Creative Conjecture: Abductive Reasoning to Generate Some Ideas in Algebra

Indriati Nurul Hidayah*, Santi Irawati, Mohammad Agung, Cholis Sa'dijah, Subanji, Sudirman

Universitas Negeri Malang, Jl. Semarang no 5, Malang, Indonesia

indriati.nurul.fmipa@um.ac.id

Abstract: Most students practice abductive reasoning in solving mathematical problems that encourage creativity. This study analyses the process of making conjectures based on abductive reasoning. This study used a qualitative design and involved 106 undergraduate mathematics students enrolled in the mathematics course Introduction to Ring. We analyzed the students' conjectures on two mathematics problems. The study was completed by grouping the types of conjectures made by students and then investigating each student's explanation of each conjecture. The results suggested two types of conjectures practised by students, namely creativity in investigating the converse of the proposition and creativity in dividing into cases.

INTRODUCTION

Creativity represents the required character for students in solving mathematical problems. Creative thinking is closely related to problem-posing and project-based learning (Ayllon et al., 2016; Wijayati et al., 2019). During this learning, students are frequently provided with problems and learning methods to build up comfortable learning that enhances students' creative thinking (Ngiamsunthorn, 2020). A previous study has reported a positive relationship between students' facts finding and problem finding with their number and originality of ideas (van Hooijdonk et al., 2020). Moreover, problem-solving correlates positively with image completion, whereas fact-finding does not (Dewijani, 2015).

Problem-solving is also a powerful evaluation tool for a person's mathematical reasoning and creativity (Ayllon et al., 2016). Students' reasoning in solving mathematical problems is divided into several types. Some researchers argued that students' reasoning in solving math problems is deductive (Leighton, 2006; Niu et al., 2007) or inductive reasoning (Haverty et al., 2000; Hozzov & Kov, 2020; Moguel et al., 2019). Meanwhile, Molnár et al. (2013) reported a link between inductive reasoning and complex mathematical problem solvency. Meanwhile, several researchers analyzed the use of deductive and inductive reasoning in solving mathematical problems (Arslan et al., 2009; Stephens et al., 2020). Previous research by Arslan et al. (2009) found that educational students prefer inductive rather than deductive reasoning. Similarly, Stephens et al. (2020) investigated the way students in the USA concluded when given a series of premises.

In the initial stage of solving problems, abductive reasoning plays a role in discovering new knowledge. Students actively construct new knowledge in problem-solving situations that may contradict their current knowledge, resulting in astonishment and obstacles (Radford, 2008). Other studies supporting abductive reasoning in forming discoveries include the studies carried out by Tschaepe (2014) and Walton (2005). Abductive reasoning also carries a role in the discovery of new rules through conjectures at the beginning of a discovery (Abe, 2003; Levin-Rozalis, 2010; Magnani, 2001; Paavola, 2006; Prendinger & Ishizuka, 2005; Woosuk, 2017). Niiniluoto (2018) also adds that abductive reasoning is required in the discovery of new concepts because abductive reasoning may produce various results that still need to be proven deductively.

Many studies have investigated abductive reasoning in solving mathematical problems. Several studies have also addressed the relationship between abductive, deductive, and inductive in solving algebraic problems (Moscoso, 2019). Additionally, previous studies have discussed the relationship between abductive reasoning and creativity (Hidayah et al., 2021; Moscoso, 2019; Tomiyama et al., 2010).

Literature Review

Creativity

Creativity in this research is defined as the process of generating ideas. In mathematics learning, ideas can be in the form of a theorem, a new solution to a problem, or further examples of a concept. Students generate ideas or connect with them through mathematical writing, mainly in solving problems. To generate ideas, students can make guesses or formulate a hypothesis as a part of their creativity (Torrance, 1965). Creativity can also be defined as a process of being sensitive to the problems at hand, deficiencies and knowledge gaps, missing parts, friction, and so forth. Creativity also includes identifying difficulties, seeking solutions, making guesses, formulating hypotheses about deficiencies, testing and retesting these hypotheses, possibly modifying and retesting the hypothesis, and finally communicating the results (Torrance, 1965).

