education Program. The requirements and course content for studying this subject are described in detail in the General Education Program in Mathematics (2018). In terms of instructional content, the textbook's 10-th-grade program's method of coordinates in a plane topic covers the following topics: (1) Vector coordinates; (2) Straight lines in the coordinate plane and applications; (3) circle in the coordinate plane and applications; and (4) three conics in the coordinate plane and applications (MoET, 2018). In terms of the prerequisites that must be fulfilled, students must: (1) Recognize the coordinates of vectors with respect to a coordinate system; find the coordinates of a vector, the length of a vector when knowing the coordinates of its two endpoints; Use coordinate expressions of vector operations in calculations; Apply knowledge of vector coordinates to solve a number of practical problems; (2) Describe the general equation and parametric equation of a straight line in the coordinate plane; explain the relationship between the graph of a first-order function and a straight line in the coordinate plane; Identify two lines that intersect, are parallel, coincident, or perpendicular to each other using the coordinate method; Establish the formula for calculating the angle formed by two straight lines; Calculate the distance from a point to a straight line using coordinates; Apply knowledge of straight line equations to solve a number of practical problems; (3) Establish the equation of a circle when knowing the coordinates of the center and radius; know the coordinates of the three points that the circle passes through; Determine the center and radius of the circle when knowing the equation of the circle; Establish the equation of the tangent to the circle when knowing the coordinates of the point of contact; Apply knowledge of circle equations to solve a number of practical problems; (4) Recognize three conics using geometry; recognize the canonical equations of three conics in the coordinate plane; solves some practical problems associated with three conics (MoET, 2018).

METHOD

The experiment aimed to determine whether using the CORE learning model to teach the method of coordinates in a plane in math textbooks for the 10th grade would help students become more proficient in solving mathematical problems. In a Vietnamese high school in Ho Chi Minh City, 96 students participated in the experiment. Of these, 47 students in class 10A1 were taught using the CORE learning model in the experimental group, and 49 students in class 10A12 used conventional methods in the control group. Subsequently, various data analysis techniques were employed to thoroughly examine the data gathered from the pre-test, post-test, classroom observation, and student surveys. The Ethics Council of Can Tho University, the Board of Directors of the High School and the parents and students of the High School in Ho Chi Minh City, Vietnam, all consented to the study.

Research design

A quasi-experimental study was conducted with a control group to answer the research objectives and questions. In the experimental design, a pre-test was given to the experimental and control groups to ascertain the participants' entry scores before the intervention and validate the equivalency between the two groups. The lessons were taught using the CORE learning model to the experimental group and conventional instruction to the control group. Specifically, participants in the control group received traditional lectures. Defining differently, they had no advantages over the experimental group from instructing through the CORE learning model. In addition, the students in this group were unaware of the subject that would be studied. The lectures had no subtopic division, and the participants were not encouraged to ask questions during the course. Also, the evaluation was conducted without the use of an inquiry-based methodology.

Each group received a post-test to see how well the students applied their new knowledge. Numerous previous studies (Arizal et al., 2018; Ardiyanto et al., 2022) on the effectiveness of the CORE model in mathematics education employed this experimental design, and there are parallels with certain studies on mathematics education. The experimental procedure took place in the following order using the above design.

A scale was created to assess students based on their proficiency in math problem-solving at each level, considering the requirements of the Mathematics General Education Program (MoET, 2018). This scale is shown in Table 1.

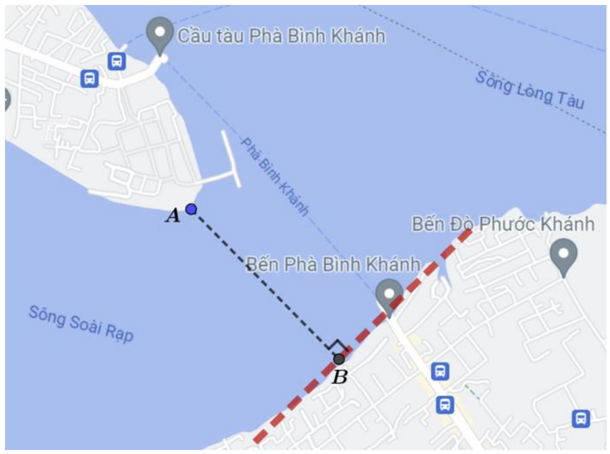
Component capacity	Student expression	Levels of expression

		0	1	2	3
Detect the problem	1. State the problem that needs to be solved in the assigned task.	Unable to raise the problem.	The problem is stated, but not fully.	Able to state the problem more fully but slowly, thanks to the teacher's guidance	Ability to raise issues fully and quickly.
Proposed Solutions	2. State relevant information.	Relevant information cannot be mentioned.	Incomplete related information.	State all relevant information.	Define all relevant information accurately and scientifically.
	3. Propose solutions to solve the problem.	No solution was proposed to solve the problem.	Propose solutions to solve the problem, but are less feasible and ineffective.	Propose possible solutions	Come up with creative solutions that can solve problems in the fastest and best way possible.
Problem-solving	4. Perform problem solving.	Unable to solve the problem, no product can be created.	Confusion when solving problems leads to creating imperfect products in both form and content.	Solve problems well and create products with good content but poor form.	Implement problem-solving to create excellent products both in content and in form.
Evaluate performance results.	5. Results of self-assess performance.	Inability to self-evaluate.	The exact advantages and limitations of the implementation results have not been stated.	The advantages and limitations of the implementation results are accurately stated, but there is no basis, and no experience has been learned.	Clearly state the advantages and limitations of the implementation results, have a valid basis, and learn from experience.

Table 1: Scale to assess students' proficiency in solving mathematical problems.

The research team then designed lesson plans for the experimental group using the CORE learning model and lesson plans using conventional methods for the control group. In CORE model-based lessons, the teacher divided learning activities for each knowledge acquisition process into four stages: connecting, organizing, reflecting, and extending. An example of the activities planned to teach the distance formula from a point to a straight line is provided below.

Stage 1: Connecting.

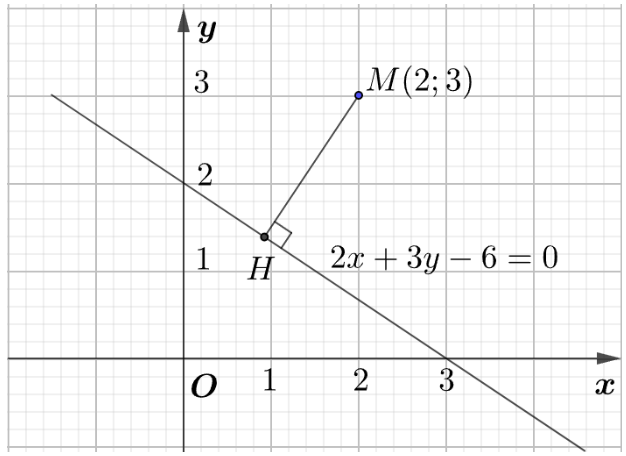


(Source: Image from Google Maps)

Teacher: From the Nha Be district, Ho Chi Minh City, you can visit the Can Gio district, Ho Chi Minh City, through the Binh Khanh ferry terminal. Assuming that the river bank (Can Gio) is a straight line, segment AB is the distance from point A to the river bank (Can Gio). At that time, the segment AB was also the shortest road connecting the two banks of the river. However, due to real conditions, we cannot go directly from A to B, but we have to make a longer journey (the journey of the Binh Khanh ferry). This is also the reason why bridges were born. Then, how is the segment length calculated? This is also the content of the next lesson.

Pedagogical intention: To create an exciting learning mindset for students through practical connections between the Nha Be and Can Gio districts in Ho Chi Minh City. This helps students feel that mathematics becomes interesting and closer to real life, and they love learning math more.

Stage 2: Organizing.



Teacher: In the coordinate plane *Oxy*, a given straight line $\Delta: 2x+3y-6=0$ and a point $M(2;3)$. *H* is called the projection of point *M* onto the line Δ.

a) Find the direction vector of the line MH.

b) Write the parametric equation of the line MH.

c) Find the coordinates of *H*. From there, calculate the length of the line segment MH.

- Call a group to come up to the board to present their group's products.

Students: Follow and comment.

Teacher: Comment. This leads to the general case of giving the distance formula from a point to a straight line.

Suggested solution:

a) MH has the direction vector $\vec{u}=(2;3)$.

b) The parametric equation of the line *MH* is $\begin{cases} x=2+2t \\ y=3+3t \end{cases}$.

c) Because $H\in MH$, we can call $H(2+2t;\ 3+3t)$. On the other hand, $H\in\Delta$, so we have the following: $2(2+2t)+3(3+3t)-6=0 \Leftrightarrow t=\frac{-7}{13}$. Inferring $H\left(\frac{12}{13};\frac{18}{13}\right)$ and

$$MH=\sqrt{(x_H-x_M)^2+(y_H-y_M)^2}=\sqrt{\left(\frac{12}{13}-2\right)^2+\left(\frac{18}{13}-3\right)^2}=\frac{7\sqrt{13}}{13}.$$

Pedagogical intention: By fulfilling the requirements and answering the teacher's purposeful questions, students form new knowledge by applying relevant old knowledge. This has shown the manifestations of problem-solving skills in students.

***Stage 3: Reflecting*.**

Teacher: Give an exact formula to calculate the distance from a point to a straight line.

In the coordinate plane *Oxy*, a given straight line Δ with its general equation of a straight line $ax+by+c=0$, satisfying the condition $a^2+b^2>0$ and a point $M_0(x_0;y_0)$. The distance from a point M_0 to a straight line Δ, denoted as $d(M_0;\Delta)$, is calculated by the formula:

$$\boxed{d(M_0;\Delta)=\frac{|ax_0+by_0+c|}{\sqrt{a^2+b^2}}}.$$

Students: Copy the above content to their memo pad.

Another example: Calculate the distance from a point $M(1;2)$ to a straight line $\Delta: 4x+3y+5=0$.

Pedagogical intention: To help students correct and systematize newly discovered knowledge and, at the same time, respond to knowledge with examples. This allows students to use newly discovered knowledge to solve mathematical problems.

***Stage 4: Extending*.**

Problem 1: In the coordinate plane *Oxy*, a triangle *ABC* whose vertex coordinates are $A(1;1), B(5;2), C(4;4)$. Calculate the length of the altitude from vertex *A* of triangle *ABC*.

Problem 2: Calculate the distance between two straight lines $\Delta_1: 3x-4y+2=0$ and $\Delta_2: 3x-4y+12=0$.

Pedagogical intention: performing the above problems will help students practice recognizing and detecting problems through reading, understanding the problem and then choosing ways and solutions to solve the problem, thus using mathematical knowledge and skills to solve problems. This is also one of the goals of developing mathematical problem-solving skills.

Before implementing the planned lessons, researchers worked with the teacher to set up the classroom using the CORE learning model. In particular, the arrangement of the classroom was adaptable. The classroom could easily rearrange chairs and tables to accommodate various learning activities. There was space in the classroom for group projects, debates, speeches, and individual study. Besides, it gave students enough room to walk around the classroom without feeling crowded. The classroom was then decorated to spark students' interest in learning. The teacher used images, pictures, and mind maps about math material to decorate the classroom. Additionally, the teacher promoted student expression and fostered a creative environment by using bulletin boards to showcase learning materials, group projects, and student accomplishments. Lastly, computers, projectors, or screens were installed in classrooms to present information, videos, and educational materials. A reliable internet connection was also available to students to access online resources.

The research team privately observed the experimental and control groups throughout the teaching process. The content of the observations in the classroom was examined based on some criteria, such as the instructional strategies used by the teacher, the student's methods of learning, the skills that the students had attained, the environment of the classroom, and most importantly, the student's ability to solve math problems both in the experimental group and the control group both before and after the intervention. Lastly, a post-test was administered to the experimental and control groups to gauge the effectiveness of enhancing their ability to solve mathematical problems.

Additionally, students from the experimental group were polled using a series of multiple-choice questions on the Likert scale, which has five levels: totally disagree, disagree, neutral, agree, and totally agree (Likert, 1922). Data on student attitudes, motivation, interests and receptivity were collected using lesson plans connected to the CORE learning model.

Experts in mathematical education at Can Tho University reviewed the experimental teaching lesson plans, and teacher colleagues validated the tests to ensure the instrument's validity and reliability. High school staff conducted experiments to ensure that lesson objectives were met. Once the expert recommendations were implemented, the tools were deemed suitable for academic purposes and could evaluate students' skills, making them suitable for experiment use. Furthermore, the reliability of the post-test questionnaires was assessed using Cronbach's Alpha reliability. The correlation between the scores of the experimental group was determined using the student attitude survey and Pearson's correlation coefficient.

Data Collection and Analysis

Data were collected from the pre-test (first-semester final exam), post-test, class observation results, and post-intervention student opinion survey results. Using JASP software, the data were examined both quantitatively and qualitatively. Table 2 shows the experimental procedure as follows:

Groups	Pre-test	Intervention	Post-test	Opinion survey
Experimental group	x	X: CORE learning model	x	x
Control group	x	-	x	-

Table 2: Quasi-experimental Design

This study used qualitative and quantitative analysis methods to evaluate the experimental results. Regarding quantitative analysis, the pre-and post-test score data of both groups were tested for normal distribution through descriptive statistics (Shapiro-Wilk test), normal probability plots (Normal Q-Q Plot), standard curve chart (Normal distribution curve), the Pearson correlation coefficient (r) between the two sets of pre-and post-test scores of the experimental class, and the effect size using the mean deviation of Cohen (1998). Independent t-test (2-tailed) was used to compare the means of the experimental and control classes. Regarding qualitative analysis, the researchers conducted classroom observations in both experimental and control groups, analyzing based on some main criteria: teaching methods, learning methods, skills acquired, learning content, and classroom atmosphere. Based on the 5-level Likert scale, eight survey questions were created to gauge students' opinions of the CORE learning model's instructional strategies used in the experimental classroom and their ability to solve problems independently.

RESULTS

Results of the pre-test

The correlation between the experimental and control groups' math learning levels was examined using the first semester's final exam. The data processing results show a normal distribution of the test scores between the two groups. The results of the Shapiro-Wilk test indicate that both groups' significance levels for the pre-test are greater than 0.05, confirming the normal distribution of the pre-test scores. Table 3 shows the results obtained.

Groups	Statistics	Sig.
Experimental group	0.982	0.689
Control group	0.956	0.065

Table 3: Pre-test results for the Shapiro-Wilk test

The hypothesis that there was no significant difference in the mean pre-test scores between the experimental and control groups was tested due to the independent t-test. The t-test and descriptive statistical results for the mean pre-test scores of the experimental and control groups are calculated in Tables 4 and 5.

Groups	N	Mean	Std Dev	Minimum	Maximum
Experimental group	47	6.809	1.458	4	10
Control group	49	6.736	1.574	3	9.5

Table 4: Descriptive statistics of scores before the intervention

Table 4 shows that the average score for 47 students in the experimental group is 6.809, while the average score for the control group is 6.736 for 49 students. The data dispersion of the experimental group (standard deviation) is 1.458. The mean and median scores for both groups are nearly identical, and the standard deviation of the control group is 1.574. Additionally, the idea that the pre-test mean scores for both groups were equal was tested using an independent t-test. Table 5 reveals the test results.

t-test			
df	t Stat	Sig. (2-tailed)	Mean difference
94	0.235	0.815	0.073

Table 5: The independent sample t-test results regarding the pre-test scores

An independent sample t-test was used to test whether there was a significant mean difference between the experimental and control groups. Consequently, the value (Sig.) is 0.815 (greater than 0.05) with a significance level of 0.05 and degrees of freedom df = 94. The mean score for the experimental and control groups did not differ according to this. In other words, the test results indicate that the qualifications of the two groups are equivalent.

Results of the post-test

The study compared the mean post-test scores of the experimental and control groups using twelve multiple-choice items and two essay items. The results of the Shapiro-Wilk test in Table 6 demonstrate that the observed significance levels of both groups are greater than 0.05, confirming the normal distribution of post-test scores for both groups.

Groups	Statistics	Sig.
Experimental group	0.966	0.178
Control group	0.957	0.071

Table 6: Results after the post-test for the Shapiro-Wilk test

The independent t-test was used to test the hypothesis that there was a statistically significant difference in the mean post-test scores between the experimental and control groups. The results of the independent sample t-test and descriptive statistics for the mean post-test scores of the experimental and control groups are shown in Tables 7 and 8.

Groups	N	Mean	Std Dev	Minimum	Maximum
Experimental group	47	7.580	1.237	4.25	10.0
Control group	49	6.776	1.468	3.50	9.50

Table 7: Descriptive statistics of post-intervention scores

The experimental group's mean score is 7.580, while the control group's is 6.776, according to the statistical analysis of post-test results in Table 7. The experimental group's standard deviation of data dispersion is 1.237, while the control group's standard deviation is 1.468. The post-test mean equality of scores for both groups was tested using an independent t-test. Table 8 shows the test results.

t-test			
df	t Stat	Sig. (2-tailed)	Mean Difference
94	2.897	0.002	0.804

Table 8: The independent sample t-test results regarding post-test scores

An independent sample t-test was used to test whether the mean difference between the experimental and control groups was statistically significant. As a result, the value (Sig.) is equal to 0.002 (less than 0.050) with a significance level of 0.050 and degrees of freedom df = 94. From this, it can be deduced that the mean score differences between the experimental and control groups are statistically significant. The experimental group was concluded to have a higher mean score in the post-test results than the control group because the mean score of the experimental group in Table 7 was higher than the control group.

Furthermore, based on the Cohen influence scale (2011), the calculated standard mean difference (SMD) is 0.591, which falls within the mean (0.5 to 0.79). Based on these findings, it can be said that the teaching process of the CORE learning model had a moderate effect on the academic performance of the experimental group's students. In contrast, a paired sample t-test was used to assess whether the intervention had improved the group's learning outcomes. The results were distributed immediately before and after the intervention, allowing for a relatively high correlation. Figure 1 illustrates the positive linear correlation between the scores of the experimental group before and after the intervention. In addition, a correlation test was conducted to validate the reliability of the results.

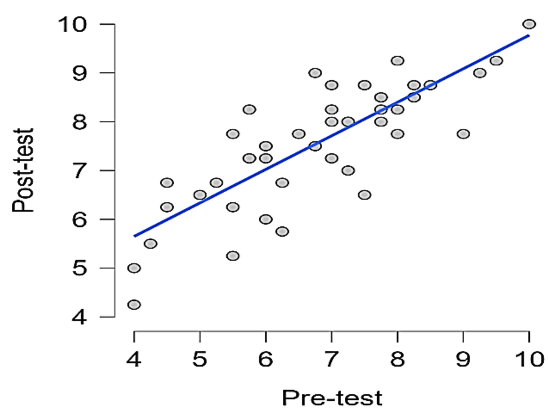


Figure 1: Q-Q plots of the scores of the experimental group before and after the intervention

	N	Correlation	Sig.
Pair of scores before and after the intervention	47	0.810	<0.001

Table 9: Results of the correlation test on the scores of the experimental group before and after the intervention

The results of Table 9 indicate that the calculated Pearson correlation coefficient (0.810) is statistically significant, with an observation value of less than 0.001. In other words, the scores acquired before and after the intervention show a significant correlation. A paired sample t-test was performed, and Table 10 reveals the results. The value obtained is less than 0.05, or < 0.001, suggesting a statistically significant difference in the scores of the experimental group before and after the intervention. In particular, it was determined that there was a difference in mean scores between the pre- and post-intervention periods. The students in the experimental group had higher learning outcomes than before the intervention.

	Mean	Sig.
Pair of scores before and after the intervention	47	<0.001

Table 10: Results of the paired sample t-test using the experimental group's pre- and post-intervention scores

Point range	Frequency	
	Experimental group	Control group
[0;1)	0	0
[1;2)	0	0
[2;3)	0	0
[3;4)	0	2
[4;5)	1	5
[5;6)	4	4
[6;7)	8	9
[7;8)	12	19
[8;9)	17	7
[9;10]	5	3
Sum	47	49

Table 11: Results of the post-test of the experimental and control groups

Table 11 shows that most of the students in the experimental group scored 5.0 points or higher (46/47 students), and no student scored less than 4.0 points. 1/47 students achieved [4;5) points, 4/47 students achieved [5;6) points, 8/47 students achieved [6;7) points, 12/47 students achieved [7;8) points, 17 /46 students achieved [8;9) points, and 5/47 students achieved [9;10] points. Meanwhile, most students in the control group scored from 3.0 points to less than 8.0 points (39/49 students), and only 10/49 students scored more than 8.0 points, of which 3/49 students achieved [9;10] points. Thus, there is a clear difference in the differentiation of scores between the experimental group and the control group. Specifically, the experimental group had an even distribution of scores, concentrated in relatively high score ranges. Meanwhile, the scores in the control group are distributed at many different high and low levels, and there is a difference between the scores, especially since the number of students who achieved scores ranging from 8.0 to higher is relatively small (10/49 students).

Evaluation of math problem-solving abilities

Based on the statistical table of student scores in the experimental and control groups, combined with the problem-solving ability evaluation scale, the ability of the students in the two groups was evaluated according to the level in Table 12:

Capacity component	Expression of students	Experimental group	Control group
1. Detect the problem	1. Identify the problems to solve in the assigned tasks.	Level 3	Level 2
2. Proposed solutions	2. State relevant information.	Level 2	Level 1
	3. Propose solutions to solve the problem.	Level 2	Level 1
3. Problem-solving	4. Perform problem solving.	Level 2	Level 1
4. Evaluate performance results	Results of the self-assess performance.	Level 2	Level 2

Table 12: Evaluation of the problem-solving skills of the students in the experimental and control groups

With the students' performance level in each component's capacity to solve problems, the students in the experimental group were at a higher level than those of the control group. From this, it can be observed that applying the CORE learning model to lesson plans contributed to developing students' problem-solving skills. In general, most of the students in the experimental group did the exercises correctly, presented them closely, discovered the problems, stated the relevant information, and proposed and solved the problems quite well. However, there were still some cases where students discovered problems but provided relevant information and did not solve the problem well. Specifically, some students discovered the problem and could state relevant information, but the conclusion was wrong, and the presentation lacked conditions. Also, most students in both classes had difficulty applying knowledge to solve real-world situations. However, a few students in the experimental group still solved the problems very well through clear and correct presentations and arguments. Furthermore, some students in the control group did not complete this item but had good ideas.

Results of classroom observations

After teaching the lessons on the method of coordinates in a plane, the results of the experimental group and the control group's observations were analyzed and compared based on the factors of the instructional approaches, learning methods, achieved skills, learning content, and students' Student attitude. The observed results are specified in Table 13.

Factors	Experimental group	Control group
Instructional approaches	Applying the CORE learning model.	The teacher gave the main presentation.

	"Connecting" phase: asking questions, making suggestions, making actual contact. "Connecting" phase: Let students participate in activities to form new knowledge. "Reflecting" phase: The teacher summarized the knowledge and gave students exercises to contemplate and reflect on the knowledge they had just learned. "Extending" phase: Students applied the newly learned knowledge to solve real-life problems.	The teacher introduced concepts and formulas on the blackboard, gave examples, and asked the students to do exercises in the textbook.
Learning methods	Individual and group work. Actively explore new knowledge with the support of teachers. Apply the learned knowledge to solve mathematical and practical problems.	Absorb the knowledge that the teacher imparts, work individually, and give opinions. Listen to the lecture and copy the content.
Achieved skills	Teamwork, presentation, and questioning skills. Skills to apply existing knowledge and experience to discover and learn new knowledge. Skills to analyze and generalize learned knowledge. Calculation skills, problem-solving skills.	Skills for personal work, comments, questions and answers, and adjusting math solutions. Interpretation-based memory and problem-solving skills.
Learning Content	Lesson 1: Vector coordinates in the coordinate plane. Lesson 2: Straight lines in the coordinate plane. Lesson 3. Circle in the coordinate plane. Lesson 4: Three conics in the coordinate plane (exercise).	Lesson 1: Vector coordinates in the coordinate plane. Lesson 2: Straight lines in the coordinate plane. Lesson 3. Circle in the coordinate plane. Lesson 4: Three conics in the coordinate plane (exercise).
Student attitude	The classroom atmosphere was cheerful; students actively participated in activities and actively thought about solving the problems that appeared during the lessons.	The class was quiet: The students listened attentively to the lecture and took notes. When the teacher asked questions, only a few students raised their hands to speak.

Table 13: Classroom observation results between the experimental and control groups

The classroom observation results above show that the teaching method according to the CORE learning model in the chapter on the method of coordinates in a plane had achieved some positive results. In terms of content, both classes ensured completeness. However, in the experimental group, students could practice more mathematical skills and abilities than in the control group.

Results of a survey of student opinions

Following the conclusion of the lesson plans in the experimental group, the research team used a Likert scale to administer multiple-choice items to the experimental group's students for their opinions. The purpose of the survey was to find out how students felt about learning using the CORE model, how they felt about the effectiveness of the instruction, and how it helped them develop their problem-solving skills after the intervention. The statistical findings of the responses are as follows.

Items	Totally disagree	Disagree	Neutral	Agree	Totally agree
1. I enjoyed the lessons on the method of coordinates in a plane.	0 0%	0 0%	7 15%	17 36%	23 49%
2. I find that the "organizing" activities in these lessons help me learn more effectively.	0 0%	0 0%	17 36%	14 30%	16 34%
3. I find that the "connecting" activities help me access and visualize new content from the lesson more easily.	1 2%	2 4%	10 21%	12 26%	22 47%
4. I find that "organizing" activities help me to be more interested, actively participate, and contribute to building lessons.	2 2%	1 2%	11 23%	16 34%	17 37%
5. I find that the "reflecting" activities help me to remember new knowledge better.	0 0%	0 0%	7 15%	10 21%	30 64%
6. The "Extending" activities help me practice analyzing and synthesizing related knowledge and better perceiving it.	4 9%	6 13%	7 15%	14 30%	16 33%
7. I am making progress in solving problems related to mathematics.	0 0%	0 0%	18 38%	13 28%	16 34%
8. I want to take similar classes on other topics.	2 4%	2 4%	4 9%	16 34%	23 49%

Table 14: Student feedback on items of the survey

Table 14 indicates that most of the students in the experimental group liked the lessons in the method of coordinates in a plane (85%). This result is consistent with the learning attitudes of the students analyzed above. Some students did not have an opinion on this (15%). Furthermore, most students expressed satisfaction with this learning process (approximately 64%). However, 17 students (36%) still felt that the learning content was vague and unclear. Through classroom observation, it can be determined that the initial cause was the group discussion process that took place quickly during class time, and the tasks were not divided among the group members. However, this can still be seen as a meaningful response to research that shows the effectiveness and feasibility of the CORE learning model.

Table 14 reveals that most of the students in the experimental group thought that the "connecting" activities in the lessons helped them become more interested (73%). Furthermore, there were still three students (6%) who disagreed, and ten students (21%) felt normal with the design of the "connecting" activities; this was also a suggestion for the design of the activities to be more intuitive and fun. Furthermore, the percentage of students in the experimental group who chose the option of totally agreeing was 37% and agreeing was 34%, showing that the students felt interested and comfortable participating in the "organizing" activities. The data in Table 14 confirm that the percentage of students who agreed was very high (85%) in the "reflecting" activities that helped them better understand concepts and the relationships between concepts, and only seven students (15%) felt normal. This shows that the designed "reflecting" activities were appropriate, a prerequisite for promoting problem-solving skills.

Table 14 reveals that 30 students (63%) agreed and totally agreed on training the ability to analyze and synthesize related knowledge and better perceive the relationship between learned knowledge and real-world problems. Furthermore, seven students (15%) felt neutral, and 10 (22%) disagreed or totally disagreed; this suggested designing, engaging, connecting and extending activities with the knowledge learned more closely and closer to practice. This particular item allowed the students to evaluate themselves. According to Table 14, most students (62%) made progress in solving the problems associated with the method of coordinates in a plane. According to Table 14, 39 students, or 83%, wanted to enroll in comparable courses on different subjects. Four students (8%) continued to dislike taking classes like this. Nevertheless, the initial cause of this issue was a fairly challenging topic; some students in the experimental group still did not understand the lesson, as evidenced by the analysis of the experimental group's post-test results, which were better than those of the control group.

DISCUSSIONS

The results of a mixed-method experiment with a control group included group observations, student opinion surveys, results of the pre-and post-test, and qualitative and quantitative analysis of the collected data. The experimental group provided a basis for determining the effectiveness and feasibility of applying the CORE learning model to enhance students' problem-solving skills in teaching the method of coordinates in a plane. The post-test results indicated a significant difference in the students' average scores in the experimental and control groups. Specifically, the t-test between the two scores shows that with sig. (2-tailed) < 0.0001, the experimental group outperformed the control group regarding average score. A correlation test was used to ensure that the students' higher scores in the experimental group were due to the effectiveness of the CORE learning model (and not due to other random factors). The results reveal that the level of correlation was very high between the two scores of the experimental group before and after the intervention, with Pearson's correlation coefficient of the scores before and after the intervention of the experimental group equal to 0.810 and the significance level of 0.810. The significance level of the test is <0.001; this level of correlation is statistically significant. In addition, the Q-Q plots also show that the students' learning outcomes in the experimental group improved when learning the method of coordinates in a plane. The results of the study are consistent with the conclusions of studies on applying the CORE learning model to promote students' problem-solving skills by the authors Arizal et al. (2018), Son et al. (2020) and Irawan and Iasha (2021).

Furthermore, the results of the observation of the experimental lessons show that the students in the experimental group were more positive and proactive in the learning process and received many opportunities to develop real-world problem-solving skills in the lessons learned. The learning activities designed according to the CORE learning model aroused curiosity and desire to learn from most students in the experimental group, and the problems that appeared in the "connecting" activities continued to be presented to the students. Group discussion aimed to generalize and summarize it into a new mathematical object. As a result, students were inspired to be enthusiastic and involved in the learning process. Furthermore, the results of the survey of students in the experimental group showed that the learning efficiency of the students in the experimental group in the lessons was designed according to the four phases of the CORE model (accounting for 64%). In particular, survey questions designed to create conditions for students to self-evaluate the effectiveness of intervention solutions show that learning with the CORE model helped students to learn actively. More extreme (agreement rate is 71%). According to the model designed to learn other topics, 83% of the students still wanted to continue their education. This result is similar to the research results of Ningsih et al. (2019), Khor et al. (2020), Ramadhani (2020), Ardiyanto et al. (2022), Farhan et al. (2022), Atiyah and Priatna (2023) and Suardani et al. (2023).

With the results achieved, this study has some implications. The research results indicate the necessity of organizing and teaching the method of coordinates in a plane based on the level of development of students' problem-solving skills. Also, researchers and educators must focus on providing students with sustainable access to this content to create long-term impact and help them learn the method of coordinates in a space more easily. Therefore, it is necessary to design a consistent and progressive mathematics education program. However, teachers' understanding of mathematical problem-solving skills is important in promoting these skills in students. Hence, mathematics teacher educators should organize training for pre-service and in-service mathematics teachers on the nature of mathematical problem-solving skills and measures to increase these skills for students. These are issues that can be considered in future studies. In addition, the research results show the effectiveness of learning activities designed according to the four important phases of the CORE learning model in enhancing students' mathematical problem-solving skills.

In addition to the results obtained, the study identified some limitations. First, the data collected by the study were not based on long-term experiments. The experimental time was not long (4 weeks), so the experiment could not observe full manifestations of the promotion of students' mathematical problem-solving skills. Therefore, research can have positive and rich results if the experiment is carried out over a long enough period so that the learning activities designed according to the CORE learning model can have a lasting enough impact and consistently improve students' mathematical problem-solving skills. From this, the research team can examine students' progress more clearly. The second limitation is that the scope of the research is restricted to instructing the method of coordinates in a plane instead of implementing it on a wider variety of mathematical topics to clarify the effectiveness of this learning model on the student's learning process. Third, with a relatively small number of students participating in the experiment, 96 students, the research results are local and limited to a narrow research scope. Additionally, because the time for group discussion activities is limited, the knowledge content is too large compared to the class distribution, and not all students can achieve the desired results. Promoting mathematical problem-solving skills requires a long-term process from which the effectiveness of the intervention solution applied is recorded. Through this, teachers must take appropriate measures based on the level of students' mathematical problem-solving skills for mathematical content and teach more effectively.

CONCLUSIONS

The study's conclusions demonstrate how the CORE learning model improves student learning outcomes, problem-solving skills, and attitudes. After analyzing the post-test results, it was discovered that the experimental group outperformed the control group by a significant level (Sig. 2-tailed < 0.001 with $\alpha = 0.05$ and degrees of freedom df = 94). Furthermore, the mean score of the experimental group increased after the intervention (the paired sample t-test revealed a significance level of < 0.001). The lesson plans were designed based on the CORE learning model, which positively impacted their learning outcomes and mathematical problem-solving skills, with an effect size (ES) of 0.591.

Some related research directions are suggested for future studies, including (1) using the CORE learning model to teach different math topics and help students improve other math skills; (2) researching the application of the CORE learning model and GeoGebra in mathematics instruction; (3) researching the influence of certain factors on the development of students' math problem-solving skills; and (4) looking into the long-term effects of using the CORE learning model. However, the research team suggests conducting new studies with sizable sample sizes and extended observation periods to assess the strengths and shortcomings of the CORE learning model in mathematics instruction.

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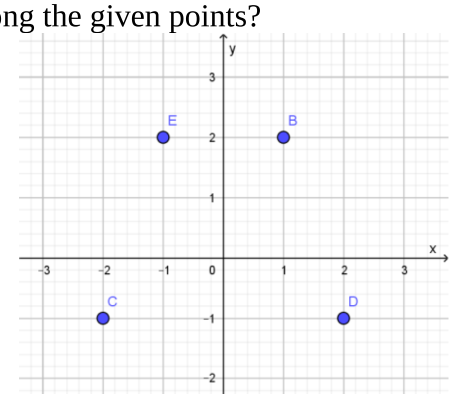
APPENDIX

POST-TEST

Allotted time: 60 minutes

A. MULTIPLE CHOICE SECTION

Item 1. In the coordinate plane Oxy, for points B, C, D, E as shown. How many points have negative coordinates among the given points?



A. 1. B. 3. C. 2. D. 4.

Item 2. Given $\vec{a} = (2;-3); \vec{b} = (-3;4)$. Then:

A. $\vec{a}+\vec{b} = (-1;1)$. B. $\vec{a}+\vec{b} = (5;-7)$.

C. $\vec{a}+\vec{b} = (1;-1)$. D. $\vec{a}+\vec{b} = (1;1)$.

Item 3. A straight line Δ has the following parametric equation: $\begin{cases} x = 2-3t \\ y = 4+2t \end{cases}$. The straight line Δ has a direction vector:

A. $\vec{u}=(2;4)$. B. $\vec{u}=(-3;2)$. C. $\vec{u}=(-3;-2)$. D. $\vec{u}=(2;-3)$.

Item 4. Which of the following points is on a straight line $x-y+3=0$?

A. $(6;12)$. B. $(4;-7)$. C. $(4;2)$. D. $(4;7)$.

Item 5. The center coordinates I , and radius of the circle $(C):(x+1)^2+(y+3)^2=36$ are:

A. $I(-1;3),\ R=6$. B. $I(-1;-3),\ R=6$.

C. $I(1;-3),\ R=36$. D. $I(-1;3),\ R=36$.

Item 6. Which of the following equations is the equation of a circle?

A. $x^2+y^2-x=0$. B. $x^2+y^2+9=0$.

C. $x^2+y^2-2xy-1=0$. D. $x^2-y^2-2x+3y-1=0$.

Item 7. An ellipse $\frac{x^2}{16}+\frac{y^2}{4}=1$ has an axis length equal to:

A. 16. B. 8. C. 2. D. 4.

Item 8. Which of the following equations is the canonical equation of the hyperbola?

A. $\frac{x^2}{16}-\frac{y^2}{9}=1$. B. $y^2=2x$.

C. $\frac{x^2}{16}+\frac{y^2}{9}=1$. D. $\frac{y^2}{16}-\frac{x^2}{9}=1$.

Item 9. A parabola (P) has a focal point $F(3;0)$. The canonical equation of a parabola (P) is:

A. $y^2=3x$. B. $y^2=-12x$. C. $y^2=12x$. D. $y^2=6x$.

Item 10. In the coordinate plane Oxy, the general equation of the straight line Δ passing through two points $A(3\,;-1)$ and $B(1;5)$ has the form:

A. $3x+y-8=0$. B. $3x-y+10=0$.

C. $3x-y+6=0$. D. $-x+3y+6=0$.

Item 11. A given circle $(C):(x-2)^2+(y+2)^2=25$. The equation of the tangent to *(C)* at the point $B(-1;2)$ is:

A. $-3x+4y-5=0$. B. $-x+2y-6=0$.

C. $3x-4y+11=0$. D. $-x+2y+6=0$.

Item 12. In the coordinate plane Oxy, given points $A(1;3)$; $B(-2;1)$; $C(4;2)$. Find the coordinates of point *D* so that quadrilateral *ABCD* is a parallelogram.

A. $D(7;4)$. B. $M(-7;-4)$. C. $D(7;-4)$. D. $D(4;7)$.

Item 13. In the coordinate plane Oxy, write the equation of a circle passing through three points $A(0;4)$, $B(2;4)$, $C(2;0)$.

A. $x^2+y^2+2x-4y=0$ B. $x^2+y^2-2x+4y=0$

C. $x^2+y^2+2x+4y=0$ D. $x^2+y^2-2x-4y=0$

Item 14. Determine the coordinates of the focus of the ellipse $4x^2+9y^2=36$?

A. $F_1\left(-\sqrt{5};0\right), F_2\left(\sqrt{5};0\right)$. C. $F_1\left(-\sqrt{5};\sqrt{5}\right), F_2\left(0;\sqrt{5}\right)$.

B. $F_1\left(\sqrt{5};0\right), F_2\left(0;\sqrt{5}\right)$. D. $F_1\left(\sqrt{5};\sqrt{5}\right), F_2\left(-\sqrt{5};-\sqrt{5}\right)$.

B. ESSAY TEST SECTION

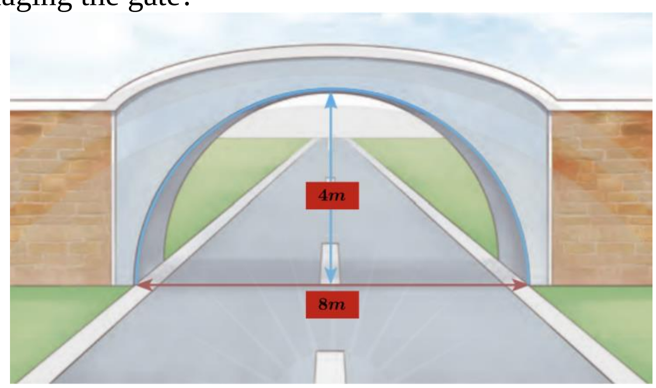
Item 15. Write the general equation of the line Δ, knowing that the line passes through a point $M(0;1)$ and is parallel to a straight line $d: x-2y+3=0$.

Item 16. Write the equation of a circle (C) with center $I(2;-1)$ and tangent to a straight line $\Delta': 3x+4y-12=0$.

Item 17. A semicircular gate is *8m* wide and *4m* high. The road under the gate is divided into two lanes for vehicles entering and exiting.

a) Write an equation to simulate the gate.

b) Can a truck *2.5m* wide and *2.9m* high traveling in the correct lane pass through the gate without damaging the gate?



Visualizing Math Word Problems: Impact on First-Grade Students' Problem-Solving Performance

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Abstract: It is important to work on verbal mathematical problem-solving skills, which is one of the high-level skills with which students have difficulty. Supporting problems with visuals has been suggested by mathematics educators to develop this skill. This study examined the effect of the visualization of mathematical problems on the problem-solving performance of primary school first grade students. The study group consisted of 41 students attending first grade in a public primary school located in a low socio-economic status region of Istanbul. Semi-structured individual interviews were conducted with 8 students. Problem-solving questions developed by the researchers were used for pretest and post-test, the results of which were analysed using the Wilcoxon Signed-Ranks test. For the experimental process, the Visually Supported Problem-Solving Worksheet was used as a data collection tool. Students' views on the implementation were analysed using a content analysis method. The results indicate that the problem-solving worksheet supported with visuals was effective in improving the mathematical problem-solving performance of the students. The majority of the students stated that the implementation contributed to improving their performance and their satisfaction.

Keywords: first grade, mathematics, mathematical word problem, problem solving, visualization

INTRODUCTION

Mathematical problem-solving, a skill that empowers students to apply mathematical concepts in diverse contexts, stands as an integral component of both mathematics and mathematics curricula (National Council of Teachers of Mathematics [NCTM], 2000). Educators hold the conviction that problem-solving not only enhances students' mathematical proficiency but also facilitates the transfer of knowledge to novel and unfamiliar problem-solving scenarios (Mršnik et al.,2023; Putri et.al, 2023). Mathematical word problem solving, with which students have difficulties at all levels of education (Fuentes, 1998; Mayer, 1998; Verschaffel et al., 2000: Verschaffel et al., 2009), requires reading and reading comprehension skills as well as mathematics. Although the roots of mathematics are observable prior to school (Sjoe et al., 2019), this skill, which is described as the heart of mathematics (Cockcroft, 1982), is first presented to children in the first grade of primary school, when students are expected to add and subtract numbers and to solve problems related to these two operations (Ministry of National Education [MoNE], 2018).

