The Situation of Mathematical Problem Solving and Higher Order Thinking Skills in Traditional Teaching Method and Lesson Study Program

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Abstract: *The purpose of this mixed-method study is to investigate and compare students' problem-solving and higher order thinking skills. This study involves two contexts: a traditional teaching program and a Lesson Study program. In qualitative phase, seven mathematics lecturers, a physics lecturer and the researcher forms the Lesson Study group and they collaboratively design five research lessons on functions topics. Also, the researcher collects all the materials taught by the lecturers individually in their classes on these topics. These topics in the textbook, individual lecturers' lessons and research lessons analyse descriptively using document analysis technique to find some understanding on the emphasis on problem-solving and higher order thinking skills. In quantitative phase, two classes choose randomly as experimental and control groups. The lecturers' developed tests use to compare the ability of students in problem-solving and higher order thinking. Data was analysed using independent samples t-test. The results of this mixed-method study show that, lecturers were teaching exactly contents of the textbook and focusing more on solving of routine exercises. Whereas, in Lesson Study program, collaborative work among lecturers in preparing suitable problem-solving activities not only improved lecturers' knowledge tremendously but also enhanced the ability of students in problem-solving and higher order thinking skills.*

1. INTRODUCTION

Teaching mathematics particularly at higher levels is more challenging than secondary school levels because of complication of the problems and concepts. Malaysian students who have completed secondary schools continue pre-university programs through pathways foundation, matriculation and A-level that lasts one year. Students choose these pre-university programs based on their results in courses Additional Mathematics, Mathematics, Physics, Chemistry and Biology at high school. Students' performance in these programs provides vast opportunity in being selected to the study majors of top universities. In these yearlong programs, the lecturers' competency in teaching mathematics greatly influences learners' performance in problem-solving. Therefore, the content knowledge and pedagogical content knowledge of lecturers are crucial in ensuring that learners knowledge have enormously enhanced to guarantee placement in competitive majors in top universities (Johannsdottir, 2013). The lecturers are recommended to prepare appropriate mathematical materials in their lesson plans to improve the ability of students in problem-solving and higher order thinking skills. Lecturers require knowledge on different pedagogical approaches and techniques for delivering each new topic thus they must constantly enhance and update their teaching knowledge to have better performance in their classes through problem-solving approach. To upgrade the teaching knowledge of educators, they should share the best practices on teaching a particular content to students through problem-solving approach and critical thinking. Fujii (2016) and Mon, Dali, and Sam (2016) highlighted that collective and cooperative teamwork among mathematics educators helps to improve their subject matter knowledge and pedagogy to acquire best practices in effective mathematics teaching through problem-solving approach.

2. LITERATURE REVIEW

The materials and lessons that design by lecturers play an important role in enhancing learners' interests in learning mathematics (Gholami et al., 2021). Doing suitable problem-solving activities based on different levels of higher order thinking skills by students help them to have higher confidence in mathematics classes and improve their abilities in mathematics. Lomibao (2016) explained that lesson planning has generally been a solitary task among mathematics teachers. Therefore, educators determine the materials individualy which are used for students with different qualities (Lomibao, 2016). Therefore, mathematics educators require having appropriate pedagogical content knowledge and subject matter knowledge about different topics in order to plan effective lessons.

In pre-university level, the central concept of function is complex and difficult (Akkus et al., 2008; Doorman et al., 2012; Ponce, 2007). Although teaching and learning function topics are deemed to be problematic and challenging, it is considered as an important topic in university programs to connect real world problems to mathematics. Modeling the real world problems is one of the most important applications of mathematical functions that explain the physical problems through mathematical language (Michelsen, 2006).

In traditional method of teaching, mathematics educators emphasize on giving lectures and solving of routine exercises in the process of teaching. However, this lecturer-centered method cannot enhance students' skills in problem-solving and higher order thinking (Khalid, 2017; Mon et al., 2016). In this method, most of learners just memorize the mathematical materials and routine solution methods to apply in solving exercises or examination questions. Traditional teaching method of mathematics is grounded on the behaviorist learning theory. This theory is according to the premise that a learner should build habit formation based on stimulus-response process and learning happens as there is a change in learner behavior (Ormord, 1995). Lesson Study is a strong professional development program for enhancing the teaching knowledge of educators (Fujii, 2016). This educational approach is based on the cognitivist learning theory and heavily emphasizes on problem-solving and higher order thinking in teaching mathematics. In mathematics teaching, problem-solving skills help learners to understand a domain of complex mathematical structures and acquire the skills to solve real life problems (Tarmizi & Bayat, 2012). The purpose of this research is to investigate the emphasis on mathematical problem-solving and higher order thinking skills in the traditional teaching method and Lesson Study program.

2.1.Mathematical Problem-Solving

Based on Xenofontos and Andrews (2014), a question is considered as a mathematics problem if the task is new and challenging for students whereas a routine task with clear process of its solution is called mathematics exercise. Problem solving approaches are used to help students learn how to think mathematically in solving problems (Purnomo et al., 2022). However, in mathematics problem-solving students engage with critical thinking and higher order thinking (Yassin & Shahrill, 2016). In this research, every mathematics problem that associated with students' everyday life, real world and other subjects such as physics and chemistry is considered as a practical problem (Gholami, 2021). For instance, the following problem is a practical problem.
Problem: The number of bacteria in a culture is $B(t)$ after t minutes. The relationship between the elapsed time t, in minutes, and the number of bacteria, $B(t)$ in the petrol dish is modelled by the function $B(t) = 10.2^{(\frac{t}{12})}$.

a. How many bacteria will make up the culture after 120 minutes?
b. After how many minutes will the population of bacteria be 5.2^{16}?

Polya (1945) had suggested four phases for mathematical problem-solving namely understanding the problem, planning a strategy, implementing the plan, and confirming the solution. It is essential that mathematics educators initially encourage and engage students in solving different levels of problems based on higher order thinking skills in order to improve their ability in learning mathematics. Mathematics educators believed that it is challenging to encourage students in solving open-ended problems and ask them to explain what problem-solving strategies they have used (Johnson & Cupitt, 2004; McDonald, 2009). However, mathematics educators require to consider the levels of difficulty in assigning open-ended problem to students and design them based on students' skills, hence every learner would be able to solve the problems to some extent (Asami-Johansson, 2015; Bergqvist, 2011). Despite different educational backgrounds in elementary and secondary mathematics education may affect student's ideas and reasoning in solving mathematics problems in foundation level, appropriate teaching method through problem-solving approach can improve the ability of students in problem-solving in yearlong foundation program (Lu & Richardson, 2018).

2.2 Higher Order Thinking Skills

Mathematics educators and the teaching materials play major roles in improving students' higher order thinking skills. Lack of content knowledge among some mathematics instructors regarding promoting higher order thinking skills and developing questioning techniques and teaching resulted in teachers employing traditional method in teaching (Alhassora et al., 2017). Malaysian Ministry of Education (2014) have a lot of emphasis on integrating abilities of higher order thinking among learners as the key factor to enable them for international competitions.

Bloom (1956) categorized skills of thinking ranging from concrete to the abstract: which are remembering, understanding, applying, analysing, evaluating and creating. Based on Thomas and Thorne (2014) higher order thinking defines as thinking skill which is beyond the memorization level. The last three levels of Bloom's Taxonomy, namely analysis, synthesis, and creativity are considered as higher order thinking skills (McBain, 2011).

Malaysian Ministry of Education (2014) defined higher order thinking skills as skills of applying knowledge, abilities of argument and reflection problem-solving, decision making, innovating and to creating something new. Based on this definition, the last four levels of the revised Bloom's Taxonomy, which are applying, analysing, evaluating and creating, are classified as higher order thinking skills, as shown in Table 1.

Table 1: Components of Higher Order Thinking Skills

Level	Explanation
Applying	Using the knowledge, skills and values in different situations to take matters.
Analysing	Breaking down the information to better understand the relationship between the divisions.
Evaluating	Making judgments and decisions using the knowledge, experience, skills and values and justify.
Creating	Produce a product or idea or create and innovative methods.

