

Editorial from Malgorzata Marciniak, Managing Editor of MTRJ



The journal has been expanding with an accelerated speed. As of today, we have eleven active members of the editorial board. Recently joined us Brian Evans, Yana Shvarstberg and Yu Gu from Pace University, NY, USA, who contribute their time, knowledge, and efforts to shape the journal.

The current volume opens with three papers about special education during the times of pandemic. We all went through tough times learning digital tools within a week and converting our classrooms to remote modality. But for teachers of students with special needs this experience was even more challenging. The first paper **Development of Interactive Media with Augmented Reality for Prospective Solution Quota-Friendly Learning and Physical Limitation in the Pandemic Era** discusses teaching geometry to hearing impaired students. The authors, Joko Lianto Buliali, Andriyani, and Yudhiakto Pramudya from *Indonesia*, validate affirmatively the augmented reality uses for teaching spatial vision.

The second paper **Impact of adoption of Information and Communication Technologies (ICTs) in Teaching Mathematics to Intellectually Disabled Children**, submitted by authors Joyti Sherawat and Poonam Punia from *India*, studies how technology impacts learning of intellectually disabled children. In their conclusions, the authors' claim that applied properly, technology can improve learning outcomes for these children.

The third paper, **Error Analysis of Dyslexic Student's Solution on Fraction Operation Tasks**, is devoted to a discussion of various erroneous ways dyslexic students may try to solve problems with fractions. Rooselyna Ekawati, et al. from *Indonesia*, analyze videos shared during professional meetings called Focus Group Discussion in which researchers displayed recorded interviews with students. The team from East Java discussed the symptoms of dyslexia by comparing the ways of thinking among dyslexic and non-dyslexic students. The authors found out that work of dyslexic students is distinctive due to unique inconsistencies within their solutions.

While digital education is very much in the center of our attention, it varies significantly for different groups of student and different mathematics courses. The paper, **Utilization of Digital Module for Asynchronous Online Independent Learning in Advanced Mathematics Education**, which was submitted by Ryan V. Dio from the *Philippines*, discusses the efficiency of a design of a learning module for independent studies in graduate mathematics education. We understand that today's education of teachers will shape the future students, so we very much appreciate the submission and follow up on the conclusions, which claim that the teaching-research methodology is very much suitable for the design and the revisions of the learning modules.

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The next paper, **Effects of Animated Instructional Packages on Achievement and Interest of Junior Secondary School Student in Algebra**, submitted by S. G. Ojo from *Nigeria*, presents results related to teaching elementary algebra with animated instructions. In the author's experiment, students exposed to the animated lesson performed better than students from the sample group, who were taught the same lesson in a traditional way. However, according to other studies, the statistical significance of the difference between the methods is negligible. Thus, the experiment may be repeated with more detail to understand the underlying principles of the nature of students' learning process and their way of thinking after learning from the animated module in comparison to other modules.

In their submission entitled, **Developing Conceptual Understanding of Irrational Numbers Based on Technology through Activity System**, Abolfazl Rafiepour et al. presents the Activity Theory in the light of digital teaching and learning of irrational numbers. The authors use WhatsApp creatively in a dual way, in one way to encourage students' collaboration and in another way for collecting the data about students' progress.

Rahmi Ramadhani, et al. from *Indonesia*, submitted an ethnomathematics study, **Exploration of Students' Statistical Reasoning Ability in the Context of Ethnomathematics: A Study of the Rasch Model**. Here the authors ask the students mathematical questions about cultural items. For example, students were asked to approximate the silk, gold, silver, and cotton threads used in making a Malay Songket displayed in a picture. Students responded with numerical values and justify their answers providing data for quantitative and qualitative analysis of their reasoning abilities.

The submission, **Specialized Content Knowledge of pre-service teachers on the infinite limit of a sequence**, submitted by Mónica Arnal-Palacián and Javier Claros-Mellado from *Spain*, displays the difficulties of understanding the concept of infinite limit. The authors study the challenges faced by future teachers while explaining the concept, which is one of the most important ideas of calculus.

The Problem Corner, edited by Ivan Retamoso, contains solutions of previous problems sent by Aradhana Kumari from Borough of Manhattan CC, USA and Jayendra Jha, and Sankalp Savaran from India. The last two contributors shared a geometrical conjecture they discovered. The next problem has been proposed by Aradhana Kumari.

Analysis of Problem Solving Process on HOTS Test for Integral Calculus, by Eko Andy Purnomo, et al. from *Indonesia*, is a paper that analyzes the process of solving application problems using integrals. Students usually experience difficulties with such problems either due to insufficient background knowledge, insufficient understanding of the mathematical concept, or, most frequently, the lack of cross association between these two. Performing a thorough analysis of the process of performance may help in identifying the challenges and finding remedies. Based on their findings, the authors suggest the statements of the questions should contain the six steps of problem solving process to help the students successfully navigate through the solution.

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Problem solving skills of word problems are discussed in the paper, **Computational, Logical, Argumentative, and Representational Thinking in the United Arab Emirates Schools: Fifth Grade Students' Skills in Mathematical Problem Solving**, submitted by Nabil Kamal Al Farra, et al. from the *United Arab Emirates*. The authors recall three problem-solving models by George Polya, by Schoenfeld, and by Verschaffel. They use these models to analyze the solutions of their students. The authors imply the models may help the students in their performance while solving word problems.

The process of thinking and constructing mathematical knowledge is the main theme of the last paper of this issue, **Constructing Students' Thinking Process through Assimilation and Accommodation Framework**, submitted by Siti Faizah, et al. from *Indonesia*. The authors divide students' schemes of thinking into assimilation and association in the context of semigroups in the abstract algebra course taught to mathematics majors. They use these findings to direct students' attention for the purpose of building their knowledge in a particular way creating a spectacular educational process.

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Development of Interactive Media with Augmented Reality for Prospective Solution Quota-Friendly Learning and Physical Limitation in the Pandemic Era

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Abstract: *The problem in studying geometry for students with normal hearing and hearing impairment is that when studying geometry, it does not only require analytical presentations but also visual presentations related to spatial visualization abilities. This problem is getting worse during the pandemic, in which teaching and learning had to be done online. For this reason, it is necessary to have learning media that accommodates the characteristics of hearing impairment students that physiological limitations, and the media must be oriented to visual presentation accompanied by verbal descriptions or sign language so that the material presented is easy to understanding. The research and development model used in this study is the Borg and Gall procedural model which is descriptive and shows systematic steps to produce interactive learning media with augmented reality. The basis for choosing this development model is related to the special characteristics of the Borg and Gall development model, namely developing products to bridge the gap between education research and education practice, as well as emphasizing specific problems related to practical problems in teaching through applied research. Interactive learning media with augmented reality can be said to meet the criteria of validity, practicality, and effectiveness based on the results of testing the feasibility of developing products. The learning media used has also proven to be a solution to the problem of hearing limitations for hearing impairment students and internet accessibility problems, especially the problem of dependence on quotas and networks. So that students become more enthusiastic about learning and have a meaningful learning experience.*

Keywords: Interactive Media, Augmented Reality, Deaf Students, Quota-Friendly Learning

INTRODUCTION

The reach of the Coronavirus pandemic that occurred quickly and massively has expanded and affected various sectors of life, as in the health, economy, tourism, and education sectors (Sintema, 2020). The implementation of various activities must be carried out from home so that the chain

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of spreading the Coronavirus is broken, including learning activities (Kamsurya, 2020). The sector of education must adapt and try to transform suddenly through a face-to-face learning system by utilizing online communication and internet technology. This is following the circular letter of the Ministry of Education and Culture of Indonesia Number 4 of 2020 concerning the implementation of education policies in the emergency period of the spread of Covid-19 so that the learning process is carried out from home through distance learning.

The application of various learning platforms and applications have even emerged as the implementation of the learning policy at home, starting from the Learning Management System (LMS), Utilization of Learning Houses, SPAD, Zenius, Cisco conference video are used to support learning from home including mathematics (Atsani, 2020; Gunawan et al., 2020; Irfan et al., 2020; Mailizar et al., 2020).

According to Atsani (2020), the problems during online learning during this pandemic are related to the unpreparedness of all components involved in the learning process, both in terms of standards and the quality of learning outcomes. Several other problems that arise along with the implementation of online learning have also been studied by several researchers such as Ali and Magalhaes (2008); Eady and Lockyer (2013); Hung and Chou (2015); and Karasavvidis (2010). As a result of these problems, it was found that students' boredom with online learning was found after the first two weeks of online learning, considerable anxiety to buy internet quota during online learning especially students whose parents had low incomes, as well as changes in students' mood due to too many assignments from the teacher (Irawan et al., 2020). In particular, the impact of Coronavirus pandemic on the online learning system was also investigated by Rasmitadila et al. (2020), Sintema, (2020), Sullivan et al., (2020), but only few studies have discussed certain about mathematics learning during pandemic (Mailizar et al., 2020). In fact, the implementation of a complex mathematics learning process during a pandemic is quite interesting to highlight because it requires accuracy delivering materials and developing mathematical skills (Santagata and Yeh, 2014).

Mathematics is a science with an abstract and hierarchical object of study, hence students need a sufficient learning experience that affects the process of acquiring knowledge to learn it (Andriyani and Maulana, 2019). One of mathematics branch whose objects is abstract, which is connected to other mathematical concepts and has a great opportunity to be understood/familiar by students is geometry (Andriyani and Dwi Juniati, 2019; Bell in Astutik, 2017; Clements and Sarama, 2011). Geometric objects are obtained through an abstraction process based on concrete objects found in daily life (Clements and Sarama, 2011; Couto and Vale, 2014). Some problems in studying geometry not only require analytical presentations but also visual presentations related to the ability of spatial visualization to understand properties and interpret two-dimensional images that represent three-dimensional objects. This spatial visualization ability is one of the geometry

activities that must be mastered by students as recommended in The National Council of Teachers of Mathematics (NCTM, 1989). Therefore, visualization has an important role in learning mathematics, especially in problem-solving that requires complex cognitive management (Puloo et al, 2018).

Following the importance of visualization in solving problems, teachers need to pay attention to the development of these abilities in learning geometry using various contexts, especially the visualization of junior high school students (Puloo et al, 2018; Utomo et al, 2017). Junior high school students are children whose cognitive development is at the formal operational stage (12 years and over) so that their understanding of mathematical concepts has begun to lead to an abstract mindset. At this age, they also should be able to think logically without having to deal with direct objects or events (George, 2017; Ryandi et al., 2018). However, facts on the ground show that there are still many students who have difficulty in terms of geometry and visualization, including plane and solid shape, measurements, polygons, geometric ratios, geometric transformations, latitude, and longitude lines, and diagrams which are topics generally considered difficult by the students and teachers (Adolphus, 2011; Poch et al., 2015; Van Garderen et al., 2013).

The difficulties in learning geometry are also experienced by students with special needs, such as the hearing impairment who have physical limitations in terms of hearing and verbal communication (Buliali et al., 2021). This communication limitation affects the mathematics learning of hearing impairment students who are slower than their mates who can hear (Gottardis et al., 2011). In addition, hearing impairment also causes other obstacles such as inadequate knowledge, deficits in social skills, language delays, limited vocabulary and literacy, the emergence of background and domain knowledge gaps, also dependence on assistive technology (Luckner et al., 2012).

With various consequences of disabilities and difficulties in learning geometry, hearing impairment students who have curriculum content that is not much different from hearing students certainly experience obstacles when learning geometry is done online. According to Serianni and Coy (2014), students with disabilities experience additional online learning challenges related to their reduced motivation and accommodation, although by learning online they have the opportunity to access learning materials freely and repeatedly. Moreover, before COVID students with disabilities already had underperformance in mathematics compared to nondisabled students, in which affect to students' opportunities (Wei et al., 2013). Students with disabilities have less access to conceptual and challenging mathematics (Jackson & Neel, 2006). The trend towards inequality in opportunities to learn mathematics is more evident during Emergency Remote Teaching (Lambert and Schuck, 2021). Facts also show that many parents are unable to provide assistance according to student needs (Schuck and Lambert, 2020). This is supported by the results

of research by Garbe et al. (2020) which shows that almost 10% of parents of non-disabled children have low motivation to engage online, thus hindering their child's learning. Several studies have found a decrease in student engagement during the online learning period in the general education (Kim et al., 2020) and among them are students with special needs (Balkist and Agustiani, 2020; Smith, 2020).

It is undeniable that online learning that has been carried out so far has not been able to fully facilitate interactive learning activities (Kamsurya, 2020), this is because communication between students and teachers during online learning is decreasing (Sintema, 2020). The lack of communication and direct interaction between teachers and hearing impairment students is an important problem that needs to be resolved and taken seriously. Moreover, hearing impairment students have difficulty interpretation formal definitions of geometry concepts, imagining spatial concepts, and relating contextual problems that contain geometry objects. The implication of this, hearing impairment students tend to memorize geometry concepts rather than understand geometry. The difficulty of students imagining abstract geometric objects certainly contradicts the condition of students' cognitive development at the high school level who should be able to reason without having to deal with direct objects. Meanwhile, the conditions of online learning and offline learning on a limited scale at this time, also make it impossible to provide direct experience to hearing impairment students. Besides the direct experience, according to Paranis and Samar (1982), the attention of students with hearing impairment is more quickly obtained by visual stimulation than others. This is reinforced by evidence that students with disabilities have better visual processing skills than hearing students (Musselman, 2000).

The existence of problems in learning geometry related to the special characteristics of hearing impairment students which have limited vocabulary in acquiring knowledge; difficulty in mental imagery to visualize the abstraction of geometry objects; and the inability to relate geometry concepts as a whole, it can affect student's ability to understand further mathematical material. Whereas referring to the Circular of the Minister of Education and Culture Number 4 of 2020, online learning must still provide meaningful learning experiences for students. This meaningful learning can be achieved by learning oriented towards providing direct experience related to the problems of everyday life. Therefore, appropriate strategies and methods of learning mathematics are needed so that students can have an understanding and ability to apply mathematics in daily life problems (Kamsurya, 2020).

To choose an appropriate learning method, teachers need to consider limitations, interest in learning, and whether the teaching materials used are interactive or not. According to Hasanah et al (2017), teaching materials used by teachers must facilitate the needs and accommodate the unique characteristics of students. As for hearing impairment students, more attention and significant changes in the learning paradigm are needed to improve the quality of their learning

(Adler et al., 2014). Learning conditions in schools which have very limited time and the use of teaching materials in the textual books form by teachers, the fact certainly influenced of hearing impairment student' learning motivation who feel increasingly different in obtaining access to educational services.

By considering the various problems of learning geometry, it is necessary to have a learning media that accommodate the characteristics of the hearing impairment who are more dependent on vision in the process of communicating and obtaining information during learning (Marschark et al., 2017). The media must be oriented towards visual presentation accompanied by verbal descriptions or sign language so that the material presented is easily understood by students. Augmented reality technology can be the best choice to facilitate hearing impairment students in understanding geometric abstract concepts through visual illustrations. According to (Buliali et al., 2021), augmented reality enriches students' perception of reality so that students' difficulties in imagining objects can be overcome by visualizing objects. In addition, augmented reality technology can also recognize physical objects to reveal their entities so that users can understand and utilize the properties of these physical objects (Ariso, 2017). This statement is reinforced by the statement of Pemberton and Winter (2009) which states that the use of augmented reality technology can support students' conceptual understanding and acquisition of information through group work and reflection on the direct experience they get. Thus, augmented reality becomes a new opportunity to support the mathematics learning process (Schallert and Lavicza, 2020). However, learning media using augmented reality technology for mathematics learning still lacking, especially in the geometry branch (Arifin et al, 2020).

Based on the problems experienced by hearing impairment students in geometry learning above, it is considered necessary and important to develop an interactive learning media that can support object visualization during geometry learning for hearing impairment students during the pandemic, but its media quota-friendly so that not cause student' anxiety. The interactive learning media developed in this study is an augmented reality technology-based learning media that contains the concepts of circles, transformations (rotations) and spheres to assist the spatial visualization of hearing impairment students in understanding the interrelationships of that three geometry concepts.

RESEARCH METHOD

This study is a research and development with the Borg and Gall procedural model, which aims to produce interactive learning media based on augmented reality that is valid, practical, and effective as a supporter of the learning model for mathematics teachers in schools with hearing impairments. This study's study procedure is part of the Borg and Gall model development procedure (Gall, 2003), consisting of five stages: developing a preliminary form of product, preliminary field

testing, primary product revision, main field testing, operational product revision. Because this research is a follow-up study, the research and information collecting stage, the planning stage, have been carried out by researchers when conducting preliminary research related to needs analysis in the first year. Meanwhile, the operational field testing, final product revision, and dissemination and implementation stages could not be carried out due to time constraints and restrictions on school activities to prevent an increase in the transmission of the Covid-19.

At the stage of developing the preliminary form of the product, the researchers develop several things. They are preparing the initial design of the product which will be developed (hypothetical design); arrange necessary instruments during the research and development process; determine the stages of design test implementation in the field; determine the task description of parties involved in the research. While the preliminary field-testing stage is the stage of assessing the feasibility of the product by an expert accompanied by a limited initial field trial involving only two hearing impairment students. At this stage, the product feasibility assessment and initial field testing are reviewed from the substance of the design in terms of materials and media. The feasibility assessment and initial field testing are carried out repeatedly to obtain a feasible design. The next stage is the main product Revision. In this step, the researchers revise the design of the instructional media based on the results of expert assessments and limited field tests. The improvement of the initial media product here is mostly done with a qualitative approach so that the improvements made are internal because the evaluation is process-oriented.

After the researchers revise the media product, then the main field testing is carried out. In this step, the learning media product will be implemented in a wider scale learning, then the practicality and effectiveness of the learning media design will be tested. The research subjects involved in this study are 17 hearing impairment students. In this study, the practicality test of learning media is carried out through the provision of student response questionnaires, while the effectiveness test was carried out through experimental techniques.

The last stage of development media is the operational product revision stage. This stage is the second improvement after conducting a wider field test than the first limited field test. The improvement of the product from the results of this wider field test further strengthens the product developed, because the previously limited field trial phase is only involved, two students. In addition to internal improvements, this product improvement is based on the evaluation of the results so that the approach used is quantitative with comparing pretest and posttest. In detail, we present this development research procedure in Figure 1 below.

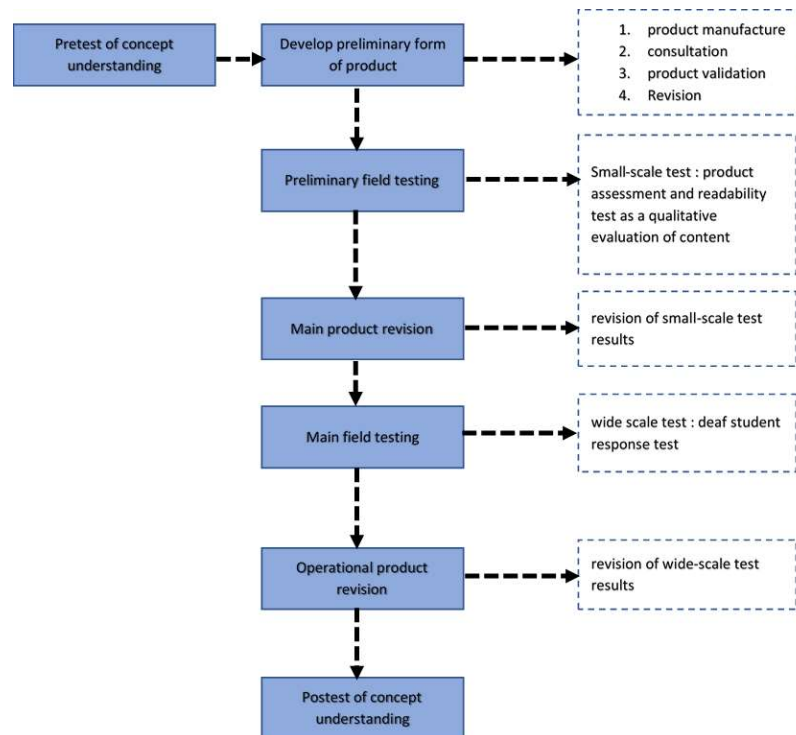


Figure 1. Research and Development section procedures of Borg and Gall

In this study, the population is all high school students with special needs for the hearing impairment, namely SLBN 2 Bantul, Yogyakarta. While the samples involved in the main field trial of this study are 17 hearing impairment students in grades VII, VIII, and IX and 2 hearing impairment students for a limited scale trial.

In this study, the media that has been developed will be assessed for feasibility in terms of validity, practicality and effectiveness. Therefore, this study data collection was carried out through test and non-test techniques. The test technique is carried out through giving pretest and posttest questions which consist of eight questions for understanding the concept of geometry to assess the effectiveness of the product as attached in Appendix 1. Non-test techniques are carried out through unstructured interviews, observation, giving student response questionnaires and giving product validation assessment questionnaires (material validation and media validation). Interviews were used to confirm and in-depth exploration, while observations were made to see student behavior during learning and taking tests. The provision of product validation assessment questionnaires and student response questionnaires resulted in two types of data, namely qualitative and quantitative.

Data analysis of questionnaires in this study was carried out qualitatively and quantitatively. The analyzed qualitative data of the validators' suggestions and students' suggestions descriptively. In line with the suggestions data, other qualitative data, namely the results of interviews and

observation also analyzed descriptively. Whereas the quantitative data from validators and students, namely the scores from filling out product assessment questionnaires by validators and scores from filling out student response questionnaires. The questionnaires contain questions to measure expert opinion and student attitudes towards the products developed in this study. Answers from the validator and students indicate their level of agreement with a series of questions posed in the questionnaire. Approval on the questionnaires was arranged in stages using a Likert scale consisting of 5 choices as attached in Appendix 2 and Appendix 3. The score for each expert or student choice answer in a row is: 1) score 'Not Good' = 1; 2) score 'Less Good' = 2; 3) score 'Good Enough'=3; 4) score 'Good'=4; and score 'Very Good'=5. Of the five answer choice scores, the highest score = 5 and the lowest score = 1.

The results of the expert assessment questionnaire and the student response questionnaire were calculated on mean, next referred to as the actual mean score. The actual mean score obtained is then converted into the form of qualitative criteria which refers to the guidelines for determining the criteria of Azwar (2010) as presented in Table 1 below.

Table 1. Conversion of Qualitative Criteria

Score Intervals	Category
$R_i + 1.5S_{d_i} < \bar{X} \leq R_i + 3S_{d_i}$	Excellent
$R_i + 0.5S_{d_i} < \bar{X} \leq R_i + 1.5S_{d_i}$	Good
$R_i - 0.5S_{d_i} < \bar{X} \leq R_i + 0.5S_{d_i}$	Enough
$R_i - 1.5S_{d_i} < \bar{X} \leq R_i - 0.5S_{d_i}$	Deficient
$R_i - 3S_{d_i} < \bar{X} \leq R_i - 1.5S_{d_i}$	Very Deficient

Description:

\bar{X} = actual mean R_i = ideal mean

= $\frac{1}{2}$ (ideal maximum score + ideal minimum score)

S_{d_i} = ideal standard deviation

= $\frac{1}{6}$ (ideal maximum score – ideal minimum score)

Ideal maximum score = number of questions \times highest score

Ideal minimum score = number of questions \times lowest score

Based on the conversion formula in Table 1, the interval difference for each actual mean score is obtained according to the number of questions on each questionnaire sheet as shown in Table 2.

Table 2. Actual Mean Score of Validator Assessment and Student Response

Score interval	Categories
Material Validation Assessment Questionnaire	
$140 < \bar{X}$	Excellent
$116.7 < \bar{X} \leq 140$	Good
$93.3 < \bar{X} \leq 116.7$	Enough
$70 < \bar{X} \leq 93.3$	Deficient
$\bar{X} \leq 70$	Very deficient
Media Validation Assessment Questionnaire	
$92 < \bar{X}$	Excellent
$76.7 < \bar{X} \leq 92$	Good
$61.3 < \bar{X} \leq 76.7$	Enough
$46 < \bar{X} \leq 61.3$	Deficient
$\bar{X} \leq 46$	Very deficient
Students Responses Assessment Questionnaire	
$64 < \bar{X}$	Excellent
$53.3 < \bar{X} \leq 64$	Good
$42.7 < \bar{X} \leq 53.3$	Enough
$32 < \bar{X} \leq 42.7$	Deficient
$\bar{X} \leq 32$	Very deficient

From the second interval the actual mean score obtained from the material and media validation assessment questionnaire, learning media products are considered valid if the actual mean score is in the minimally 'Good' category. The criteria for the practicality of learning media are also assessed in the same way namely, the actual average score obtained from the student response questionnaire is in the minimally 'Good' category.

In a different way, the analysis of the effectiveness of learning media products was carried out by comparing the pretest and posttest scores using SPSS. The comparison was carried out through the paired sample t-test mean, where the first mean showed the test results before the students were given the learning media and the second mean showed the test results after the students were given the learning media. The test used here is the statistical paired sample t-test. Before testing using the t-test statistical test, it is necessary to test for normality using the Shapiro Wilk test. After the

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data meet the normality requirements, the paired sample t-test is performed by comparing the significance values. If the significance value of the paired sample t-test < 0.05 , then H_0 is rejected, so that augmented reality-based learning media is effectively used because there are differences in students' understanding of geometry concepts before and after using learning media. H_0 is a condition where the mean pretest score is the same as the posttest mean score or there is no significant difference between the pretest and posttest mean scores. The opposite condition occurs if H_0 is rejected.

Furthermore, to find out whether the use of learning media can be said to be effective or not, the researchers calculated the difference between the pretest and posttest scores (Gain score) using the N-Gain formula as a determinant of the category of the increasing students' conceptual understanding (Hake, 1999). The researchers also consider the mean posttest and pretest scores of students, which claims that an increase in the mean understanding of students' geometric concepts from before to after being given learning media occurs if the mean posttest score $>$ the mean pretest score. The two N-Gain calculations and the consideration of pretest and posttest scores are used to base conclusions that the use of augmented reality-based learning media can be said to be effective in improving the understanding of geometry concepts for students with hearing impairment.

RESULT AND DISCUSSION

1. Develop Preliminary Form of Product Stage

At the stage of developing the preliminary form of the product, the researchers also compiled the instruments needed to see the feasibility of the learning media including namely, a material validation assessment instrument, a media validation assessment instrument, a student response questionnaire instrument, and an understanding test instrument. In addition, the researchers also compiled the initial design of the learning media that contained the circle material, the elements of the circle, its rotation, and the sphere. The initial design of learning media was based on the results of the needs analysis conducted in the initial research. Based on the results of needs analysis conducted by the researcher through observations and interviews in the initial research, there are some information obtained as follows:

- a) Students with hearing impairment have difficulty understanding the material of a circle and its elements, so that when they learn the concept of a circle rotation they are increasingly constrained. As a result, the students have low understanding of several mathematical concepts related to the basic concept of a circle and it has implications in learning geometric concepts separately.
- b) Students have difficulty translating formal definitions of circle and circle elements into pictures, so they always fail to visually imagine these abstract geometric objects.

- c) The physical limitations of students with hearing impairment related to their hearing impairment make it difficult for them to communicate freely with anyone, without having to be constrained in mastering certain sign languages. In the end, these communication limitations have impact to their limited vocabulary and knowledge, especially mathematical knowledge that contains many symbols and abstract objects.
- d) Mathematics teachers in special schools generally only use textual teaching materials as a source of student learning, such as textbooks. In the textbook there are many terms that students with hearing impairment do not understand when they present a formal definition of geometry. This learning resources that contain a series of writings do not attract students' interest in learning.
- e) Geometry learning activities in class do not encourage student activity and are not related to students' daily life problems. In fact, many geometric models are found in the environment around students, so that in learning geometry such as circles, there should be a great opportunity to explore the surrounding environment by involving students' active participation through inquiry.
- f) Students with hearing impairment need a learning media that can illustrate the formal definition of a circle and its elements, without having to load a series of texts and can visualize the abstract geometric objects so the students can easily imagine them.

From the results of the needs analysis in the initial research, the researchers developed a learning media designed with an image visualization display and oriented to students' interaction to facilitate the construction of knowledge. The learning media developed in this study contains a formal definition of a circle, elements of a circle, rotation of an object, and some results of the rotation associated with a 3-dimensional object, namely a sphere. The elements of a circle involved here are radius, diameter, arc, chord, sector, segment, and apothem. This learning media connects a concept of a 2-dimensional shape (circle and its elements) with a concept of the transformation geometry (rotation) of the 2-dimensional shape. Furthermore, the 2-dimensional shapes that undergo transformation are associated with the formation of 3-dimensional shapes due to the transformation events experienced.

To help students' spatial imagery, the learning media displays digital elements that can visualize these geometric abstract materials using markerless augmented reality technology. This markerless AR technology tracks real objects around students as marker objects, thus the media displays animated geometric objects in real time. Designing of learning media into application prototypes is carried out by Java programming language and openJL es, so the application can be implemented on Android. Furthermore, learning media in this form of applications can be used without internet connection access (offline). With application development using markerless augmented reality technology, the use of the application is free without having to create patterns or barcodes such as tracking objects. Specifically, the use of markerless augmented reality technology in the visual

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application of a circle and its elements uses the Hough Circle Transform method to detect circle shapes and their elements. In addition to compiling research instruments and designing the initial learning media, the researchers also made a description of task plans that must be carried out in the next stages of development, including determining the stages of implementing the design test in the field as follows: 1) testing the feasibility of learning media with trials initial field with a limited number of subjects; 2) doing revision the design of instructional media based on expert advice and limited field testing; 3) providing a pre-test of concept understanding; 4) implementing learning media in learning on a wider scale; 5) providing response questionnaires to students; 6) making improvements based on student responses in a wide field test; 7) providing post-test understanding of the concept.

2. Preliminary field testing stage

This stage began with an assessment of the feasibility of the product in terms of the substance of material and media design by the validator. The assessment of the feasibility of each material and media product was carried out by three validators who have relevant expertise in the fields of learning materials, learning media and practitioners. The assessment of validity of the learning material by the material validators is presented in Table 3 below.

Table 3: Description of Learning Material Validity Assessment by Material Experts

Aspects	Validator 1	Validator 2	Validator 3
Introduction	36	34	40
Content	98	95	105
Closure	7	6	7
Actual Mean	141	135	152
Overall Actual Mean	142,67		

Based on analysis of material expert validation result, it is known that the actual mean score of the material validator 1 is 141 with the "Excellent" category, the actual mean score of the material validator 2 is 135 with the "Good" category, while the actual mean score of the material validator material 3 is 152 with the category "Excellent". The overall actual mean score of the three material validators is 142.67 with the "Excellent" category. Thus it can be concluded that the learning media developed in terms of material is declared valid or feasible to use. The assessment of validity of the learning media by the media validators is presented in Table 4 below.

Table 4: Description of Learning Media Validity Assessment by Media Experts

Aspects	Validator 1	Validator 2	Validator 3
Introduction of Application	10	10	12
User control	23	20	25

Application View	50	40	52
Principles of Multimedia Design	19	18	15
Actual Mean	102	88	104
Overall Actual Mean		98	

Judging from the actual mean score of media validator 1 is 102 in the "Excellent" category, the actual mean score of media validator 2 is 88 in the "Good" category, while the actual mean score of media validator 3 is 104 with the category "Excellent". The overall mean score of the three media validators is 98 in the "Excellent" category. Thus, it can be concluded that the learning media developed in terms of media is declared valid or feasible to use. In terms of materials and media, it can be concluded that the augmented reality-based interactive learning media developed in this research has met the criteria for validity of a development product.