When the first-grade objectives of the curriculum related to verbal problem solving of the mathematics are examined, there are found to include objectives such as "Students are able to solve problems that require addition with natural numbers" and "Students are able to solve problems that require subtraction with natural numbers." It has been stated in the explanations regarding these objectives that problems with these operations should only have one step (MoNE, 2018). When first grade mathematics textbooks are examined, it is apparent that addition and subtraction problems are generally presented to students with visuals, and students are asked to solve problems by writing mathematical sentences corresponding to the given images (e.g., Bahçivancı et al., 2021).

The related literature has noted that students' have obstacles related to mathematical problem solving (Fuentes, 1998; Hudson & Miller, 2006; Mayer, 1998; Khoshaim, 2020).). The results of national and international exams of Turkey, which generally include the application of verbal mathematics problems to daily life, also support these findings. Looking at the 2019 results of the Trends in International Mathematics and Science Study (TIMSS), in which the science and mathematics achievements of fourth and eighth grade students are evaluated by the International Association for the Evaluation of Educational Achievement (IEA) over four-year periods, Turkish students performed significantly higher than the mean of the scale (500 points) for the first time. It was also observed that, although they showed higher performance in questions at the application level, they showed lower performance in verbal mathematics problems that require reasoning, which is one of the higher learning skills (MoNE, 2019). In 2018, the results of the PISA (Programme for International Student Assessment), another international assessment exam conducted by the Organization for Economic Co-operation and Development (OECD) in three-year periods were published. Although Turkish students increased their average score to 454 in mathematical literacy, it remained below the average score (459) of the participating countries (MoNE, 2019). In the national high school entrance exam, it has been observed that students have difficulty in solving mathematical problems and can only answer some of the questions (Karip, 2017; MoNE, 2019).

It is important to conduct research to try to solve the problems related to verbal mathematical problem-solving skills in the first years of primary school. Supporting problems with visuals is one of the strategies suggested by mathematics educators in the related literature to improve these skills (Cankoy & Özder, 2011; Krawec, 2014; van Garderen & Montague, 2003). Using images helps to eliminate the dependency on reading skills by making the problem more concrete or realistic. Visualization facilitates verbal problem solving by helping mental model formation (Múñez et al., 2013). For this reason, problems supported by visuals are frequently used in mathematics textbooks (Bahçivancı et al., 2021). Supporting problems with visuals is also an important aspect of Polya's (1957) problem-solving stages, which is emphasized in both the understanding and planning stages. According to Smith (1994), the visualization process matures between the ages of 8 and 11. Visualization occurs both internally (e.g., as a mental image) and externally (e.g., via the individual's use of a pencil or a different material) (Zimmerman & Cunningham, 1991; as cited in van Garderen & Montague, 2003, p. 246).

When the studies in the literature are examined, it appears that the effects of visualization on mathematical problem solving can differ. The reason for this difference may be related to the type of visualization used. Researchers have tried to explain the definition and types of visual images exactly. According to Arcavi (2003, p. 217), "visualization is the ability, the process and the product of creation, interpretation, use of and reflection upon pictures, images, diagrams, in our minds, on paper or with technological tools, with the purpose of depicting and communicating information, thinking about and developing previously unknown ideas and advancing understandings" (as cited in Presmeg, 2020). One of the earlier classification of visualization was defined by Presmeg (1986a, 1986b), who mentioned five different types of visualization: concrete imagery (a "picture in the mind"); kinesthetic imagery (of physical movement, e.g., "walking" several vectors head to tail with the fingers); dynamic imagery (the image itself is moved or transformed); memory images of formulae; and pattern imagery (pure relationships stripped of concrete details). A more recent classification has been made by Elia and Phillippou (2004), who identified four types of visualization: "(a) decorative, (b) representational, (c) organizational, and (d) informational. Decorative pictures do not give any actual information concerning the solution of the problem. Representational pictures represent the whole or a part of the content of the problem, while organizational pictures provide directions for drveawing or written work that support the solution procedure. Finally, informational pictures provide information that is essential for the solution of the problem; in other words, the problem is based on the picture (p. 328)".

Hoogland et al. (2016) presented problems to 31,842 students between the ages of 10 and 20 with depictive representations. They mostly used photographs—defined as decorative according to Elia and Philippou (2004)—and it was found that this had a positive effect on students' problem-solving performance. Cankoy and Özder (2011) found that fifth grade primary school students' performance in solving problems that included decorative visual expression was higher than their performance in solving problems that did not include such visual expression. In addition, adding a visual representation to problems with a context unfamiliar to the students increased their performance in solving these problems. Therefore, visual representations appear to be an important resource in making problems easier for students to solve (Cankoy & Özder, 2011). A study in which the quantities in the problem were visualized found that representational pictures had a strong and significant effect on the mathematical problem-solving performance of kindergarten students for addition and subtraction problems (Elia, 2020).

Matalliotaki (2012) compared oral and visual representation to support mathematical problems presented to five- to six-and-a-half-year-old children and found that visualization is more efficient than oral presentation in helping the children to solve problems. On the other hand, in studies conducted with primary school students, it has been observed that decorative visuals are not supportive in solving mathematical problems and the communication process. In the same study, other types of visualization were effective on mathematical problem-solving performance and communication skills (Elia & Philippou, 2004; Gagatsis & Elia, 2004).

There are also studies that indicate visualization has no effect on mathematical problem-solving performance. For example, Dewolf et al. (2017) found that visualization of problems did not have a positive effect on the performance of students aged 9–12. Indeed, Berends and van Lieshout (2009) showed that adding visuals to a verbal mathematical problem had a negative effect on the performance of students in the 9–10 age group. In their study, Hegarty and Kozhevnikov (1999) emphasized two different types of visuals: schematic and pictorial. They said that these two different types of visualization have different relations with mathematical problem solving. Like decorative visuals, pictorial visualization is defined as only coding the appearance of the object in the problem, while schematic visualization shows spatial relationships in parts of the problem, including spatial transformations. As a result of their study with sixth grade male students, they concluded that schematic visualization was positively related to mathematical problem solving, and pictorial visualization was negatively related. According to the results of another study conducted to examine the visual images used by students at different levels (gifted, medium, and with learning difficulties) while solving mathematical problems, it was concluded that gifted students used statistically significantly more visual spatial visualization than the students in the other two groups. Individuals with learning disabilities used more pictorial representations than other students. In other words, it was concluded that successful problem solving was positively related to visual spatial representations and negatively related to pictorial representations (van Garderen & Montague, 2003).

In a recent study conducted with students attending first grade in primary school (van Lieshout & Xenidou-Dervou, 2018) mathematics problems were supported pictorially, auditory, and both pictorially and auditory, and the effect of this support on student performance in solving addition and subtraction problems was examined. They found that combining pictorial information with auditory information reduced the cognitive load and consequently increased performance. In addition, these effects were observed to be most prevalent in children who scored below the average in the general mathematics test.

While verbal mathematical problem solving is first encountered by students in kindergarten, word problems involving reading comprehension first appear in the first grade of primary school. Solving addition and subtraction problems that require a one-step operation is among the first-year objectives for mathematics. The use of visualization in the first grade, when students are just learning to read, can be effective in helping students to acquire this skill by reducing the cognitive load in mathematical problem-solving. As Smith (1998) has pointed out, the visualization skills of children begin to mature in and after the second grade. In this context, the present research examined the effect of visualizing mathematical word problems on the problem-solving performance of primary school first-grade students. The visuals used in this research were created to explain the mathematical sentences that the students had to write. For this purpose, answers to the following questions were sought. (1) Do mathematical word problems supported by visuals have an effect on students' problem-solving performance? (2) What are the students' views on the implementation of mathematical word problems supported by visuals?

METHOD

Research Design

This research used the one-group pre-test–post-test pre-experimental model, which is an experimental research design (Karasar, 2013). In addition, an exploratory sequential design, a mixed research design, was used to answer the research questions. In exploratory sequential design, quantitative and qualitative data take place in two stages and sequentially. First, quantitative data, which are prioritized for answering the study's questions, were collected and analyzed. In the second phase, qualitative data were collected and analyzed to complement the initial data (Creswell & Clark, 2008).

Study Group

The homogeneous sampling method, a purposive sampling type, was used to determine the study group for this research. The study group consisted of 58 students attending the first grade in a public primary school located in a low socio-economic status region of Istanbul. According to the pre-test results, 12 students were successful in the test (100%) and 5 students did not want to participate in the study and were therefore not included. A total of 41 volunteer students, 25 boys and 16 girls, constituted the study group. The parents of these students were informed by the classroom teacher and their consent was obtained. Semi-structured individual interviews were conducted with 8 (5 boys and 3 girls) students who are volunteer to participate. One student's mathematical achievement was above the average, five were at an average level, and two of them were below the average.

Data Collection Tools

Problem-solving questions (pre-test–post-test)

The problem-solving questions developed by the researchers consisted of six problems related to the addition of natural numbers. The objective of these problems was that "Students will be able to solve problems that require addition with natural numbers" (MoNE, 2018). Based on this objective, three different types of problems requiring addition operations were presented to the students. According to the action in the content of the problem, the problems involving addition operations were grouped under three main headings. The unknown items in these headings are named as result unknown, start unknown, or change unknown (Olkun & Toluk Uçar, 2012), and two of each type were developed for the measurement tool. According to these headings, examples of problems related to the addition of natural numbers are given in Table 1 below.

	Example of Symbolic Model	Sample Problem	Number of Questions
Addition Result Unknown (ARU)	7+4=?	There were 7 fish in Nehir's aquarium. His mother bought 4 more fish for Nehir. How many fish were in the Nehir's aquarium altogether?	2
Addition Change Unknown (ACU)	8+?=15	Alp had 8 marbles. With the marbles his father gave as a gift, Alp had 15 marbles. In this case, how many marbles did his father give Alp as a gift?	2
Addition Start Unknown (ASU)	?+7=16	There were some cookies in the jar. By the time my mom added 7 more cookies, the number of cookies in the jar was 16. How many cookies were in the jar in the beginning?	2

(ARU: Addition Result Unknown, ACU: Addition Change Unknown, ASU: Addition Start Unknown)
Table 1. Examples of the Types of Problems

The rubric used in scoring the problems is as follows: 0 points, If the question is not solved at all; 1 point, if an attempt is made to solve the question; 2 points, if the problem is half solved or a computation error is made; 3 points, if there is only an answer without any calculation; and 4 points, if the question is completely correct. In this case, the minimum score that students can get is 0, and the maximum score is 24.

Expert opinions regarding the problem-solving questions and scoring were obtained from one primary school teacher, one mathematics educator, and one specialist with a doctorate in primary education. An expert opinion related to the language and expression of the problems was sought from an expert with a doctorate in Turkish education. This tool was used as both a pretest and a post-test. It was developed according to the expert opinions sought, and a pilot study was carried out with three students who are not in the study group. At the end of the pilot study, the tool was finalized. For the reliability of the problem-solving questions, the answers given by the students were scored according to rubric and the reliability coefficient between the raters was calculated. Reliability among researchers was calculated according to Miles and Huberman's (1994) formula (reliability = agreement / (agreement + disagreement) × 100), which takes into account the number of agreements and disagreements between the researchers. The inter-rater reliability coefficient was calculated as .97. A consensus was reached by the researchers for the questions scored differently. The Cronbach's alpha reliability coefficient for the reliability of the test was calculated as .83.

The visually supported problem worksheet

The visually supported problem worksheet developed by the researchers consisted of 10 mathematical word problems supported by visuals. The visuals illustrated in the worksheet are used to explain the mathematical sentence that the student has to write. Because the students' ARU (Addition Result Unknown) average was high (6.06 out of 8), it was decided that the worksheets supported by visuals should consist of 5 ASU (Addition Start Unknown) and 5 ACU (Addition Change Unknown) problems. Students were asked to solve the questions and fill in the boxes under each problem on the worksheet. The visuals were illustrated by a visual design expert. For the validity of the problem worksheet supported by visuals, expert opinions were obtained from one primary school teacher, one mathematics educator, one specialist with a doctorate in primary education, and one education technologist. An expert opinion related to the language and expression of the problems was sought from an expert with a doctorate in Turkish education. The answers given by the students to the worksheet were scored with the rubric prepared for the pre-test (Problem-Solving Questions). The Cronbach's alpha reliability coefficient, which checked the reliability of the questions, was calculated as .97. Examples of change unknown and start unknown problems with worksheet visuals are given in Figure 1.

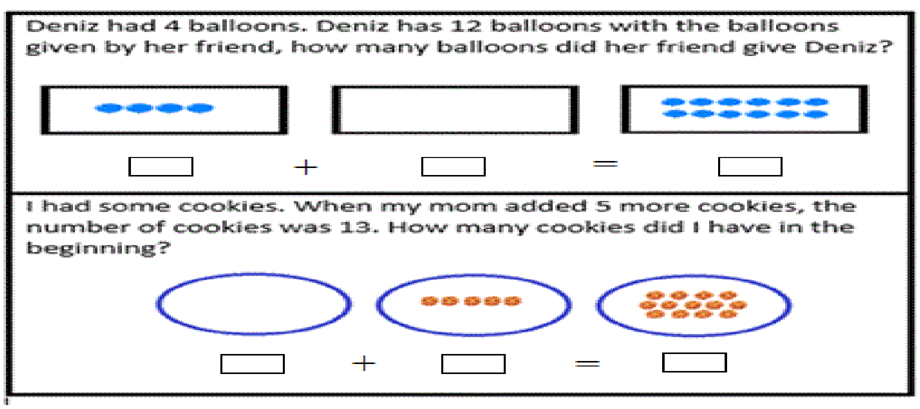

Figure 1. Examples of Change Unknown and Start Unknown Problems.

Interview questions

A semi-structured interview form with three questions was prepared by the researchers to get the students' views on the implementation. The opinions of a classroom teacher, a mathematics educator, and an expert with a doctorate in primary school education were obtained to assess the validity of the interview questions, and the interview form was finalized with three questions. With the necessary permissions, the interviews were audio-recorded. While the audio recordings were being analyzed by the two researchers, all of the words, pauses, and exclamations of the participants were included and converted into a written transcript. One of the audio recordings was randomly selected and transcribed separately by the researchers and compared. The similarity in the transcripts was determined as 100%. Voice recording analyses were performed separately by two researchers, and the inter-coder reliability coefficient was calculated as 98%. A consensus was reached by discussing a small number of encodings that were thought to be different.

Data Collection Process

The data collection process started with the application of the Problem-Solving Questions (pre-test) to the students by one of the researchers in the middle of the spring semester. The students were given 25 minutes for the six questions in the pre-test. According to the results of this test, 12 students were successful (100%), and 5 students who did not come to school on the application day were not included in the study. A Visually Supported Problem Worksheet was prepared, consisting of questions with change unknown and start unknown, since the remaining 41 students had a success rate greater than 60% for the result unknown questions. One of the researchers divided the 41 students into two groups and applied the Visually Supported Problem Worksheet over 40 minutes in two separate lessons. The researcher explained to the students that the questions in the prepared worksheet were supported by visuals, and the students answered the questions individually. A post-test was given to the students 3 weeks after the implementation. During these three weeks, students worked on data analysis and measurement issues in the curriculum. During this period, no problem solving studies involving addition and subtraction of natural numbers were carried out. To get students' opinions on the application, semi-structured interviews were conducted with 8 students who were not successful in the pre-test but were successful in the post-test and agreed to be interviewed voluntarily. In order not to distract the students, the interviews were carried out individually by one of the researchers in a room outside of the classroom. Each interview lasted approximately 15 minutes.

Data Analysis

Before testing whether there was a significant difference between the students' pre-test and post-test problem-solving scores, it was tested whether the distribution of the data was normal. Because the data were not normally distributed, the Wilcoxon Signed Rank test was used to compare the students' pre-test and post-test scores. The opinions of the students about the intervention were analysed using content analysis. Instead of the names of the students, numbers were given to the students, and they were coded as Student (S; e.g., the code S5 means student number five). The mathematics achievement of one of these students was above the average, five of them are at an average level, and two of them were below the average according to the opinion of the teacher who is also one of the researchers of this study. During the interviews, three questions were asked about the intervention. The codes and frequencies of the answers given by the students to the questions and examples from the answers of the students are explained in order.

RESULTS

The findings related to answering each of the research questions are presented separately. Before looking for an answer to the first research question—Do mathematical word problems supported by visuals have an effect on students' problem-solving performance? —the distribution of the data was checked, and it was concluded that the distribution of the data was not normal for both pre-test and post-test. Before the Wilcoxon Signed-Ranks test, the descriptive statistics for both pre-test and post-test scores are included and explained in Table 2.

	n	Min.	Max.	X	s
ACU - pre-test	41	2.00	8.00	2.78	1.75
ACU - post-test	41	0.00	8.00	4.39	2.84
ASU - pre-test	41	0.00	8.00	2.59	1.86
ASU - post-test	41	0.00	8.00	4.07	2.71
ARU - pre-test	41	1.00	8.00	6.07	2.45
ARU - post-test	41	5.00	8,00	7.46	1.00

Table 2. Descriptive Statistics for the Students' Problem-Solving Performance

Examination of the results shown in Table 2 reveals that scores improved from pre-test to post-test for the problem-solving questions. The average score was 6.07 for ARU in the pre-test, which rose to 7.46 in the post-test. The pre-test ACU average was 2.78, which rose to 4.39 in the post-test, and the pre-test ASU average was 2.59, which rose to 4.07 in the post-test. The Wilcoxon Signed-Ranks test results on whether students' problem-solving skills differed significantly before and after the intervention are shown in Table 3.

	n	Mean rank	Sum of ranks	z	p

ARU(pre-test post-test)	Negative ranks	4a	7.63	30.50	−2.98	0.003
	Positive Ranks	17b	11.79	200.50		
	Ties	20c				
ACU (pre-test − post-test)	Negative Ranks	7d	7.86	55.00	−2.54	0.01
	Positive Ranks	16e	13.81	221.00		
	Ties	18f				
ASU (pre-test − post-test)	Negative ranks	4g	7.63	30.50	−3.15	0.002
	Positive Ranks	18h	12.36	222.50		
	Ties	19i				

Table 3. Wilcoxon Signed-Ranks Test Results for Problem-Solving Test

The results of the analysis shown in Table 3 indicate that there is a statistically significant difference between the pre-test and post-test among the students participating in the study for all three problem types (result unknown, $z=-2.98$, $p < 0.01$; change unknown, $z=-2.54$, $p < 0.05$; and start unknown, $z=-3.15$, $p < 0.01$). When the mean rank and totals of the difference scores are considered, this difference appears to be in favour of the positive ranks—that is, the post-test score.

The second research question was: What are the students' views on the implementation of mathematical word problems supported by visuals? The analyses of the semi-structured interviews with eight students are explained below, including the codes and frequencies of the answers given, as well as example answers. The answers to the question of whether the visuals provided with the problems contributed to problem-solving are given in Table 4.

View		f	Student Number
Contribute	understand the problem more easily	3	S1, S4, S7
	solve the problem by counting the images	3	S3, S6, S8
Did not contribute	confused me	1	S5
	made it difficult for me to solve the problem	1	S2

S1, S2, S3, S7, and S8 average; S4 and S5 below average; S6 above average.

Table 4. Contribution of the Visuals to Problem Solving

In response to the first interview question, six students stated that presenting the problems with visuals contributed to the solution. Three of these students stated that the reason for the contribution was that the visuals made it easier to understand the problem, while the other three stated that they solved the problem by counting the images and benefited in this way. One of the two students who stated that they saw no contribution stated that it confused them, and the other stated that it made it difficult to solve the problem. One of the students who stated that they saw no contribution but gave more correct questions when given the visuals had low mathematics achievement and the other an average level. Some of the example answers include: "I did it by looking at the balloons and cookies given with the problems. These helped me to understand" (S1); "I counted the items given while solving the problem. Counting it that way was easy. I did it" (S6); and "[the] cookies and pencils confused me while solving the problem. I did not understand the problem" (S5). Based on these results, it appears that the students thought that presenting the problems with visuals contributed to more easily finding the solution to the problem. The opinions of the students regarding the presentation of the problems with the visuals are given in Table 5.

View	f	Student Code
I like that being given the images	6	S1, S3, S4, S6. S7, S8

I didn't like being given the images	2	S2, S5

Table 5. Student Views on Giving Problems with Visuals

In the student interviews, six students stated that they were satisfied with the presentation of the problems with the visuals, while two students were not satisfied. Sample answers included: one student said, "I like the fact that there are pens and cookies within the problems" (S1), while another student said, "I didn't like having visuals within the problems" (S5). It can thus be said that the students were mostly satisfied with the presentation of the problems with the visuals.

Finally, the opinions of the students regarding the use of visuals in future problems they may solve and their reasons are given in Table 6.

Preference	Reason	f	Student Code
I prefer it to be given with images.	It is easy to solve the problem when the problem is presented with visuals.	4	S1, S3, S7, S8
	When the problem is presented with visuals, it provides support for solving the problem.	2	S4, S6
I do not prefer it to be given with images.	When the problems are presented with visuals, the solution to the problems becomes even more difficult.	1	S2
	When the problems are presented with visuals, it becomes difficult to understand the problems.	1	S5

Table 6. Reasons for Preferences Related to Visually Presented Problems

According the results displayed in Table 6, six students stated that they preferred see the problems with visuals, because it provided ease of problem solving (n=4) or supported problem solving (n=2). Two students did not prefer problems with visuals and argued that the visuals make the solution and understanding difficult. For example, one student said, "It is very nice that problems are presented like this, and they are solved very easily. I would like it to be like this" (S7), while another student said, "I want it to be plain. When there are visuals, I get confused" (S5). Based on the overall results, it appears that most of the students prefer to see the problems presented with visuals.

DISCUSSION AND CONCLUSIONS

This study examined the effect of visuals to support mathematical word problems on the problem-solving performance of first grade students. First, in this context, addition problem-solving questions containing three different problem types (ARU, ACU and ASU) were applied to the students as a pretest. When the student answers were examined, it was found that the mean score for ARU was 6.07 (out of 8), while the means for ASU and ACU were much lower (2.78 and 2.58, respectively, out of 8). Prior studies have found that students are generally more successful in ARU problems than other types of addition problems (Peterson et al., 1989; Tarım, 2017). In a study conducted with children attending kindergarten and primary school in the United States, all children were successful in problems with ARU. In the problems with ACU, 61% of the kindergarten children and 56% of the first-grade children were successful, and in the ASU problems, 9% of the kindergarten students and 26% of the first grade students were successful (Riley et al., 1983 as cited in Erdoğan and Özdemir Erdoğan, 2009, p.39). Because the students in the present study already appeared, as in other studies, to be confident with ARU problems, the focus of the worksheets supported with visuals was on ASU and ACU problems.

According to the post-test results, the problem-solving worksheet supported with visuals appeared to be effective in improving the performance for both ACU and ASU problems. In fact, although ARU problems were not used in the worksheet, an increase was also observed in the students' ARU performance in the post-test. The students' views on the implementation also support this finding. According to the results of the interviews with eight students (who were not successful in the pre-test but succeeded in the post-test), most of the students (n=6) said that presenting the problems with visuals contributed to solving the problem, and they were pleased with the visual presentation of the problems, which facilitated finding the solution. Although the mathematical problem-solving performance of other the two interviewed students improved, they expressed a negative opinion about the intervention. Despite the negative opinions of these two students, the fact that their performance improved may indicate that the study achieved its purpose.

In a study in which the quantities in the problem were visualized, it appeared that visualization had a strong and significant effect on the performance of kindergarten students in addition and subtraction problems (Elia, 2020). Matalliotaki (2012) has also shown that visualization is more efficient than verbal presentation in solving verbal or visualized problems presented to five- to six-and-a-half-year-old children. Similarly, in studies conducted with primary school students, it has been observed that all kinds of visuals, except decorative ones, help mathematical problem solving and communication processes (Elia & Philippou, 2004; Gagatsis & Elia, 2004). There are also studies stating that even adding decorative visuals to the problems may contribute to student performance. For example, 31,842 students aged 10–20 were presented with decorative representations—mostly photographs—and it was found to have a positive effect on their problem-solving performance (Hoogland et al., 2018). Similarly, Cankoy and Özder (2011) have argued that adding decorative visuals to problems with a context that students are not familiar with increases student performance in solving these problems.

According to the related literature, it has been seen that the ways to visualize problems differ from each other, and this affects the study results. In their study with sixth grade students, Hegarty and Kozhevnikov (1999) concluded that (decorative) pictorial visualization and mathematical problem solving were negatively related, while schematic visualization was positively related to mathematical problem-solving success. Dewolf et al. (2017) also found that adding visuals to mathematical word problems did not have a positive effect on the performance of students aged 9–12. In a similar study, Berends and van Lieshout (2009) showed that visualization had no effect on the problem-solving process for students aged 9–10, and may even slow it down. In addition, there are also studies emphasizing that this kind of (decorative) visualization is negatively related to mathematical problem-solving success (Hegarty & Kozhevnikov, 1999; van Garderen & Montague, 2003). Although the results of the studies on problem visualization differ from each other, it can be said that visualization, which contributes to the understanding and solution of the problem, has a positive effect on the performance in solving mathematical word problems.

To increase students' mathematical word problem-solving performance, it is important to use visualization, especially in the first grade, when students first encounter word problems. The results of this research indicate that adding visuals that model the mathematical sentence necessary to solve the word problems improves student performance. In fact, when we look at the books prepared by the MoNE, such examples are often included (e.g., Bahçivancı et al., 2021). However, problems supported by visuals related to ACU and ASU word problems as given during the implementation process in this study are not encountered in MoNE textbooks (Olkun & Toluk Uçar, 2012; Peterson et al., 1989; Tarım, 2017). It is important to have questions supported by visuals that include all types of additional problems. This may be one of the reasons why students have difficulties, especially in ACU and ASU type problems. Because there may be difficulties in reading comprehension in the first grade, giving word math problems with visuals can reduce students' cognitive load. Future studies should consider monitoring the change in students' cognitive load. In addition, studies in which the control group is included in testing the effectiveness of the study and which has a longer duration could also be recommended for future research.

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Geometric Reasoning to Reinventing Quadratic Formula: The Learning Trajectory on Realistic Mathematics Education Principles

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Abstract: This study aims to establish a knowledge base on how to support students in learning. We develop an initial hypothetical learning trajectory by formulating learning activities and predicting the development of students' thinking and understanding. The methodological framework employed in this study is design research, which seeks to generate actionable knowledge for achieving various educational goals through design. The design process incorporates the Hypothetical Learning Trajectory (HLT) thought experiment and the teaching experiment based on the six Realistic Mathematics Education (RME) principles. By integrating individual learning activities with the context and principles of the RME approach, students can construct their knowledge and rediscover the quadratic equation formula. Through comparing the HLT with observed learning outcomes, we redesign the process, revise our HLT, and provide answers to research questions regarding the attainment of specific learning objectives. Similar to our retrospective data analysis, we create a revised HLT to reinforce the concept of the completing the square and reinvent the quadratic formula.

Keywords: Quadratic formula, geometric reasoning, realistic mathematics education, design research

INTRODUCTION

Students struggle to solve quadratic equations, for they do not adequately develop their understanding of the concept, leading to the lack of justification of their knowledge (Allaire & Bradley, 2020; Fachrudin et al., 2014; Gözde & Kabar, 2018; Thomas & Mahmud , 2021; Utami & Jupri, 2021; Yahya & Shahrill, 2015; Zakaria et al., 2010). These may include struggles in solving symbolic problems, challenges in manipulating operations, and weaknesses in mastering related concepts, such as arithmetic, which we provide further details in the literature review section below. Furthermore, students also encountered challenges with comprehending the quadratic formula as a viable approach to solving quadratic equations (Mahmood, 2021; Picciotto, 2008; Yahya & Shahrill, 2015; Zakaria et al., 2010). The quadratic formula has long been a fundamental component of the introductory Algebra course. However, it is unfortunate that students are introduced to quadratic formulas as their initial encounter with relatively complex formulas they memorize (Loh, 2019). Many often study it as a systematic alternative to the guess-and-check method, limited to factoring certain quadratic equations. These circumstances contribute to our assumption that there is a lack of understanding regarding the construction and utilization of the quadratic formula.

For many students, developing algebraic skills in the classroom often marks the beginning of their sense of mathematical inadequacy. This is because students frequently struggle to transition from arithmetic to algebra (Star et al., 2015). Moreover, they find formalistic or early abstract approaches challenging and uninteresting, whereas delving into context-related subjects proves more meaningful (Wittmann, 2021). Such difficulties also arise when learning quadratic equations and quadratic formulas, which are more effectively understood when presented with a geometric approach (Allaire & Bradley, 2020; Fachrudin et al., 2014; Maharaj & Takisa, 2012). Besides, from a historical standpoint, the foundation for solving quadratic equations is rooted in geometric principles (Güner & Uygun, 2016; Hungerbühler, 2020; Irving, 2020; Mehri, 2017; Milne, 1927; Owen, 1991). In his book “Hisob Al-Jabr wa'l Muqabalah,” Al-Khwarizmi (in Ben-Ari, 2022; Hungerbühler, 2020; Mehri, 2017) describes the geometric proof of solving quadratic equations. It has also been popular that students use the Babylonian geometry method to solve quadratic equations, a concept identified by J. Høyrup as Naïve Geometry (Høyrup, 1990; Radford & Guérette, 2000).

In the discursive processes of expanding knowledge, explaining concepts, and providing proof, Duval (1998) presents a cognitive model of geometric reasoning. According to this model, visualization can assist reasoning by aiding in the search for evidence. The model comprises three distinct cognitive processes: visualization, construction (using tools), and reasoning. Geometric reasoning, as described by the figural concept, involves the interaction between two aspects: figural and contextual. Therefore, geometric reasoning facilitates student learning by allowing them to expand their knowledge through visualizing and exploring geometric situations and engaging in construction processes. Furthermore, as geometric reasoning enables students to deduce and draw conclusions based on the properties of geometric shapes, this approach can be utilized in student learning to help them rediscover the quadratic formula. In practice, we can present instructional designs incorporating integrated learning activities, enabling the application of geometric reasoning throughout the student learning process.

The domain-specific instruction theory Realistic Mathematics Education (RME, den Heuvel-Panhuizen, 2020; Freudenthal, 1973; Gravemeijer, 2020; Gravemeijer & Stephan, 2002; Treffers, 1987) suggests that students learn mathematics most effectively when actively building their understanding through problem-solving activities and mathematical discourse. RME theory provides a broader perspective and meaning on real-world problems, going beyond the challenges students encounter in everyday life. Instead, students are presented with problem situations they can relate to, such as the imaginative realm of fairy tales or the formal world of mathematics, as long as the problem holds experiential significance. This principle, referred to henceforth, pertains to how geometric approximation is presented while preserving all other RME principles.

This study aims to contribute knowledge on supporting students in learning. In this study, we present a novel approach to geometric reasoning by focusing on the use of not only visual but interactive squares. This approach deviates from the previous existing procedure, which relied on a geometric approach and visualization of the quadratic formula using the static completing the square procedure (Fachrudin & Putri, 2014; Utami & Jupri, 2021; Zakaria et al., 2010) . We emphasize the importance and urgency of fostering independent student learning as the issue of quadratic equations, particularly the quadratic formula, persists. We encourage students to develop their understanding of mathematics by formulating quadratic equation formulas through teaching experiments guided by RME principles. Initially, we constructed a hypothetical learning trajectory (HLT, Akker et al., 2006; Bakker & Smit, 2018; Simon, 1995) by formulating learning activities and predicting how students' thinking and understanding would evolve. This HLT incorporates assumptions about students' potential to develop quadratic formulas through approaches that facilitate geometric reasoning and how teachers would support them in rediscovering quadratic equation formulas. Subsequently, the modified HLT was derived based on the conclusions drawn from a retrospective analysis of the teaching experiments.

LITERATURE REVIEW AND THEORETICAL FRAMEWORK

Challenges Faced by Students in Quadratic Equations and the Quadratic Formula

In learning situations, several problems related to quadratic equations have been identified. Despite facing difficulties in solving symbolic quadratic equations and quadratic word problems, students demonstrated better performance when dealing with symbolic equations as compared to word problems (Szczerba & Krygowska, 2007). Students' challenges in solving symbolic problems stem from arithmetic and algebraic manipulation errors. Gözde & Kabar (2018) revealed that students had a limited understanding of the quadratic equation concept and were predominantly influenced by the idea of factoring. French (2002) discovered that common mistakes made by students included assuming that $(a + b)^2$ is equal to $a^2 + b^2$. Zakaria et al., (2010) found that most students made errors in transformation and process skills while solving quadratic equations. These errors were attributed to weaknesses in grasping fundamental concepts such as algebra, fractions, negative numbers, and algebraic expansion. Yahya & Shahrill (2015) conducted research that analyzed students' errors in solving quadratic equations, highlighting patterns and causes such as selecting an incorrect multiplication factor when factoring quadratic expressions. (Akgul & Yilmaz, 2023) discovered that advanced participants struggled with interpreting the square root of a squared number when presented in exponential form.

In this study, we examine various sources of error, including the inability to recall the correct quadratic formula while solving quadratic equations due to a flawed comprehension of the formula. Additionally, we address the issue of incorrectly manipulating operations when attempting to change the subject of a given formula.

Geometric Reasoning

Fischbein (1993) argues that a geometrical figure, such as a square, is not only a concept but also an image. He states that these figures possess both conceptual and figural characteristics. Figural properties pertain to mental representations of space, unique to images, and not found in ordinary concepts. Thus, all geometric figures are mental constructions with conceptual and figural properties. Fischbein's notion of figural concepts suggests that geometric reasoning involves the interaction between these two aspects: figural and conceptual. Another extensively researched model describing the development of geometric reasoning is van Hiele's model of thought in geometry (Usiskin, 1982; Van Hiele, 1986). Additionally, we incorporated Duval's framework ((Duval, 1998)) in this study since it encompasses a process for knowledge extension, accommodating the formation of other understandings, including algebra. The framework proposes that geometric reasoning involves three cognitive processes that serve specific epistemological functions. These processes are as follows:

1. Visualization processes, such as visually representing a geometrical statement or heuristically exploring a complex geometrical situation.
2. Construction processes, involving the use of tools.
3. Reasoning processes, particularly discursive processes for knowledge extension, explanation, and proof.

Duval emphasized that the different processes involved in geometry can be conducted independently. For instance, visualization is not necessarily dependent on construction, and the construction process relies solely on the relationship between relevant mathematical properties and the tools' limitations, even if it leads to visualization. While visualization can assist in reasoning and evidence finding, it can sometimes be misleading. Nevertheless, these three cognitive processes are closely interconnected.

By visualizing various geometric representations of the quadratic formula, students are expected to develop a better understanding of the underlying mathematical properties and relationships. Through this reasoning process, students can approach the study of the quadratic formula with a more comprehensive and integrated perspective, thereby enhancing their understanding of the topic.

The RME Theory

RME is a domain-specific instructional theory for mathematics education (den Heuvel-Panhuizen, 2020; Freudenthal, 1973; Gravemeijer, 2020; Gravemeijer & Stephan, 2002; Treffers, 1987). Initially conceptualized by Treffers (1987), the theory aimed to distinguish the realistic approach from other approaches, such as structuralistic, empiricistic, and mechanistic. Treffers intended RME to be a descriptive theory but has since expanded to include instructional design heuristics, guided reinvention, didactic phenomenology, and emergent modeling.

The development of RME theory was based on Freudenthal's ideas, which argue that mathematics education must be viewed as a human activity. One of the main principles of RME is guided reinvention, which emphasizes that teachers and assignments should guide students to reinvent mathematics and experience it as a human activity. This heuristic aims to help students develop a deep understanding of mathematical concepts and foster their creativity and problem-solving skills. Another central instructional design heuristic in RME theory is didactic phenomenology. This heuristic is based on Freudenthal's notion that organization is a key characteristic of mathematical activity. Students are encouraged to analyze which phenomena are organized and how they are organized through their mathematical thinking. This analysis can lead to situations that require regulating such phenomena and generating appropriate thinking. The third heuristic in RME theory is the emergent modeling design heuristic, which aims to support the incremental process by which mathematical models and concepts develop together. The central idea of this heuristic is the use of a series of sub-models that collectively strengthen the overall model. This overarching model evolves from an informal model of mathematical activity to a more formal model of mathematical reasoning. These three heuristics are also represented in six core principles: activity, reality, level, intertwinement, interactivity, and guidance. (Heuvel-panhuizen et al., 2020).

The adjective “realistic” accurately reflects the approach to teaching and learning mathematics in RME, but it can also be confusing. In Dutch, the verb “zich realiseren” translates to “imagine,” meaning that “realistic” refers more to the idea that students should be presented with problem situations they can imagine rather than the authenticity of the problem. However, this does not diminish the importance of relating mathematics to real-life scenarios. The problem context need not be limited to real-world situations; even fantasy worlds or formal mathematics are suitable as long as they are perceived as "real" by students. It emphasizes the importance of creating a mental connection between students and the problems presented.

We provide comprehensive background information on RME theory and its role in this research. Our discussion emphasizes two key mathematization methods that are integral to RME, the various levels of understanding that shape the learning process, the potential for students to participate in model development actively, and the dynamic nature of models throughout teaching and learning to enhance higher levels of understanding. We apply this foundational knowledge to the specific domain of quadratic equation formulas, examining how geometric models can effectively bolster geometric reasoning in constructing these concepts.

METHOD

Research Design

This study used design research as a methodological framework to develop actionable knowledge. This knowledge was condensed into design principles, conjecture maps, and hypothetical learning trajectories (Akker et al., 2006; Bakker, 2018; Cobb et al., 2017; Confrey & Maloney, 2015 ; Plomp, 2013). Throughout the study, we formulate the findings in the form of a revised hypothetical learning trajectory

The study consisted of three phases: (1) Preparation and implementation of teaching experiments: Anticipatory thought experiments were conducted to envision how the proposed instructional activities could be utilized in the classroom and predict what students might learn through participation. We aimed to anticipate students' learning processes during a pilot experiment. Subsequently, we determined the design of the learning activities and the type of knowledge to be developed in the initial hypothetical learning trajectory (HLT). (2) Teaching experiments—Trials with individuals: Weekly one-hour sessions were held outside of the regular school schedule, where the teaching experiments took place. The process was carefully documented and recorded. A total of six teaching experiment sessions focused on fostering an understanding of quadratic equations by reinventing the quadratic formula. At this stage, the HLT primarily guided the implementation of teaching experiments and data collection regarding the learning process, mechanisms, and the revised HLT. (3) Retrospective analysis: The actual learning process of the students was compared to the HLT. Transcripts were read aloud, and video footage was reviewed chronologically, episode by episode. Using the HLT and research questions as a guide, conjectures about student learning and perspectives were formulated, documented, and tested against other episodes and data materials such as student work, field notes, and assignments.

Participants and Data Collection

We recruited two students, Kenia and Rara (pseudonyms), from different junior high schools (SMP) in Indonesia to participate in one pilot experiment and three teaching experiments. They performed above average compared to their peers based on their semester grades. However, they showed apparent weaknesses in their understanding of mathematics, particularly in algebra and quadratic equations, specifically factoring quadratic equations and quadratic formulas. We outline their initial knowledge in the results section. We selected them as subjects in an assessment of their prior knowledge, which then guided the development of the initial HLT.

The data we present were collected during three separate learning sessions, each involving Kenia and Rara. Prior to the teaching experiments, we engaged in anticipatory thought experiments, imagining how the proposed instructional activities could be used in the classroom and what students might learn from participating. Additionally, video recordings of the teaching experiments and student worksheets serve as primary data that are retrospectively analyzed according to the HLT. We analyze all of these elements simultaneously in the results and discussion section.

Data Analysis

We present and analyze the three phases in parallel order for each activity performed. First, we outline each initial HLT component and then analyze it based on the implementation in the lesson. We examine why and how our design worked and use our findings to evaluate and revise the HLT. The main results of this study depend not on the design itself but on the underlying principles that explain how and why the design works as the goal of the study is preserved.

In addition, we analyze the activities carried out by students based on the three components of the geometric reasoning framework: visualization processes, construction processes (using tools), and reasoning processes. These components are integral to the geometric reasoning experienced by students.

RME as Design Principle and Heuristic

We employ six RME principles as the foundation for our design process, encompassing the HLT thought experiment and the teaching experiment.

To grasp quadratic equations, students participate in a series of learning activities. These activities are practically implemented by utilizing geometric shapes that can be adjusted to meet the objectives of each activity. Furthermore, students actively engage in collaborative exercises with the teacher, demonstrating how geometric reasoning can be applied. This process facilitates the progression of students' comprehension from a situational understanding to a formal knowledge throughout the activities. Additionally, we ensure that each geometry concept is interconnected with a concept in quadratic equations, promoting an intertwined understanding. Lastly, as part of the teaching experiments, we guide students in constructing knowledge of quadratic equations and encourage them to rediscover the quadratic formula as a guiding principle.

TASK, RESULT, AND ANALYSIS

Prior Knowledge of Kenia and Rara.

Prior to engaging in instructional interventions, and to develop the initial HLT, we assessed the mathematical abilities of Kenia and Rara in solving quadratic equations. We provided a quadratic equation, $2x^2 + 4x - 12 = 0$, which would later be used in teaching experiments. Based on the findings and our interviews, conjectures were made about each's knowledge of quadratic equations. Kenia was found to have limited understanding of constructing quadratic formulas and struggled with algebraic operations, but she showed proficiency in geometry. On the other hand, Rara also lacked knowledge of quadratic formulas and focused more on geometric concepts but demonstrated better skills in algebraic manipulation.

Utami and Jupri (2021) discovered that most students used a structure sense strategy, although some were categorized as having partial structure sense. Building upon these findings, the researchers developed HLT, which included instructional objectives, hypothesized learning processes, and task sequences. The following sections will discuss HLT and the experiments in more detail.