In addition, four components of creativity have been proposed, including fluency, flexibility, elaboration, and originality (Torrance, 1965) Fluency is the number of relevant responses from the subject, while flexibility is thinking of different questions, causes, or consequences. Meanwhile, originality represents the statistical infrequency of these questions, reasons, or effects, as well as the extent to which the response represents a mental leap departure from the apparent and commonplace. Lastly, elaborate is the detail and specificity incorporated into the questions.

In mathematics learning, creativity has particular criteria. Mathematical creativity is the ability to formulate mathematical objectives and find inherent relationships among them (Ervynck, 1991 in Tall, 2002). Mathematics learning focusing on creativity will improve students' representational ability, strategic fluency, and flexibility, as well as an appreciation for new problems or solutions (Silver, 1997). Therefore, dimensions of creativity, namely fluency, flexibility, and novelty, are a core part of mathematics.

Silver (1997) details the indicators of creativity dimensions in solving mathematics problems, as presented in Table 1.

Table 1. The Indicators of Creativity Dimensions

Dimensions of Creativity	Indicators
Fluency	• generating multiple mathematical ideas, • generating various answers to a mathematical problem (if it exists), • exploring mathematical situations.
Flexibility	• generating new mathematical solutions when at least one has already been produced.
Novelty	• exploring many solutions to a mathematical problem and generating a new one.

Another aspect of creativity is divergent and convergent thinking. Divergent thinking includes finding patterns, breaking fixed mindsets, formulating mathematical conjectures, evaluating original mathematical ideas, identifying missing components, and moving from general to specific concepts. Meanwhile, convergent thinking enables students to answer without requiring significant creativity (Mann, 2006). In terms of supporting a statement, there is little difference between divergent and convergent topics (Pamungkas et al., 2018).

Student creativity can be enhanced through open-ended problems since solving mathematical problems in multiple ways is closely related to personal mathematical creativity and suggests evaluating mathematical creativity (Yaftian et al., 2011). Open-ended tasks also promote students' mathematical creativity (Fatah et al., 2016; Molad et al., 2020; Rahayuningsih et al., 2019). Besides, lecturers may adopt schemes from other lessons with a sufficient challenge to increase students' creativity (Diezmann & Watters, 2002).

Abductive Reasoning

In solving mathematical problems, students practice reasoning. Reasoning is a thinking process that connects known facts or realities to conclusions (Krawczyk, 2017), leading to problem solvency. It is also regarded as an incredibly logical or analytical thinking process (Subanji & Supratman, 2015). Thus, reasoning can be interpreted as a logical process for reaching conclusions based on the available information.

Several studies reported that students' reasoning in solving problems is deductive (Lachmy & Koichu, 2014; Niu et al., 2007; Stephens et al., 2020) and inductive (Bisanz et al., 2013; Haverty et al., 2000; Hozzov & Kov, 2020). Other researchers have also analyzed deductive and inductive reasoning in their studies (Arslan et al., 2009; Nickerson, 2010; Rivera, 2008; Stephens et al., 2020).

The reasoning that facilitates creativity is abductive reasoning. It is classified as the first phase of new ideas generation. Abductive reasoning contributes to improving creative knowledge in the learning process (Moscoso, 2019; O'Reilly, 2016). Abductive reasoning consists of examining and using facts to propose a conjecture (Peirce, 1960) with unproven validity. Conjecture can also be referred to as the origin of a new idea in mathematics learning. Further, once the conjecture is verified by deductive reasoning, then it is determined as a theorem.

People are frequently perplexed by the many styles of reasoning, particularly the distinctions between deductive, inductive, and abductive reasoning. To describe abductive reasoning, the examples of deductive, inductive, and abductive reasoning proposed by Peirce (1960) are presented in the following.

Examples of deductive reasoning

Rule: All marbles in this bag are white.
Case: These marbles come from inside this bag.
∴Result: These marbles are white.

Examples of inductive reasoning

Case: These marbles come from inside this bag.
Result: These marbles are white.
∴Rule: All the marbles in this bag are white.

Examples of abductive reasoning

Rule: All marbles in this bag are white.
Result: These marbles are white.
∴Case: These marbles come from inside this bag.

In addition, the conclusion made based on abductive reasoning in mathematics learning is presented in the following.