Beside of providing an assessment to the validator, the researchers tested the learning media on a limited basis to get input from the test subject. The mathematics teacher at SLBN 2 Bantul randomly selected two students with hearing impairment as subjects for a limited trial. Researchers tested the media as long as students participated in learning activities using augmented reality-based interactive learning media. After the students had a learning experience with the learning media, the researcher required them to fill out an assessment sheet and students' impressions while using augmented reality-based interactive learning media. From the results of the assessment and impressions of students in the small-scale trial class, it is known that there are no significant problems with the use of augmented reality-based interactive learning media. Students found it a little difficult to distinguish writing from real backgrounds in the form of objects in the surrounding environment, so the researcher needs to improve the color contrast of writing on learning media.

3. Main product revision stage

At this stage the researchers revised the design of the learning media based on the results of expert assessments and limited field tests. Based on the suggestions of the two students in the small-scale trial, the researchers changed the color of the writing that accompanies the description of the object image display of the learning media so that it is easy for students to read. The researchers also revised the design of learning media based on some suggestions of the validators. The improvement of the initial media products here is more related to the size of the letters that appear on smartphones and the delay in presentation between materials.

4. Main field testing stage

After revising the media product according to the suggestions of the validators and the test subjects, the researcher implemented the learning media on a wider scale to test the practicality and effectiveness of the instructional media design. The research subjects involved in the implementation of this development product were 17 students with hearing impairment.

Researchers used augmented reality-based interactive learning media in learning activities of circles, elements of circles and their rotations. The researchers provided a series of implementations of the implementation of learning media in the seventeenth grade students as below.

First Meeting Activities

The researchers gave pretest questions to find out the students with hearing impairment' initial understanding of the circle material, its elements and, rotations before learning using learning media. The pretest consisted of 8 essay questions with an allocated time of 90 minutes. Students worked on pretest questions at school by implementing a strict health protocol by the school.

Second-Third Meeting Activities

The second meeting was held at SLBN 2 Bantul by implementing a strict health protocol. The researchers started the lesson by motivating students before learning the basic concept of a circle by explaining the importance of learning the material related to everyday life. The purpose of learning at this meeting is to understand the formal definition of a circle which was previously only able to be memorized by students. Before starting the student activity to understand the definition of a circle, the researchers divided the students into several groups with each group consisting of three students. Then, the researchers distributed one smartphone each that had the markerless augmented reality application installed to group representatives and introduced the use of the learning media to students.

The learning activity began with exploring objects around students to detect which ones were circular and not circular. This activity is to stimulate student activity through investigation and concept discovery with direct experience. Figure 2 (a)-(b) shows the exploration of surrounding objects by students in groups.



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Figure 2 (a)-(b). Students are exploring objects around them using augmented reality media. A display of circular objects recognition on real objects explored by students is presented as shown in Figure 3 below.

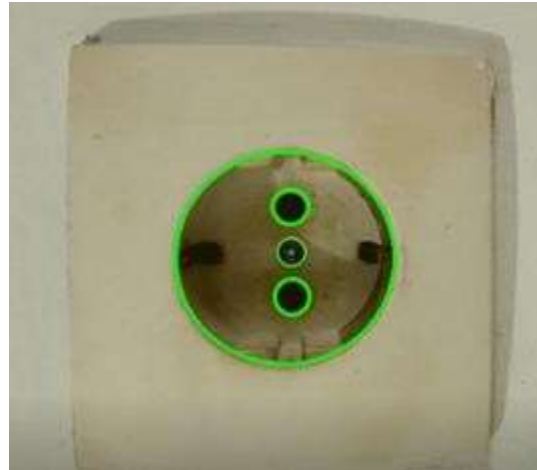


Figure 3. Recognition of circular objects with markerless augmented reality

If the object detected by the student is in the form of a circle, the learning media will automatically display a green circle marker as shown in Figure 3 above. Detection of a circle shape is followed by the presentation of a formal definition of a circle consisting of two definitions. The first definition relates to a circle as a simple closed curve as shown in Figure 4 below.

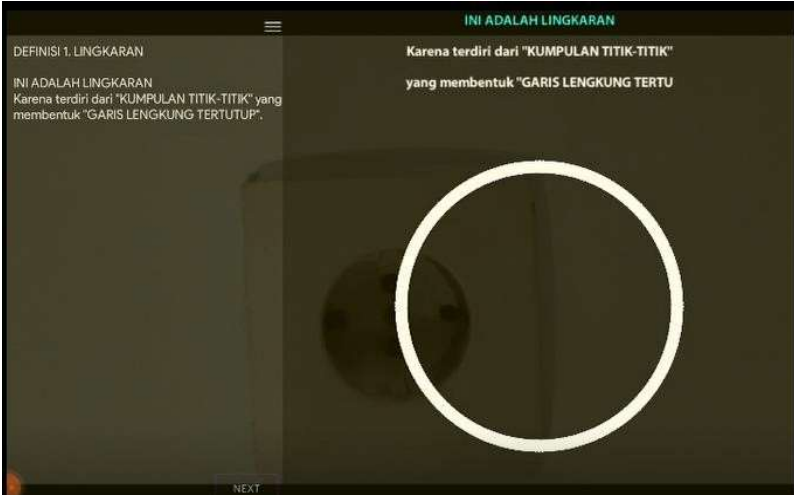
	<p>DEFINITION 1: CIRCLE</p> <p>THIS IS A CIRCLE Because it consists of a "SET OF POINTS" that constructed a "CLOSED CURVED LINE"</p> <hr/> <p>THIS IS A CIRCLE Because it consists of a "SET OF POINTS" that constructed a "CLOSED CURVED LINE"</p>
---	---

Figure 4. The first definition relates to a circle as a simple closed curve

The second definition relates to a circle as a set of points that are equidistant from the center as shown in Figure 5 below.



DEFINITION 2: CIRCLE

THIS IS A CIRCLE

because the distance of "POINTS" that constructed a curved line to "CENTER/MIDDLE POINT" is the same

PAY ATTENTION TO CHANGES OF NUMBERS/VALUES !!!

Figure 5. The second definition of circle relates to a set of points equidistant from the circle center

After presenting the definition of a circle, students can continue learning by choosing what circle elements they wanted to learn. The selection of the circle element that appears on the student's smartphone screen is accompanied by an example of a concept and not an example of a concept, while an example of circle element display is shown in Figure 6 (a)-(d).



CIRCLE ELEMENTS. ARC

THIS IS ARC

"ARC" is "CURVED LINE" on the circle relating "2 (TWO) RADIUS" passing through the circle center/middle point

Figure 6 (a). Visualization of an example of an arc as a circle element



Figure 6 (b). Visualization is not example of an arc

**CIRCLE ELEMENTS.
ARC**

THIS IS NOT ARC

because the "CURVED LINE" doesn't connect "2 (TWO) RADIUS" and doesn't pass through the circle center/middle point

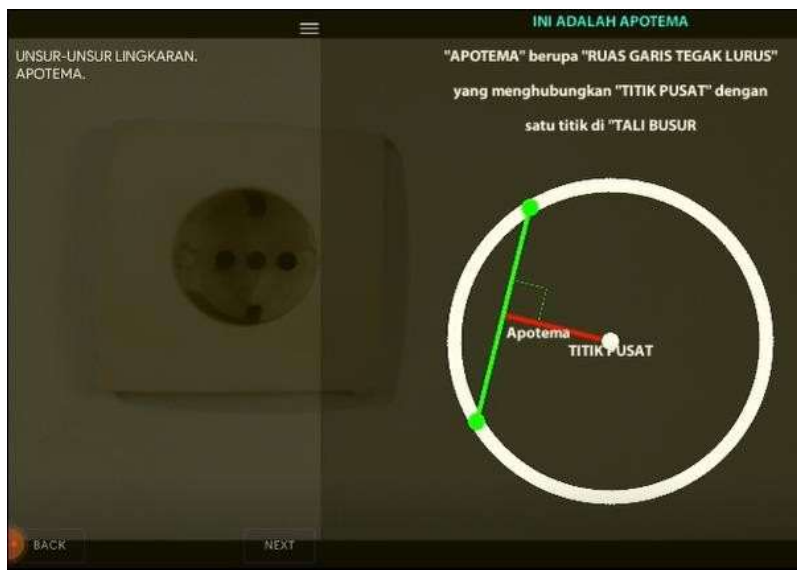


Figure 6 (c). Visualization of an example of an apothem as a circle element

**CIRCLE ELEMENTS.
APOTHEM**

THIS IS APOTHEM

"APOTHEM" has the shape of "LINE SEGMENT" of "PERPENDICULAR" relating "CENTER" with one point in "ARC POINT"



**CIRCLE ELEMENTS.
APOTHEM**

THIS IS NOT APOTEMA

Because the "LINE SEGMENT" that connecting "CENTER" with "A POINT on THE CHORD" is not "PERPENDICULAR"

Figure 6 (d). Visualization is not example of an apothem

The presentation of examples and not examples of concepts presented in this learning media is in accordance with the achievement of indicators of understanding exemplifying and interpreting as presented by Anderson & Krathwohld (2001).

After exploring what objects were circle-shaped and observing the appearance of the elements of the circle, students wrote down all the results of their investigation and concept discovery with the guidance of the teacher as shown in Figure 7 (a)-(b) below.



Figure 7 (a)-(b). Students are exploring objects around them using augmented reality media

Fourth Meeting Activities

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At this fourth meeting, students continued learning about the rotation of a circle and a sphere. By choosing any object that was detected as a circle, students selected rotation menu and observed the rotation display of a circle with a certain angle and a sphere on the smartphone screen that had been distributed by the researcher. The view of the rotation of object (circle) and the sphere is presented as in Figure 8 (a)-(b) below.

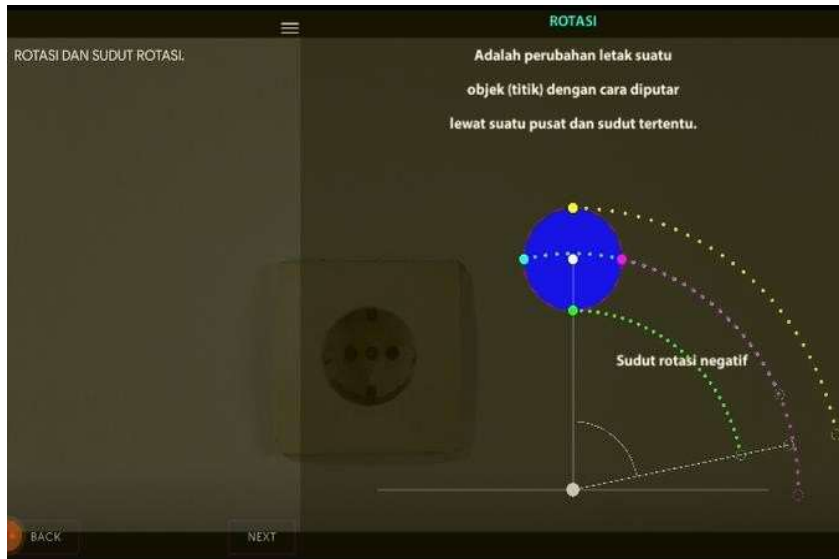


Figure 8 (a). The rotation of object and angle of rotation

ROTATION AND ANGLE OF ROTATION

ROTATION

It is a location change of an object (point) by rotating through a center of rotation and a particular center angle.



Figure 8 (b). The circle rotation become a sphere

CIRCLE ROTATION THAT FOLLOWS A SPECIFIC AXIS

For example, a Gasing toy in the shape of a CIRCLE (Plane figure/2-dimensional) in which rotated following the axis (line) will become a SPHERE (building a 3-dimensional space). The SPHERE also rotates.

CIRCLE ROTATION THAT FOLLOW A SPECIFIC AXIS

Example: GASING TOU (Freesbee)

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Fifth Meeting Activities

After learning the circle material, elements of circles, rotations and sphere, the researcher gave a response questionnaire to the seventeen students with hearing impairment. The provision of student response questionnaires was carried out to test the practicality of learning media. Based on the results of the student response questionnaire, it is known that there is one student who categorized the interactive learning media with augmented reality as "Good" and there are sixteen other students who categorized the media as "Excellent". The actual mean score of the results of the responses of students with hearing impairment who became users of learning media is 67.66. This means that the mean is in the "Excellent" category, so that the learning media can be said to have met the criteria for the practicality of a product development.

5. Operational product revision stage.

At this stage the researchers made improvements based on student responses in a wide field test. The product improvement from the results of this wider field test was used to further strengthen the product being developed. Next, the researcher gave a posttest related to students' understanding of the concept after being given learning media. The results of the posttest carried out by the researcher would then be compared with the results of the pretest that had been carried out before the main field testing stage was carried out. Comparison of the results of the pretest and posttest of the seventeen students was carried out to assess the effectiveness of the learning media by using the test questions that can be seen in Appendix 1. Before testing the two test results, the researcher tested the normality of the data using Shapiro Wilk test. From the results of testing the normality data for the pretest, it is obtained significance value (p-value) = 0.643 and for the posttest data it is obtained a significance value (p-value) = 0.464. Because the p-value of the second test is > 0.05 , the data meets the requirements for normality of the distribution data. Then, paired data test was performed using paired sample t-test with SPSS software.

Based on the paired sample t-test, it is obtained that the significance value (p-value) = 0.004. Because p-value < 0.05 then H_0 is rejected. In other words, there is a significant difference between students' understanding of geometry concepts before and after using learning media according to the results of testing the pretest and posttest scores. Besides, the researcher also compared the average understanding of students at 57.50 and the posttest at 76.38. Because the mean posttest score $>$ the mean pretest score, it can be said that there was an increase in students' understanding after using learning media. Next, the researcher calculated the N-Gain value of the students with hearing impairment ' understanding. From the results of the calculation on the pretest and posttest, students obtained a score of 0.45. This shows that the increase in students' understanding is in the moderate category. In other words, the learning media developed in this study is effective in increasing the concepts understanding of students with hearing impairment who previously had difficulties and were constrained in learning due to physiological and accessibility to the internet network limitation.

The effectiveness of Augmented Reality learning media in improving students' understanding is in line with the research results of Coimbra et al (2015) which shows that augmented reality can encourage higher motivation, understanding and interaction with the material being studied. This effectiveness is an indicator of the breadth of use of augmented reality and the magnitude of its potential in education (Garzón et al., 2017; Yu et al., 2009). In particular, the effectiveness of augmented reality-based learning media in geometry learning is also seen through increasing of students' mathematical spatial abilities (Arifin et al, 2020).

Regarding to the practicality of learning media, which refers to the results of the student response questionnaire (very good), it shows that students with hearing impairment accept the use of augmented reality-based learning media in their learning very well. Based on the results of observations and interviews, the students even seem more enthusiastic about learning because they no longer have to depend on the existence of internet quotas. The enthusiasm of students with hearing impairment in learning, which is also seen from student responses during learning, shows that learning media innovation with augmented reality can be accepted because it utilizes smartphones as a technology that is close to students. This is supported by Setyaningrum and Waryanto (2018) which states that the use of smartphones in classroom learning increases students' motivation and learning outcomes. One of the applications that are suitable to be developed on smartphones is augmented reality (Guntur et al, 2019).

Enthusiasm and increasing understanding of the concept of students with hearing impairment when using augmented reality-based learning media, also shows the important role of technology in student interaction and better performance during mathematics learning as stated by Panthi et al (2021). The use of such technology also reduces the abstraction of learning and creates an environment that is suitable for students' life situations (Dikovic, 2009). With this, Dikovic (2009) adds that the exploration of Information and Communication Technology (ICT) encourages student engagement and motivates them to leave rote-based learning. Likewise, the appropriate use of ICT positively encourages more interaction between teachers and students, resulting in better collaborative outcomes (Koc, 2005). Furthermore, Koc (2005) clearly explains that the usefulness of ICT enables the students to communicate, share knowledge, and work collaboratively anytime and anywhere. In this case, students not only get knowledge together but also mutually share diverse learning experiences with each other. Moreover, if the use of ICT is applied to learning during a pandemic, then its use supports effective learning from home whose teaching requires mastery and the application process of many parents who are not necessarily able to do so (Gay, 2002).

Seeing the significant role of augmented reality-based interactive learning media, the use of this media as a complement to the learning of students with a hearing impairment needs to continue even though it is not in a pandemic condition. In fact, it can also be used by students with normal

hearing as an alternative learning tool. The use of augmented reality as an additional learning tool for all students is also in line with the research results of Guntur et al. (2019). Furthermore, in his research, Guntur also stated that most of the teachers who participated in the augmented reality development training agreed with applying this technology in classroom learning because it would help teachers improve their students' affective and cognitive abilities. Thus, the use of augmented reality-based interactive media is one of the answers to the implementation of a non-discriminatory information and communication technology-based learning system.

CONCLUSIONS

Interactive learning media with augmented reality can be said to meet some criteria of validity, practicality and effectiveness based on the results of product development feasibility testing. The validity of the learning media is shown by the fulfillment of the excellent category in terms of material with the mean validation score of 142.67 and the excellent category in terms of media with the mean validation score of 98. The practicality of the learning media is shown by the fulfillment of the excellent category in terms of student responses with the mean score is 67.66. For the effectiveness of learning media, it is shown by the significant difference between the pretest and posttest scores for the understanding ability of hearing impairment students. Finally, the mean posttest score is greater than the mean pretest score, which can also be seen from the increase between the pretest and posttest scores in the moderate category. Therefore, the learning media used has proven to be a solution to the problem of limited hearing for students with hearing impairment and internet accessibility problems related to the availability of internet data balance and networks.

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APPENDIX 1. PRETEST and POSTTEST

Answer the following questions well!

1. What is circle in your opinion? Give an illustration and explanation about the circle!
2. Give examples of objects that are circle models and models that aren't circles according to your knowledge and give the reason!
3. Which of the following images contains circle model?



(a)



(b)



(c)



(d)



(e)



(f)



(g)



(h)

4. Mention what you know about elements or parts of circle based on books you have read?
5. Take a look at the following bike picture.



(source: pulsks.com)

(a)



(source: dawaihati.com)

(b)

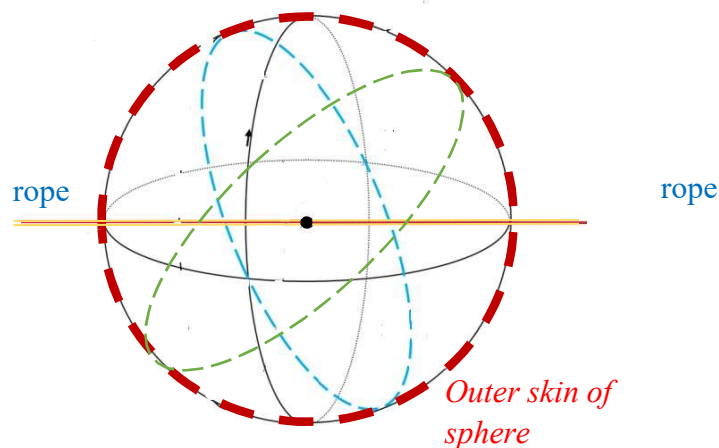
Which of the two bikes do you think you would choose to ride? Give me a reason for choosing this bike!

6. A coin rolled on a flat floor will turnaround. What do you know about the rotation of a circle?
7. An object that is rotated at a certain direction and angle, will definitely change the position of the object. What do you think about its position after the object has been rotated? Try to show it by your own way!
8. Suppose a color circle is rotated with its center point is the center point of the circle connected by a rope as in the following image.



(source: kesekolah.com)

If the rope is moved in various directions on a regular basis, then the circle will rotate and leave a trace of a red sphere skin as in the image below.



What can you infer from this experiment?

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APPENDIX 2.

**AUGMENTED REALITY (AR) INTERACTIVE LEARNING MEDIA
 VALIDATION SHEET BY MEDIA EXPERT**

A. PURPOSE

To measure validity of Augmented Reality Interactive Learning Media of application introduction quality, user control, application view, and principles of multimedia by media experts.

B. INSTRUCTIONS

1. To Messrs./Mmes. please give an assessment by giving a tick (√) in the column that has been provided, in accordance with the following assessment criteria:
 - 1: Not Good
 - 2: Less Good
 - 3: Good Enough
 - 4: Good
 - 5: Very Good
2. To Messrs./Mmes. please give advice for improvement by writing on the comment line and suggestions that have been provided.

No	Assessment Criteria	Score				
		1	2	3	4	5
Introduction of Application						
1	Ease of application title in providing an overview of the application					
2	Clarity of operating guidance and the displayed menu					
3	How attracting the view of the learning media design					
User control						
4	Control sequence accuracy					
5	Consistency of navigation button layout					
6	Smooth use without hang, crash or lag					
7	Use of media on the Android platform flexibility of time and place of usage					
8	Interactive AR learning media					
Application View						
9	Consistency of layout proportions (text and image layout)					
10	Accuracy of background selection					
11	Consistency of colors use					

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12	Consistency of selection of text types and fonts presented					
13	Consistency of text size selection presented					
14	Icons and navigation buttons are easy to understand					
15	Consistency of use of icons as navigation buttons					
16	Suitability of the animation used in the material					
17	Accuracy of presentation of audio replacement writing					
18	Video display quality and length of video duration					
19	Suitability of video use with material					
Principles of Multimedia Design						
20	Presentation of material using more than one medium					
21	Presentation of material using words and images/animations/videos side by side (not separately)					
22	Use of images, writing and animations that are interrelated only (negating unrelated and relevant information)					
23	Presentation of material using media in a non-excessive manner					

Comment and Suggestion:

.....

C. CONCLUSION

In terms of media aspects, Augmented Reality Interactive Learning Media, stated:

1. Worth using
2. Worth using after revision
3. Not worth using

Please circle the choice of numbers that have been provided in accordance with the conclusion as a whole.

....., 2021
 Validator,

.....



**AUGMENTED REALITY (AR) INTERACTIVE LEARNING MEDIA
 VALIDATION SHEET BY MATERIAL EXPERT**

A. PURPOSE

To measure the validity of The Interactive Learning Media of Augmented Reality based on the aspect of introduction, content, and aspect to closure by material experts.

B. INSTRUCTIONS

- To Messrs./Mmes. please give an assessment by giving a tick (√) in the column that has been provided, in accordance with the following assessment criteria:
 - 1: Not Good
 - 2: Less Good
 - 3: Good Enough
 - 4: Good
 - 5: Very Good
- To Messrs./Mmes. please give advice for improvement by writing on the comment line and suggestions that have been provided.

No	Assessment Criteria	Score				
		1	2	3	4	5
Aspect of Introduction						
1	Clarity of learning instructions					
2	Clarity of learning achievement					
3	Clarity of the description of the concept map of the material studied					
4	Clarity of 3D images in introducing the concept of circles and their transformations					
5	Clarity of the appearance of a 3D image of a circle and its transformation in the user's point of view in real time					
6	Learning media contains pattern recognition on objects					
7	The suitability of a 3D image to represent a model of a circle and its transformation					
8	Conformity of material in the learning media with Core Competencies and Basic Competencies					

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9	Conformity of the material with the purpose of learning					
10	Conformity of material content with standard concepts					
Aspect of Content						
11	Traceable contents/material description					
12	Coverage (breadth/depth) of material					
13	Factual material					
14	Actualization of material					
15	Example clarity included to clarify content					
16	Clarity and suitability of the relevance of the language used					
17	Material suitability for students' cognitive characteristics and development					
18	Material contained in the media provides new knowledge and stimulates the mathematical ability of students.					
19	Clarity of instructions for completing a learning task					
20	The attractiveness of the instructions presentation to stimulate students' thinking activities					
21	Accuracy of giving feedback on learning achievements					
22	Conformity of sentence use with the intellectual level of the student					
23	Ease of use of the terms					
24	Consistency usage of text					
25	AR learning media facilitates the introduction of abstract concepts that are difficult to realize in real / direct					
26	AR learning media attracts user attention and motivation					
27	The function of AR learning media is to reinforce students' understanding and spatial abilities					
28	Learning material directs students to do exploration of around objects students					
29	Learning material directs students to do knowledge discovery					
30	Efficiency usage of language					

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31	Suitability the sentence with good and correct language rules					
32	Completeness of information					
33	Detail and completeness of content					
Aspect of Closure						
34	Clarity of interrelationships between concepts as a whole					
35	Clarity of summary as looping material					

Comment and Suggestion:

.....

C. CONCLUSION

In terms of material aspects, Augmented Reality Interactive Learning Media, stated:

1. Worth using
2. Worth using after revision
3. Not worth using

Please circle the choice of numbers that have been provided in accordance with the conclusion as a whole.

....., 2021
 Validator,

.....



APPENDIX 3.

INTERACTIVE LEARNING MEDIA ASSESSMENT SHEET AUGMENTED REALITY (AR) BY STUDENTS

A. PURPOSE

To measure the practicality of Augmented Reality Interactive Learning Media through student response after using it.

B. INSTRUCTIONS

- To students, please provide an assessment by giving a check mark (√) in the column that has been provided, in accordance with the following assessment scale criteria:
 - 1: Not Good
 - 2: Less Good
 - 3: Good Enough
 - 4: Good
 - 5: Very Good
- To the students, please give an impression by writing on the column that has been provided.

No	Assessment Criteria	Score				
		1	2	3	4	5
Ease of Application Usage						
1.	Clarity of application usage instructions					
2.	Symbols and buttons are easy to understand					
3.	Ease of running the application					
Application View						
4.	Color composition does not interfere.					
5.	How interesting the application background color					
6.	Text layout accuracy and background with text color					
7.	Compatibility and harmony of the selection of typefaces, size and spacing of spaces between writings					
8.	Suitability and attractiveness the use of image and video					
9.	Balance between text and video use					
10.	Regularity of the location of components (icons, navigation) of applications					

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Ease of Application to Learn Its Contents						
11.	Clarity of learning achievement and goal					
12.	Attractiveness and the systematic of material presentation					
13.	Use of sentences and grammar to support an understanding of the material					
14.	Text readability in terms of text type and size					
15.	Setting examples to support an understanding of the material					
16.	Use of text, images, and video/animation to support an understanding of the material					

C. IMPRESSION

Yogyakarta,2021
 Student

.....



Impact of adoption of Information and Communication Technologies (ICTs) in Teaching Mathematics to Intellectually Disabled Children

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Abstract: This study examined the effect of information and communication technologies (ICTs) on intellectually disabled children's academic achievement in teaching mathematics. Hundred children with mild to moderate levels of intellectual disability participated in a four-week experiment in two groups. The results showed that (1) the experimental group (taught with the help of ICT based instruction) performed significantly better than the control group (taught with a conventional method); (2) a significant effect of the level of intellectual disability was observed on the academic achievement in mathematics, and (3) while no significant difference was observed on the basis of gender. Results of regression analysis showed that out of three, two variables-treatment and level of intellectual disability added significance to the prediction. The study proposed that the use of ICT-based instruction could prove helpful in teaching mathematical concepts to children with intellectual disability.

Keywords: ICT, mathematics, intellectually disabled children.

INTRODUCTION

Intellectual disability implies the condition of ceased or incomplete development of a person's mind, notably marked by natural intelligence (Persons with Disabilities act, 1995). Individuals with Disabilities Education Act (2004) describes intellectual disability as a considerable decline in average intellectual function that exists simultaneously with adaptive behavior deficits and negatively impacts children's academic performance. Due to societal and political compulsions, the term used to define this condition has changed several times throughout the years. In the late 20th century, the term "mental retardation" has now been replaced in most nations by "Intellectual

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Disability (ID)" as mentioned in the "Diagnostic and Statistical Manual 5th Revision (DSM-V)". The International Classification of Diseases (ICD-11) 11th revision has suggested the name of "Intellectual disability" to "Disorders of Intellectual Development" (DID). It has been classified as a health condition rather than a disability in ICD-11 (Girimaji & Pradeep, 2018).

Intellectual disability has various causes, and its prevalence is impacted by social, economic, ethnic, cultural, and other environment-related variables, including age and gender. The prevalence of intellectual disability has consistently been linked to the low socioeconomic status of people in many research studies (Durkin, Hasan & Hasan, 1998). It is more common in underdeveloped nations due to increased birth trauma, oxygen deprivation, and brain infections during early childhood. As per Nicholas (2003), the major causes of intellectual disability among individuals are genetic, biological, and environmental. Often, genetic and biological factors cause a severe Level of Intellectual disability.

Students with intellectual disability should be provided with a different curriculum, unlike other students with special needs, and therefore, extra efforts are required to be put in as per their cognitive ability (Myreddi & Narayan, NIMH, 1998). Children with intellectual disability must learn functional academic skills to become self-sufficient and employable (Manavalan, *Functional Academics for Students with Mild Intellectually disability*, NIMH, 1998). Teaching mathematics to intellectually disabled pupils, on the other hand, entails learning based on concrete experiences and applying the learned skills in real situations. Furthermore, classroom learning should be blended with community exercises in a way that allows the generalization of learned skills (Myreddi & Narayan, NIMH, 1998). The mathematical curriculum provided to pupils should consider their future learning needs that are likely to be used in their real-life contexts (Polloway, Patton, Epstein & Smith, 1989). Numbers play a crucial part in our daily lives, and the subject of negotiating quantities comes up in our daily conversations. Over the last few decades, India has experienced a paradigm shift in teaching-learning. Children with intellectual disability have become engaged learners due to tailor-made individualized programs.

In the last few decades, technological advancement has leaped enormously. It has woven the world together as globalization and constituted further avenues for various sectors and institutions of society; ICT is one of them. DeSimone & Parmar (2006) underlined the benefits of visual technologies in engaging learners in concrete representation and helping them comprehend abstract concepts without difficulty. Instructional technology features like spell-checking, word predictions, touchscreen technology, navigation, etc., can help reduce the cognitive load required in learning (Ciampa, 2017; Kennedy & Deshler, 2010; Hasselbring & William Glaser, 2000). Virtual learning spaces in e-learning (Skillshare, 2018) help children with disabilities to work on concepts at their own pace through course material (Braddock et al., 2004; Fichten et al., 2009).

Over time, the use of technologies in learning boosts the productivity of children (Ciampa, 2017) by enabling them to become more adaptable, self-reliant, more confident, and self-esteemed (Stock et al., 2004). Children with disabilities are usually inhibited in schools due to fear of failing. However, virtual technologies and e-resources provide them with opportunities to learn from errors (Cromby et al., 1996). E-learning platforms help them keep away from social anxieties and provide an individualized form of instructional materials, enhancing learners' confidence (Bühler & Fisseler, 2007; Fichten et al., 2009). Several technologies can be effectively utilized for the development of intellectually disable individuals (Carey et al., 2005; Sowers et al., 1985; Wehmeyer, 1998). UNESCO (2011) stressed how ICTs could be effectively utilized for the disabled population and their education. Many frameworks have focused on the positive effects of various technologies on the development of children with disabilities in diverse classroom contexts (Bowser & Reed, 1995; Parette, 1997; Parette & Wojcik, 2004; Zabala, 1995). Nonetheless, teachers still feel lack of adequate knowledge and skills to adopt appropriate technological devices (Anderson & Petch-Hogan, 2001; Derer et al., 1996; Edyburn, 2004). Also, educators and students face challenges in accessing technological devices for teaching and learning (Wehmeyer, 1998), which may result in causing frustration and unwillingness among teachers, parents, and students in the integration of technologies into the learning process (Lahm et al. 2001; Lesar, 1998). Children with intellectual disability are considered to have significant difficulty in learning basic

mathematical skills (Noffsinger & Dobbs, 1970). Many research studies have shown that multimedia education boosts pupils' academic achievement in science, mathematics, and literacy (Gee, 2003). However, it has been observed that these facilities are not available at ground level to people with disabilities in all parts of the country.