Activity 1: Let's Complete the Square.

Activity 1 focuses on solving quadratic equations using diagrams. The approach involves using the concept of equal areas of squares to simplify the equations by stating that the lengths must be equal.

Picciotto (2008) presented how the elements of quadratic formulas can be derived from graphs or images. Using geometric ideas (reshaping into squares) can assist students in performing symbolic operations (Fachrudin et al., 2018; Gözde & Kabar, 2018). In the visualization process, students use the interactive tool to construct the shape find a solution by changing the size of the square on the right and considering its area (Figure 2). Students are invited to consider that in order to create an almost square total area, certain shapes can be rearranged using the small circular button located in the corner, although it may seem as though a piece is missing. This process eventually leads to the completion of the square, and showing how one side of the square equals to the other.

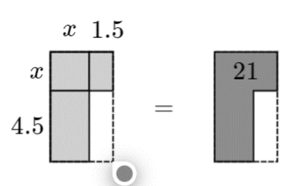


Figure 1. Interactive button

In this learning activity, students are invited explore how to use an interactive tool to solve equations involving squares and their areas. For example, they consider the equation $x^2 = 30$ (Figure 2a) and discuss how to find a number that, when multiplied by itself, yields 30. They note that there is no integer solution to this equation, but a different type of number that works. Specifically, the solution is the side length of a square whose area is equal to 30. The interactive tool helps students visualize this and find an approximate solution to the equation.

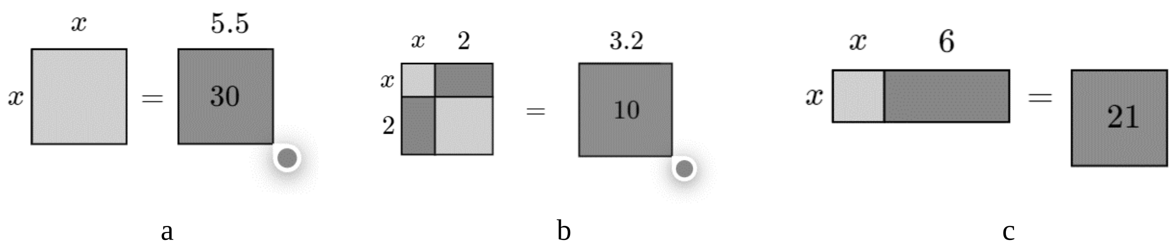


Figure 2. The phase of Activity 1

The activity also covers another problem involving areas and squares of the equation $(x + 2)^2 = 10$ (Figure 2b). Students are also asked to determine the equation represent from the diagram that is not a square (Figure 2c), that eventually become a square. This time they were given multiple choices giving them help that they have to finally emerge conjecture the $6x$ rectangle to be moved half.

Our prediction for Activity 1 is that students will learn how to reason about the area of a square using two different visuals side by side. This will help them determine the value of one side of a square in algebraic notation. By recognizing that the lengths must be equal, they can simplify the problem and find a solution, as shown in Figure 3. However, relying solely on geometry may not be sufficient for all problems. Students may struggle to understand that the area of a square is determined by the product of x, resulting in x^2. They may only estimate the area based on one side of x and one side of the square, which is acceptable since it yields the correct value of x.

Nevertheless, they will also need to comprehend square roots because they will encounter variables rather than numbers at the end of the activity.

In this activity, Kenia first determines the relationship between a square with a side length of x and another square with an area of 30. Initially, Kenia used a circle to adjust the area to 30. Then, we asked what the approximate value of x is. (Note: "R" stands for the researcher, "K" for Kenia, and "A" for Rara).

R: *Please use the mouse to adjust the button to change the size of the square. Based on the current situation, what is the value of* x*?*

K: [Adjusting the button to make the area of the square 30] *Looking at the square with side* x*, the value of x changes as I move the button. Therefore, the value of x is* 5.5.

R: *How do you calculate the area of the square with side* x*?*

K: *The area of the square is* x *multiplied by* x*, which is* x^2.

Meanwhile, Rara gives a more specific conjecture, namely $\sqrt{30}$, which shows how he derived the value of x from an area of 30. Here is how he solved the problem:

A: [Adjusts the button to make the area of the square 30] *The value of* x *is* $\sqrt{30}$*, which is* 5.5.

R: *Can you explain how you arrived at* $\sqrt{30}$*?*

A: *Sure. The area of the square is x multiplied by* x*, which is* x^2*. We know that the area of the square is* 30*, so* $x^2 = 30$*. By taking the square root of both sides, we get* $x = \sqrt{30}$*, which is* *equal to* 5.5.

At this stage, although students arrive at the quadratic form in different ways, they both obtain similar results for the value of x. However, in the next scenario, where the square has an area of $(x+2)^2$, they both make an error in determining the value of x. To resolve this, they manipulate the circle until the area of the square on the right is 10, creating a visual representation of the equation $(x+2)^2 = 10$. From this, they determine that the side length of the square is approximately 3.2 when $x+2$ is equal to the square's side length. By visually inspecting the square, students can see that the value of 3.2 represents the length of one side of the square, which is equivalent to $x+2$ in the other square. Therefore, x can be calculated as approximately 1.2, that is by $3.2 - 2$. We confirm their answer and reinforce the concept of the length of the other side of a square. They arrived at the same conclusion when they realized the relationship between the length of 3.2 in one of the squares and the length of $x+2$ in the other square.

R (to both): *Are you certain that the length of this square is $x+2$ and that x has a value of approximately 3.2?*

K: [pausing to examine the picture] *Uh...no, I apologize. The value of x is actually 3.2-2, which is equal to 1.2.*

Meanwhile, in a separate teaching experiment, Rara states:

A: *It's 1.2,* [pausing briefly to examine the picture] *I just realized.*

R: *How did you arrive at that conclusion?*

A: *Since the length of the square is* $x + 2$*, and we know it equals* 3.2*, we can subtract* 2 *from* 3.2 *to get* x*, which equals* 1.2.

We continue working on determining the equation of the third shape (Figure 2c). One of the challenges students face is dealing with non-square areas and attempting to make them more square-like.

This situation resembles the one observed by Zakaria et al. (2010) in which students demonstrated a tendency to make errors in transformation and process skills while solving quadratic equations. Reshaping a non-square area into a square or a shape that closely resembles a square can aid in the understanding of the relationship between side lengths and areas. To accomplish this, students may need to rearrange some parts of the shape until the total area is nearly a square. Previous research has shown that students often struggle due to the teacher's insufficient focus on fostering a deep understanding of mathematical language and the necessary skills (Zakaria et al., 2010).

In this instance, students are required to adjust the button to create an almost square shape and determine the missing area. Subsequently, they were tasked with determining the equation resulting from the sum of the two squares. During the process, the two students had no trouble reshaping the non-square shape into a more square-like one. However, they encountered difficulties answering the equation question. The following describes the situation experienced by both of them.

K: *I'm not entirely sure, but what is the equation supposed to look like?*

R: *Think about this*: *the area of the square on the left is equal to the area of the square on the right.*

K: [pauses] *So, can I just multiply it and get* $6x^2$*?*

R: *Well, there are two shapes there, so you need to make it correct.*

K: *Let me see, this one* [points to the square with side x] *must be* x^2*, and this one* [points to the rectangle] *is* $6x$*, so it's* $x^2 + 6x$.

R: *Equal to?*

K: *Equal to* 21.

R: *When you finally make it into a square, what is the area of the missing square?* [points at the square]

K: *It's* 9*, from* 3×3.

R: *And what is the total area and the new side length?*

K: *The new side length is* $x + 3$*, so* x *is* 6*. The total area is...* 30.

Rara worked diligently on this case. He immediately started reshaping the rectangle into an almost square shape, and only then did he determine the equation by squaring both sides, resulting in $(x + 3)^2$. However, he encountered the same problem here and was initially confused by the resulting equation. Our approach remains the same, by providing instructions in the form of a left square being equal to a right square.

R: *I give you an idea, the area of square in the left* ***is equal to*** *the one in the right.*

A: *So, it is equal to* 21*?*

R: *Take it carefully, you have your new shape, there is a missing square, no?*

A: *Yes, but I have no idea about the area of the missing square.*

R: *Look at the left, there is a length in it.*

A: .... [pausing briefly], *maybe it is* 9

R: *Why?*

A: 3×3.

It was also noted that a significant number of students obtained an inaccurate final solution. At this point, we need to add instructions to our HLT that explain how the two shapes are related to each other so that students can generate their ideas more effectively. However, both students were generally able to complete this activity well, and we also discovered a new conjecture about the area of $6x^2$ that we had yet to anticipate before. The trajectory of how to complete the square in Activity 1 is illustrated in Figure 3 below. Additionally, we have found that we may need to emphasize to students the importance of "dividing by half" when they modify and reshape a non-square shape into the HLT, as it affects their geometric reasoning in the next activity. To arrange a non-square shape into a square, students must divide it into two parts.

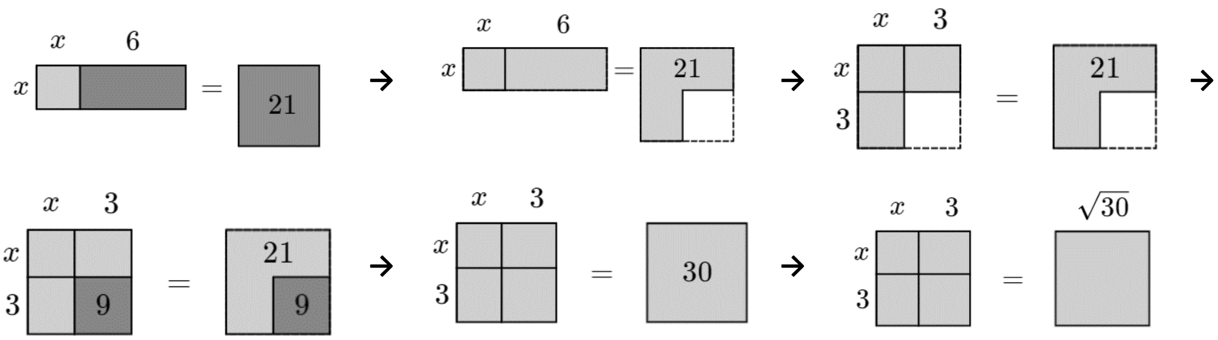


Figure 3. Activity 1 on completing the square

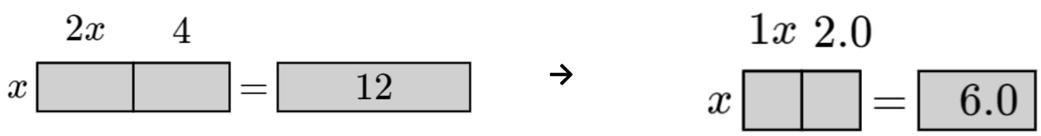
Although students may not realize it yet, we just come up with our goal for solving lots of quadratic equations —completing the square— and reinventing the quadratic formula.

Activity 2: How about different shape?

Activity 2 involves solving an equation that is still quadratic but slightly different from the one they solved earlier, that is $2x^2 + 4x - 12 = 0$.

The purpose of employing these diverse equations is to enhance students' comprehension by training them on more intricate problems, thereby elevating their understanding to a more advanced level. According to Utami and Jupri (2021), the capacity to perceive patterns and solve quadratic equations is more intricate compared to factoring algebraic expressions. In this section, the visualization process remains connected to the preceding activity, namely, the utilization of interactive boxes.

Here, the students are encouraged to apply the same technique they used on the previous problem to solve the current equation. However, this time, the students are given multiple choice options that lead them to divide every rectangle's width by 2, while ensuring that the equation remains balanced (see Figure 4). While manipulating them, students explore these geometric scenarios to ascertain the equilibrium between algebraic and geometric shapes.



A B

Figure 4. Illustration of dividing rectangle into a half.

Upon constructing in this manner, this division modifies the equation, while the value of x remains constant. Subsequently, the instructor directs the students to complete the square as used in the previous task to solve the current equation logically. Finally, the students are expected to approximate the value of x, similar to their approach in activity 1.

During this phase, students may encounter fewer difficulties than before, provided that they have fully understood the entire procedure of activity 1. The potential challenge they might face is when they come across division of the rectangle, prompting them to question why the side of the rectangle is being divided by two (Figure 4). However, we anticipate that this is simply a way to determining the value of x.

We began by posing this task (R stands for the researchers, and K and R for the students, Kenia and Rara). Kenia and Rara started by looking at a non-square shape (Figure 4a). Both felt something was different but they still made the same equation: $2x^2 + 4x = 12$. Our anticipation makes them think with the question whether determining x will be easier or more difficult if what we have is $2x$ instead of x, they have the same answer which is more difficult. So, here are the ideas they have for the problem:

R (to both): *So, what do you think is the solution of this?*

K: *We should make it into a square first, so we may be squaring or dividing two times here. All dividing by* 2.

Meanwhile, Rara sticks with his square root idea.

A: *The length of the side must be* x *because we need to find the square root of* x^2

R: *How do you do that.*

A: *Just…divide them into the half of it.*

They were unable to complete the task correctly or provide an explanation of the steps until the value of x was determined. Based on this analysis, our evaluation in Activity 2 serves more as a guide for what students should do at the beginning, leading to a tendency for the given rectangles to not have a length of x. As a result, students are immediately encouraged to solve these problems without going through interviews.

Furthermore, Kenia presented an interesting idea stating that "I think we may have to divide by two here," indicating that she needs to square the first rectangle to turn $2x$ into x, along with the other value. She also needs to reshape the rectangle with the side x to make it a square. We have this conjecture into our revised HLT.

Finally, the path of how to complete the square in activity 2 is illustrated in Figure 5 below.

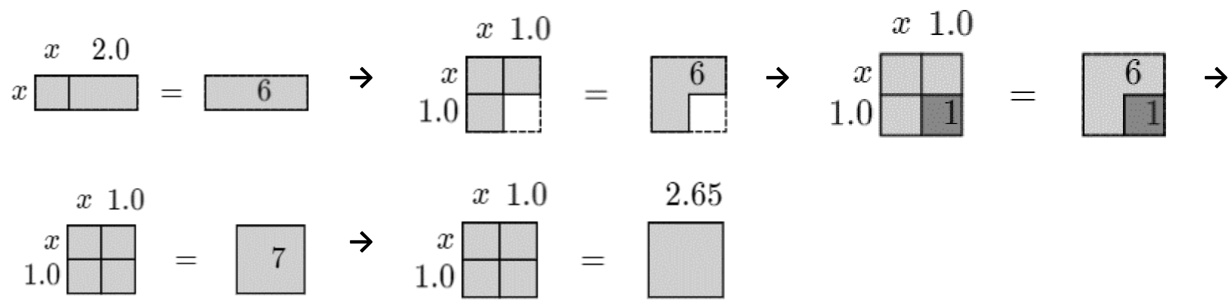


Figure 5. Activity 2 on completing the square

Activity 3: Let's generalize.

Gözde & Kabar (2018) emphasized the significance of knowing and comprehending the formal definition of a quadratic equation to gain a conceptual understanding of the relationships involved beyond mere symbolic calculations when solving quadratic equations. This activity invites students to solve any quadratic equation containing an x^2 term and an x and a constant term. Although Activity 3 requires a deeper understanding, it still relies on the student's ability to complete the square. Students asked to represent all quadratic equations simultaneously by rearranging the equation to the form $ax^2 + bx + c = 0$, where $a, b,$ and c are numbers. The students are then tasked with determining the values of $a, b,$ and c and retracing their steps to solve $ax^2 + bx + c = 0$. Although we presented the algebraic manipulations, we only showed the changes in geometric approach and algebra culminating in the quadratic formula, as the operations are too advanced for their grade level.

Start by subtracting c from both sides and dividing every horizontal length by a (Figure 5), to determine the side lengths of the left side shape when it becomes a square. The students then need to find $x + \frac{b}{2a}$ by splitting the $x + \frac{b}{a}$ rectangle in half. The missing square's side length is then $\frac{b}{2a}$, so its area is $\left(\frac{b}{2a}\right)^2$. Adding $\frac{b^2}{4a^2}$ makes the left-hand side of the equation a square, which lets the students finish solving for x. The full steps are presented in Figure 5.

We conjecture students may not have any difficulty in determining the values of $a, b,$ and c. However, the process of deriving the quadratic formula using a geometric approach may raise some questions for students, particularly in the algebraic expressions involved. Therefore, we aim to assess their understanding by asking about the changes in the geometric and algebraic aspects of the process (see Figure 5).

Both individuals did not encounter any difficulties in determining the values of $a, b,$ and c as they progressed toward the general form of the quadratic equation. Although they initially failed to recognize that they had halved before, in activity 2, both students halved again in this activity. However, they faced an obstacle in the last question. When Rara was asked to determine the length of side $x + \frac{b}{a}$ after it was squared, they responded with $\frac{x+\frac{b}{a}}{2}$, indicating their need to understand the halving concept introduced in the previous activities. When we intervened, they immediately recognized their mistake. As a result, we consider this error to be a form of inaccuracy. On the other hand, Kenia provided the correct answer, which was $x + \frac{\frac{b}{a}}{2}$.

One noteworthy aspect was their realization of how the quadratic formula is derived after knowing that the sides of a square are divided by two (the complete process is shown in Figure 5). We also asked them to share their insights on the process and how they interpreted the plus or minus notation in the quadratic formula.

R: *Did you find this process meaningful? What do you think the plus-or-minus notation means there?*

K: *The value may be either positive or negative.*

R: *The value of what?*

K: *I mean… this operation, basically you can just use the minus one or just the plus one, so after "$-b$" we do it with plus or minus.*

R: *How about using both?*

K: *I don't know.*

Meanwhile, unlike Kenia, Rara has actually understood the meaning of "or" there.

A: [paying attention to the process in Figure 5]

R: *What do you think of it, why does it have* $\frac{b^2}{4a^2}$*?*

A: *That's the multiplication of* $\frac{b}{2a}$ *times* $\frac{b}{2a}$

R: *What do you think the plus-or-minus notation means there?*

A: *It will be used later when determining the final result, use it with plus or minus.*

R: *So, if for example* $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$*, how many operations are there? [pointing at the plus-or-minus* *or operation at the quadratic formula].*

A: *Two, plus and minus*

R: *When you mention "and", does that mean you have to operate both?*

A: *No, only one.*

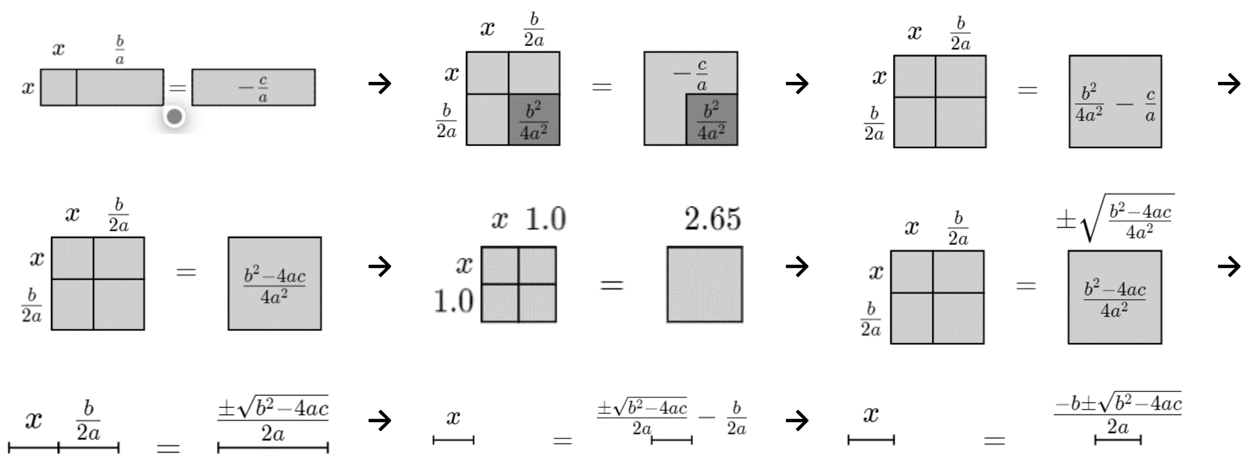


Figure 5. Activity 3 on completing the square and reinventing the quadratic formula

Our analysis indicates that an important aspect of this lesson, algebraic notation, is missing.

Tall et al. (2014) stated that teachers also highlight the quadratic formula, as it is applicable to all quadratic equations. When we present the real form of the area of a square and ask students to determine the length of one side (x), they often assume that finding the value of $x^2 = a$ involves taking the square root. However, there are actually two different results, and we take the positive one because it represents the length of the side of the square. Therefore, we need to add steps related to determining the result of $x^2 = a$ through factorization to follow the process that leads to the positive and negative results. Nonetheless, students may still see the plus-or-minus notation as representing two operations, even though only one of the results is used.

CONCLUSION

Concluding Remarks

Recalling this study aims to examine the impact of learning activities on students' ability to formulate a quadratic equation, the analysis of the z presented above and the explanation of the teaching experiment processes provide evidence of a promising trajectory for students learning. The study design emphasizes students' active engagement in learning and its correlation with quadratic equation concepts. Through individualized learning activities that integrate the context and principles of the RME approach, students can construct their knowledge and reinvent the quadratic equation formula.

In conclusion, activities designed in such a way resulted in students developing their knowledge. The results indicate that students can learn according to the designed learning trajectory. Likewise, the situation that occurs in this study may well be different depending on students' initial abilities.

Trajectory Experience, Conjectures, and the Revised HLT

From the comparison of HLTs and observed learning, we do the redesign process, revising our HLT, and affords answers to research questions that ask how particular learning goals could be reached. Following our retrospective analysis of the data, we created a revised HLT for fostering quadratic equation concept and reinventing quadratic equation formula.

Activity 1 — This activity focuses on solving quadratic equations using diagrams. The approach involves using the concept of equal areas of squares to simplify the equations by stating that the lengths must be equal. Students use the interactive tool to find a solution by changing the size of the square on the right and considering its area. The two shapes are related to each other so that students can generate their ideas more effectively. Students are invited to explore how to use an interactive tool to solve equations involving squares and their areas. For example, they consider the equation $x^2 = 30$ and discuss how to find a number that, when multiplied by itself, yields 30. Next, the activity covers another problem involving areas and squares of the equation, that is $(x + 2)^2 = 10$.

Our prediction for Activity 1 is that students will learn how to reason about the area of a square using two different visuals side by side. This will help them determine the value of one side of a square in algebraic notation. By recognizing that the lengths must be equal, they can simplify the problem and find a solution. However, relying solely on geometry may not be sufficient for all problems. Students may struggle to understand that the area of a square is determined by the product of x, resulting in x^2. They may only estimate the area based on one side of x and one side of the square, which is acceptable since it yields the correct value of x. Nevertheless, they will also need to comprehend square roots because they will encounter variables rather than numbers at the end of the activity.

Activity 2 — In this activity, the aim is to solve a equation that is still quadratic but slightly different from the one they solved earlier, that is $2x^2 + 4x - 12 = 0$. Students are asked to apply the technique they used in the previous activity to solve it. The student presented with options that require them to divide every rectangle's width by 2 while maintaining a balanced equation. After that, the student is guided to use the same method of completing the square as they did in the previous activity to solve the current equation. They will be expected to approximate the value of x as they did in the first activity. This will help them consolidate their understanding of the process and how it works.

During this activity, the student may encounter fewer difficulties than before, especially if they have fully understood the entire procedure of the previous activity. However, the potential challenge they might face is when they come across the division of the rectangle, prompting them to question why the side of the rectangle is being divided by two. Therefore, the student will need to ask the instructor to explain the rationale behind this step to ensure that they understand its purpose.

Activity 3 — This activity invites students to solve any quadratic equation containing an x^2 term and an x and a constant term. Activity 3 still relies on the student's ability to complete the square. Students are asked to represent all quadratic equations simultaneously by rearranging the equation to the form $ax^2 + bx + c = 0$, where $a, b,$ and c are numbers. The students are then tasked with determining the values of $a, b,$ and c and retracing their steps to solve $ax^2 + bx + c = 0$. Students only showed the changes in geometric approach and algebra culminating in the quadratic formula, as the operations are too advanced for their grade level. We conjecture students may not have any difficulty in determining the values of $a, b,$ and c. After that, students doing steps related to determining the result of $x^2 = a$ through factorization need to follow the process that leads to the positive or negative results.

Students may still see the plus-or-minus notation as representing two operations, even though only one of the results is used. We acknowledge that this may raise some questions for students, particularly in the algebraic expressions involved. The process of deriving the quadratic formula using a geometric approach may be challenging for some students. Therefore, additional support and guidance is required to help students understand the process.

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Unlocking the Future: Mathematics Teachers' Insight into Combination of M-learning with Problem-Based Learning Teaching Activities

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Abstract: The rapid advancement of information technology has significantly facilitated modern human life. However, despite the swift progress in digital technology, there has been limited headway in leveraging technology to support mathematics education, particularly in the domain of problem-solving. Mobile learning (M-learning) platforms offer promising avenues to address this gap, providing accessible and interactive tools that can enhance problem-solving skills in mathematics through engaging activities, real-time feedback, and personalized learning experiences. Thus, this study aimed to evaluate the implementation of a combination of M-learning and problem-based learning (M-PBL) teaching activities among mathematics teachers. The M-PBL teaching activities were developed based on the M-learning model, Problem-Based Learning model, and Social Constructivism theory, resulting in 17 pertinent activities eligible for execution by mathematics teachers. The evaluation phase was conducted through interviews with three mathematics teachers. Data collection involved interview transcripts and document analysis, with subsequent content analysis based on the TUP Usability Evaluation Model's three facets: technology, usability, and pedagogy. Overall, the participating teachers expressed positive perspectives on the technological, usability, and pedagogical aspects of the M-PBL teaching activities. These activities provide teachers with a valuable resource to creatively teach problem-solving in mathematics using mobile technology, fostering the development of creative and innovative students aligned with the aspirations of the Fourth Industrial Revolution.

Keywords: M-learning, problem-based learning, teaching activities, interview, TUP usability evaluation model

INTRODUCTION

The emergence and evolution of the Fourth Industrial Revolution have inaugurated a transformative epoch, imprinting an indelible mark on the trajectory of technological and scientific advancements globally. This epochal phase signifies a seismic shift in the way societies interact with technology, characterized by the integration of cutting-edge digital innovations, sophisticated automation, and the seamless exchange of data. According to Bonfield et al. (2020), this revolution epitomizes a state of change and progress in terms of human civilization and culture. It is intricately linked to the introduction of modern communication, technological applications, and information control. Alaloul et al. (2020) elucidates that this fourth revolution markedly diverges from its three predecessors. This revolution seamlessly integrates the biological, physical, and digital realms. As a consequence, a myriad of new technologies emerges, exerting an impactful influence across various disciplines.

In response to these transformative developments, teachers are urged to embrace technology as an integral tool for pedagogical enrichment (Payadna et al., 2020). By effectively leveraging technology, teachers not only enhance the quality of their teaching but also cultivate a learning environment that resonates with the dynamic needs of modern students. The dissolution of traditional boundaries ensures that the delivery of knowledge becomes a flexible and immersive experience, fostering an educational landscape that extends beyond the conventional classroom setting (Ehsanpur & Razawi, 2020). Moreover, the profound impact of technological advancements, exemplified by the Internet of Things (IoT) in the context of IR 4.0, has left an indelible mark on both teachers' pedagogical approaches and students' learning patterns. The strategic integration of online content systems, such as mobile learning (M-learning), significantly contributes to sustaining an advanced and efficient education system responsive to the demands of the digital age (Qashou, 2021).

The primary goal of introducing M-learning is to enhance students' motivation to delve deeply into a particular field of study (Chantaranima & Yuenyong, 2021). This is because learning applications on mobile devices have encouraged teachers and students to explore information from various perspectives, generating new ideas. Moreover, freely available, multifunctional, and easily accessible learning applications have attracted the interest of teachers and students in using them optimally (Kamaghe et al., 2020). For instance, during the period of the Movement Control Order (MCO), learning applications such as Google Classroom as an information-sharing platform and Kahoot and Quizizz as assessment platforms were widely used (Alsharida et al., 2021). These learning applications facilitated the implementation of teaching and learning at home during that period. As a result, students were able to maintain a continuity of education despite the challenges posed by the circumstances.

Despite the availability of various learning applications for teachers, it is undeniable that the implementation of technology in mathematics education, especially in problem solving, is still at a low level (Verschaffel et al., 2020). Teachers still predominantly employ one-way communication and are limited to existing learning resources (Le et al., 2022), emphasis on uniform learning methods and top-down teaching approaches (Bakker et al., 2021). Such practices should not persist because contemporary students are exposed to technological advancements in their lives (Luritawaty et al., 2024). The current generation of students, known as Generation Z, is considered techno-savvy and requires interactive, collaborative, and hands-on learning (Hamidi & Jahanshaheefard, 2019). Recognizing and addressing these factors becomes imperative for teachers to bridge the gap between traditional teaching methods and the preferences of a generation deeply immersed in technology.

To enable such changes, a transformation is needed in the field of mathematics education (Engelbrecht et al., 2020). This change must occur to ensure that the current generation of students evolves alongside technological advancements. This study focused on the implementation of a new teaching method involving a combination of M-learning and Problem-Based Learning (M-PBL) that mathematics teachers can use to improve their teaching practices. After the implementation process, there is a need to obtain teachers' perspectives on the effectiveness, efficiency, and satisfaction with the implementation of M-PBL teaching activities. To this end, the researcher chose the interview method to obtain feedback from mathematics teachers on the usability evaluation of M-PBL teaching activities. This evaluation was conducted based on the TUP Usability Evaluation Model, which involves three aspects: technology, usability, and pedagogy (Bednarik et al., 2004).

THE M-PBL TEACHING ACTIVITIES

As a guide, the researchers conducted literature surveys on the theories and models that support the study's constructs. The development of the theoretical framework of this study involves three main components namely the M-learning Model, Problem-Based Learning Model and Social Constructivism Theory. To this end, there are 17 eligible and relevant M-PBL teaching activities that can be conducted by mathematics teachers as shown in Figure 1 until Figure 17. The following is stated in summary as the models and theory used to develop the combination of M-learning with Problem-Based Learning (M-PBL) teaching activities.

- M-learning Model: In this study, the researcher selected the M-learning model by Brown (2005). In this regard, two main components were chosen, flexible learning and learning with electronic devices.
- Problem-Based Learning Model (PBL): In this study, the researcher adopted Wee's (2004) PBL Model to conduct problem-solving lessons. The researcher selected this model as a guide since it evolved from a more fundamental paradigm, namely PBL Model by Barrow (1980). In addition, the model includes various simple activities for teachers to comprehend and implement.
- Social Constructivism Theory: In this study, the researcher selected Lev Vygotsky's 1978 theory of social constructivism. Three components are involved, active student learning, scaffolding, and the Proximal Development Zone (ZPD).

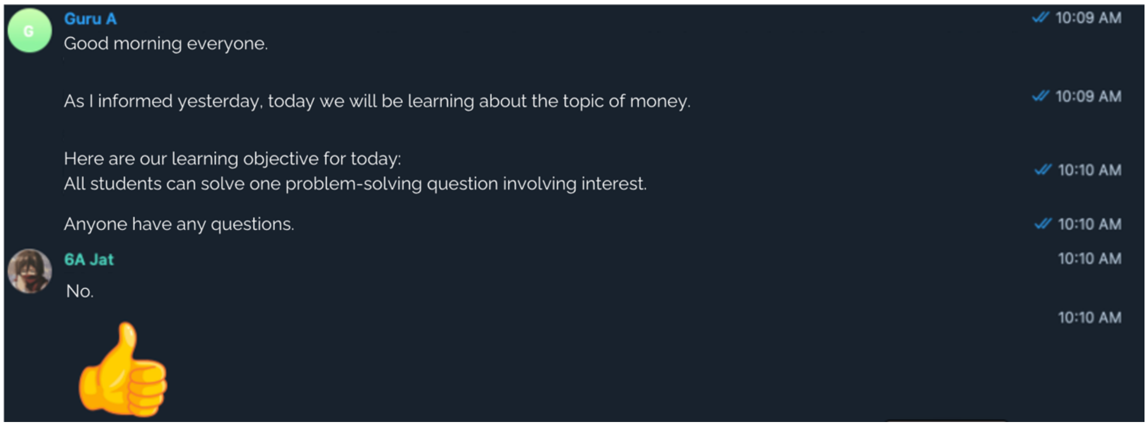

Figure1: Teacher shares the learning objective that the pupils need to achieve using the telegram

Figure 2: Pupils are guided by the teacher to form groups for various levels of ability using the telegram

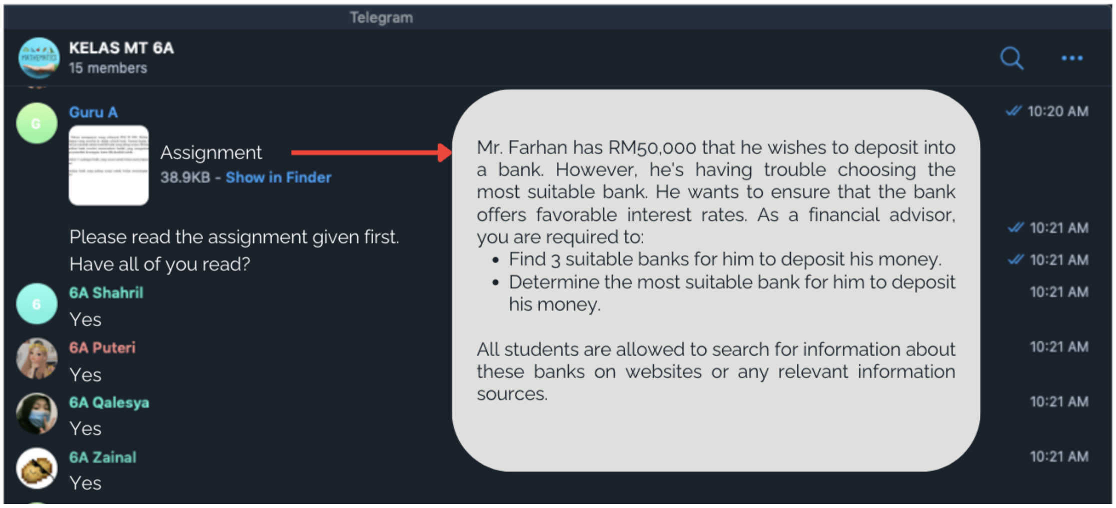

Figure 3: The teacher sets out the tasks (problems) that each group needs to solve using the telegram

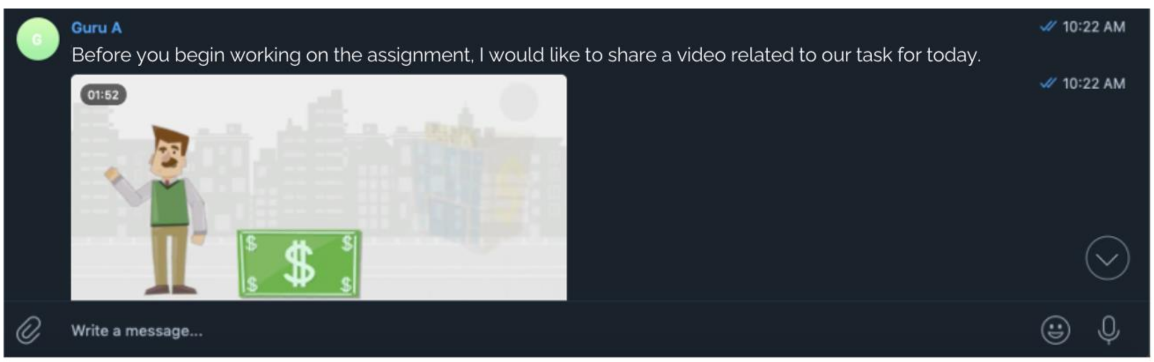

Figure 4: Teacher share several stimulus materials consisting of various forms of media using the telegram

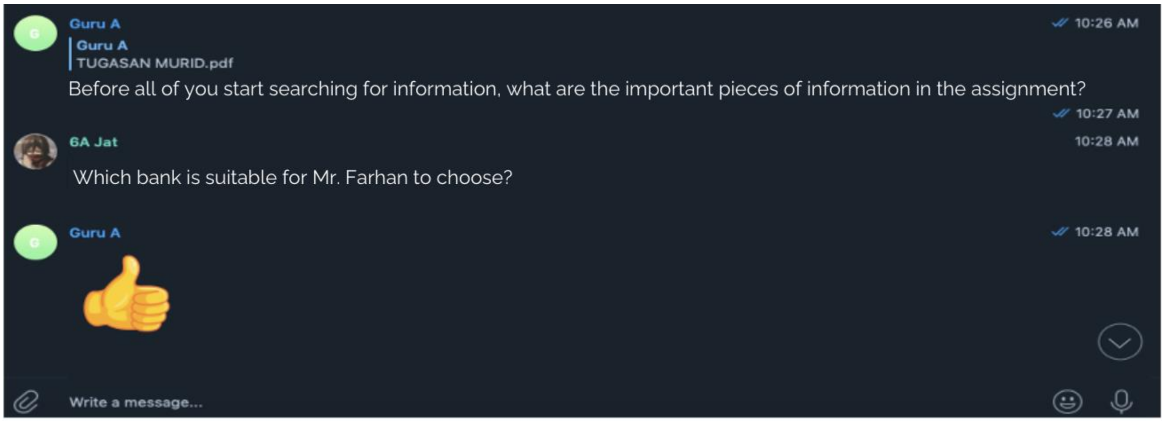

Figure 5: Each group discusses the tasks assigned in the context of their daily living situations using the telegram

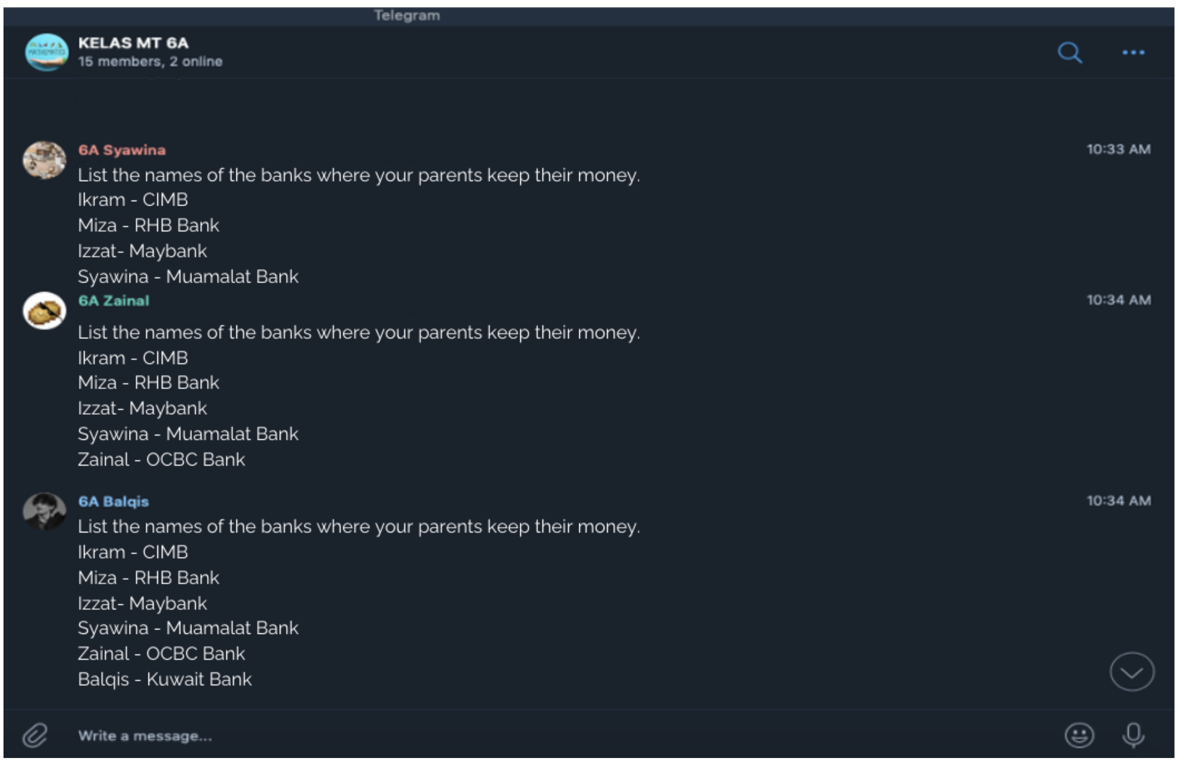

Figure 6: Each group understands and talks about the stimuli material shown using the telegram

