Mathematical Problem Design to Explore Students' Critical Thinking Skills in Collaborative Problem Solving

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Abstract: Social interactions, including collaborative problem-solving situations, can trigger critical thinking skills. Giving questions that are not routine can trigger students' critical thinking skills in solving problems collaboratively. This research aims to develop non-routine mathematics problems that can be used to explore students' critical thinking abilities in collaborative problem-solving. The research results show that questions with problem criteria that require justification for the solution provided and questions with a graphical analysis approach can be used to explore students' critical thinking skills in collaborative problem-solving. This is proven by solving the problems; each group member contributed to the solution-finding process. The contribution of each group member shows the high intensity of interaction between members. Interaction in the form of exchanging opinions, giving suggestions, and evaluating each other's ideas or answers significantly impacts students' critical thinking abilities. This is seen by the emergence of several students' critical thinking skills (analysis, synthesis, argumentation, evaluation, self-regulation) triggered by suggestions or ideas put forward by other group members. The research results can be a reference for researchers or practitioners exploring critical thinking skills as a guide in developing research instruments.

Keywords: Critical Thinking Skills, Non-Routine Problem, Collaborative Problem Solving, Mathematics

INTRODUCTION

Mathematics education focuses on increasing the use of acquired mathematical knowledge and skills in daily problem-solving activities (Stacey & Turner, 2015). Critical thinking skills are needed to solve complex problems. Yee et al. (2011) state that critical thinking skills play a role in determining decisions in the problem-solving process. Someone who has critical thinking skills has a high level of sensitivity to problems so that they can quickly formulate problems, review problems from several perspectives, and evaluate every step of solving problems that have been solved (Maričić et al., 2016).

Critical thinking skills are one of the competencies that need to be developed because they predict one's success (Butler et al., 2017; Haynes et al., 2016). In addition, critical thinking skills can mediate several competencies that need to be mastered in the 21st century, such as collaborative skills, creative thinking, algorithmic thinking, and problem-solving (Kocak et al., 2021). Although critical thinking skills are cognitive processes, some experts define indicators that can be used to represent critical thinking skills. Perkins & Murphy (2006) formulated four indicators of critical thinking skills, namely clarification, assessment, inference, and strategy. Facione (2015) mentions six indicators of critical thinking skills: interpretation, analysis, inference, explanation, evaluation, and self-regulation. Furthermore, four indicators of critical thinking skills were formulated by Ennis (2016), namely essential clarification, bases for decision, inference, and advanced clarification. Reynders et al. (2020) created a rubric to assess critical thinking skills based on four indicators: analyzing, synthesizing, forming arguments, and evaluating. At the same time, Cortazar et al. (2021) used six aspects as indicators of critical thinking skills: interpretation, analysis, inference, arguments, evaluation, and metacognition. From the results of the studies of several experts, the researchers formulated five indicators of critical thinking skills: analysis, synthesis, argumentation, evaluation, and self-regulation.

The development of a person's critical thinking skills is influenced by social interaction. Collaboration catalyzes critical thinking skills (Waite & Davis, 2006) because collaboration encourages students to think deeply (Ebiendele Ebosele Peter, 2012; Hussin et al., 2019). Collaborative Problem Solving (CPS) is a problem-solving activity that requires interaction between group members during the problem-solving process. Hagemann & Kluge (2017) state that CPS is an interdependent activity of group members in the context of turning an input into output through cognitive, verbal, and behavioral activities to regulate task completion to achieve common goals. The interdependence attitude manifests in two roles: *explainer/solver* and *checker* (Westermann & Rummel, 2012). *Explainers* trigger cognitive processes such as elaborating knowledge, and *checkers* monitor explanations and reflect on understanding. In other words, CPS facilitates cognitive and metacognitive processes, supporting a person to become a good critical thinker (Maynes, 2015).

CPS emphasizes the interdependence between group members. To foster this attitude of interdependence, the given in CPS are problematic for each group member (Hagemann & Kluge, 2017; Westermann & Rummel, 2012). Complicated tasks involve problems involving several mathematical concepts, and solving them requires critical thinking skills, namely the ability to analyze, synthesize, and evaluate (Westermann & Rummel, 2012; Williams, 2000). This problematic task's characteristics follow the characteristics of non-routine mathematics tasks. In mathematics, non-routine tasks are characterized by not having an immediate solution, requiring productive thinking (Kolovou et al., 2009), involving unexpected solutions (Yeo, 2009), requiring strategic thinking, and containing various mathematical concepts (Mabilangan et al., 2011).

The development of mathematical tasks that can trigger critical thinking skills has been carried out by (Kuntze et al., 2017) in a particular context. In the context of collaboration, there still needs to be more development of tasks that can trigger students' critical thinking skills. In learning activities, students often work collaboratively due to the demands of collaboration skills in the 21st century (Barron, 2000; Chew et al., 2020; Sofroniou & Poutos, 2016). So for educators or researchers who support collaborative performance to explore critical thinking skills, it is necessary to know what characteristics of the problem can trigger students' critical thinking skills. The accuracy of the problem design will affect the accuracy of the critical thinking skills data obtained. The teacher and researcher can appropriately determine the next step if the data is accurate. Based on the literature review results mentioned, this study aims to develop non-routine mathematics problems that can be used to explore students' critical thinking skills in collaborative problem-solving. The urgency of developing non-routine problems is shown in Figure 1.