Teaching mathematical concepts and exercises to children with intellectual disability is a great challenge. The use of various technological devices by intellectually disabled people helps to increase their level of self-sufficiency, self-determination, and the ability to integrate all acquired skills (Wehmeyer, 1998). Bennett et al. (2013) observed the positive effect of a computer-based memory training program on Down Syndrome-affected children in the United States. Intellectually disabled children are considered to have significant difficulty learning basic mathematical skills (Noffsinger & Dobbs, 1970). Advocates of computer-based instructional programs say that this method of instruction has many benefits for exceptional children (Schmidt, Weinstein, Niemic & Walberg, 1986). Lin et al. (1994) studied the effect of CAI on mathematical abilities among children with and without intellectual disability and found an increased response rate using technologies. The use of a computer-based program for teaching addition to intellectually disabled children was found to be more beneficial than traditional methods (Leung, 1994). Computer-aided software program assists in improving the reading abilities of children with disabilities and shows the positive attitude of children towards the program (Kim et al., 2006). Also, computerized programs help in developing mathematical skills of addition and subtraction among children with mental disabilities, including children with intellectual disability (Al Rassees, 2003; Anitha, 2005; Podell, Tournaki-Rein & Lin, 1992; Kumar, 2012; Leung, 1994; Mary & Premila, 2019; Pang, 2005; Patra & Rath, 2000; Raouf, Alenizi & Attiya, 2016; Sharma, 2004). The efficacy of CAI and behavioral approaches for teaching mathematics to intellectually disabled children have been validated by these findings. Tudela and Ariza (2006) carried out research showing a significant positive effect of using computer-assisted instruction with multimedia-based programs in improving children's mathematical skills with Down syndrome. Campbell et al. (2008) have shown the effectiveness of smartboard technology for children with learning difficulties in letter-sound

training. These children can benefit from computers in learning mathematics (Küçükalkan, Beyazsaçlı, and Öz (2019). Haugland (1992) found that computer-assisted instruction is useful in the medicinal program on dialect improvement for those youngsters who have cognitive delayed development or are 'at risk' for school disappointment. Mioduser et al. (2000) investigated the role of computer-based instruction in improving phonological awareness, word and letter recognition among students with learning disabilities, compared to conventional instruction through printed resources and formal reading practices.

Further, Moore & Calvert (2000) reconnoitered the usefulness of computer-based technologies to enhance the language skills and vocabulary abilities of children with autism. Technology usage also helps children with intellectual disability develop self-help skills (Rai, 2008; Turan, Yilma, Sakalar & Ucan (2016) worked on designing a digital-based education model for the development of auditory and visual perception in educable children with intellectual disability. Learning will occur more effectively and permanently when auditory and visual elements are implemented together. Kumar (2012) recommended integrating CAI-based programs for teaching children with special needs. It was also suggested that CAI could prove to be an effective tool for special educators.

Further, Dandashi et al. (2015) proved the positive effect of the edutainment gaming program on the cognitive and motivational levels, specifically when students were more physically active in their classrooms. Technological games tend to simulate the process of learning but are incapable of replacing the natural mode of teaching for children with disabilities (Kwon, 2012). However, serious games can also teach basic job skills to individuals with developmental disabilities (Kwon & Lee, 2016). Agarwal & Yash (2012) supported the usage of computer games in the rehabilitation of students with intellectual disability. Davies & Wehmeyer (2004) conducted a pilot test on an internet-based multimedia assessment system that uses audio, video, and image assistance to help people with intellectual disability. Stock, Parette, and Wojcik (2004) constructed an assistive technology-based toolkit for students with intellectual disability that would benefit the professionals working in special education. Also, various studies have been conducted to propose

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ways of utilizing augmented reality technology for future development (Karamanoli, Tsinakos & Karagiannidis, 2017).

Moreover, some studies are conducted in the area of technology use in special education. However, these studies are either at trial or merely about awareness. Investigators found the dire need for more specific and scientific research to be conducted in using technology in special education to make it work in real-life situations rather than merely on a small scale of research. Studies could be done to acquire and assess the efficiency of using ICT-based instructional packages for various target groups. With a subsequent review of related literature, the researcher felt that very few empirical studies had been conducted on the present theme in India. Therefore, comprehensive research is crucial to studying information and communication technology's effect on children with intellectual disability in teaching basic mathematical functions. After reviewing all the previous research studies done on similar areas, the investigators gained assistance in developing a program based on information and communication technology and recognizing the skills that would be established. The investigators observed that the ICT-based programs utilized in previous research focused on developing mathematical skills and limited arithmetic functions among students with mental or intellectual disabilities, as in the research conducted by Dyab (2001) and Al Kashef (2002). At the same time, the present study attempts to find the effect of information and communication technology on children with intellectual disability in developing mathematical abilities related to concepts of numbers, addition, subtraction, and time and money.

The investigator realized that there is a lack of Indian studies that worked on Children with intellectual disability and mathematical abilities, which is believed to be a novel additive. Also, the sample size taken in previous studies is relatively small, making it difficult to generalize the results. This study has been planned to fill the gap by examining the effect of ICT-based instruction on teaching mathematical concepts to intellectually disabled (MR) children. This study has built a link and thrived in augmenting the gaps left by previous research and can shape the way special and mainstream educators teach mathematical concepts to intellectually disabled children. In other programs that serve intellectually disabled children, employing ICTs can be repeated. Recent

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advancements in technology have created an ideal atmosphere for creating individual training programs. After reviewing all the previous research done on similar areas, the investigators conceptualized the present study to investigate the effect of ICT on Children with intellectual disability in developing mathematical abilities related to concepts of numbers, addition, subtraction, and the concepts of time and money. The study's primary objectives were to develop an ICT package for teaching Mathematics to intellectually disabled children and compare the effectiveness of ICT-based teaching with that of conventional teaching in terms of academic achievement of intellectually disabled children in Mathematics. The study also aimed to investigate the main effects and interaction effects of treatment, gender, and Level of Intellectual disability on the academic achievement of Children with intellectual disability in mathematics. This study seeks answers to the following research questions: (1) What is the effect of ICT-based instruction on academic achievement in mathematics in the case of MR children? (2) Do MR children differ in their academic achievement in mathematics based on gender? (3) What is the effect of the Level of Intellectual disability on the performance of MR children?

METHOD

However, true experimental research is almost impossible in social sciences, as a result, the current study used a quasi-experimental two-group pre and post-test design. The sample was divided into experimental and control groups (details are given in Table 2). The experimental group was taught mathematics with the help of ICT, while the control group was taught using the conventional method. For the present study, gender, Level of Intellectual disability, and "treatments" (ICT-based teaching and conventional teaching) provided to children with intellectual disability were taken as independent variables, while academic achievement in mathematics was considered as a dependent variable. Academic achievement scores were assessed twice during the process of research. Firstly, before the start of experimental treatment, i.e., at the pretest stage, and then after the end, i.e., at the post-test stage.

Participants

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The present study population selected children with intellectual disability from Haryana state. A multistage random sampling technique was applied to select a sample of 100 (72 male, 28 female) children having mild or moderate levels of intellectual disability from special schools in Gurgaon and Rewari district (refer to Table 1 for more details), and subjects were assigned into one of the two groups randomly. The age group of the subject ranges between 10-22 years. Before approaching the participants, consent from the parents and schools included in the study was obtained and students were informed about the purpose of the study, and their willingness to participate in the study was sought. Investigators tried to build rapport with the students before starting the experiment, and it was the most challenging task during the study as these students took time to connect with a new teacher. Moreover, it was not easy to maintain their attention during the experiment as they did not follow instructions easily.

		Frequencies (f)	Percentage (%)
Gender	Male	72	72%
	Female	28	28%
Level of Retardation	Mild	29	29%
	Moderate	71	71%
Group	Experimental	50	50%
	Control	50	50%

Table 1: Detail of Sample

Investigators applied some controls in the study to minimize the errors during the experiment viz: (1) only students studying in special schools were considered for the study (Selected schools were Khushboo Welfare Society, Navprerna Shikshan Kendra, and Manav Vikas SewaSamiti); (2) only mild and moderate level of children with intellectual disability were selected; investigator herself taught (3) children of both the group; (4) same Mathematics concepts were taught to both the groups for four weeks; (5) To nullify the effect of non-equivalence of groups, ANCOVA statistical technique was employed as perfect dichotomization and randomization were inconceivable for the present study (Winer, Brown and Michels,1991).

Experimental group (ICT based Teaching)		Control Group (Conventional Teaching)	
Male	Female	Male	Female
35	15	35	15
Level of Intellectual disability			
Mild	Moderate	Mild	Moderate
12	38	17	33

Table 2 Sample in Experimental and Control Group

Tools Used

An academic achievement test was constructed and standardized on a sample of 50 intellectually disabled children. This test was reviewed by three experts working in special education for more than five years. Item analysis of the test was completed by determining each item's difficulty value and discriminating power. Also, the test's reliability was calculated using the split-half method and was found to be 0.73. The investigator prepared an ICT package for teaching mathematics for intellectually disabled children, focusing on concepts of mathematics related to numbers, time, and money. This test was used to obtain pretest and post-test academic achievement scores in mathematics.

Development of an ICT-based instructional package

The researchers followed the following steps to develop an ICT-based instructional package in mathematics for children with intellectual disability.

1. **Content Selection:** The investigator referred to "Behavioral Assessment Scales for Indian Children with Mental retardation (BASIC-MR)" prepared by R. Peshawariya and S. Venkatesan (first edition, 1992 and reprint 2000). This scale is designed to collect information about all the functional skills of persons with intellectual disability, which can be used for individualized program planning. These functional skills include motor skills, activities of daily living, language, reading-writing, numbers-time, domestic-social, prevocational-money skills. The investigators also visited schools of intellectually disabled children and, after consultation with the teachers,

selected content related to essential mathematical functions of numbers, shapes, addition, and subtraction, and time and money concepts; ICT-based packages were prepared.

2. ***Defining the Entry Behavior of the Target Group*** According to Russell (1974), the entry behavior of learners refers to the precondition set of knowledge, skills, or attitudes that are acquired for the new learning, which also incorporates their previous learning and experiences. The primary objective of preparing such modules is to observe learners' progress from their entry behavior to terminal behavior (mastery over learning/instructional objectives). In the present study, the entry behavior of the children was observed with the help of their special educators. The latter assisted in determining students' current learning level of mathematics concepts, which further helped the investigator select the related concepts of mathematics.
3. ***Task Analysis:*** According to Alberto and Troutman (2003), task analysis can be defined as the process of splitting up a complex task into easier and smaller steps. These many smaller steps or components are further divided into phases that can be used for teaching purposes. Children with intellectual disability may not learn the concepts in the same way as children without disabilities. Therefore, the investigator found it essential to split up the whole content into smaller sections according to the learning needs of the learners. Mathematical numbers, shapes, time, and money-related concepts were further split into smaller sections. Details are given in Table 3.
4. ***Scriptwriting and Storyboard:*** As per the views of Kumar, 2006, the script can be defined as a comprehensive plan of action and events envisioned in a teaching program. It also includes a list of all activities in all visuals and words spoken in a storyboard format. Writing a script is the beginning stage of developing any ICT-based multimedia package. The script was written while keeping in mind the needs of all learners, and all pertinent information related to content was included in the script. The entire content was divided into smaller chunks or sub-topics. Afterward, the opinion of the subject experts was sought for modification or rewording of the script.
5. ***Story Board:*** According to Varvel and Lindeman, 2005, "Storyboards are a means to graphically represent layout, organization, content and linkages of information within a multimedia to create a conceptual idea of the information location, meaning and appearance." Thus, a storyboard, a general outline in the form of a working document, was prepared, highlighting major headings and

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captions and showcasing all the critical points of the necessary information. A sample storyboard on the topic 'Numbers 1-10' is shown in Table 3.

6. **Collection and Development of Resources:** After preparing the storyboard, appropriate resources pertinent to the selected content were placed together before starting the program development. In the present study, the investigator recorded the videos herself using a platform like Screencast-o-Matic, an Active presenter. Open resource videos were also obtained from YouTube and Teacher Tube sites. Images were also downloaded from open resources.
7. **Sequencing and Integrating:** Proper sequencing of the content with the integration of various multimedia components like images, text, graphics, audio, video clips, etc., in a storyboard, helps prepare an effective ICT-based program. Thus, the researcher collected various e-resources from the web and self-prepared components, arranged sequentially and integrated after discussion with experts in the related field.
8. **Editing:** The process of revising and reorganizing the content to develop modified new work is called editing. Here, the researcher, at this step, attempted to edit the unwanted content or footage of the video, add effects, narration, graphics, music, audio, using user-friendly and readily available software such as MS PowerPoint with updated features, Audacity, Filmora, Active presenter etc. In general, the same concept is required to teach intellectually disabled children several times over and with this instructional program based on ICT, this can certainly be possible. Multiple media such as text, images, audio, video, and animations can make it more interesting.
9. **Evaluation:** After completing the ICT-based instructional package editing, the investigator sent it to subject experts to evaluate and provide feedback on its content and technology used. The program was evaluated along with its content accuracy, ease of use, and appeal to learners. Based on the feedback, the required modifications were made. This step helped the investigators to avoid ambiguity, insignificances, and other shortfalls in the program.

Content	Visuals	Effect
Counting package from 1 to 10	Researcher's introductory video	Video



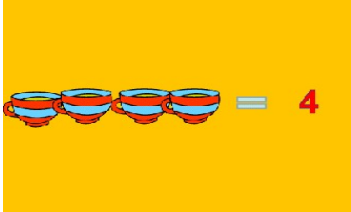
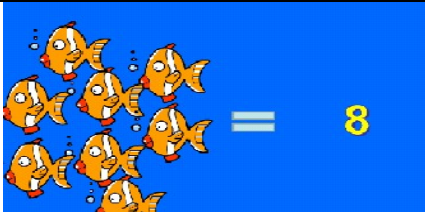
		
<p>Stage 1: Minor stage Concept of a number is introduced</p>	1	Ppt, pictures
<p>Numerals 1 to 10 are introduced this way</p>		Pictures, animations
<p>Stage 2: Major Stage Identification of taught numeral (Straight line) Learner has to match the particular numeral with number of objects in pictures in straight line</p>		Pictures, animations
<p>Cluster Counting Learner has to match the particular numeral with same no. of objects presented in cluster</p>		Pictures, animations
<p>Recapitulations with different objects and pictures</p>		Pictures, Video

Table 3: Sample Storyboard for Teaching Numbers from 1 to 10

Procedure

The experiment was conducted for four weeks in each school. Following the formation of the control and experimental groups, the investigator established a positive rapport with the children

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and provided them with information about the study's tools and significance. Before conducting the pretest, necessary instructions related to the test were given to the children of both groups. Then, lastly, the pretest of standardized achievement test in mathematics was administered to both the groups of children with intellectual disability as per the plan. Once the pretest was administered at stage I, treatment was presented to both groups. The experimental group was taught concepts of mathematics through an ICT-based multimedia package prepared by the investigators herself. Simultaneously, the control group was taught the same mathematics concepts using conventional teaching methods like lecture and demonstration. The program was conducted for four weeks. The following concepts of mathematics were covered.

Phases	Experimental Groups	Control Groups
Pretest Stage	Administration of pretest of achievement test in Mathematics prepared for intellectually disabled children.	Administration of achievement test in Mathematics prepared for intellectually disabled children.
Experimental Stage	Teaching mathematical concepts through ICT based instructional package	Teaching mathematical concepts through conventional method of teaching
Posttest Stage	After completion of experimental stage, administration of post -test achievement test in Mathematics prepared for intellectually disabled children.	After completion of experimental stage, administration of post -test achievement test in Mathematics prepared for intellectually disabled children.

Table 4: Stages of Experiment

The program was conducted for four weeks, following concepts of mathematics were covered. ICT-based Instructional program included demonstrations and practice exercises on the related concepts. For example, in 'Single Digit Addition', first students were shown the demonstrations with the help of a small story using images, animations. Then, the concept of addition was

introduced with numerals on the screen. The first addend was displayed first, followed by the '+' sign, and then the second addend. Then, a horizontal line was drawn underneath the '+' and the second addend. Finally, the answer is presented with the addends, blinked for 2 seconds to highlight. The children were asked to read the entire procedure aloud. For example, the sum '3 + 2 = 5' can be interpreted as 'three plus two equals five.' Similarly, other examples were used for their better understanding.

The researcher noticed the children's interactions throughout teaching sessions by observing them in mathematics class. The ICT-based instructional program resulted in positive interactions between experimental group children. Children were more joyful, interactive, and focused during the teaching-learning process.

During the intervention programme, children from both the groups were given worksheets on the related concepts of mathematics which were taught in the classroom, and parents were given instructions on how to assist their wards in completing their homework tasks. Parents of intellectually disabled children were also asked about their opinion on the use of ICTs in the classroom. Most of them supported the idea that ICTs make the instruction process more appealing due to the use of images, audio, ease of use, and multisensory approach of the medium. As one of the responses says *“My nine years old son easily spend 50 minutes daily on completing mathematics worksheets as he says, ‘I want to learn mathematics’. That's quite satisfactory for me because he's rapidly expanding his mathematical understanding and interest”*. Another excerpt from the parent, *“I believe that children from today's generation are born into a digital world of images and animations. As a result, use of computers and technology piques their interest, and they are able to watch more clearly”*. With the use of ICTs, children don't just memorize mathematical facts, instead, they obtain a conceptual comprehension of mathematical abilities with the help of drills and exercises. Hence, the use of technologies has the potential to enhance intellectually disabled children's classroom mathematical experiences, but teachers and parents must carefully plan the integration of technological resources and implement them into learning tasks.

Number-related concepts	Shapes	Time-related	Money-related
Numbers from 1 to 20	Different types of shapes	Concept about days of the week, months in a year	Concept of Money with illustrations

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Single-digit addition and subtraction	Concept of Big-Small, Tall-Short, More-Less	Concept of time: Hour hand and minute hand	Familiarity with coins and notes of Indian currency
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Table 5 Concepts of mathematics covered during the treatment stage

The instructional program was completed for both groups, i.e., the experimental and control group. Their performances were measured to find the effectiveness of ICT-based teaching treatment and conventional teaching methods on teaching mathematics to intellectually disabled children. Post-test was administered on both groups using achievement test constructed and standardized by the investigators.

Results and Discussion

After measuring the mean and standard deviation of the achievement in Mathematics in the pretest, the experimental group was given the treatment of ICT-based teaching. The mean scores of academic achievements in Mathematics of the experimental and control group after the intervention treatment (Post-test) were 15.78 and 14.58, respectively. The estimated t-value is 2.027, which is significant at 0.05 level of significance. It shows that children taught through the ICT-based approach performed better than children who were taught using the conventional method. This result agrees with the findings of Kumar (2017), who revealed the significant positive effect of computer-assisted instruction programs on the development of motor, academic, and communication level of intellectually disabled children.

Additionally, Singh & Aggarwal (2013) concluded that computer games helped in better gain in Mathematics concepts among intellectually disabled children. Similar results have been shown by Parette, 1997; Silver & Oakes, 2001; Forgen & Weber, 2002; Weber, Forgan, Schoon & Singler, 1999). Mary & Premila (2019) found out the more significant effect of computer-assisted instruction than teacher-centric instruction on developing mathematical skills among primary level students with mild intellectual disabilities.

	N	Mean	SD	t-value	df	p	Effect size
Control Pre-test Score	50	8.44	2.60	0.937	49	0.353	0.133
Experimental Pretest Score	50	8.00	2.52				
Pre-test Score	100	8.22	2.56	30.341	99		

Post-test Score	100	15.20	3.03			< .001	3.034
Experimental Post-test Score	50	15.78	2.78	24.827	49	< .001	3.511
Experimental Pretest Score	50	8.00	2.52				
Control Post-test Score	50	14.62	3.18				
Control Pre-test Score	50	8.44	2.60	20.653	49	< .001	2.921
Experimental Post-test Score	50	15.78	2.78			0.05	0.278
Control Post-test Score	50	14.62	3.18	1.969	49		

Table 6: Effect of ICT on academic achievement of intellectually disabled children in mathematics

As per the objectives, the main effect of treatment on the academic achievement of intellectually disabled children in mathematics was verified. To nullify the effect of non-equivalence of groups, ANCOVA as a statistical technique was employed. The pretest and post-test scores were analyzed for both the groups, and pretest scores were considered co-variate. It can be observed that the main effect of treatment is significant ($F=4.898$, $p<0.05$) at 0.05 level of significance. This means that the group of intellectually disabled children taught using ICT has shown greater improvement in academic achievement in mathematics than those taught using traditional methods. This finding agrees with Zarei & Gharibi (2012), who found that the learning and retention of fourth grade educable intellectually disable girl students were significantly higher in the experimental group (who were given training through multimedia software) compared to the control group students.

Tests of Between-Subjects Effects (Dependent Variable: Post-test Score)					
Source	Type III Sum of Squares	Df	Mean Square	F	Sig.
Pre-test Score	25.783	1	25.783	6.054	.016
Gender	7.887	1	7.887	1.852	.177
Treatment	20.859	1	20.859	4.898	.029*
Leve of MR	28.175	1	28.175	6.616	.012*
Gender * Treatment	2.343	1	2.343	.550	.460
Gender * Level of MR	2.755	1	2.755	.647	.423

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Treatment * Level of MR	2.811	1	2.811	.660	.419
Gender * Treatment* Level of MR	.990	1	.990	.232	.631
Error	387.537	91	4.259		
Total	24012.000	100			

*Significant at 0.05 level

Table 7: ANCOVA for Academic Achievement in Mathematics

From Table 7, it can be observed that the main effect of gender is found to be insignificant ($F=1.852$, $p > 0.05$). This implied that male and female intellectually disable students' academic achievement in mathematics did not differ significantly. It concludes that male and female intellectually disabled children performed equally well in mathematics. The findings are in congruence with the observations of Singh & Aggarwal (2013), who reported that gender does not affect significantly the acquisition of mathematical concepts of time and number-related skills by intellectually disabled children using computer games. However, males benefitted more than females on one concept of money-related skills.

From Table 7, it can be observed that the main effect of the Level of Intellectual disability is found to be significant ($F= 6.616$, $p<0.05$) at 0.05 level of significance. Also, Table 10 shows the adjusted means of academic achievement of mild and moderately intellectually disabled children in mathematics to elucidate the direction of the difference, which are 18.379 and 13.901, respectively. The result shows that children with mild levels of intellectually disability ($M=18.379$) performed better than moderately intellectually disabled children ($M=13.901$) in terms of academic achievement in mathematics.

One of the research purposes was to study the interaction effect of treatment and gender on the academic achievement of intellectually disabled children in mathematics. It is evident from Table 7 that the F value (0.550 , $p > 0.05$) for the interaction effect of treatment and gender on academic achievement in mathematics is found to be insignificant. This result agrees with Aladwan (2013) study, who found no statistically significant interaction effect between the technology-based

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program and gender on the mathematical skills of addition and subtraction. Similarly, Singh & Aggarwal (2013) found no interaction between treatment and gender in time-related mathematical skills.

Multiple regression was run to predict children's academic achievement from gender, treatment, and Level of Intellectual disability. From Table 8, the R-value (0.73) predicts the level of prediction, and adjusted R square (0.5236) shows that gender, treatment, and Level of Intellectual disability explain 52.36% of the variability in the academic achievement of children in mathematics. The model statistically significantly predicted academic achievement $F(37.27)$, $p < 0.05$, R Square (.538). Out of three variables, only two, treatment ($p = .000$) and Level of Intellectual disability ($p = 0.000$) contributed significantly to predicting academic achievement scores. However, gender ($p = 0.1281$) is not significant, and therefore no substantial contribution of gender was observed in explaining the academic achievement of intellectually disabled children when the variables treatment and Level of Intellectual disability are present in the model.

Regression Statistics				
Multiple R	0.733534136			
R Square	0.538072329			
Adjusted R Square	0.523637089			
Standard Error	2.090231061			
Observations	100			
ANOVA	df	SS	MS	F
Regression	3	488.5696746	162.8565582	37.27491467
Residual	96	419.4303254	4.369065889	
Total	99	908		
	Coefficients	Standard Error	t Stat	P-value
Intercept	26.46672478	1.213580619	21.80878993	0.0000
Gender	-0.718669445	0.468334634	-1.534521244	0.12819069
Treatment	-1.649161172	0.420843157	-3.918707349	*0.000
Level of MR	-4.604143939	0.465794965	-9.884486276	* 0.000

Table 8: Model Summary, ANOVA and Coefficients

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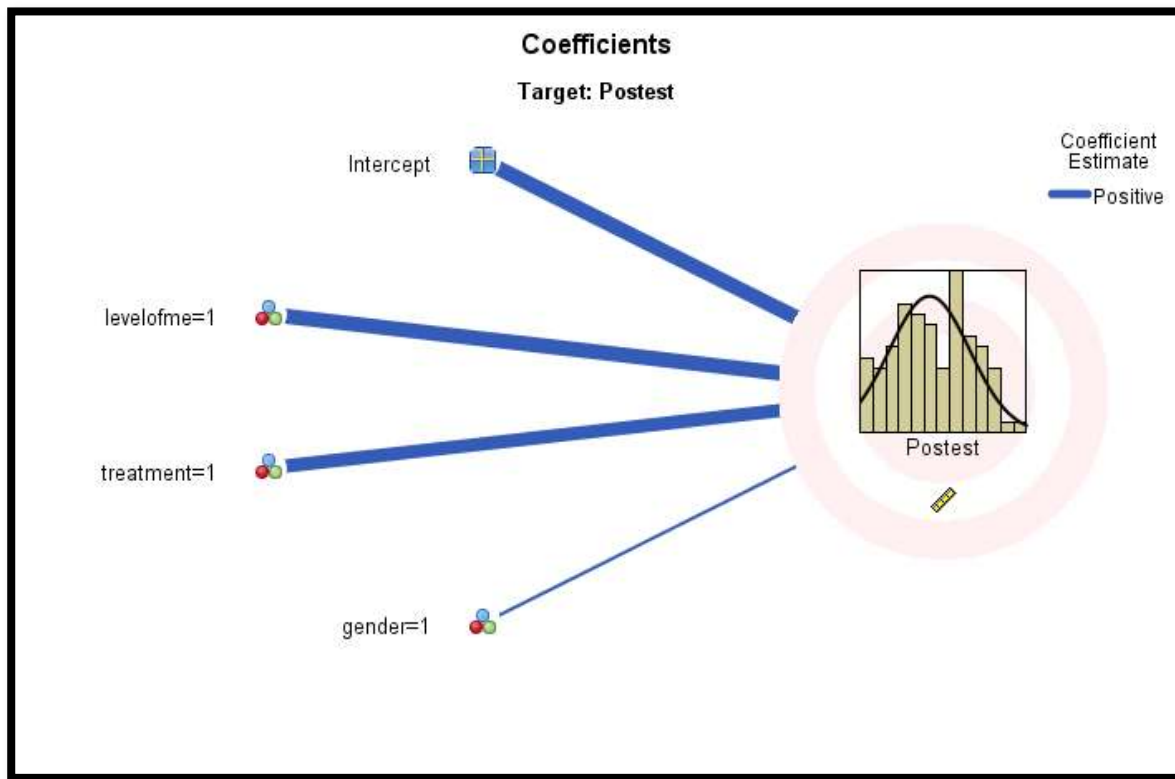


Figure. 1 Graph Showing Contribution of Treatment, Gender, and Level of Intellectual disability on the academic achievement.

Conclusions

Based on the findings of the analysis and interpretation of the obtained data, it can be inferred that the post-test score of the experimental group is significantly different from that of the control group in terms of the academic achievement of intellectually disabled children in mathematics. It follows that children with mild and moderate intellectual disability who were taught using ICTs showed better performance in mathematics than those taught using the conventional method of teaching. The role of gender was found insignificant in terms of the academic achievement of intellectually disabled children, whereas children with mild levels of intellectual disability performed better in mathematics after treatment than moderately intellectually disabled children. Also, after regression analysis, it was found that no substantial contribution of gender is present in explaining the

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academic achievement of intellectually disabled children when the variables treatment and Level of Intellectual disability are present in the model. Based on the outcomes of the present study and previous research, it can be stated that the use of information and communication technologies significantly improves the academic achievement of Children with intellectual disability at mild and moderate levels in mathematics. There may be several reasons for the dominance of ICT-based teaching over traditional methods, such as that ICTs provide various platforms for children to rehearse the same content repeatedly without wasting any content with the help of joyful drills and practice exercises. Also, graphics, animation, immediate feedback, and different multimedia lead to better learning of mathematical concepts through ICTs. Another factor contributing to its favorable impact might be the children's enthusiasm and focus during the intervention program. Support provided by the teachers and parents to the students for engaging in the training program may also be crucial in determining the positive and beneficial effects of the ICT-based intervention program. Children with intellectual disability can benefit from information and communication technologies (ICTs) by overcoming the barriers to autonomy and inclusion. Support provided by the teachers and parents to the students for engaging in the training program may also be a crucial element in determining the positive and beneficial effects of the ICT-based intervention program. Usage of technology should be offered to intellectually disabled children as early as possible as it can help compensate for their functional limitations. Technology accessibility, usage, and how the user is instructed to operate the devices should be consistent.

Educational Implications of the Study

This research analysis was intended to determine the effects and advantages of ICT integration into the educational process for students with intellectual disability in mathematics. The present conclusive research analysis would have immense ripple effects on students with intellectual disability welfare. That will be visible through educational facilities for children with disabilities. Research has highlighted the advantages of successful consolidation of ICT into the educational process for children with mental disabilities and emphasized how ICT integration can supplement the overall efficacy of all the stakeholders involved, be it the teachers, students, and their parents, towards the augmentation of the settlement and rehabilitation of intellectually disabled children

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and other disabled people in society. This research analysis has reflected how imperative it is to incorporate specialized assistive programs into the daily classroom, especially for students with special needs. It will be a guiding principle for institutions and teachers on implementing technology-assisted programs into the educational process for children with disabilities.

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Error Analysis of Dyslexic Student's Solution on Fraction Operation Tasks

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Abstract: *How dyslexia students solve number operations is still challenging to unravel. This study aimed at revealing the types of errors conveyed by a dyslexic student in performing fractional operations on mathematical tasks that combined non-verbal text (symbols and pictures) and verbal text. The data were collected using a task-based interview with a 13-year-old dyslexic student that was recorded and focused on the types of errors on fractional operations (addition, subtraction, multiplication, and division). Results depicted that the student overgeneralized whole number operations when adding two fractions with different denominators although she successfully converted the area model of fraction into a correct fraction operation, identified a common denominator but failed to change the fractions into equivalent form when subtracting two mixed fractions, failed to interpret a multiplication word problem into subtraction operation, and applied only part of the "invert and multiply" algorithm on a word problem. The assumption of the phonological disturbances that was found in the student participant's performance was not found consistently in all the given word problem.*

INTRODUCTION

There is a shred of emerging evidence that dyslexia is linked to mathematics difficulties. Although dyslexia is often understood as a reading and writing disorder, studies have reported that dyslexic children and adults are slower and less accurate in remembering arithmetic facts than those non-dyslexic children and adults (Simmons & Singleton, 2006). This is due to the fact that dyslexic children's phonological processing deficits have an adverse effect on the development of arithmetic fact memory (Simmons & Singleton, 2008). Similarly, Simmons and Singleton (2008) report that the main difficulty of dyslexic children is the ability to remember number facts so that they are slow in calculating or verifying sums of numbers. In this case, the memory footprint for an

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arithmetic question may deteriorate before the answer is calculated. In addition, slow computations can exacerbate this problem as it increases the required times to store the problem in working memory while the answer is computed. In particular, Cornoldi et al (2021) show that a dyslexic student not only has difficulty in reading and writing in terms of alphabetic material, but also numerical material such as symbols.