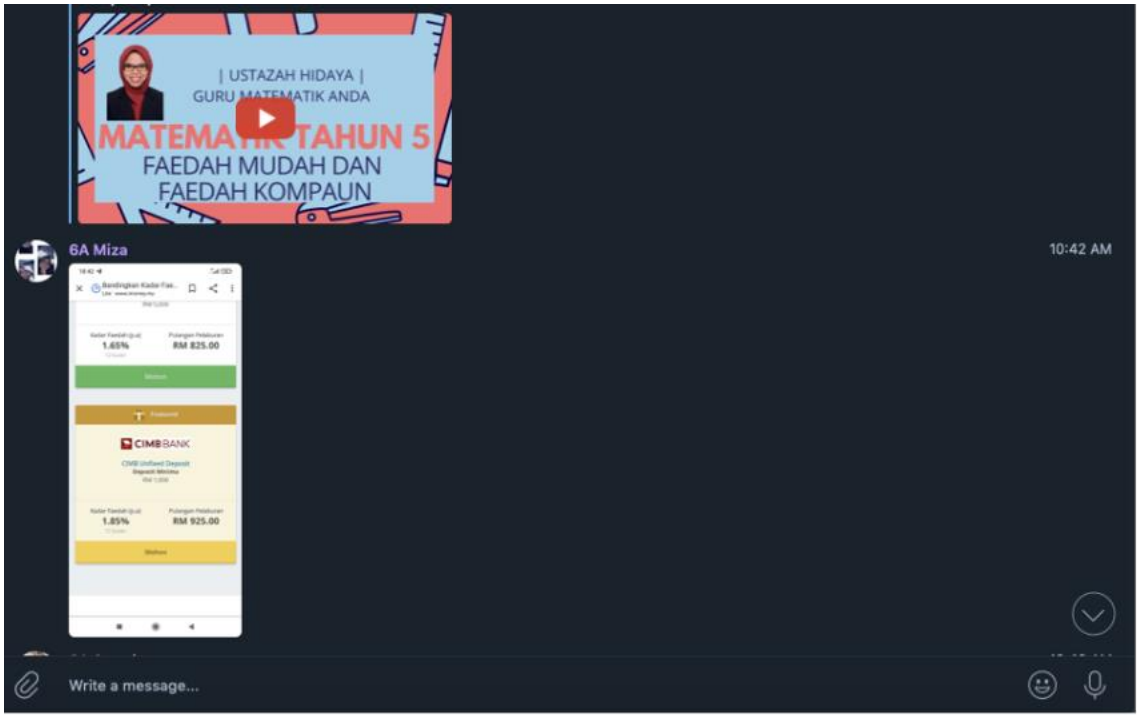

Figure 7: Each group is given time to explore the various applications and other learning media available on the mobile devices to get solution ideas

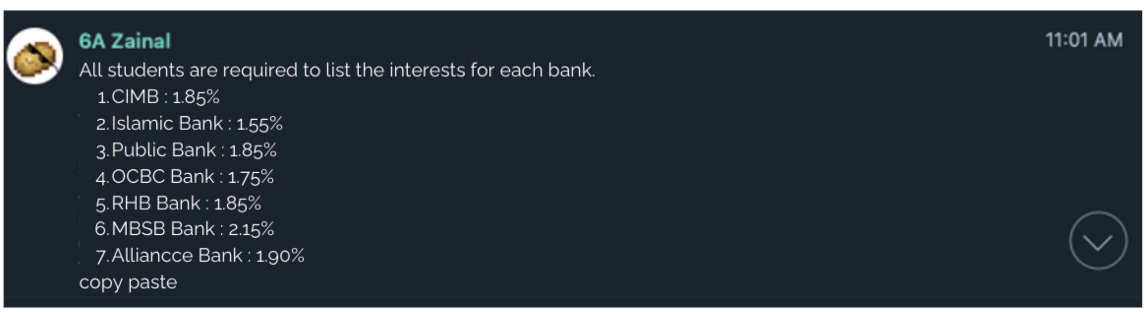

Figure 8: Each group shares information that has been obtained with the teacher and other groups through the telegram

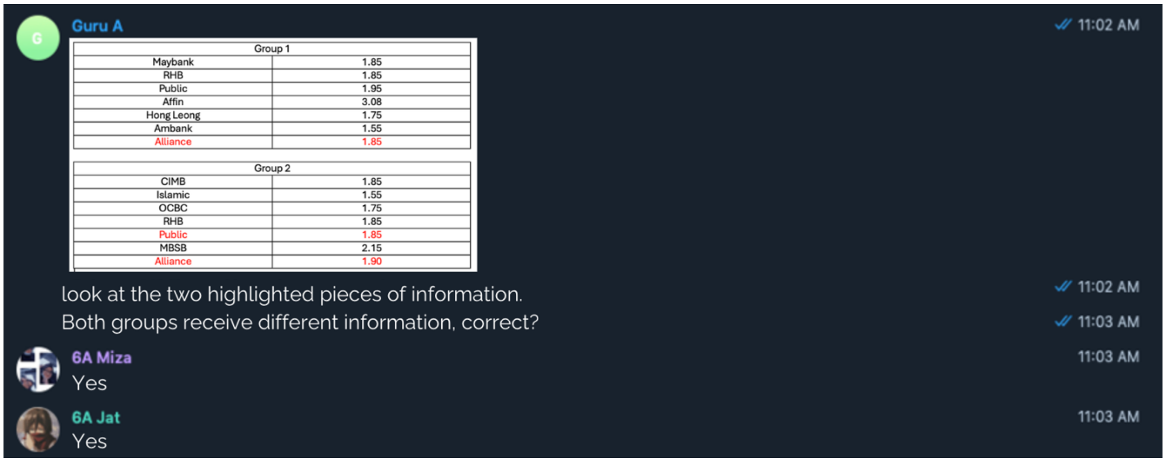

Figure 9: Teacher classify the information obtained from each group according to their preferences through the telegram

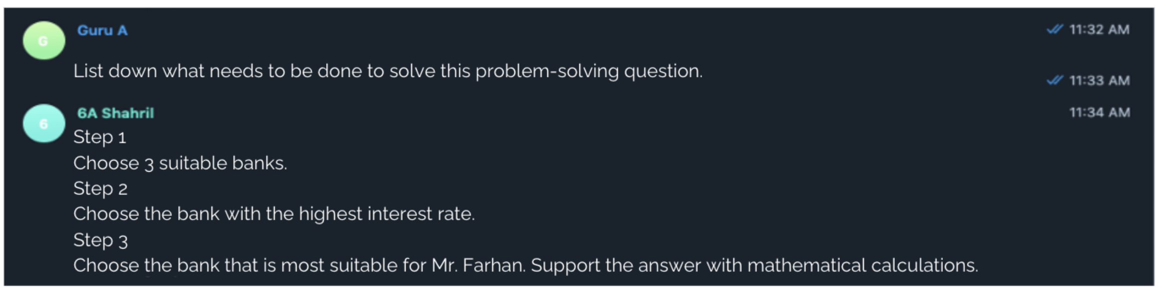

Figure 10: Each group discusses the steps that can be used to complete the tasks through the telegram

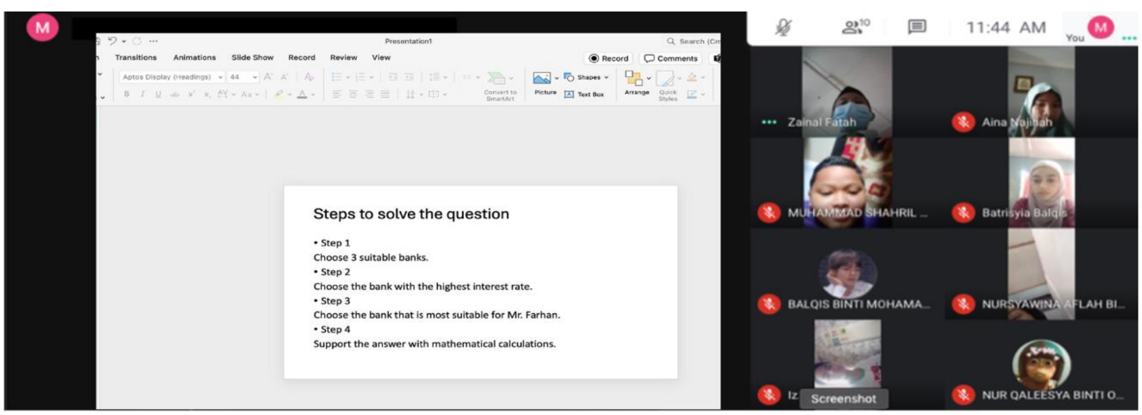

Figure 11: Each group discusses the most appropriate steps to use to complete the tasks given through the Google Meet

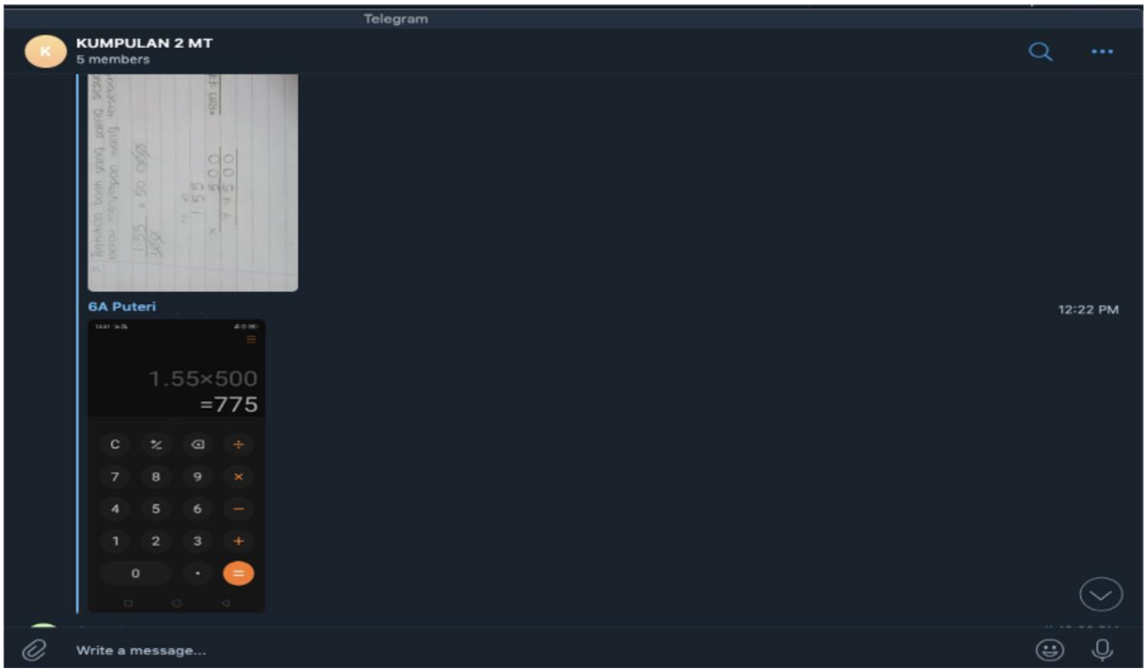

Figure 12: The teacher guide each group to revise the steps that the solutions have selected using the telegram

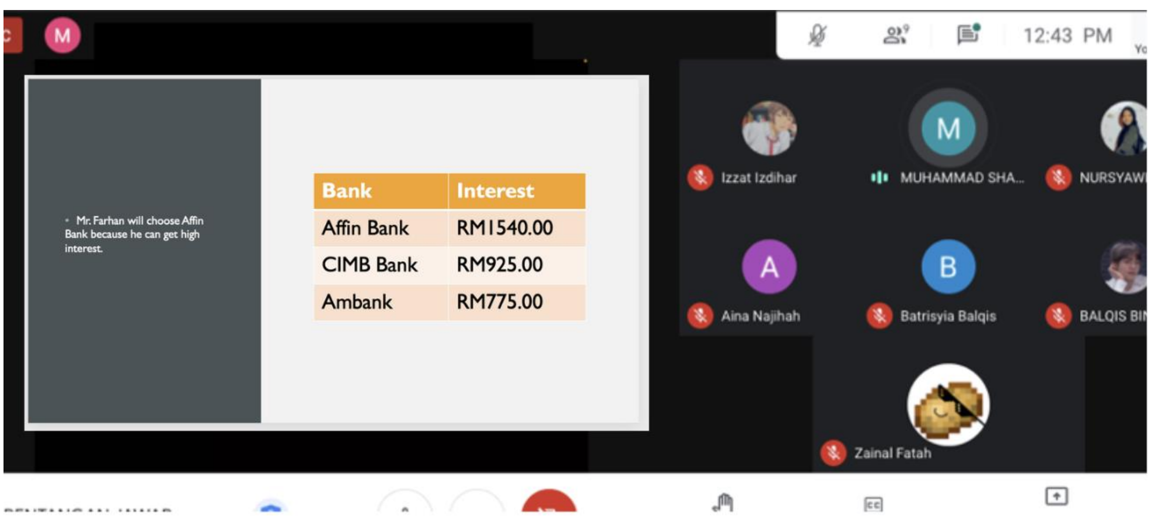

Figure 13: Each group presents the final solution in various forms of graphic media using the Google Meet

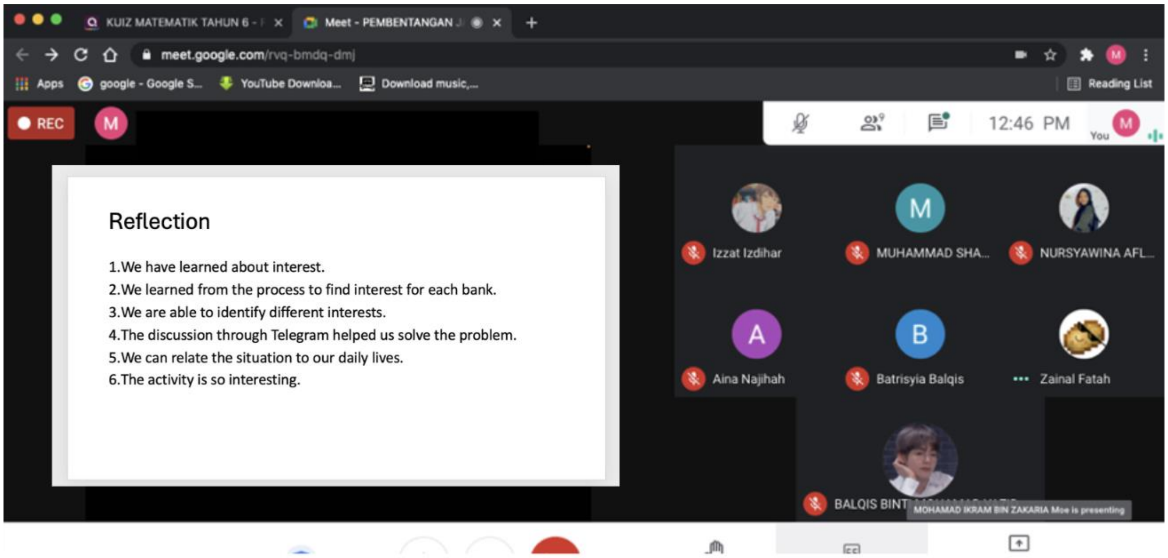

Figure 14: Each group concludes on the learning process and shares it with the teacher and other groups through the Google Meet

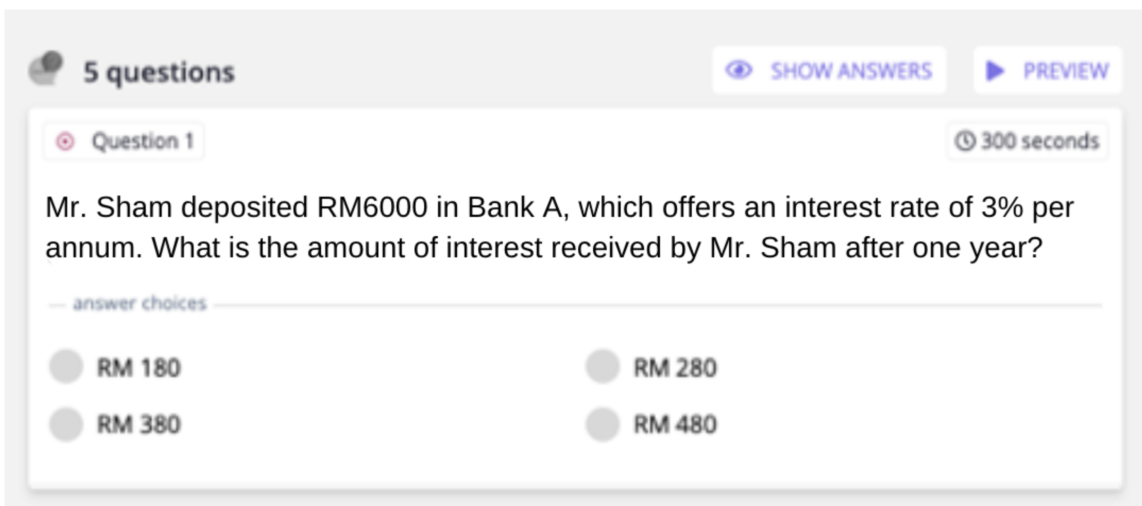

Figure 15: Pupils answer the quiz questions assigned by the teachers through the Quizizz application

Figure 16: The teacher gives feedback based on the marks obtained on Quizizz through the Google Meet session

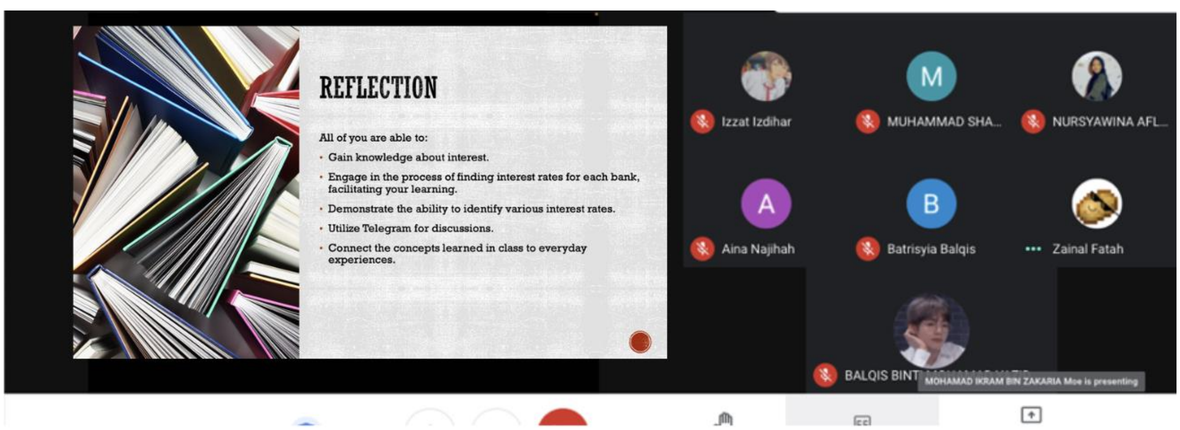

Figure 17: The teacher concludes on the learning using the Google Meet

METHODOLOGY

Research design

The researchers have applied a qualitative approach to assess the usability of the M-PBL teaching activities. This approach is a form of descriptive study involving the direct collection of data in the field (Cilesiz & Greckhamer, 2020). Qualitative data can be described as a form of description involving spoken or written words about human behavior that can be understood (Johnson et al., 2020). Qualitative data can be obtained through observation, interviews, and written materials (Cheung & Tai, 2021). After careful consideration, the researchers chose the technique of data collection through interviews. The interview was conducted according to the TUP Usability Evaluation Model, involving three aspects: technology, usability, and pedagogy (Bednarik et al. 2004). This technique can help the researchers assess the suitability and effectiveness of the M-PBL teaching activities involving minimum access, cost, and time while not compromising individual privacy.

TUP usability evaluation model

The TUP model serves as a guide that researchers can use to assess a model, module, product, or teaching activity that integrates technology into the educational environment (Laurah Markus, 2022). This is because the model was developed with the aim of evaluating technology-based learning environments (Bednarik et al., 2004). Therefore, this model facilitates users who wish to assess the implementation of technology in the development of models, modules, products, or teaching activities. The name of this model is derived from the acronym "TUP," which refers to three aspects: technology (T), usability (U), and pedagogy (P). The model focuses on a checklist that serves as an evaluation tool (Hamid et al., 2023). This checklist has been developed based on Nielsen's Usability Model (1994), involving elements of technology and usability, and the theoretical framework of constructivism theory (Soloway, 1996), involving pedagogical elements in the learning environment (Idris & Md Nor, 2022). Both have been combined to form a theoretical framework in Bednarik's TUP Usability Evaluation Model (Bednarik et al., 2004).

Participants

In this study, the participants are specifically selected through the purposive sampling method. The researcher has identified the characteristics for selecting the participants, such as (1) possessing knowledge and experience in teaching mathematics; (2) having good communication skills; (3) being willing to participate in the study; (4) having the availability to participate in the study; (5) owning at least one mobile device; and (6) having high skills in using mobile devices. Simultaneously, the number of participants is also an important consideration. However, past researchers have stated that there are no specific criteria or numbers for determining the number of participants in qualitative research (Bogdan & Knopp 2003; Delamont & Jones 2012). Therefore, the researchers have chosen three participants (Teacher A, Teacher B, Teacher C). This selection is based on the characteristics established by the researcher. Additionally, these teachers have expressed interest in and willingness to participate in this field study.

Instrument

There are several types of instruments that can be used to collect data in qualitative research. Among them is the interview protocol, which is relevant to qualitative studies. There are three types of interviews that can be conducted: open-ended interviews, informal interviews, and formal interviews (Busetto et al., 2020). Additionally, Engward et al. (2022) explain that there are three types of interviews: unstructured interviews, semi-structured interviews, and structured interviews that can be employed in qualitative research. In the context of this study, the researchers have opted for a semi-structured interview. The items in this semi-structured interview protocol were developed based on the TUP Usability Evaluation Model (Bednarik et al., 2004). This semi-structured interview is administered with the aim of obtaining feedback from teachers regarding the usability assessment of the M-PBM teaching model in real-world contexts.

Validity and reliability of the data

Triangulation is a process that involves using multiple sources of data, researchers, data collection methods, and analyses with the aim of providing the researchers with an opportunity to confirm findings more robustly and reliably (Santos et al., 2020). This technique aims to enhance the validity of research findings by reinforcing them with other methods. In the context of this study, the researchers have collected data using the semi-structured interview technique, and these findings have been corroborated through document analysis, specifically the analysis of lesson plans. Simultaneously, the researchers have sought confirmation from the participants to validate the data and interpretations by asking them to review the data (Craig et al., 2021). Participant review serves to verify and improve the accuracy of the research information obtained. Moreover, this method also serves to enhance the credibility and reliability of the research findings.

Research procedure

The study was conducted from October 1 to October 31, 2023. In each session, the researchers took an active part as observers, while the participants were given autonomy to carry out the teaching activities. Each teaching session was conducted once a week to ensure that teachers could make adequate preparations and that students were better prepared to engage in the subsequent teaching and learning sessions. Following the completion of the M-PBL teaching activities, the researcher conducted semi-structured interviews with all participants. The purpose of this interview was to gather feedback from the participants regarding the teaching activities. The interview session was conducted simultaneously, involving all participants together. This simultaneous approach was employed to further enhance the validity of the interview findings that were obtained (Craig et al., 2021).

Data analysis

In qualitative research, there are various techniques for analyzing data, such as phenomenology, ethnography, and content analysis (Bengtsson 2016). In the context of this study, the researcher employed a content analysis approach to analyze interview data. This aligns with Roller's perspective (2019), which suggests that content analysis can be a suitable approach for analyzing interview data. This approach helps the researcher identify themes, concepts, and meanings. Furthermore, the coding system aids the researcher in addressing the research questions (Elo et al. 2014). Therefore, the researcher undertook the following steps: (1) manually transcribing the interviews; (2) Organizing the data; (3) constructing categories and themes; and (4) coding the data.

RESULTS

This section discussed the usability evaluation of the combination of M-Learning with Problem-Based Learning (M-PBL) teaching activities. This evaluation was conducted through a semi-structured interview based on the TUP Usability Evaluation Model, covering technological, usability, and pedagogical aspects (Bednarik et al., 2004). Overall, participants were satisfied with the technological, usability, and pedagogical aspects of M-PBL teaching activities.

Technological evaluation

This section discussed the theme of technology. There were two categories within this theme. Firstly, the suitability of mobile device usage, and secondly, the use of learning applications. Findings from interviews with participants indicated that the mobile devices and learning applications used in M-PBL teaching activities were appropriate and user-friendly.

> *"Hmmm... I have no issues at all using a tablet or phone. Because we always use them. So... it's really suitable to use these gadgets, both of them." (TB1/R1, p. 1, lines 21-23).*

> *"Google meet, telegram, those are all things I've been using regularly. Our class group used telegram before. So far... I'm okay with using all of them" (TB1/R2, p. 2, lines 32-33).*

Usability evaluation

This section discussed the theme of usability. There were four categories within this theme. First, suggestions for M-PBL teaching activities. Second, the suitability of M-PBL teaching activities. Third, improvement in students' understanding through M-PBL teaching activities. Fourth, the enjoyment of conducting M-PBL teaching activities. Findings from interviews with participants indicated that teachers agreed with the M-PBL teaching activities. At the same time, they felt that all these teaching activities were suitable and capable of enhancing students' understanding. Besides, participants also enjoyed implementing M-PBL teaching activities.

> *"Really like it, sir... because before this... I had to skip teaching problem-solving. The reason is... well, it's difficult to teach that skill during class... now, when there's a method on how to teach, it becomes a bit easier... right?" (TB1/R1, p. 2, lines 44-46).*

> *"Suitable... very much so. Students also learn when they want to solve questions. Moreover... they all have phones anyway." (TB1/R2, p. 2, lines 55-56).*

> *"The kids nowadays... are experts. They can find all the information. It's the information that helps them understand and complete task. Moreover... even if they don't understand, they can ask me or other teachers." (TB1/R3, p. 3, lines 69-71).*

> *"Very enjoyable, sir... I get all sorts of answers... this is the best. Even for getting the students to achieve Band 6, it's possible. Because it produces various responses." (TB1/R1, p. 3, lines 74-75).*

Pedagogical evaluation

This section discussed the theme of pedagogy. There were four categories within this theme. First, exploration activities were part of the M-PBL teaching activities. Second, activities involving the use of learning applications. Third, there were discussion activities in the M-PBL teaching activities. Fourth, assessment activities were part of the M-PBL teaching activities. Interview findings with participants indicated that exploration activities were suitable. At the same time, participants also believed that learning applications such as Google Classroom, Google Meet, and Quizizz were appropriate. Additionally, the teaching activities conducted also encouraged discussions, while assessment activities involving reflection and online quizzes were deemed suitable, engaging, and enjoyable.

> *"This activity is very enjoyable. Students have the opportunity to generate various answers because they search for different information. The best part is when they compare answers. There are groups that tease their friends because their friend used the wrong information." (TB1/R3, p. 5, lines 135-138).*

> *"All those apps are easy to use... access and everything." (TB1/R2, p. 5, line 142).*

> *"Overall... it's okay, sir. The students just discuss. But, you know, their language... it's diverse... even during Google Meet sessions, they respond. When discussing, various ideas can come out." (TB1/R1, p. 5, lines 153-155).*

> *"I see there are two things we assess... first, reflections. Then... answering quiz questions, right? The reflections are okay. It's just... at the beginning, you know. The students don't understand how to do it... when I mentioned something like a parking lot, then they understand. They share what they learned. The quiz is really enjoyable. Plus, the quiz helps me assess the students. The reflections help me know what the students learned." (TB1/R1, p. 6, lines 165-170).*

DISCUSSION

It was found that teachers involved in this evaluation phase were satisfied with the combination of M-learning with problem-based learning (M-PBL) teaching activities. Specifically, teachers expressed positive views on three evaluated aspects: technology, usability, and pedagogy. This indicates that these teaching activities are suitable and can be further developed in a real educational context. Technological aspect findings show that teachers had no issues using smartphones, iPads, and laptops for implementing M-PBL teaching activities. They felt that mobile devices were suitable for teaching, influenced by factors such as convenience, suitability, and skills. These factors motivated teachers to use mobile devices in their teaching (Al-Rahmi et al., 2021).

Additionally, teachers' skills influenced their ability to conduct M-PBL teaching activities. The study findings indicate that teachers possess high skills in using mobile devices. These skills helped them determine the suitability of mobile devices for various teaching activities (Sophonhiranrak, 2021). It also facilitated their access to learning materials available in various learning applications (Huang et al., 2020). These views align with the study's findings, where teachers' skills, experience, and knowledge eased their use of various learning applications like Google Classroom and Google Meet.

Usability aspects findings also indicate that teachers had no issues implementing suggested M-PBL teaching activities. Teachers found that the proposed teaching activities were suitable for the context of mathematics education. These findings support the views of Sjöberg and Brooks (2022) suggesting that mobile technology's use can guide teachers in problem-solving teaching. At the same time, teachers also found that the suggested teaching in the M-PBL teaching activities could enhance students' understanding of learning (Mapile & Lapinid, 2023). This is because students have knowledge and skills in using mobile devices, impacting their learning. They can use their knowledge and skills to seek various information in learning applications available on their mobile devices (Criollo-C et al., 2021).

Through these activities, students could generate various new ideas (Kacetl & Klímová, 2019) using the gathered information. Furthermore, they could build knowledge and understanding of the desired learning content (Mughal et al., 2018). Additionally, M-PBL teaching activities also increased teachers' enjoyment of problem-solving teaching. Teachers felt that these activities could capture students' interest in learning. One particularly engaging activity was exploration. In this activity, students were given the opportunity to explore learning using various learning applications on mobile devices. Thus, students had the chance to access various information and new ideas through this activity (Kunwar et al., 2023).

Pedagogical aspects findings also show that teachers had positive views on learning activities in the M-PBL teaching activities. Teachers believed that exploration activities were suitable. Nikolopoulou (2018) suggests that exploration activities through mobile devices are efforts made to promote active learning among students. At the same time, exploration activities through this mobile technology can help students build knowledge and enhance their understanding (Holenko Dlab et al., 2020). This activity also helps students build various new and authentic ideas through information obtained from various sources and media (Bernacki et al., 2019).

At the same time, the recommended learning applications were also suitable according to their functions and intended activities. For example, the Google Meet application served as a platform for conducting video conferences. Chinaza's study (2021) shows that this application is highly suitable for immediate use during the COVID-19 pandemic. This application has helped teachers conduct live teaching to deliver learning content, discussions, and reinforcement. Additionally, teachers found that this teaching could encourage discussions and collaboration between teachers and students and among students (Baker et al., 2020). The use of suitable learning applications can create a collaborative approach among students (Ansari & Khan, 2020).

Moreover, teachers also found that the assessment activities conducted were suitable, interesting, and enjoyable. This is because these activities provide information to teachers about the teaching and learning process experienced by both teachers and students. Through this feedback, teachers can assess students' objective achievements (Bachelor & Bachelor, 2016). Thus, teachers can improve the teaching process and subsequently enhance the quality of teaching in the future. At the same time, these activities can also help students increase their self-confidence (Choi et al., 2017), encourage students to think critically, help students identify mistakes in learning, improve students' skills in identifying creative ideas, and monitor learning progress from the initial stage to the final stage (Cunningham, 2018). In conclusion, the usability evaluation of the M-PBL teaching activities shows that teachers have positive views on the technological, usability, and pedagogical aspects.

CONCLUSION

This study holds significant implications for the continual evolution of teaching methodologies, emphasizing the pivotal role of usability and pedagogical considerations in the successful incorporation of technology. In the future, other researchers could delve into the impacts of M-PBL teaching activities on students' mathematics learning outcomes, broadening the scope to encompass diverse educational contexts and subjects. To facilitate the replication of the study, researchers are encouraged to follow all the activities as suggested in the M-PBL Teaching Activities (Table 1). Additionally, researchers should document their procedures meticulously to ensure transparency and reproducibility, thereby contributing to the advancement of knowledge in this area. This includes providing detailed descriptions of each activity, specifying the intended learning outcomes, outlining the technological tools used, and offering guidance on the facilitation of problem-solving discussions. By adhering to these guidelines, researchers can enhance the reliability and validity of their findings and promote the replication of the study in diverse educational contexts. Furthermore, an exploration of potential challenges and a proactive response to the evolving needs of teachers engaged in technology-integrated teaching practices will contribute to the ongoing enrichment of the educational research landscape. This iterative process of refinement and exploration will be pivotal in shaping the future landscape of mathematics education and its profound impact on student learning and teacher practices.

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Mathematical Problem Design to Explore Students' Critical Thinking Skills in Collaborative Problem Solving

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Abstract: Social interactions, including collaborative problem-solving situations, can trigger critical thinking skills. Giving questions that are not routine can trigger students' critical thinking skills in solving problems collaboratively. This research aims to develop non-routine mathematics problems that can be used to explore students' critical thinking abilities in collaborative problem-solving. The research results show that questions with problem criteria that require justification for the solution provided and questions with a graphical analysis approach can be used to explore students' critical thinking skills in collaborative problem-solving. This is proven by solving the problems; each group member contributed to the solution-finding process. The contribution of each group member shows the high intensity of interaction between members. Interaction in the form of exchanging opinions, giving suggestions, and evaluating each other's ideas or answers significantly impacts students' critical thinking abilities. This is seen by the emergence of several students' critical thinking skills (analysis, synthesis, argumentation, evaluation, self-regulation) triggered by suggestions or ideas put forward by other group members. The research results can be a reference for researchers or practitioners exploring critical thinking skills as a guide in developing research instruments.

Keywords: Critical Thinking Skills, Non-Routine Problem, Collaborative Problem Solving, Mathematics

INTRODUCTION

Mathematics education focuses on increasing the use of acquired mathematical knowledge and skills in daily problem-solving activities (Stacey & Turner, 2015). Critical thinking skills are needed to solve complex problems. Yee et al. (2011) state that critical thinking skills play a role in determining decisions in the problem-solving process. Someone who has critical thinking skills has a high level of sensitivity to problems so that they can quickly formulate problems, review problems from several perspectives, and evaluate every step of solving problems that have been solved (Maričić et al., 2016).

Critical thinking skills are one of the competencies that need to be developed because they predict one's success (Butler et al., 2017; Haynes et al., 2016). In addition, critical thinking skills can mediate several competencies that need to be mastered in the 21st century, such as collaborative skills, creative thinking, algorithmic thinking, and problem-solving (Kocak et al., 2021). Although critical thinking skills are cognitive processes, some experts define indicators that can be used to represent critical thinking skills. Perkins & Murphy (2006) formulated four indicators of critical thinking skills, namely clarification, assessment, inference, and strategy. Facione (2015) mentions six indicators of critical thinking skills: interpretation, analysis, inference, explanation, evaluation, and self-regulation. Furthermore, four indicators of critical thinking skills were formulated by Ennis (2016), namely essential clarification, bases for decision, inference, and advanced clarification. Reynders et al. (2020) created a rubric to assess critical thinking skills based on four indicators: analyzing, synthesizing, forming arguments, and evaluating. At the same time, Cortazar et al. (2021) used six aspects as indicators of critical thinking skills: interpretation, analysis, inference, arguments, evaluation, and metacognition. From the results of the studies of several experts, the researchers formulated five indicators of critical thinking skills: analysis, synthesis, argumentation, evaluation, and self-regulation.

The development of a person's critical thinking skills is influenced by social interaction. Collaboration catalyzes critical thinking skills (Waite & Davis, 2006) because collaboration encourages students to think deeply (Ebiendele Ebosele Peter, 2012; Hussin et al., 2019). Collaborative Problem Solving (CPS) is a problem-solving activity that requires interaction between group members during the problem-solving process. Hagemann & Kluge (2017) state that CPS is an interdependent activity of group members in the context of turning an input into output through cognitive, verbal, and behavioral activities to regulate task completion to achieve common goals. The interdependence attitude manifests in two roles: *explainer/solver* and *checker* (Westermann & Rummel, 2012). *Explainers* trigger cognitive processes such as elaborating knowledge, and *checkers* monitor explanations and reflect on understanding. In other words, CPS facilitates cognitive and metacognitive processes, supporting a person to become a good critical thinker (Maynes, 2015).

CPS emphasizes the interdependence between group members. To foster this attitude of interdependence, the given in CPS are problematic for each group member (Hagemann & Kluge, 2017; Westermann & Rummel, 2012). Complicated tasks involve problems involving several mathematical concepts, and solving them requires critical thinking skills, namely the ability to analyze, synthesize, and evaluate (Westermann & Rummel, 2012; Williams, 2000). This problematic task's characteristics follow the characteristics of non-routine mathematics tasks. In mathematics, non-routine tasks are characterized by not having an immediate solution, requiring productive thinking (Kolovou et al., 2009), involving unexpected solutions (Yeo, 2009), requiring strategic thinking, and containing various mathematical concepts (Mabilangan et al., 2011).

The development of mathematical tasks that can trigger critical thinking skills has been carried out by (Kuntze et al., 2017) in a particular context. In the context of collaboration, there still needs to be more development of tasks that can trigger students' critical thinking skills. In learning activities, students often work collaboratively due to the demands of collaboration skills in the 21st century (Barron, 2000; Chew et al., 2020; Sofroniou & Poutos, 2016). So for educators or researchers who support collaborative performance to explore critical thinking skills, it is necessary to know what characteristics of the problem can trigger students' critical thinking skills. The accuracy of the problem design will affect the accuracy of the critical thinking skills data obtained. The teacher and researcher can appropriately determine the next step if the data is accurate. Based on the literature review results mentioned, this study aims to develop non-routine mathematics problems that can be used to explore students' critical thinking skills in collaborative problem-solving. The urgency of developing non-routine problems is shown in Figure 1.

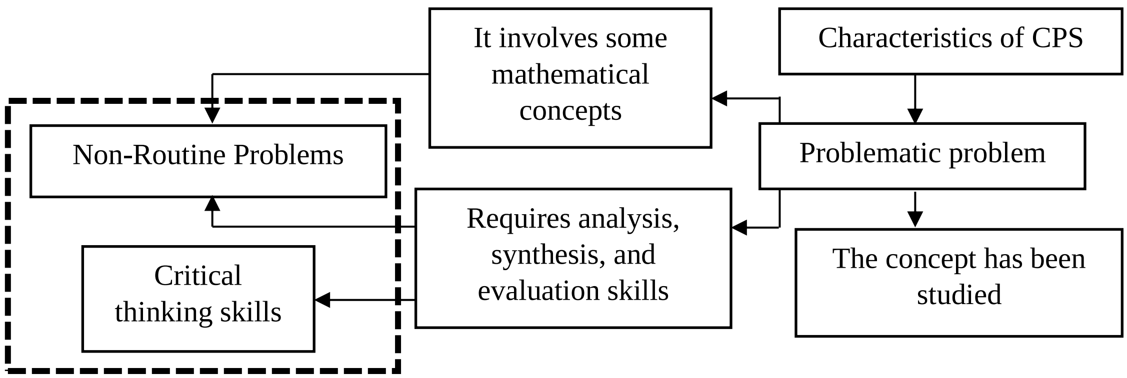


Figure 1. The Urgency of Developing Non-Routine Mathematical Tasks

LITERATURE REVIEW

Critical Thinking Skills in Collaborative Problem Solving

Vygotsky's sociocultural theory and the Zone of Proximal Development (ZPD) model state that conversations with peers will expand students' ZPD to think critically (Wass et al., 2011). Furthermore, Wass et al. (2011) stated that in Vygotskian's view, critical thinking involves the collaboration of several mental functions, such as memory, imagination, analysis, and evaluation taught through conversation. Therefore Wait & Davis (2006) stated that collaboration is a skill catalyst for critical thinking. In this study, collaboration settings were facilitated by Collaborative Problem-Solving (CPS) activities.

The indicators for critical thinking skills in this study use critical thinking indicators from Reynders et al. (2020), namely analysis, synthesis, argumentation, and evaluation, coupled with another aspect, namely self-regulation from Facione (2015). Self-regulation is deemed necessary to add because, in collaborative work, there will be an interaction between group members. Someone who thinks critically will check his understanding to respond to analysis, synthesis, argumentation, and evaluation activities carried out by others (Facione, 2015). The five indicators are then adjusted to the stages of solving collaborative problems proposed by Hesse et al. (2015):

problem identification, planning and exploring, execution, and verification. This adjustment is based on the opinion put forward by Lester (2013), which implied that critical thinking skills play a role in every problem-solving activity. Analysis skills play a role in simplifying the context of the problem, and synthesis is needed when selecting the mathematical concepts to be used, argumentation is needed to execute the selected mathematical concepts, and evaluation is needed when checking the suitability of the problem with the solutions found. Figure 2 is a visual representation of the role of critical thinking skills in solving collaborative mathematical problems.

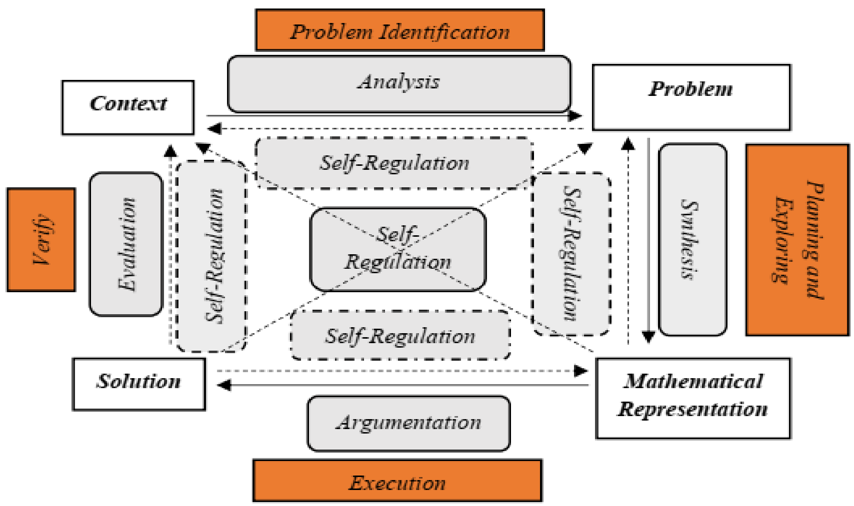


Figure 2. The Role of Critical Thinking Skills in Solving Collaborative Mathematical Problems

Descriptions of adjustments to the stages of problem-solving and indicators of critical thinking skills are explained in Table 1. Combines the results of the theoretical studies put forward by Reynders (2020); Facione (2015); Hesse et al. (2015); dan Lester (2013), indicators of critical thinking skills in collaborative problem solving used by researchers are presented in Table 2.