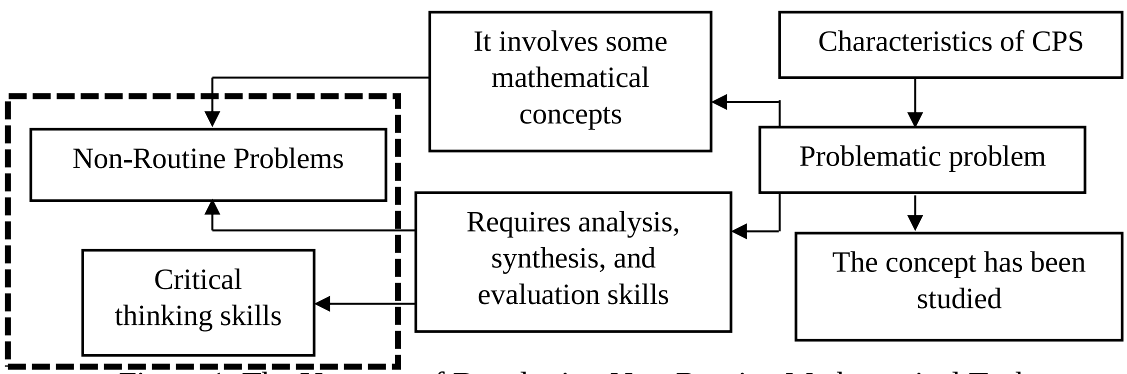


Figure 1. The Urgency of Developing Non-Routine Mathematical Tasks

LITERATURE REVIEW

Critical Thinking Skills in Collaborative Problem Solving

Vygotsky's sociocultural theory and the Zone of Proximal Development (ZPD) model state that conversations with peers will expand students' ZPD to think critically (Wass et al., 2011). Furthermore, Wass et al. (2011) stated that in Vygotskian's view, critical thinking involves the collaboration of several mental functions, such as memory, imagination, analysis, and evaluation taught through conversation. Therefore Wait & Davis (2006) stated that collaboration is a skill catalyst for critical thinking. In this study, collaboration settings were facilitated by Collaborative Problem-Solving (CPS) activities. The indicators for critical thinking skills in this study use critical thinking indicators from Reynders et al. (2020), namely analysis, synthesis, argumentation, and evaluation, coupled with another aspect, namely self-regulation from Facione (2015). Self-regulation is deemed necessary to add because, in collaborative work, there will be an interaction between group members. Someone who thinks critically will check his understanding to respond to analysis, synthesis, argumentation, and evaluation activities carried out by others (Facione, 2015). The five indicators are then adjusted to the stages of solving collaborative problems proposed by Hesse et al. (2015): problem identification, planning and exploring, execution, and verification. This adjustment is based on the opinion put forward by Lester (2013), which implied that critical thinking skills play a role in every problem-solving activity. Analysis skills play a role in simplifying the context of the problem, and synthesis is needed when selecting the mathematical concepts to be used, argumentation is needed to execute the selected mathematical concepts, and evaluation is needed when checking the suitability of the problem with the solutions found. Figure 2 is a visual representation of the role of critical thinking skills in solving collaborative mathematical problems.

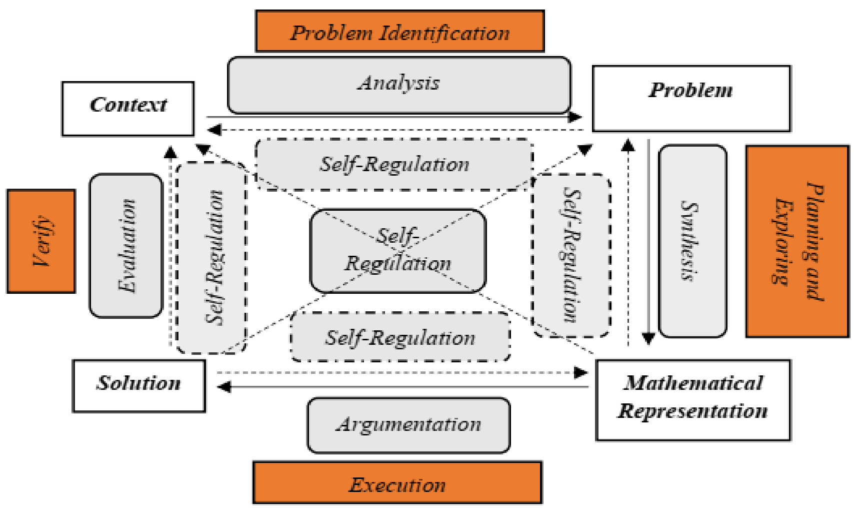


Figure 2. The Role of Critical Thinking Skills in Solving Collaborative Mathematical Problems

Descriptions of adjustments to the stages of problem-solving and indicators of critical thinking skills are explained in Table 1. Combines the results of the theoretical studies put forward by Reynders (2020); Facione (2015); Hesse et al. (2015); dan Lester (2013), indicators of critical thinking skills in collaborative problem solving used by researchers are presented in Table 2.

Table 1. Description of CPS Stages and Critical Thinking Skills Indicators

Stages of CPS (Hesse et al., 2015)	Critical Thinking Skills Indicator (Facione, 2015; Reynders et al., 2020)
Problem Identification (PI) Identifying problematic problem elements by communicating opinions or information based on different roles.	*Analysis* Describe and explore the meaning of data based on existing knowledge.
	Self-Regulation Check the quality of your thinking.
Planning and Exploring (PE) Determining mathematical ideas that can support solving complex	*Synthesis* Identify the relationship of some information or concepts.

Stages of CPS (Hesse et al., 2015)	Critical Thinking Skills Indicator (Facione, 2015; Reynders et al., 2020)
problems by accommodating the various perspectives of team members based on different roles.	*Self-Regulation* Check the quality of your thinking.
Execution (EX) Implementing problematic problem-solving ideas that team members have agreed upon based on differences in roles.	*Argumentation* Provide a systematic explanation in responding or providing information.
	Self-Regulation Check the quality of your thinking.
Verify (VF) Checking the suitability of complex problems with solutions found by team members.	*Evaluation* Assess the credibility of the claims and arguments that have been generated.
	Self-Regulation Check the quality of your thinking.