Source: Ministry of Education (2014)

2.3 Lesson Study

Lesson Study approach, as a beneficial method for enhancing educators' professional development, has been used by Japanese teachers since the 1950's (Abiko, 2011). Yoshida (1999) translated the Japanese term "*Jugyo Kenkyu*" into Lesson Study and these two Japanese words *Jugyo* and *Kenkyu*, means lesson and study respectively. However, Lesson Study has been most popular since 1999 among mathematics teachers and researchers. Lesson Study refers to collaborative work of educators (Lesson Study group) on some mathematics topics to plan and design a lesson, teach and observe the lesson and to reflect and discuss on the taught lesson. The purpose of this educational approach is to improve students' ability in problem-solving by presenting effective teaching (Matanluk et al., 2013). The lessons that prepared in the process of Lesson Study program, in Japanese language are called *gakushushido-an*, and translated into research lessons (Fujii, 2016) or study lessons (Yoshida, 1999). Lesson Study focuses on pedagogical progress among educators in which the research lesson considered as a central component (Lewis, 2002).

School-based, district-based, and national-level Lesson Study are three popular kinds of Lesson Study in Japan (Fujii, 2016). The process of all forms of Lesson Study is essentially similar however; the only difference is related to the range of students. School-based Lesson Study is merely suitable for learners in the specific school; district-based Lesson Study is more applicable in the district; and national-level Lesson Study is mostly focused on the learners around the country

(Fujii, 2016; Takahashi & McDougal, 2016). Several Lesson Study models have been developed by Yoshida (1999), Takahashi (2001), Richardson (2004) and Fuji (2014). Fujii (2014) suggested the following five steps for implementation of Lesson Study program:

a. Goal setting: Mathematics instructors focus on the long-standing targets to improve the ability of students in problem-solving and higher order thinking skills
b. Lesson planning: Teachers through collaborative work try to prepare suitable mathematical materials based on the different levels of higher order thinking.
c. Research lesson: The members of Lesson Study group provide a proper research lesson, one of them teaches the research lesson and other members observe and collect data for next step.
d. Post-lesson discussion: Educators in post-lesson discussion discuss on students' misconceptions, students' learning, variety of solutions and the levels of higher order thinking for the given problems to improve the quality of research lesson.
e. Reflection: In this phase, teachers collaboratively consider some new problems in the research lesson and they discuss on likely solutions for next cycle of Lesson Study. Finally, they prepare a report on their output.

In Lesson Study approach, educators by considering the ability of students, engage them with suitable problem-solving activities based on different levels of higher order thinking. It seems that the school-based Lesson Study is the most effective form of Lesson Study because educators in a specific educational center plan and design the research lessons according to their students' abilities in problem-solving (Takahashi & McDougal, 2016).

2.4 Research Question and Hypothesis

The research question (qualitative part) and hypothesis (quantitative part) of this research are as follows: Is there a difference between the Lesson Study program and the traditional teaching approach in terms of problem-solving and higher order thinking skills?

Hypothesis: There is significant statistical difference in problem-solving and higher order thinking skills between experimental (Lesson Study program) and control (traditional method) groups.

3. METHOD

The present mixed method study was conducted in a foundation center that offer pre-university program in a public university of Malaysia. Students are selected in foundation programs to continue their studies, based on their good results in high school. There has not been any research which compares learners' problem-solving and higher order thinking skills between those who undergone Lesson Study program and traditional teaching method for foundation level students. After obtaining permission from the director of the foundation center, all lecturers and students who were involved in the study signed the disclosure letter.

3.1.Qualitative Phase (Case Study)

In this foundation center, nine mathematics lecturers (four male and five female) were teaching 20 classes with 952 students (326 males and 626 females). Eight mathematics lecturers and two physics lecturers volunteered to participate in this research, however, a mathematics lecturer and a physics lecturer later withdrew from the study due to their time constraint. Thus, the Lesson Study group consisted of seven mathematics lecturers, a physics lecturer and the researcher (nine lecturers participated in this part of study). The physics lecturer assisted the mathematics lecturers in constructing practical problems for their research lessons and to identify application of function in physics problems. Meanwhile, in this study, the researcher not only was the coordinator and discussion leader but also participated in discussion meetings as a member of the Lesson Study group.

Before the meeting for each research lesson, the researcher introduced the topic to the lecturers and asked them to provide appropriate materials, problems, applications of contents and the suitable pedagogy to implement the contents. This process was followed with a two hour session by the Lesson Study group members to plan, discuss and prepare the research lesson. In the second session, one of the members of the Lesson Study group taught the prepared research lesson and other lecturers observed the teaching. Therefore, they further improved the research lessons based on problem-solving approach and higher order thinking skills.

In foundation program, lecturers teach two textbooks namely Mathematics 1 and Mathematics 2 during first and second semesters respectively. These textbooks comprise of chapters related to algebra, calculus, trigonometry, geometry, probability and statistics. In the Mathematics 1 textbook, about two fifths of contents are allocated to the mathematical functions. In this study, the researcher chose the topic on functions because it is problematic concept for lecturers to teach and for students to learn (Oehrtman & Carlson, 2008). The members of Lesson Study group through collaborative work planned, designed, discussed and improved five research lessons as shown by Table 2.

Table 2: The Topics of the Research Lessons

Research lesson	Topic
1	Relation and function concepts
2	Domain and range of the functions and algebraic combination
3	Composite function, inverse function, odd and even functions
4	Trigonometric functions
5	Exponential and logarithmic functions

For all 20 classes, the researcher had earlier requested some students (two students in each class) to jot down all the materials related to these five topics of this study taught by the lecturers in their classes. This issue allowed the researcher to compare the situation of problem-solving and higher order thinking between the traditional method and Lesson Study program. Research lessons and the individual lecturers' original lessons were analysed descriptively using document analysis technique. Initially, the Mathematics 1 textbook was analysed to find the situation of higher order thinking and problem-solving in this textbook. The tasks of textbook were compared with each other to determine that each task is mathematics exercise or mathematics problem. For instance, in Mathematics 1 textbook there were 18 similar routine exercises on topic of composite function. After that, the research lessons and the lecturers' lessons were compared with the contents flow and approach in the textbook. Lastly, the research lessons and individual lecturers' lessons were compared to find the extent of the implement of problem-solving in traditional method and Lesson Study Program. Through similar process the status of higher order thinking skills compared between traditional teaching method and Lesson Study program based on the levels of higher order thinking skills of Malaysian Ministry of Education (2014). Meanwhile, after analyzing the mathematical problems and higher order thinking in the research lessons, the materials in the textbook and the individual lessons of the mathematics lecturers were analysed, these results were studied by three professors from the mathematics department of one of the public universities in Malaysia. The researcher applied their comments and used the obtained results. Figure 1 represents the process of data analysis in this study.

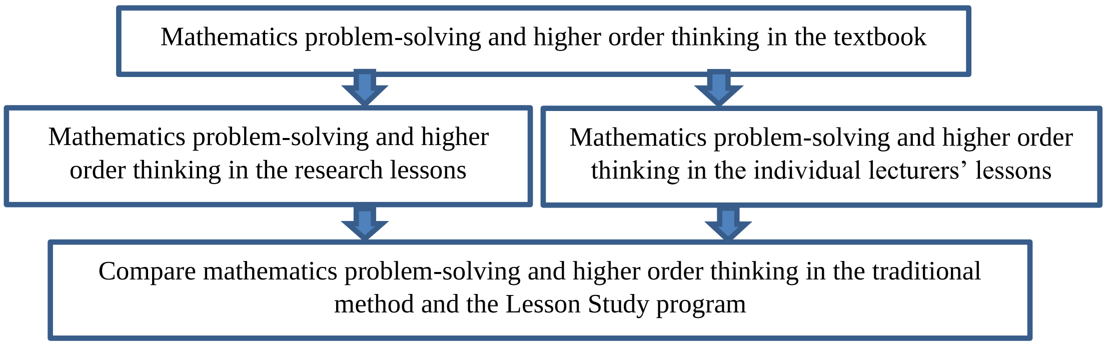


Figure 1: The Process of Data Analysis in this Study
Source: Designed by the Author