Table 1. Description of CPS Stages and Critical Thinking Skills Indicators

Stages of CPS (Hesse et al., 2015)	Critical Thinking Skills Indicator (Facione, 2015; Reynders et al., 2020)
Problem Identification (PI) Identifying problematic problem elements by communicating opinions or information based on different roles.	*Analysis* Describe and explore the meaning of data based on existing knowledge.
	Self-Regulation Check the quality of your thinking.
Planning and Exploring (PE) Determining mathematical ideas that can support solving complex	*Synthesis* Identify the relationship of some information or concepts.

Stages of CPS (Hesse et al., 2015)	Critical Thinking Skills Indicator (Facione, 2015; Reynders et al., 2020)
problems by accommodating the various perspectives of team members based on different roles.	*Self-Regulation* Check the quality of your thinking.
Execution (EX) Implementing problematic problem-solving ideas that team members have agreed upon based on differences in roles.	*Argumentation* Provide a systematic explanation in responding or providing information.
	Self-Regulation Check the quality of your thinking.
Verify (VF) Checking the suitability of complex problems with solutions found by team members.	*Evaluation* Assess the credibility of the claims and arguments that have been generated.
	Self-Regulation Check the quality of your thinking.

Table 2. Indicator for Critical Thinking Skills in Collaborative Problem Solving

Code	**Critical Thinking Skills Indicator in Collaborative Problem Solving**
(An)	*Analysis* Describe and explore the meaning of data to understand and identify problematic problem elements by communicating opinions or information.
(Sy)	*Synthesis* Identifying the relationship between some information or concepts by accommodating the various perspectives of team members based on different roles to determine ideas that can support solving complex problems.
(Ar)	*Argumentation* Provide systematic explanations to apply ideas to solve complex problems that team members have agreed upon.
(Ev)	*Evaluation* Assess the credibility of the claims and arguments generated to check the suitability of problematic issues with the solutions that team members have found.
(Sr)	*Self-Regulation* Checking the quality of one's thinking during the problem-solving process

Non-Routine Mathematical Problem

Types of mathematics problems are divided into routine and non-routine problems (Jäder et al., 2017). Routine problems are problems that are often encountered by students and have algorithms that are ready to be used to solve problems. In contrast, non-routine problems require high-level thinking and are rarely found in learning materials (Kablan & Uğur, 2020). Non-routine problems require students to use cognitive processes such as critical thinking to find solutions (Asman & Markovits, 2009; Thomas et al., 2013). In the context of mathematics, non-routine problems are mathematical problems that do not have a straightforward solution (Elia et al., 2009), require productive thinking (Kolovou et al., 2009), and require strategic thinking (Mabilangan et al., 2011). In other words, non-routine math problems are math problems that do not have a unique algorithm, so they require strategic thinking to solve them.

Furthermore, mathematical problems, according to their purpose, are divided into two, namely "problem to find" and "problem to prove" (Polya, 1945). At the advanced level, the given math problem can be a "problem to prove," while at the intermediate to a basic level, the problem given is a "problem to find" (Stylianou et al., 2015). The participants in this study were high school students. Thus, "problem to find" was more appropriate to be developed into a research instrument. Problems related to quadratic functions were chosen to be developed in this study.

METHOD

This study aims to develop non-routine mathematics problems to explore students' critical thinking skills in collaborative problem-solving. A non-routine mathematical problem on quadratic function material is developed. Problem requires students to analyze graphs (Table 4). Function material was chosen because based on research conducted by Marzuki et al (2021) and Endrawati & Aini (2022) stated that problems related to functions can be used to explore students' critical thinking skills. Problem development also refers to the opinion of Rott (2021), which states that using math problems with clear but wrong solutions can explore students' critical thinking skills. This problem will trigger students to evaluate the stages of problem-solving thoroughly. Finding the right solution to a problem will trigger students to think critically. More specifically, the research results of Korres & Tsami (2010) and Ariza et al. (2021) also state that in material related to function, misuse of definitions can be detected and recognized by students through the use of graphical representations to present concepts, for example, by describing various graphic positions and connecting them with definition or concept. Therefore, a non-routine mathematical problem related to functions developed by researchers, namely, asking students to analyze graphs of functions. Table 3 shows the developed grid of non-routine mathematical problems.

Table 3. Lattice of Non-Routine Mathematical Problem

Problem Criteria	Problem
Purpose of Problem	Problem to Find

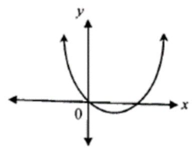
Material	Quadratic Function
Approach	Chart Analysis
Type of Problem	Require Justification Mathematics Problem

In addition to the problem items, the researcher also made guidelines for solving each problem item.

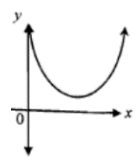
Table 4. Developed Non-Routine Mathematical Problems

Problem

The function curve $f(x) = x^2 + kx$ is as follows.



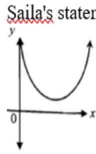
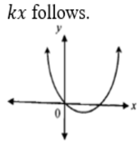
Based on this information, Saila was asked to draw a curve which is a function curve $f(x) = x^2 - kx + 5$. Next, Saila draws the following curve and states that the curve is a function curve $f(x) = x^2 - kx + 5$.



Is Saila's statement true? Explain your reasons.

Validation activities are carried out to check the validity of the non-routine math problems that have been developed. Validation was carried out by three expert validators who are lecturers in Mathematics Education at Surabaya State University. One of the validators is a professor of Mathematics Education, the second validator is a lecturer in Mathematics Education whose research focuses on secondary education, and the third is a lecturer in Mathematics Education whose research focuses on critical thinking skills. Two drafts are given to the validator, namely drafts of non-routine math problems and guidelines for solving them. The validation results show that there are several suggestions from the validator, namely; 1) provide instructions for working on questions that can condition groups to complete tasks collaboratively from start to finish; 2) create an alternative data analysis document that shows; a) part of the problem that can trigger the emergence of aspects of critical thinking skills; b) the possibility of the emergence of critical thinking skills when students solve problems collaboratively. The conclusion from the results of the validation is that the non-routine math problems that have been developed can be used to explore students' critical thinking skills after being revised according to the suggestions of the validator. Based on suggestions from the validator, Table 5 shows the results of improvements to non-routine math problems that can be used to explore students' critical thinking skills in collaborative problem-solving.

Table 5. Improvement of Non-Routine Mathematical Problems based on the Validator's Suggestions

<table>
<tr><th>Validator's Suggestion</th><th>Improvement of Non-Routine Math Tasks</th></tr>
<tr><td>Provide instructions for working on questions that can condition groups to complete tasks collaboratively from start to finish.</td><td>Work on the questions in collaboration with group members;
Each number is done together;
Please ensure the written answers result from group discussion agreements; Double-check answers with group members before collecting them.</td></tr>
<tr><td>Create an alternative data analysis document that shows;
a) part of the problem that can trigger the emergence of aspects of students' critical thinking skills;</td><td rowspan="2"><table>
<tr><th>Solution</th><th>Trigger Critical Thinking Skills on the Problem</th><th>Possible Forms of Appearance of Indicators of Critical Thinking Skills in the Problem</th></tr>
<tr><td>Saila's statement which mentions that the curve

is a function graph $f(x) = x^2 - kx + 5$ is not true. Look at the function curve $f(x) = x^2 + kx$ follows.

For example $T(p, q)$ is the vertex of the curve $f(x) = x^2 + kx$. Because T is on the right of the Y axis, $p > 0$. (An, Ar, Sr)</td><td>Analysis (An)
Students explore function graphs $f(x) = x^2 + kx$ by looking at the top of the curve $f(x) = x^2 + kx$ to determine value k.</td><td>Analysis (An)
• After reading the questions, through discussions with group members and by repeatedly checking the information contained in the questions, the groups try to check the information contained in the questions. e.g:
Explainer: "Do we need to find the point of intersection of the curve $f(x) = x^2 - kx + 5$ about the X axis?".
Checker: "can it be searched? Because if it's like that there are still two variables that can't be determined yet".</td></tr>
</table></td></tr>
<tr><td>b) the possibility of developing critical thinking skills when students solve problems collaboratively.</td></tr>
</table>

After the problem is validated well, the researcher implements the problem on selected participants. The participants in this study were two groups of two 10th-grade students. Zuniga et al (2021) stated that working in pairs can increase the activity of negotiating, interacting, reaching agreements, and evaluating between group members. Thus, CPS will likely run well. The participants were chosen based on their mathematics ability, determined by their final exam scores on quadratic function material. Group 1 consisted of students with high (T1) and average (S1) mathematics abilities. Group 2 consisted of students with average (S2) and low (R1) mathematics abilities. We chose the combination of group members based on the type of group that we believe exists in a regular class. A week before implementation, the plan and topic of the assignment were informed to participants. The two groups were given assignments at different times so that the researcher could focus on seeing the critical thinking skills that emerged during the assignment. In the implementation, each group is given a maximum work duration of 90 minutes. To support collaboration conditions, each group is only given one sheet of answer paper and one sheet of calculation paper, used together during the problem-solving process. During the problem-solving process, students cannot consult with researchers or teachers. This is done so that students' critical thinking activities emerge naturally without the influence of other parties. The activities of each group in solving problems were recorded using audio-visual material. At the end of the session, all working papers are collected. Work discussions were transcribed and coded based on Table 2.

RESULTS

This section analyses the potential of non-routine mathematical problems developed in exploring students' critical thinking skills in CPS. Analysis of potential problems was carried out by comparing the activities of the two groups in solving problem based on CSP stages. The presentation of group activities was accompanied by critical thinking ability indicator codes are shown in Table 2.

Group 1 (T1S1)

1. *Problem Identification*

After reading the problem, S1 identifies $f(x) = x^2 + kx$ and $f(x) = x^2 - kx + 5$ **(An-S1)**. S1 conveys to T1 that the function $f(x) = x^2 + kx$ is known data, and the function $f(x) = x^2 - kx + 5$ is what is being asked. At the same time, T1 identifies the graph corresponding to the function $f(x) = x^2 + kx$ **(An-T1).** T1 shows S1 the graph of the function $f(x) = x^2 + kx$. Next, S1 explains the relationship between the T1 identification results and the things asked in the question **(An-S1)**. According to S1, the results of T1's identification, namely the function $f(x) = x^2 + kx$ and the graph $f(x) = x^2 + kx$ can be used as a reference to determine the correctness of the graph $f(x) = x^2 - kx + 5$ and drawn by Saila. At this stage, T1 and S1 agree with what is asked in the question.

2. *Planning and Exploring*

This stage begins with T1 connecting the function graphs $f(x) = x^2 + kx$ and $f(x) = x^2 - kx + 5$ with the discriminant concept to find the value of k and finding that $0 < k^2 < 20$ **(Sy-T1)**. T1 asks S1 whether it is true that zero is one of the solution sets of the inequality $0 < k^2 < 20$. S1 states that zero is included in the inequality $0 < k^2 < 20$ solution set. S1 looks again at the inequality $0 < k^2 < 20$. S1 states that zero is not included in the solution set of the inequality $0 < k^2 < 20$ **(Sr-S1)**. In the end, the finding of the inequality $0 < k^2 < 20$ was not used by T1 and S1 because it did not find a specific k value.

3. Problem Identification

This stage occurs after T1 and S1 do not find a specific k value. S1 re-identified the function $f(x) = x^2 + kx$ graph and found that the function $f(x) = x^2 + kx$ had an intersection point on the X axis. Next, S1 connected this finding with the roots of the equation $x^2 + kx = 0$ to determine another way to find the value of k **(An-S1)**. In this case, T1 corresponds to the meaning of the graph $(x) = x^2 + kx$ discovered by S1.

4. Planning and Exploring

To implement the idea agreed with S1, T1 connects the intersection point of the graph of the function $f(x) = x^2 + kx$ (S1's idea) with the roots of the quadratic equation $x^2 + kx = 0$ and finds that $x_1 = 0$ or $x_2 = -k$ **(Sy-T1)**. Next, T1 checks the effectiveness of the synthesis results found, namely $x_1 = 0$ and $x_2 = -k$, by asking S1's opinion about the value of $x_2 = -k$ **(Sr-T1).** S1 states that if $x_2 = -k$, then the graph of the function $f(x) = x^2 + kx$ given in the problem is not correct. T1 rejects S1's statement because the function $f(x) = x^2 + kx$ graph is the information given in the problem. This makes T1 aware of T1's mistake in the problem-solving process (not by what was asked in the question, namely identifying the truth of the function f $f(x) = x^2 - kx + 5$ **(Sr-T1)**. S1 relates the results of T1's examination to the solution steps. The identification step that T1 has carried out is by identifying the roots of the equation $x^2 - kx + 5 = 0$ and finding that $x_1 = \frac{-k+\sqrt{k^2-20}}{2}$ or $x_2 = \frac{-k-\sqrt{k^2-20}}{2}$ **(Sy-S1)**. S1 suggests returning to using the root values of the function $f(x) = x^2 + kx$ after examining the synthesis results that S1 has obtained **(Sr-S1).** T1 rejected the suggestion from S1 because it was inconsistent, so T1 proposed re-observing the questions given.

5. Problem Identification

S1 again identified $f(x) = x^2 - kx + 5$ (the information asked for in the question) before proposing to redraw the graph $f(x) = x^2 - kx + 5$ to check its suitability with the graph drawn by Saila **(An -S1)**. At the same time, T1 identifies the elements of the function $f(x) = x^2 - kx + 5$ to determine whether or not the function $f(x) = x^2 - kx + 5$ can be drawn (idea S1) **(An-T1)**. T1 rejected S1's idea because the value of k had yet to be found. Next, T1 identifies the relationship between the function $f(x) = x^2 + kx$ and the function $f(x) = x^2 - kx + 5$. T1 finds that the value of k in the function $f(x) = x^2 + kx$ and the function $f(x) = x^2 - kx + 5$ is the same **(An-T1)**. S1 identifies the elements of the function $f(x) = x^2 + kx$ and the function $f(x) = x^2 - kx + 5$ before agreeing with T1's statement **(An-S1).** T1 and S1 agree that the value of k in the function $f(x) = x^2 + kx$ and the function $f(x) = x^2 - kx + 5$ is the same.

6. Planning and Exploring

T1 checks the correctness of the information given in the problem by drawing a graph of the function $f(x) = x^2 + kx$ before further identification **(Ev-T1).** This was triggered by T1's statement, which stated that the graph provided in the question was wrong at the second PE stage (Number 4). T1 connects the location of the intersection point of the function graph $f(x) = x^2 + kx$ with the value of the roots of the quadratic equation $x^2 + kx = 0$ and finds that the intersection point of the function graph is (0,0)and $(-k, 0)$ **(Sy-T1)**. S1 approves the results of T1's synthesis. T1 tries to connect the roots of the quadratic equation $x^2 - kx + 5 = 0$, the intersection point of the function graph $f(x) = x^2 - kx + 5$ and the drawn graph of the function $f(x) = x^2 + kx$ by Saila. However, T1 realized that the relationship must be corrected because no conclusion could be drawn **(Sr-T1).** T1 connects the intersection point of the graph of the function $f(x) = x^2 + kx$ with the location of the graph of the function $f(x) = x^2 + kx$ in the Cartesian plane and finds that the value of k is negative **(Sy-T1)**. T1 connects the synthesis results, namely the inequality $0 < k^2 < 20$ and the synthesis results of negative k values and finds that the possible k values are $-1, -2, -3$, and -4 **(Sy-T1)**. S1 agrees with T1's synthesis results, which state that the possible k values are $-1, -2, -3$, and -4.

7. *Execution*

T1 explained to S1 the application of the solution idea by assuming $k = -n$ to obtain $f(x) = x^2 + nk + 5$ **(Ar-T1)**. T1 realized that the analysis did not solve the problem because it contained variable n **(Sr-T1)**. S1 proposes to find the discriminant value. T1 explained to S1 that the discriminant value cannot determine the value of k because the discriminant function $f(x) = x^2 - kx + 5$ contains the form k^2 **(Ar-T1)**. Applying ideas from S1 and T1 did not find a solution, so T1 looked for other alternative solutions.

8. *Planning and Exploring*

T1 connects the function $f(x) = x^2 - kx + 5$ with the concept of intercept and finds that whatever the value of k, the function $f(x) = x^2 - kx + 5$ will still be tangent to the X axis **(Sy-T1)**. S1 approves the results of T1's synthesis. T1 re-explained the agreed solution idea by visualizing it in the cartesian plane **(Ar-T1).**

9. *Verify*

S1 re-examines the point drawn by T1. According to S1, the location of a point if $x = 0$ is on the Y axis **(Ev-S1)**. T1 re-examines the solution ideas that have been put forward and connects them with the concept of intersection to find where the ideas that have been put forward are wrong **(Sr-T1)**. T1 agrees with S1's rebuttal. S1 re-examined the synthesis results, which found $k^2 < 20$ based on the fact that the discriminant value of the function graph $f(x) = x^2 - kx + 5$ drawn by Saila did not touch the X axis **(Ev-S1)**. T1 rechecked the synthesis results, which found $k^2 < 20$ based on algebraic facts and a negative k value based on the function graph $f(x) = x^2 + kx$ **(Ev-**

T1). T1 realized that the graph drawn by Saila was not necessarily correct after T1 reread the question **(Sr-T1)**. S1 rechecks the question's meaning to check the truth of T1's statement **(Ev-S1)**.

10. Planning and Exploring

T1 again observes the location of the intersection point of the function $f(x) = x^2 + kx$ graph to ensure the correctness of the synthesis result, namely that k must be negative. S1 stated to T1 that from the start, the value of k had been agreed to be negative. S1 connects the synthesis results, namely the negative k value, the known data, namely the graph of the function $f(x) = x^2 + kx$, and the data in question, namely the truth of the graph $f(x) = x^2 - kx + 5$. Finally, S1 found an idea for a solution: checking the suitability of the negative k value on the graph $f(x) = x^2 - kx + 5$ **(Sy-S1)**. T1 agrees with the results of S1's synthesis. T1 proposed using the X-axis intersection point. S1 explained to T1 that T1's idea, namely using the X-axis intersection formula, was inappropriate because it still contained the form k^2 **(Ar-S1)**. T1 lists several elements related to the quadratic function that fulfils the solution idea proposed by S1 and finds that the turning point abscissa formula is the most suitable **(Sy-S1)**. T1 states that the graph drawn by Saila should be different from the graph $f(x) = x^2 + kx$.

11. Verify

S1 rechecks the suitability of the abscissa value of the turning point of the function $f(x) = x^2 - kx + 5$, namely $\frac{k}{2}$, with the location of the turning point of the graph of the function $f(x) = x^2 - kx + 5$ drawn by Saila (synthesis process T1) before agreeing to the conclusion stated by T1 **(Ev-S1)**. S1 stated to T1 that the graph drawn by Saila was correct because the final value of $\frac{k}{2}$ found was positive and corresponded to the location of the turning point of the graph of the function $(x) = x^2 - kx + 5$ drawn by Saila (T1's synthesis was not correct). S1 re-observes the agreed value of k and states that the abscissa value of the turning point of the function $f(x) = x^2 - kx + 5$ is negative (agrees with T1's opinion) **(Sr-S1)**. T1 explained the problem-solving process to S1, where Saila should draw the function graph $(x) = x^2 - kx + 5$ based on the abscissa value obtained **(Ar-T1)**. S1 explains the problem-solving process to T1, namely comparing the abscissa value of the turning point of the function $f(x) = x^2 + kx$ and the function $f(x) = x^2 - kx + 5$ (problem-solving process according to S1) **(Ar-S1)**. S1 and T1 agreed that the graph drawn by Saila was incorrect.

Group 2 (S2R1)

1. Problem Identification

After reading the problem, R1 identified the element $f(x) = x^2 - kx + 5$ and then connected it to the graph drawn by Saila **(An-R1)**. R1 conveyed to S1 that Saila's statement was correct because the graph drawn by Saila corresponded to the coefficient value x^2. On the other hand, after reading the problem, S1 proposed another idea for a solution.

2. *Planning and Exploring*

S2 connects the differences between the graph $f(x) = x^2 + kx$ and the graph drawn by Saila and finds that the concept of discriminant can be used to solve the problem **(Sy-S2)**. Meanwhile, R1 connects the graph elements $f(x) = x^2 + kx$ and the general form of the quadratic function and finds that the value of k must be found first through the vertex $f(x) = x^2 + kx$ **(Sy-R1)**. R1 applies the idea to the function $f(x) = x^2 + kx$ to check the proposed idea **(Sr-R1)**. R1 dropped the idea. S2 connects the graph $f(x) = x^2 + kx$ with the concept of discriminant and finds that $k^2 > 0$ **(Sy-S2)**. Thus, the value of k is obtained from $f(x) = x^2 + kx$. R1 agrees with S2's idea of finding the value of k from the function $f(x) = x^2 + kx$.

3. *Execution*

S2 explained to R1 the process of finding k systematically and based on the intersection point of the graph $f(x) = x^2 + kx$ **(Ar-S2)**. However, the S2 explanation produces a k value that cannot be precisely determined.

4. *Verify*

R1 checks why k has not been found. R1 checks the k value based on the graph position $f(x) = x^2 + kx$ regarding the X axis **(Ev-R1)**. R1 finds the value $k > 0$ because $k^2 > 0$. S2 refutes R1's opinion by providing a counter-example, namely $k > 0$ also results in $k^2 > 0$ **(Ev-S2)**. R1 receives a rebuttal from S2.

5. *Problem Identification*

S2 re-explores the graph $f(x) = x^2 + kx$ to complete the undiscovered intersection point **(An-S2)**. S2 has not been able to determine the use of analysis results in solving problems. R1 describes the meaning of the task given, namely finding two intersection points of the graph $f(x) = x^2 + kx$, which can be used to find the truth of the graph drawn by Saila **(An-R1)**. S2 agrees with the analysis results from R1.

6. *Planning and Exploring*

S2 connects the intersection point $(x) = x^2 + kx$ with the function and graph $f(x) = x^2 - kx + 5$ based on the results of R1 analysis. S2 concluded that Saila's answer was correct **(Sy-S2)**.

7. *Verify*

S2 connects the intersection point $f(x) = x^2 + kx$ with the function and graph $f(x) = x^2 - kx + 5$ based on the results of R1 analysis. S2 concludes that the value of $f(x)$ at $f(x) = x^2 - kx + 5$ is never zero, so Saila's answer is correct **(Sy-S2)**. R1 asked the logic of the method used by S2 to connect the function $f(x) = x^2 + kx$ with the function $f(x) = x^2 - kx + 5$ **(Ev-R1)**. S2 explained to R1 the synthesis stage and the underlying reasons based on the task's meaning **(Ar-**

S2). R1 refutes S2's explanation by suggesting a more appropriate alternative using the discriminant value **(Ev-R1)**. S2 re-examines the problem-solving process that has been carried out using descriminants **(Sr-S2)**. S2 found the discriminant value of the function $f(x) = x^2 - kx + 5$, namely $k^2 - 20$ (did not find the specific discriminant value used to check the correctness of the graph drawn by Saila.

8. Planning and Exploring

R1 connects the peak point with the data in question, namely the truth of the graph drawn by Saila and finding the peak point can be used to determine the truth of the graph drawn by Saila **(Sy-R1)**. S2 agrees with R1's idea and determines the peak point using the formula $x = \frac{-b}{2a}$. R1 checks the effectiveness of the concept chosen by S2 before rejecting S2's idea. R1 proposed using the formula for the ordinate value of the vertex, namely $y = \frac{-D}{4a}$ **(Ev-R1).** S2 checks the effectiveness of the concept chosen by R1 before rejecting R1's idea. R1 states that the formula $y = \frac{-D}{4a}$ still contains a discriminant value that has yet to be found **(Ev-S2)**. S2 and R1 agreed to return to the results of the answer written by S2; namely, Saila's answer was correct because they did not find any other alternative method.

The results of the analysis of potential problem in exploring students' critical thinking skills based on the results of video-audio recordings and observations during collaborative problem-solving are presented in Table 6.

Table 6. Visible Group Critical Thinking Skills Indicators

Code	Group 1		Group 2	
An	T1	- Identify relationships between known data to understand the problem. - Identify known data elements to determine whether or not the S1 idea can be implemented. (*Trigger*: new idea resulting from analysis proposed by S1).	S2	- Exploring known data to complete parts of data that have yet to be identified. (*Trigger*: group condition that has not found a solution by the agreed solution idea).
	S1	- Identifying relationships between known data to understand the problem. - Describe the relationship between the results of the T1 analysis and the information asked for in the question. (*Trigger*: T1 analysis results). - Identifying known data using other concepts that have never been proposed. (*Trigger*: group condition	R1	- Identifying known data and connecting it with the data being asked. - Identify the suitability of the data found with the facts. - Describe the meaning of the data being asked. (*Trigger*: S2 has yet to find the use of the analysis results in solving the problem).

Code	Group 1		Group 2	
		that has not found the right solution idea). - Identify the information asked for in the question. (*Trigger*: T1 statement stating that the S1 synthesis results are inconsistent). - Identify known data elements to determine whether idea T1 is possible. (*Trigger*: new idea resulting from analysis submitted by T1)		
Sy	T1	- Connecting known data using a concept that T1 already knows. (*Trigger*: results of S1's analysis of the information asked for in the question). - Connecting S1's ideas with concepts that T1 already knows. (*Trigger*: results of S1 analysis on data known to use other concepts). - Connecting the synthesis results with concepts that T1 already knows. - Connect several synthesis results that have been found to determine a solution idea. - Connecting known data with concepts that T1 already knows. (*Trigger*: implementation of the agreed solution idea does not find a solution). - List several elements related to the concept that fulfil the idea of completion. (*Trigger*: S1's argument stating the weakness of the idea T1 proposed).	S2	- Connecting known data based on concepts that S2 already knows. - Connecting known data with concepts that S2 already knows. (*Trigger*: cancellation of the problem-solving idea carried out by R1). - Connect the results of their analysis with the data in question. (*Trigger*: results of analysis carried out by R1).
	S1	- Connect the results of the T1 examination in the problem-solving process with the concepts that S1 already knows. (*Trigger*: T1 check on own statement). - Connecting several synthesis results to determine a solution idea. (*Trigger*: The group has yet to find the right solution.)	R1	- Relate known data elements to definitions. - Connecting known data with other concepts not proposed in the forum. (*Trigger*: S2 check result that finds R1's idea unusable). - Connect the analysis results found with the results of the S2 examination. (*Trigger*:

Code	Group 1		Group 2	
				results of examining the problem-solving process carried out by S2).
Ar	T1	- Simplify the explanation of solution ideas to S1 using algebra. (*Trigger*: T1 synthesis result agreed upon by S1). - Explain the reasons for rejecting S1's idea by showing where it is inaccurate. (*Trigger*: new idea proposed by S1). - Explain the solution idea to S1 by visualizing the synthesis results obtained. - Explain the problem-solving process to S1 based on the agreed T1 synthesis results. (*Trigger*: S1 agrees with T1's evaluation results after re-examining his statement).	S2	- Explain to R1 the synthesis process, which is carried out systematically and is based on data known in the assignment. - Explain to R1 the synthesis process based on the meaning of the task given. (*Trigger*: R1, who asks the logic of the method used by S2).
	S1	- Explain the reasons for rejecting idea T1 by showing where it is inaccurate. (*Trigger*: new idea proposed by T1). - Explain the problem-solving process to S3 by comparing the synthesis results found. (*Trigger*: T1 explains the process of solving the problem based on the agreed but incomplete results of T1's synthesis).	R1	***Not Visible***
Ev	T1	- Draw graphs based on known concepts to check the correctness of the information given in the question. (*Trigger*: S1's statement stating that the information given in the question is incorrect based on the results of T1's synthesis). - Check again the synthesis results that have been found.	S2	- Refute R1's opinion by providing counterexamples. (*Trigger*: results of evaluation carried out by R1). - Check the effectiveness of the concept chosen by R1 before rejecting the idea proposed by R1. (*Trigger*: idea proposed by R1). - Check the problem-solving process that has been carried out to find the causes of inconsistencies in facts with calculation results. (*Trigger*: the result of R1 synthesis activity).

Code	Group 1		Group 2	
	S1	- Checking T1's arguments using concepts that S1 knows. (Trigger: argumentation carried out by T1). - Check again the synthesis results that have been found. - Check the meaning of the question to check the truth of the T1 statement. (*Trigger*: results of examination (self-regulation) carried out by T1). - Examine T1's synthesis process before agreeing to the conclusions stated by T1. (*Trigger*: conclusion stated by T1).	R1	- Checking the S2 explanation using other data. (*Trigger*: S2's explanation of the synthesis results but cannot yet be applied to solve the problem). - Asking the logic of the method used by S2 in synthesizing. (*Trigger*: synthesis process carried out by S2). - Refute S2's explanation by suggesting a more appropriate concept to the answer. (*Trigger*: S2's explanation of the problem-solving process, which, according to R1, is inappropriate).
Sr	T1	- Checking the effectiveness of the synthesis results that T1 has found in solving problems. - Recheck the suitability of the problem-solving process that has been carried out in T1 with the information asked for in the question. (*Trigger*: S3's statement states that the T3 synthesis results cause the information given in the question to be incorrect). - Recheck the relationship between the synthesis results and the information in the questions T1 has created. (*Trigger*: T1 did not find the conclusion used as a resolution idea). - Re-examine the effectiveness of the proposed solution ideas for the problem-solving process. - Check where there are errors in problem-solving ideas proposed using concepts that T1 already knows. (*Trigger*: evaluation carried out by S1). - Re-read the questions to investigate the accuracy of the agreed meaning of the questions. (*Trigger*: evaluation carried out by S1).	S2	- Re-examine the problem-solving process that has been carried out using the R1 idea. (*Trigger*: R1's evaluation of the argument S2 put forward). - Recalculate the calculations that have been carried out to check the findings. (*Trigger*: results of analysis carried out by R1).

Code	Group 1		Group 2	
	S1	- Checking the synthesis results again using concepts that S1 already knows. - Review the synthesis results before deciding to return to using old ideas. - Checking the solution that S1 has found by connecting it to the agreed synthesis results.	R1	- Apply the idea to known data to check whether the idea can be implemented. (Trigger: R1's synthesis).

Based on Table 6, several indicators of critical thinking skills need to be visible based on the results of video-audio recordings and observations during problem-solving, namely *argumentation in R1*. Based on these results, there are indications that although the problems developed can trigger collaborative problem-solving conditions, the problems developed cannot be used to see students' critical thinking skills fully. Triangulation is needed by conducting interviews after the group has completed the task so that indicators of critical thinking skills can be seen and explored through interviews. Therefore, auxiliary instruments are still needed as guidelines for group and individual interviews. Individual interview guidelines are needed if there is a group where one member dominates when solving problems or conducting group interviews. The interview guide is semi-structured by adjusting the results of problem-solving that students have worked on. In this research, researchers conducted group interviews with group 2 to explore argumentation indicators in R1. Group interviews were chosen because S2 and R1 did not dominate each other when solving problems. This can be seen at every problem-solving stage; both S2 and R1 contribute to the problem-solving process. The following is an interview conducted by researchers.

P : Now, try to observe the location of the intersection point you found with this $f(x) = x^2 + kx$.

S2 : (write down the two points on the graph $f(x) = x^2 + kx$ provided). The coordinate here should be $(k, 0)$ because it is the coordinate with x being positive. But based on the calculation, the k value is negative.

R1 **: We choose negative. In terms of location, the k value should be positive. If we choose negative k, the position and the equation you find, namely $x = -k$, can be the same. Note $x = -k = -(-k) = k$. So that's positive—the same as the positive position.**

Based on the interview excerpt, R1 explained the problem-solving process to S2 based on the relationship between existing data. Thus, the argumentation indicator in R1 was found when a semi-structured interview was conducted by adapting the results of problem-solving that had been carried out by group 2.

DISCUSSION

Critical thinking skills can be triggered by social interaction. One of them is collaborative problem-solving settings because, in collaborative problem-solving settings, there will be cognitive and verbal activities that are interdependent in the context of problem-solving. This is also reinforced by Vygotsky's sociocultural theory and the Zone of Proximal Development (ZPD) model, which states that conversations with peers will expand students' ZPD to think critically (Wass et al., 2011). Appropriate instruments are needed to explore students' critical thinking skills in collaborative problem-solving to obtain accurate and in-depth data. The potential of non-routine mathematical problems developed in exploring students' critical thinking skills in CPS was discussed from two perspectives: students' interaction and students' critical thinking skill that emerged in the process. The interaction between group members in completing tasks collaboratively was visible in both groups. Responding to this, students admitted that the task was difficult but could be done because each student succeeded in contributing to the group and completing each other's steps. Several studies state that the questions' difficulty level (Chiu, 2008; Graesser et al., 2017; Westermann & Rummel, 2012) can encourage interaction in CPS. Paying attention to the problem-solving activities carried out by both groups, the task does not have the potential to trigger a division of labor to obtain a solution. Each stage completed is completed collaboratively and recorded in one shared workspace.

Using questions with a graphical analysis approach triggers students to explore graphs by being given various concepts. The results of observations in both groups showed that several graphic explorations had been carried out, namely checking the location of the intersection of the X and Y axes, checking discriminant values, checking the location of the peak point, checking the axis of symmetry of the graph and determine the possible direction of shift of the graph. Bezanilla et al (2019) stated that resource exploration activities like graphs would trigger students to think critically. Apart from that, in this graphic exploration activity, group members provide opinions to each other based on their knowledge. They give each other ideas that help check the solutions' correctness. This activity of sharing understanding allows for debate to criticize other people's thoughts and one's thoughts (Häkkinen et al., 2017). Thus, this aligns with research results showing that critical thinking activities that emerge in students are triggered not only by the questions given but also by ideas, statements, or problem-solving processes carried out by other group members. This mutually triggering activity also shows that CPS impacts the development of students' critical thinking activities.

The results showed that more than giving non-routine math problems was needed to explore students' critical thinking skills in collaborative problem-solving. Another auxiliary instrument is still needed namely task-based interview guidelines. Interviews are needed to examine indicators of critical thinking skills in collaborative problem-solving that cannot be explored by giving non-routine problems. Several previous studies also used interview guidelines to collect data on student's critical thinking skills (Ariza, 2021; Dolapcioglu & Doğanay, 2022; Setiana et al., 2021). Li & Ren (2020) research states that interviews will provide more precise results in exploring students' critical thinking skills. Further research is needed regarding the use of interviews to stimulate students' critical thinking.

The research results will add valuable insight for researchers and practitioners in designing non-routine mathematics problems that can be used to explore students' critical thinking abilities in collaborative problem-solving. However, the results of this research still have limitations. These limitations include the number of participants used in only two groups. It would be better if there were more participants so that the potential of the task could be explored more. In addition, participants in this study were selected based on high, medium and low mathematics abilities. This ability is determined using student report scores. Discrepancies between student-reported scores and standardized tests may result in the selection of different participants and subsequently influence research findings.

CONCLUSION

Based on the results of the analysis that has been done, it can be concluded that giving non-routine mathematics problems with problem criteria that require justification for the solutions given and problems with a graphical analysis approach can be used to explore students' critical thinking skills in collaborative problem-solving. This is proven by solving the problems; each group member contributed to the solution-finding process. The contribution of each group member shows the high intensity of interaction between members. Interaction in the form of exchanging opinions, giving suggestions, and evaluating each other's ideas or answers significantly impacts students' critical thinking abilities. This is seen by the emergence of several students' critical thinking skills (analysis, synthesis, argumentation, evaluation, self-regulation) triggered by suggestions or ideas put forward by other group members. Thus, the non-routine questions developed can explore students' critical thinking skills in CPS. However, the analysis results also show that more than giving non-routine math problems with the abovementioned criteria are needed to explore students' critical thinking skills in collaborative problem-solving. An auxiliary instrument is still needed, namely an interview guide. This shows that in addition to giving non-routine math problems, triangulation methods are still needed in conducting interviews to explore students' critical thinking skills in collaborative problem-solving.

Another thing that needs to be considered is the closeness between students in a group. Interaction between students will run well if one student avoids dominating the other. The closeness between students in a group needs to be considered mainly if the group consists of members with significant differences in cognitive abilities, for example, high and low abilities. Lastly, the researcher hopes that the results of this research can be helpful for researchers or practitioners as a reference in developing or researching critical thinking skills, especially in collaborative settings.

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The Situation of Mathematical Problem Solving and Higher Order Thinking Skills in Traditional Teaching Method and Lesson Study Program

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Abstract: *The purpose of this mixed-method study is to investigate and compare students' problem-solving and higher order thinking skills. This study involves two contexts: a traditional teaching program and a Lesson Study program. In qualitative phase, seven mathematics lecturers, a physics lecturer and the researcher forms the Lesson Study group and they collaboratively design five research lessons on functions topics. Also, the researcher collects all the materials taught by the lecturers individually in their classes on these topics. These topics in the textbook, individual lecturers' lessons and research lessons analyse descriptively using document analysis technique to find some understanding on the emphasis on problem-solving and higher order thinking skills. In quantitative phase, two classes choose randomly as experimental and control groups. The lecturers' developed tests use to compare the ability of students in problem-solving and higher order thinking. Data was analysed using independent samples t-test. The results of this mixed-method study show that, lecturers were teaching exactly contents of the textbook and focusing more on solving of routine exercises. Whereas, in Lesson Study program, collaborative work among lecturers in preparing suitable problem-solving activities not only improved lecturers' knowledge tremendously but also enhanced the ability of students in problem-solving and higher order thinking skills.*

1. INTRODUCTION

Teaching mathematics particularly at higher levels is more challenging than secondary school levels because of complication of the problems and concepts. Malaysian students who have completed secondary schools continue pre-university programs through pathways foundation, matriculation and A-level that lasts one year. Students choose these pre-university programs based on their results in courses Additional Mathematics, Mathematics, Physics, Chemistry and Biology at high school. Students' performance in these programs provides vast opportunity in being selected to the study majors of top universities. In these yearlong programs, the lecturers' competency in teaching mathematics greatly influences learners' performance in problem-solving. Therefore, the content knowledge and pedagogical content knowledge of lecturers are crucial in ensuring that learners knowledge have enormously enhanced to guarantee placement in competitive majors in top universities (Johannsdottir, 2013). The lecturers are recommended to prepare appropriate mathematical materials in their lesson plans to improve the ability of students in problem-solving and higher order thinking skills. Lecturers require knowledge on different pedagogical approaches and techniques for delivering each new topic thus they must constantly enhance and update their teaching knowledge to have better performance in their classes through problem-solving approach. To upgrade the teaching knowledge of educators, they should share the best practices on teaching a particular content to students through problem-solving approach and critical thinking. Fujii (2016) and Mon, Dali, and Sam (2016) highlighted that collective and cooperative teamwork among mathematics educators helps to improve their subject matter knowledge and pedagogy to acquire best practices in effective mathematics teaching through problem-solving approach.

2. LITERATURE REVIEW

The materials and lessons that design by lecturers play an important role in enhancing learners' interests in learning mathematics (Gholami et al., 2021). Doing suitable problem-solving activities based on different levels of higher order thinking skills by students help them to have higher confidence in mathematics classes and improve their abilities in mathematics. Lomibao (2016) explained that lesson planning has generally been a solitary task among mathematics teachers. Therefore, educators determine the materials individualy which are used for students with different qualities (Lomibao, 2016). Therefore, mathematics educators require having appropriate pedagogical content knowledge and subject matter knowledge about different topics in order to plan effective lessons.

In pre-university level, the central concept of function is complex and difficult (Akkus et al., 2008; Doorman et al., 2012; Ponce, 2007). Although teaching and learning function topics are deemed to be problematic and challenging, it is considered as an important topic in university programs to connect real world problems to mathematics. Modeling the real world problems is one of the most important applications of mathematical functions that explain the physical problems through mathematical language (Michelsen, 2006).

In traditional method of teaching, mathematics educators emphasize on giving lectures and solving of routine exercises in the process of teaching. However, this lecturer-centered method cannot enhance students' skills in problem-solving and higher order thinking (Khalid, 2017; Mon et al., 2016). In this method, most of learners just memorize the mathematical materials and routine solution methods to apply in solving exercises or examination questions. Traditional teaching method of mathematics is grounded on the behaviorist learning theory. This theory is according to the premise that a learner should build habit formation based on stimulus-response process and learning happens as there is a change in learner behavior (Ormord, 1995). Lesson Study is a strong professional development program for enhancing the teaching knowledge of educators (Fujii, 2016). This educational approach is based on the cognitivist learning theory and heavily emphasizes on problem-solving and higher order thinking in teaching mathematics. In mathematics teaching, problem-solving skills help learners to understand a domain of complex mathematical structures and acquire the skills to solve real life problems (Tarmizi & Bayat, 2012).

The purpose of this research is to investigate the emphasis on mathematical problem-solving and higher order thinking skills in the traditional teaching method and Lesson Study program.

2.1.Mathematical Problem-Solving

Based on Xenofontos and Andrews (2014), a question is considered as a mathematics problem if the task is new and challenging for students whereas a routine task with clear process of its solution is called mathematics exercise. Problem solving approaches are used to help students learn how to think mathematically in solving problems (Purnomo et al., 2022). However, in mathematics problem-solving students engage with critical thinking and higher order thinking (Yassin & Shahrill, 2016). In this research, every mathematics problem that associated with students' everyday life, real world and other subjects such as physics and chemistry is considered as a practical problem (Gholami, 2021). For instance, the following problem is a practical problem.
Problem: The number of bacteria in a culture is $B(t)$ after t minutes. The relationship between the elapsed time t, in minutes, and the number of bacteria, $B(t)$ in the petrol dish is modelled by the function $B(t) = 10.2^{(\frac{t}{12})}$.

a. How many bacteria will make up the culture after 120 minutes?
b. After how many minutes will the population of bacteria be 5.2^{16}?