Table 2. Indicator for Critical Thinking Skills in Collaborative Problem Solving

Code	**Critical Thinking Skills Indicator in Collaborative Problem Solving**
(An)	*Analysis* Describe and explore the meaning of data to understand and identify problematic problem elements by communicating opinions or information.
(Sy)	*Synthesis* Identifying the relationship between some information or concepts by accommodating the various perspectives of team members based on different roles to determine ideas that can support solving complex problems.
(Ar)	*Argumentation* Provide systematic explanations to apply ideas to solve complex problems that team members have agreed upon.
(Ev)	*Evaluation* Assess the credibility of the claims and arguments generated to check the suitability of problematic issues with the solutions that team members have found.
(Sr)	*Self-Regulation* Checking the quality of one's thinking during the problem-solving process

Non-Routine Mathematical Problem

Types of mathematics problems are divided into routine and non-routine problems (Jäder et al., 2017). Routine problems are problems that are often encountered by students and have algorithms that are ready to be used to solve problems. In contrast, non-routine problems require high-level thinking and are rarely found in learning materials (Kablan & Uğur, 2020). Non-routine problems require students to use cognitive processes such as critical thinking to find solutions (Asman & Markovits, 2009; Thomas et al., 2013). In the context of mathematics, non-routine problems are mathematical problems that do not have a straightforward solution (Elia et al., 2009), require productive thinking (Kolovou et al., 2009), and require strategic thinking (Mabilangan et al., 2011). In other words, non-routine math problems are math problems that do not have a unique algorithm, so they require strategic thinking to solve them.

Furthermore, mathematical problems, according to their purpose, are divided into two, namely "problem to find" and "problem to prove" (Polya, 1945). At the advanced level, the given math problem can be a "problem to prove," while at the intermediate to a basic level, the problem given is a "problem to find" (Stylianou et al., 2015). The participants in this study were high school students. Thus, "problem to find" was more appropriate to be developed into a research instrument. Problems related to quadratic functions were chosen to be developed in this study.

METHOD

This study aims to develop non-routine mathematics problems to explore students' critical thinking skills in collaborative problem-solving. A non-routine mathematical problem on quadratic function material is developed. Problem requires students to analyze graphs (Table 4). Function material was chosen because based on research conducted by Marzuki et al (2021) and Endrawati & Aini (2022) stated that problems related to functions can be used to explore students' critical thinking skills. Problem development also refers to the opinion of Rott (2021), which states that using math problems with clear but wrong solutions can explore students' critical thinking skills. This problem will trigger students to evaluate the stages of problem-solving thoroughly. Finding the right solution to a problem will trigger students to think critically. More specifically, the research results of Korres & Tsami (2010) and Ariza et al. (2021) also state that in material related to function, misuse of definitions can be detected and recognized by students through the use of graphical representations to present concepts, for example, by describing various graphic positions and connecting them with definition or concept. Therefore, a non-routine mathematical problem related to functions developed by researchers, namely, asking students to analyze graphs of functions. Table 3 shows the developed grid of non-routine mathematical problems.

Table 3. Lattice of Non-Routine Mathematical Problem

Problem Criteria	Problem
Purpose of Problem	Problem to Find

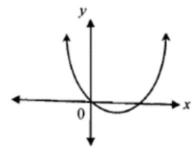
Material	Quadratic Function
Approach	Chart Analysis
Type of Problem	Require Justification Mathematics Problem

In addition to the problem items, the researcher also made guidelines for solving each problem item.

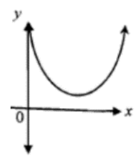
Table 4. Developed Non-Routine Mathematical Problems

Problem

The function curve $f(x) = x^2 + kx$ is as follows.



Based on this information, Saila was asked to draw a curve which is a function curve $f(x) = x^2 - kx + 5$. Next, Saila draws the following curve and states that the curve is a function curve $f(x) = x^2 - kx + 5$.



Is Saila's statement true? Explain your reasons.

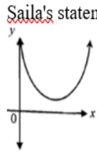
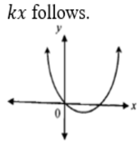
Validation activities are carried out to check the validity of the non-routine math problems that have been developed. Validation was carried out by three expert validators who are lecturers in Mathematics Education at Surabaya State University. One of the validators is a professor of Mathematics Education, the second validator is a lecturer in Mathematics Education whose research focuses on secondary education, and the third is a lecturer in Mathematics Education whose research focuses on critical thinking skills. Two drafts are given to the validator, namely drafts of non-routine math problems and guidelines for solving them. The validation results show that there are several suggestions from the validator, namely; 1) provide instructions for working on questions that can condition groups to complete tasks collaboratively from start to finish; 2) create an alternative data analysis document that shows; a) part of the problem that can trigger the emergence of aspects of critical thinking skills; b) the possibility of the emergence of critical thinking skills when students solve problems collaboratively. The conclusion from the results of the validation is that the non-routine math problems that have been developed can be used to explore students' critical thinking skills after being revised according to the suggestions of the validator. Based on suggestions from the validator, Table 5 shows the results of improvements to non-routine math problems that can be used to explore students' critical thinking skills in collaborative problem-solving.

Table 5. Improvement of Non-Routine Mathematical Problems based on the Validator's Suggestions

<table>
<tr><th>Validator's Suggestion</th><th>Improvement of Non-Routine Math Tasks</th></tr>
<tr><td>Provide instructions for working on questions that can condition groups to complete tasks collaboratively from start to finish.</td><td>Work on the questions in collaboration with group members;

Each number is done together;

Please ensure the written answers result from group discussion agreements; Double-check answers with group members before collecting them.</td></tr>
<tr><td>Create an alternative data analysis document that shows;

a) part of the problem that can trigger the emergence of aspects of students' critical thinking skills;</td><td rowspan="2"><table>
<tr><th>Solution</th><th>Trigger Critical Thinking Skills on the Problem</th><th>Possible Forms of Appearance of Indicators of Critical Thinking Skills in the Problem</th></tr>
<tr><td>Saila's statement which mentions that the curve

is a function graph $f(x) = x^2 - kx + 5$ is not true. Look at the function curve $f(x) = x^2 + kx$ follows.