Nevertheless, some studies indicate insignificant correlation between students' mathematical performance and the symptoms of dyslexia. For example, Simmons' (2002) study showed that there was a statistically significant relationship between non-verbal reasoning ability and place value understanding, but there was no significant relationship between phonological circle function and place value understanding in children aged 7 to 11 years. This finding motivated some researchers to investigate further whether dyslexic weaknesses in processing numbers in mathematical tasks were mainly related to language processing weaknesses (e.g., problems with number facts and exact calculations) or weaknesses in performing mathematical processes, such as comparing a quantity and estimating the results of calculations. This simple question remains a source of controversy. Simmons and Singleton (2009) found that dyslexic children have slower and less accurate memory of numerical facts than those non-dyslexic children, but it has an undisturbed understanding of place value. In addition, Simmon and Singleton (2008) concluded that the existence of the dyslexic group's arithmetical weakness could not be attributed to their dyslexic difficulties or due to their weaker intellectual abilities. More specifically, they added that the aspects of mathematics that are less dependent on verbal codes (e.g., estimation, subitizing) are not impaired. This is reinforced by the findings of Träff and Passolunghi (2015) that dyslexic students performed worse than students in the control group on number fact-taking, multi-step arithmetic problem solving, and multi-digit computation. Their scored arithmetical approximations and conceptual understanding such as place value and principles in count operations did not differ from those in the control group.

There are several research on investigating dyslexic students' number processing skills, however, it is still less and underreported. Place value understanding becomes the main factor affecting students' success in giving solutions on number processing tasks. In relation to dyslexic students' performance on place value, there is evidence that dyslexic students are less accurate and slower in multiplying two single-digit numbers in non-verbal tasks (Boets & De Smedt, 2010). Another finding with the non-verbal task is reported by Koerte et al (2016) that there is no significant difference between the group of dyslexic and non-dyslexic students regarding their performance on nonverbal number line tasks, which is still linked to place value understanding.

While researchers have focused on the number processing skill on whole or natural numbers (e.g. Träff, Desoete, & Passolunghi, 2017; Teixeira & Moura, 2019), research on how dyslexic students performed place value understanding on fraction-related tasks is not reported yet, whereas place

value is important as a basis for understanding fraction operations. Place value understanding is important since it can be used to figure out that the numerator and denominator of a fraction were not made up of different groups of place value. Therefore, research on number processing skills that specifically discuss dyslexic students' performance on fraction operation needs to be further studied.

In performing a solution on any fraction-related task, a solver, including a dyslexic student, needs to be aware of the existence of errors when providing the solution. According to Siyepu (2013), an error is an incorrect answer due to planning, where this error is done systematically because it is applied regularly in the same situation as a symptom of the conceptual structure that causes the error. It may be found from students' previous learning, either in mathematics class or from their interactions with the social and physical world (Smith et al., 1993). More specifically, errors in fraction operation have been identified by Brown and Quinn (2006) and organized into six main categories, namely algorithmic applications, applications of basic concept on fraction operation in a word problem, elementary algebraic concept, specific arithmetic skills for algebraic understanding, comprehension of the structure of rational number, and computational fluency. The first two categories become crucial aspects who learn fraction in primary school, need to be proficient as they are frequently found in the students' solution strategies. Regarding algorithmic application as the basic skill on solving fraction operation task, Ashlock (2006) also identified four types of errors, namely incorrectly writing a fraction representing a shaded area of a figure, failing to simplify fraction into the simplest form, incorrectly dealing with numerator and denominator of a fraction when adding or subtraction two fractions. Hwang and Riccomini (2021) also identified the most common errors in students' solutions to the fraction operation task, namely failing to decompose mixed numbers into integers and fractional parts or converting mixed numbers into ordinary fractions when performing addition operations.

This study tried to unpack dyslexic student errors on fractions through a fraction operation task covering both verbal and non-verbal information. Thus, the aim of this study is to analyze the errors of dyslexic student in performing solution strategies on fraction operation tasks covering addition, subtraction, multiplication, or division.

RESEARCH METHOD

Research Design

The present study used a case study research design, which was used to investigate contemporary phenomena in-depth and in the context of the real world (Yin, 2014, p. 237). It was to answer the "how" and "why" questions (Yin, 2014, p. 2), which were relevant to the present study. This study aimed to uncover how a dyslexic student solved problems related to fraction operations by focusing

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on investigating the types of errors that might occur and investigating why they occurred in a Forum Group Discussion activity with students' parents, mathematics education experts, and outside education experts, ordinary students, and teacher of the participating students. In addition, the researchers had little or no control over the events that occurred during the interview (Yin, 2014).

The student participant, i.e., the dyslexic student, was recruited by means of a letter of consent that the children gave to their parents. At the interview times, she was 13 years old and had normal or corrected-to-normal visual acuity with no hearing loss.

A task-based interview was prepared by writing a semi-structured interview guideline and a set of fraction operation tasks. The set of fraction operation tasks was designed and developed by focusing on the combination of text types, i.e., verbal and non-verbal for every task. A group discussion consisting of ten teachers, the researchers, and an expert in mathematics education was involved in a forum group discussion to review the initial draft of the task. The fraction operation tasks were designed and developed by focusing its feature on four basic fraction operations: addition, subtraction, multiplication, and division. Some of them were in the non-verbal text (symbolic and figural) or verbal text only, while others combined non-verbal and verbal text.

Fraction operation	Types of text
Addition	Verbal and non-verbal (figural & symbolic)
Subtraction	Non-verbal (symbolic)
Multiplication	Verbal
Division	Verbal and non-verbal (figural)

Table 1: Feature of task

Table 1 indicates the distribution of tasks regarding the types of text resulted from the revision of the initial draft after the review process. Figure 1 depicts an example of the task of fraction addition and subtraction. The addition task asked student to represent two fraction models as two different fractions and further added those two fractions, while the subtraction task asked the student to subtract two mixed number with different denominators.

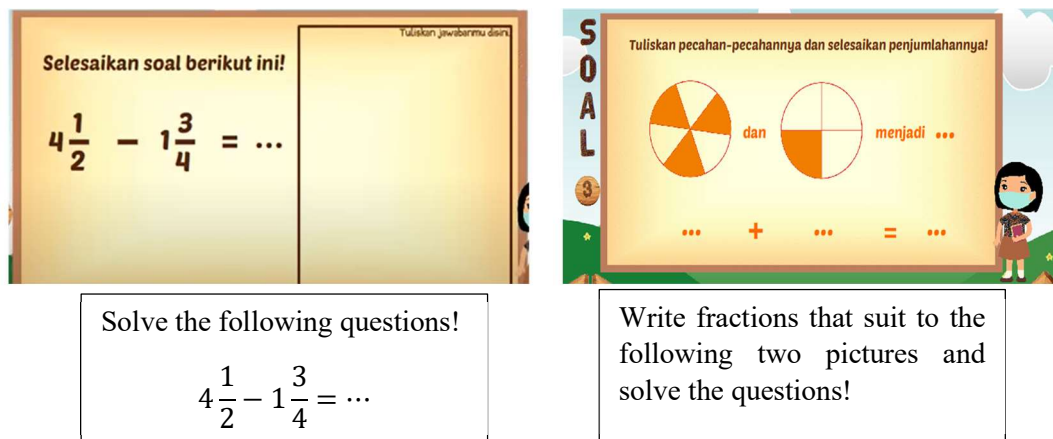


Figure 1: Example of addition and subtraction task

The interview activity was conducted through an online platform synchronously and had been recorded for about an hour. During the interview, the student participant explained how to answer the given question with the guidance of the interviewer. Through the interview, the interviewer got the student participant's thinking process in the fraction operation task.

Data Analysis

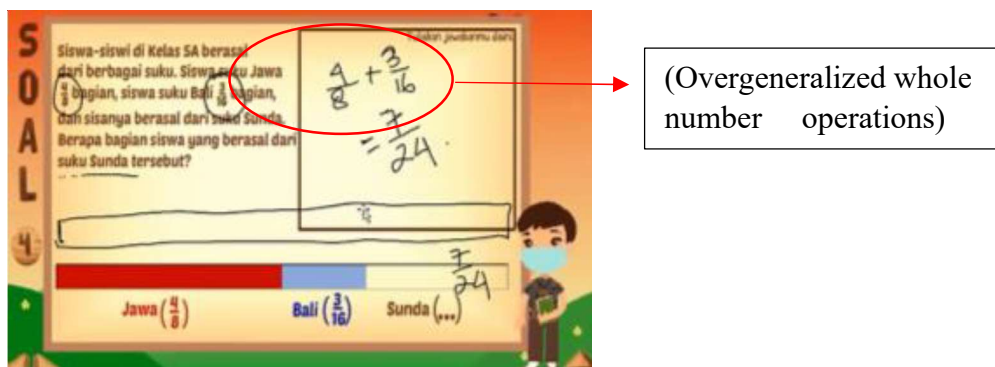
Data analysis was conducted through a Focus Group Discussion (FGD) of several experts in East Java, Indonesia. In general, the FGD aimed to explore the level of consensus of the participants on the interpretation of the work carried out by students. Technically, the researchers presented the recorded video of interviewing the student participant and showed some student participant's responses on the task. On the other hand, the FGD was also used to collect student participant's opinions, ideas, and beliefs of the FGD participants on topics related to how normal students compared with dyslexic students in terms of solving fraction operation problems, how the symptoms of dyslexia on the mathematical ability of dyslexic children, and issues related to relevant to the research discussion.

RESULTS AND DISCUSSION

Errors in Addition and Subtraction

The feature of the task for addition of two fractions was verbal and non-verbal (figural & symbolic). A word problem, "The students in class 5A come from various ethnic groups, $\frac{4}{8}$ of Javanese students, $\frac{3}{16}$ of Balinese students, and the rests are Sundanese. How many students are Sundanese?" did not

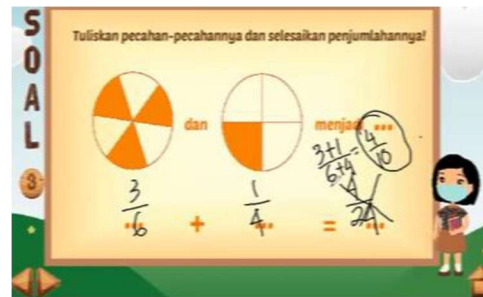
lead the participant to do subtraction, for example, by subtracting 1 as whole by the sum of $\frac{4}{8}$ and $\frac{3}{16}$. Instead, she added the two numbers (symbolic code) emerging in the written text without any further consideration of the contextual meaning of the word problem. She wrote the sum of $\frac{4}{8}$ and $\frac{3}{16}$ was $\frac{7}{24}$. This could be considered as overgeneralized whole number operations in which the numerator as well as the denominator in two fractions were added. When the interviewer asked her, “Do you use this bar to guide you understand the problem?”, she said, “Yes. I did”. However, she could not explain how she used the bar as part of her solution by concerning the size of the bar representing $\frac{4}{8}$ and $\frac{3}{16}$. Therefore, the students solved the problem without understanding the problem as a whole.



Students in grade 5A came from various ethnic. $\frac{4}{8}$ of them are Javanese, $\frac{3}{16}$ of them are Balinese, and the rest are Sundanese. How many students are Sundanese?

Figure 2: Participant's solution on fraction addition task

This finding was consistent with her performance on another addition/subtraction task, namely adding two fractions by writing the fraction of two area models (see Figure 3). Moreover, the student participant solved the problem without understanding the problem as a whole. She immediately added up the numbers that appeared in the problem. While, she successfully represented the area circle model with correct fraction, she failed to add up the two fractions by adding the numerator and denominator without changing to the same denominator first.



Write fractions that suit to the following two pictures and solve the questions!

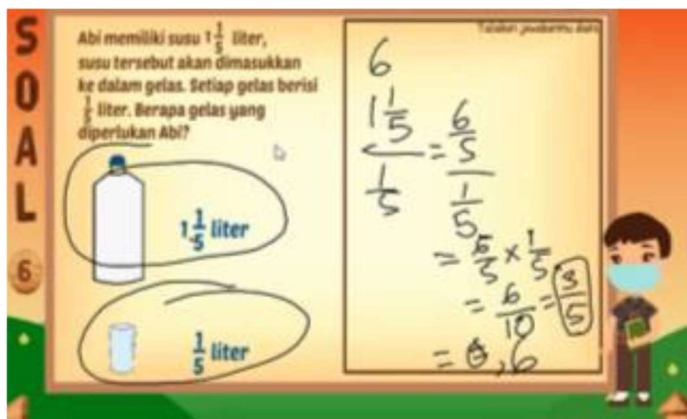
Figure 3. Participant's solution on determining and adding fraction

As with most children, the participant overgeneralized operations on fractions such as the fact that the numerator was operated alone with the numerator, and the denominator was also operated independently with the denominator although she successfully converted the shaded figures into appropriate fractions. In this case, no clue indicating the existence of part of the task dealing with the written text as the phonological structure that makes her difficult to understand. From a cognitive perspective, it was possible to process fractions either componential — as two separate integers (3 and 16) or holistically as one (rational) number with one overall magnitude (i.e., the numeric value $3/16$). This distinction between component and holistic processing was useful for understanding why people had difficulty in solving fractional problems: many of the errors that students made in fractional problems appeared to be due to their dependence on component processing in problems that required holistic processing.

The student was given word problems, as well as direct computation such as “solve the following problem; $4\frac{1}{2} - 1\frac{3}{4}$ ”. In this problem, the student participant was asked to determine the result of subtracting two mixed fractions. To solve the problem, the student participant converted mixed fractions into common fractions correctly, then she got $\frac{9}{2}$ and $\frac{7}{4}$. In the subtraction operation, the student realized that the denominators were different so it took another step before subtracting. But when the student tried to convert to the same denominator, the student failed to convert it into an equivalent form. She multiplied the first fraction by $\frac{9}{9}$ and the second fractions by $\frac{7}{4}$, so the operation became $\frac{81}{18} - \frac{49}{28}$. Then, she subtracted the numerator and denominator separately without concerning that the denominator was still different. By subtracting the numerator, the student participant got 32. But she incorrectly subtracted the denominator (see Figure 4). She subtracted the smaller number from the larger number but forgot to put negative sign in the result and got the wrong answer. Thus, a hypothesis was rising due to the student participant's attempt to extend the subtraction algorithm for natural numbers and to apply it directly to fractions. In this regard, similar findings of Brown and Quinn (2006) reveal that students in their study subtracted the numerators and subtracted the denominators. Interestingly, the algorithm that the student participant applied

Figure 5 shows that this problem was supposed to be a multiplication operation problem, but the student participant failed to understand that the solution required the use of multiplication. Instead, the student participant solved this problem using the subtraction operation. In this subtraction operation, the student added the denominator and numerator without converting it to a form with the same denominator. In addition, the student performed a subtraction operation by subtracting a large number by a small number regardless the location of the number. Thus, again, she tried applying commutative rule of natural number operation incorrectly as found in her work on the subtraction task (Figure 4). She also got the wrong answer for $2 - 5$, wrote 2 as the solution that led multiple errors, did not understand the operation required, and had not internalized the activity applied.

On the other hand, the student participant succeeded understanding the meaning of a word problem, “Abi has $1\frac{1}{5}$ liter of milk. The milk will be poured in some glasses. Each glass contains $\frac{1}{5}$ liter of milk. How many glasses does Abi need?”. This problem was a division operation problem and the student participant was able to take the first step as indicated in the Figure 6.



Abi has $1\frac{1}{5}$ liter of milk. The milk will be poured in some glasses. Each glass contains $\frac{1}{5}$ liter of milk. How many glasses does Abi need?

Figure 6. Participant’s solution on division of fractions

According to Figure 6, the first step was right. The interviewer asked, “which part of the question that helped you solving the problem?” and the student participant answered, “the picture, because there is a big bottle and a small glass”. Then the interviewer asked, “what about if there is no picture? Will you understand the problem easily?”, she said “if there is no picture, I will try to use my imagination”. It indicated that dyslexic students often used picture to understand something and had difficulty in understanding a word problem to make any abstraction.

Based on the division of fractions' rules that we needed to convert it into multiplication, the student participant succeeded to convert it into multiplication. However, student participant forgot to invert the second fraction. It should be $\frac{5}{1}$ but she writes $\frac{1}{5}$. In the multiplication operation, the numerator was multiplied correctly, but the student treated 5×5 as $5 + 5$ in the denominator. This finding was quite interesting since most errors within a multiplication problem found when a solver should keep the denominator the same before multiplying corresponding numerators and denominators. According to Yin (2014), this finding was related to the fact that students might believe that if denominators were equal. They should keep the denominator in the solution, otherwise, the denominators should be combined using the operation provided. There was a student's belief that if the size of the denominator of a fraction was the same, then the denominator must still appear in the final answer regardless of the fraction operation used such as multiplication.

Based on the student participant's solution on several fraction operation tasks, it showed her inconsistent behavior in solving fraction operations that led to errors in performing fraction operations. Many researchers argued that such errors occurred because of insufficient understanding of fraction concepts (e.g., Newton, 2014). The possible reason that caused the student participant to think inconsistently in solving fraction operation problems was due to the symptoms of dyslexia that made her difficult in understanding a word problem. This was linear with the student participant's statement that she always looked for numbers or pictures that helped her in understanding a word problem and if there was no picture, she tried to imagine the problem visually. On the other hand, after failed to understand the problem, the student participant often failed to do the fraction operation. This was not caused by her inability to do the operation. Similarly, Singleton (2008) stated that dyslexic students were slow in the calculation because they had difficulty remembering number facts.

In regard to phonological processing during her mathematical computation skills, the moment where the student participant failed to convert all the word problem into a precise mathematical procedure indicated that she might find difficulties in phonological processing when interpreting the written text. It could be explained that there was a relationship between reading skills and general computational skills (Newton, 2014; Yang et al. 2021), which explained the possibility that reading and mastery of mathematics, including number processing skill, might influence the growth of phonological processing (Hecht, 2001).

The results of this study also addressed to other unanswered questions to be further studied regarding the cognitive processed performed by a dyscalculic student to solve fraction operation tasks. It was interesting when the student participant tried to focus on the symbolic information (e.g., finding any number within the whole text) instead of the written text information. It challenged to understand whether her preference was due to the symptoms of dyslexia or her weak number processing skills. Thus, her actual cognitive processes needed to be investigated through

another method such as eye-tracking. In relation with fraction, for instance, eye-tracking could examine whether an individual could solve a fraction comparison task using componential strategies, which relied on the fraction numerators or denominators, or a combination of both, or holistic strategies that concerned on the magnitude of fraction (Obersteiner & Tumpek, 2016). In normal people, eye-tracking had also been reported as a tool to measure the amount of fixation concentrated on the denominator or numerator of a fraction when comparing fractions or even adding fractions (Huber, et al 2014). Thus, processing the denominator of a fraction tended to require more cognitive effort than processing the numerator of a fraction. How the implications of this finding with the alleged performance of a dyslexic student when comparing or adding two fractions needed to be investigated further.

CONCLUSIONS

The present study shows dyslexic students' difficulty in solving fraction operation tasks. The tasks consist of addition, subtraction, multiplication, and division problem that are presented in a word problem or calculation task. Findings also suggest that a dyslexic student often thinks inconsistently in doing fraction operations, such as generalizing integer operations when adding two fractions with different denominators and applying incomplete parts of the "inversion and multiplication" algorithm. Moreover, she experienced difficulties in understanding a word problem indicated by her failure to translate the textual information into appropriate mathematical symbols and operations. However, when some prompts are given, they seem to understand the meaning of the task more easily. For some cases, they need pictures and numbers to help them understand the problem. It is speculated that the phonological glitches seen in student participants' performances in word tasks are not consistently seen in all given word tasks. This reinforces the previous finding that understanding of values does not seem to rely much on phonological processing, including those related to fractional operations.

Although the findings of this study may add relevant literature towards the insights on how dyslexic students deal with the number operation, a weakness was the limited space and time to work with the student participant due to the challenge of having an interview with her in this Covid-19 pandemic situation that may affect the internal validity of the findings. We need to understand the correct moment of dyslexic students working and explaining their solution. The potential future research in connection with this study finding is the incorporating eye-tracker tools and its developed software to accurately investigate the dyslexic students' cognitive process.

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Utilization of Digital Module for Asynchronous Online Independent Learning in Advanced Mathematics Education

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Abstract: *The exponential increase in the number of cases and number of affected countries of the novel coronavirus disease 2019 (COVID-19) brought significant change in the mode of instruction from face-to-face to distance learning among the different levels of education around the world. This descriptive-developmental method of study adopted the ADDIE (Analysis, Design, Develop, Implement, Evaluate) model of instructional system design (ISD) framework in the development and evaluation of digital learning module intended for asynchronous online independent learning among students at the advanced mathematics education level. The adopted instructional materials evaluation instrument and the 10 items open-ended teacher-made test were used to describe the learning outcomes of the 13 enrolled graduate mathematics education students (4Male, 9Female) during the COVID-19 pandemic period in Sorsogon, Philippines. Findings of the study revealed that the designed digital learning modules covering the required content topics of modern algebra arranged in increasing complexity and ensuring the presence of the basic elements of discussions, definition, examples, and practice drills (worksheets) provide a significant learning experience among the students in times of pandemic as exhibited by their very satisfactory level of evaluation (Mean=4.42±2.58) on its characteristics. The utilization of the digital module available for asynchronous online independent learning maximizes learning outcomes which showed an increase in the mean score of 8.10 equivalent to 53.97 percentage score indicating a highly significant improvement ($t=5.034$, $p<0.05$) in the level of students' understanding of the content topics. Investment in strong internet connectivity is necessary to strengthen the asynchronous learning experiences of the students with the use of digital modules through the regular conduct of virtual conferences for monitoring and immediate feedbacking.*

Keywords: *Digital Module, Asynchronous Online Learning, Independent Learning, Advanced Mathematics Education, Instructional System Design*

INTRODUCTION

The pandemic phenomenon as declared by the World Health Organization (WHO) on March 11, 2020, due to an exponential increase in the number of cases and number of affected countries of the novel coronavirus disease 2019 (COVID-19) (Cucinotta & Vanelli, 2020; Ducharme, 2020) showed a significant impact not only in the health sectors (Xiong et. al, 2020; Giusti et. al., 2020; Berardi, Antonini, Genie, Cotugno, Lanteri, Melia & Paolucci, 2020) but also in the socio-economic activities (Nicola et. al., 2020; Martin, Markhvida, Hallegatte & Walsh,

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2020; Iacus, Natale, Santamaria, Spyrtos & Vespe, 2020; Sharifi & Khavarian-Garmsir, 2020) including the education sectors (Chandasiri, 2020; Marinoni, Van't Land & Jensen, 2020) around the world. Similarly, in the Philippines, the Proclamation No. 929 declaring the state of calamity throughout the country due to the COVID-19 outbreak (Gita-Carlos, 2020; Merez, 2020; Kabling, 2020; Vallejo & Ong, 2020) brought drastic changes in the mode of instruction at a different level of education (Tria, 2020; Toquero & Talidong, 2020; Reimers & Schleicher, 2020).

The restriction of face-to-face interaction paved the way towards the adoption of online learning among the basic, higher, and advanced levels of education in the Philippines. One of the most salient features of the educational delivery during the pandemic period was the implementation of synchronous and asynchronous online learning which requires stronger internet connectivity among the teachers and learners. Each educational leader including classroom teachers was engaged in the creation of the best instructional mode either print or non-print for learners' utilization.

The introduction of this new mode of instruction became a challenging task for most of the teachers in bringing active participation and involvement among the students. Swan (2001) found out that the clarity of design, interaction with instructors, and active discussion among course participants significantly influenced students' satisfaction from asynchronous online learning. Moreover, the task of designing appropriate instructional material is necessary to support the needs of the students and thus they will become more engaged in asynchronous online learning (Alrajeh & Shindel, 2020). The students in advanced education level (or graduate students) have different learning styles that need to be supported with suitable teaching approaches and strategies. Graduate students who are generally independent learners desired more of a mentoring relationship with faculty where they could seek guidance and information about their professional development.

Holzweiss, Joyner, Fuller, Henderson & Young (2014) reveals in their investigation that the best learning experiences of graduate students in an online class are the activities that allowed for the creation and/or sharing of knowledge such as problem-solving assignments, research, writing, journal reflection, discussion forums, video lesson creation and virtual conferencing. In addition, students enrolled in online education demonstrate strong preferences for asynchronous mode of learning because of convenience and favored individual assignments (Butler & Pinto-Zipp, 2005). Swan, Shen, and Hiltz (2006) explored collaborative activities such as discussion, small group sessions, and collaborative exams as a form of assessing students' learning in an online class which can be possibly made by an explicit learning goal with an explicit evaluation criterion available at the beginning of the course.

The digital learning module has been recognized as one of the common instructional deliveries being utilized in the implementation of asynchronous online independent learning at the advanced education level during the pandemic period. The development of the digital module requires time to plan out the learning activities appropriate to the learners covering the course objectives aligned to the program goal and performance indicators as reflected in the course syllabus. The digital module shall be designed to capture advanced education students' independence in learning mathematics concepts (Setiyani, Ferdianto & Fauji, 2020).

OBJECTIVES

This study evaluated the utilization of the designed digital module in an asynchronous online independent learning at the advanced mathematics education level amid the COVID-19 pandemic period. The following were the specific objectives: (1) design a digital module for asynchronous online learning, (2) describe the learning experiences of the students in the utilization of the digital module, and (3) test the effectiveness of the developed digital module in student's understanding of the subject content topics.

METHODOLOGY

The study utilized a descriptive-developmental method of research combined with a qualitative approach to analysis. The developmental nature of this study adopted the ADDIE model of instructional system design (ISD) framework which is composed of the five phases: Analysis, Design, Develop, Implement, and Evaluate.

Analysis. This phase involved the review of the set program standards including the instructional objectives for the Master of Arts in Education (MAEd) major in Mathematics. The analysis of the approved course syllabus for the semester focusing on the content and coverage, course objectives, and the performance standards was executed to define and meet the needs of the advanced education mathematics students in the course during the COVID-19 pandemic period.

Design. This phase involved the logical arrangements of content topics of the course with due consideration to the prior knowledge and needs of the enrolled graduate education students. The listed topics were based on the previous coverage of the course subjects as reflected in the approved course syllabus. The learning module for each of the identified topics contains the following elements: discussions, definition, example, and practice drills to check their understanding. The learning modules were designed in a manner that will be available in digital format to be utilized by the students in an asynchronous online independent learning.

Development. This phase involved the creation of the learning modules for each of the identified topics to be covered in the course. The content and discussions of each topic came from different sources both print and non-print materials. Each learning module contains the worksheet as drill exercises to check the student's understanding of the topic using the developed learning materials. The developed worksheets ensure their alignment to the content and examples provided in the learning modules. There was a total of 22 learning modules developed with the corresponding worksheets to be accomplished by the students before the semester ends.

Implement. This phase involved the utilization of the developed learning modules. Before its utilization, general directions were provided to the whole class which includes the manner of weekly distribution of each learning module virtually as well as the submission of the worksheets. The accomplishment of the worksheets was designed on weekly basis. The synchronous learning feature of the course was made through virtual discussion of the learning modules once a week to

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follow up students' understanding of the content. Students are free to ask any questions regarding the topic coverage specific for the week as reflected in the digital learning module during the virtual discussion. The weekly format of implementation in the course follows: Digital module distribution, utilization (asynchronous independent learning), accomplishments of worksheets, submission of worksheets, and virtual discussions for feedbacking and deepening of skills.

Evaluate. This phase was an integral component of each stage of development from the Analysis phase involving the two faculty members of mathematics education of the institution to ensure alignment of the module content with the set program standards. This phase guarantees the accuracy and reliability of the developed digital learning module appropriate for graduate education mathematics students. The evaluation in terms of the content and coverage, theoretical considerations, appearance/visual appeal, and language of the materials by the students who utilized the digital module in an asynchronous online independent learning were also executed. The evaluation phase includes the qualitative approach to the analysis of students' experiences with the use of a digital learning module in an asynchronous online independent learning.

Moreover, the evaluation phase of the current study also involved the test for the effectiveness of the digital learning module in understanding the learning content of the course through a one-group pre-test post-test design. The study involved 13 officially enrolled students of the regular class (4 Males, 9 Females) under the Master of Arts in Education (MAEd) major in Mathematics during the First Semester (August – December) of AY 2020-2021 in Sorsogon, Philippines.

Instrument

The Board of Trustees (BOT) approved Instructional Materials Evaluation instrument of the institution was used in the assessment of the developed digital learning module. The study made use of the interview guide focusing on their learning experiences in the asynchronous online independent learning with the use of a digital module. The questions include: How is your experience with this subject? How does the digital module help you in meeting your learning needs with this subject course?

Moreover, the teacher-made test for the pre-test and post-test was used in the assessment of students' understanding of the content topics in the subject offered during the semester. The test is an open-ended test question that requires a graduate student who is pursuing the MAEd degree major in mathematics to show solutions and/or explanations to justify their answer in a particular item. The 10-item teacher-made open-ended test item requires a substantial response expected from a graduate mathematics education student to get a maximum score of 15 points. This type of test item will ensure assessment of student attainment of the performance standards and program outcomes required for a graduate education student.

Data Collection Procedures

The data were collected through virtual/online surveys and interviews. The teacher-made test was conducted before (pretest) and after (posttest) utilization of the digital module to test

students' conceptual understanding. The test was conducted virtually with a specific time allotment of submission within the day. The students were explicitly provided with the learning goals, targets, and performance indicators that they need to accomplish in the course which is made available through the distribution of the course syllabus at the beginning of the semester. This was made to ensure clarity of the students' tasks and deliverables before the study implementation.

Data Analysis Procedures

Descriptive statistical measures such as frequency count, mean, and standard deviation were used in the evaluation of the digital learning module. Cronbach's alpha (α) was also used in the assessment of the consistency of students' evaluation of the digital module. This was supported with a qualitative approach to the analysis of the textual responses and information from the respondent's learning experiences in the utilization of the digital module. Coding of responses was used to evolve the themes or categories of the experiences in the asynchronous online independent learning.

The t-test was used to test the effectiveness of the digital learning module in students' understanding of the required mathematics content topics after its utilization in asynchronous online independent learning. Test of normality of scores in the pretest ($D = 0.265$, $p=0.541$) and posttest ($D= 0.217$, $p = 0.506$) were confirmed through the Kolmogorov-Smirnov (K-S) test statistic (D). Both the pre-test and post-test scores of the students were translated into percentage scores (PS) to show the difference in students' level of understanding of the required content before and after utilization of the digital module. The responses of the students in the pretest and posttest were analyzed to further show evidence of learning of the identified content topics.

RESULTS AND DISCUSSIONS

The Designed Digital Module in Learning Mathematics Course for Graduate Students

The Modern Algebra course, also known as Abstract Algebra, has been identified by the mathematics education major students as one of the most challenging subjects because of its symbolic features and structures which made its concept foreign to study (Ko & Knuth, 2013; Mowahed, Song, Xinrong, & Changgen, 2019). The course intends to enhance logical and analytical reasoning and symbolic thinking of the students in the appreciation of basic algebraic structures: groups, semigroups, and rings. The *New Normal* phenomenon made the teaching and learning of the subject more challenging because of the absence of the usual face-to-face classroom interaction among the higher education institutions (HEIs) worldwide. The digital learning module on selected 22 primer topics in modern algebra has been conceived as one of the self-learning materials intended for the mathematics major students of the Teacher Education Institutions (TEIs) in response to the *New Normal* teaching and learning approach brought by the pandemic due to the Corona Virus Disease (COVID -19).