Polya (1945) had suggested four phases for mathematical problem-solving namely understanding the problem, planning a strategy, implementing the plan, and confirming the solution. It is essential that mathematics educators initially encourage and engage students in solving different levels of problems based on higher order thinking skills in order to improve their ability in learning mathematics. Mathematics educators believed that it is challenging to encourage students in solving open-ended problems and ask them to explain what problem-solving strategies they have used (Johnson & Cupitt, 2004; McDonald, 2009). However, mathematics educators require to consider the levels of difficulty in assigning open-ended problem to students and design them based on students' skills, hence every learner would be able to solve the problems to some extent (Asami-Johansson, 2015; Bergqvist, 2011). Despite different educational backgrounds in elementary and secondary mathematics education may affect student's ideas and reasoning in solving mathematics problems in foundation level, appropriate teaching method through problem-solving approach can improve the ability of students in problem-solving in yearlong foundation program (Lu & Richardson, 2018).

2.2.Higher Order Thinking Skills

Mathematics educators and the teaching materials play major roles in improving students' higher order thinking skills. Lack of content knowledge among some mathematics instructors regarding promoting higher order thinking skills and developing questioning techniques and teaching resulted in teachers employing traditional method in teaching (Alhassora et al., 2017). Malaysian Ministry of Education (2014) have a lot of emphasis on integrating abilities of higher order thinking among learners as the key factor to enable them for international competitions.

Bloom (1956) categorized skills of thinking ranging from concrete to the abstract: which are remembering, understanding, applying, analysing, evaluating and creating. Based on Thomas and

Thorne (2014) higher order thinking defines as thinking skill which is beyond the memorization level. The last three levels of Bloom's Taxonomy, namely analysis, synthesis, and creativity are considered as higher order thinking skills (McBain, 2011).

Malaysian Ministry of Education (2014) defined higher order thinking skills as skills of applying knowledge, abilities of argument and reflection problem-solving, decision making, innovating and to creating something new. Based on this definition, the last four levels of the revised Bloom's Taxonomy, which are applying, analysing, evaluating and creating, are classified as higher order thinking skills, as shown in Table 1.

Table 1: Components of Higher Order Thinking Skills

Level	Explanation
Applying	Using the knowledge, skills and values in different situations to take matters.
Analysing	Breaking down the information to better understand the relationship between the divisions.
Evaluating	Making judgments and decisions using the knowledge, experience, skills and values and justify.
Creating	Produce a product or idea or create and innovative methods.

Source: Ministry of Education (2014)

2.3.Lesson Study

Lesson Study approach, as a beneficial method for enhancing educators' professional development, has been used by Japanese teachers since the 1950's (Abiko, 2011). Yoshida (1999) translated the Japanese term "*Jugyo Kenkyu*" into Lesson Study and these two Japanese words *Jugyo* and *Kenkyu*, means lesson and study respectively. However, Lesson Study has been most popular since 1999 among mathematics teachers and researchers. Lesson Study refers to collaborative work of educators (Lesson Study group) on some mathematics topics to plan and design a lesson, teach and observe the lesson and to reflect and discuss on the taught lesson. The purpose of this educational approach is to improve students' ability in problem-solving by presenting effective teaching (Matanluk et al., 2013). The lessons that prepared in the process of Lesson Study program, in Japanese language are called *gakushushido-an*, and translated into research lessons (Fujii, 2016) or study lessons (Yoshida, 1999). Lesson Study focuses on pedagogical progress among educators in which the research lesson considered as a central component (Lewis, 2002).

School-based, district-based, and national-level Lesson Study are three popular kinds of Lesson Study in Japan (Fujii, 2016). The process of all forms of Lesson Study is essentially similar however; the only difference is related to the range of students. School-based Lesson Study is merely suitable for learners in the specific school; district-based Lesson Study is more applicable in the district; and national-level Lesson Study is mostly focused on the learners around the country (Fujii, 2016; Takahashi & McDougal, 2016). Several Lesson Study models have been developed by Yoshida (1999), Takahashi (2001), Richardson (2004) and Fuji (2014). Fujii (2014) suggested the following five steps for implementation of Lesson Study program:

a. Goal setting: Mathematics instructors focus on the long-standing targets to improve the ability of students in problem-solving and higher order thinking skills
b. Lesson planning: Teachers through collaborative work try to prepare suitable mathematical materials based on the different levels of higher order thinking.
c. Research lesson: The members of Lesson Study group provide a proper research lesson, one of them teaches the research lesson and other members observe and collect data for next step.
d. Post-lesson discussion: Educators in post-lesson discussion discuss on students' misconceptions, students' learning, variety of solutions and the levels of higher order thinking for the given problems to improve the quality of research lesson.
e. Reflection: In this phase, teachers collaboratively consider some new problems in the research lesson and they discuss on likely solutions for next cycle of Lesson Study. Finally, they prepare a report on their output.

In Lesson Study approach, educators by considering the ability of students, engage them with suitable problem-solving activities based on different levels of higher order thinking. It seems that the school-based Lesson Study is the most effective form of Lesson Study because educators in a specific educational center plan and design the research lessons according to their students' abilities in problem-solving (Takahashi & McDougal, 2016).

2.4. Research Question and Hypothesis

The research question (qualitative part) and hypothesis (quantitative part) of this research are as follows:

Research question: Is there a difference between the Lesson Study program and the traditional teaching approach in terms of problem-solving and higher order thinking skills?

Hypothesis: There is significant statistical difference in problem-solving and higher order thinking skills between experimental (Lesson Study program) and control (traditional method) groups.

3. METHOD

The present mixed method study was conducted in a foundation center that offer pre-university program in a public university of Malaysia. Students are selected in foundation programs to continue their studies, based on their good results in high school. There has not been any research which compares learners' problem-solving and higher order thinking skills between those who undergone Lesson Study program and traditional teaching method for foundation level students. After obtaining permission from the director of the foundation center, all lecturers and students who were involved in the study signed the disclosure letter.

3.1. Qualitative Phase (Case Study)

In this foundation center, nine mathematics lecturers (four male and five female) were teaching 20 classes with 952 students (326 males and 626 females). Eight mathematics lecturers and two physics lecturers volunteered to participate in this research, however, a mathematics lecturer and a physics lecturer later withdrew from the study due to their time constraint. Thus, the Lesson Study group consisted of seven mathematics lecturers, a physics lecturer and the researcher (nine lecturers participated in this part of study). The physics lecturer assisted the mathematics lecturers in constructing practical problems for their research lessons and to identify application of function in physics problems. Meanwhile, in this study, the researcher not only was the coordinator and discussion leader but also participated in discussion meetings as a member of the Lesson Study group.

Before the meeting for each research lesson, the researcher introduced the topic to the lecturers and asked them to provide appropriate materials, problems, applications of contents and the suitable pedagogy to implement the contents. This process was followed with a two hour session by the Lesson Study group members to plan, discuss and prepare the research lesson. In the second session, one of the members of the Lesson Study group taught the prepared research lesson and other lecturers observed the teaching. Therefore, they further improved the research lessons based on problem-solving approach and higher order thinking skills.

In foundation program, lecturers teach two textbooks namely Mathematics 1 and Mathematics 2 during first and second semesters respectively. These textbooks comprise of chapters related to algebra, calculus, trigonometry, geometry, probability and statistics. In the Mathematics 1 textbook, about two fifths of contents are allocated to the mathematical functions. In this study, the researcher chose the topic on functions because it is problematic concept for lecturers to teach and for students to learn (Oehrtman & Carlson, 2008). The members of Lesson Study group through collaborative work planned, designed, discussed and improved five research lessons as shown by Table 2.

Table 2: The Topics of the Research Lessons

Research lesson	Topic
1	Relation and function concepts
2	Domain and range of the functions and algebraic combination
3	Composite function, inverse function, odd and even functions
4	Trigonometric functions
5	Exponential and logarithmic functions

For all 20 classes, the researcher had earlier requested some students (two students in each class) to jot down all the materials related to these five topics of this study taught by the lecturers in their classes. This issue allowed the researcher to compare the situation of problem-solving and higher order thinking between the traditional method and Lesson Study program. Research lessons and the individual lecturers' original lessons were analysed descriptively using document analysis technique. Initially, the Mathematics 1 textbook was analysed to find the situation of higher order thinking and problem-solving in this textbook. The tasks of textbook were compared with each other to determine that each task is mathematics exercise or mathematics problem. For instance, in Mathematics 1 textbook there were 18 similar routine exercises on topic of composite function. After that, the research lessons and the lecturers' lessons were compared with the contents flow and approach in the textbook. Lastly, the research lessons and individual lecturers' lessons were compared to find the extent of the implement of problem-solving in traditional method and Lesson Study Program. Through similar process the status of higher order thinking skills compared between traditional teaching method and Lesson Study program based on the levels of higher order thinking skills of Malaysian Ministry of Education (2014). Meanwhile, after analyzing the mathematical problems and higher order thinking in the research lessons, the materials in the textbook and the individual lessons of the mathematics lecturers were analysed, these results were studied by three professors from the mathematics department of one of the public universities in Malaysia. The researcher applied their comments and used the obtained results. Figure 1 represents the process of data analysis in this study.

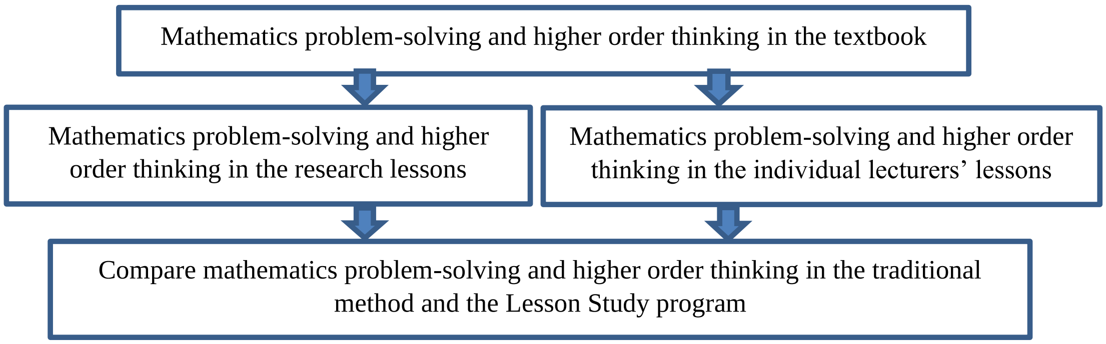


Figure 1: The Process of Data Analysis in this Study

Source: Designed by the Author

3.2. Quantitative Phase (Quasi-experimental Research)

The participants of this part of the study were a total of 86 students, comprising of 44 learners in the experimental class (Lesson Study program) and 42 learners in the control class (traditional method). The teaching videos of these lessons were recorded in both groups. The measurement used in this part of study was the mathematics tests that developed by the lecturers involved in the Lesson Study program. Although in foundation center, textbooks used are in English and all subjects are taught in English, the mathematics tests also includes the Malaysian language (Bahasa Melayu) translation so as to avoid ambiguity for learners. Thus back to back translation from English to Bahasa Melayu was done by two lecturers of English Language from one of the faculties from the same university. The final version was confirmed by two experts in mathematics education in order to ensure there was no problem in the translation. The mathematics tests contain 12 open-ended problems, which was tested for its contextual validity and reliability and was proved by using Equivalent-forms Method. Six experts in mathematics and mathematics education from a public university confirmed the validity and suitability of these problems. The Pearson correlation significant for pre-test with 31 and post-test with 40 participants outside this research were 0.78 and 0.74 respectively. However, these tests were different because new concepts were taught in these five weeks. Also, post-test was conducted one month after finished the study again as follow-up test. A sample of the pre-test is "The graph of the function $f(x) = \frac{ax+3}{x-b}$ passes from two points (-1, 0) and (1,-2). Find the values of a and b". Also, two samples of the post-test are "Let m be a non-zero constant. Find the two x-values where the graphs of the functions $y = 10^{6m}x$ and $y = \frac{x^2}{10^{5m}}$ intersect" and "Determine whether the function $h(x) = \sqrt{x} + \sqrt{-x}$ is even or odd". Lastly, mathematics tests were confirmed by some experts in the Ethical Committee of the Research Management Center (RMC) of a public university in Malaysia as suitable instruments for the study. The problems in each test were categorised according to the three levels of higher order thinking skills in the revised Bloom's Taxonomy, namely applying (four questions), analysing (four questions) and evaluating (four questions) based on Table 1 and was confirmed by three mathematics experts in the same university. Therefore, the researcher compared the results of the students in problem-solving and higher order thinking based on the three levels of revised Bloom's Taxonomy, between the experimental and control classes.

The students' answers in the tests were scored by two lecturers, based on Polya's problem-solving model (Brijlall, 2015). If learner wrote illogical and incorrect answer or no answer given, then the question is given a score 0. If some phases are given in the solution that shows the learner understand the problem, the considered score is 1 (first phase of Polya's model). If learner understand and provide a method for solving the problem but made some errors, the score is 2 (first and second phases of Polya's model) and lastly, a complete answer is given a score of 3 (all phases of Polya's model). Therefore, the possible minimum and maximum marks for the mathematics tests were 0 and 36 respectively.

A mathematics lecturer, who was a member of the Lesson Study group, was chosen randomly and his classes were randomly considered as experimental and control groups (each of the mathematics lecturers taught in two classes, except for two of them who had three classes). The Lesson Study was implemented for five weeks, which covered five topics on functions. In the experimental group, the student-centered teaching approach was applied and the lecturer gave some problems and practical problems that students did individually and in teams. It was meant to improve students' abilities in mathematics problem-solving and higher order thinking skills. In teaching the research lessons, the lecturer walked around the class to provide guide, to assess, encourage and engage students in problem-solving activities. In contract, the control group was lecturer-centered and the same lecturer taught the exact same topics but using traditional teaching method, where students are worked individually with emphasis on mathematics exercise solving. In fact, in the control class, the lecturer taught exactly the provided materials of the textbook. Independent sample t-test was used to analyse the data for this part of research.

4. FINDING

4.1. Qualitative Part

The situation of mathematical problem-solving and higher order thinking skills in the foundation level is discussed based on the sources the textbook, individual lecturers' lessons and research lessons.

4.1.1. Textbook

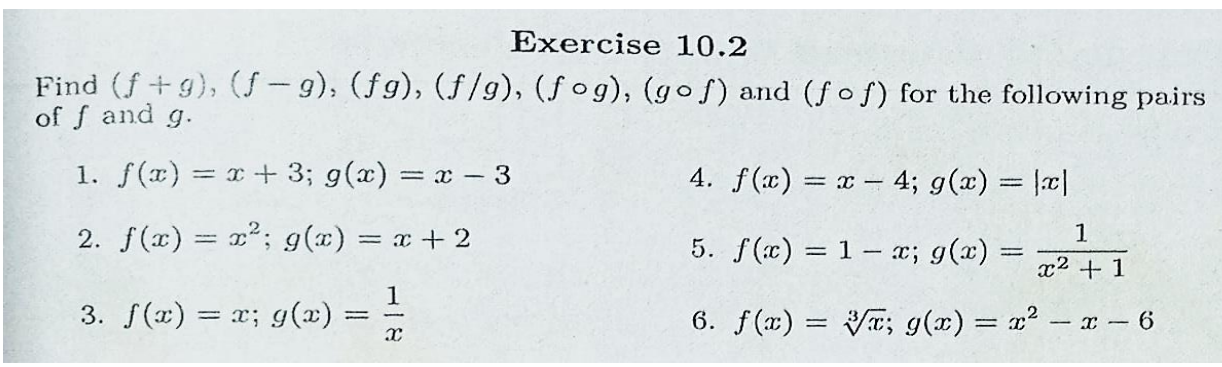
The analysis of contents related to these five topics in the Mathematics 1 textbook showed that the textbook does not emphasises much on problem-solving. There are only a few mathematics problems in this textbook and there are not any practical problem-solving tasks given. In fact, the textbook emphasises more on solving of routine mathematics exercises. The contents of the textbook seem to encourage memorization of formulas, theorems, methods and shortcuts that students can apply in solving other mathematics exercises. Apparently, the textbook approaches do not promote students' abilities in problem-solving. The number of mathematics exercises, mathematics problems and practical problems is showed in Table 3.

Table 3: Textbook's Materials

No.	Topic	Mathematics Exercise	Mathematics Problem	Practical Problem
1	Relation and function concepts	16	1	0
2	Domain and range of the functions and algebraic combination	19	1	0
3	Composite function, inverse function, odd and even functions	26	2	0
4	Trigonometric functions	10	1	0
5	Exponential and logarithmic functions	18	1	0
6	Total	89	6	0

Based on Table 3, only six percentages of tasks in these five topics are mathematics problems whereas other ninety four percentages are merely mathematics exercises.

Many of mathematics exercises that considered in each subtopic are similar in terms of teaching approach and content. For example, in the Topic 3 of this research (composite function, inverse function, odd and even functions), there are 18 exercises related to the composite function that just the rule of functions changed in the tasks. One of these tasks is as follows:
Exercise: If $f(x) = 1 - x$ and $g(x) = \frac{1}{x^2+1}$ find the function fog.
Figure 2 shows some of these exercises on page 157 of the textbook.


Exercise 10.2

Find $(f+g)$, $(f-g)$, (fg), (f/g), $(f \circ g)$, $(g \circ f)$ and $(f \circ f)$ for the following pairs of f and g.

1. $f(x) = x + 3;\ g(x) = x - 3$
2. $f(x) = x^2;\ g(x) = x + 2$
3. $f(x) = x;\ g(x) = \frac{1}{x}$
4. $f(x) = x - 4;\ g(x) = |x|$
5. $f(x) = 1 - x;\ g(x) = \frac{1}{x^2+1}$
6. $f(x) = \sqrt[3]{x};\ g(x) = x^2 - x - 6$

Figure 2: Some Exercises of the Textbook about Composite Function

However, the number of problems is very limited in each subtopic. For example, on page 159 of the Topic 4 (trigonometric functions), only one problem was given (Figure 3), which is as follows:
Problem: Find all angles $(0 \leq \theta \leq \pi)$ which satisfy the equation $4sec^2\theta = 3\tan\theta + 5$.

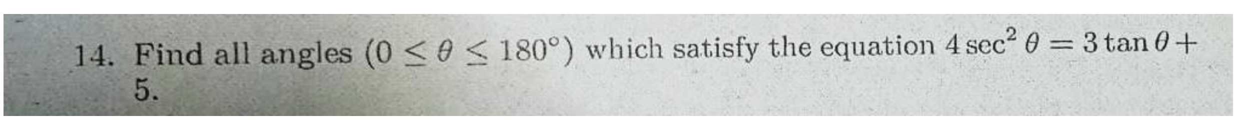

14. Find all angles $(0 \leq \theta \leq 180°)$ which satisfy the equation $4\sec^2\theta = 3\tan\theta +$ 5.

Figure 3: An Example of a Problem in the Textbook

The materials in the Mathematics 1 textbook were also categorised based on the revised Bloom's Taxonomy. Table 4 shows the categories of textbook materials based on the Bloom's taxonomy.

Table 4: The Categorisation of Textbook Materials Based on the Bloom's Taxonomy

Topic	Remembering	Understanding	Applying	Analysing	Evaluating	Creating
1	5	11	1	0	0	0
2	7	12	1	0	0	0
3	14	13	1	1	0	0
4	4	5	0	1	0	0
5	6	12	1	0	0	0
Total	36	53	4	2	0	0

As respect to Table 4, about 93.5 percentages of the materials only require lower order thinking skills and 6.5 percentages of tasks are associated with higher order thinking skills. Therefore, the materials in the textbook do not help much in improving students' higher order thinking skills.

4.1.2. Lecturers' Lessons

The researcher received all the lecturers' lessons of the topics as in Table 2 and compared the tasks applied in each lesson with the contents of the Mathematics 1 textbook to determine the quality of problems posed in each lesson. Although the lecturers H and I did not participate in this research, but in order to get better results, the researcher was coordinated with them that their individual lessons on five topics of this study were also analysed. The results of this part showed that the mathematics lecturers taught the exact same materials of the textbook. According to Table 3, the textbook that was used only include a few mathematics problems. Thus, by relying very much on the materials of textbook, the lecturers were not promoting much of problem-solving in their classes. Meanwhile, none of the lecturers considered practical problem in their lessons. Table 5 shows the number of problems posed by each of the lecturers in their teaching based on the topics of this research.

Table 5: The Number of Mathematics Problems Posed in the Lecturers' Lessons

Lecturer	Highest Degree	Mathematics Problem					Total
		Topic 1	Topic 2	Topic 3	Topic 4	Topic 5	
A	Master	1	1	3	1	1	7
B	Master	1	1	2	1	1	6
C	PhD	1	2	2	2	1	8
D	Master	1	1	2	1	1	6
E	PhD	1	1	2	1	1	6
F	Master	1	1	2	1	1	6
G	Master	1	1	2	1	1	6
H	PhD	1	1	3	1	1	7
I	PhD	1	1	2	1	1	6

Similarly, the levels of thinking that were used by the lecturers in their lessons were also guided by the textbook. Table 6 shows the questions posed in five lessons conducted by each lecturer and categorised based on the revised Bloom's Taxonomy.

Table 6: Number of Tasks Posed in the Teaching Based on the Bloom's Taxonomy

Lecturer	A	B	C	D	E	F	G	H	I
Applying	5	4	5	4	4	4	4	5	4
Analysing	2	2	3	2	2	2	2	2	2

Evaluating	0	0	0	0	0	0	0	0	0
Creating	0	0	0	0	0	0	0	0	0
Total	7	6	8	6	6	6	6	7	6

For example, Figure 4 shows an example of student work. In this session, several similar routine exercises were discussed by the lecturer.

Example: Find fog and gof for the functions $f(x) = 5x + 2$ and $g(x) = \frac{4}{x-3}$.

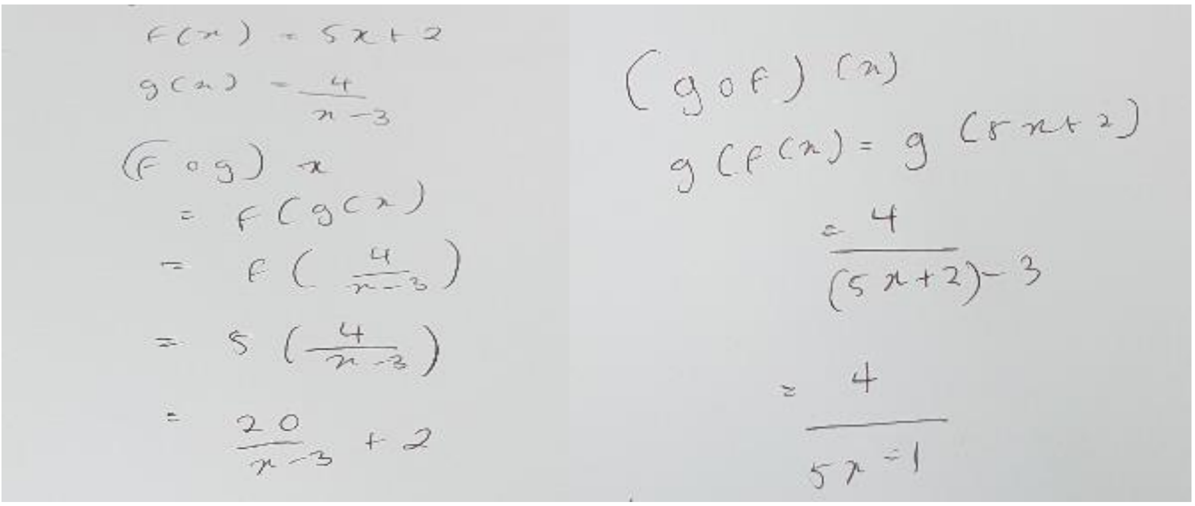


Figure 4: A Student Work about Composite Function

Also, the following two examples that adopted from a lecturer's lesson illustrates he/she taught exactly the same materials in the textbook.

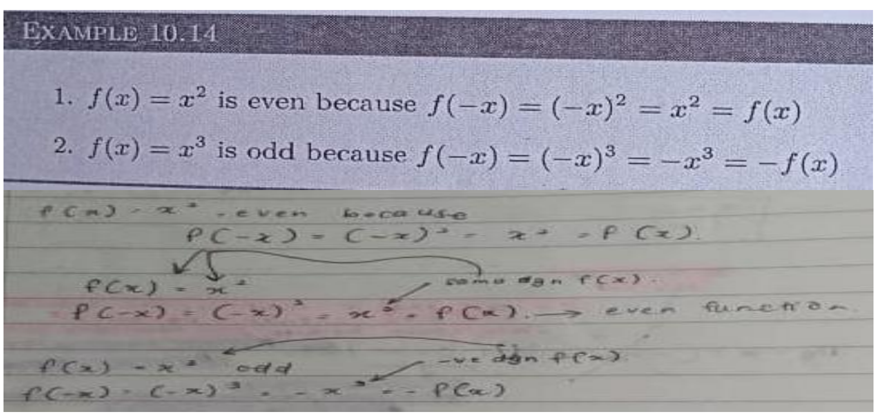
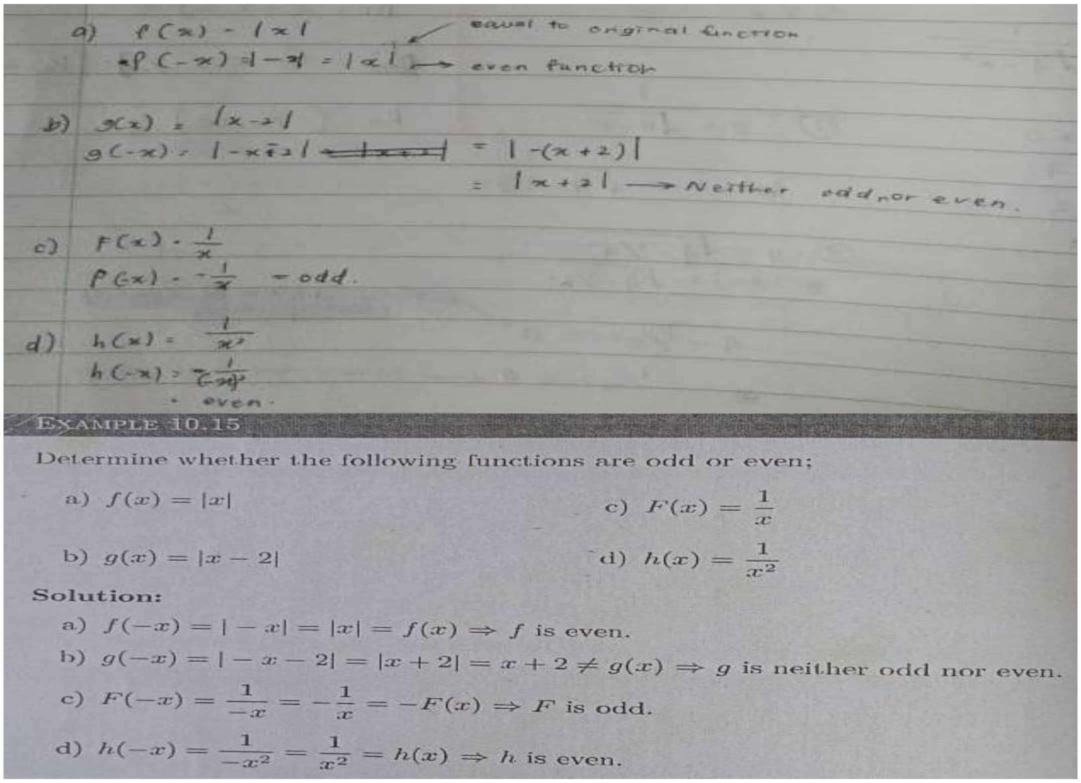


Figure 5: An Example of the Textbook in the Classroom



a) $f(x) = |x|$ — equal to original function
$f(-x) = |-x| = |x|$ → even function

b) $g(x) = |x-2|$
$g(-x) = |-x-2| = |-(x+2)|$
$= |x+2|$ → Neither odd nor even.

c) $F(x) = \frac{1}{x}$
$F(-x) = -\frac{1}{x}$ = odd.

d) $h(x) = \frac{1}{x^2}$
$h(-x) = \frac{1}{(-x)^2}$
= even.

EXAMPLE 10.15

Determine whether the following functions are odd or even;

a) $f(x) = |x|$

b) $g(x) = |x-2|$

c) $F(x) = \frac{1}{x}$

d) $h(x) = \frac{1}{x^2}$

Solution:

a) $f(-x) = |-x| = |x| = f(x) \Rightarrow f$ is even.

b) $g(-x) = |-x-2| = |x+2| = x+2 \neq g(x) \Rightarrow g$ is neither odd nor even.

c) $F(-x) = \frac{1}{-x} = -\frac{1}{x} = -F(x) \Rightarrow F$ is odd.

d) $h(-x) = \frac{1}{-x^2} = \frac{1}{x^2} = h(x) \Rightarrow h$ is even.

Figure 6: A Textbook Example in the Classroom

4.1.3. Research Lessons

During the Lesson Study program, the lecturers had considered suitable mathematics problems into each research lesson. Furthermore, they believed that the integration of practical problems into the mathematics curricula play a major role in encouraging learners in solving mathematical problems. In fact, Lesson Study had helped the lecturers to enhance ability to guide learners in improving their abilities in problem-solving. The analysis in terms of number of tasks, which were categorised based on types of tasks (mathematics exercises, mathematics problems, and practical problems) of the research lessons, is showed in Table 7.

Table 7: The Number of Mathematics Tasks in the Research Lessons

No.	Title of Research Lesson	Mathematics Exercise	Mathematics Problem	Practical problem
1	Relation and function concepts	7	8	3
2	Domain and range of the functions and algebraic combination	14	5	2
3	Composite function, inverse function, odd and even functions	9	12	3
4	Trigonometric functions	6	11	1
5	Exponential and logarithmic functions	11	5	2
6	Total	47	41	11

According to Table 7, the percentages of mathematics exercises, mathematics problems and practical problems are 48, 41 and 11 respectively. In other words, 52 percentages of mathematics tasks in these topics are related to problem-solving. When learners are engaged in the process of problem-solving, not only they learnt mathematics conceptually but lecturers also improved their pedagogical content knowledge and content knowledge since they are confronted with different ideas, methods and solutions from the peers and students. The levels of tasks in these five research lessons are also classified based on the revised Bloom's Taxonomy that showed by Table 8.

Table 8: Tasks Based on Bloom's Taxonomy for all Research Lessons

Topic	Remembering	Understanding	Applying	Analysing	Evaluating	Creating
1	2	5	5	3	2	1
2	5	9	3	2	1	1
3	3	6	6	5	2	2
4	1	5	5	3	2	2
5	3	8	3	2	1	1
Total	14	33	22	15	8	7

Based on Table 8, about 47 percentages of tasks in all topics are related to the lower order thinking and 53 percentages are related to higher order thinking. Therefore, these collaborative lessons not only enhanced the ability of lecturers and students in problem-solving but also learners experienced the beauty of mathematical materials through engaging with appropriate problems and practical problems. In fact, lecturers considered some problems in the research lessons that improve the critical thinking and higher order thinking among students. Figure 7 represents a student work regarding the composite function. This student work is the answer of the following problem.

Problem: Consider the functions $f(x) = \sqrt{x-1}$ and $g(x) = \sqrt{x-3}$. Find the rule and domain of the function fog.

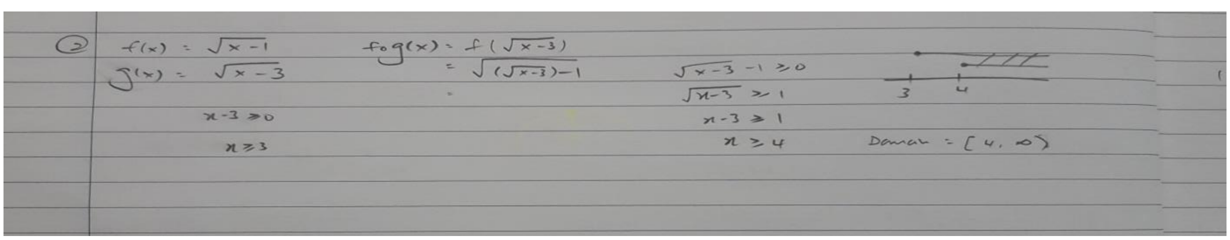


Figure 7: A Student Work about the Composite Function and Its Domain

The rule of the function fog is as below.

$$fog(x) = f\big(g(x)\big) = f\big(\sqrt{x-3}\big) = \sqrt{\sqrt{x-3} - 1} \Rightarrow fog(x) = \sqrt{\sqrt{x-3} - 1}.$$

Although the lecturer in the class explained the following method to find the domain of composite function fog, this student found the domain of composite function through a creative method.

$$f(x) = \sqrt{x-1} \Rightarrow D_f = [1, +\infty[.$$

$$g(x) = \sqrt{x-3} \Rightarrow D_g = [3, +\infty[.$$

$$D_{fog} = \{x \in D_g \mid g(x) \in D_f\} = \{x \in [3, +\infty[\mid \sqrt{x-3} \in [1, +\infty[\}.$$

After solving the inequality $\sqrt{x-3} \geq 1$, the domain of the function fog obtains as $D_{fog} = [4, +\infty[$.

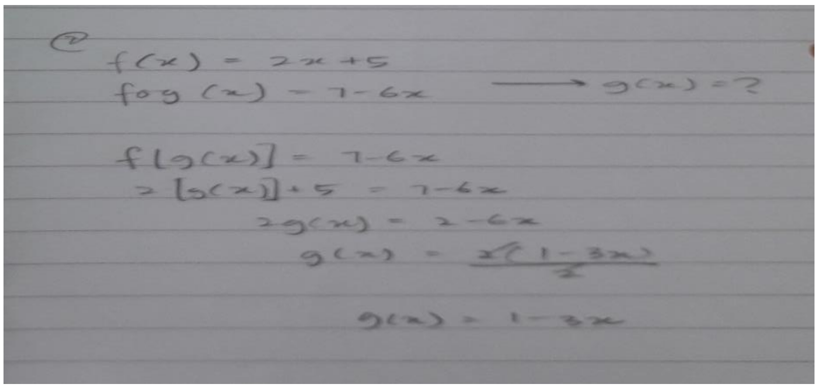


Figure 8: A Student Work about a Problem

Figure 8 illustrates a student work regarding a problem of a research lesson. There is no any similar task in the textbook. This problem that is suitable task in improving the higher order thinking skills among students is as follows.
Problem: If $f(x) = 2x + 5$ and $fog(x) = 7 - 6x$, find the rule of the function g.

One of the most important advantages of Lesson Study program is discussion about the variety of solution methods for the given problems. In Figure 9, there are some solution methods for the following trigonometric problem.
Problem: If $A = \sin 15 + \cos 15$, find the value of A^2.

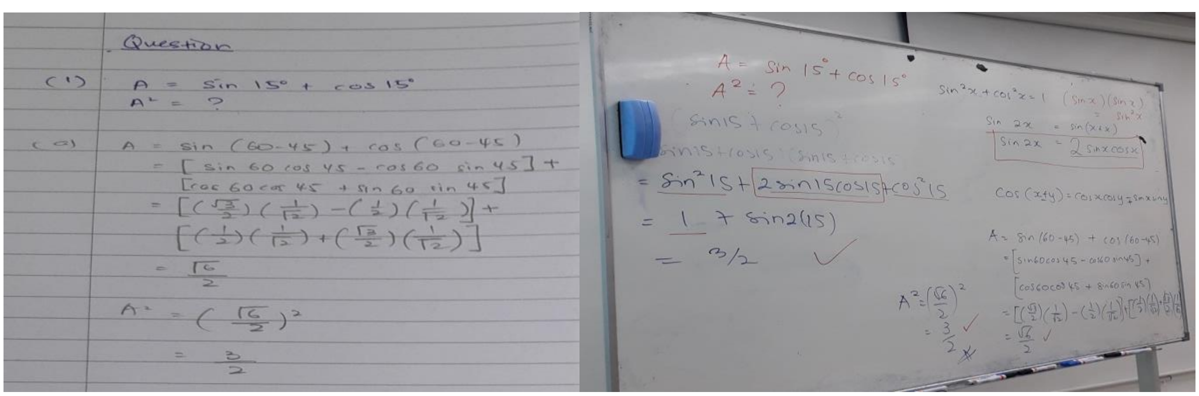


Figure 9: Different Solution Methods for a Given Trigonometric Problem

In the Lesson Study program, the lecturers considered some application of mathematical concepts. Considering these tasks in the research lessons not only helped students to understand the relation between mathematical concepts meaningfully but also showed them some applications of function topics in the real-world. For instance, in Figure 10, the lecturer highlighted the application of inverse function in determining the range of functions. The discussed problem is as below.

Problem: Find the range of the function $f(x) = \frac{2}{x-3}$.

It is difficult for students to find the range of the function f directly. The lecturer explained to students that one of the applications of inverse function is to find the range of some functions. There is a relation between the domain and range of the functions f and f^{-1} as follows.

$$R_{f^{-1}} = D_f \text{ and } D_{f^{-1}} = R_{f.}$$

The inverse function of $f(x) = \frac{2}{x-3}$ is $f^{-1}(x) = \frac{2+3x}{x}$ and students easily can find the range of the function f as $R_f = D_{f^{-1}} = (-\infty, 0) \cup (0, +\infty)$.

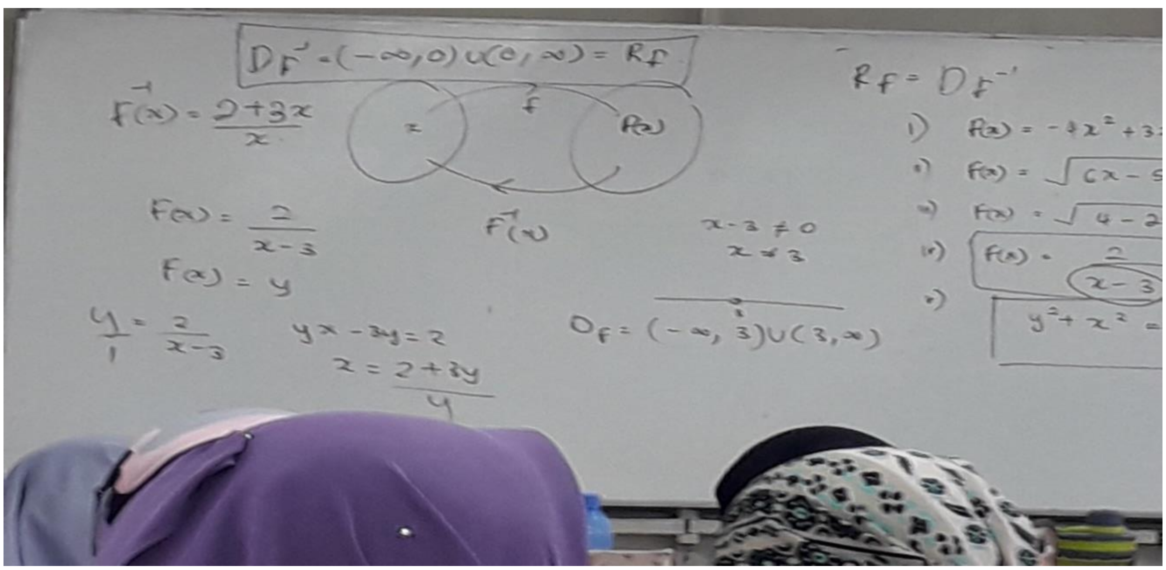


Figure 10: Determining the Range of a Function Using Its Inverse Function

The lecturer explained to students that one of the most common applications of functions is modeling the real-world problems. In other words, mathematical functions use to determine the relationship between variables in the human life. Figure 11 represents two examples of functions that apply in the real-world. The first function is the area value for circles that shows the domain and range of this function is positive real numbers. The second physical formula is related to the earthquake and shows that scientists are able to model complex real-world problems using mathematical functions.

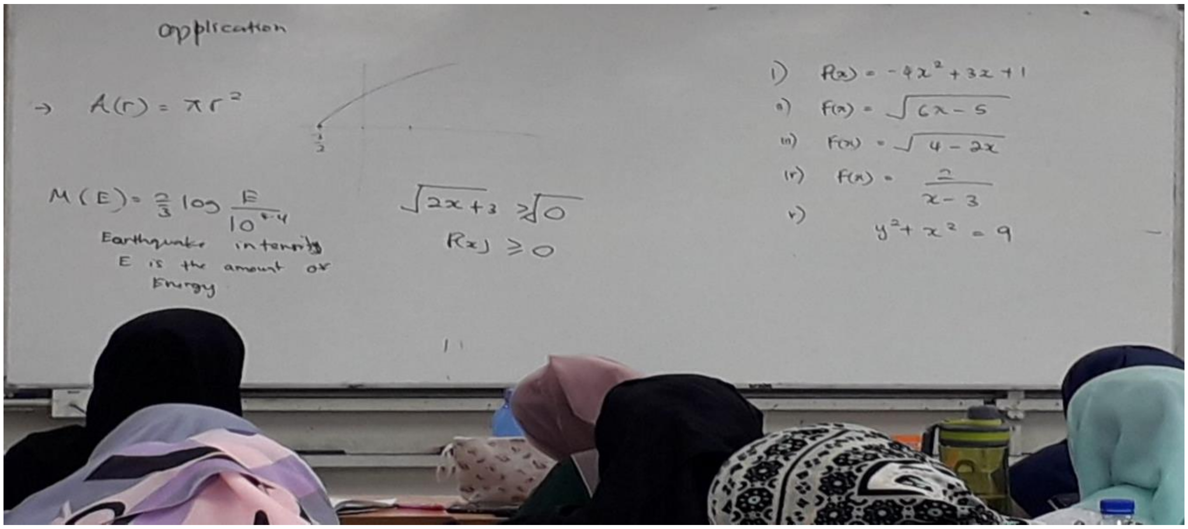


Figure 11: Applications of Mathematical Functions in the Real-world

Lecturers considered some fun problems (related to the topic of research lessons) in the research lessons that were interesting for students. For example, Figure 12 shows a fun function that give students the number of squares in a chess plane.