For example $T(p, q)$ is the vertex of the curve $f(x) = x^2 + kx$. Because T is on the right of the Y axis, $p > 0$. (An, Ar, Sr)</td><td>Analysis (An)
Students explore function graphs $f(x) = x^2 + kx$ by looking at the top of the curve $f(x) = x^2 + kx$ to determine value k.</td><td>Analysis (An)
• After reading the questions, through discussions with group members and by repeatedly checking the information contained in the questions, the groups try to check the information contained in the questions. e.g:
Explainer: "Do we need to find the point of intersection of the curve $f(x) = x^2 - kx + 5$ about the X axis?".
Checker: "can it be searched? Because if it's like that there are still two variables that can't be determined yet".</td></tr>
</table></td></tr>
<tr><td>b) the possibility of developing critical thinking skills when students solve problems collaboratively.</td></tr>
</table>

After the problem is validated well, the researcher implements the problem on selected participants. The participants in this study were two groups of two 10th-grade students. Zuniga et al (2021) stated that working in pairs can increase the activity of negotiating, interacting, reaching agreements, and evaluating between group members. Thus, CPS will likely run well. The participants were chosen based on their mathematics ability, determined by their final exam scores on quadratic function material. Group 1 consisted of students with high (T1) and average (S1) mathematics abilities. Group 2 consisted of students with average (S2) and low (R1) mathematics abilities. We chose the combination of group members based on the type of group that we believe exists in a regular class. A week before implementation, the plan and topic of the assignment were informed to participants. The two groups were given assignments at different times so that the researcher could focus on seeing the critical thinking skills that emerged during the assignment. In the implementation, each group is given a maximum work duration of 90 minutes. To support collaboration conditions, each group is only given one sheet of answer paper and one sheet of calculation paper, used together during the problem-solving process. During the problem-solving process, students cannot consult with researchers or teachers. This is done so that students' critical thinking activities emerge naturally without the influence of other parties. The activities of each group in solving problems were recorded using audio-visual material. At the end of the session, all working papers are collected. Work discussions were transcribed and coded based on Table 2.

RESULTS

This section analyses the potential of non-routine mathematical problems developed in exploring students' critical thinking skills in CPS. Analysis of potential problems was carried out by comparing the activities of the two groups in solving problem based on CSP stages. The presentation of group activities was accompanied by critical thinking ability indicator codes are shown in Table 2.

Group 1 (T1S1)

1. *Problem Identification*

After reading the problem, S1 identifies $f(x) = x^2 + kx$ and $f(x) = x^2 - kx + 5$ **(An-S1)**. S1 conveys to T1 that the function $f(x) = x^2 + kx$ is known data, and the function $f(x) = x^2 - kx + 5$ is what is being asked. At the same time, T1 identifies the graph corresponding to the function $f(x) = x^2 + kx$ **(An-T1).** T1 shows S1 the graph of the function $f(x) = x^2 + kx$. Next, S1 explains the relationship between the T1 identification results and the things asked in the question **(An-S1)**. According to S1, the results of T1's identification, namely the function $f(x) = x^2 + kx$ and the graph $f(x) = x^2 + kx$ can be used as a reference to determine the correctness of the graph $f(x) = x^2 - kx + 5$ and drawn by Saila. At this stage, T1 and S1 agree with what is asked in the question.

2. *Planning and Exploring*

This stage begins with T1 connecting the function graphs $f(x) = x^2 + kx$ and $f(x) = x^2 - kx + 5$ with the discriminant concept to find the value of k and finding that $0 < k^2 < 20$ **(Sy-T1)**. T1 asks S1 whether it is true that zero is one of the solution sets of the inequality $0 < k^2 < 20$. S1 states that zero is included in the inequality $0 < k^2 < 20$ solution set. S1 looks again at the inequality $0 < k^2 < 20$. S1 states that zero is not included in the solution set of the inequality $0 < k^2 < 20$ **(Sr-S1)**. In the end, the finding of the inequality $0 < k^2 < 20$ was not used by T1 and S1 because it did not find a specific k value.

3. Problem Identification

This stage occurs after T1 and S1 do not find a specific k value. S1 re-identified the function $f(x) = x^2 + kx$ graph and found that the function $f(x) = x^2 + kx$ had an intersection point on the X axis. Next, S1 connected this finding with the roots of the equation $x^2 + kx = 0$ to determine another way to find the value of k **(An-S1)**. In this case, T1 corresponds to the meaning of the graph $(x) = x^2 + kx$ discovered by S1.

4. Planning and Exploring

To implement the idea agreed with S1, T1 connects the intersection point of the graph of the function $f(x) = x^2 + kx$ (S1's idea) with the roots of the quadratic equation $x^2 + kx = 0$ and finds that $x_1 = 0$ or $x_2 = -k$ **(Sy-T1)**. Next, T1 checks the effectiveness of the synthesis results found, namely $x_1 = 0$ and $x_2 = -k$, by asking S1's opinion about the value of $x_2 = -k$ **(Sr-T1).** S1 states that if $x_2 = -k$, then the graph of the function $f(x) = x^2 + kx$ given in the problem is not correct. T1 rejects S1's statement because the function $f(x) = x^2 + kx$ graph is the information given in the problem. This makes T1 aware of T1's mistake in the problem-solving process (not by what was asked in the question, namely identifying the truth of the function f $f(x) = x^2 - kx + 5$ **(Sr-T1)**. S1 relates the results of T1's examination to the solution steps. The identification step that T1 has carried out is by identifying the roots of the equation $x^2 - kx + 5 = 0$ and finding that $x_1 = \frac{-k+\sqrt{k^2-20}}{2}$ or $x_2 = \frac{-k-\sqrt{k^2-20}}{2}$ **(Sy-S1)**. S1 suggests returning to using the root values of the function $f(x) = x^2 + kx$ after examining the synthesis results that S1 has obtained **(Sr-S1).** T1 rejected the suggestion from S1 because it was inconsistent, so T1 proposed re-observing the questions given.