Every two even numbers, when added together, will produce an even number
Two numbers add to an even number
So, maybe the two numbers are both even

By using abductive reasoning, we can conclude many conclusions based on known rules and results.

Abductive reasoning is closely related to the form of logic. In logic, the sentence which has a truth value, true or false, is called a statement, symbolized by p, q, r, and so forth. A statement can contain universal quantifiers or existential quantifiers with symbols $\forall x$ and $\exists x$, respectively. The example of the symbol for a statement with a quantifier is $(\forall x)\, p(x)$. The example of symbol creation for a statement is shown in the following.

Let $p(x)$: x be an even number

$q(x)$: $5x$ is an even number

Then the statement "For every x, if x is an even number, then 5x is also an even number" can be denoted by $\forall x, p(x) \rightarrow q(x)$

Using the abductive form proposed by Peirce (Niiniluoto, 2018), the statement can be written as

$$(\forall x)F(x) \rightarrow G(x)$$
$$(\exists x = a)G(a)$$
$$\therefore F(a)$$

in a simple form,

$$A \rightarrow B$$
$$B$$
$$\therefore A$$

When compared with the modus ponens and the modus tolens in deductive reasoning, it becomes

$$A \rightarrow B$$
$$A$$
$$\therefore B$$

and

$$A \rightarrow B$$
$$-B$$
$$\therefore -A$$

As observed in the example, the modus ponens and modus tolens rules do not encompass abductive reasoning. In abductive reasoning, the inference attempts to modify the modus ponens and the modus tolens. The validity of deductive reasoning has been confirmed by the arguments $'(A \rightarrow B) \wedge A \rightarrow B'$ and $'(A \rightarrow B) \wedge -B \rightarrow A'$ form a tautology. Although, abductive reasoning is not completely valid, it opens opportunities to come up with creative conclusions (Niiniluoto, 2018).

Several studies have investigated types of students' abductive reasoning in problem-solving. Four abductive reasoning based on fact were reported, including creative conjecture, fact optimization, factual error, and mistaken facts (Hidayah et al., 2020). In the creative conjecture, to solve the problem, students have to use every piece of information inside the problem, understand the questions, and use "actual" facts from outside the problem. Students create conjectures from facts by writing, describing, or drawing problem-solving designs and composing a new conjecture on a problem related to the question (Hidayah et al., 2020).

In this study, we investigated students' abductive reasoning in solving two mathematics problems. Specifically, we analyzed the students' answers to two mathematics problems. The analysis was carried out by grouping the types of conjectures made by students, followed by an examination of each student's explanation based on their reasoning. For the answers using abductive reasoning, the explanation was observed based on the use of facts and the form of logic.

RESEARCH METHOD

The participants of this study were 106 undergraduate mathematics students consisting of 86 female and 20 male students in the fourth semester of the mathematics course introduction to ring at the Department of Mathematics, Faculty of Science, Universitas Negeri Malang, Indonesia. In the pandemic era, learning was carried out online. In this study, the researcher acted as a lecturer in the classes. During the learning, the lecturers often gave questions and assignments that enhanced students' curiosity or creativity. Besides, the learning also facilitated students to discuss material they had not understood or convey new ideas to lecturers and friends. Sometimes lecturers also asked students to form small discussion groups to discuss assignments.

The instruments in this research were two mathematical problems facilitating students to make some conjecture. Moreover, the mathematical problems also improved students' creativity because they aided students in developing many conjectures along with explanations.

The instrument validity was checked through discussion with an expert group. In detail, two mathematics experts were involved in the content validity test on the mathematics questions and interview sheets. The validity test included the eligibility of the item test, the concept's authenticity, multiple interpretations, and appropriate instructions for abductive reasoning. Additionally, the questions provided several facts, and the participants were asked to check whether the provided statement was true or not based on their existing knowledge, as shown in Figure 1.

Let R be a commutative ring with unity, $x, y, z \in R$.

1. If x is a unit, does $y \mid z$ imply $xy \mid z$?
2. Did your answer in part (1) still hold if x is not a unit? Justify your answer!

Figure 1. Open-ended Problem to Observe Students' Abductive Reasoning

The problem is about a commutative ring in Algebra. The commutative ring R is a non-empty set with two binary operations, addition (denoted by $a + b$) and multiplication (denoted by ab), such that for all a, b, c in R (Gallian, 2016), as presented in the following.