3.2.Quantitative Phase (Quasi-experimental Research)

The participants of this part of the study were a total of 86 students, comprising of 44 learners in the experimental class (Lesson Study program) and 42 learners in the control class (traditional method). The teaching videos of these lessons were recorded in both groups. The measurement used in this part of study was the mathematics tests that developed by the lecturers involved in the Lesson Study program. Although in foundation center, textbooks used are in English and all subjects are taught in English, the mathematics tests also includes the Malaysian language (Bahasa Melayu) translation so as to avoid ambiguity for learners. Thus back to back translation from English to Bahasa Melayu was done by two lecturers of English Language from one of the faculties from the same university. The final version was confirmed by two experts in mathematics education in order to ensure there was no problem in the translation. The mathematics tests contain 12 open-ended problems, which was tested for its contextual validity and reliability and was proved by using Equivalent-forms Method. Six experts in mathematics and mathematics education from a public university confirmed the validity and suitability of these problems. The Pearson correlation significant for pre-test with 31 and post-test with 40 participants outside this research were 0.78 and 0.74 respectively. However, these tests were different because new concepts were taught in these five weeks. Also, post-test was conducted one month after finished the study again as follow-up test. A sample of the pre-test is "The graph of the function $f(x) = \frac{ax+3}{x-b}$ passes from two points (-1, 0) and (1,-2). Find the values of a and b". Also, two samples of the post-test are "Let m be a non-zero constant. Find the two x-values where the graphs of the functions $y = 10^{6m}x$ and $y = \frac{x^2}{10^{5m}}$ intersect" and "Determine whether the function $h(x) = \sqrt{x} + \sqrt{-x}$ is even or odd".

Lastly, mathematics tests were confirmed by some experts in the Ethical Committee of the Research Management Center (RMC) of a public university in Malaysia as suitable instruments for the study. The problems in each test were categorised according to the three levels of higher order thinking skills in the revised Bloom's Taxonomy, namely applying (four questions), analysing (four questions) and evaluating (four questions) based on Table 1 and was confirmed by three mathematics experts in the same university. Therefore, the researcher compared the results of the students in problem-solving and higher order thinking based on the three levels of revised Bloom's Taxonomy, between the experimental and control classes.

The students' answers in the tests were scored by two lecturers, based on Polya's problem-solving model (Brijlall, 2015). If learner wrote illogical and incorrect answer or no answer given, then the question is given a score 0. If some phases are given in the solution that shows the learner understand the problem, the considered score is 1 (first phase of Polya's model). If learner understand and provide a method for solving the problem but made some errors, the score is 2 (first and second phases of Polya's model) and lastly, a complete answer is given a score of 3 (all phases of Polya's model). Therefore, the possible minimum and maximum marks for the mathematics tests were 0 and 36 respectively.

A mathematics lecturer, who was a member of the Lesson Study group, was chosen randomly and his classes were randomly considered as experimental and control groups (each of the mathematics lecturers taught in two classes, except for two of them who had three classes). The Lesson Study was implemented for five weeks, which covered five topics on functions. In the experimental group, the student-centered teaching approach was applied and the lecturer gave some problems and practical problems that students did individually and in teams. It was meant to improve students' abilities in mathematics problem-solving and higher order thinking skills. In teaching the research lessons, the lecturer walked around the class to provide guide, to assess, encourage and engage students in problem-solving activities. In contract, the control group was lecturer-centered and the same lecturer taught the exact same topics but using traditional teaching method, where students are worked individually with emphasis on mathematics exercise solving. In fact, in the control class, the lecturer taught exactly the provided materials of the textbook. Independent sample t-test was used to analyse the data for this part of research.

4. FINDING

4.1. Qualitative Part

The situation of mathematical problem-solving and higher order thinking skills in the foundation level is discussed based on the sources the textbook, individual lecturers' lessons and research lessons.

4.1.1. Textbook

The analysis of contents related to these five topics in the Mathematics 1 textbook showed that the textbook does not emphasises much on problem-solving. There are only a few mathematics problems in this textbook and there are not any practical problem-solving tasks given. In fact, the textbook emphasises more on solving of routine mathematics exercises. The contents of the textbook seem to encourage memorization of formulas, theorems, methods and shortcuts that students can apply in solving other mathematics exercises. Apparently, the textbook approaches do not promote students' abilities in problem-solving. The number of mathematics exercises, mathematics problems and practical problems is showed in Table 3.

Table 3: Textbook's Materials

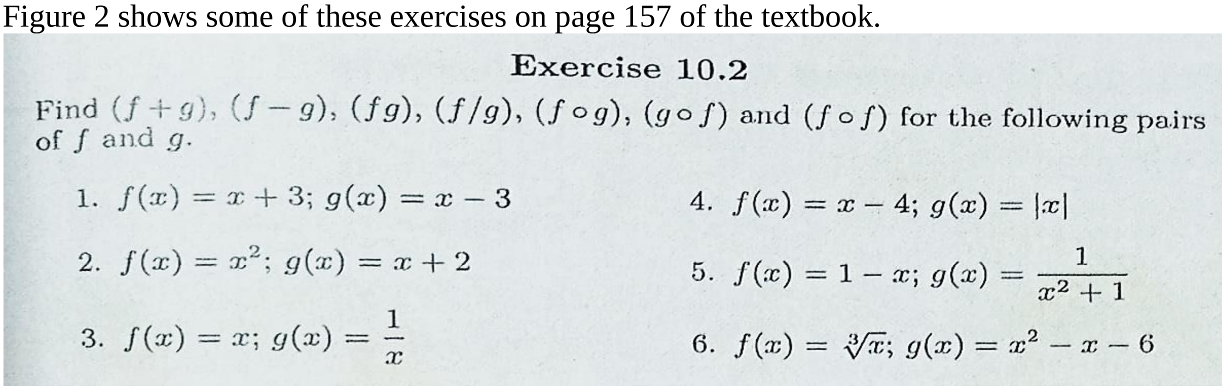
No.	Topic	Mathematics Exercise	Mathematics Problem	Practical Problem
1	Relation and function concepts	16	1	0
2	Domain and range of the functions and algebraic combination	19	1	0
3	Composite function, inverse function, odd and even functions	26	2	0
4	Trigonometric functions	10	1	0
5	Exponential and logarithmic functions	18	1	0
6	Total	89	6	0

Based on Table 3, only six percentages of tasks in these five topics are mathematics problems whereas other ninety four percentages are merely mathematics exercises.

Many of mathematics exercises that considered in each subtopic are similar in terms of teaching approach and content. For example, in the Topic 3 of this research (composite function, inverse function, odd and even functions), there are 18 exercises related to the composite function that just the rule of functions changed in the tasks. One of these tasks is as follows:

Exercise: If $f(x) = 1 - x$ and $g(x) = \frac{1}{x^2+1}$ find the function fog.

Figure 2 shows some of these exercises on page 157 of the textbook.


Exercise 10.2

Find $(f+g)$, $(f-g)$, (fg), (f/g), $(f \circ g)$, $(g \circ f)$ and $(f \circ f)$ for the following pairs of f and g.

1. $f(x) = x + 3$; $g(x) = x - 3$
2. $f(x) = x^2$; $g(x) = x + 2$
3. $f(x) = x$; $g(x) = \frac{1}{x}$
4. $f(x) = x - 4$; $g(x) = |x|$
5. $f(x) = 1 - x$; $g(x) = \frac{1}{x^2+1}$
6. $f(x) = \sqrt[3]{x}$; $g(x) = x^2 - x - 6$

Figure 2: Some Exercises of the Textbook about Composite Function

However, the number of problems is very limited in each subtopic. For example, on page 159 of the Topic 4 (trigonometric functions), only one problem was given (Figure 3), which is as follows:

Problem: Find all angles $(0 \leq \theta \leq \pi)$ which satisfy the equation $4sec^2\theta = 3\tan\theta + 5$.