5. Problem Identification

S1 again identified $f(x) = x^2 - kx + 5$ (the information asked for in the question) before proposing to redraw the graph $f(x) = x^2 - kx + 5$ to check its suitability with the graph drawn by Saila **(An -S1)**. At the same time, T1 identifies the elements of the function $f(x) = x^2 - kx + 5$ to determine whether or not the function $f(x) = x^2 - kx + 5$ can be drawn (idea S1) **(An-T1)**. T1 rejected S1's idea because the value of k had yet to be found. Next, T1 identifies the relationship between the function $f(x) = x^2 + kx$ and the function $f(x) = x^2 - kx + 5$. T1 finds that the value of k in the function $f(x) = x^2 + kx$ and the function $f(x) = x^2 - kx + 5$ is the same **(An-T1)**. S1 identifies the elements of the function $f(x) = x^2 + kx$ and the function $f(x) = x^2 - kx + 5$ before agreeing with T1's statement **(An-S1).** T1 and S1 agree that the value of k in the function $f(x) = x^2 + kx$ and the function $f(x) = x^2 - kx + 5$ is the same.

6. Planning and Exploring

T1 checks the correctness of the information given in the problem by drawing a graph of the function $f(x) = x^2 + kx$ before further identification **(Ev-T1).** This was triggered by T1's statement, which stated that the graph provided in the question was wrong at the second PE stage (Number 4). T1 connects the location of the intersection point of the function graph $f(x) = x^2 + kx$ with the value of the roots of the quadratic equation $x^2 + kx = 0$ and finds that the intersection point of the function graph is $(0,0)$and $(-k, 0)$ **(Sy-T1)**. S1 approves the results of T1's synthesis. T1 tries to connect the roots of the quadratic equation $x^2 - kx + 5 = 0$, the intersection point of the function graph $f(x) = x^2 - kx + 5$ and the drawn graph of the function $f(x) = x^2 + kx$ by Saila. However, T1 realized that the relationship must be corrected because no conclusion could be drawn **(Sr-T1).** T1 connects the intersection point of the graph of the function $f(x) = x^2 + kx$ with the location of the graph of the function $f(x) = x^2 + kx$ in the Cartesian plane and finds that the value of k is negative **(Sy-T1)**. T1 connects the synthesis results, namely the inequality $0 < k^2 < 20$ and the synthesis results of negative k values and finds that the possible k values are $-1, -2, -3$, and -4 **(Sy-T1)**. S1 agrees with T1's synthesis results, which state that the possible k values are $-1, -2, -3$, and -4.

7. Execution

T1 explained to S1 the application of the solution idea by assuming $k = -n$ to obtain $f(x) = x^2 + nk + 5$ **(Ar-T1)**. T1 realized that the analysis did not solve the problem because it contained variable n **(Sr-T1)**. S1 proposes to find the discriminant value. T1 explained to S1 that the discriminant value cannot determine the value of k because the discriminant function $f(x) = x^2 - kx + 5$ contains the form k^2 **(Ar-T1)**. Applying ideas from S1 and T1 did not find a solution, so T1 looked for other alternative solutions.

8. Planning and Exploring

T1 connects the function $f(x) = x^2 - kx + 5$ with the concept of intercept and finds that whatever the value of k, the function $f(x) = x^2 - kx + 5$ will still be tangent to the X axis **(Sy-T1)**. S1 approves the results of T1's synthesis. T1 re-explained the agreed solution idea by visualizing it in the cartesian plane **(Ar-T1).**

9. Verify

S1 re-examines the point drawn by T1. According to S1, the location of a point if $x = 0$ is on the Y axis **(Ev-S1)**. T1 re-examines the solution ideas that have been put forward and connects them with the concept of intersection to find where the ideas that have been put forward are wrong **(Sr-T1)**. T1 agrees with S1's rebuttal. S1 re-examined the synthesis results, which found $k^2 < 20$ based on the fact that the discriminant value of the function graph $f(x) = x^2 - kx + 5$ drawn by

Saila did not touch the X axis **(Ev-S1)**. T1 rechecked the synthesis results, which found $k^2 < 20$ based on algebraic facts and a negative k value based on the function graph $f(x) = x^2 + kx$ **(Ev-T1)**. T1 realized that the graph drawn by Saila was not necessarily correct after T1 reread the question **(Sr-T1)**. S1 rechecks the question's meaning to check the truth of T1's statement **(Ev-S1)**.

10. Planning and Exploring

T1 again observes the location of the intersection point of the function $f(x) = x^2 + kx$ graph to ensure the correctness of the synthesis result, namely that k must be negative. S1 stated to T1 that from the start, the value of k had been agreed to be negative. S1 connects the synthesis results, namely the negative k value, the known data, namely the graph of the function $f(x) = x^2 + kx$, and the data in question, namely the truth of the graph $f(x) = x^2 - kx + 5$. Finally, S1 found an idea for a solution: checking the suitability of the negative k value on the graph $f(x) = x^2 - kx + 5$ **(Sy-S1)**. T1 agrees with the results of S1's synthesis. T1 proposed using the X-axis intersection point. S1 explained to T1 that T1's idea, namely using the X-axis intersection formula, was inappropriate because it still contained the form k^2 **(Ar-S1)**. T1 lists several elements related to the quadratic function that fulfils the solution idea proposed by S1 and finds that the turning point abscissa formula is the most suitable **(Sy-S1)**. T1 states that the graph drawn by Saila should be different from the graph $f(x) = x^2 + kx$.

11. Verify

S1 rechecks the suitability of the abscissa value of the turning point of the function $f(x) = x^2 - kx + 5$, namely $\frac{k}{2}$, with the location of the turning point of the graph of the function $f(x) = x^2 - kx + 5$ drawn by Saila (synthesis process T1) before agreeing to the conclusion stated by T1 **(Ev-S1)**. S1 stated to T1 that the graph drawn by Saila was correct because the final value of $\frac{k}{2}$ found was positive and corresponded to the location of the turning point of the graph of the function $(x) = x^2 - kx + 5$ drawn by Saila (T1's synthesis was not correct). S1 re-observes the agreed value of k and states that the abscissa value of the turning point of the function $f(x) = x^2 - kx + 5$ is negative (agrees with T1's opinion) **(Sr-S1)**. T1 explained the problem-solving process to S1, where Saila should draw the function graph $(x) = x^2 - kx + 5$ based on the abscissa value obtained **(Ar-T1)**. S1 explains the problem-solving process to T1, namely comparing the abscissa value of the turning point of the function $f(x) = x^2 + kx$ and the function $f(x) = x^2 - kx + 5$ (problem-solving process according to S1) **(Ar-S1)**. S1 and T1 agreed that the graph drawn by Saila was incorrect.