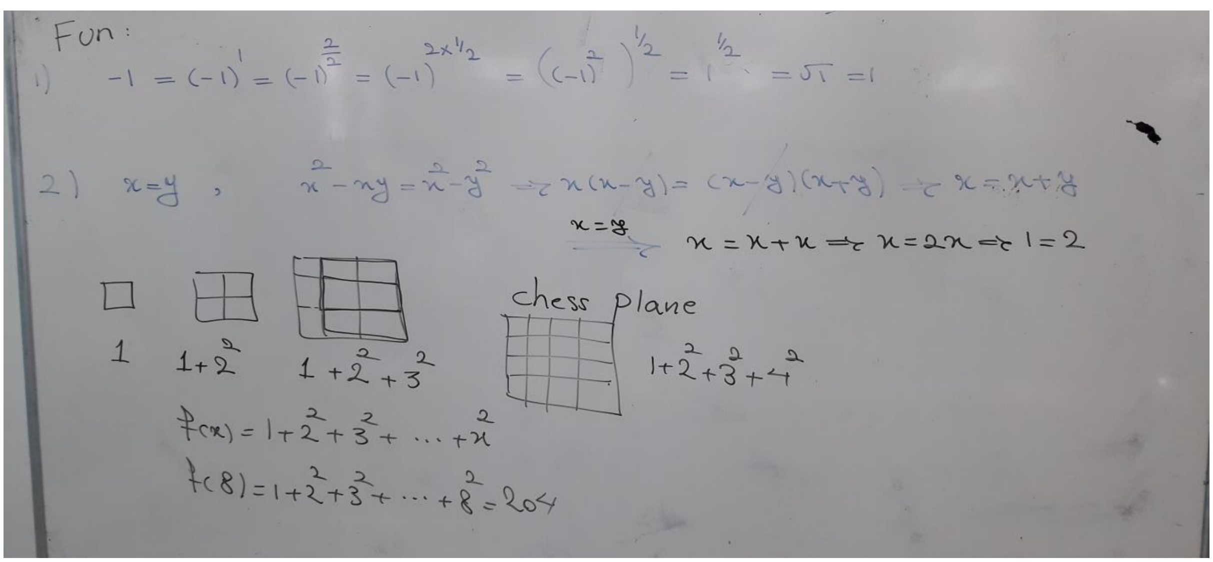


Figure 12: A Fun Function to Calculating the Number of Squares in the Chess Plan

4.2.Quantitative Part

In the experimental class, the lecturer used research lessons that provided by the Lesson Study group members that focused on problem solving ability and higher order thinking skills. This class was student-centered and students solved the given problems individually or in team. In fact, based on Figure 1 by engaging students with problems with different learning levels and discussing their different solutions, the teacher increased the students' motivation to solve problems. The contents of the textbook included repeated examples and exercises, while in the experimental class, the lecturer used problems that each of them taught new techniques to the students. Students' misunderstandings, their weaknesses and strengths were discussed. In this part, the researcher refers to two students' answers of experimental group with different scores for the given problem "Let m be a non-zero constant. Find the two x-values where the graphs of the functions $y = 10^{6m}x$ and $y = \frac{x^2}{10^{5m}}$ intersect". The following four samples show the considered score for the answers of students. As respect to Figure 13, student A solved this problem as follows:

$10^{6m}x = \frac{x^2}{10^{5m}} \Rightarrow 10^{-5m}x^2 = 10^{6m}x \Rightarrow x = 0$ or $x = 10^{6m+5m} = 10^{11m}$.

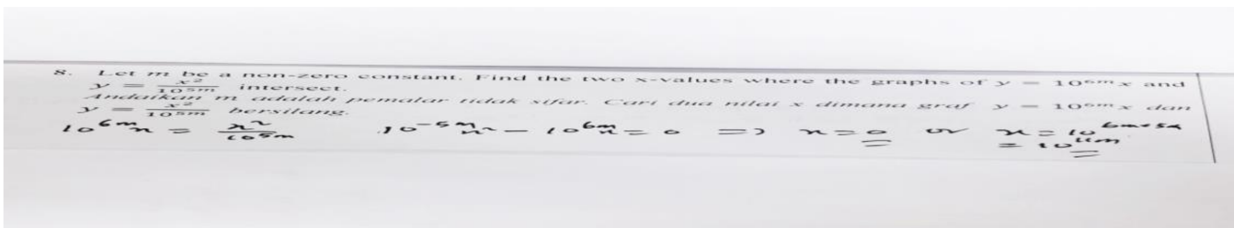


Figure 13: The Solution Method of Student A

The solution of student A that scored 3 shows student A had high skills in solving this problem. Student B used another method to solve the given problem.

$$10^{6m}x = \frac{x^2}{10^{5m}} \Rightarrow \log 10^{6m} + \log x = \log x^2 - \log 10^{5m}$$

$$6m \log 10 + \log x = 2 \log x - 5m \log 10$$

$$6m + \log x = 2 \log x - 5m$$

$$11m = 2 \log x - \log x$$

$$\log x = 11m \Rightarrow x = 10^{11m}.$$

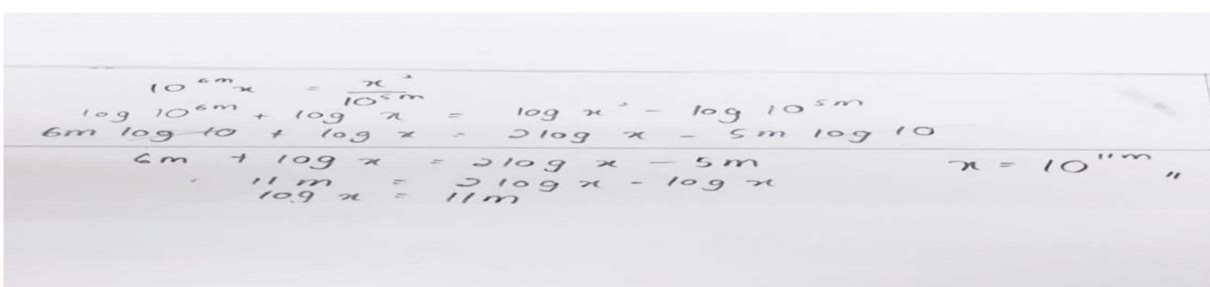


Figure 14: The Solution Method of Student B

Although student B had presented a creative solution (Figure 14) for this problem, he/she was given a score of 2 because the zero root of the problem was not calculated. The solution of student C that was shown by Figure 15 scored 1, because he/she provided an equation but the solution is not logical.

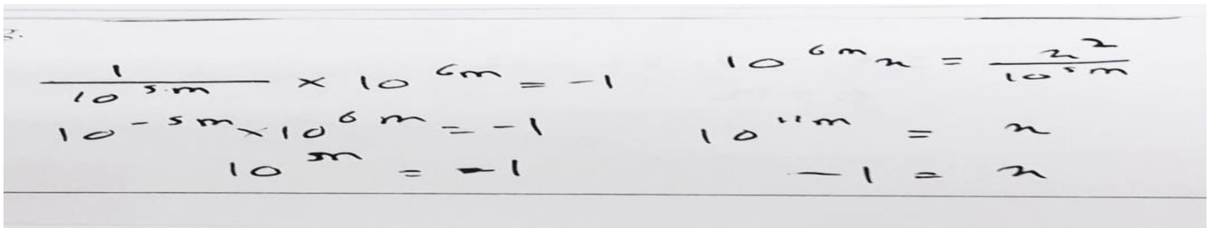


Figure 15: The Solution Method of Student C

The solution method of student D that showed by Figure 16, scored 0, because the response shows an insufficient understanding of the problem's essential mathematical concepts.

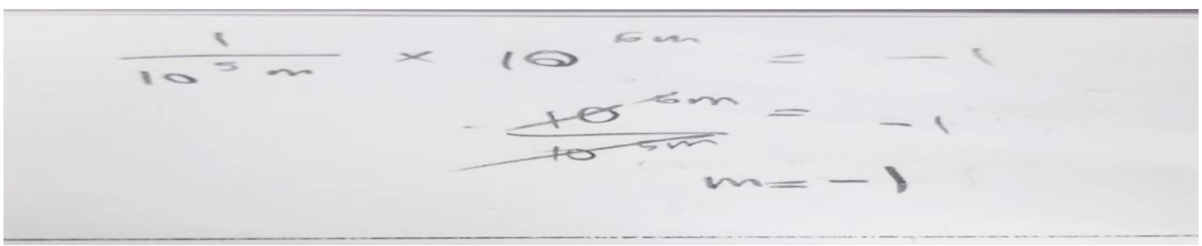


Figure 16: The Solution Method of Student D

The analysing of students' performance represents that the quality of teaching methods had important role in students' learning. For example, the definitions of even and odd functions in the textbook were as follows:
"A function f is said to be even if and only if $f(-x) = f(x)$ for all x".
"A function f is said to be odd if and only if $f(-x) = f(x)$ for all x".
The members of Lesson Study group defined the even and odd functions as the following:
A function f with the following two properties is called an even function:

1. Domain f is symmetric with respect to the origin
2. $\forall x \in D_f, f(-x) = f(x)$

A function g with the following two conditions is called an odd function

1. Domain g is symmetric with respect to the origin
2. $\forall x \epsilon D_g, g(-x) = -g(x)$

The textbook's definition of even and odd functions that used in the traditional group did not refer to the domain of the functions, so students ended up superficially memorising the two properties $f(-x) = f(x)$ and $f(-x) = -f(x)$ to identify whether a function is odd or even. However, in the Lesson Study group, the lecturers discussed regarding the domain of the even and odd functions and improved the definitions of even and odd functions in the textbook. For the problem "Determine whether the function $h(x) = \sqrt{x} + \sqrt{-x}$ is even or odd" in the post test, the majority of students in the traditional group argued that this is an even function because this function satisfies in the condition $h(-x) = h(x)$ as $h(-x) = \sqrt{-x} + \sqrt{x} = \sqrt{x} + \sqrt{-x} = h(x)$ without understanding the properties of odd and even functions conceptually. In the Lesson Study group, the majority of students would first identify the domain of the function h as $D_h = \{0\}$ and found that $h = \{(0, 0)\}$, so they argued this zero-function is both odd and even.

The normality of mathematics scores of students is shown in Table 9. Since the value of p for all tests are greater than 0.05, the scores are normally distributed.

Table 9: The Normality of Tests Scores

Group	Test	Kolmogorov-Smirnov		
		Statistic	df	Sig
	Pre-test	0.120	44	0.115
Lesson Study	Post-test	0.102	44	0.200
	Follow-up	0.119	44	0.135
	Pre-test	0.100	42	0.200
Control	Post-test	0.107	42	0.200
	Follow-up	0.093	42	0.200

As respect to Table 10, the result of independent sample t-test shows that there is no significant statistical difference between means of the experimental group (M = 18.22, SD = 3.99) and the control group (M = 19.83, SD = 5.08) in pre-test t(84) = -1.632, p = 0.106.

Table 10: Comparing the Mean of Mathematics Scores in the Pre-test

Group	Number	Mean	Standard Deviation	T	df	Sig
Lesson Study	44	18.22	3.99	-1.632	84	0.106
Control	42	19.83	5.08			

Table 11 shows the results of independent sample t-test for the post-test and the follow-up test. There is significant statistical difference between means of the experimental class (M = 24.02, SD = 4.64) and the control class (M = 19.07, SD = 3.92) in the post-test t(84) = 5.326, p = 0.000. Also, there is significant statistical difference between means of the experimental class (M=23.52, SD=3.75) and the control class (M = 19.28, SD = 3.92) in the follow-up test t(84) = 5.117, p = 0.00.

Table 11: Comparing the Mean of Scores in the Post-test and the Follow-up Test

Test	Group	Number	Mean	Standard Deviation	T	df	Sig
Post-test	Lesson Study	44	24.02	4.64	5.326	84	0.00
	Control	42	19.07	3.92			
Follow-up	Lesson Study	44	23.52	3.75	5.117	84	0.00
	Control	42	19.28	3.92			

Since the p value is smaller than 0.05 the null hypothesis was rejected for both post-test and follow-up test. Therefore, Lesson Study program enhanced the students' skills in problem-solving and higher order thinking as compared to the traditional teaching method.

5. DISCUSSION AND CONCLUSIONS

Function is a very important topic and it is used in many mathematics courses at the university level. Students in the foundation level require having sound knowledge of functions so that they are able to apply the concept in any fields of study. Specially in the real world, learners need to be able to solve higher order problems and apply the concept to real world situations (Michelsen, 2006). The results of this study showed that in this foundation centre, problem-solving and higher order thinking are not greatly emphasised by the lecturers. The classes were lecturer-centred and lecturers transferred contents to students through traditional teaching method. Meanwhile, they emphasised on solving of routine mathematics exercises in their teaching. Students did not seem to learn mathematics conceptually and hence, not experiencing the beauty of mathematics. It is an important question "how can learners learn mathematical contents without engaging in problem-solving?" They may gust end up memorising the mathematics materials such as definitions, theorems and methods and later apply them in solving similar mathematics exercises or exam questions. In the lesson study program, the lecturers through collaborative work planned the research lessons to minimise the need to memorise and to allow students to learn the mathematics concepts meaningfully through problem-solving activities. For example, for the question "If $(gof)(x) = -2x^2 + 4x + 1$and $g(x) = 3 - 4x$ then find the rule of the function f" students in the Lesson Study group showed better performance rather than students in traditional group. Because in the traditional group, the lecturer taught exactly the textbook' materials and for instance, in the textbook there are 18 similar questions such as "If $f(x) = 1 - 5x^3$ and $g(x) = 2 + \sqrt{2x^2 + 3x}$ then find the function $(fog)(x)$". In fact, in traditional teaching method the lecturer only focused on mathematics exercise solving. Whereas, in the Lesson Study group, the lecturer more focused on problem solving and higher order thinking skills based on the rich materials that prepared in the research lessons by lecturers collaboratively.

Mathematics lecturers seem to teach according to the flow and the approach of the textbook that was provided. In teaching different groups of students in twenty classes conducted by several lecturers that work towards a common examination, the practice of teaching according to the textbook may be less risky because the lecturers need to ensure that all students acquire the same materials. Practicing using routine exercises may be an option that most lecturers use, especially if the exam format is at a similar level to previous exam questions. This has been the predicament of many lecturers. If they focus more on higher order thinking, their students may not be able to do many of practice questions that need to form a pattern for the solution. On the other hand, the over dependency on the textbook may also resulted from their lack of pedagogical content knowledge and it could be more prevalent among those that do not have teaching certification.

Lesson Study as a professional development program had helped many lecturers to enhance their teaching knowledge, specifically their pedagogical content knowledge and content knowledge. Participatory educational environments for mathematics teaching, provide learners with effective opportunities to improve their learning, thereby enhancing their chances to succeed (Moreno & Rutledge, 2020). The lecturers would be able to improve their lessons to emphasise more on problem-solving and higher order thinking. In this foundation centre, most lessons are being taught through traditional method and only a small percentage of tasks were related to the problem-solving. In the Lesson Study approach, the focus is more on problem-solving. It has been found that collaborative work among lecturers is very beneficial in increasing the quality of mathematics teaching. Furthermore, collaborative work through Lesson Study provides opportunity for lecturers to improve students' higher order thinking skills. It also helps improve the quality of mathematics materials which help learners to engage in suitable problem-solving at different levels of thinking, based on the revised Bloom's Taxonomy. The results of quantitative part of this study show that students in the experimental group had better performance in problem-solving and higher order thinking in post-test and follow-up test. It seems Lesson Study program was able to improve the ability of students in problem-solving and higher order thinking compare to traditional teaching method. In fact, in Lesson Study program lecturers prepared appropriate mathematics problem for students and transferred the mathematical materials to students through better pedagogical methods so they improved the ability of students in problem-solving. Whereas students in control class received the mathematics materials from textbook that emphasises on routine exercise solving. If lecturers were able to collaboratively write their textbooks and teaching materials, definitely they can design and develop better output. Based on the results of this study, Lesson Study is a strong approach for lecturers' professional development and students' outcomes in problem-solving and higher order thinking. Therefore, Lesson Study is suitable program in teaching mathematics for foundation centres, and not just schools.

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Development of Student Self-Efficacy for Mathematics Learning in Indonesia

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Abstract: The purpose of this development research is to create a self-efficacy instrument to assist students learning mathematics that is valid, practical, and reliable. A self-efficacy questionnaire is the format of the instrument that was created. The main driving force behind this research is the scarcity of empirically validated self-efficacy surveys for studying mathematics in different parts of Indonesia. Students' self-efficacy is important since it affects how well they learn. This research used the ADDIE development technique as its methodology, comprising five stages: Analysis, Design, Development, Implementation, and Evaluation. Readability, validity, and reliability tests were used to test the data empirically and theoretically through expert evaluation. Both qualitative and quantitative analyses were done on the data; expert validation results and readability test results were subjected to a qualitative analysis, while validity and reliability test results were subjected to a quantitative analysis. Students from different schools in Indonesia took the quantitative test. At least ten times as many statements were included in the sample size that was used. 36 statement items were originally developed throughout the development phase; however, item 33 was split into two distinct statements, resulting in 37 questionnaire items that represent students' self-efficacy in learning mathematics. This designed questionnaire can be used to evaluate Indonesian students' self-efficacy in learning mathematics at the junior and senior high school levels.

Keywords: questionnaire, self-efficacy, learning mathematics

INTRODUCTION

Mathematics is a topic that many pupils dislike (Kamarullah, 2017). When most students hear the word of mathematics, they appear to want to avoid it. This dread is exacerbated by parents who struggled with mathematics during their school years and frequently relay the message that mathematics is a tough subject, adding psychological stress to their children learning mathematics. This condition persists, almost becoming a fiction that mathematics is terrifying.

One of the reasons mathematics is difficult is because it is abstract and loaded with symbols (Kurniawan, 2017). Many figures and educators in mathematics education have observed and experienced that if these symbols are not understood, it becomes difficult to connect with the subject. Furthermore, solving mathematical problems or equations needs following accurate and logical procedures, which needs conceptual competence. As a result, understanding and mastering mathematics frequently necessitates the assistance of experts.

The affective part of mathematics learning is very important and plays a key impact in learning success. Beliefs, attitudes, and emotions are three affective elements that might influence students' learning processes. When students engage in an investigation process, their behaviors, efforts, tenacity, flexibility in differences, and goal realization are influenced by belief elements. Student beliefs include their mathematical self-confidence or self-efficacy, which is the student's assessment of their capacity to achieve a desired or defined performance level, impacting subsequent behaviors (Bandura, 1997). As a result, considerable self-efficacy in mathematics is required for students to succeed in the mathematics learning process.

Albert Bandura, a psychologist, pioneered the concept of self-efficacy in 1982. The concept was suggested by Bandura (1982) as a personal assessment of "how well one can execute actions required to deal with prospective situations." Bandura (1997) defined self-efficacy as a person's belief in their ability to succeed in a certain scenario or complete a task. Every aspect of human endeavor is influenced by self-efficacy. Beliefs regarding one's power to control situations have a substantial impact on a person's ability to handle challenges properly and the choices they are most likely to make. The consequences of self-efficacy are most noticeable and persuasive in human activities such as health (Luszczynska and Schwarzer, 2005; Buckley, 2014), education (Krishnan and Krutikova, 2013; Schunk, 1991; Chemers et al., 2001; Morton et al., 2014), and industry and organization (Akhtar et al., 2012), and agriculture (Wuepper and Lybbert, 2017).

Self-efficacy is frequently misunderstood as a sense of personal efficacy. This sense is critical in how a person handles objectives, tasks, and obstacles. An individual's self-efficacy, according to Bandura (1997), will: (1) influence their decision-making and actions; (2) determine the extent of their effort in an activity, how long they persevere when faced with difficulties, and their flexibility in unfavorable situations; and (3) affect their thought patterns and emotional reactions.

Mathematical self-efficacy positively adds to and plays a vital part in the achievement of mathematics learning that students can achieve (Sunaryo, 2017). It is also important in solving mathematical difficulties (Ayotola and Adedeji, 2009). For example, Ramlan (2013) noticed that when asked to comment or solve problems verbally, students frequently look left and right as if seeking assistance from their peers, showing a lack of confidence or fear in responding and expressing their ideas.

Most teachers concentrate solely on imparting knowledge to pupils, whereas many students struggle with non-cognitive aspects such as self-efficacy or a negative attitude toward mathematics. These characteristics can impede mathematics learning, as the application of emotive aspect assessments has been shown to improve learning (Qadar et al., 2015: 9). Gurefe and Bakalim (2018: 157) discovered that self-efficacy in mathematics favorably improves performance in their research with Faculty of Education students.

In terms of the function of self-efficacy in mathematics, knowing how to do mathematical operations is not enough; one must also have self-efficacy in the credibility of their conceptions and procedures (Garfield and Ben-Zvi, 2009: 1). For example, while using a formula, students must be confident in their application. Students' mathematical self-efficacy is not fixed, but it can be enhanced. Yuliarti et al. (2016), for example, investigated the growth of student self-efficacy through generative learning methodologies, discovering that students' self-efficacy can be increased even from a low starting position.

According to Finney and Schraw (2003), students' self-efficacy develops as a result of their own circumstances and environment. Teachers, textbooks, learning methodologies, and, in particular, the usage of everyday problems in the students' environment all influence the change in students' self-efficacy. Thus, all of these aspects must be examined in order to boost self-efficacy (Ulpah, 2019).

An instrument that reliably evaluates students' self-efficacy is required to understand their self-efficacy and lead them successfully in the learning process for optimal outcomes. Novrianto et al. (2019) used a sample of 585 students from UIN Sultan Syarif Kasim Riau to examine the construct validity of the General Self Efficacy Scale (GSES) converted to Indonesian. The confirmatory factor analysis (CFA) approach was employed with Lisrel 8.80 software to indicate that the GSES's ten items are unidimensional and measure a single factor, supporting the GSES's one-factor model. This instrument is of a generic character.

Betz and Hackett (1983) established the Mathematics Self-Efficacy Survey (MSES) for mathematics in 1983. This instrument has been widely used to study the effects of mathematical self-efficacy in a variety of contexts, including high school students (O'Brien et al., 1999), older students (Gatobu et al., 2014), science and technology students (Lent et al., 1991), first-year college students (Hall and Ponton, 2005), and pre-service teachers (Bates et al., 2011). However, using it by other researchers without adapting and validating it diminishes the validity of study conclusions, especially when translated into multiple languages and applied to diverse cultures (Chávez and Canino, 2005; Walther, 2014).

Usher and Pajares (2009) have created a mathematical self-efficacy questionnaire for Northern American middle school students. They developed a 24-item questionnaire measuring self-efficacy across four dimensions: mastery experiences, social persuasion, physiological states, and vicarious experiences after three stages including students from varied ethnic backgrounds (White, Black, Asian-American, and Mixed). The findings revealed that it was appropriate for students in various situations. However, because self-efficacy is strongly influenced by the learning environment, validation for multiple ethnic groups and places is required.

Meanwhile, Chan and Abdullah (2018) sought to construct a self-efficacy questionnaire for primary school pupils, testing it on 100 fifth-grade students from Penang Island, Malaysia's government schools. The research yielded 14 valid and dependable items for a mathematical self-efficacy questionnaire. However, this study was limited to public elementary schools and was not examined in private or international elementary schools. To investigate if different types of schools' learning environment rules influence students' self-efficacy, the study subjects should be enlarged to include both public and private schools.

Furthermore, Riboroso (2020) created a questionnaire in the Philippines to assess first-year college students' mathematics self-efficacy, resulting in 46 valid and useful items from an initial 76. The questionnaire assesses self-efficacy in four areas: mathematical modeling, representation, communication, and learning technology use. However, given the importance of technology in current education, the item distribution warrants more examination.

Muhtarom et al. (2017) designed a mathematics learning belief questionnaire rather than a self-efficacy questionnaire in the setting of Indonesia. Yoga et al. (2020) created a math self-efficacy questionnaire for elementary school students. Continuous research demonstrates a scarcity of Indonesian researchers' self-efficacy questionnaires. Most studies in Indonesia use existing questionnaires established outside Indonesia, either directly or with adjustments, like in Somakim and Risnanosanti's doctoral research, which adjusted mathematics learning self-efficacy surveys. Modified self-efficacy surveys were also utilized by Ramlan (2013), Yuliarti et al. (2016), Sunaryo (2017), Ulpah (2019), and others.

As a result, there is a substantial limitation in the availability of self-efficacy questionnaires for assessing student self-efficacy in mathematics learning within the environment and culture of Indonesia, particularly for students in junior high and senior high school. This scarcity presents difficulties for mathematics educators in assessing students' self-efficacy. To promote the quality of mathematics education in Indonesia, the creation of a self-efficacy instrument in mathematics learning is regarded required. Both theoretical and empirical testing are required to confirm the instrument's validity and reliability. Theoretical testing should include highly qualified experts, while empirical testing should include a diverse spectrum of responders from across Indonesia. The created instrument is expected to help teachers improve and enhance the mathematics learning process in the classroom. The designed self-efficacy instrument is based on Bandura's Theory.

METHODOLOGY

This is a development study to create a valid, practical, and reliable student self-efficacy measure. The ADDIE development approach is used in the study, which stands for Analysis, Design, Development or Production, Implementation or Delivery, and Evaluation. Dick and Carey created the ADDIE model in 1996 to help them construct learning systems. The ADDIE model's research and development steps are divided into five parts. The following are descriptions of each step in the ADDIE development paradigm (Mulyatiningsih, 2019; Aldoobie, 2015):

a) *Analysis*

At this stage, the emphasis is on examining student needs, curriculum, and characteristics. The needs analysis is carried out to identify existing problems in the field so that the construction of the self-efficacy questionnaire is in line with the students' circumstances. The curriculum analysis is used to understand mathematics learning in schools and the suitability of the utilized questionnaire. The results of the student characteristics study are then used to personalize the questionnaire that is being prepared.

b) *Design*

A theoretical study is undertaken at this stage, which includes researching self-efficacy theory, learning about questionnaire design theory, making an initial draft of the questionnaire, and creating the questionnaire sheets to be developed. At this point, the instruments for the questionnaire's validation are also being produced.

c) *Development*

A questionnaire is constructed at this step based on the previously investigated theories (Prototype 1). Following the development of the product, it is subjected to theoretical testing by validating Prototype 1 with specialists in psychology and learning. The prototype evaluation format is consistent with construct, content, and language elements. Prototype 2 of the questionnaire is obtained after adjustments depending on the validation procedure.

The procedure is then followed by empirical testing, specifically a practicability test. A limited readability test with four students is used to evaluate the questionnaire's practicality in this case. The purpose is to see if the statements in the questionnaire, as written, are intelligible and comprehensible to students. After detecting Prototype 2's weaknesses it is updated, resulting in Prototype 3.

d) *Implementation*

At this stage, additional empirical testing is carried out with a larger subject group. Prototype 2 is being tested in junior high schools (SMP) and senior high schools (SMA) in many Indonesian provinces. Students in grades VII, VIII, and IX of junior high schools (SMP/MTs) and students in grades X, XI, and XII of senior high schools take the self-efficacy questionnaire.

e) *Evaluation*

Following the empirical trial, an evaluation is carried out to determine the quality of the created self-efficacy questionnaire (Prototype 3). There are quantitative and qualitative analyses carried out. This stage tries to make adjustments if the produced self-efficacy questionnaire still has limitations.

This research was conducted over two years, from September 2020 to September 2022. Data collection took place in several junior high schools (SMP) and senior high schools (SMA) across various provinces in Indonesia. The research areas included both urban and rural regions, to ensure that the developed instrument was not solely targeted at urban students who generally have access to better educational facilities. The goal was for the instrument resulting from this research to be suitable for use by all students.

Although this is a qualitative study, considering that the produced instrument is intended for use by all junior high school and equivalent senior high school students, the selection of trial subjects had to adhere to the sampling rules of quantitative research. Therefore, purposive sampling was used. This technique was chosen because the samples were selected based on certain considerations and characteristics of the known population (Malik and Chusni, 2018). In line with the type of questionnaire developed, namely a self-efficacy questionnaire for students in mathematics learning, the sample selection was filtered through the following criteria: (1) Students who have studied mathematics, as those who have not will not have experienced the difficulties of learning mathematics; (2) Students who understand the questionnaire statements, given the relatively high level of comprehension required, thus making it more suitable for students of at least seventh grade in junior high school; (3) Students from various locations/regions, to ensure representation from every area. Therefore, the respondents were junior high school equivalent students (grades VII, VIII, and IX) and senior high school equivalent students (grades X, XI, and XII) from various areas within Indonesia.

Regarding sample size, it is statistically stated that a larger sample size is expected to yield better results. According to Krejcie and Morgan (Schreiber and Asner-Self, 2011: 92), for a population below 100, all should be sampled; for a population of 500, 50% should be sampled; for a population of 5000, 357 respondents should be sampled; and for a population of 100,000, only 384 respondents need to be sampled. In this study, samples were taken based on how many respondents filled out the questionnaire, as long as they met the respondent criteria, by asking for help in distributing the questionnaire through teacher colleagues in various places. Once enough data met the minimum requirements, the distribution of the trial questionnaire was stopped. The minimum requirement was adjusted according to Nunnaly's suggestion (Alwi, 2012), stating that the size of respondents in a trial should be ten times the number of items in the measuring instrument. Since there were 36 statement items, the minimum number of respondents was 360 students for each grade level.

The variable in this study is a valid, practical, and reliable student self-efficacy questionnaire. The questionnaires in this study included: (1) a questionnaire for expert validation, (2) a questionnaire for readability testing, and (3) a self-efficacy questionnaire to test validity and reliability quantitatively. The dimensions measured to observe student self-efficacy are based on Bandura's (1997) concept, namely Magnitude (How an individual can overcome their learning difficulties), Strength (How strongly students believe in their ability to overcome learning difficulties), and Generality (Indicating whether efficacy beliefs will persist in a specific domain or apply across a variety of activities and situations).

Data analysis techniques were employed to produce a high-quality self-efficacy questionnaire that meets the criteria of being valid, practical, and reliable. The steps in analyzing the quality criteria of the developed self-efficacy questionnaire are as follows:

(1) Validity Analysis

a) Based on the validation data from the questionnaire assessment by learning and psychology experts, the validity of the questionnaire can be determined by examining the qualitative evaluation results (theoretical validity). These experts were asked to review and theoretically evaluate the questionnaire, providing comments on incorrect statements and suggestions for their replacement.

b) Subsequently, by conducting a trial of the self-efficacy questionnaire with a large number of students, its validity is also examined quantitatively (empirical validity). The calculation of the trial results uses the product moment formula. An instrument is considered valid if the calculated correlation coefficient (r_calculated) is greater than the table correlation coefficient (r_table) at a 5% significance level (Sugiyono, 2017).

(2) Practicality Analysis

The content of the questionnaire statements was also tested for readability with 4 students, specifically 3 junior high school students and 1 sixth-grade elementary school student. This readability test was conducted to determine if the statements created were understandable and comprehensible to students. A questionnaire was also designed to assess students' readability of the statements. From the scores obtained in the readability test questionnaire, the practicality value of the self-efficacy questionnaire was calculated. The questionnaire is considered good if the students' responses meet the minimum practicality criteria, referring to the following Table 1:

Interval score	Category
$x > 3,4$	Highly practical
$2,8 < x \leq 3,4$	Practical
$2,2 < x \leq 2,8$	Moderately practical
$1,6 < x \leq 2,2$	Slightly practical
$x \leq 1,6$	Not practical

Table 1. Criteria for the practicality of the questionnaire

(Riduwan, 2018)

(3) Reliability Analysis

The questionnaire was trialed with many students to quantitatively assess its reliability. The testing of the instrument's reliability utilized Cronbach's Alpha formula, appropriate for research instruments in the form of questionnaires and scales (Ananda and Fadhli, 2018). A Cronbach's Alpha value greater than 0.7 indicates sufficient reliability. Meanwhile, an Alpha value above 0.80 suggests that all items are reliable and that the entire test consistently exhibits strong reliability (Wahyuni, 2014). Mehrens and Lehmann (Retnawati, 2017) stated that although there is no general consensus, it is widely accepted that for tests used to make decisions about individual students, a minimum reliability coefficient of 0.85 should be achieved.

RESULTS

The results of the development research on the student self-efficacy instrument in mathematics learning are outlined according to the ADDIE stages, as follows:

1) *Analysis*

Needs Analysis: Understanding students' self-efficacy is as important as teaching the material because students' readiness and confidence in learning will affect their learning outcomes. To effectively teach students and produce good learning outcomes, it is necessary to consider their self-efficacy in learning mathematics. It was found that there are few self-efficacy questionnaires in mathematics learning developed by Indonesian researchers, necessitating the creation of a new self-efficacy questionnaire that is field-tested directly with many respondents and tailored to the Indonesian context.

Curriculum Analysis: The curriculum mostly applied in schools where students were asked to fill out the questionnaire is the revised 2013 curriculum. This curriculum integrates four elements: character education reinforcement (PPK), Literacy, the 4Cs (Creative, Critical Thinking, Communicative, and Collaborative), and HOTS (Higher Order Thinking Skills).

Therefore, it is indeed necessary to look at students' self-efficacy to identify where students' weaknesses in learning mathematics may lie due to the implementation of this curriculum.
Student Analysis: Based on the ability to understand sentences and recognize feelings, it was deemed best to measure the self-efficacy of junior high school equivalent and senior high school equivalent students.

2) *Design*

a) Collection of References

Credible reference sources were sought to create an accurate and effective self-efficacy questionnaire. Materials on self-efficacy and examples of questionnaires were collected. Researchers studied the theory of self-efficacy. This activity took place from November to December 2020.

b) Creation of Questionnaire and Validation Sheet

Researchers discussed and created a self-efficacy questionnaire for students in mathematics learning. The content of the questionnaire was aligned with the self-efficacy framework. The dimensions of self-efficacy referred to were magnitude, strength, and generality (Bandura, 1997). A total of 36 questionnaire statements were created. Validation sheets for lecturers and readability test sheets for students were also developed. Creating statements that truly reflect self-efficacy was challenging, leading researchers to spend a considerable amount of time studying the theory of self-efficacy and designing the questionnaire, and consulting with experienced colleagues about self-efficacy. This activity took place from January to February 2021. The initial draft of the questionnaire statements based on dimensions and derived indicators is as follows:

Dimension	Operational definition	Indicators and their statements
Magnitude	How an individual can overcome their learning difficulties	1. Optimistic Outlook in Working on Lessons and Assignments. – I feel that I can always solve mathematical problems or complete math assignments on my own (+) – I cannot succeed in learning mathematics without help from others (-) 2. Level of Interest in Lessons and Assignments – For me, learning mathematics is enjoyable (+) – I am not at all interested in mathematics (-) 3. Developing Skills and Achievements – Learning mathematics is important for training the mind (+) – For me, there is no benefit in learning mathematics (-)

		4. Planning in Task Completion – Correct step-by-step solutions are necessary to solve mathematical problems (+) – When I have a mathematics problem, I just work on the core issue immediately without first writing down the preliminary steps (-) 5. Confidence in Performing and Completing Tasks – I can solve mathematical problems accurately (+) – Mathematical problems are always difficult to solve (-) 6. Viewing Difficult Tasks as a Challenge – I feel challenged when completing more difficult math problems (+) – When the math problems become harder, I start to feel overwhelmed (-) 7. Studying According to a Set Schedule – I can study mathematics independently when the time is calm, such as at night (+) – In any situation, I find it hard to learn and understand mathematics (-)) 8. Selective Action in Achieving Goals (Statement not provided)
Strength	How High is a Student's Confidence in Overcoming Their Learning Difficulties	1. Effort Can Lead to Improved Performance – If we diligently work, we can solve even the difficult math tasks (+) – No matter how much effort I put into solving math problems, the results are always unsatisfactory (-) 2. Commitment to Completing Assigned Tasks – I will try to complete every math task or problem that is given(+) – I am unwilling to work on any math tasks or problems (-) 3. Belief in and Awareness of One's Strengths – I know that I am capable of learning mathematics (+) – I believe that I have many weaknesses in mathematics (-)

		4. Persistence in Completing Tasks – I will continuously try my best to complete the math tasks given by my teacher (+) – If the task given by the teacher turns out to be difficult, I start to avoid it (-) 5. Having a positive purpose in doing various things – I aim to become smarter by studying mathematics (+) – Studying mathematics is not beneficial (-) 6. Having good self-motivation for personal development – I always push myself to keep liking and learning mathematics because I think it's important – I always try to follow mathematics lessons because they can train my thinking ability – I feel that learning mathematics is useless
Generality	Indicates whether efficacy beliefs will persist in a specific domain or apply across various activities and situations	1. Responding well to different situations and thinking positively – If I study different mathematical materials than before, I remain happy to increase my thinking skills (+) – The more varied the mathematics materials I study, the more confused I become (-) 2. Using life experiences as a path to success – I always try to correct my previous mistakes in solving mathematical problems – From past experiences, I have never been able to learn mathematics 3. Enjoy seeking new situations – Once I finish studying one mathematics topic, I move on to another – Every mathematics topic is always difficult for me to learn 4. Able to effectively handle all situations – I prefer to solve each mathematical problem in a simple way without needing to use formulas – I often encounter obstacles when solving (+) mathematical problems and sometimes go in circles without reaching a solution (-)

	5. Trying new challenges – I feel happy when there are more challenging mathematical problems (+) – Studying even easy mathematics topics feels difficult, let alone the harder ones (-)

Table 2. Initial design framework for the Self-Efficacy questionnaire

3) *Development*

a) Validity Test through Expert Validation

The researcher created and designed the self-efficacy questionnaire, which passed the expert validation test. Several experts then validated the questionnaire. Attempts were made to find specialists who are very knowledgeable about self-efficacy. The following criteria were used in selecting experts: (1) comprehension and mastery of learning theory, (2) comprehension and mastery of psychological theories, (3) involvement in the field of psychology, (4) comprehension and mastery of questionnaire theory, and (5) comprehension of sentence purpose.

The validation by experts employed a panel technique, where validators reviewed each item of the questionnaire based on the rules of writing questionnaire items. This review considered aspects of material, construction, language/culture, and the accuracy of answer keys/scoring guides. The process involved several reviewers who were given the questionnaire items to assess, a format for the review, and guidelines for evaluation. The reviewers worked independently in different locations. They were allowed to make direct corrections to the questionnaire text, provide comments, and assign a rating to each item based on criteria such as: good, needs improvement, or needs replacement (Alwi, 2012).

Experts who were asked to theoretically test the content of the self-efficacy questionnaire included three lecturers from the Guidance and Counseling Program at Universitas PGRI Palembang and one lecturer from the Islamic Psychology Study Program at the Faculty of Psychology, UIN Raden Fatah Palembang. Three out of these four lecturers also worked as psychological consultants. The validation testing period was from March 8, 2021, to July 16, 2021. The following is a summary of all comments and suggestions from the validation of the self-efficacy questionnaire content by experts:

No.	Comments and suggestions from the experts
1.	Overall, the construction, content, and language are good and accurate. However, it's essential to consider the meaning of self-efficacy in its context.
2.	Self-efficacy refers to the ability to produce outcomes (the belief that students can create, not just solve problems).
3.	It also encompasses self-trust, which is the confidence in one's ability to perform tasks according to their competence.

4. The word 'feel' is not suitable for self-efficacy, as it is emotional and abstract, while self-efficacy should be a tangible product.
5. There are still some words that need to be replaced to be appropriately reflective of self-efficacy.
6. The statements made do not sufficiently reflect the students' attitudes.
7. The statements are not formulated clearly and definitively.
8. Some statements have ambiguous meanings.
9. It seems there are sentences that are not yet robust.
10. Every expert judgment or academic work should be scientific, so please improve the language, such as changing 'not' to 'less.' This is because if 'not' is used, it implies complete ignorance, whereas every student should at least understand basic addition and subtraction for future use.
11. Overall, this questionnaire is suitable, but it needs only minor improvements.
12. There are a few statement items that still need refinement.
13. There appears to be one overlapping item.
14. On the header of the second sheet, alternative answer options should be placed again.
15. Some words still need to be improved.
16. Is this research qualitative or quantitative? It seems to be a combination of both.
17. The word ' I ' is unnecessary because the subject fills it out themselves.
18. There must be a distinction between attitude questionnaires and survey questionnaires

Table 3. Comments and suggestions from the experts

After validation and identification of deficiencies and weaknesses in the questionnaire, the researchers proceeded to improve the content of the self-efficacy questionnaire. The revised version was reconfirmed with the validators to ensure that the improvements met the correct criteria. Once accepted and deemed appropriate by the validators, the next step was taken.

b) Practicality Test through Readability Test

The purpose of the readability test was to ensure that the text of the statements created could be understood and were appropriate for the comprehension of the targeted students. The selection of students as subjects for the readability test met the following criteria: (1) a minimum of seventh-grade junior high school students, as the questionnaire was aimed at junior high school students, but there was one sixth-grade elementary school student included as a subject because this student was considered capable of understanding and comprehending the text statements; (2) able to feel what the text states, (3) at the time of testing, still an active student, not a dropout or unemployed. The results of the readability test were as follows:

1) Subject 1, initials AMP, a ninth-grade student at SMP Negeri 1 Inderalaya, South-Sumatera. Tested on Tuesday, September 7, 2021, from 14:00 - 15:00, by the third researcher as the examiner. Tested by giving a questionnaire sheet and the subject filled it out independently. No obstacles encountered. Conclusion: The statement sentences were understandable.
2) Subject 2, initials NFI, a sixth-grade student at SD Negeri 68 Palembang, South-Sumatera. Tested on Sunday, September 12, 2021, from 17:00 - 18:00, by the second

researcher as the examiner. Tested through face-to-face questioning. No obstacles encountered. Conclusion: The long sentences were still quite understandable, did not fully understand the word "material," some words should be replaced with easier equivalents, and some words should be simplified as they were too verbose.