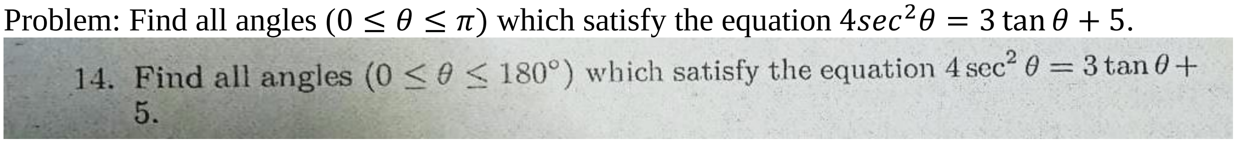

14. Find all angles $(0 \leq \theta \leq 180°)$ which satisfy the equation $4\sec^2\theta = 3\tan\theta + 5$.

Figure 3: An Example of a Problem in the Textbook

The materials in the Mathematics 1 textbook were also categorised based on the revised Bloom's Taxonomy. Table 4 shows the categories of textbook materials based on the Bloom's taxonomy.

Table 4: The Categorisation of Textbook Materials Based on the Bloom's Taxonomy

Topic	Remembering	Understanding	Applying	Analysing	Evaluating	Creating
1	5	11	1	0	0	0
2	7	12	1	0	0	0
3	14	13	1	1	0	0
4	4	5	0	1	0	0
5	6	12	1	0	0	0
Total	36	53	4	2	0	0

As respect to Table 4, about 93.5 percentages of the materials only require lower order thinking skills and 6.5 percentages of tasks are associated with higher order thinking skills. Therefore, the materials in the textbook do not help much in improving students' higher order thinking skills.

4.1.2. Lecturers' Lessons

The researcher received all the lecturers' lessons of the topics as in Table 2 and compared the tasks applied in each lesson with the contents of the Mathematics 1 textbook to determine the quality of problems posed in each lesson. Although the lecturers H and I did not participate in this research, but in order to get better results, the researcher was coordinated with them that their individual lessons on five topics of this study were also analysed. The results of this part showed that the mathematics lecturers taught the exact same materials of the textbook. According to Table 3, the textbook that was used only include a few mathematics problems. Thus, by relying very much on the materials of textbook, the lecturers were not promoting much of problem-solving in their classes. Meanwhile, none of the lecturers considered practical problem in their lessons. Table 5 shows the number of problems posed by each of the lecturers in their teaching based on the topics of this research.

Table 5: The Number of Mathematics Problems Posed in the Lecturers' Lessons

Lecturer	Highest Degree	Mathematics Problem					Total
		Topic 1	Topic 2	Topic 3	Topic 4	Topic 5	
A	Master	1	1	3	1	1	7
B	Master	1	1	2	1	1	6
C	PhD	1	2	2	2	1	8
D	Master	1	1	2	1	1	6
E	PhD	1	1	2	1	1	6
F	Master	1	1	2	1	1	6
G	Master	1	1	2	1	1	6
H	PhD	1	1	3	1	1	7
I	PhD	1	1	2	1	1	6

Similarly, the levels of thinking that were used by the lecturers in their lessons were also guided by the textbook. Table 6 shows the questions posed in five lessons conducted by each lecturer and categorised based on the revised Bloom's Taxonomy.

Table 6: Number of Tasks Posed in the Teaching Based on the Bloom's Taxonomy

Lecturer	A	B	C	D	E	F	G	H	I
Applying	5	4	5	4	4	4	4	5	4
Analysing	2	2	3	2	2	2	2	2	2

Evaluating	0	0	0	0	0	0	0	0	0
Creating	0	0	0	0	0	0	0	0	0
Total	7	6	8	6	6	6	6	7	6

For example, Figure 4 shows an example of student work. In this session, several similar routine exercises were discussed by the lecturer.

Example: Find fog and gof for the functions $f(x) = 5x + 2$ and $g(x) = \frac{4}{x-3}$.

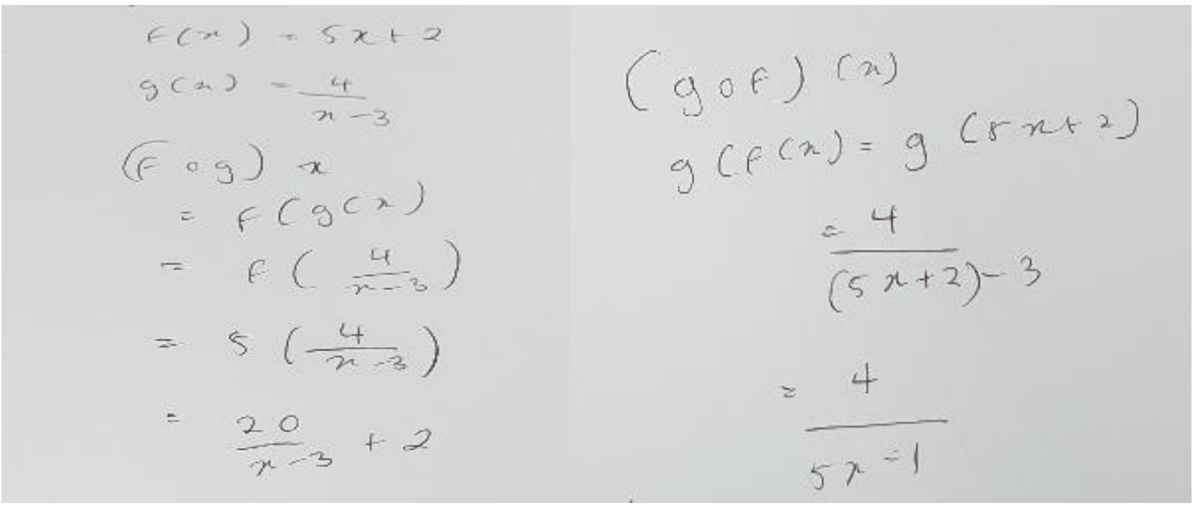


Figure 4: A Student Work about Composite Function

Also, the following two examples that adopted from a lecturer's lesson illustrates he/she taught exactly the same materials in the textbook.

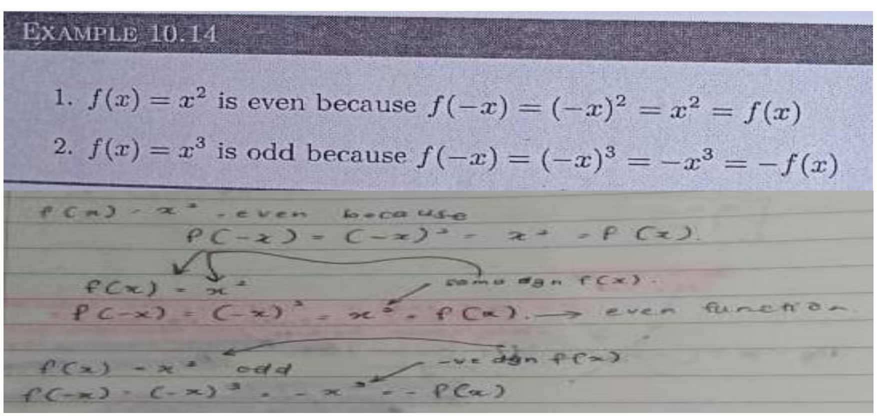
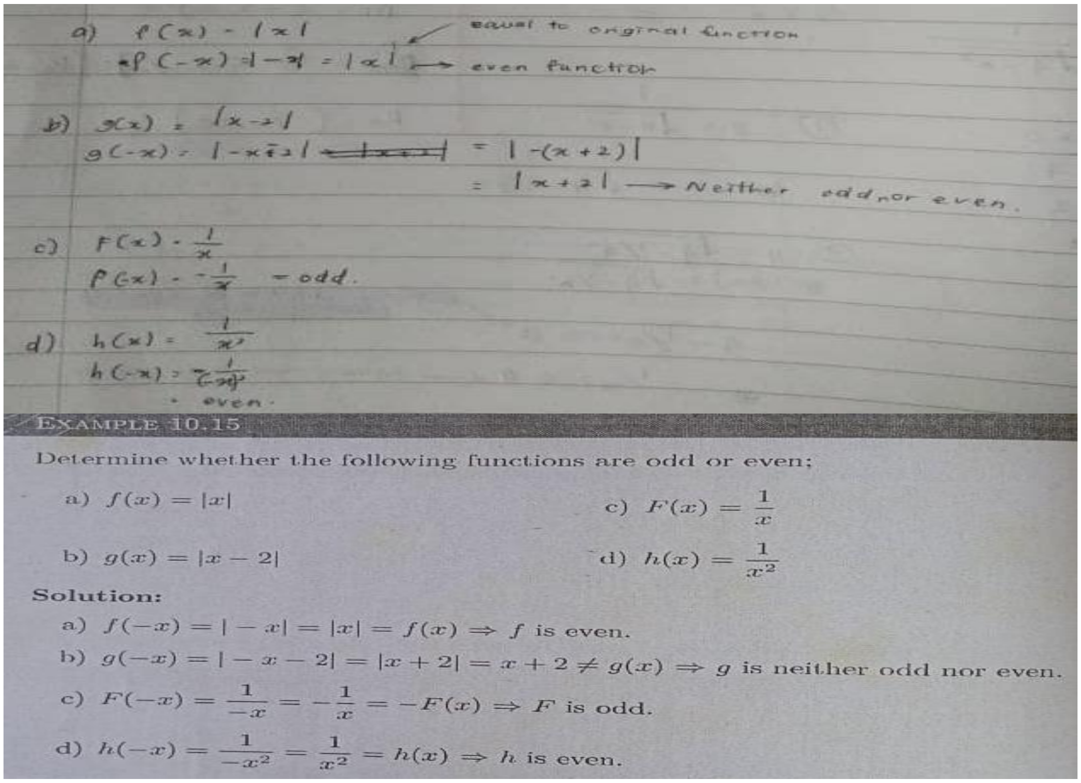