The primary reason for designing the digital module is to make possible the attainment of the set of educational objectives for a one-semester modern algebra (abstract algebra) course. Students were assumed that they have learned the most essential foundational topics and concepts of the set theory, linear algebra, number theory, and probability theory to better understand the content of this course materials. They are expected to exert time and effort to learn every topic which were arranged in a manner that prerequisites are considered first. The material hopes to help students learn the salient and essential topics of the subject, both in synchronous and asynchronous blended teaching-learning approach.

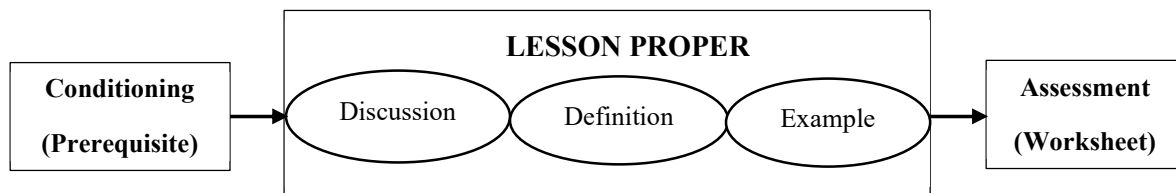


Figure 1. Manner of Utilization of the Digital Module in Asynchronous Learning

The learning modules were designed in a manner that will be available in digital format to be utilized by the students in an asynchronous online independent learning. With the intent of ensuring that the material would cater to independent learning, topics were arranged from simple to increasing complexity so as prerequisites are discussed first and deepened through an integrative approach between and among content topics as shown in Figure 1. The user of the module is encouraged to have some review of the topics on properties of real numbers, properties involving equations, and inequalities to have a better understanding of the course. These topics can be found in the appendices (Appendices A to C) of the compiled format of the module for easy reference which was distributed at the beginning of the semester. Each module corresponds to a specific identified topic of Modern Algebra provided to students every week with a corresponding assigned task to be accomplished and to be returned in the succeeding week. The topics included are divided into two main parts; part I deals with the preliminary topics intended for those students who have a little background, if none, in Abstract Algebra, Number Theory, and Probability Theory. On other hand, part II deals with the basics of algebraic structures which will lead students to deepen their understanding of group theory and ring theory as illustrated in Figure 2.

The learning module for each of the identified topics contains the following basic elements: discussions, definition, example, and practice drills (worksheets) to check their understanding. The *discussion* component of the module contains the basic concepts and ideas about the topic being introduced in the module. This serves as the backgrounder leading towards the understanding of the content topic and how it is related to their prior and acquired knowledge and skills in the previous lessons to connect with the new lesson. The *definition* component of the module will strengthen student understanding of the new mathematics concepts introduced in the lesson.

Essential mathematics concepts which bear significance in students' understanding of the new concepts are defined in the lesson.

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Topic No. 11: ODD AND EVEN PERMUTATIONS

This section considers permutation which will be classified as odd or even. An even permutation can be obtained as the composition of an even number and only an even number of exchanges (called transpositions) of two elements, while an odd permutation can be obtained by (only) an odd number of transpositions. Thus, it is important to discuss the concept of transposition and recall the cycle form of the permutation.

Definition: An element of S_n is called a **transposition** if and only if it is a 2-cycle. A transposition is a permutation which exchanges two elements and keeps all others fixed; for example $(1\ 3)$ is a transposition.

Example 1: Consider the set $\{1, 2, 3, 4, 5\}$. Then the transposition $(1\ 2)$ is given by:

$$(1\ 2) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 3 & 4 & 5 \end{pmatrix}$$

furthermore, the transposition $(1\ 3)$ is given by:

$$(1\ 3) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 2 & 1 & 4 & 5 \end{pmatrix}$$

Every permutation can be written as a product of transpositions; for instance, the permutation $g = (1\ 2\ 3)$ can be written as $g = (1\ 3)(1\ 2)$ (You may verify this by getting the result of the composition of $(1\ 3)(1\ 2)$ as $(1\ 2\ 3)$). The **number of transpositions** in a permutation is important as it gives the minimum number of 2 element swaps required to get this arrangement from the identity arrangement: $1, 2, 3, \dots, n$. The parity of the number of such 2 cycles represents whether the permutation is even or odd.

Example 2: The cycle $(5\ 1\ 2\ 4\ 3)$ can be written as $(5\ 3)(5\ 4)(5\ 2)(5\ 1)$. Therefore, there are 4 transposition.

Example 3: The cycle $(5\ 1\ 2)(4\ 3)$ can be decompose as $(5\ 2)(5\ 1)(4\ 3)$. Therefore, there are three transposition in the given permutation.

Figure 2. Screenshots of Some parts of the Digital Module

The *example* component of the module further elaborates the defined mathematics concepts through illustrations. This element of the module may contain computations, techniques, algorithms, approaches, among others which will expound the mathematics concepts and procedures to fully demonstrate understanding by the students. The students will see the pattern and the techniques in applying the definition on how certain mathematics expressions are converted in another form. The *definition* and *example* components are integral parts of the *discussion* component of the module which further explains and provides a concrete representation of the content topic being introduced.

Moreover, the *practice drills* component of the module will provide a venue for the student to further explore and test their understanding of the content topics being discussed. Each of the lessons (content topic) has a corresponding worksheet which serves as the practice drill. It contains several forms of assessment depending on the nature of the topic and the objectives of the lesson which may vary from multiple-choice, True or False, Short Answer test, Essay, and problem-solving. Answer key for each of the worksheets is provided to check their understanding of the lesson following the principle of independent learning. Further discussions and follow-up are held during the conduct of the virtual meeting via google meet, queries and questions from the students were entertained for clarification of the module content, see Figure 4.

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Characteristics	Cronbach's			
	Mean (Sd)	Description	Alpha (α)	Reliability
Content and Coverage	4.56 (1.66)	Outstanding	0.95	High
Theoretical Consideration	4.26 (1.27)	VS	0.81	High
Appearance/Visual Appeal	4.27 (1.48)	VS	0.96	High
Language	4.51(1.13)	Outstanding	0.87	High
Over-all	4.42 (2.58)	VS	0.95	High

Table 1. Users (n=13) Evaluation on the Characteristics of the Digital Module

Table 1 shows the results of the students' evaluation after a semester of the utilization of the digital module which illustrates the characteristics of the digital module along with content and coverage, theoretical consideration, appearance/visual appeal, and language used which represents the content validity and construct validity of the materials. The data reveals that the digital learning module has obtained an overall very satisfactory rating (Mean = 4.42 ± 2.58) from the students with a corresponding Cronbach's alpha (α) value of 0.95 which signifies a high internal consistency rating from among the student evaluators. An enrolled male graduate mathematics education student teaching in a private school at the basic education level expresses his appreciation in the utilization of the digital module, he stated that "*The provided instructional materials helped me greatly for the acquisition of learning even during the pandemic where we can learn anytime and anywhere*". This expression is supported by the statement from a female mathematics teacher of a public school who enrolled in the same course expounded that "*the provision of the digital module is an effective tool for continuous learning despite the pandemic since face-to-face is not possible at the moment*". This only means that the students are generally and consistently satisfied with the utilization of the digital learning module along with its characteristics.

Moreover, the students have an outstanding rating of the content and coverage (Mean = 4.56 ± 1.66) as well as the language (Mean = 4.51 ± 1.13) used in the digital module exceeding their learning needs and requirements on their asynchronous learning utilization. One of the newly enrolled graduate mathematics education male students in Sorsogon City mentioned that "*the content and approach of the module arranged from very simple ideas to increasing complexity together with the worksheets for practice assisted me in learning the subject*". Another manifestation of a female student from Masbate Province commented that "*Though it was a difficult subject, I am satisfied with the content of the module because I learned new topics*". The result of evaluation signifies that the material can provoke and sustain students' understanding of the content through the language used appropriately to their level of thinking.

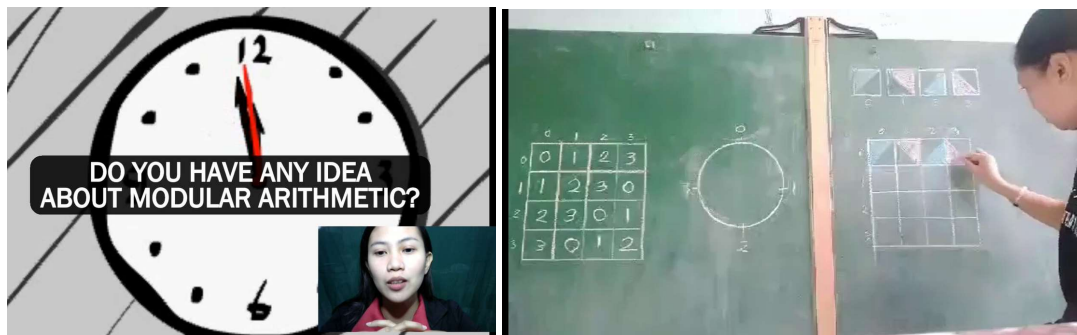


Figure 3. Screenshots of the Sample Lecture Videos Created by the Graduate Students

The very satisfactory rating along with the theoretical considerations (Mean = 4.26 ± 1.27) and appearance (Mean = 4.27 ± 1.48) also indicates that the digital module can sustain their interests in learning the content of the lesson with corresponding high internal consistency Cronbach's alpha (α) values exceeding the acceptable value of 0.70 (George and Mallery, 2003; Hair, Black, Babin & Anderson, 2010). The qualitative and quantitative data revealed that the presentation of the important concept suitable to the level of student understanding by building on their previous knowledge is necessary to capture student interest in the subject. The newly enrolled male student in Sorsogon City also expounded his appreciation of the particular topic by saying "I enjoyed learning the mathematical concepts behind the modulo art ... we were asked to do modulo art design since in our elementary years without even knowing the reasons behind the patterns". The graduate students' appreciation of the mentioned topic can be further expounded by their output on the creation of lecture video as one of the proofs and outcome of their learning as demonstrated in Figure 3.

Learning Experiences of the Students on the Use of the Digital Module

The sudden change of the mode of instruction in any level of education brought a significant effect on the teaching-learning situation. There were some challenges encountered by both the students and teachers during the pandemic in the flexible learning environment (Laguador, 2021) limiting face-to-face interactions and promoting online distance learning through synchronous and asynchronous learning approaches. The utilization of the developed digital module for the asynchronous mode of learning boosted the learning experiences of graduate education students.

Table 2 summarizes graduate mathematics education students learning experiences based on their feedbacks and written responses in the utilization of the digital module for asynchronous independent learning showing their identified challenges encountered. The corresponding features of the digital module were designed together with their learning strategies as an adaptive mechanism to minimize, if not eliminate, the challenges encountered. The feedbacks of the

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students show that their challenges can be grouped into three broad categories: (1) the nature of the subject itself, (2) online learning facilities, and (3) the learning modalities.

Challenges Encountered	Features of the Digital Module Utilization	Students' Adaptive Mechanism
Complex nature of the subject with a new set of topic/lessons encountered by the students	The module is comprehensive and informative with a complete discussion of the topics.	Eagerness to learn and upgrade their content knowledge in their area of specialization.
Some new encountered content topics/lessons need further elaborations.	Topics are arranged in increasing complexity bridging students' prior knowledge with supplemental learning materials.	Students explored available online resources such as e-books and video lectures on YouTube as supplemental materials/lessons on challenging topics.
Unstable internet facilities	Availability of digital module for asynchronous learning modality.	Students look for a place with a strong internet connection and/or available Wi-Fi.
Modular Distance Learning (MDL) is time-consuming on the part of graduate students who are working at the same time.	The digital module is available at their most convenient time for independent learning.	Students find time in reading the module and answering the worksheets to beat the agreed schedule of submission (Time management skills).
Teachers hardly provide and/or get immediate feedback on students' difficulties.	Worksheets are attached in each lesson to check student understanding of the lesson. Constant communication and monitoring of student progress through the conduct of weekly synchronous online teaching for feedbacking.	Students challenge themselves to discover and perform higher-order learning tasks.
Limited interaction among teachers and classmates.	Each module was designed in an interactive manner featuring the lesson discussions, definition, example, and practice drills (worksheets) to check their understanding.	Students develop independent learning skills in critical thinking and analyzing information.

Table 2. Learning Experiences of the Graduate Students

Nine out of 13 enrolled students in the subject have just encountered some set of topics specified in the subject since most of them are teaching at the basic education level and have not been teaching algebraic structures in general. *"I am not familiar with most of the topics, I am just*

starting to learn the concepts on my own,” said one of the newly enrolled female students in the program. While the other four students are fresh graduates at the undergraduate level and able to recall some of the prerequisite contents included in the course.

Though students agreed that the digital module provided them with comprehensive inputs and information about the topics which are arranged from simple to complex, they still looked for other available online learning resources to supplement their understanding of the topics. A female graduate mathematics education student in the Municipality of Irosin mentioned that to further validate and deepen her understanding of the concepts discussed in the developed digital module she tried to search for more examples from the available materials online such as the video lecture on the YouTube website. This is supported by the feedback of her classmate in Sorsogon City who said that “*I enjoyed the given worksheets for it has driven my curiosity to read articles and watch videos on YouTube*”.

This is an indication that the students at the graduate level are independent learners capable of looking for additional learning resources coupled with an eagerness to learn new things and upgrade their knowledge in their area of specialization. The module has been designed in a manner that will provide the graduate students with the overview and ideas regarding the concepts presented building on what they have learned already that would aggravate them to learn further and deepen their understanding about the abstract concepts of mathematics such as group, subgroup, and ring. The students will not able to determine whether $G = \{a, b, c, d\}$ with operation $*$ defined by the table below, as excerpted from the problems in Worksheet number 8 (Group),

*	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	b	a
d	d	c	a	b

is a group without understanding the concept and definition of a group as well as the binary operation as reflected in the digital module which they explored during the asynchronous independent learning. The students at the advanced education level after a walk-through of the specific topic in the digital module were given a chance to further explore the available learning resources whether print or non-print materials with an already preconceived idea about the topic, e.g., group, for their verification and deepening of conceptual understanding. This will make them more confident with the completion of the task given in the worksheets and apply the mathematics concept in solving problems in the relevant field of study. Teachers, therefore, need to design well the learning materials and activities that would provoke students’ willingness to adopt the principles of independence in online distance learning despite the complex nature of the subject.

The Province of Sorsogon, together with its neighboring Provinces such as Masbate, is in the southernmost tip of Luzon Island in the Philippines experienced unstable internet connectivity especially those in the remote area as supported by the feedbacks from most of the students (8 out

of 13 students). The limitations during the online synchronous lessons such as video conferencing due to poor internet connectivity are strengthened by the utilization of the provided digital module. The provided module available on weekly basis according to the scheduled topic helped the graduate student a lot for self-learning at their most convenient time. Students were given enough time to study the module and answer the worksheets to check their understanding anytime they want. This is also supported by the feedback of an enrolled graduate student from the rural area of the Province of Masbate as follows *“For me, it is more convenient to have modules/worksheets as a mode of instruction while I am working at the same time. I can learn anytime”*.

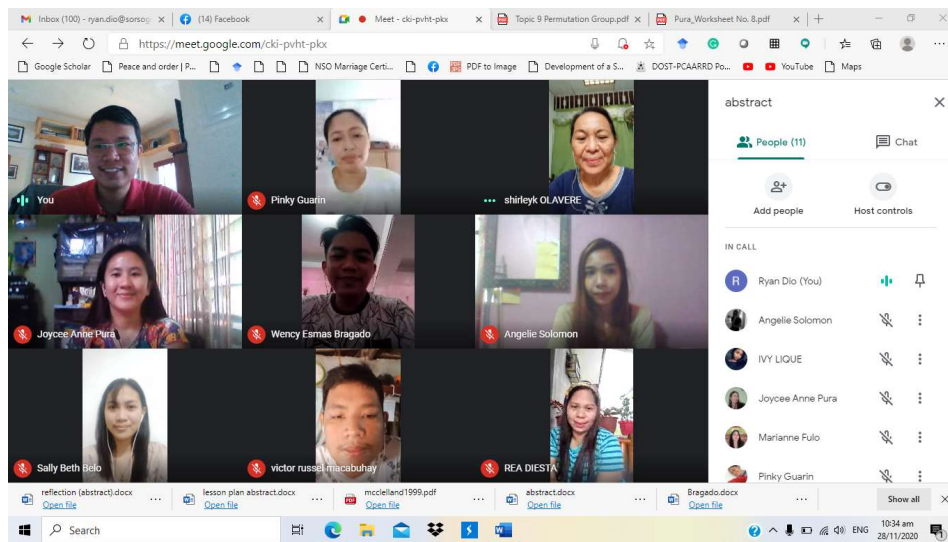


Figure 4. Captured Moments during the Video conferencing with the students

Feedback mechanism on students’ outputs and learning is provided during the conduct of the online synchronous teaching where students find ways to look for a place with strong internet connectivity in their respective area once a week. Feedback from another student in Masbate Province highlighted that *“... since I am aware of poor internet connectivity in our area as one of the reasons of not participating in our video conference via google meet, I find a place where I could stay with strong internet connection for me to attend our online meeting”*. The virtual sessions as shown in Figure 4 provided the students an opportunity to interact and share their experiences with their classmates. The students felt the need to attend the virtual session utilizing any available facilities, equipment, or devices such as personal computers and mobile phones with the internet connection either through Wi-Fi or prepaid load so that areas needing improvement along with the content of the module are discussed to unlock any difficulties encountered.

It is, therefore, necessary that the teachers should be vigilant on the weekly need of the students and see to it that appropriate instructions and feedbacking is provided during the conduct of online teaching. Professors may also provide for the extension of the submission of weekly outputs, if necessary, especially for those students who are working at the same time and residing

in an area with poor internet connectivity. The unexpected change of the mode of instruction implementing online distance learning for the first time has to make adjustments on the part of the professors towards accomplishing the task within the semester without sacrificing the quality of instruction.

Moreover, the independent learning modalities with the utilization of the digital module brought some big adjustments on the part of the students such as the time management skills, need for immediate feedback mechanism, and limited interactions with teachers and classmates. *“The distance learning limits interaction, and meaningful learning experience between the teachers and students which led us to rely the information on the provided module/worksheets,”* said a female student who is a fresh graduate from her undergraduate education. The conditioning mechanism on the very first day of the semester which includes orientations and leveling of expectations is necessary to implement independent online learning modalities. This conditioning mechanism is an important strategy to assess students’ needs and readiness to adopt the new approach to learning. This led to the design of the digital module as a teaching-learning approach to implement asynchronous online independent learning coupled with a weekly scheduled online synchronous teaching for feedbacking. Consistency of submitting students’ outputs per week is necessary, with some considerations to students’ needs, to check their progress through active participation during the scheduled online teaching.

Generally, students’ reflections revealed that the digital module provided them a guide on what to learn, what to do, and what to accomplish per week which developed their time management skills, creativity, and independent learning skills which eventually improved their critical thinking skills and problem-solving skills. The graduate students enrolled in online education demonstrate a strong preference for an asynchronous mode of learning because of convenience and favored individual assignments (Butler & Pinto-Zipp, 2005). One male student mentioned that he can save a lot of money, time, and effort in traveling at a most 2-hour distance from home every weekend.

Effectiveness of the Digital Module in Asynchronous Online Independent Learning

Table 3 displays the differences between the posttest and pretest scores and the corresponding equivalent percentage scores (PS) of the students. It can be noted that eight out of the 13 enrolled students were considered in the analysis of the pretest scores since these are the only students who were able to satisfy the requirements of submitting the test within the allotted time duration during the conduct of the test. There were three out of eight students who has no sufficient knowledge of the content topics before the utilization of the digital module with an overall mean score of 3.75 or 25% level of understanding.

The pretest answer sheet of student 4 referred to in Table 3 reflected the statements as *“I humbly apologized that I have not been able to answer any of the questions because I forgot already the concepts which made me difficult to deal with the problems. I do not teach these lessons*

since my first year of teaching but I am much eager to learn and appreciate them again.” This is an indication that the graduate students in this particular educational institution have different needs along with their acquired knowledge and skills of the course content of Modern Algebra including the pre-requisites of the course. These are some of the areas of concern to address in times of pandemic through the conduct of online distance learning.

Through the utilization of the digital module featuring the basic elements of discussion, definition, examples, and practice drill (worksheets) during the asynchronous learning coupled with the constant checkup and follow-up during the synchronous online learning via google meet, the aforementioned student can obtain a score of 13 or 86.7 PS during the posttest. He was able to properly execute the requirement in a problem on “Determine whether the product of the following permutation in cycle notation form (1 3 5 2 4), (1 3 5) and (2 4) in S_5 is odd or even” he obtained the product which is (1 5 3 2) that gave him the idea that the product is an odd permutation. Many of the learners of Modern Algebra find difficulty in finding a product of permutation of n especially when it is written in cycle notation format which he was able to perform properly the operation. After performing the given operation, the student has executed his knowledge about the number of transpositions of the given permutation, so he has able to determine whether it is odd or even.

Student	Pretest		Posttest		Difference (Post – Pre)	
	Score	PS	Score	PS	Score	PS
1	0	0.0	6	40.0	6	40.0
2	4	26.7	14	93.3	10	66.7
3	1	6.7	13	86.7	12	80.0
4	0	0.0	13	86.7	13	86.7
5	9	60.0	14	93.3	5	33.3
6	0	0.0	11	73.3	11	73.3
7	9	60.0	14	93.3	5	33.3
8	7	46.7	12	80.0	5	33.3
9	-	-	14	93.3	-	-
10	-	-	11	73.3	-	-
11	-	-	7	46.7	-	-
12	-	-	12	80.0	-	-
13	-	-	13	86.7	-	-
Mean Score	3.75	25.00	11.85	78.97	8.10	53.97
Sd	4.06	27.08	2.61	17.39	-	-

Table 3. Difference between Students’ Posttest and Pretest Score

Generally, the students obtained more than thrice their pretest mean scores ($MS = 3.75 \pm 4.06$) in the post-test ($MS = 11.85 \pm 2.61$) with an equivalent of 78.97 MPS. The data in Table 3 also revealed that the group of graduate students involved in this investigation have a closer level

understanding of the mathematics content topics after utilization ($MPS = 78.97 \pm 17.39$) of the digital module as compared to their pretest percentage scores ($MPS = 25 \pm 27.08$). Moreover, all eight students showed a significant improvement in their test scores in the posttest with a mean score (MS) gain of 8.10 equivalent to 53.97 MPS.

Figure 5 reveals the way student 7 (who obtained a Pretest Score of 9 or 60.0 PS, Posttest score of 14 or 93.3 PS) answers a problem on subgroups content topic where they were asked to show the subgroup diagram of the cyclic group $(Z_9, +)$. It can be seen in the figure the big difference of how the student responded to the problem with some maturity of response during the posttest. During the pretest, the student was able to show the set generators of the cyclic group $(Z_9, +)$ including the set generated by each of the elements, however, student 7 did not able to show the subgroup diagram. On the other hand, the student 7 posttest response showed all the subgroups of the given cyclic group together with the description as trivial, set generator, or proper subgroup and was able to show the subgroup diagram. The illustration indicates that the use of the digital module with its basic elements of discussions, definition, and examples helped any student, whether has prior knowledge or not about the content topic, to further expand their learning and understanding of the mathematics concepts including its principles and processes.

Pretest Response	Posttest Response
<p>Given: $Z_9 = \{0,1,2,3,4,5,6,7,8\}$ $\langle 0 \rangle = \{0\}$ $\langle 1 \rangle = \{0,1,2,3,4,5,6,7,8\}$ $\langle 2 \rangle = \{0,1,2,3,4,5,6,7,8\}$ $\langle 3 \rangle = \{0,3,6\}$ $\langle 4 \rangle = \{0,1,2,3,4,5,6,7,8\}$ $\langle 5 \rangle = \{0,1,2,3,4,5,6,7,8\}$ $\langle 6 \rangle = \{0,3,6\}$ $\langle 7 \rangle = \{0,1,2,3,4,5,6,7,8\}$ $\langle 8 \rangle = \{0,1,2,3,4,5,6,7,8\}$</p> <p>Note: I cannot create a subgroup diagram. I have no past knowledge about it.</p>	<p>Given: $(Z_9, +)$</p> <p>The subgroups of $(Z_9, +)$</p> <ul style="list-style-type: none"> • $\langle 0 \rangle$ generates $\{0\}$, trivial subgroup • $\langle 1 \rangle, \langle 2 \rangle, \langle 4 \rangle, \langle 5 \rangle, \langle 7 \rangle,$ and $\langle 8 \rangle$ generate $(Z_9, +)$ itself, set generators • $\langle 3 \rangle$ and $\langle 6 \rangle$ generate $\{0,3,6\}$, proper subgroup <p>The subgroup diagram:</p> $\begin{array}{c} \langle 1 \rangle \\ \\ \langle 3 \rangle \\ \\ \langle 0 \rangle \end{array}$

Figure 5. Comparison of the Sample of a Student Pretest-Posttest Response

The presented illustrations above can be supported by the gain in mean score after exposure to the digital module indicating a highly significant improvement ($t=5.034, p<0.05$) in the level of students' understanding of the content topics. The statistical result signifies that the utilization of the digital module in the asynchronous online independent learning of the graduate students provides them with a better understanding of the content topics in Modern Algebra. The graduate

students' experiences also support that the features and basic elements of the designed digital module for asynchronous online learning are more effective when combined with a regular schedule of virtual conferencing for monitoring and evaluation of their gained knowledge and skills. The findings of the current investigation affirmed that any instructional materials designed for students' online learning will be more effective and engaging when there is constant communication with the instructor (Swan, 2011; Alrajeh & Shindel, 2020) and provide them with activities for the creation and sharing of knowledge (Holzweiss, Joyner, Fuller, Henderson & Young, 2014) and experiences via video conference.

CONCLUSIONS

The designed digital learning modules cover major topics arranged in increasing complexity for the utilization of the graduate mathematics students in an asynchronous online independent learning. The basic elements and features of the module ensuring the presence of discussions, the definition of important terms, examples, and practice drills (worksheets) to check their understanding made students satisfied with the utilization of the digital learning module. The learning experiences of the graduate students are boosted and learning outcomes are maximized when the provision of the digital module for asynchronous online independent learning is supported with the regular conduct of virtual conferences as an opportunity for feedbacking, evaluation, and discussions. The regular conduct of virtual conferences allows the students to share their ideas and thoughts responding to their needs and identified challenges along with the nature of the subject, online learning facilities, and the learning modalities during the COVID-19 pandemic. Moreover, the utilization of the developed digital module in online learning of graduate education level is an effective modality to better understand the mathematics content topics and better provide them with an independent learning experience.

Teachers and instructional practitioners at a different level of education are therefore recommended to involve themselves in the creation of the appropriate instructional materials available to students for online asynchronous independent learning coupled with regular monitoring for immediate feedbacking and evaluation. The teaching-research methodologies performed in this investigation in the creation of the digital module featuring the essential elements of discussions, definitions, examples, and practice drills (worksheets) may be further explored and replicated to support its applicability to the larger class at a different level of education. Investment along with the provision of strong internet connectivity and facilities is necessary in the provision of the best learning experiences and optimizing learning outcomes while not sacrificing the safety and health conditions of both teachers and students in times of pandemic.

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(MAEd) major in Mathematics students at the School of Graduate Studies of the institution for the entire duration of the study.

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Effects of Animated Instructional Packages on Achievement and Interest of Junior Secondary School Student in Algebra

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Abstract: This study explored the use of animation in the classroom by investigating whether or not students taught with animation will achieve better than recorded teaching. This study adopted non-equivalent control group, quasi experimental design. Eighty (80) Junior Secondary School 2 students (40 males, 40 females) from two co-educational private junior secondary schools in Bwari Area Council of the Federal Capital Territory (in Nigeria) were involved. Gender of the students and availability of functional computers determined the choice of schools hence purposive sampling was used to select six schools that have functional computer laboratories out of which two schools were randomly sampled using simple balloting with replacement. The purposive sampling means the sampling that adequately addresses the purpose of research which means in this case means schools with boys and girls and having functional computer systems. Two instruments were used for this study, namely Algebra Achievement Test (AAT) and Interest Inventory (MII). The findings showed that teaching algebra with animation can enhance achievement and interest of students in algebra. Gender had no significant influence on the performance of students exposed to animation. The implication of these findings is that teachers now have alternative strategy to teach a concept in junior secondary school. Based on the findings, it was recommended that workshops, seminars and conferences should be organized for teachers to implement the findings of this study in the classroom.

Keywords: Animation, Instructional packages, Achievement, Interest

INTRODUCTION

The importance of mathematics in the modern society is overwhelming. The importance of mathematics has long been recognized all over the world, and that is why all students are required to study mathematics at the primary and secondary school levels, whether they have the aptitude for it or not (Ojo, 2015; Adebayo, 2008; Adeleke, 2007). However, despite the importance of mathematics in the Nigerian education system, the students' performances continue to deteriorate year after year. Several reasons have been proffered for the high failure rate in mathematics. Given the options, many students will not offer mathematics (Agwagah, 2013). Agwagah noted that an increasing number of students find it difficult to do well in mathematics

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because of the teaching method used in teaching most of the themes are not interesting. In other words, the prime position accorded mathematics in the society is at risk because of persistent poor performance. According to Kurumeh and Achor (2008), the difficulties of students in learning mathematics could be attributed to the approach to which the contents are being presented to the students, the abstractness of mathematical concepts, and poor foundation, among others. Most students, especially, at the Junior Secondary School (JSS) have difficulties in understanding mathematics because of the language of instruction (which is English Language) (Ifeanacho, 2012). Majority of the students who are highly proficient in performing mathematical operations, in solving symbolic problems are less proficient in solving problems when it is algebraic (Iji, Abakpa, & Takor. 2015).

The usefulness and vitality of algebra are generally seen as arising from its concepts and pattern of reasoning as they have been adapted in the attempt to solve ongoing human activities. As good as algebra appears to be solving human problems, the symbols, language and expressions commonly in this branch of have continued to be a source of problems to the students (Odili, 2006). This accounts for why Recorded Conventional Instruction (RCI) was considered as a better option to conventional ‘chalk and talk’ strategy.

In RCI strategy, the lessons are recorded and students participate in the lesson by listening to the computer and carrying out the activities as directed by the computer. The use of innovative strategies as well as instructional media that will bring meaningful learning of mathematics becomes imperative (Ubah & Uzoechi, 2018; Usman & Ezeh 2011). The method being advocated in this study is the one which involves the use of animated instructional package (AIP).

Animation instruction is the use of computer in the delivery of text materials that appear to move (Yalcinalp, Geban & Ozkan, 2005). This delivery of instruction is computer based. Computer-based instruction (CBI) is a method, which uses computer in learning media, strengthening students’ motivation and education process (Kulik, 2013; Serin, 2011; Frenzel, Goetz, Pekrun, & Watt, 2010). CBI may enhance interest and students’ achievement in algebra.