3) Subject 3, initials PR, a ninth-grade student at SMP Negeri 6 Palembang, South-Sumatera. Tested on Wednesday, September 22, 2021, from 18:50 - 19:25, by the third researcher as the examiner. Tested through direct face-to-face questioning. No obstacles encountered. Conclusion: Many sentences caused doubt in subject 3's understanding, and several sentences required more clarity.
4) Subject 4, initials DAV, a seventh-grade student at SMP IT Iqro' Bengkulu. Tested on Thursday, October 7, 2021, from 14:15 - 15:05, by the second researcher as the examiner. Tested through online questioning via Google Meet. No obstacles encountered. Conclusion: There were words that subject 4 did not understand, and the long statements were understandable as long as the words were easy to understand.

The results of the readability test were totaled and recapitulated for calculation and conclusion, and the results are presented in the following Table 4:

No.	Subject	Score				Total	Mean
		1	2	3	4		
1.	Subject 1	-	-	4	32	4×3+32×4=12+128 = 140	140/36 = 3,89
2.	Subject 2	-	2	6	28	2×2+6×3+28×4 = 4+18+112 = 134	134/36 = 3,72
3.	Subject 3	-	12	13	11	12×2+13×3+11×4 = 24+39+44= 107	107/36 = 2,97
4.	Subject 4	-	4	9	23	4×2+9×3+23×4 = 8+27+92= 127	127/36 = 3,53
		Mean				504/4 = 127	14,11/4 = 3,53

Table 4. Recapitulation of readability test results

The recapitulation results from the readability test conducted on four students yielded an average score of 3.53. This score indicates that the questionnaire tested on these four students falls into the category of being very practical.

Furthermore, from the results of this readability test, the evaluators' assessment of the students' responses to the created self-efficacy questionnaire is as follows:

No.	Evaluators' assessment from the readability test results
1.	All students understood the meaning of the content in 18 statements.
2.	The phrase 'achieving success' was somewhat confusing for the students.
3.	Two students were quite confused by the words 'build and train.'
4.	The word 'challenged' was confusing for the students.

5.	The sentence 'When there's a math problem, I just work on the crux of the issue without first stating the introduction' was confusing for the students.
6.	The phrase 'in any situation' was confusing for the students.
7.	The sentence 'Every effort I make in solving math problems always ends up being unsatisfactory' was not effective.
8.	The phrase 'withdraw' was confusing for the students.
9.	The sentence 'Studying mathematics is less beneficial for my life' was confusing for the students.
10.	There were several other words that confused the students, so their wording was simplified

Table 5. Evaluators' assessment from the readability test results

From the results of the readability test, it was found that certain statement sentences needed to be revised. After making these revisions, the final version of the questionnaire was established, which was a result of revising the questionnaire based on the readability test outcomes.

4) *Implementation*

At this stage, the questionnaire was distributed online in a Google Form format. The Google Form included the connecting teacher's name, the student's school name and address, the student's name and grade, the date the questionnaire was filled out, and the questionnaire statements.

For the distribution of the questionnaire, assistance was requested from teachers spread across several provinces in Indonesia. These teachers had participated in the Teacher Professional Education (PPG) Program for Mathematics Education at Universitas PGRI Palembang, including the cohorts of 2018 (in-person), 2019 (in-person), 2020 (online), and 2021 (online). Assistance was also provided by teachers who are alumni of the Mathematics Education Study Program at Universitas PGRI Palembang. Specifically, for teachers outside the Southern Sumatra region, they had participated in the PPG program online, so there were teachers from areas such as Toraja and Bulukumba (South Sulawesi), East Java, Bali, East Nusa Tenggara, and West Papua.

5) *Evaluation*

The questionnaire was distributed in a partisan manner by asking teachers voluntarily to disseminate it to their students for empirical testing in a quantitative manner (validity and reliability testing). A total of 113 teachers from various areas across 17 provinces agreed to help, including 16 from Riau, 2 from the Riau Islands, 5 from West Sumatra, 7 from Jambi, 18 from South Sumatra, 6 from Bangka Belitung, 4 from Bengkulu, 8 from Lampung, 20 from West Java, 3 from Central Java, 1 from East Java, 1 from Bali, 5 from West Kalimantan, 3 from East Kalimantan, 11 from South Sulawesi, 1 from East Nusa Tenggara, and 2 from West Papua.

Students were asked by their teachers to fill out the questionnaire online. The selection of students as samples was left to the teachers distributing the questionnaire. The only requirement was to ask for their willingness to fill out the questionnaire, without coercion. The overall timeframe for completing the questionnaire ranged from January 19, 2022, to March 16, 2022. The schools whose students completed the questionnaire included 29 junior high schools, 1 Islamic junior high school, 31 senior high schools, and 7 vocational high schools from 17 provinces.

No.	SMP/MTs	School accreditation	Grade			Number of students
			VII	VIII	IX	
1	SMPN 30 Batam City of Riau Islands	A	2	74	0	76
2	SMPN 2 Bukit Batu Bengkalis Riau	A	13	13	30	56
3	SMPN 7 Muaro Jambi	A	40	17	48	105
4	SMPN 12 Muaro Jambi	A	46	0	1	47
5	SMPN 6 Mesuji OKI South Sumatera	B	6	2	38	46
6	SMPN 59 Palembang	C	22	17	15	54
7	SMPN 39 Palembang	A	64	50	3	117
8	SMP Islam Al Azhar Sriwijaya Palembang	B	18	15	7	40
9	SMPN 31 Palembang	A	0	14	1	15
10	SMPN 7 Palembang	A	0	22	21	43
11	SMPN 5 Keluang Musi Banyuasin South Sumatera	C	0	7	26	33
12	SMPN 7 Satu Atap Sungai Selan Bangka	B	34	21	35	90
13	SMPN 3 Bakam Bangka Belitung	B	0	20	20	40
14	SMPN 4 Kaur Bengkulu	B	25	0	18	43
15	SMPN 22 Kota Bengkulu	A	15	13	21	49
16	MTs Qaryatul Jihad Center Bengkulu, Bengkulu	B	8	3	14	25
17	SMPN 3 Bandar Lampung	A	8	18	21	47
18	SMP Datarajan Tanggamus Lampung	B	4	7	6	17
19	SMPN 2 Kalianda Lampung	B	43	0	0	43
20	SMPIT Al-Ukhuwah Subang West Java	A	13	16	15	44
21	SMP Pesantren Ciwaringin Cirebon West Java	A	14	0	2	16
22	SMP Muhammadiyah Cilongok Banyumas Center Java	A	9	23	21	53
23	SMPIT Insan Cendekia Banyuwangi East Java	C	8	10	1	19
24	SMPN 4 Matan Hilir Selatan Ketapang West Kalimantan	A	11	7	8	26
25	SMP Muhammadiyah 2 Samarinda East Kalimantan	B	0	26	32	58
26	SMP Kristen Gandangbatu Tana Toraja South Sulawesi	B	0	10	24	34
27	SMPN 47 Bulukumba South Sulawesi	C	0	8	5	13
28	SMP Jembatan Budaya Badung Bali	A	48	0	0	48
29	SMPN 1 Poco Ranaka Manggarai Timur NTT	C	7	4	19	30
30	SMPN 5 Kota Sorong West Papua	B	0	0	23	23
	Total		458	417	475	1.350

Table 6. Some junior high schools/islamic junior high schools (SMP/MTs) where students filled out the questionnaire

No.	SMA/SMK	School accreditation	Grade			Number of students
			X	XI	XII	
1	SMAN 1 Bunguran Timur Natuna of Riau Islands	A	14	17	3	34
2	SMKN 1 Tapung Kampar Riau	A	18	36	15	69
3	SMAN 4 Bangko Pusako Rokan Hilir Riau	A	21	16	2	39
4	SMAN 1 Rengat Indragiri Hulu Riau	A	36	29	31	96
5	SMAN 10 Pekanbaru Riau	A	29	12	7	48
6	SMAN 1 Pangkalan Kuras Pelalawan Riau	A	2	8	0	10
7	SMAN 1 Ranah Pesisir West Sumatera	A	60	25	0	85
8	SMAN 4 Solok Selatan West Sumatera	A	0	15	2	17
9	SMAN 11 Padang West Sumatera	A	0	0	27	27
10	SMAN 2 Jambi	A	31	35	0	66
11	SMA Xaverius 1 Jambi	A	0	33	23	56
12	SMK PP Negeri, Batanghari Jambi	A	21	0	10	31
13	SMAN 3 Bungo, Jambi	A	0	0	50	50
14	SMAN 1 Tiga Dihaji OKU South Sumatera	C	26	27	24	77
15	SMAN 8 Palembang	A	20	0	0	20
16	SMAN South Sumatera	A	0	0	31	31
17	SMAN 3 Banyuasin III South Sumatera	B	0	0	5	5
18	SMAN 1 Sanga Desa Musi Banyuasin South Sumatera	A	19	26	1	46
19	SMAN 1 Namang Bangka Tengah	A	34	18	25	77
20	SMAN 1 Pangkalan Baru Bangka Tengah	A	23	9	0	32
21	SMKN 1 Sragi Lampung	A	25	2	0	27
22	SMAN 1 Gedong Tataan Pesawaran Lampung	A	16	35	11	62
23	SMAN 1 Gunung Pelindung East Lampung	B	0	0	8	8
24	SMKN 2 Lebong Bengkulu	B	6	0	3	9
25	SMAN 3 Lebong Bengkulu	A	0	10	0	10
26	SMA Swasta Pasundan 1 Bandung West Java	A	8	7	7	22
27	SMAN 1 Palimanan Cirebon East Java	A	36	12	8	56
28	SMAN 1 Ciwaru Kuningan East Java	A	22	41	8	71
29	SMK Muhammadiyah 1 Ajibarang, Banyumas Center Java	A	0	22	0	22
30	SMA Muhammadiyah 2 Samarinda East Kalimantan	B	13	10	8	31
31	SMAN 1 Suhaid Kapuas Hulu West Kalimantan	C	23	18	20	61
32	SMA Swasta Advent Singkawang West Kalimantan	C	6	2	7	15
33	SMAN 1 Belitang Hulu Kabupaten Sekadau West Kalimantan	B	24	25	25	74
34	SMAN 13 Makassar South Sulawesi	A	35	51	24	110
35	SMK Muhammadiyah Bungoro District Pangkajene and Kepulauan South Sulawesi	B	10	0	0	10
36	SMAN 5 Kabupaten Enrekang South Sulawesi	A	0	11	14	25
37	SMAN 5 Barru Kabupaten Barru South Sulawesi	A	0	97	2	99
38	SMKN 1 Kota Sorong West Papua	A	0	30	0	30
	Total		578	679	401	1.658

Table 7. Some senior high schools/vocational high schools (SMA/SMK) where students filled out the questionnaire.

After the questionnaire was distributed and filled out by students online, the results were obtained in an Excel format. A total of 3,020 students filled out the questionnaire voluntarily, but only 3,008 were selected, comprising 1,350 junior high school students and 1,658 senior high school students. The data of 12 students were not included due to unclear responses. After the students completed the questionnaire, the results in the Excel format were sorted into six groups based on grade criteria, namely Grade VII, VIII, and IX for junior high school, and Grade X, XI, and XII for senior high school.

No.	Item	Field test result											
		Junior high school						Senior high school					
		Grade VII		Grade VIII		Grade IX		Grade X		Grade XI		Grade XII	
		rcal	CA	rcal	CA	rcal	CA	rcal	CA	rcal	CA	rcal	CA
1.	Item 1	0,575	0,915	0,510	0,914	0,520	0,905	0,582	0,920	0,576	0,908	0,593	0,925
2.	Item 2	0,367		0,379		0,333		0,356		0,413		0,415	
3.	Item 3	0,598		0,562		0,590		0,645		0,563		0,611	
4.	Item 4	0,547		0,498		0,555		0,583		0,550		0,520	
5.	Item 5	0,468		0,425		0,511		0,492		0,488		0,468	
6.	Item 6	0,515		0,448		0,431		0,455		0,430		0,463	
7.	Item 7	0,359		0,392		0,374		0,406		0,386		0,378	
8.	Item 8	0,383		0,392		0,377		0,380		0,302		0,392	
9.	Item 9	0,506		0,493		0,528		0,584		0,499		0,596	
10.	Item 10	0,550		0,585		0,501		0,616		0,552		0,609	
11.	Item 11	0,386		0,465		0,464		0,502		0,475		0,514	
12.	Item 12	0,437		0,461		0,348		0,378		0,395		0,448	
13.	Item 13	0,410		0,372		0,392		0,399		0,451		0,472	
14.	Item 14	0,617		0,647		0,541		0,645		0,563		0,642	
15.	Item 15	0,515		0,490		0,530		0,557		0,471		0,465	
16.	Item 16	0,521		0,562		0,555		0,552		0,550		0,594	
17.	Item 17	0,532		0,560		0,601		0,590		0,556		0,573	
18.	Item 18	0,524		0,558		0,451		0,525		0,486		0,492	
19.	Item 19	0,574		0,607		0,594		0,627		0,583		0,583	
20.	Item 20	0,458		0,523		0,397		0,432		0,458		0,556	
21.	Item 21	0,511		0,544		0,525		0,539		0,503		0,529	

22.	Item 22	0,604		0,568		0,526		0,604		0,545		0,594	
23.	Item 23	0,500		0,522		0,475		0,522		0,459		0,478	
24.	Item 24	0,524		0,513		0,449		0,444		0,473		0,416	
25.	Item 25	0,583		0,529		0,530		0,594		0,575		0,573	
26.	Item 26	0,522		0,521		0,458		0,418		0,439		0,406	
27.	Item 27	0,570		0,628		0,567		0,590		0,580		0,620	
28.	Item 28	0,549		0,609		0,530		0,505		0,520		0,576	
29.	Item 29	0,506		0,514		0,457		0,510		0,476		0,543	
30.	Item 30	0,624		0,570		0,567		0,614		0,584		0,609	
31.	Item 31	0,563		0,505		0,567		0,550		0,487		0,546	
32.	Item 32	0,570		0,600		0,591		0,616		0,628		0,628	
33.	Item 33	0,065		0,017		0,069		0,049		0,001		0,194	
34.	Item 34	0,606		0,604		0,513		0,589		0,588		0,660	
35.	Item 35	0,534		0,521		0,571		0,632		0,558		0,617	
36.	Item 36	0,588		0,529		0,524		0,521		0,541		0,617	
		r table = 0,093	0,85	r table = 0,098	0,85	r table = 0,088	0,85	r table = 0,081	0,85	r table = 0,074	0,85	r table = 0,098	0,85

Table 8. Validity and reliability test results for junior high school and senior high school students

Notes:
rcal: calculated r
CA: Cronbach's Alpha value

From the data in Table 8, it is concluded that:

(1) For Grades VII, VIII, IX, X, and XI, there are 35 statements with a calculated r (rhit) greater than the table r (r tabel), making these 35 statements valid. Additionally, for Grades VII, VIII, IX, X, and XI, there is 1 statement with a calculated r (rhit) less than the table r (r tabel), making this statement invalid. This invalid statement is item 33.

(2) For Grade XII, there are 36 statements with a calculated r (rhit) greater than the table r (r tabel), so all statements are declared valid

(3) Meanwhile, the Cronbach's Alpha value for all statements is greater than 0.85, making all statements reliable.

Statement item 33, declared invalid, reads: 'I prefer to solve mathematical problems in a simple way without using formulas'

DISCUSSION

For a broader empirical test, namely a field test, the questionnaire was distributed to junior high school (SMP sederajat) and senior high school (SMA sederajat) students. The selection of students to fill out the questionnaire was not based on their ability level; the main criterion was their willingness to participate. By having a large number of students participate, all ability levels were naturally included. The schools whose students filled out the questionnaire had accreditation levels of A, B, and C. These schools were located in both large cities and districts, as well as in rural areas.

By examining the data from the schools where students who filled out the questionnaire were enrolled, it was evident that the questionnaire was completed by students from various ethnic groups, reflecting the diversity of Indonesia. These included the Malay, Batak, Minang, Rejang, Komering, Lampung, Sundanese, Javanese, Bugis, Duri, Enrekang, Maiwa, To Balo, Toraja, Dayak, Balinese, Manggarai, and Papuan ethnic groups.

The questionnaire designed to assess self-efficacy, prepared for deployment in field testing, consists of 36 statements. After distributing the questionnaire to students in various regions of Indonesia, it was concluded that for students in Grades VII, VIII, IX, X, and XI, 35 statements were deemed valid and 1 statement was invalid, which was item 33, stating, 'I prefer to solve mathematical problems in a simple way without using formulas'. All statements were found to be reliable, with a reliability coefficient greater than 0.85. However, for Grade XII, all 36 statements were valid, and no statements were found to be invalid. All statements were reliable, with a reliability coefficient greater than 0.85.

These results indicate that statement item 33 was disliked by most respondents. The wording of the statement implies that 'the respondent prefers to solve mathematical problems in a simple way without using formulas'. This can be interpreted in two ways: (1) students prefer solving math problems in a simple way, and (2) students prefer solving math problems without using formulas. In actual classroom learning, students indeed prefer simpler methods if there are easier alternatives available. However, students may face difficulties in solving mathematical problems without using formulas, as they may not know the direction to take for solving the problem without formulas.

Therefore, statement item 33 should be split into two separate statements, as its content indeed represents two different conditions as mentioned in the previous paragraph. The revised wording for statement item 33 should be 'I prefer to solve mathematical problems in a simple way' and 'I prefer to solve mathematical problems using formulas'.

Although for Grade XII senior high school students, statement item 33 can still be used to assess self-efficacy, as it was also found valid when tested on these students. However, for consistency, for Grade XII as well, statement item 33 should be split into two statements as mentioned above.

Finally, the self-efficacy questionnaire produced herein completed a comprehensive development process using the ADDIE model, going systematically through each phase of the ADDIE framework, and its validity and reliability were rigorously validated. The questionnaire was originally comprised of 36 items, but after intensive field testing and content analysis, the item count was refined and expanded to 37. As a result, the findings confirm that the self-efficacy questionnaire is valid, practical, and reliabel, culminating in a finalized instrument of 37 items, as shown below:

No.	Statement
1.	I can solve mathematical problems or assignments
2.	I find it difficult to succeed in learning mathematics without someone else's help
3.	Learning mathematics is enjoyable
4.	I am not at all interested in learning mathematics
5.	Learning mathematics is important for training the mind
6.	I have not yet found the benefit of learning mathematics
7.	Correct steps are needed to solve mathematical problems or tasks
8.	If there's a math problem, I just write the answer without stating 'given' and 'asked'
9.	I can solve mathematical problems accurately
10.	I always face difficulties in solving mathematical problems
11.	I want to finish quickly when there is a more difficult math problem
12.	If the math problems become harder, I start to feel overwhelmed
13.	I can study mathematics on my own when it's quiet, like at night
14.	I always find it hard to learn and understand mathematics
15.	If I work hard, I can complete even the difficult math tasks
16.	Every time I try to solve a math problem, the result is always unsatisfying
17.	I will try to complete every math task or problem given
18.	I am not willing to work on any math tasks or problems
19.	I realize that I am capable of learning mathematics
20.	I am aware that I have many weaknesses in learning mathematics
21.	I keep trying to complete the math tasks given by the teacher as best as I can
22.	If the math task given by the teacher turns out to be difficult, I start to avoid it
23.	I want to be smart by learning mathematics
24.	Learning mathematics is not very beneficial
25.	I always try to follow the math lessons because they can train my abilities
26.	I don't like learning mathematics because it's not important
27.	Even if I learn different math materials than before, I am still happy, as it enhances my thinking ability
28.	The more varied the math materials I study, the more confused I become
29.	I try to correct my previous mistakes in learning mathematics
30.	I have never been able to learn mathematics
31.	If I finish studying one math topic, I will study another
32.	I always find it difficult to learn every math topic
33.	I prefer to solve math problems in a simple way
34.	I prefer to solve math problems by using formulas
35.	I always struggle when solving math problems, sometimes even unable to finish them
36.	I am happy when there are new, more challenging materials
37.	Even learning easy math topics feels difficult, let alone the hard ones

Table 9. Self-Efficacy questionnaire valid, practical, and reliable

During this research, several challenges were encountered that may have affected the validity of the study, as follows:

1. Creating questionnaire statements that truly reflect self-efficacy. In this process, the researcher spent a considerable time designing the questionnaire, frequently consulting with experienced colleagues about self-efficacy, and also sought validators who were highly competent in this area.
2. Testing readability for junior high school students, as they sometimes did not understand the meaning of the sentences they read.
3. When administering the questionnaire to students, especially junior high school students, it was estimated that some did not fully understand its purpose, particularly those with lower IQs.
4. The self-efficacy questionnaire was created and distributed using Google Forms, and for the Likert scale, only the far left and far right statement options were written, potentially confusing students about the middle options. Therefore, the researcher asked teachers distributing the questionnaire to explain this.
5. Many teachers helping to distribute the questionnaire faced signal issues, especially those in remote areas like Bulukumba and Tanah Toraja.
6. Many students were not allowed to bring mobile phones to school, forcing them to fill out the questionnaire at home. If completed at school, teachers could provide explanations, but at home, if students were confused, they might just click options randomly without further thought.
7. Some students in urban areas, and especially in district and rural areas, did not have mobile phones and thus could not fill out the questionnaire, resulting in fewer data from those regions.
8. Some teachers asked their students to fill out the questionnaire on Google Forms, but only a few responded, with some classes having only one student participating.

Despite these challenges, considering that 3,008 students filled out the questionnaire, with each grade level having a minimum of 400 respondents, this meets the recommendations of Crocker and Algina (Alwi, 2012), who stated that a minimum of 200 respondents is needed for stability. It also aligns with Nunnaly's (Alwi, 2012) suggestion that the sample size in a trial should be ten times the number of items in the measuring instrument. In this study, with 36 questionnaire items, $10 \times 36 = 360$ respondents were needed, while the actual number of respondents was at least 400. Thus, the challenges mentioned above were overcome with the number of respondents meeting these criteria.

CONCLUSIONS

This development research has produced a valid, practical, and reliable self-efficacy questionnaire for students in mathematics learning. This questionnaire has undergone a development process using the ADDIE model. In this development process, the questionnaire was subjected to both theoretical and empirical testing. The questionnaire statements were reviewed and commented on by 4 expert validators, tested for readability on 4 students (1 elementary and 3 junior high school students), and quantitatively tested on 1,350 junior high school students and 1,658 senior high school students across Indonesia.

Initially, there were 36 statements. Of these, 35 statements were declared valid after empirical validity testing, and 1 statement was declared invalid (item 33), leading to the modification of this statement into 2 separate statements (items 33 and 34). All statements were declared reliable based on field test results. Ultimately, the self-efficacy questionnaire produced consists of 37 statements.

The self-efficacy questionnaire developed through this process can be used to assess students' self-efficacy in mathematics learning in Indonesia. According to the research subjects, the questionnaire is suitable for junior high school students and senior high school students. However, it can potentially also be used by university students.

This developed questionnaire can be further researched to make it more perfect, and it can also be retested on students outside Indonesia to broaden its applicability. Of course, this would involve adapting the developed self-efficacy questionnaire to suit the conditions of other countries

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The Problem Corner

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The Purpose of ***The Problem Corner*** is to give Students and Instructors working independently or together a chance to step out of their "comfort zone" and solve challenging problems. Rather than in the solutions alone, we are interested in methods, strategies, and original ideas following the path toward figuring out the final solutions. We also encourage our Readers to propose new problems. To submit a solution, type it in Microsoft Word, using math type or equation editor, however PDF files are also acceptable. Email your solution as an attachment to The Problem Corner Editor iretamoso@bmcc.cuny.edu stating your name, institutional affiliation, city, state, and country. Solutions to posted problem must contain detailed explanation of how the problem was solved. The best solution will be published in a future issue of MTRJ, and correct solutions will be given recognition. To propose a problem, type it in Microsoft Word, using math type or equation editor, email your proposed problem and its solution as an attachment to The Problem Corner Editor iretamoso@bmcc.cuny.edu stating your name, institutional affiliation, city, state, and country.

Hello, problem solvers!

I'm pleased to report that we've received correct and insightful solutions for Problem 26 and Problem 27 in *The Problem Corner*. These submissions have not only met our accuracy requirements but also demonstrated thoughtful and effective problem-solving strategies. Our goal is to present exemplary solutions that inspire and advance mathematical knowledge worldwide.

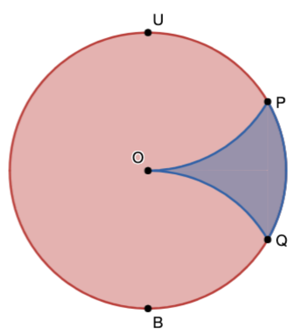
Solutions to **Problems** from the Previous Issue.

“Curvy” slice of a circle problem.

Problem 26

Proposed by Ivan Retamoso, BMCC, USA.

The diagram illustrates a circle with a radius of 6 inches and center O, with UB as its diameter. Points P and Q are positioned on the circle so that OP and OQ are arcs of circles with a radius of 6 inches and centers at U and B, respectively. Determine, in exact form, the area of the "blue" region OPQ.



First solution to problem 26

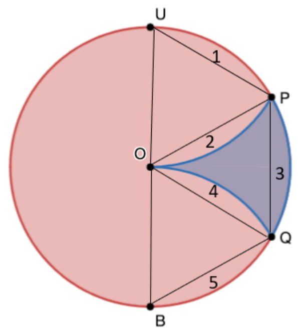
By Dr. Hosseinali Gholami, University Putra Malaysia, Serdang, Malaysia

Our solver delivered two equivalent solutions: one using only geometric formulas and symmetry, and the other using integral calculus. Each solution is explained in detail, with accompanying graphs that enhance understanding.

Solution 1:

As respect to the following shape, the surface of areas 1, 2, 3, 4 and 5 are equal, because each of these three triangles is an equilateral triangle. The surface of area 3 is calculated as below.

$$S = \frac{1}{6}\pi r^2 - \frac{r^2\sqrt{3}}{4} = \frac{1}{6}\pi 6^2 - \frac{6^2\sqrt{3}}{4} = 6\pi - 9\sqrt{3}.$$



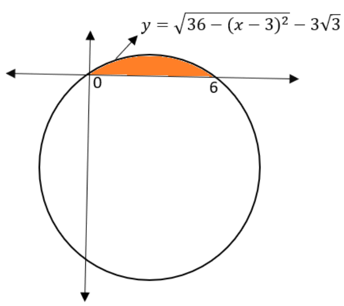
Therefore, the area of the "blue" region OPQ is determined as follows.

$$S_{OPQ} = \frac{1}{6}\pi r^2 - 2S = \frac{1}{6}\pi 6^2 - 2\left(6\pi - 9\sqrt{3}\right) = 18\sqrt{3} - 6\pi.$$

Solution 2:

Based on the figure above, we transfer the area 3 on the coordinates axes as shown in the following figure.

The equation of this circle with the center $(3, -3\sqrt{3})$ is $(x-3)^2 + (y - 3\sqrt{3})^2 = 36$.



This area is calculated as $S = \int_0^6 (\sqrt{36-(x-3)^2} - 3\sqrt{3})\, dx$. The value of S is determined according to the formula

$$\int \sqrt{36-(x-3)^2}\, dx = \frac{1}{2}(x-3)\sqrt{36-(x-3)^2} + 18\sin^{-1}(\frac{x-3}{6}) - 3\sqrt{3}x + c$$

as below.

$$S = \left(\frac{3}{2} \times 3\sqrt{3} + 18 \times \frac{\pi}{6} - 3\sqrt{3} \times 6\right) - \left(\frac{-3}{2} \times 3\sqrt{3} - 18 \times \frac{\pi}{6}\right) = 6\pi - 9\sqrt{3}.$$

Therefore, the answer of this problem is obtained as follows.

$$S_{OPQ} = \frac{1}{6}\pi r^2 - 2S = \frac{1}{6}\pi 6^2 - 2\left(6\pi - 9\sqrt{3}\right) = 18\sqrt{3} - 6\pi.$$

Second solution to problem 26

By Dr. Aradhana Kumari, Borough of Manhattan Community College, USA.

This solution relies solely on Integral Calculus and symmetry. Our solver cleverly uses two curves to compute the area of the requested region. The diagrams and labeling make the solution appealing and easy to follow.

Solution: Consider the figure 1. Below

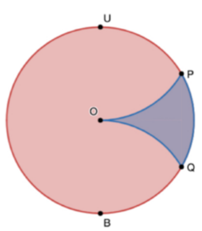


Figure 1.

Let's put the Figure 1. given in the problem to the rectangular coordinate system.

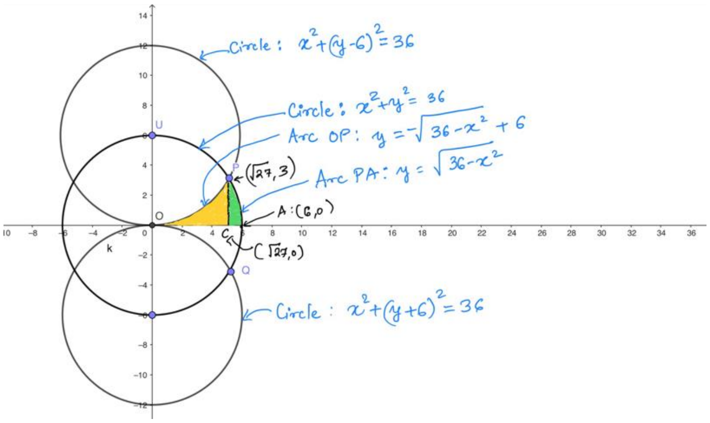


Figure 2.

Point P is the intersection of the circle given by equation $x^2 + y^2 = 36$ and the circle $x^2 + (y-6)^2 = 36$. The coordinates of the point P can be obtained by solving

$x^2 + y^2 = x^2 + (y-6)^2$,

i.e $y^2 = (y-6)^2$

Hence $-12y = -36$

$y = 3$

Substituting the value of $y = 3$ in the equation $x^2 + y^2 = 36$ we get $x = \sqrt{27}$

Hence the coordinates of the point P $(\sqrt{27}, 3)$.

The required area is 2 times the area of yellow shaded region plus the area of green shaded region.

The equation of circle with center O $(0,0)$ and radius 6 is $x^2 + y^2 = 36$..................(1)

The equation of circle with center U $(0,6)$ and radius 6 is $x^2 + (y-6)^2 = 36$ ....(2)

The equation of circle with center B $(0,-6)$ and radius 6 is $x^2 + (y+6)^2 = 36$.......(3)

To find the area of yellow shaded region, we need to know the equation of curve passing through O to P in the first quadrant. This curve lies on the circle with center U and radius 6. Using equation given by (2) we get $y = -\sqrt{36 - x^2} + 6$

The area of the yellow shaded region is $\int_0^{\sqrt{27}} \{-\sqrt{36 - x^2} + 6\}\, dx$

$= \int_0^{\sqrt{27}} \{-\sqrt{36-x^2}\}\, dx + \int_0^{\sqrt{27}} 6\ dx = \frac{-9\sqrt{3}}{2} - 6\pi + 6\sqrt{27}$

To find the area of the green shaded region, first we need to find the equation of the curve PC in the first quadrant. This curve lies on the circle with center O and radius 6. Using equation given by (1) we get $y = \sqrt{36-x^2}$

The area of the green shaded region is

$\int_0^{\sqrt{27}} \{\sqrt{36-x^2}\}dx = \int_{\sqrt{27}}^{6} \{\sqrt{36-x^2}\}\, dx = 3\pi - \frac{9\sqrt{3}}{2} - 6\pi$

The required area of the blue region $OPQ = 2\left(\frac{-9\sqrt{3}}{2} - 6\pi + 6\sqrt{27} + 3\pi - \frac{9\sqrt{3}}{2}\right)$

$= 2\left(-9\sqrt{3} - 3\pi + 18\sqrt{3}\right) = 2\left(9\sqrt{3} - 3\pi\right) = 18\sqrt{3} - 6\pi$

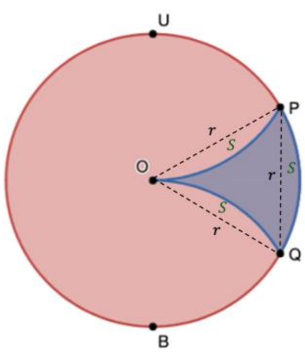
Third solution to problem 26

By Dr. Abdullah Kurudirek, Mathematics Ed. Department, Tishk International University.

By combining the proportions of circle sectors derived from central angles with the area formula for an equilateral triangle, our solver produces a solution that is both concise and elegant.

Solution:

We can join the points O, P, and Q by auxiliary lines (each has the same length $r = 6$ inches) and then we get an equilateral triangle ΔOPQ and its area equals $\frac{\sqrt{3}}{4}r^2$



Area of segment $S = \frac{\frac{\pi}{3}}{2\pi}\pi r^2 - \frac{\sqrt{3}}{4}r^2 = \frac{1}{6}\pi 6^2 - \frac{\sqrt{3}}{4}6^2 = 6\pi - 9\sqrt{3}$

And the area of the blue region OPQ will be as follows:

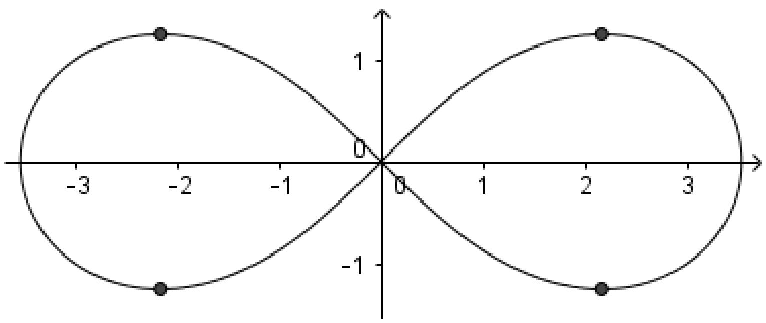
$$OPQ = Area\ of\ Sector - 2S = 6\pi - 2\left(6\pi - 9\sqrt{3}\right) = 18\sqrt{3} - 6\pi$$

The "bow tie" problem

Problem 27

Proposed by Ivan Retamoso, BMCC, USA.

Consider the lemniscate curve $2(x^2 + y^2)^2 = 25(x^2 - y^2)$.



a. Find the slope of the tangent line to the lemniscate in terms of the variables x and y.

b. The four points on the lemniscate where the tangent line is horizontal are all on the intersection of the lemniscate with circle $x^2 + y^2 = k$, find the value of k.

First solution to problem 27

By Dr. Abdullah Kurudirek, Mathematics Ed. Department, Tishk International University.

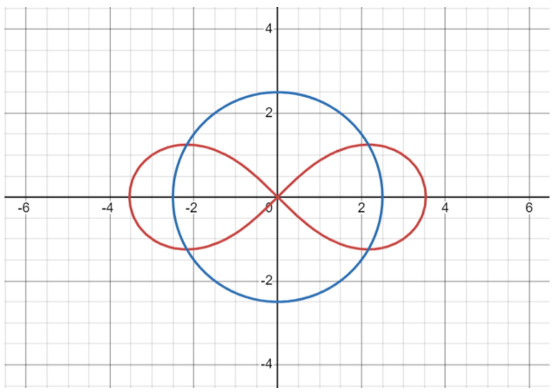
The approach employed by our solver makes excellent use of implicit differentiation and partial derivatives. The final DESMOS graph plays a crucial role in visualizing and comprehending the results.

a) If we consider the lemniscate curve $2(x^2+y^2)^2 = 25(x^2 - y^2)$ as $2x^4 + 4x^2y^2 + 2y^4 - 25x^2 + 25y^2 = 0$ then $\frac{dy}{dx} = -\frac{diff.\ w.r.t\ x}{diff.\ w.r.t\ y} = -\frac{x(4x^2+4y^2-25)}{y(4x^2+4y^2+25)}$ by using implicit differentiation, and this is the slope of the tangent line to the lemniscate in terms of the variables x and y.

b) The four points on the lemniscate where the tangent line is horizontal means $\frac{dy}{dx} = -\frac{x(4x^2+4y^2-25)}{y(4x^2+4y^2+25)} = 0$ so $4x^2+4y^2-25=0$ and $x \neq 0$ (not one of the four points)

$4x^2+4y^2-25=0$ then $x^2+y^2=\frac{25}{4}$ so $k=\frac{25}{4}$, and by the way, if we substitute $x^2+y^2=\frac{25}{4}$ in the original equation, we find $x=\pm\frac{5\sqrt{3}}{4}$ and $y=\pm\frac{5}{4}$.

On the other hand, by using Desmos graphing calculator we can see the intersection of the lemniscate with the circle as follows.



Second solution to problem 27

By Dr. Aradhana Kumari, Borough of Manhattan Community College, USA.

By first applying implicit differentiation to compute dydx\frac{dy}{dx}dxdy, our solver then uses the property that horizontal lines have a slope of 0 to smartly determine the value of kkk. The step-by-step solution is clear and engaging to follow.

Solution a). Consider the lemniscate curve given by the equation below

$2\,(x^2+y^2)^2 = 25\,(x^2-y^2)$............(1)

To find the slope of the tangent line to the lemniscate curve in terms of x and y we differentiate the equation given by (1) we get

$$\frac{d}{dx}2\,(x^2+y^2)^2 \;=\frac{d}{dx}25\,(x^2-y^2)$$

$4(x^2+y^2)\frac{d}{dx}(x^2+y^2) = \frac{d}{dx}25\,(x^2-y^2)$

$4(x^2+y^2)\left(2x+2y\frac{dy}{dx}\right) = 25\left(2x-2y\frac{dy}{dx}\right)$

$4(x^2+y^2)\left(x+y\frac{dy}{dx}\right) = 25\left(x-y\frac{dy}{dx}\right)$

$4(x^2+y^2)x + 4\mathrm{y}(x^2+y^2)\frac{dy}{dx} = 25x - 25y\frac{dy}{dx}$

$4\mathrm{y}(x^2+y^2)\frac{dy}{dx} + 25y\frac{dy}{dx} = 25x - 4(x^2+y^2)x$

$[4\mathrm{y}(x^2+y^2)+25y]\frac{dy}{dx} = x[(25-4(x^2+y^2)]$

$$\frac{dy}{dx} = \frac{x[25-4(x^2+y^2)]}{\mathrm{y}[25+4(x^2+y^2)]}$$

The slope of the tangent line at the point (x, y) is given by $\frac{x[25-4(x^2+y^2)]}{\mathrm{y}[25+4(x^2+y^2)]}$.

Solution b). If the tangent line is horizontal, then $\frac{dy}{dx} = 0$ that means

$$\frac{dy}{dx} = \frac{x[25-4(x^2+y^2)]}{\mathrm{y}[25+4(x^2+y^2)]} = 0$$

Hence $x[(25-4(x^2+y^2)] = 0$

Either $x = 0$ or $[25-4(x^2+y^2)] = 0$

Consider $[25-4(x^2+y^2)] = 0$,

$25 = 4(x^2+y^2)$

Hence $x^2+y^2 = \frac{25}{4}$, therefore the required value of k is $\frac{25}{4}$.

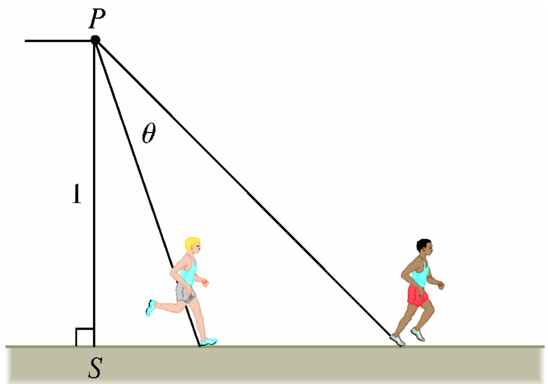
Dear fellow problem solvers,

I'm thrilled you enjoyed working through problems 26 and 27 and that you've expanded your mathematical strategies. Let's dive into the next set of problems to further develop your skills.

Problem 28

Proposed by Ivan Retamoso, BMCC, USA.

An observer is positioned at point P, one unit away from a track. Two runners begin at point S, which is illustrated in the diagram, and move along the track. One of the runners runs at a speed three times faster than the other. Determine the maximum angle θ that the observer's line of sight forms between the two runners.



Problem 29

Proposed by Ivan Retamoso, BMCC, USA.

A regular octagon $ABCDEFGH$ has sides that are 2 units in length. The points W, X, Y, and Z are the midpoints of the sides $\overline{AB}$, $\overline{CD}$, $\overline{EF}$, and $\overline{GH}$, respectively. Find the probability that a point chosen uniformly at random from inside the octagon $ABCDEFGH$ will be located inside the quadrilateral $WXYZ$. Give your answer in exact form.