Figure 5: An Example of the Textbook in the Classroom



EXAMPLE 10.15

Determine whether the following functions are odd or even;

a) $f(x) = |x|$

b) $g(x) = |x-2|$

c) $F(x) = \frac{1}{x}$

d) $h(x) = \frac{1}{x^2}$

Solution:

a) $f(-x) = |-x| = |x| = f(x) \Rightarrow f$ is even.

b) $g(-x) = |-x-2| = |x+2| = x+2 \neq g(x) \Rightarrow g$ is neither odd nor even.

c) $F(-x) = \frac{1}{-x} = -\frac{1}{x} = -F(x) \Rightarrow F$ is odd.

d) $h(-x) = \frac{1}{-x^2} = \frac{1}{x^2} = h(x) \Rightarrow h$ is even.

Figure 6: A Textbook Example in the Classroom

4.1.3. Research Lessons

During the Lesson Study program, the lecturers had considered suitable mathematics problems into each research lesson. Furthermore, they believed that the integration of practical problems into the mathematics curricula play a major role in encouraging learners in solving mathematical problems. In fact, Lesson Study had helped the lecturers to enhance ability to guide learners in improving their abilities in problem-solving. The analysis in terms of number of tasks, which were categorised based on types of tasks (mathematics exercises, mathematics problems, and practical problems) of the research lessons, is showed in Table 7.

Table 7: The Number of Mathematics Tasks in the Research Lessons

No.	Title of Research Lesson	Mathematics Exercise	Mathematics Problem	Practical problem
1	Relation and function concepts	7	8	3
2	Domain and range of the functions and algebraic combination	14	5	2
3	Composite function, inverse function, odd and even functions	9	12	3
4	Trigonometric functions	6	11	1
5	Exponential and logarithmic functions	11	5	2
6	Total	47	41	11

According to Table 7, the percentages of mathematics exercises, mathematics problems and practical problems are 48, 41 and 11 respectively. In other words, 52 percentages of mathematics tasks in these topics are related to problem-solving. When learners are engaged in the process of problem-solving, not only they learnt mathematics conceptually but lecturers also improved their pedagogical content knowledge and content knowledge since they are confronted with different ideas, methods and solutions from the peers and students. The levels of tasks in these five research lessons are also classified based on the revised Bloom's Taxonomy that showed by Table 8.

Table 8: Tasks Based on Bloom's Taxonomy for all Research Lessons

Topic	Remembering	Understanding	Applying	Analysing	Evaluating	Creating
1	2	5	5	3	2	1
2	5	9	3	2	1	1
3	3	6	6	5	2	2
4	1	5	5	3	2	2
5	3	8	3	2	1	1
Total	14	33	22	15	8	7

Based on Table 8, about 47 percentages of tasks in all topics are related to the lower order thinking and 53 percentages are related to higher order thinking. Therefore, these collaborative lessons not only enhanced the ability of lecturers and students in problem-solving but also learners experienced the beauty of mathematical materials through engaging with appropriate problems and practical problems. In fact, lecturers considered some problems in the research lessons that improve the critical thinking and higher order thinking among students. Figure 7 represents a student work regarding the composite function. This student work is the answer of the following problem.

Problem: Consider the functions $f(x) = \sqrt{x-1}$ and $g(x) = \sqrt{x-3}$. Find the rule and domain of the function fog.

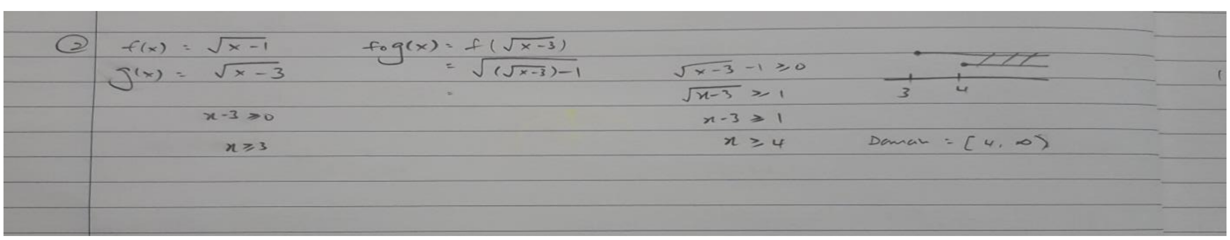


Figure 7: A Student Work about the Composite Function and Its Domain

The rule of the function fog is as below.

$$fog(x) = f\big(g(x)\big) = f\big(\sqrt{x-3}\big) = \sqrt{\sqrt{x-3}-1} \Rightarrow fog(x) = \sqrt{\sqrt{x-3}-1}.$$

Although the lecturer in the class explained the following method to find the domain of composite function fog, this student found the domain of composite function through a creative method.

$$f(x) = \sqrt{x-1} \Rightarrow D_f = [1, +\infty[.$$
$$g(x) = \sqrt{x-3} \Rightarrow D_g = [3, +\infty[.$$

$$D_{fog} = \{x \in D_g \mid g(x) \in D_f\} = \{x \in [3, +\infty[\mid \sqrt{x-3} \in [1, +\infty[\}.$$

After solving the inequality $\sqrt{x-3} \geq 1$, the domain of the function fog obtains as $D_{fog} = [4, +\infty[$.

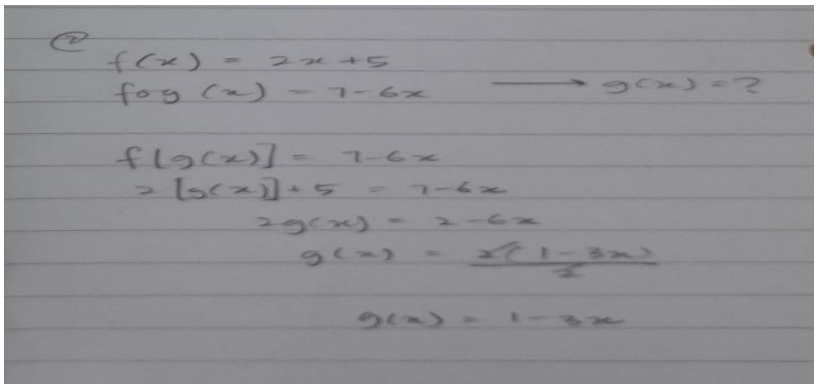


Figure 8: A Student Work about a Problem

Figure 8 illustrates a student work regarding a problem of a research lesson. There is no any similar task in the textbook. This problem that is suitable task in improving the higher order thinking skills among students is as follows.
Problem: If $f(x) = 2x + 5$ and $fog(x) = 7 - 6x$, find the rule of the function g.