Achievement is the result, the successfulness, the extent or ability, the progress in learning educational experiences that the individual indicate in relation with his/her educational learning (Olga, 2008). Achievement in mathematics, which is the focus of this study, is viewed as a very important factor in teaching and learning of mathematics and it refers to students’ cognitive achievement and psychomotor skills, which are measured in terms of pass or fail (Adebayo, 2008; Popoola & Ajani, 2011). When achievement is below expectation, it is referred to as under-achievement or poor achievement. When students are successful in examination, they will have the feeling of pride that they made success with their own efforts and skill. Small success can give students sense of achievement. On the other hand, persistent failure in dampens students’ interest.

Interest is a zeal or willingness to participate in an activity for which one derives some pleasure (Harbor – Peters, 2002). The influence of interest on students’ learning may vary across

gender. Gender has been identified as one of the factors influencing students' interest and achievement in mathematics (Popoola & Ajani, 2011). Gender issues as a factor or variable are not yet skewed to any direction. There are different findings on gender matters, some in favor of males, others in favor of females and sometimes no gender differences are found (Ifeanacho, 2012).

From the foregoing, the researcher considers it necessary to search for an appropriate approach that is student-centered and that can enhance academic achievement and interest in mathematics. Therefore, this study intends to find if animation could be used to improve students' achievement and interest.

Statement of the Problem

Literature is filled with evidence that teachers are using ineffective methods and strategies in teaching mathematics, which among other factors, have contributed to the students' poor achievement in mathematics especially at the Junior Secondary School Certificate Examination (JSSCE) (Usman and Eze, 2011). Research efforts over the years have not only indicated poor performance in mathematics among senior and junior secondary school students but have shown that the traditional teaching method (like chalk-and-talk) has proved ineffective in achieving the desired achievement of students in algebra (Usman and Eze, 2011). The need to find ways of improving students' interest in mathematics is obvious. Teaching strategies have known to influence students' interest in mathematics. Perhaps, the use of AIP with animation can enhance students' interest in algebra. Can the use of AIP affect the academic achievement and interest of male and female JSS students in algebra?

Objectives of the Study

Objectives of this study were to determine the effect of animated and recorded conventional instructional packages on achievement and interest of junior secondary school students in algebra. Specifically, the researcher sought to:

1. Ascertain the mean achievement scores of students taught with animated and recorded conventional instructional packages.
2. Investigate the effect of animated instructional package on junior secondary school students' interest in Algebra.
3. Compare the influence of gender on mean achievement scores of students taught with animation as measured by the Algebra Achievement Test (AAT).
4. Find out the influence of gender on mean interest scores of students taught with animated instructional package.

Research Questions

The following research questions were raised to guide this study.

1. What is the mean achievement score of junior secondary school students taught mathematics using animated instructional and those taught using recorded conventional packages?

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2. What is the mean interest score of junior secondary school students taught mathematics using animated instructional and those taught mathematics using recorded conventional packages?
3. What is the influence of gender on mean achievement scores of male and female students taught mathematics with animated instructional package?
4. What is the influence of gender on mean interest scores of male and female students taught mathematics with animated instructional package?

Hypotheses

The following null hypotheses were formulated and tested at 0.05 level of significance.

- H0₁:** There is no significant difference in the mean achievement scores of junior secondary school students taught mathematics using animated instructional and those taught using recorded conventional packages.
- H0₂:** There is no significant difference in the mean interest scores of Junior Secondary School (JSS) students taught mathematics using animated instructional and those taught using recorded conventional packages.
- H0₃:** Gender is not significant in the mean achievement scores of male and female junior secondary school students taught mathematics using animated instructional package.
- H0₄:** Gender is not significant in the mean interest scores of male and female junior secondary school students taught mathematics using animated instructional package.

Methodology

Design of the Study

The design of this study is quasi-experimental. Specifically, the non - equivalent control group design involving two groups will be adopted for the study. This design can be used when it is not possible for the researcher to randomly sample the subject and assign them to treatment groups without disrupting the academic programmes of the schools involved in the study (Anaeke, 2007). This design is considered suitable to conduct this study because intact classes (non-randomized groups) were assigned to the two different techniques of AIP and RCI in order to determine comparative effect on achievement and interest of junior secondary school students in algebra.

Population

The population of the study was made up of all JSS2 students in the 135 private schools in FCT numbering 10,979 (Male=4,899 and Female=6,080).

Sample and Sampling Techniques

The sample for the study was made of 80 students randomly selected from two out of the six schools that met the conditions of having functional computers and allows individual use of the computers in the course of the lesson. The six schools were selected through purposive random sampling technique based on the criteria. The two schools were randomly sampled using simple balloting with replacement. All the students in the two arms of JSS 2 in the selected schools

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participated in the study. The two schools are co-educational because gender is a variable. Students in one of the schools were taught using animated instructional package while those from the other school were taught with recorded conventional instructional package.

Instrument for Data Collection

Two instruments for data collection in this study were:

1. Algebra Achievement Test (AAT)
2. Interest Inventory (MII)

The AAT had 40 multiple choice test items developed by the researcher. Each of the multiple choice test instruments had four options (A-D) as possible answers to each question. The items were developed to reflect the concepts treated and in reference to the objectives of the lesson on which the instruction was based. Students were expected to answer the questions in the multiple choice section and the essay section. Each question in the multiple choice section carried one mark while five marks were awarded for each question in the essay section. The questions were drawn from Junior Secondary School Certificate Examination (JSSCE) set by National Examinations Council (NECO). The items measured only the three objectives in the cognitive domain of Bloom's (2014) taxonomy of educational objectives.

The MII was a 24-item interest scale developed by the researcher. It had a four point Likert-type response scale namely: Strongly Agree (SA); Agree (A); Disagree (D); and Strongly Disagree (SD). There were 12 positive and 12 negative statements in MII.

Validation

The 40-item multiple choice with four essay test was validated by five experts. The test items were corrected and modified on the basis of suggestions and recommendations of the experts.

The lesson plans were vetted by two experts in Education. Reliability coefficient of the AAT was determined with Kuder- Richardson formula 21 (K-R 21) and Kendall's coefficient of concordance techniques. The procedure helps to establish the internal consistency of the AAT items. The students' scores were used to compute the coefficient of internal consistency of the AAT which was found to be 0.87 for the multiple choice part. Inter-rater method was used for the essay part and the Kendall's coefficient of concordance was found to be 0.89. Copies of MII filled during trial-testing were scored by the researcher. Cronbach alpha was used to determine the internal consistency of the items and the reliability coefficient. The internal consistency index for interest inventory was found to be 0.75.

Experimental Procedure

The researcher trained the teachers for 10 days while the students were trained for five days. After the training, the researcher observed the teachers in practice session for necessary corrections. The lesson plan covered four weeks on four topics used for the study namely: indices, linear inequalities, probability, and algebraic expressions (expansion and factorization). A total of 16 lesson periods of 40 minutes duration was taught to cover the mathematical concepts used for

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the study. Each student was requested to use the computer system to participate in the lesson. The regular mathematics teachers were instructed on how to use the AIP and RCI. On the first day of the experiment, the test instruments – AAT and MII typed in white coloured paper were administered as pre-test to all the students in both the experimental and control groups. After this, both groups were taught mathematics for a period of four weeks. At the end of four weeks, the questions in the AAT were reshuffled, typed in blue-coloured paper and administered to the students as post-test. The scores of both experimental and control groups in pre-test and post-test were computed for data analysis.

Data Analysis and Results

The research questions were answered using means and standard deviation. The hypotheses were tested at 0.05 alpha level using a 2x2 (mode of instruction and gender) analysis of covariance (ANCOVA). The Scientific Package for Social Sciences (SPSS) program of version 17.00 was used to analyze the data.

Research Question 1

What is the mean achievement score of junior secondary school students taught mathematics using animated instructional package and those taught with recorded conventional instruction?

Table 1: Mean and Standard Deviation of achievement scores of students taught mathematics using animated instructional package and those taught with recorded conventional instruction

Approach	Pretest			Posttest		Mean diff.
	N	\bar{x}	SD	\bar{x}	SD	
AIP	24	10.08	2.98	34.96	12.30	24.88
RCI	56	9.79	3.22	13.11	5.85	3.32

Results in Table 1 show that the group taught mathematics with animated instructional package had a pretest mean of 10.08 with a standard deviation of 2.98 and posttest mean of 34.96 with a standard deviation of 12.30. The difference between the pretest and posttest mean for the group taught with animation is 24.88. The RCI group had a pretest mean of 9.79 with a standard deviation of 3.22 and a posttest mean of 13.11 with a standard deviation of 5.85. The difference between the pretest and posttest mean for the RCI group is 3.32. However, for each group, the posttest means are greater than the pretest means with the animated group having the higher mean difference. This is an indication that animation has some effect on achievement of junior secondary students in mathematics.

Research Question 2

What is the mean interest scores of junior secondary school students taught mathematics using animated instructional package and those taught with recorded conventional instruction?

Table 2: Mean and Standard Deviation of students' interest score taught mathematics using animated instructional package and those taught with recorded conventional instruction

Approach	Pre-interest			Post-interest		Mean diff.
	N	\bar{x}	SD	\bar{x}	SD	
AIP	24	2.40	0.82	2.52	0.85	0.12
RCI	56	2.55	0.91	2.54	0.93	-0.01

Results in Table 2 show that the group taught using AIP had pre-interest mean of 2.40 with a standard deviation of 0.82 and a post-interest mean of 2.52 with a standard deviation of 0.85. The difference between the pre-interest and post-interest mean interest for the animated group is 0.12. The RCI group had a pre-interest mean of 2.55 with a standard deviation of 0.91 and a post-interest mean of 2.54 with a standard deviation of 0.93. The difference between the pre-interest and post-interest mean for the RCI is -0.01. This result indicates that teaching mathematics using AIP significantly increased interest of the students.

Research Question 3

What is the influence of gender on mean achievement scores of male and female students taught mathematics with animated instructional package?

Table 3: Mean and Standard deviation of achievement scores of the Male and Female students who were taught mathematics with animated instructional package

Gender	Pre-test			Post-test		Mean diff.
	N	\bar{x}	SD	\bar{x}	SD	
Male	9	9.22	1.64	42.56	14.44	33.34
Female	15	10.60	3.50	30.40	8.37	19.80

Results in Table 3 show the pretest and posttest mean and standard deviations of students who were taught with AIP. The male students had pretest mean achievement of 9.22 and posttest mean of 42.56 while the female students had a pretest mean achievement of 10.60 and a posttest mean of 30.40. The male students had a standard deviation of 1.64 in the pretest and 14.44 in the posttest while the female students had a standard deviation of 3.50 in the pretest and 8.37 in the posttest.

The difference between pretest and posttest means score for the male students is 33.44 while the difference between the pretest posttest means score for the female students is 19.80. However, for both male and female students, the posttest means are greater than the pretest means with the male students having the higher mean difference. This result shows that animation may have influence on achievement of students.

Research Question 4

What is the influence of gender on mean interest scores of male and female students taught with animated instructional package?

Table 4: Mean and Standard Deviation of interest scores of Male and Female students who were taught mathematics with animated instructional package

Approach	Pre-interest			Post-interest		Mean diff.
	N	\bar{x}	SD	\bar{x}	SD	
Male	9	2.39	0.87	2.53	0.97	0.14
Female	15	2.41	0.81	2.52	0.80	0.11

Results in Table 4 show the pre-interest and post-interest mean scores of male and female students exposed to teaching with the use of AIP. The result shows that the male students had pre-interest mean of 2.39 with a standard deviation of 0.87 and a post-interest mean of 2.53 with a standard deviation of 0.97. The difference between the pre-interest mean and post-interest mean for the male students was 0.14. Result also shows that female students had pre-interest mean of 2.41 with a standard deviation of 0.81 and a post-interest mean of 2.52 with a standard deviation of 0.80. The difference between the pre-interest mean and post-interest mean for the female is 0.11. However, for both male and female students, the post-interest mean scores are greater than the pre-interest scores. This result shows that gender may have some effects on male and female students when exposed to teaching with animation.

Hypothesis 1

There is no significant difference in the mean achievement scores of students taught mathematics using AIP and those taught RCI.

Table 5: Summary of ANCOVA Table of Students in Algebra Achievement Test (AAT)

Source	Sum of Squares	Df	Mean Square	F	Sig.
Corrected Model	8920.66	4	2230.16	37.44	.000
Intercept	4374.67	1	4374.67	73.45	.000
Pre Test	1.51	1	1.51	.03	.874
Group	8649.36	1	8649.36	145.21	.000
Sex	397.26	1	397.26	6.67	.012
Group * Sex	775.74	1	775.74	13.02	.001
Error	4467.23	75	59.56		
Total	44317.00	80			
Corrected Total	13387.89	78			

Table 5 shows that with respect to the mean scores of students exposed to AIP and those exposed to RCI, an F-ratio of 145.21, corresponding to Group, is obtained with associated exact probability value of 0.00. Since the associated probability value (0.00) is less than 0.05 set as level of significance, the hypothesis is rejected. Hence, there is significant difference in the mean achievement score of students taught mathematics using AIP and those taught using RCI.

Hypothesis 2

There is no significant difference in the mean interest scores of Junior Secondary School (JSS) students taught mathematics using animated instructional package and those taught with recorded conventional instruction.

Table 6: Summary of ANCOVA Table of Interest Inventory (MII) scores

Source	Sum of Squares	Df	Mean Square	F	Sig.
Corrected Model	63.25	4	15.81	1887.08	.000
Intercept	.00	1	.000	.05	.824
Pre Interest	63.14	1	63.14	7535.01	.000
Group	.31	1	.31	36.77	.000
Sex	.00	1	.004	.46	.502
Group * Sex	.004	1	.004	.46	.499
Error	.63	75	.008		
Total	577.30	80			
Corrected Total	63.88	78			

The result in Table 6 shows that with respect to interest mean rating scores of students exposed to AIP and RCI, an F-ratio of 36.77 is obtained with associated exact probability value of 0.00. Since the associated probability value (0.00) is less than 0.05 set as level of significance, the hypothesis which states there is no significant difference in the mean rating scores of students exposed to AIP and RCI is rejected. This means that there is a significant difference in the mean interest ratings of students taught with AIP and RCI. Hence, Table 6 shows that the use of computer in teaching is a significant factor in the mean interest rating scores of students who were taught mathematics with AIP and RCI.

Hypothesis 3

Gender is not a significant in the mean achievement scores of male and female junior secondary school students taught mathematics using AIP.

Table 7: Summary of ANCOVA Table of Male and Female Students' scores in Achievement Test (MAT) for those taught mathematics with AIP

Source	Sum of Squares	Df	Mean Square	F	Sig.
Corrected Model	833.357	2	416.678	3.305	.057
Intercept	2548.448	1	2548.448	20.214	.000
Pre Test	2.221	1	2.221	.018	.896
Sex	768.569	1	768.569	6.096	.062
Error	2647.601	21	126.076		
Total	32811.000	24			
Corrected Total	3480.958	23			

The result in Table 7 shows that with respect to the mean achievement of male and female students exposed to AIP an F-ratio of 6.10 is obtained with associated probability value of 0.06. Since the associated probability value (0.06) is higher than 0.05 set as level of significance, the hypothesis which states that gender is not significant in the mean achievement of male and female junior secondary school students taught using AIP is not rejected. Thus, inference drawn is that, gender is not significant in the mean achievement scores of students who were taught mathematics with AIP.

Hypothesis 4

Gender is not significant in the mean interest score of male and female junior secondary school students taught mathematics using AIP.

Table 8: Summary of ANCOVA Table of Male and Female Students' mean interest scores of students exposed to animated instructional package (AIP)

Source	Sum of Squares	Df	Mean Square	F	Sig.
Corrected Model	16.346	2	8.173	724.478	.000
Intercept	.005	1	.005	.455	.507
Pre Interest	16.346	1	16.346	1448.921	.000
Sex	.005	1	.005	.487	.49
Error	.237	21	.011		
Total	169.304	24			
Corrected Total	16.583	23			

Table 8 shows the mean interest scores of male and female students exposed to AIP. Table 8 shows that the value of significance of F (0.49) on the mean interest scores of students due to gender is 0.49 against the level of $P \leq 0.05$ level for one degree of freedom. Since the value of F is higher than that of the alpha set at $P \leq 0.05$, the null hypothesis of no significant difference in the mean interest scores of male and female students exposed to AIP is therefore not rejected. Thus, inference drawn is, gender is not significant in the mean interest scores of students who were taught mathematics with AIP.

Summary of Findings

The major findings of this work are summarized based on the results of the analysis of data.

1. The result of the study showed that the students who were taught mathematics using AIP achieved higher than those taught with RCI.
2. The interest of the students who were taught mathematics using AIP increased while the interest of those who were taught with RCI waned.
3. Male students who were exposed to AIP achieved higher than their female counterparts. However, gender is not statistically significant in achievement.
4. There is a significance difference in the mean achievement and interest scores of students who were taught using AIP and those who were taught with RCI.

Discussion of the Findings

On the basis of result presented, instructional strategy is a significant factor in students' achievement in JSS mathematics. This may generally imply that students' achievement in mathematics is related to the strategy of presenting the concepts.

The mean achievement scores in the post AAT of the students taught with AIP was higher than post AAT mean achievement scores of the students taught with RCI. This shows that the AIP strategy used in teaching mathematics content improved the students' achievement. The difference in achievement of male and female students is higher among the students exposed to AIP. This finding is again confirmed by the result of ANCOVA test, which shows that strategy of instruction is a significant factor on students' achievement in mathematics. As a result, students who were taught mathematics using AIP achieved better than those taught mathematics by using RCI. This may be justified by the fact that the components AIP are highly structured. Moreover, the animation, graphics and sound attract the attention of the students towards learning. The implication is that, mode of instruction used in teaching mathematics is capable of producing effect on students' achievement.

This result is in line with Ubah & Uzoechi (2018), Iji, Abakpa & Takor (2015), Ojo (2015), Ifeanacho (2012), Usman & Eze (2011), Yalcinalp, Geban and Ozkan (2005), Olga (2008), Olusi (2008), and Adebayo (2008). These researchers discovered that exposing students to computer-assisted instruction can improve students' achievement in mathematics. This result also agrees with Usman and Ezech (2011) and Lodree (2005), whose studies revealed a significant difference in mean achievement scores of students taught with two modes of computer – aided instruction. The result revealed that students exposed to teaching with AIP achieved higher than those exposed to teaching with RCI. This result may have been due to the innovative strategy used to convey mathematical concepts which involves 'hands-on and minds-on' on the part of the students. However, students who were exposed to teaching with RCI may have continued with their rote memorization syndrome of learning mathematics which they were used to. They may not have seen any significant change in their teacher's style of teaching at present compared to what they have been used to in the time past even though computer was introduced.

On the other hand, students exposed to teaching with animation were eager to learn considering the way they participated by asking questions and carrying out activities recommended in the package. This implies that the AIP teaching-learning strategy is efficacious in enhancing students' achievement in mathematics. However, this study is contrary to some studies which claimed that there was no significant difference between AIP and conventional teaching methods (Serin, 2011). The result of this study is contrary to consistent findings in literature on mathematics interest which documented that boys are more interested in mathematics than girls (Frenzel, Goetz, Pekrun & Watt, 2010). The result of this study agrees with Adeleke (2007) whose work showed non-significance in interest of boys and girls exposed to two teaching strategies.

Conclusion

Based on the findings of this study the following conclusions are made. The result of this study provided empirical evidence that computer assisted instruction with animation enhanced students' achievement and interest in more than the use of computer to play recorded lessons. Male and female students who were taught mathematics with AIP performed better than their counterparts that were taught mathematics with RCI. Male students taught mathematics with animation performed higher than female students. The interest of the students who were taught mathematics using AIP increased while the interest of those taught with RCI reduced. Male students who were exposed to AIP w achieved higher than their female counterparts. However, the mean achievement scores of male and female students were not statistically significant. There was a significant difference in the mean achievement scores of students who were taught mathematics using AIP and those taught with RCI. There was a significant difference in the mean interest scores of students who were taught mathematics using AIP and those taught with RCI.

Recommendations

The study recommended among other things that since the use of AIP enhances achievement and interest of students in mathematics:

1. Mathematics teachers should use AIP as one of the strategies to be employed in classroom teaching and learning.
2. Parents should provide computers and animated instructional packages on mathematics topics for their children to use at home after school hours.
3. Seminars and workshops should be organized by State and Federal Ministries of Education for mathematics teachers to enable them to learn how to develop software packages and also learn how to use computer in teaching mathematics.
4. Curriculum planners should embrace and include AIP strategies that will bring about improvement in learning, acquisition of creative thinking, problem solving and performance skills in students into the curriculum

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Developing Conceptual Understanding of Irrational Numbers Based on Technology through Activity System

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Abstract: The main purpose of this study is to develop a conceptual understanding of the irrational number of the square root of 2 ($\sqrt{2}$). Participants in the study were 20 ninth-grade male students. Activity Theory was used as a framework to show the development of the conceptual understanding. Since this study was conducted during the COVID-19 pandemic; online teaching method was adopted. In this teaching method, WhatsApp messaging and calculator were used as our basic technology. Virtual education lasted 2 sessions (120 minutes) for the development conceptual understanding of the irrational number of square root of 2. To produce data, WhatsApp Export Capability was used. For data analysis, the online teaching activity system was used. By analyzing this activity system, three tensions were understood. Modifying these tensions, has led to their students making the concept of the irrational number of the square root of 2 ($\sqrt{2}$) and reach a single definition.

INTRODUCTION

In mathematics education, concepts have historical roots. The studies show that teachers and students have problems in understanding irrational numbers such as *square root* (Sirotic & Zazkis (2007b). As many students are not able to provide a uniform and accurate definition of square root (Sirotic & Zazkis, 2007a), because school mathematics has focused on problem-solving techniques. This has caused irrational numbers, such as square root, to have been neglected in school mathematics (Fischbein, 1995).

A review of studies on teaching irrational numbers shows that teachers use a variety of methods to teach it. One of the methods that teachers use to introduce an irrational number is to use the example $\sqrt{2}$ and π (Zazkis, 2005). The second method that studies show is to use the definition

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of the perfect root, in other words, the irrational number is a number that does not have a perfect root (Shinno, 2018; Patel & Varma, 2018). But the formal definition of the irrational number that most people use is as follows:

A real number that is not a rational number is said to be irrational (Rosenthal, Rosenthal, & Rosenthal (2014), p.66)

On the other hand, the irrational number $\sqrt{2}$ can be considered an irrational number by the triangle chord, where the point of intersection of a circular arc with radius 1 with the axis of real numbers indicates its geometric location that is shown in Fig. 1.

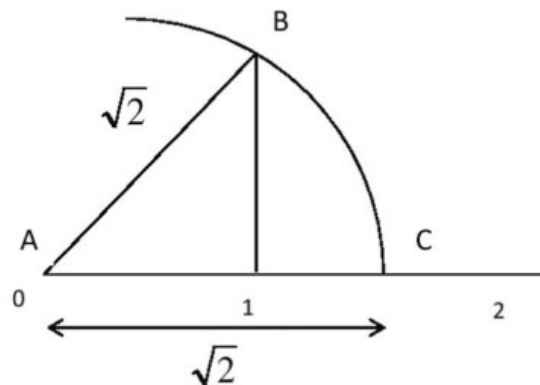


Figure 1. Geometric drawing of the irrational number $\sqrt{2}$ (Kidron, 2018, p.66)

In other words, school mathematics places more emphasis on instrumental understanding and less emphasis on conceptual understanding of mathematical concepts. This focus has led research to show that students have difficulty understanding this set of irrational numbers. In addition to students, teachers have difficulties understanding the concept of irrational numbers. In this regard, Sirotic & Zazkis (2007b) in their study showed that even mathematics teachers have a misunderstanding of the concept of irrational numbers. Hence, the concept of irrational numbers is perceived to be a challenging concept for teachers and students. On the other hand, the importance of irrational numbers can be described in such a way that without understanding it, one cannot understand the meaning of a set of real numbers (Sirotic & Zazkis, 2007a). Therefore, understanding the concept of irrational numbers is essential for students, especially in junior and high school (Voskoglou & Kosyvas, 2012). Given the importance of this issue, the present study focuses on understanding one of the irrational numbers, the *square root* of the number 2 ($\sqrt{2}$).

From what has been said in the previous section, it can be concluded that teaching the concept of irrational numbers is a fundamental issue for mathematical education. But the most important issue that is gripping most societies today is the prevalence of the Covid-19 pandemic. The Covid-19 has affected the political, technical, and pedagogical aspects of education worldwide (Breda et al., 2020; Cahapay, 2020). The challenge posed by the Covid-19 pandemic has affected face-to-face teaching, causing teachers and educators turning to online teaching. This issue has opened a new field in education (Borba, 2021). Education in this area has become very difficult due to its complexities and has become an important issue for teachers and students (Cahapay, 2020). Hence, teaching concepts such as irrational numbers, in which teachers and students themselves have difficulty, has doubled the difficulty of teaching during COVID-19 pandemic era. On the other hand, the threat posed by education due to the spread of the Coronavirus has become an opportunity to use technology that can be used to teach mathematical concepts (Attard & Holmes, 2020). Therefore, the issue of technology is very important learning tool in this era. In this regard, the importance of technology has led NCTM (2000) to consider it as one of the important principles and standards of school mathematics and to express it as follows:

***Technology:** Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning (p.11).*

What highlights the role of technology in this era is its interactive nature between teachers and students (Kaput & Thompson, 1994) that can be used to engage students and improve education. Students are perceived to have a positive attitude towards the use of technology (Eyyam & Yabatan, 2014). This positive attitude can be led to more interaction. In addition to a positive attitude, technology can improve students' understanding of mathematical concepts. In this regard, Dogruer and Akyuz (2020)'s study showed that using technology could improve students' understanding of difficult mathematical concepts and improve their learning comprehension. For example, one of the simplest technologies that students use is the calculator, according to Satianov (2015), review of studies about the calculator, shows that few studies have used calculator to teach mathematical concepts. But with the outbreak of the Coronavirus, real-world classroom space has given way to virtual spaces (Mulenga & Marbn, 2020). Teachers and students in virtual space use chat-enabled software to interact with each other. The latest published statistics related to the most popular global mobile messaging apps 2021 shows WhatsApp Messenger with 2 billion users, has the most users among messengers in the figure below:

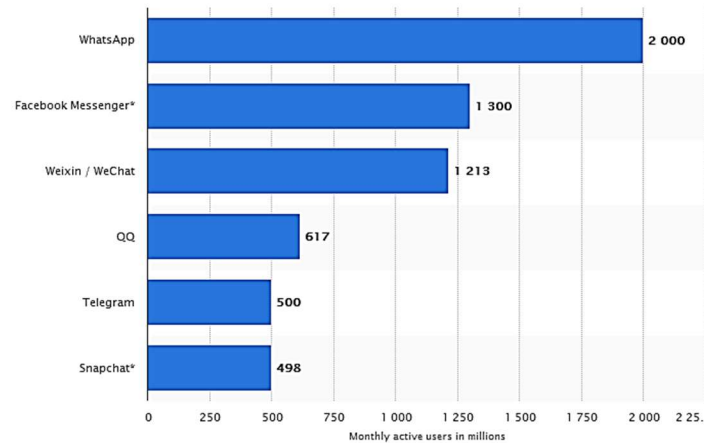


Figure 2. Percentage of users of the most famous messengers in the world (cited *in web site*: <https://www.statista.com/statistics/258749/most-popular-global-mobile-messenger-apps>)

In this regard, Barhoumi (2015) showed that WhatsApp Messenger has good features for teaching such as interaction, creating a class group, sharing. It can be concluded that there is a good interactive space in this messenger that can be used for educational purposes. Therefore, the main purpose of this study is to answer the following question:

- ❖ How can technology be used in a mathematics classroom to develop a better understanding of the concept of *square root* of 2 ($\sqrt{2}$) during the Covid-19 era?

Theoretical Framework

As mentioned in the previous section, the purpose of this study is to develop a conceptual understanding of the irrational number. For this purpose, it is necessary to examine the structure of students' collective activity. In this regard, Engstrom 1987 defines the structure of collective activity in the form of activity system of Activity Theory. The activity system consists of 6 components: Tools, Subject, Rules, Community, and Division of labor and Object. The result of the interaction of these 6 components with each other is the output of the activity system. These 6 components, along with the system output, can be seen in the following Figure:

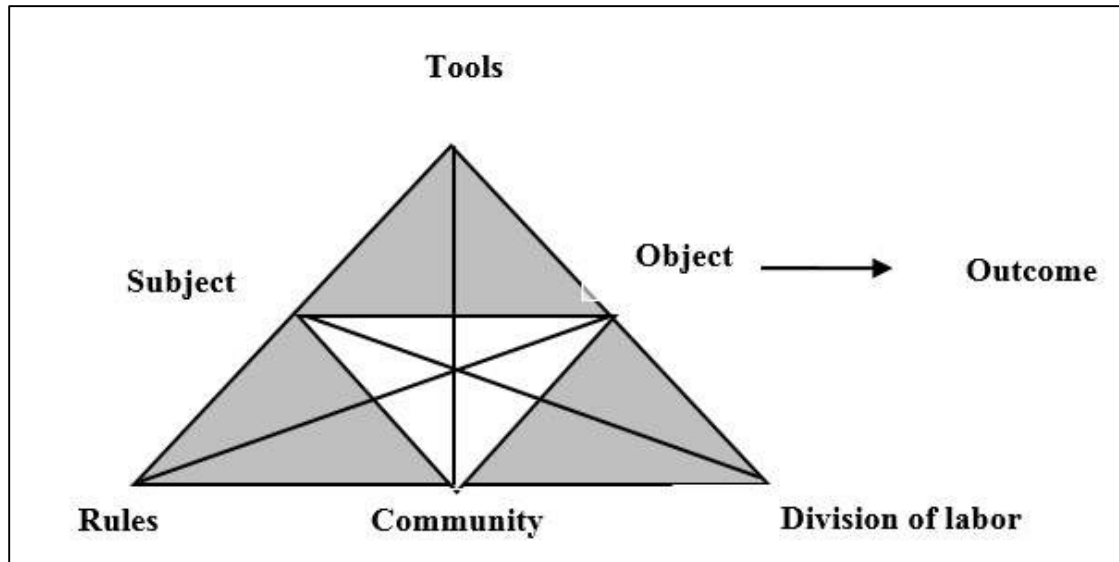


Figure 3. The structure of human activity (Engestrom, 1987 cited in Jurdak, 2006, p. 288).