Group 2 (S2R1)

1. Problem Identification

After reading the problem, R1 identified the element $f(x) = x^2 - kx + 5$ and then connected it to the graph drawn by Saila **(An-R1)**. R1 conveyed to S1 that Saila's statement was correct because the graph drawn by Saila corresponded to the coefficient value x^2. On the other hand, after reading the problem, S1 proposed another idea for a solution.

2. Planning and Exploring

S2 connects the differences between the graph $f(x) = x^2 + kx$ and the graph drawn by Saila and finds that the concept of discriminant can be used to solve the problem **(Sy-S2)**. Meanwhile, R1 connects the graph elements $f(x) = x^2 + kx$ and the general form of the quadratic function and finds that the value of k must be found first through the vertex $f(x) = x^2 + kx$ **(Sy-R1)**. R1 applies the idea to the function $f(x) = x^2 + kx$ to check the proposed idea **(Sr-R1)**. R1 dropped the idea. S2 connects the graph $f(x) = x^2 + kx$ with the concept of discriminant and finds that $k^2 > 0$ **(Sy-S2)**. Thus, the value of k is obtained from $f(x) = x^2 + kx$. R1 agrees with S2's idea of finding the value of k from the function $f(x) = x^2 + kx$.

3. Execution

S2 explained to R1 the process of finding k systematically and based on the intersection point of the graph $f(x) = x^2 + kx$ **(Ar-S2)**. However, the S2 explanation produces a k value that cannot be precisely determined.

4. Verify

R1 checks why k has not been found. R1 checks the k value based on the graph position $f(x) = x^2 + kx$ regarding the X axis **(Ev-R1)**. R1 finds the value $k > 0$ because $k^2 > 0$. S2 refutes R1's opinion by providing a counter-example, namely $k > 0$ also results in $k^2 > 0$ **(Ev-S2)**. R1 receives a rebuttal from S2.

5. Problem Identification

S2 re-explores the graph $f(x) = x^2 + kx$ to complete the undiscovered intersection point **(An-S2)**. S2 has not been able to determine the use of analysis results in solving problems. R1 describes the meaning of the task given, namely finding two intersection points of the graph $f(x) = x^2 + kx$, which can be used to find the truth of the graph drawn by Saila **(An-R1)**. S2 agrees with the analysis results from R1.

6. Planning and Exploring

S2 connects the intersection point $(x) = x^2 + kx$ with the function and graph $f(x) = x^2 - kx + 5$ based on the results of R1 analysis. S2 concluded that Saila's answer was correct **(Sy-S2)**.

7. Verify

S2 connects the intersection point $f(x) = x^2 + kx$ with the function and graph $f(x) = x^2 - kx + 5$ based on the results of R1 analysis. S2 concludes that the value of $f(x)$ at $f(x) = x^2 - kx + 5$ is never zero, so Saila's answer is correct **(Sy-S2)**. R1 asked the logic of the method used by S2 to connect the function $f(x) = x^2 + kx$ with the function $f(x) = x^2 - kx + 5$ **(Ev-R1)**. S2 explained to R1 the synthesis stage and the underlying reasons based on the task's meaning **(Ar-S2)**. R1 refutes S2's explanation by suggesting a more appropriate alternative using the discriminant value **(Ev-R1)**. S2 re-examines the problem-solving process that has been carried out using descriminants **(Sr-S2)**. S2 found the discriminant value of the function $f(x) = x^2 - kx + 5$, namely $k^2 - 20$ (did not find the specific discriminant value used to check the correctness of the graph drawn by Saila.

8. Planning and Exploring

R1 connects the peak point with the data in question, namely the truth of the graph drawn by Saila and finding the peak point can be used to determine the truth of the graph drawn by Saila **(Sy-R1)**. S2 agrees with R1's idea and determines the peak point using the formula $x = \frac{-b}{2a}$. R1 checks the effectiveness of the concept chosen by S2 before rejecting S2's idea. R1 proposed using the formula for the ordinate value of the vertex, namely $y = \frac{-D}{4a}$ **(Ev-R1).** S2 checks the effectiveness of the concept chosen by R1 before rejecting R1's idea. R1 states that the formula $y = \frac{-D}{4a}$ still contains a discriminant value that has yet to be found **(Ev-S2)**. S2 and R1 agreed to return to the results of the answer written by S2; namely, Saila's answer was correct because they did not find any other alternative method.

The results of the analysis of potential problem in exploring students' critical thinking skills based on the results of video-audio recordings and observations during collaborative problem-solving are presented in Table 6.

Table 6. Visible Group Critical Thinking Skills Indicators

Code	**Group 1**		**Group 2**	
An	T1	- Identify relationships between known data to understand the problem. - Identify known data elements to determine whether or not the S1 idea can be implemented. (*Trigger*: new idea resulting from analysis proposed by S1).	S2	- Exploring known data to complete parts of data that have yet to be identified. (*Trigger*: group condition that has not found a solution by the agreed solution idea).
	S1	- Identifying relationships between known data to understand the problem. - Describe the relationship between the results of the T1 analysis and the	R1	- Identifying known data and connecting it with the data being asked.