One of the most important advantages of Lesson Study program is discussion about the variety of solution methods for the given problems. In Figure 9, there are some solution methods for the following trigonometric problem.
Problem: If $A = \sin 15 + \cos 15$, find the value of A^2.

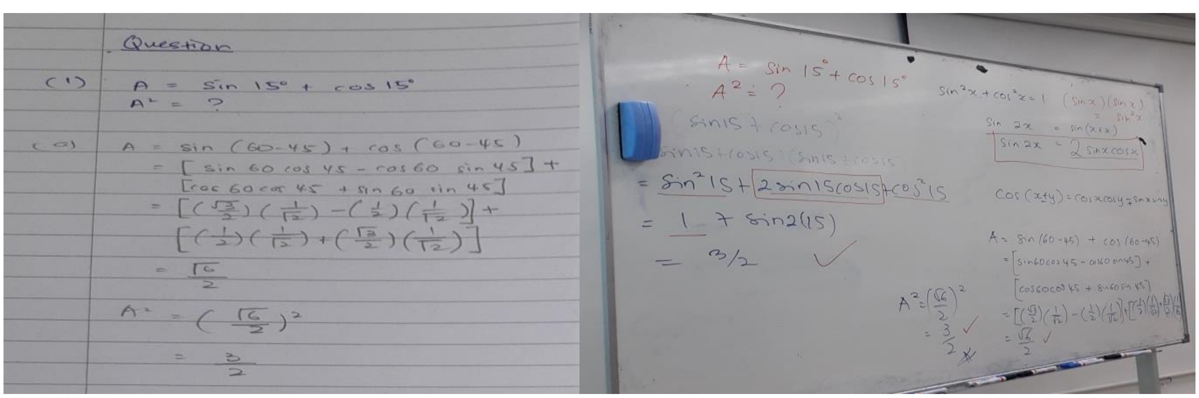


Figure 9: Different Solution Methods for a Given Trigonometric Problem

In the Lesson Study program, the lecturers considered some application of mathematical concepts. Considering these tasks in the research lessons not only helped students to understand the relation between mathematical concepts meaningfully but also showed them some applications of function topics in the real-world. For instance, in Figure 10, the lecturer highlighted the application of inverse function in determining the range of functions. The discussed problem is as below.

Problem: Find the range of the function $f(x) = \frac{2}{x-3}$.

It is difficult for students to find the range of the function f directly. The lecturer explained to students that one of the applications of inverse function is to find the range of some functions. There is a relation between the domain and range of the functions f and f^{-1} as follows.

$$R_{f^{-1}} = D_f \text{ and } D_{f^{-1}} = R_{f.}$$

The inverse function of $f(x) = \frac{2}{x-3}$ is $f^{-1}(x) = \frac{2+3x}{x}$ and students easily can find the range of the function f as $R_f = D_{f^{-1}} = (-\infty, 0) \cup (0, +\infty)$.

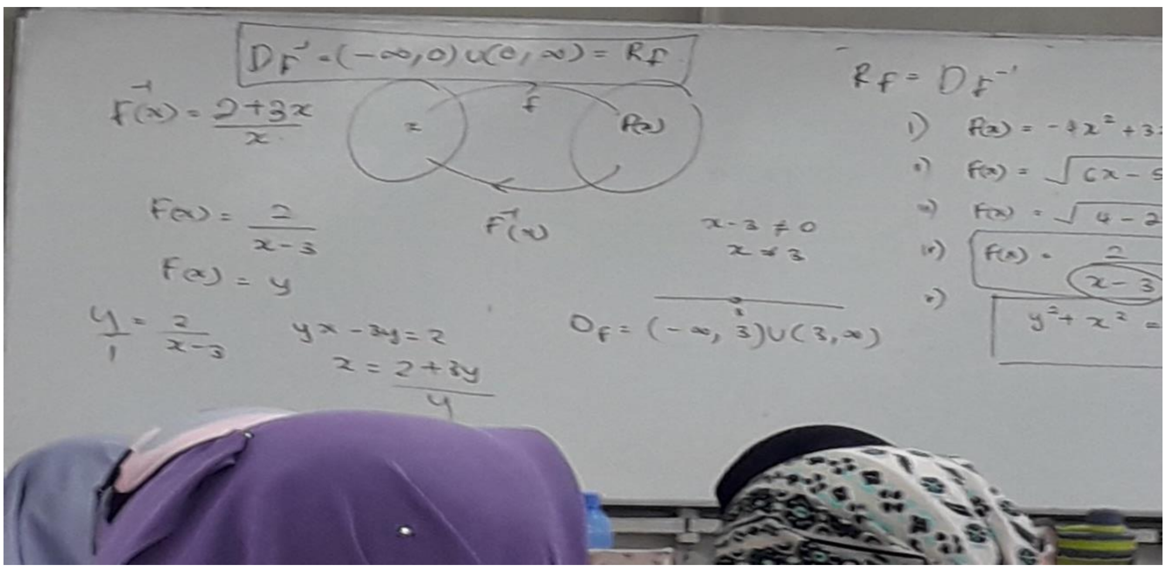


Figure 10: Determining the Range of a Function Using Its Inverse Function

The lecturer explained to students that one of the most common applications of functions is modeling the real-world problems. In other words, mathematical functions use to determine the relationship between variables in the human life. Figure 11 represents two examples of functions that apply in the real-world. The first function is the area value for circles that shows the domain and range of this function is positive real numbers. The second physical formula is related to the earthquake and shows that scientists are able to model complex real-world problems using mathematical functions.

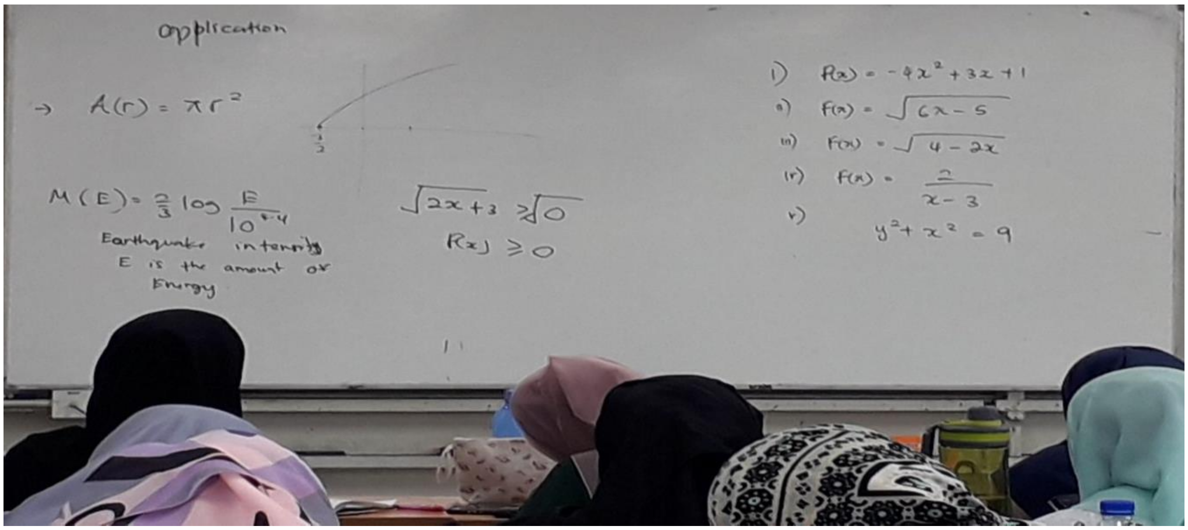


Figure 11: Applications of Mathematical Functions in the Real-world

Lecturers considered some fun problems (related to the topic of research lessons) in the research lessons that were interesting for students. For example, Figure 12 shows a fun function that give students the number of squares in a chess plane.