As mentioned in the introduction, this study was conducted during the Coronavirus period, in which teaching was temporarily conducted online. On the other hand, studies show that the activity system can be used to analyze classroom interactions (Salloum and BouJaoude, 2020, Huang & Lin, 2012)

Therefore, it is necessary to define an activity system for online teaching that includes classroom interactions. In this regard, Barhoumi (2015) considered three levels of activity system for online teaching: The technological level, the individual level, and the community level which showed in below Figure:

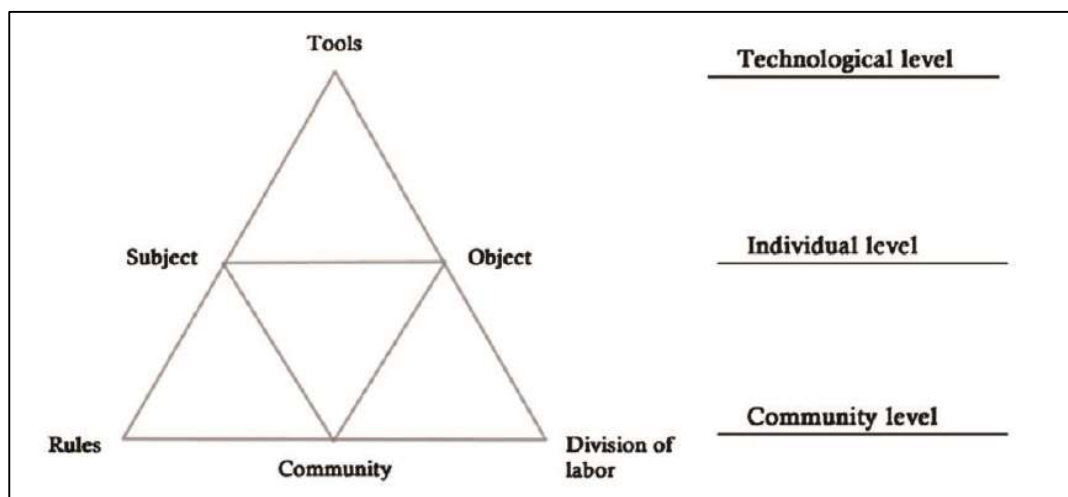


Figure 4. three levels of Activity Theory for Online teaching (Barhoumi, 2015, p.221)

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On the other hand, before defining these three levels, it is necessary to define the 6 components of the activity system of Figure 4. The definition of these 6 components is defined as the following 6 principles:

- *The first principal is the orientedness of the object. The objective of the activity system has social and cultural properties in the system, such as collaborative or cooperative learning in an online course.*
- *Subjects are actors engaged in activities. This is considered the individual level of Activity Theory; students are contextual subjects engaged in collaborative learning.*
- *Community or externalization is considered a social context of the system and a community level of Activity Theory; all actors are involved in the activity system (e.g., a group of students engaged in learning based on social interaction for constructing and sharing of knowledge is an example of a learning community).*
- *Tools are considered a technological level of Activity Theory. In the system, communication between communities is mediated by tools that transmit social knowledge. It includes the artifacts used by actors in the system. Tools influence actor-structure interactions and are influenced by culture.*
- *The division of labor is a considered a hierarchical structure of activity or the division of activities among actors in the system.*
- *Rules are the conventions and guidelines regulating activities in the system, such as rules of discussion between students in collaborative learning (Barhoumi, 2015, p.221)*

Having defined the components of the activity system, it is necessary to define the three levels mentioned in Figure 4 in the field of online teaching that is the subject of the study. In Figure 4, the first level is the technology level. In the present study, this level was defined as the use of WhatsApp Messenger to conduct virtual classes and calculator to perform mathematical calculations. The second level of Figure 4 is the individual level. In fact, the individual level is the same activity that students do in the WhatsApp virtual classroom. The third level is the collective level. Students can benefit from each other's comments, questions, and whole classroom discussions to better develop an understanding of the concept of irrational numbers and make it a comprehensible concept. Given these levels that were defined for Online teaching activity system, then, this activity system needs a driving force to use it to develop this concept of irrational number. In studies of Activity Theory, researchers cite tension as the driving force behind the system and see it as a source of development.

Given that the topic of this study is the irrational number and this particular topic is perceived to be challenging, and thus it is necessary to analyze the tensions that occur in the operating system. In an activity system, researchers state that tensions occur either within the components of the

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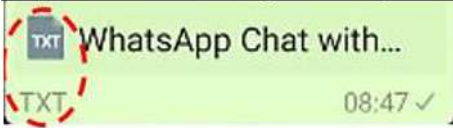
system or between its components (Engestrom, 2001). The tensions that occur within the components of the activity system are called level one tension and, the tensions that occur between the components of the activity system are called level two tensions (Engstrom, 2015). Therefore, in this study, students are involved with the concept of irrational number by creating level 1 and 2 tensions of the activity system (*square root*).

Methodology

The theoretical framework of the study is based on Activity Theory. On the other hand, Activity Theory is the nature of qualitative work and is used to analyse in qualitative methodological issues (Yamagata-Lynch, 2010). Hence, Activity Theory is a framework for qualitative data analysis (Hashem & Jones, 2007). Researchers use the method of collecting interview data, observing ethnographic methods, and case studies for the activity systems of Activity Theory (Russell, 2009). They provide evidence for perceived tensions in operating systems by referring to interview quotes and descriptive narratives (Hashem & Jones, 2007). On the other hand, data collection method is related to the time when face-to-face training was available to the research participants. But nowadays, with the spread of the coronavirus, conventional data collection methods cannot be used. Instead, the capacity of cyberspace can be used to collect data. In this regard, WhatsApp Messenger has a great ability to generate text data with images. This is how the Export chat WhatsApp is more, the production process by which data will be described. In this regard, first, the virtual classroom environment is described as follows:

In this virtual class, the first author of the research was a teacher who has 9 years of experience teaching mathematics in junior and high school. The teaching method adopted in this virtual class was based on the constructivist learning method. In other words, in this virtual classroom, the classroom environment was such that students built their knowledge by talking, discussing, and typing their views on mathematics. All students' activities in the group were recorded by WhatsApp Messenger. Students responded to the teacher's question in a written format (using symbols, letters, and pictures). All of the students' responses to the teacher's questions were documented as a source of data collection. The data was generated as a Word File under the following process by WhatsApp export chat messenger. The stages of production of study data were identified step by step with the following numbers:

 <p>Step 1 In this step you click on the three dot symbol in the right corner, this image appears with 6 items that you click on more</p>	 <p>Step 2 In this step, click on option 4, ie Export chat</p>
 <p>Step 3 In this step, you can choose one of two options, which is usually the first option</p>	 <p>Step 4 At this point, it will ask you how to save data, you can choose the same WhatsApp.</p>



Step 5
In this step, text data with txt extension is generated

3/28/21, 21:05 - Student8:I sent

3/28/21, 21:05 - Teacher: Send others

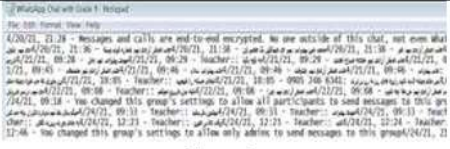
3/28/21, 21:06 - Student:

1.420,1.410
21:41


3/28/21, 21:06 - Student5>: 1/41421356

3/28/21, 21:06 - Teacher: Excellent

3/28/21, 21:06 - Student2>



Step 6
In this step, by opening the text file, select all the text data and copy it to the Word file.



Step 7
In this section in Office Word software, you can view the text data and add the images shown in the text to generate text data with pictures.

Figure 5. The process of generating text data with images by using WhatsApp Export

Since the study uses the activity system of Activity Theory as a tool of analysis, in this regard, each activity system is defined as a unit of analysis (Engstrom, 2001). The process of data analysis by activity systems can be described as follows:

- **Step 1:** First, the elements of the activity system are defined according to the topic of the study (definition of the components of the activity system).
- **Step 2:** After defining the components of the activity system, the researchers refer to the study data to understand the tensions that occur in the activity system. Finally, tensions are defined at this stage (definition of tensions).
- **Step 3:** In this step, the sources of tensions are identified (identify sources of tensions)
- **Step 4:** After identifying the sources of tensions, the researchers refer to the study data again to find clues from the tensions modification so that they can refer to it in the study findings (finding clues from the tensions modification).

Therefore, in this study, these steps were performed in the order shown above:

- **Step 1:** First, according to the study topic, the study activity system in the field of online teaching was defined.
- **Step 2:** At this stage, the entire study data were read several times to understand the tensions that had occurred in the classroom for the students.
- **Step 3:** In the next step, the sources of tensions were identified.

- **Step 4:** Tensions, evidence related to their correction was collected. Given that the subject of the study is the development of the conceptual understanding of the irrational number. In this regard, evidence of modification of tensions was collected, which in the findings of the study shows how students using They were able to modify these tensions and develop the concept of the irrational number.

After stating how to analyze the study data, it is necessary to explain the details of the research participants, the virtual class (time, number of sessions, subject of teaching, etc.) through which the study data was generated. Participants in this study were 20 ninth-grade male students from three schools who volunteered to participate and their work be used for the study. Students attended this virtual class for one hour from 9 pm to 10 pm for 14 nights from April 3 to 17, 2021. A total of 840 minutes of virtual class was held for the total irrational numbers. In these 14 sessions, students were taught the concept of irrational numbers: Square and Cube roots, addition and subtraction of them. Since the volume of study data was very high, this study has focused only on a part of this data related to the development of the concept of *square root* of 2. The development of this concept took two sessions. These two sessions are related to April 3 and 4, 2021. The times of these two sessions are illustrated minute by minute in the study findings section. In this regard, the output of the conversations that the teacher and the students had at that time, turned into a 120-page Word Document, which due to the high volume of this data, only a selective part is selected in the study.

Results

In this section, the findings related to the analysis of perceived tensions and their modifications related to the online teaching activity system of the concept of *square root* of 2 are presented. In this regard, the elements of this system of activity are defined as follows:

- **Tools:** The technological tool in this activity consisted of both WhatsApp Messenger and students' calculator.
- **Community:** the community of this system is all classroom that all the people in the class together form.
- **Subject:** In this system, the subject is the students of the class who, by dividing the work, advance the classroom in a participatory manner.
- **Object:** The object is the understanding of the concept of the irrational number by all students of the system.
- **Rules:** The teacher used WhatsApp to control locking and opening the group at pre-determined intervals).
- **Division of labor:** WhatsApp Messenger as a technology tool enabled the classroom teacher to be able to manage the virtual classroom teaching, meaning, who when students are able to contribute writing in the group chat and when the chat was momentarily disabled.

4-21:08-Students 3, 1, 15,10, 20 and 8: 2, -2 and 2^2

5-21:09 - Teacher: Excellent. So far, you have noticed that the answer is both positive and negative. Our focus is on positive numbers:

$$(2) \times (2) = 4 \quad , (-2) \times (-2) = 4$$

6-21:12 - Teacher, think more about the next question and do not answer too soon. Find a number that when it is multiplied by itself twice, will result in 2?

7-21:16 - Student3: 2×1

8-21:16 -Teacher: Very well done. This number is good, but since the two numbers are not the same, the answer is incorrect. The question is whether the two numbers are the same. Please others say their view of point about this question, even if it is not true.

9-21:16- Student8: $(-2) \times (-2)$

10-21:17 - Studentt20: 1.2×1.2

11-21:17 - Students 1 and 8: 4, 2^1

12-21:21 - Teacher: These 4 and 2^1 are good. But the problem is that multiplying by itself twice is 4, if the question is asked; the product of 2 numbers is 2.

13-21:21 - Student 1: 1.3×1.3

14-21:22 - Student20: 2^0

15-21:22 - Teacher: It is good hearing all your answers. I just observed that among the answers that are given, someone said we shall multiply 1.2 by 1.2. What is the product of this?

16-21:23 - Student20: 1.04

17-21:24 - Teacher: You can use your calculator

18-21:24 – Students 8,20,3 and 1: 1.44

19-21:25 - Teacher: This is a good number, but find a number whose product is closer to 2

20-21:26 – Students 8, 20, 15 and 3: $1.4 \times 1.4 = 1.96$

21-21:27 - Student1: $1.2 \times 1.2 = 1.44$

22-21:28 - Studen8: $1.5 \times 1.5 = 2.25$

23-21:28 - Teacher: Congratulations to all. So far, most of you have said 1.4 and one person has said 1.5. ($1.4 \times 1.4 = 1.96$ $1.5 \times 1.5 = 2.25$)

24-21:29 - Teacher: Now tell me what is the next number in between. I will lock the group and open it for another two minutes. You can send your answer once the group is opened

25-21:29 - Close to this group

26-21:31 - Open to this group

27-21:31 - Studen8: This number is 1.42 .

28-21:31 - Student 1: 1.41

29-21:31 - Teacher: Let everyone to express their opinion.

30-21:31 – Students 3 and 20: it is between 1.40 and 1.50.

31-21:31 -Student15: What do you mean? I do not understand anything.

- 32-21:32** - Student20: I mean the next number that we multiply twice by itself is the result of two is between two numbers 1.40,1.50.
- 33-21:32** - Student3: Multiply by itself, see it approaching two?
- 34-21:32** -Student15: Ah!
- 35-21:32** - Teacher: So the next number is between1.40,1.50 . Now choose a number from one point four to one point five. Multiply it twice by itself to get two.
- 36-21:32** - Studen10:, this number $1.41 \times 1.41 = 1.9881$ is closer to two
- 37-21:32** - Studen8: I suggest these two numbers 1.412,1.416 :
 $1.416 \times 1.416 = 2.005056$ $1.412 \times 1.412 = 1.993744$
- 38-21:32** - Teacher: For now, say up to two decimal places. You do not need three decimal places now.
- 39-21:33** – Students 20, 3 and 10: 1.41
- 40-21:33** - Teacher: Why did you all choose one number? I choose 1.43
- 41-21:34** -Student15: This message was deleted
- 42-21:34** - Studen10: Because the other numbers that are multiplied by themselves are greater than2
- 43-21:34** - Student20: Because the product of other numbers is more than two.
- 44-21:34** - Teacher: So now, calculate the product of the numbers and see which of these numbers are closer to two? $1.41 \times 1.41 = ?$ $1.42 \times 1.42 = ?$ $1.43 \times 1.43 = ?$
- 45-21:34** - Close to this group
- 46-21:35** - Open to this group
- 47-21:36** – Students 3, 20 and 1: $1.41 \times 1.41 = 1.9881$
- 48-21:36** - Teacher: Send the answer to all these three multiplications above
- 49-21:37** - Student20: $1.42 \times 1.42 = 2.0164$
- 50-21:37** – Students8 and 5: $1.43 \times 1.43 = 2.0449$
- 51-21:37** - Teacher: Now that you multiplied these three numbers, the product of which number is closer to two?
- 52-**(21 : 37,21 : 38) - All students: The product of the first number is closer to the number 2
 $1.41 \times 1.41 = 1.9891$ $1.42 \times 1.42 = 2.0164$ $1.43 \times 1.43 = 2.0449$
- 53-21:38** - Close to this group
- 54-21:39** Teacher: Now tell me, the product of which two numbers gives us 2?? I will open the group in two minutes
 $1.41 \times 1.41 = 1.9891$ $1.42 \times 1.42 = 2.0164$ $1.43 \times 1.43 = 2.0449$
- 55-21:41** - Open to this group
- 56-21:41** - Teacher: Please let everyone know, what are the two numbers whose product is the closest to 2?
- 57-**(21 : 41,21 : 32) All students: Between these two numbers 1.41,1.42

58-21:44 - Teacher: So choose a number between these two numbers. Multiply twice by it so that its product is even closer to two. Use your mobile calculator: 1.410,1.420

59-21:47 – Students 8, 20 and 3: $1.414 \times 1.414 = 1.999396$

60-21:48 – Students 1 and 15: $1.415 \times 1.415 = 2.002225$

61-21:48 - Student 4: $1.4142 \times 1.4142 = 1.99996164$

62-21:49 - Teacher: So now multiply 1.415 by itself, the result is as follows?

$$1.415 \times 1.415 = 2.002225$$

63-21:50 - Student 1: gets bigger than 2

64-21:50 -Teacher: Well, $1.415 \times 1.415 = 2.002225$ and $1.414 \times 1.414 = 1.999396$. Let us choose the number 1.414. We are examining a number that is slightly bigger than 1.414 that when we multiply it by itself would result us in obtaining a number even closer to 2. Now let us discover the fourth digit of this number? What could it possibly be?

65-21:52 – Student 3: 1.4142

66-21:54 - Teacher: Others please feel free to express opinion. We are not looking for the right answer. We want all your opinions to reach a conclusion. Remember your opinion is important for all of us

67-21:55 - Students 1, 15, 20 and 6: $1.4142 \times 1.4142 = 1.99996164$

$$1.4142 \times 1.4142 = 2.00034449$$

68-21:56 - Teacher: Well, then you all got the fourth digit. Excellent, amazing. Can we still get closer to 2 by finding the fifth digit?

69-21:56 – Students 10, 20, 1, 15, 3: one or two

70-21:57 - Teacher: Now let everyone try test their guess

71-21:57 -Student3: If we choose the fifth digit of be two, then the product $1.41422 \times 1.41422 = 2.0000182084$ will be more than two

72-21:57 Student 6: I got the same answer

73-21:57 Students 3,1 and 20:The fifth decimal place $1.41421 \times 1.41421 = 1.99998992$ becomes 1

74-21:59-Student15:If the fifth decimal place is doubled, this $1.41422 \times 1.41422 = 2.0000182084$ product is greater than two, so the fifth decimal place becomes one, that is 1.41421

75-21:59 - Teacher: Excellent. So the fifth decimal place (1.41421) is the number one

76-22:00 - Teacher: Now can you guess the sixth digit? I will open the group chat in two minutes

77-22:02 -Student15: $1.414211 \times 1.414211 = 1.99999275$. The sixth digit becomes one

78-22:02 Student 6: the number becomes 2 ($1.414212 \times 1.414212 = 1.99999558$)

79-22:02 - Student20: $1.414213 \times 1.414213 = 1.99999841$

80-22:02 - Student 1: So the sixth digit of the decimal point gets even closer to two

81-22:02 – Students 8 and 3: The sixth digit becomes three

82-22:03 - Teacher: So the sixth digit is 3: 1.414213

83-22:03 - Teacher: Find the seventh decimal place?

84-(22 : 00,22 : 08). All students: The seventh digit of the decimal is 5:

$$1.4142135 \times 1.4142135 = 1.99999982$$

85-22:08 - Teacher: 1.4142135. Calculate the eighth digit in the next session

$$1.4142135 \times 1.4142135 = 1.99999982$$

Session 2: at 4 April 2021

86-(21 : 00,21 : 06) : All students in the eighth digit become 6:

$$1.41421356 \times 1.41421356 = 1.99999999$$

87-21:06 - Teacher: Excellent. Now find the ninth digit of this number

88-(21 : 07,21 : 11).All students: The ninth decimal digit is two:

$$1.414213562 \times 1.414213562 = 1.999999989$$

89-21:11 - Teacher: Excellent. You are all coming up with an answer, fine

90-21:12 - Teacher: The ninth digit was obtained. Now, what would be the tenth digit?

91.(21 : 11,21 : 13) All students: The tenth digit of the decimal number becomes 3:

$$1.4142135623 (1.4142135623 \times 1.4142135623 = 1.9999999979325598129)$$

92-21:15 - Teacher: Excellent, very well! Now, find the eleventh digit of 1.4142135623

93-21:15 - Student4: No

94-21:15 - Student10: The calculator gives an error

95-21:15 - Student4: I cannot count with the calculator, the calculator gives an error

96-21:15 -Student15: No more than me. He says enough.

97-21:15 - Student3: I cannot calculate more than ten decimal places with this calculator

98-21:15 - Teacher: How was the calculator?

99-21:16 -Student15: the calculator tired!!!!

100-21:16 - Student3: No more than 10 digits

101-21:16 - Student4: No, not the eleventh digit. Unless we have a different program to utilize.

102-21:16 - Teacher: Did your calculators give an error?

103-21:16 - Student 1: Yes, the calculator stopped me

104-21:16 -Student15: made worse than me

105-21:17 - Teacher: Well, you finally challenged the calculator

106-21:17 - Student 1: The calculator gives an error

107-21:17 - Teacher: Now how do we get the eleventh digit?

108-21:18 - Student3: I do not know. This is an imaginary number

109-21:18 - Student10: We've got to use our minds

110-21:18 -Student15: How many decimal places do you want to get?

111-21:18 - Student4: Let's go, we do not have the patience to calculate this number

112-21:19 - Teacher: Try calculating it without a calculator

113-21:19 - Student 1: No matter how much we multiply, we will not reach two

114-21:19 -Student15: By the time we finish multiplying will be morning.

115-21:19 - Student4:?

116-21:19 - Student3: Very hard. How do we find the eleventh decimal place?

117-:20 -Teacher: Do you think the eleventh digit of the number 1.4142135623 can be guessed or not? What is this number called when no matter how much depth we go into, we cannot multiply it by itself to get 2? Think for two minutes

118-21:22 - Student3: No, it can't

119-21:23 -Student15: irrational number

120-21:23 - Studen4: I think this number is an irrational number

121-21:23 - Teacher: What does an irrational number mean?

122-21:23 - Student 1: We cannot talk about it with certainty

123-21:24 - Student20: It means a number whose decimal point becomes infinite

124-21:24 - Student4: irrational number means unknown or does not tell you the exact meaning.

124-21:24 -Teacher: Do you think this number is the last decimal place of that number 1.4142135623?"

125-21:25 – Students 4, 20, 15, 3 and 1: No

126-21:25 - Student4: Because it is an irrational number

127-21:26 - Student4: to think for the rest of our live

128-21:26 - Student17: This number is vague

129-21:26 -Student15: Not known. This number is vague for us

130-21:27 - Teacher: The result. Multiply a number by itself. The result is two. An unknown number. We only know its decimal places. We have ten decimal places for this account number. The calculator can no longer count this number, as some calculators cannot count more than ten decimal places. So this number is unknown or it is the same as the irrational number. Write the symbol of this irrational number.

131-21:31 – Students 4, 1,6, 15,17 and 20: *Square root of 2*

132-All students say *Square root of 2*: $\sqrt{2}$

Figure 6. Creating tension and modifying it in the online teaching activity system the concept of the irrational number of $\sqrt{2}$

In this section, tensions and their modification are discussed in detail by mentioning the evidence. Students gradually understood and constructed the concept of irrational number, which is described in the process illustrated in Figure 6. At the beginning of this process in line 1, the teacher asks the students the first question, "**Find a number that when multiplied by itself result in 1?**". Asking this question engaged students to collaborate in the virtual classroom, and to came up with the answer of the number one and the negative one. Next, the class teacher raises the level of the previous question a little, and this time asks the question, "**What is 'the number' that when multiplied by itself would give us 4?**" (line3). Students offered their answers in line 4.

The first tension is created by the teacher's question "Find a number that when it is multiplied by itself twice, will result in 2?" (line6). This tension is between students, which is level 1 from

tensions of the activity system. This question engages students in the classroom and causes students to come up with different answers that are not close to the answer to the question. Students express various answers such as 2^0 , 2^1 , 1.3 and 1.2. Among the answers given by the students, the answer of 1.2 (line10) provides a clue to make the square root of 2, which the teacher uses it to ask students to multiply 1.2 by 1.2 "It is good hearing all your answers. I just observed that among the answers that are given, someone said we shall multiply 1.2 by 1.2 "What is the product of this?" (line15). Students calculate the product of these two numbers as 1.44. In line 19, the teacher asks the students to "**find a number whose product is closer to 2**". Most students have chosen 1.4 as their answer with the exception of only one student obtaining 1.5 (line22). The same different answers of the students⁸ helped to complete the class discussion and redirected the whole class's attention to get closer to the answer of Q_1 question which subsequently reduced the intensity of the tension that this question had created among the students.

In line 29, the teacher asks the student to get the second digit of 1.4. Students give answers of 1.41, 1.42. "I do not understand anything," says one of the students (student 15, line31). This opposite answer of the student helps to clarify the answer of the question and causes one of the students to guide the above student, which means the next number, the number between 1.40 and 1.50 (line32). The same guidance causes the student to notice and say the word "Ah" (line34). Letting students talk can address any mis-conceptions or clarify their understandings (Farsani, 2015).

This causes other students to express the three numbers 1.41, 1.42 and 1.43. Next, the teacher asks them to calculate the product of these three numbers 1.41, 1.42 and 1.43 when multiplied by themselves and to explore which product is closer to 2 (line44). All students calculated (using calculator) and suggested that only the product of 1.41 is the closest to 2, and the products of both 1.42 and 1.43 are greater than 2. Therefore, all students on line 52 choose 1.41. So far, students have discovered two decimal places in the answer to the question. Next, for the third decimal place, students present two numbers 1.415 and 1.414, which by calculating the multiplication of these two numbers by themselves and comparing their answers; students choose the next number to be 1.414. Students then obtained the fourth decimal place of this number in 5 minutes (lines65 to 68), which became 1.4142. As can be seen, the students were able to calculate the fourth decimal place in less time by collaborating.

The teacher asked students to test their guess to get the fifth decimal place of this number. Most students suggest the numbers 1 and 2 for the fifth decimal place, which was eventually, reported the number 1 as the fifth decimal place. Calculations were a key to the students in obtaining the fifth place value (lines71 to 75). By getting the number 1.41421, the teacher asked the students to find the sixth decimal place. The students, with the spirit of their collaborative participation achieved the seventh, eighth, ninth and tenth digits in a much shorter time. It is important to note that students obtained the first 4 digits (first, second, third, and fourth) in 39 minutes, while they obtained the next four digits (fifth, sixth, seventh, and eighth) in 20 minutes. Student's flow of progression consists of their use of collaboration, the use of WhatsApp messenger as a platform

where students could both express and view other peers' responses. As a consequence, they could evaluate their responses in relation to their peers' responses:

- Students gain the number 5(1.4142135) as the seventh digit of the number 1.414213, in 8 minutes (22 : 00, 22 : 08).
- Students gain the number 6(1.41421356) as the eighth digit of the number 1.4142135, in 6 minutes (21 : 00, 21 : 06).
- Students gain the number 2(1.414213562) as the ninth digit of the number 1.41421356, in 4 minutes (21 : 07, 21 : 11).
- Students gain the number 3(1.4142135623) as the tenth digit of the number 1.414213562, in 2 minutes (21 : 11, 21 : 13).

Students were able to overcome the first tension (T_1) created by question Q_1 and obtain the approximate answer to question Q_1 at 1.4142135623 with the help of a calculator. This answer shows that students have been able to gain a greater understanding of the concept of square root of 2 by modifying this tension to a large extent. Next, the teacher created the second tension (T_2) in the system by asking the second question (Q_2), "Calculate the eleventh digit of 1.4142135623" (line137). This tension (T_2) is of the first level tension, e.g. it is of the intra-component tension of the activity system. This tension has challenged technology. When the teacher asks the students to count the eleventh digit of the number as 1.4142135623, the answer given by the students indicates the tension in the technology itself.

For example, phrases such as "the calculator give an error, the calculator tired! (line99). The calculator gives an error (line106), the calculator stopped me, etc. (line103)" illustrates that the calculator as an instrument of technology itself is challenged and which subsequently created a technological tension. On the other hand, the design of the second question (Q_2), in addition to creating a second tension in technology, has caused in Figure 6-1 the third tension (T_3) between students and technology (calculator) which is a type of second level tension of the activity system. This tension can be expressed in sentences such as "I can't calculate more than ten decimal places with this calculator, I can't count with the calculator," (lines95) is apparent throughout the lesson. The teacher uses the same sentences of the students to correct the tensions and asks them to answer the question, "Do you think the eleventh digit of the number 1.4142135623 can be guessed or not? (line117)". By asking this question enabled showing students' understanding by expressing comments like:

- 21:22 - Student3: No, it can't (line 118)
- 21:23 -student15: irrational number (line 119)
- 21:23 - Studen4: I think this number is irrational number (line 120)
- 21:23 - Teacher: What does an irrational number mean? (line121)
- 21:23 - Student 1: We cannot talk about it with certainty (line122)

- 21:24 - student20: It means a number whose decimal point becomes infinite (line1623)
- 21:24 - Studen4: irrational number means unknown or does not tell you the exact meaning. (line124)

Students refer to an important point in these sentences: the word "irrational number" which they explicitly refer to and define. Students define this number as follows: "An irrational number is an unknown number that cannot be talked about with certainty; its decimal number becomes infinite" (line123). Next, the teacher asks the students the question, "Do you think this number is the last decimal place of that number 1.4142135623?" (line124), causes students to complete the concept of an irrational number that is unknown (lines125). In general, in this process, the student concluded that the answer to question Q₁ is an irrational number. In the last part of this process, the teacher asks the students to specify the mathematical symbol of this irrational number (line130). The symbol that the students introduce is the symbol of the square root of two ($\sqrt{2}$). Students were empowered to construct the conception of the irrational number of $\sqrt{2}$.

Discussion and Conclusion

This study explored the concept of the *square root* of two during the Coronavirus pandemics that he algebraic proof is as follows:

Agarwal, & Agarwal (2021) use inverse reasoning to prove it. Suppose radical two is an expressive number. Therefore, there are two integer numbers, p/q , where q is zero and p, q are co-primes that have the following relationships.

$$\sqrt{2} = \frac{p}{q} \Rightarrow p = \sqrt{2}q \Rightarrow p^2 = 2q^2 \Rightarrow \begin{cases} p = 2k \\ q = 2k \end{cases} \quad (1)$$

Given that Equation (1), p, q are multiples of two numbers, this is in contradiction with their being prime. Therefore, radical two is an irrational number.

For this purpose, technology was used to explore this concept. In this regard, students using the concept of irrational numbers (lines1, 3, and 44) could determine the approximate amount of the square number of two. This is consistent with the findings of Agwu (2007). He used the upper and lower bounds to teach the irrational number in his study. However, the difference between the two studies can be expressed as follows: In the present study, the upper and lower bounds were used to develop a conceptual understanding of the irrational number, but in Agwu (2007), the upper and lower bounds were used to develop the procedural understanding of the irrational number.

As mentioned in the introduction, researchers use different methods to teach irrational numbers (Sirotic & Zazkis, 2007a, Shinno, 2018; Patel & Varma, 2018; Protasov et al., 2009; Zazkis, 2005). Despite such showed teaching methods, students have difficulty with irrational numbers (Sirotic & Zazkis, 2007a). But the findings of this study showed that with the help of each other and with

the teacher's guidance, students can reach a comprehensive and uniform understanding of the definition of the irrational number.

The findings showed that by combining the two technologies of calculator and WhatsApp messenger, the first in the field of mathematical calculations and the second in the field of virtual classroom and class discussion, can teach hard and complex mathematical concepts. This finding is consistent with Dogruer and Akyuz (2020)'s the results of the study. Their study showed that the use of technology can improve students' understanding of difficult mathematical concepts and improve their learning comprehension. This finding is in line with Attard & Holmes, (2020)'s the results of the study. In their study, they found that using technology provides an opportunity for students to interact with each other in a variety of ways.

In this study, the Activity Theory activity system was used to analyze classroom interactions in order to better classroom interactions that is formed in the development of the concept of an irrational number. A review of other studies shows that researchers used the activity system to analyze classroom interactions (Salloum & BouJaoude 2020; Huang & Lin, 2012). In their study, they used Activity Theory to examine the understanding of classroom interactions and showed that through these activity systems of this theory, classroom interactions can be analyzed and understood, and developed in the field of education can be created. Huang & Lin (2012) used Activity Theory to analyze classroom activities.

Mulenga & Marbn (2020) showed that Covid-19 is the gateway to digital learning in mathematics education. But the problem with teaching during Covid-19 pandemic is that teachers have limited skills in using technology such as mobile phones and other Medias such as WhatsApp for the teaching and learning of mathematics. Therefore, addressing the issue of technology in teacher education is a necessity that should be addressed to improve the skills of teachers. Another point that was shown in the methodology of the study was the use of WhatsApp messaging software to generate study data. With the outbreak of the Coronavirus, it is practically impossible to use the traditional form of research collection that was in the past today, and it is not possible to use this method to collect research data. For example, researchers used to meet face-to-face with research participants to conduct data and conduct audio and video interviews in the past. In addition, in this method, the researcher had to spend a long time producing study data to be able to produce textual study data from audio and video interviews. But this study introduced a new way of generating study data in which all classroom conversations between teacher and student, along with time and image, were presented in a coherent text. Therefore, this study showed that technology can be used to teach mathematical concepts, but it can also be used to study data collection and speed up the research process.