Code	Group 1		Group 2	
		information asked for in the question. (*Trigger*: T1 analysis results). - Identifying known data using other concepts that have never been proposed. (*Trigger*: group condition that has not found the right solution idea). - Identify the information asked for in the question. (*Trigger*: T1 statement stating that the S1 synthesis results are inconsistent). - Identify known data elements to determine whether idea T1 is possible. (*Trigger*: new idea resulting from analysis submitted by T1)		- Identify the suitability of the data found with the facts. - Describe the meaning of the data being asked. (*Trigger*: S2 has yet to find the use of the analysis results in solving the problem).
Sy	T1	- Connecting known data using a concept that T1 already knows. (*Trigger*: results of S1's analysis of the information asked for in the question). - Connecting S1's ideas with concepts that T1 already knows. (*Trigger*: results of S1 analysis on data known to use other concepts). - Connecting the synthesis results with concepts that T1 already knows. - Connect several synthesis results that have been found to determine a solution idea. - Connecting known data with concepts that T1 already knows. (*Trigger*: implementation of the agreed solution idea does not find a solution). - List several elements related to the concept that fulfil the idea of completion. (*Trigger*: S1's argument stating the weakness of the idea T1 proposed).	S2	- Connecting known data based on concepts that S2 already knows. - Connecting known data with concepts that S2 already knows. (*Trigger*: cancellation of the problem-solving idea carried out by R1). - Connect the results of their analysis with the data in question. (*Trigger*: results of analysis carried out by R1).
	S1	- Connect the results of the T1 examination in the problem-solving process with the concepts that S1	R1	- Relate known data elements to definitions. - Connecting known data with other concepts not proposed in

Code	Group 1		Group 2	
		already knows. (*Trigger*: T1 check on own statement). - Connecting several synthesis results to determine a solution idea. (*Trigger*: The group has yet to find the right solution.)		the forum. (*Trigger*: S2 check result that finds R1's idea unusable). - Connect the analysis results found with the results of the S2 examination. (*Trigger*: results of examining the problem-solving process carried out by S2).
Ar	T1	- Simplify the explanation of solution ideas to S1 using algebra. (*Trigger*: T1 synthesis result agreed upon by S1). - Explain the reasons for rejecting S1's idea by showing where it is inaccurate. (*Trigger*: new idea proposed by S1). - Explain the solution idea to S1 by visualizing the synthesis results obtained. - Explain the problem-solving process to S1 based on the agreed T1 synthesis results. (*Trigger*: S1 agrees with T1's evaluation results after re-examining his statement).	S2	- Explain to R1 the synthesis process, which is carried out systematically and is based on data known in the assignment. - Explain to R1 the synthesis process based on the meaning of the task given. (*Trigger*: R1, who asks the logic of the method used by S2).
	S1	- Explain the reasons for rejecting idea T1 by showing where it is inaccurate. (*Trigger*: new idea proposed by T1). - Explain the problem-solving process to S3 by comparing the synthesis results found. (*Trigger*: T1 explains the process of solving the problem based on the agreed but incomplete results of T1's synthesis).	R1	***Not Visible***
Ev	T1	- Draw graphs based on known concepts to check the correctness of the information given in the question. (*Trigger*: S1's statement stating that the information given in the question is incorrect based on the results of T1's synthesis). - Check again the synthesis results that have been found.	S2	- Refute R1's opinion by providing counterexamples. (*Trigger*: results of evaluation carried out by R1). - Check the effectiveness of the concept chosen by R1 before rejecting the idea proposed by R1. (*Trigger*: idea proposed by R1).

Code	Group 1		Group 2	
				- Check the problem-solving process that has been carried out to find the causes of inconsistencies in facts with calculation results. (*Trigger*: the result of R1 synthesis activity).
	S1	- Checking T1's arguments using concepts that S1 knows. (Trigger: argumentation carried out by T1). - Check again the synthesis results that have been found. - Check the meaning of the question to check the truth of the T1 statement. (*Trigger*: results of examination (self-regulation) carried out by T1). - Examine T1's synthesis process before agreeing to the conclusions stated by T1. (*Trigger*: conclusion stated by T1).	R1	- Checking the S2 explanation using other data. (*Trigger*: S2's explanation of the synthesis results but cannot yet be applied to solve the problem). - Asking the logic of the method used by S2 in synthesizing. (*Trigger*: synthesis process carried out by S2). - Refute S2's explanation by suggesting a more appropriate concept to the answer. (*Trigger*: S2's explanation of the problem-solving process, which, according to R1, is inappropriate).
Sr	T1	- Checking the effectiveness of the synthesis results that T1 has found in solving problems. - Recheck the suitability of the problem-solving process that has been carried out in T1 with the information asked for in the question. (*Trigger*: S3's statement states that the T3 synthesis results cause the information given in the question to be incorrect). - Recheck the relationship between the synthesis results and the information in the questions T1 has created. (*Trigger*: T1 did not find the conclusion used as a resolution idea). - Re-examine the effectiveness of the proposed solution ideas for the problem-solving process.	S2	- Re-examine the problem-solving process that has been carried out using the R1 idea. (*Trigger*: R1's evaluation of the argument S2 put forward). - Recalculate the calculations that have been carried out to check the findings. (*Trigger*: results of analysis carried out by R1).

Code	Group 1		Group 2	
		- Check where there are errors in problem-solving ideas proposed using concepts that T1 already knows. (*Trigger*: evaluation carried out by S1). - Re-read the questions to investigate the accuracy of the agreed meaning of the questions. (*Trigger*: evaluation carried out by S1).		
	S1	- Checking the synthesis results again using concepts that S1 already knows. - Review the synthesis results before deciding to return to using old ideas. - Checking the solution that S1 has found by connecting it to the agreed synthesis results.	R1	- Apply the idea to known data to check whether the idea can be implemented. (Trigger: R1's synthesis).