1. $a + b = b + a$
2. $(a + b) + c = a + (b + c)$
3. There is an additive identity 0 in R such as $a + 0 = a$ for all a in R.
4. There is an element $-a$ in R such as $a + (-a) = 0$
5. $a(bc) = (ab)c$
6. $a(b + c) = ab + ac$ and $((b + c)a = ba + ca$
7. $ab = ba$

In the first question, if the student answers 'yes,' then their statement became 'If x unit and $y|z$, then $xy|z$.' The students who answer 'no' produced a statement '$\exists x, y, z \in R,\ x$ units or $y|z$, but $xy|z$.' Additionally, the students could also present other answers to generate many conjectures. By answering this problem, students' creativity is expected to increase.

From the first question, we constructed the second question by modifying the first question of "If x is not a unit, does $y \mid z$ implies $xy \mid z$?". Through students' answer to this question, we investigated the conjectures developed by students and their explanations. Further, we analyzed the students' answers that contained new ideas. The analysis was performed by investigating the students' responses, relating them to the logical form of the conjecture's proposition, and discussing them based on creativity criteria.

RESULTS AND DISCUSSION

Through data reduction, only 83 out of 106 students' answers could be analyzed. Besides, nine of 83 students did not answer the second question. The analysis was completed by grouping the types of conjectures proposed by students and examining each student's conjecture based on the reasoning. For the students who use abductive reasoning, the explanation was observed based on facts and logic.

In solving the first question, three different conjectures were made by students. Fifty-three students created the conjecture 'If $y \mid z$ and x unit then $xy \mid z'$ and used deductive reasoning to their answers' validity and authenticity. The students' valid conjecture and justification represent their proper reasoning enhancing their discovery (Folger & Stein, 2017; Niiniluoto, 2018; Peirce,1960). In mathematics, deductive reasoning is used to justify some theorem (Ayalon & Even, 2010; Ellis, 2007; Leighton, 2006).

Our obtained data suggested that many students presented the same conjecture with different explanations. Meanwhile, twenty students proposed incomplete facts, and they took $x = 1$ and $x = -1$ as a unit element in R. These facts allow the production of a conjecture, but they remain insufficient for the justification of a statement in mathematics. Further, other students present different conjectures with an explanation.

Two students created two creative conjectures, namely 'If $y \mid z$ and x unit then $xy \mid z'$ and 'If $xy \mid z$ then $y \mid z$.' Then, they attempted to converse with the true proposition. The student's creative conjecture is listed in Table 2.

Table 2. Conjecture Proposed by the Student for the First Question

Conjecture Proposed by the Student	Description	Number of Students
Let R be a commutative ring with unity, $x, y, z \in R$. • If $y \mid z$ and x unit, then $xy \mid z$	• Using deductive reasoning • Using incomplete facts • Using the wrong form of logic • Assuming the questionable concept as a given fact	53 20 4 2
Let R be a commutative ring with unity, $x, y, z \in R$. • If $y \mid z$ and x unit, then $xy \nmid z$	• Using incomplete facts • Using the wrong form of logic	1 1
Let R be a commutative ring with unity, $x, y, z \in R$. • If $y \mid z$ and x unit, then $xy \mid z$ • If $xy \mid z$, then $y \mid z$	• Using deductive reasoning	2

For the second question, we observed three different conjectures and their explanation. Nine students answered, 'If $y \mid z$ and x non-unit then $xy \mid z$ is false' and used deductive reasoning, shown from $\exists x, y, z \in R$, $y \mid z$, and x non-unit, but $xy \nmid z$. Students use abductive reasoning to create conjectures and justify them with deductive reasoning (Nandasena et al., 2018; Peirce, 1960.). As reported in a previous study, many students provide one counterexample to refute a false conjecture, but others give some counterexamples (Zeybek, 2017). Besides, numerous students create the same conjecture with different explanations.