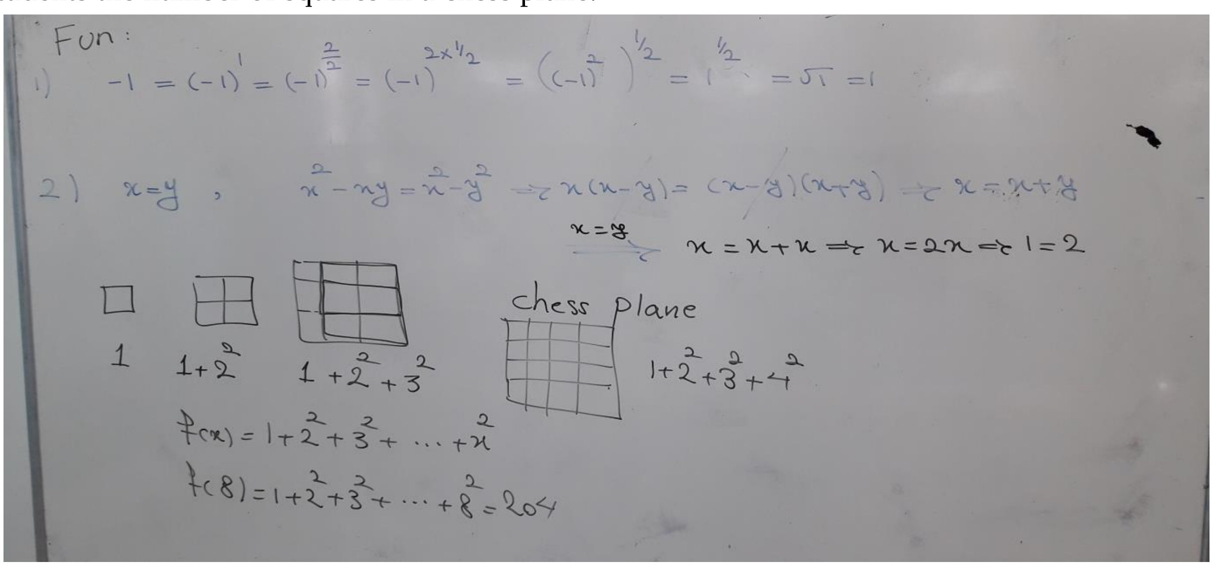


Figure 12: A Fun Function to Calculating the Number of Squares in the Chess Plan

4.2.Quantitative Part

In the experimental class, the lecturer used research lessons that provided by the Lesson Study group members that focused on problem solving ability and higher order thinking skills. This class was student-centered and students solved the given problems individually or in team. In fact, based on Figure 1 by engaging students with problems with different learning levels and discussing their different solutions, the teacher increased the students' motivation to solve problems. The contents of the textbook included repeated examples and exercises, while in the experimental class, the lecturer used problems that each of them taught new techniques to the students. Students' misunderstandings, their weaknesses and strengths were discussed. In this part, the researcher refers to two students' answers of experimental group with different scores for the given problem "Let m be a non-zero constant. Find the two x-values where the graphs of the functions $y = 10^{6m}x$ and $y = \frac{x^2}{10^{5m}}$ intersect". The following four samples show the considered score for the answers of students. As respect to Figure 13, student A solved this problem as follows:

$10^{6m}x = \frac{x^2}{10^{5m}} \Rightarrow 10^{-5m}x^2 = 10^{6m}x \Rightarrow x = 0$ or $x = 10^{6m+5m} = 10^{11m}$.

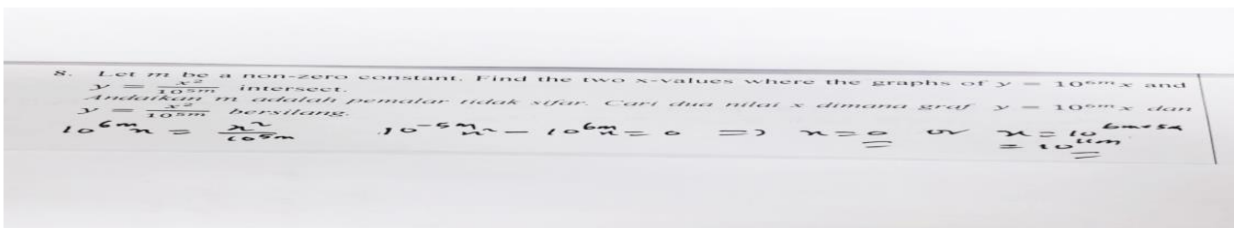


Figure 13: The Solution Method of Student A

The solution of student A that scored 3 shows student A had high skills in solving this problem. Student B used another method to solve the given problem.

$$10^{6m}x = \frac{x^2}{10^{5m}} \Rightarrow \log 10^{6m} + \log x = \log x^2 - \log 10^{5m}$$

$$6m \log 10 + \log x = 2 \log x - 5m \log 10$$

$$6m + \log x = 2 \log x - 5m$$

$$11m = 2 \log x - \log x$$

$$\log x = 11m \Rightarrow x = 10^{11m}.$$

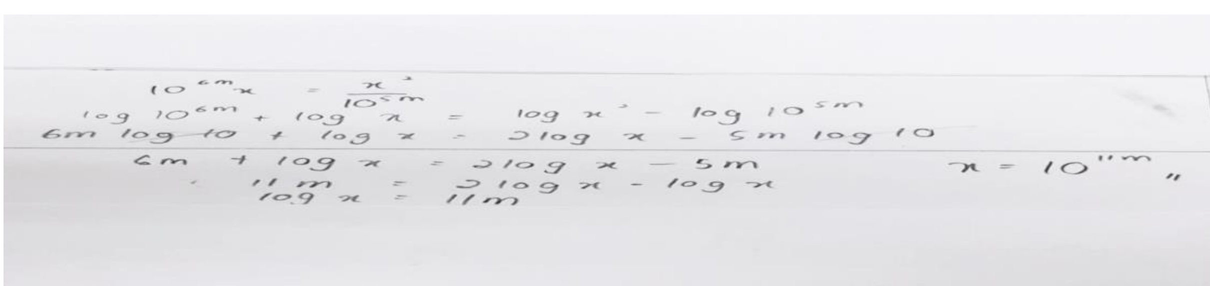


Figure 14: The Solution Method of Student B

Although student B had presented a creative solution (Figure 14) for this problem, he/she was given a score of 2 because the zero root of the problem was not calculated. The solution of student C that was shown by Figure 15 scored 1, because he/she provided an equation but the solution is not logical.

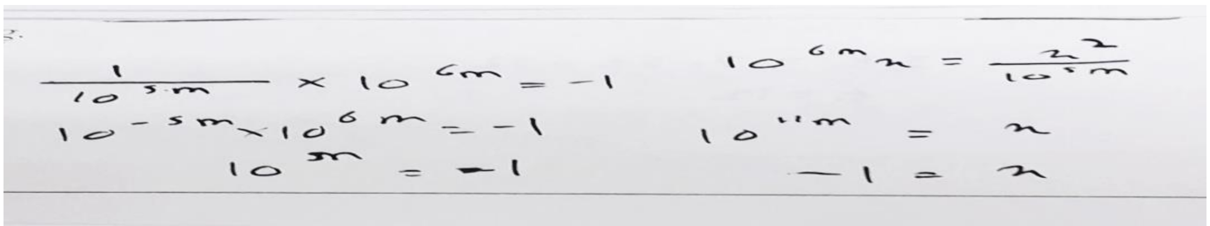


Figure 15: The Solution Method of Student C

The solution method of student D that showed by Figure 16, scored 0, because the response shows an insufficient understanding of the problem's essential mathematical concepts.