Based on Table 6, several indicators of critical thinking skills need to be visible based on the results of video-audio recordings and observations during problem-solving, namely *argumentation in R1*. Based on these results, there are indications that although the problems developed can trigger collaborative problem-solving conditions, the problems developed cannot be used to see students' critical thinking skills fully. Triangulation is needed by conducting interviews after the group has completed the task so that indicators of critical thinking skills can be seen and explored through interviews. Therefore, auxiliary instruments are still needed as guidelines for group and individual interviews. Individual interview guidelines are needed if there is a group where one member dominates when solving problems or conducting group interviews. The interview guide is semi-structured by adjusting the results of problem-solving that students have worked on. In this research, researchers conducted group interviews with group 2 to explore argumentation indicators in R1. Group interviews were chosen because S2 and R1 did not dominate each other when solving problems. This can be seen at every problem-solving stage; both S2 and R1 contribute to the problem-solving process. The following is an interview conducted by researchers.

P : Now, try to observe the location of the intersection point you found with this $f(x) = x^2 + kx$.

S2 : (write down the two points on the graph $f(x) = x^2 + kx$ provided). The coordinate here should be $(k, 0)$ because it is the coordinate with x being positive. But based on the calculation, the k value is negative.

R1 **: We choose negative. In terms of location, the k value should be positive. If we choose negative k, the position and the equation you find, namely $x = -k$, can be**

the same. Note $x = -k = -(-k) = k$. So that's positive—the same as the positive position.

Based on the interview excerpt, R1 explained the problem-solving process to S2 based on the relationship between existing data. Thus, the argumentation indicator in R1 was found when a semi-structured interview was conducted by adapting the results of problem-solving that had been carried out by group 2.

DISCUSSION

Critical thinking skills can be triggered by social interaction. One of them is collaborative problem-solving settings because, in collaborative problem-solving settings, there will be cognitive and verbal activities that are interdependent in the context of problem-solving. This is also reinforced by Vygotsky's sociocultural theory and the Zone of Proximal Development (ZPD) model, which states that conversations with peers will expand students' ZPD to think critically (Wass et al., 2011). Appropriate instruments are needed to explore students' critical thinking skills in collaborative problem-solving to obtain accurate and in-depth data. The potential of non-routine mathematical problems developed in exploring students' critical thinking skills in CPS was discussed from two perspectives: students' interaction and students' critical thinking skill that emerged in the process. The interaction between group members in completing tasks collaboratively was visible in both groups. Responding to this, students admitted that the task was difficult but could be done because each student succeeded in contributing to the group and completing each other's steps. Several studies state that the questions' difficulty level (Chiu, 2008; Graesser et al., 2017; Westermann & Rummel, 2012) can encourage interaction in CPS. Paying attention to the problem-solving activities carried out by both groups, the task does not have the potential to trigger a division of labor to obtain a solution. Each stage completed is completed collaboratively and recorded in one shared workspace.

Using questions with a graphical analysis approach triggers students to explore graphs by being given various concepts. The results of observations in both groups showed that several graphic explorations had been carried out, namely checking the location of the intersection of the X and Y axes, checking discriminant values, checking the location of the peak point, checking the axis of symmetry of the graph and determine the possible direction of shift of the graph. Bezanilla et al (2019) stated that resource exploration activities like graphs would trigger students to think critically. Apart from that, in this graphic exploration activity, group members provide opinions to each other based on their knowledge. They give each other ideas that help check the solutions' correctness. This activity of sharing understanding allows for debate to criticize other people's thoughts and one's thoughts (Häkkinen et al., 2017). Thus, this aligns with research results showing that critical thinking activities that emerge in students are triggered not only by the questions given but also by ideas, statements, or problem-solving processes carried out by other group members.

This mutually triggering activity also shows that CPS impacts the development of students' critical thinking activities.

The results showed that more than giving non-routine math problems was needed to explore students' critical thinking skills in collaborative problem-solving. Another auxiliary instrument is still needed namely task-based interview guidelines. Interviews are needed to examine indicators of critical thinking skills in collaborative problem-solving that cannot be explored by giving non-routine problems. Several previous studies also used interview guidelines to collect data on student's critical thinking skills (Ariza, 2021; Dolapcioglu & Doğanay, 2022; Setiana et al., 2021). Li & Ren (2020) research states that interviews will provide more precise results in exploring students' critical thinking skills. Further research is needed regarding the use of interviews to stimulate students' critical thinking.

The research results will add valuable insight for researchers and practitioners in designing non-routine mathematics problems that can be used to explore students' critical thinking abilities in collaborative problem-solving. However, the results of this research still have limitations. These limitations include the number of participants used in only two groups. It would be better if there were more participants so that the potential of the task could be explored more. In addition, participants in this study were selected based on high, medium and low mathematics abilities. This ability is determined using student report scores. Discrepancies between student-reported scores and standardized tests may result in the selection of different participants and subsequently influence research findings.

CONCLUSION

Based on the results of the analysis that has been done, it can be concluded that giving non-routine mathematics problems with problem criteria that require justification for the solutions given and problems with a graphical analysis approach can be used to explore students' critical thinking skills in collaborative problem-solving. This is proven by solving the problems; each group member contributed to the solution-finding process. The contribution of each group member shows the high intensity of interaction between members. Interaction in the form of exchanging opinions, giving suggestions, and evaluating each other's ideas or answers significantly impacts students' critical thinking abilities. This is seen by the emergence of several students' critical thinking skills (analysis, synthesis, argumentation, evaluation, self-regulation) triggered by suggestions or ideas put forward by other group members. Thus, the non-routine questions developed can explore students' critical thinking skills in CPS. However, the analysis results also show that more than giving non-routine math problems with the abovementioned criteria are needed to explore students' critical thinking skills in collaborative problem-solving. An auxiliary instrument is still needed, namely an interview guide. This shows that in addition to giving non-routine math problems, triangulation methods are still needed in conducting interviews to explore students' critical thinking skills in collaborative problem-solving.

Another thing that needs to be considered is the closeness between students in a group. Interaction between students will run well if one student avoids dominating the other. The closeness between students in a group needs to be considered mainly if the group consists of members with significant differences in cognitive abilities, for example, high and low abilities. Lastly, the researcher hopes that the results of this research can be helpful for researchers or practitioners as a reference in developing or researching critical thinking skills, especially in collaborative settings.

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