Our data also showed that forty-two students misunderstood the negation of the statement. They assumed that 'If $y \mid z$ and x non-unit then $xy \nmid z$' is a negation form of 'If $y \mid z$ and x unit then $xy \mid z'$. From the first question, the students comprehended 'If $y \mid z$ and x unit, then $xy \mid z'$ is true. Therefore, they assumed 'If $y \mid z$ and x non-unit, then $xy \mid z'$ was false. As reported in a previous study, merely denying the meaning of the statements was less successful than employing symbolic principles of negation in a recursive style (Piatek-Jimenez, 2010). Besides, nine students provided no answer to the second question.

In addition, one student presented two conjectures creatively. They divided case by case for conjecture, from 'Let R be a commutative ring with unity, $x, y, z \in R, y \mid z \leftrightarrow z = yc$ with the first case, If c prime and x non-unit then $xy \nmid z$ and the second case, If c is non-prime and x non-unit then $xy|z$, as summarized in Table 3.

The students who practiced abductive reasoning concluded based on facts and provided reasoning. Students' conclusions are referred to as conjectures since they have not been proven true. However, in their explanation, 53 and nine students used deductive reasoning in justifying their conjectures for the first and second questions, respectively. Meanwhile, the remaining students presented other explanations. As described by Peirce (1960), the construction of conjectures and justification in abductive reasoning is part of the inquiry process.

Table 3. Conjecture Made by Students for Second Question

Conjectures Presented by Students	Explanation	Number of Students
Let R be a commutative ring with unity, • $\exists x, y, z \in R$, $y \mid z$ or x non-unit but $xy \nmid z$	• Using deductive reasoning • Using incomplete fact	9 12
Let R be a commutative ring with unity, $x, y, z \in R$. • If $y \mid z$ and x are non-unit, then $xy \nmid z$	• Using incomplete facts • Using the wrong form of negation • Assuming the questionable thing as a given fact	8 42 2
Let R be a commutative ring with unity, $x, y, z \in R, y \mid z \leftrightarrow z = yc$	• Dividing into cases	1

- If c prime and x non-unit, then $xy \nmid z$
- If c is non-prime and x non-unit, then $xy|z$

Open-ended problem facilitates students to make different conjectures (Fatah et al., 2016; Molad et al., 2020; Rahayuningsih et al., 2019; Suyitno et al., 2018). The open-ended problem can improve students' mathematical reasoning (Bernard & Chotimah, 2018). Generating conjectures is an essential mathematical habit helping students develop their mathematic skills (Meagher et al., 2020)

Although the students' conjecture is not necessarily true, it is a candidate for the new theorem that requires some corrections. Making conjectures is the initial stage in developing a new mathematical theory. Further, those conjectures can be a theorem if equipped with valid proof steps, known as deductive reasoning. However, sometimes students cannot practice these valid steps, as shown in Tables 2 and 3.

Two students (coded as S1 and S2) answered the first question, and one student (coded as S3) answered the second question by generating a creative conjecture because they presented a new conjecture correlated to the problem in the question (Hidayah et al., 2020). The students practising creative conjectures generate different conjectures from the existing questions, as illustrated in Figures 2 and 3.

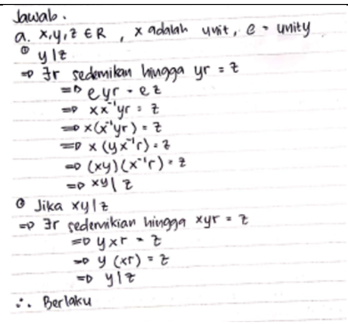
Creativity in Investigating the Converse of Proposition

Students S1 and S2 produced creative conjectures (Hidayah et al., 2020) because, in the first question, they presented "If $y \mid z$ and x unit then $xy \mid z$" and used deductive reasoning to justify the proposed conjecture. Besides, these students also made another conjecture. S1 and S2 attempted to generate the converse of the previously made propositions, as shown below.

Proposition: If $y \mid z$ and x unit, then $xy \mid z$

Converse: If $xy \mid z$ then $y \mid z$ and x unit

In the converse form, S1 and S2 eliminated or ignored the x-unit condition so that for any x, the converse still applies. This conjecture is excellent as it extends the sufficient condition of a theorem. The student's creative answer in investigating the converse of the proposition is illustrated in Figure 2.