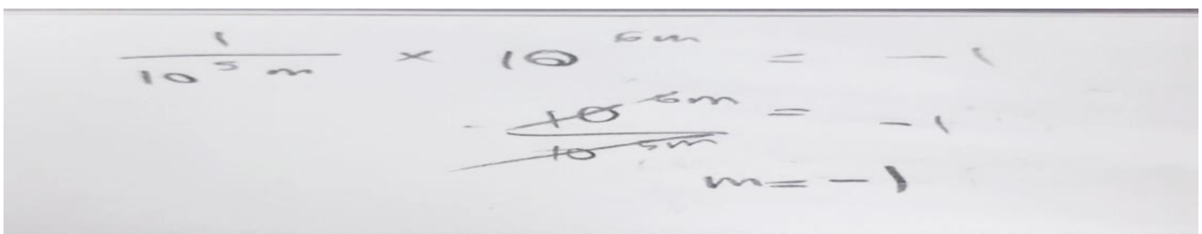


Figure 16: The Solution Method of Student D

The analysing of students' performance represents that the quality of teaching methods had important role in students' learning. For example, the definitions of even and odd functions in the textbook were as follows:
"A function f is said to be even if and only if $f(-x) = f(x)$ for all x".
"A function f is said to be odd if and only if $f(-x) = f(x)$ for all x".
The members of Lesson Study group defined the even and odd functions as the following:
A function f with the following two properties is called an even function:

1. Domain f is symmetric with respect to the origin
2. $\forall x \in D_f, f(-x) = f(x)$

A function g with the following two conditions is called an odd function

1. Domain g is symmetric with respect to the origin
2. $\forall x \epsilon D_g, g(-x) = -g(x)$

The textbook's definition of even and odd functions that used in the traditional group did not refer to the domain of the functions, so students ended up superficially memorising the two properties $f(-x) = f(x)$ and $f(-x) = -f(x)$ to identify whether a function is odd or even. However, in the Lesson Study group, the lecturers discussed regarding the domain of the even and odd functions and improved the definitions of even and odd functions in the textbook. For the problem "Determine whether the function $h(x) = \sqrt{x} + \sqrt{-x}$ is even or odd" in the post test, the majority of students in the traditional group argued that this is an even function because this function satisfies in the condition $h(-x) = h(x)$ as $h(-x) = \sqrt{-x} + \sqrt{x} = \sqrt{x} + \sqrt{-x} = h(x)$ without understanding the properties of odd and even functions conceptually. In the Lesson Study group, the majority of students would first identify the domain of the function h as $D_h = \{0\}$ and found that $h = \{(0,0)\}$, so they argued this zero-function is both odd and even.

The normality of mathematics scores of students is shown in Table 9. Since the value of p for all tests are greater than 0.05, the scores are normally distributed.

Table 9: The Normality of Tests Scores

Group	Test	Kolmogorov-Smirnov		
		Statistic	df	Sig
Lesson Study	Pre-test	0.120	44	0.115
	Post-test	0.102	44	0.200
	Follow-up	0.119	44	0.135
Control	Pre-test	0.100	42	0.200
	Post-test	0.107	42	0.200
	Follow-up	0.093	42	0.200

As respect to Table 10, the result of independent sample t-test shows that there is no significant statistical difference between means of the experimental group (M = 18.22, SD = 3.99) and the control group (M = 19.83, SD = 5.08) in pre-test t(84) = -1.632, p = 0.106.

Table 10: Comparing the Mean of Mathematics Scores in the Pre-test

Group	Number	Mean	Standard Deviation	T	df	Sig
Lesson Study	44	18.22	3.99	-1.632	84	0.106
Control	42	19.83	5.08			

Table 11 shows the results of independent sample t-test for the post-test and the follow-up test. There is significant statistical difference between means of the experimental class (M = 24.02, SD = 4.64) and the control class (M = 19.07, SD = 3.92) in the post-test t(84) = 5.326, p = 0.000. Also, there is significant statistical difference between means of the experimental class (M=23.52, SD=3.75) and the control class (M = 19.28, SD = 3.92) in the follow-up test t(84) = 5.117, p = 0.00.

Table 11: Comparing the Mean of Scores in the Post-test and the Follow-up Test

Test	Group	Number	Mean	Standard Deviation	T	df	Sig
Post-test	Lesson Study	44	24.02	4.64	5.326	84	0.00
	Control	42	19.07	3.92			
Follow-up	Lesson Study	44	23.52	3.75	5.117	84	0.00
	Control	42	19.28	3.92			

Since the p value is smaller than 0.05 the null hypothesis was rejected for both post-test and follow-up test. Therefore, Lesson Study program enhanced the students' skills in problem-solving and higher order thinking as compared to the traditional teaching method.

5. DISCUSSION AND CONCLUSIONS

Function is a very important topic and it is used in many mathematics courses at the university level. Students in the foundation level require having sound knowledge of functions so that they are able to apply the concept in any fields of study. Specially in the real world, learners need to be able to solve higher order problems and apply the concept to real world situations (Michelsen, 2006). The results of this study showed that in this foundation centre, problem-solving and higher order thinking are not greatly emphasised by the lecturers. The classes were lecturer-centred and lecturers transferred contents to students through traditional teaching method. Meanwhile, they emphasised on solving of routine mathematics exercises in their teaching. Students did not seem to learn mathematics conceptually and hence, not experiencing the beauty of mathematics. It is an important question "how can learners learn mathematical contents without engaging in problem-solving?" They may gust end up memorising the mathematics materials such as definitions, theorems and methods and later apply them in solving similar mathematics exercises or exam questions. In the lesson study program, the lecturers through collaborative work planned the research lessons to minimise the need to memorise and to allow students to learn the mathematics concepts meaningfully through problem-solving activities. For example, for the question "If $(gof)(x) = -2x^2 + 4x + 1$and $g(x) = 3 - 4x$ then find the rule of the function f" students in the Lesson Study group showed better performance rather than students in traditional group. Because in the traditional group, the lecturer taught exactly the textbook' materials and for instance, in the textbook there are 18 similar questions such as "If $f(x) = 1 - 5x^3$ and $g(x) = 2 + \sqrt{2x^2 + 3x}$ then find the function $(fog)(x)$". In fact, in traditional teaching method the lecturer only focused on mathematics exercise solving. Whereas, in the Lesson Study group, the lecturer more focused on problem solving and higher order thinking skills based on the rich materials that prepared in the research lessons by lecturers collaboratively.

Mathematics lecturers seem to teach according to the flow and the approach of the textbook that was provided. In teaching different groups of students in twenty classes conducted by several lecturers that work towards a common examination, the practice of teaching according to the textbook may be less risky because the lecturers need to ensure that all students acquire the same materials. Practicing using routine exercises may be an option that most lecturers use, especially if the exam format is at a similar level to previous exam questions. This has been the predicament of many lecturers. If they focus more on higher order thinking, their students may not be able to do many of practice questions that need to form a pattern for the solution. On the other hand, the over dependency on the textbook may also resulted from their lack of pedagogical content knowledge and it could be more prevalent among those that do not have teaching certification.

Lesson Study as a professional development program had helped many lecturers to enhance their teaching knowledge, specifically their pedagogical content knowledge and content knowledge. Participatory educational environments for mathematics teaching, provide learners with effective opportunities to improve their learning, thereby enhancing their chances to succeed (Moreno & Rutledge, 2020). The lecturers would be able to improve their lessons to emphasise more on problem-solving and higher order thinking. In this foundation centre, most lessons are being taught through traditional method and only a small percentage of tasks were related to the problem-solving. In the Lesson Study approach, the focus is more on problem-solving. It has been found that collaborative work among lecturers is very beneficial in increasing the quality of mathematics teaching. Furthermore, collaborative work through Lesson Study provides opportunity for lecturers to improve students' higher order thinking skills. It also helps improve the quality of mathematics materials which help learners to engage in suitable problem-solving at different levels of thinking, based on the revised Bloom's Taxonomy. The results of quantitative part of this study show that students in the experimental group had better performance in problem-solving and higher order thinking in post-test and follow-up test. It seems Lesson Study program was able to improve the ability of students in problem-solving and higher order thinking compare to traditional teaching method. In fact, in Lesson Study program lecturers prepared appropriate mathematics problem for students and transferred the mathematical materials to students through better pedagogical methods so they improved the ability of students in problem-solving. Whereas students in control class received the mathematics materials from textbook that emphasises on routine exercise solving. If lecturers were able to collaboratively write their textbooks and teaching materials, definitely they can design and develop better output. Based on the results of this study, Lesson Study is a strong approach for lecturers' professional development and students' outcomes in problem-solving and higher order thinking. Therefore, Lesson Study is suitable program in teaching mathematics for foundation centres, and not just schools.

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