Jawab.
a. x,y,z ∈ R , x adalah unit, e = unity
① y|z
⇒ ∃r sedemikian hingga yr = z
⇒ eyr = ez
⇒ xx⁻¹yr = z
⇒ x(x⁻¹yr) = z
⇒ x(yx⁻¹r) = z
⇒ (xy)(x⁻¹r) = z
⇒ xy|z
② Jika xy|z
⇒ ∃r sedemikian hingga xyr = z
⇒ yxr = z
⇒ y(xr) = z
⇒ y|z
∴ Berlaku

Translate:

a. $x, y, z \in R$, x is a unit, e unity

If $y|z \rightarrow \exists r$ such that $yr = z$

$$\rightarrow eyr = ez$$
$$\rightarrow xx^{-1}yr = z \rightarrow x(x^{-1}yr) = z$$
$$\rightarrow x(yx^{-1}r) = z$$
$$\rightarrow (xy)(x^{-1}r) = z$$
$$\rightarrow xy|z$$

If $xy|z \rightarrow r$ such that $xyr = z$

$$\rightarrow yxr = z \rightarrow y(xr) = z$$
$$\rightarrow y|z$$

Figure 2. Students' Creative Conjecture in Investigate Converse of Proposition

In the first question, S1 and S2 made conjectures. Then, they used deductive reasoning to prove their conjectures, as shown in Figure 2. Students S1 and S2 were not satisfied with the answer, then they tried to investigate the converse of the true proposition in part (i). Through deductive proof, S1 found the true proposition "If $xy \mid z$ then $y \mid z$." They modified this form of converse, such as S1 omitted x as a unit in the ring R. So x applies to any element in the ring R.

Students' ability to construct different conjectures is essential. Although the justification for the conjecture is still lacking, creativity in making new propositions should be appreciated. However, the justification provided by these two students is still lengthy, and there are more effective justification steps, as written in the following.

Given that $y \mid z$, so $z = ky$ for some $k \in R$. Since x is a unit, we have

$$z = ky$$
$$z = k.1y$$
$$z = k(x^{-1}x)y$$
$$z = (kx^{-1})xy$$

S1 and S2 present excellent creativity because they explore mathematical situations within the problems to generate multiple conjectures. Their answers represent their high level of flexibility. From its first proposition, S1 generates conversion of implication for flexibility criteria. The conversion of the implication becomes a mathematic rule used by S1 and S2 in their abductive reasoning.

In addition to making two new conjectures, their implications, and conversations, S1 and S2 also provided explanations using deductive reasoning. As their explanation is valid, the conjectures from S1 and S2 are included in the novelty criteria.

Creativity in Dividing into Cases

The second creativity type in solving a mathematical problem is creativity in dividing into cases. One of our respondents, coded as S3, presented a conjecture, 'If $y \mid z$ and x unit then $xy \mid z$' and practised deductive reasoning to explain the conjecture. So, this student has a true proposition. Further, S3 discovered a piece of new knowledge through abductive and deductive reasoning in solving the problem (Folger & Stein, 2017; Nandasena et al., 2018; Żelechowska et al., 2020)

Student S3 answered the second question by dividing two cases based on primary property. S3 explored the mathematical situation in the problem and obtained the possible value c divided into two cases. The first case was c is non-primary; x is not a factor of c, while the second case was c is non-primary; x is a factor of c, as presented in Figure 3.

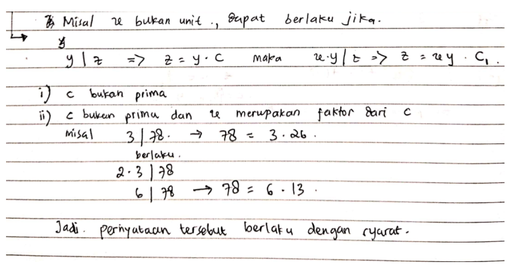
<table>
<tr>
<td>
</td>
<td>Translate:
(2) if x is not a unit, is the conclusion in (1) hold?
Let x is not a unit; the statement will behold, if
$y|z \rightarrow z = yc$ then $xy|z \rightarrow z = xy$
i) c is not prime
ii) c is not prime, and x is a factor of c
For the example
$3|78 \rightarrow 78 = 3.26$
So
$2.3|78$
$6|78 \rightarrow 78 = 6.13$
So, the proposition is held if we add the sufficient condition</td>
</tr>
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Figure 3. S3 Exploration of the Mathematical Situation

In S3's answer, he added the sufficient condition in the proposition, namely, c was not prime, x was not a factor of c, c was not prime, and x was a factor of c. So, S3 divided sufficient condition in two cases, namely, c was not prime, while x was not a factor of c, *and* c was not prime, while x was a factor of c.

So, the conjectures made by S3 are presented below,

(i) if $y|z \rightarrow z = yc$, and c is not prime, x is not a factor of c, then $xy \nmid z$;

(ii) If $y|z \rightarrow z = yc$, and c is not prime, x is a factor of c, then $xy|z$.

S3 justified the conjecture by using an example, as presented in the following.

In the example of $y|z \rightarrow z = yc$, and c is not prime, x is not a factor of c

$$y = 3, z = 78, c = 26$$

$$3|78 \rightarrow 78 = 3.26$$

There is $x = 7$ is not a factor of 26, so that $7.3 \nmid 78$

In the example of $y|z \rightarrow z = yc$, and c is not prime, x is a factor of c

$$y = 3, z = 78, c = 26$$

$$3|78 \rightarrow 78 = 3.26$$

If $x = 2$, is a factor of 26, then $2.3|78 \rightarrow 6|78 \rightarrow 78 = 6.13$

S3 justified his conjecture by using an example. In mathematics, the example is just one case from a proposition, so it can not be proof of some theorem or proposition. Further, S3 must prove his conjecture by deductive reasoning to preserve it. However, S3 creativity in generating conjectures deserves to be appreciated, though it cannot be proven.

One method for proving a theorem in mathematics is to divide it into cases. One commonly used method for verifying a statement of the implication form is breaking up the proof into several cases(Bloch, 2011; Hammack, 2013). By dividing it into cases, the process of finding proof is simplified. This method is often advantageous in splitting the problem into many minor problems (Stefanowicz et al., 2014).

From the mathematical situation presented in Figure 3, S3 obtained $z = yc$, and then S3 constructed some conjectures by dividing c into two cases. S3 generated new ideas when at least one has already been produced, so the conjecture remains to be in creative criteria. Based on the answer of S3, novelty criteria are held because S3 generates a new idea, different from other students. Creativity leads to a novel and useful outcome idea, product, or expression (Schubert, 2021).

CONCLUSIONS

One sort of abductive reasoning connected to the use of facts while solving algebraic problems is creative conjecture. In the creative conjecture, students use all the facts inside the problem to solve it. Besides, the students must know the question's meaning and use authentic facts outside the problem to solve the problem. Therefore, students develop conjectures based on facts by writing, describing, or drawing problem-solving designs and writing a new conjecture outside the question but still related to the problem in the question (Hidayah et al., 2020). In this research, three students presented creative conjecture.

In the data collection process, we provided two interrelated problems that facilitate the making of conjectures. Our obtained data showed two types of creativity in constructing conjectures. The first type is creativity in investigating the converse of the proposition, and the second type is creativity in dividing into cases. For the first type, the students made two conjectures in implication form and converse from the implication. The conjectures were 'If $y \mid z$ and x unit, then $xy \mid z$ and If $xy \mid z$ then $y \mid z'$. Further, they completed their answer with deductive reasoning to justify the conjecture. The student also divided into two cases based on the primary property for the second type. He explored the mathematical situation in the problem and obtained the possible value of c divided into two cases. The cases are c primary, and c is non-primary.

In addition, those students also presented a high level of creativity because they have fluency, flexibility, and novelty in solving open-ended problems. They explored the mathematical situation in the problem and used abductive reasoning to make some conjectures. Some students even used deductive reasoning to justify conjectures, but one student did not. Even so, they found a piece of knowledge that would be useful for increasing their creativity.

Therefore, as lecturers are obligated to enable students to think creatively, they have to provide open-ended problems that stimulate students to create conjectures. The type of reasoning that increases students' creativity is abductive reasoning. Abductive reasoning, completed by deductive reasoning, is essential to discovery learning.

This study of abductive reasoning was carried out on mathematics students who learned pure mathematics. The results would be slightly different from the studies involving junior high students with cognitive abilities that require the usage of natural objects. Therefore, future investigation is encouraged to examine the abductive reasoning of students in junior high school.

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