Another important point that this study showed was the challenge of technology. The findings of this study showed that when students could not obtain the eleventh decimal digit of the radical number two ($\sqrt{2} = 1.4142135623\dots$) using the multiplier of the calculator. In this regard, this study used the same issue to create tension among students and tension in technology (calculator). Subsequently, with the guidance of this teacher, modification of tensions was directed towards the

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development of an understanding of the concept of square root. Eventually, students were able to construct the concept of the irrational number. In constructing the concept of the irrational number, students constantly tested their conjectures about radical two decimal places using a calculator. This process continued up to ten decimal places of the irrational number ($\sqrt{2} = 1.4142135623\dots$), but from the eleventh decimal place of this number, that is $\sqrt{2} = 1.4142135623?$, the students realized that due to the inability of the calculator to multiply, this number and the next digits of this number cannot be guessed and tested. Hence, the students realized that the answer to the math problem, "Find a number that is multiplied twice by itself, is the result of two" is an irrational number whose whole numbers cannot be guessed and shown. In the following, the students' definition of the irrational number is similar to what is found in the research literature as the definition of the irrational number. According to Patel and Varma (2018), the irrational number is a number that, unlike the rational numbers, cannot be measured; its decimal digits are non-repetitive and infinite. Similar to this definition, students in the study concluded that the decimal number of this number is continuous or in other words infinite. On the other hand, when students realized that this number could not be guessed, they realized that this number could not be measured. Therefore, it can be said that the interactive and constructive atmosphere in the virtual classroom was provided to students by technology, which played a significant role in the development of this mathematical concept.

In the end, the challenges we faced in using the technology were as follows: the limitation of the calculator as a technology tool in multiplying numbers above 11 decimal places, the limitation of WhatsApp messenger in showing images of study participants, the limitation students have access to a computer and use a cell phone instead, not seeing how students feel when they cannot do the calculation.

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Exploration of Students' Statistical Reasoning Ability in the Context of Ethnomathematics: A Study of the Rasch Model

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Abstract: *Statistical reasoning ability is one of the essential skills in developing competence, which is one of the Sustainable Development Goals (SDGs). This study aims to explore the statistical reasoning ability of junior high school students in descriptive statistics learning. The investigation directs students to determine their level of statistical reasoning ability. The Rasch model analysis analyzes whether there is a bias in the item questions used in exploring these abilities based on demographic factors such as gender, initial mathematical ability, and ethnic background. This study uses a mix-method (a combination of quantitative and qualitative methods). Based on the Rasch Model analysis, quantitative and qualitative data were obtained from 22 students utilizing a statistical reasoning ability test instrument based on the ethnomathematics context, which was valid and reliable. The results of data analysis show that students have different levels of statistical reasoning abilities and are related to students' initial mathematical ability. Students with low initial mathematical skills are at level 1 idiosyncratic, while students with high initial mathematical skills are at procedural level 4. The results of the DIF (Differential Item Functioning) analysis showed no significant difference between the use of the ethnomathematics context-based test instrument and the demographic factors of the research subjects. This study concludes that students can explore statistical reasoning ability through ethnomathematics-based problem solving but still pay attention to other demographic factors.*

Keywords: *Statistical Reasoning Ability, Ethnomathematics, Descriptive Statistics, Rasch Model*

INTRODUCTION

Currently, the development of society in a country is supported by the development of advances in Science and Technology. This is in line with the emergence of the Industrial Revolution 4.0

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which brings innovation and technology as part of the development of life. The factor that gives the biggest role in the advancement of science and technology is the factor of educational development. The education factor is also included in one of the Sustainable Development Goals (SDGs) which are used as indicators for countries in their efforts to develop society and sustainable economic development. Education provides space for people to develop innovation and technology in higher-order thinking processes. This can be seen with the increasing importance of understanding data which will later be applied in the development of technology-based on the Internet of Things (IoT) and Artificial Intelligence (Yaqoob et al., 2019). Understanding the data ultimately leads to the development of analytical skills on data that is closely related to statistics. Data analysis using statistics aims to filter information that appears massively so that later it can be concluded that the right and significant information is used as further recommendations regarding the development of a system (Setiawan & Sukoco, 2021).

Based on this situation, the education factor clearly has an important role in creating a society that is critical of information, quick to collect valid data, so that later it can test and analyze valid data for conclusions and applications in a broad scope. This important role requires good statistical reasoning ability. In this context, statistical reasoning ability is defined as a general conclusion expressed with uncertainty but can be proven using available data sets (Ben-Zvi et al., 2015).

Statistical reasoning emphasizes how students use their reasoning in gathering information taken from statistical random data (Pfannkuch, 2006). Statistical reasoning also helps students understand statistical data and can describe the data obtained in various forms of data presentation, interpret data to draw conclusions from the collection of statistical data obtained (Pfannkuch, 2005; Rubin et al., 2006). Statistical reasoning ability also focuses on how to find the meaning of the statistical data set that has been identified, whether there are data deviations, and then decide whether the data analysis result can be concluded or not (Makar, 2013). The importance of statistical reasoning skills based on the key concepts of NCTM did not yield positive outcomes in field studies. Facts on the ground show that high school students still have difficulty describing data in various forms of data presentation, as well as reasoning and statistical thinking processes in the context of statistical problems (Ramadhani & Fitri, 2020; Ramadhani & Narpila, 2018).

The results of this study are also supported by other findings, where students still make mistakes in answering questions related to the size of the central value, variability, and standard deviation (Chan et al., 2016; Rufiana et al., 2018) which have an impact on students' low reasoning abilities related to literacy and mathematics (especially in the context of statistics). The level of statistical reasoning ability of students on average is still at the level of idiosyncratic reasoning (Level 1) (Chan et al., 2016; delMas, 2002; Garfield, 2002). Students at Level 1 can employ a variety of statistical terms and symbols, but they cannot fully interpret and apply them to the correct data. As a result, their responses are frequently incorrect. For example, such students may be familiar with

the term 'standard deviation,' but cannot correctly apply it (Chan et al., 2016). In general, idiosyncratic, level 1 replies are not helpful in classifying thinking. For example, a student's argument would be classed as idiosyncratic if it was entirely focused on the context of the work and did not refer to the data. Based on the confirmation, we found the results of the world assessment where Indonesia is ranked in the bottom 6 (72 out of 79 countries) for reading, mathematics, and science assessments (PISA 2018), while for the 2015 TIMSS assessment, Indonesian students did not achieve the average score. determined by TIMSS (TIMSS Scale Centrepoint) which is 397 out of 500 points on average (Mullis et al., 2016; OECD, 2019).

Another fact is that most students in Indonesia, namely 75.7% of students do not reach Level 2 which is set as the basic level in understanding mathematical literacy. These students can only solve math problems using familiar contexts, clear questions, and present all relevant information. On the other hand, students are actually able to identify relevant information and carry out routine mathematical procedures only if explicit instructions are given (Zulkardi et al., 2020). Based on this, students at Level 2 (Statistical Reasoning Level) are better able to employ a variety of statistical phrases and symbols since they are more familiar with the meanings of various statistical ideas. They still don't know how to use them properly. Students may, for example, select the proper quartile explanation yet fail to answer conceptual questions. These findings and facts are reinforced by the statement of Maryati & Priatna (2018) where students show difficulties in reading the statistical data presented, understanding statistical problems, and solving problems related to the average value of a data. Likewise, the results obtained by Mahdayani (2016) where as many as 54.6% of high school students have difficulty in reading and understanding statistical data; 83.5% of students have difficulty in transforming data; and as many as 91.7% of students have difficulty in processing data and concluding the data presented. Based on the result, we can find of one possible explanation is that students tend to memorize so it has an impact on the process of understanding and statistical reasoning (Nuralam & Gadeng, 2018). Statistical reasoning for high school students is an important skill, because it affects the understanding process in other scientific fields (English, 2014; Sharma, 2017). Students' difficulties in reasoning statistical problems will affect students' abilities in spatial and numerical terms. Students will feel anxious and confused when faced with simple numerical problems, describe spatial problems to count objects on a graphic display which is the basis of statistical reasoning abilities (Ching et al., 2020).

Based on the facts and findings obtained, it is necessary to diagnose what makes students tend to have low statistical reasoning abilities. In response to this, we can explore students' statistical reasoning abilities using five levels of statistical reasoning ability in descriptive statistics based on the statistical reasoning model initiated by Garfield (2002), namely idiosyncratic (level 1), verbal (level 2), transitional (level 3), procedural (level 4), and investigative process reasoning (level 5). Students with level 1 and 2 statistical reasoning skills continue to struggle with symbols, terminology, statistical concepts, and their application in real-life situations. They can understand

one or two components of the statistical process, but they cannot use these principles to get answers, unlike students at level 3. Such students, for example, can recognize the shape, central tendency measures, and variability of graphical representations but are unable to integrate them into their solutions. Students at Level 4 can identify statistical processes with accuracy, but they are unable to completely comprehend or integrate them. These students, for example, may understand the concept of averages but are unable to interpret it fully. Level 5 students have a thorough understanding of statistical processes and are capable of synchronizing rules and behavior as well as explaining the function in their own words. Students at Level 1 (idiosyncratic reasoning) would display pre-structural traits in the iconic style. Meanwhile, students at Level 2 (verbal reasoning) would exhibit characteristics of the uni-structural level using the concrete symbolic mode. Students at Level 3 would likewise use the concrete figurative approach to depict components of the multi-structural level (transitional reasoning). Students at Level 4 (procedural reasoning) would show tangible symbolic representations of relationship level traits. Finally, Level 5 students formally reveal extended abstract level features. (Reasoning based on integrated processes).

In addition to Garfield's five levels of statistical reasoning ability; Biggs & Collins (1982) also introduced the taxonomic model Structure of Observed Learning Outcomes (SOLO) which was used as a cognitive model for the development of statistical reasoning, where junior high school students should have been in a formal function mode. Garfield & Ben-Zvi (2008) further identified that there are 9 important things in the development of statistical reasoning abilities, including data (including the nature of data and types and various data sources), statistical models (regression and normal distribution), distribution (involving ideas about, form, centering, and dispersion of data), center (includes the difference between the median and mean), variability (including measurement and spread of distribution), comparing groups based on concentration and dispersion, sampling (such as the effect of sample size), inferential statistics (testing hypothesis or confidence interval), and covariation with scatterplots, correlation, and linear regression.

Statistical reasoning abilities can also be investigated by taking into account the characteristics of the learning provided, including (1) an inquiry-based learning environment that builds collaborative norms among students; (2) statistical concepts and tools to support and expand students' informal statistical constructions in concluding the data presented; and (3) providing data-rich tasks that trigger conflicts with beliefs (contextual and/or statistical) to encourage students to seek deeper insights and explanations (Makar, 2013; Makar & Fielding-Wells, 2011). Wild & Pfannkuch (1999) also stated that statistical learning which emphasizes the development of statistical reasoning abilities is carried out through real situations. Students who obtain data from real situations will gain new knowledge to provide information and make decisions and actions on evidence-based situations that will later have an impact on similar problems that exist in people's lives. The ability to philosophize on data, argue about patterns, and provide suggestions regarding

solutions to problems encountered depends on a good foundation of statistical and contextual knowledge. This is because statistics demand data-based and social argumentation skills (Pfannkuch, 2011).

Several types of tests used to measure students' statistical reasoning abilities have been developed, such as The Reasoning about P-values and Statistical Significance (RPASS) which was developed in the form of an application and focused on measuring advanced inferential statistics (Lane-Getaz, 2013), the Comprehensive Assessment of Outcome in Statistics (CAOS) (DelMas et al., 2007; Zieffler et al., 2010), Statistical Reasoning Assessment (SRA) (Garfield, 2002), Statistical Concept Inventory (SCI) (Stone et al., 2003) which was developed focusing on measuring various introductory statistical concepts, and The Assessment Resource Tools for Improving Statistical Thinking (ARTIST) which focused on the topic of test significance. Zieffler et al. (2008) also developed tests that focus on types of tests such as: estimating and graphing a population, comparing two data samples, and making an assessment of the two data samples being compared to draw conclusions and final decisions on the two data samples owned. The same thing was also done by Huey & Jackson (2015) using a test instrument that focuses on inferential, unstructured, open-ended, context-based dimensions, and represents visual displays (such as tables, graphs, plots, diagrams, and others).

Based on the test instruments developed as a whole using data derived from contextual problems. The real problems used in developing students' statistical reasoning abilities can be integrated with the cultural context and traditions inherent in students' daily lives. Real experiences that are contextual and come from cultures and traditions that are close to students will increase students' motivation and interest. The use of culture in the context of mathematics is not new, because mathematics is a product of culture (Marsigit et al., 2018) and mathematics is part of local wisdom (Madusise, 2015). Culture-based mathematics known as ethnomathematics makes it easy for students to do mathematical modeling based on ideas, methods, and techniques from what has been developed by the surrounding community and can be used as an alternative to introduce students to be closer to the phenomena that occur in their lives (Prahmana et al., 2021). Investigation of data originating from phenomena that occur in students' lives will help develop students' mathematical reasoning abilities, to students' creativity (D'Ambrosio & Rosa, 2017; Rosa & Orey, 2017), and this can also be integrated into the test instruments used in measuring students' statistical reasoning ability. Incorporating the ethnomathematics framework in testing statistical reasoning abilities further supports the SOLO model's definition of cognitive thinking. Students who respond to their physical environment are at the sensorimotor stage in the SOLO model. Students that can internalize actions through images are at the iconic moment. Students in the concrete symbolic stage are interested in symbolic systems such as number schemes, maps, and the written word. Students can also translate abstract concepts (informal mode) into formal thoughts (formal mode). Post-formal students are those who comprehend the fundamental

structure of theories and disciplines. Based on this, junior high school students according to the SOLO thinking model, are in a formal model (Chan et al., 2016), so that measurements using the ethnomathematics context can help students optimize their statistical reasoning abilities.

This study aims to analyze the statistical reasoning ability possessed by junior high school students which focuses on descriptive statistics measured using the level of statistical reasoning ability developed by Garfield. This study also wants to see how the pattern of answers given by students, is classified hierarchically based on student demographics (gender, ethnicity, and initial mathematical ability). We will also compare the answers given by students to see bias or differences based on student demographics, as well as consistent student answer patterns. This study uses a test instrument that was developed based on the ethnomathematics context. The ethnomathematics context used is Malay culture which will later be integrated into simple descriptive statistical problems. This study has several research questions to be explored, including:

1. How do students demonstrate statistical reasoning ability using ethnomathematics-based problems?
2. What is the level of students' statistical reasoning ability using ethnomathematics-based problems based on Rasch Model analysis?
3. Are there biases of gender, ethnicity, and students' initial mathematical abilities on students' statistical reasoning abilities using ethnomathematics-based problems based on Rasch Model analysis?

RESEARCH METHODOLOGY

This study uses a combination of qualitative and quantitative approaches (mix-method) to gain a clearer understanding of the exploration of students' statistical reasoning abilities in descriptive statistics learning as well as to triangulate findings (Creswell & Clark, 2018). The subjects of this study were junior high school students who were in the final level, totaling 22 people. Descriptions of research subjects can be seen in Table 1 below:

Table 1.

Demographic Background of Research Sample

Demographic Background		Number of Participants	Percentage
Gender	Male	15	68.18%
	Female	7	31.82%
Age	13 years old	3	13.64%

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	14 years old	17	77.28%
	> 15 years old	2	9.08%
Initial Mathematics Ability	Low	4	18.18%
	Average	14	63.64%
	High	4	18.18%
Cultural Tribal Background	Pure (Malay ethnic)	17	77.28%
	Mix (Malay ethnic and Javanese ethnic)	5	22.72%

Based on Table 1 above, the demographic indicators on the research subjects used refer to the gender of the students, the students' initial mathematics ability, and the cultural tribal background of each student. Information related to ethnic and cultural backgrounds possessed by students is important to ensure that students understand the cultural context that will be integrated into descriptive statistics learning. This is also done to ensure that the findings of this study do not have bias related to differences in cultural backgrounds, even though the research was conducted in areas that have a Malay cultural background. The three demographic indicators are used on research subjects with the aim that the analysis carried out on students' statistical reasoning abilities can be carried out at the individual level of students and the level of the items used.

Multi-Tier Test Instrument of Statistical Reasoning Ability

The test instrument used in this study was a test instrument in the form of a multi-tiered essay test, which consisted of 3-tiered essay questions (the total number of tests was 10 questions). The research test instrument used to analyze students' statistical reasoning abilities based on the answer process carried out by students referred to the results of statistical learning tests (limited to the concept of data collection and calculation of central values on date data). The assessment of the test instrument is based on a rating scale that is adjusted to the indicators of statistical reasoning ability on each item (ie idiosyncratic, verbal, transitional, procedural, and investigative process reasoning). The highest score for each test is 5, and the lowest score for each test is 1. If the student does not answer the given test, then no score is given (no assessment).

Item Response Test (IRT) using Rasch Model Measurement

The test instrument in analyzing students' statistical reasoning abilities was developed based on constructs in descriptive statistics learning and the level of statistical reasoning abilities. The description of the test instrument used is based on the descriptive statistics learning construct and the level of statistical reasoning ability which can be seen in Table 2 and Table 3 below:

Table 2.

Description of Developed Test Instruments Based on the Descriptive Statistics Learning Construct

Constructs	Code	Sub-Process	Items
Describing data	A1	Extraction and generation of information from data or graphs depending on a contextual situation in Malay culture	How much cotton, silk, gold, and silver thread is needed in the weaving of the traditional Malay Songket based on the graph?
	A2	Demonstrating sensitivity to the graphical representation's exhibited qualities	Do you agree, based on the graph above, that silk thread has the fewest uses in the Malay Songket production process compared to other types of thread?
	A3	Recognizing the graphical representation's general features	What is the difference in the usage of silk and cotton, silk and gold-silver, cotton and gold-silver threads in the production of a typical Malay Songket, according to the graph above?
Organizing and reducing data	B1	Making a table out of the data	Arrange the tabular data acquired and adjust it to the weavers' production and number of Okik used (based on the information presented related to the weaving culture)!
	B2	Using the measure of center to reduce the data, either by calculation or analysis	In which month is the highest number of Okik used
	B3		Determine the number of Okik in the middle of the sequence if you order the number of Okik from least to most!
Representing data	C1	Recognizing multiple representations of the same data set	Analyze the outcomes of these sales to determine which month saw the

			highest profit from Putu Mayam sales based on the graph!
	C2		Re-examine, which month saw the greatest growth in profit?
Analyzing and interpreting data	C3	Comparing data from the same data set	What is the total profit earned over the course of a year based on the findings of your analysis?
	C4	Using data or graphs to make predictions, inferences, or draw conclusions	Determine the average profit obtained from the sale of Putu Mayam!

Table 3.

Description of Developed Test Instruments Based on Statistical Reasoning Ability Level

Construct	Level	Level 1 Idiosyncratic	Level 2 Verbal	Level 3 Transitional	Level 4 Procedural	Level 5 Integrated Process
Describing data		A1L1	A1L2	A1L3	A1L4	A1L5
		Do not extract or generate idiosyncratic or meaningful information from graphs.	Some information is extracted and generated from the graph, but the interpretation is imprecise or uncertain.	Extracts and creates information from a graph in one or two dimensions.	Correctly extracts and creates information from the graph	Completely extracts and creates information from the graph
		A2L1	A2L2	A2L3	A2L4	A2L5
		Does not demonstrate awareness of the graphical representation's displayed properties.	Shows understanding of the displayed properties of pictorial representation in writing, but is only partially right.	Displays minimal knowledge of the graphical representation's displayed features.	Demonstrates some understanding of the graphical representation's displayed attributes.	Demonstrates comprehensive understanding of the graphical representation's displayed attributes

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	A3L1	A3L2	A3L3	A3L4	A3L5
	Does not identify the graphical representation's general properties.	Recognize general characteristics of the pictorial representation in words, however, it is only partially accurate	Recognizes one or two broad characteristics of the graphical depiction	Accurately recognizes general elements of the graphical representation	Completely recognizes general elements of the graphical depiction
Organizing and reducing data	B1L1	B1L2	B1L3	B1L4	B1L5
	The data could not be organized into a table.	Provides oral assertions when putting data into a table, but is only partially correct.	Organizes the data into a table with notable errors	Organizes the data into a table with minimal errors	Correctly organizes data into a table
	B2L1	B2L2	B2L3	B2L4	B2L5
	Incapable of reducing the data using central tendency measures through calculation or analysis	Reduces data by employing central tendency measures in words, either by computation or analysis, but is only partially correct.	Data is reduced using measures of central tendency with considerable errors, either by calculation or analysis.	Reduces the data by calculating or analyzing metrics of central tendency with minor errors.	Completely reduces the data utilizing central tendency metrics, either by calculation or analysis
B3L1	B3L2	B3L3	B3L4	B3L5	
Unable to reduce the data using central tendency metrics via math or analysis	Reduces data by employing central tendency measures in words, either by computation or analysis, but is only partially correct.	Reduces data using measures of central tendency with significant errors, either by computation or analysis	Reduces the data by calculating or analyzing metrics of central tendency with minor errors.	Completely reduces the data utilizing central tendency metrics, either by computation or analysis	

Representing data	C1L1	C1L2	C1L3	C1L4	C1L5
	Does not distinguish between alternative representations of the same data set.	Identifies many word representations for the same data set, but is only partially right.	Identifies one or two features shared by multiple representations of the same data set.	Identifies different representations of the same data collection correctly.	Determines the effectiveness of two distinct representations for the same data collection.
	C2L1	C2L2	C2L3	C2L4	C2L5
	Does not distinguish between alternative representations of the same data set.	Identifies many word representations for the same data set, but is only partially right.	Identifies one or two aspects of multiple representations of the same data set.	Recognizes different representations of the same data set.	Determines the effectiveness of two distinct representations for the same data gathering.
Analyzing and interpreting data	C3L1	C3L2	C3L3	C3L4	C3L5
	Do not compare data sets from the same source.	Compares multiple data sets verbally, however, the comparisons are imperfect.	Performs one or two comparisons within the same data sets	Accurately compares data sets from the same source.	Completely compare data sets inside the same data sets.
	C4L1	C4L2	C4L3	C4L4	C4L5
	Make no predictions, judgments, or conclusions based on the graphics.	Predictions, judgments, and conclusions derived from graphs are conveyed in words, yet they are insufficient.	One or two predictions, inferences, or conclusions are drawn from the graphs.	Predicts, infers, or draws suitable conclusions from graphs	Completely and comprehensively predicts, infers, or draws conclusions from graphs

Modified from: Chan et al. (2016)

The test instrument was developed in accordance with Table 2 above, then tested for validation and reliability using the Rasch Model Measurement analysis assisted by the Winstep application. Rasch Model Measurement is a measurement analysis based on Item Response Theory (IRT) developed by George Rasch (a Danish Mathematician) using the Joint Maximum Likelihood Estimation (JMLE) equation which converts raw data into the interval (logit) data (Soeharto, 2021). Rasch's estimation is based on the interaction of item-person and Probability estimations.

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The interaction between items and persons (in this case, students) may be expressed mathematically using logit values (log odd units). The measurement's probability is governed by the difficulty of the item and person at the same time, yielding the logit item value (using the odd probability of each item) and the logit person value (using the odd probability of each respondent). In other words, the probability is closely proportional to the gap between item difficulty and student abilities (Boone, 2016; Boone et al., 2014). This means that the student's ability to measure remains the same regardless of the difficulty level of the item, but the item's difficulty level remains unchanged regardless of the student's ability. In this work, Rasch analysis was utilized to overcome some of the limitations of Classical Test Theory (CTT). In describing the measuring model, the CTT has four limitations: (a) the measurements are constructed using ordinal data results rather than an interval (logit) scale; (b) the items and people in the dependent measurement are highly dependent on the sample; (c) the nature of the instrument's measurement in terms of reliability and validity is highly dependent on the sample; and (d) data is centered on group-centered statistics but is insufficient for explaining individual respondent measurements (Barbic & Cano, 2016).

The logit scale in Rasch Model Measurement can indicate a person's skill and item difficulty from positive infinity to negative infinity. The results of the validation show that the multi-tiered description questions which the total number of questions used are 10 questions (statistical reasoning ability test) developed are valid (according to the valid item criteria regarding item response theory), namely the MNSQ OUTFIT value is in the range .5 to 1.5, the OUTFIT Z-STANDARD (ZSTD) value is in the range of -2 to +2 and the Point Measurement Correlation (Pt. Measure Corr) value is in the range of .4 to .85. The mean item size (logit) is .00, and the standard deviation (SD) is 1.36, indicating that the variance of the item measurements in item difficulty is wide across the logit scale. For the person (students), the average size is -.45 logit, which indicates that if the average value is less than the logit value of .00. Based on this logit value, we concluded the student's ability tends to be smaller than the difficulty level of the question. However, the SD on the person (students) is 1.75 which indicates that the variation of person is suitable for data analysis. The overall test instrument developed is also declared reliable by referring to the results of the summary statistical test (Rasch Model Measurement) on the Reliability value ($\alpha = .95$) in the special category and the Reliability value ($\alpha = .88$) in the good category for persons or students (Sumintono & Widhiarso, 2015). The summary of test items and person statistics can be seen in Table 4 below.

Table 4.

The Summary of the Statistics Based on Pearson and Items

Statistic Test	Test Group	
	Person	Item Test
N	22	10
Measure	-.45	.00
Mean	27.6	60.8
SD	1.75	1.36
SE	.98	.45
Mean Outfit MNSQ	.92	.92
Mean Outfit ZSTD	-.02	.08
Separation	2.76	4.28
Reliability	.88	.95
Cronbach's Alpha	.90	

After the test instrument for statistical reasoning ability is valid and reliable, then the test instrument can be given to the research subject to explore the abilities possessed by the research subject (junior high school students). The results of the answers given by students were analyzed based on the process of answers given in accordance with the indicators of statistical reasoning ability and descriptive statistics learning constructs (see Table 3). The answering process was then analyzed to see if there is a gender bias, initial mathematical ability, and cultural background that students have on the answer process given by students on each item of the question. The bias analysis was carried out by measuring Rasch using Differential Item Functioning (DIF). This test is carried out to answer the research problem formulation which allows that an instrument or item has a bias if it is found that one individual with certain characteristics is more advantageous than individuals with other characteristics. Demographic indicators (demographic variables) used to analyze the existence of bias in the results of answers made by students are gender factors, students' initial mathematical abilities, and cultural background. Furthermore, the DIF analysis can also provide information on whether there are students who have a tendency to answer questions in an unusual way (such as students with low initial abilities being able to answer difficult questions, but still unable to answer easy questions) (Sumintono & Widhiarso, 2015).

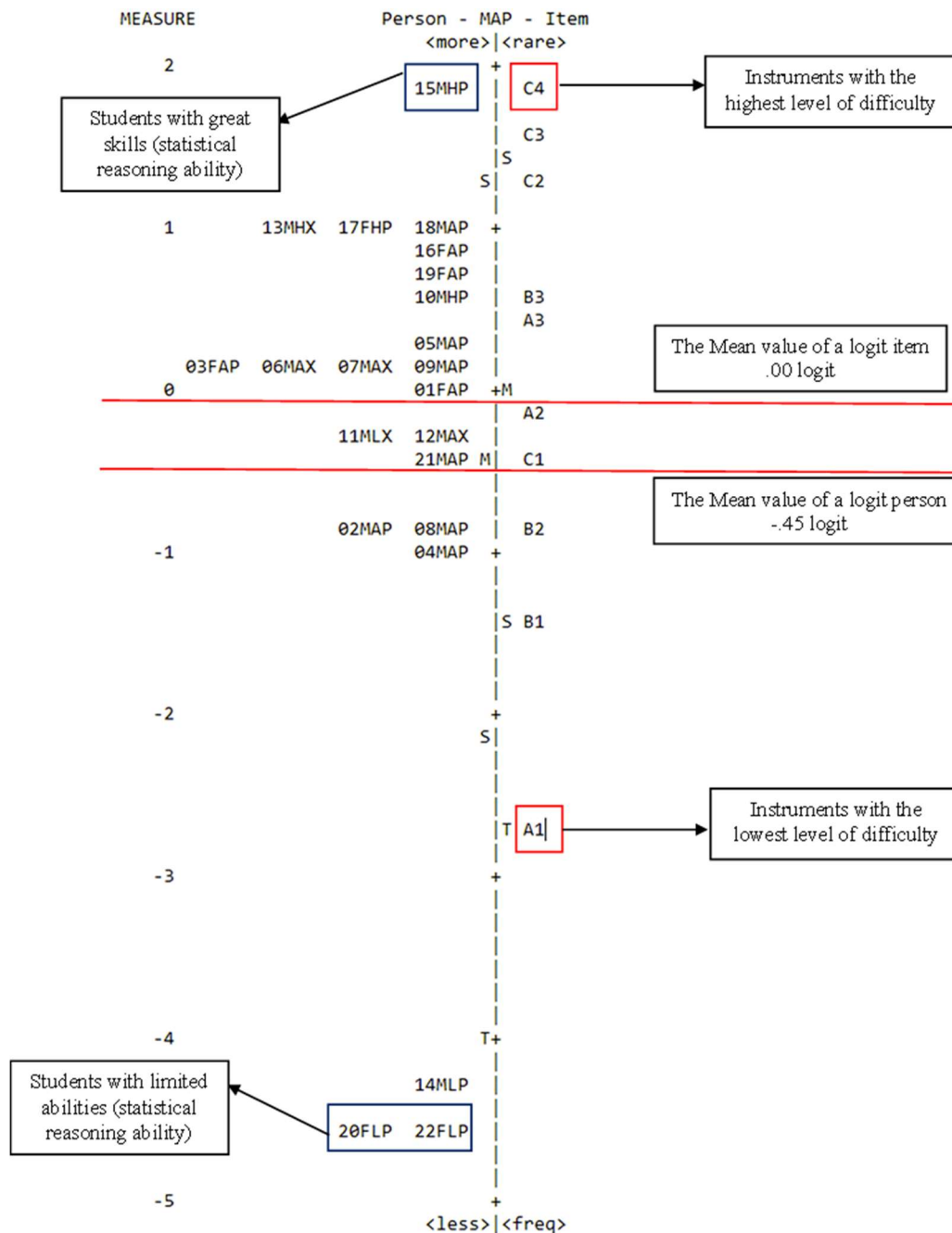
RESULT AND DISCUSSION

Students' Statistical Reasoning Ability Level and Difficulty Level of Test Instruments Based on Rasch Measurement Model Analysis

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Based on the results of the analysis of the answers given by 22 students on the multi-tier statistical reasoning ability test, the results obtained as shown in the Wright Map below:



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Figure 1.

Wright's Map Showing Students' Statistical Reasoning Ability Level and Test Instrument Difficulty Level Based on Logit Score

Based on Figure 1 above, it could be seen that student with code 15MHP (Male, High Initial Mathematics Ability, Pure Ethnic) are students with demonstrates high statistical reasoning abilities. This can be seen from the position of students on Map Wright who are in the top position with logit values approaching +2.00 which is +1.82, while students with low statistical reasoning abilities are students with codes 20FLP and 22FLP (Female, Low Initial Mathematics Ability, Pure Ethnic) with a logit value of more than -4.00 i.e., -4.64. Figure 1 above also shows that more than 50% of students are above the average logit person value (.00 logit), which is 13 students. In addition to the level of students' statistical reasoning abilities, the Wright Map also shows that the test instruments fall into the categories of difficult tests and easy tests. The test instrument with code C4 which has the construct of Analyzing and interpreting data is in the category of difficult test, while the test instrument with code A1 which has the construct of Describing data is included in the category of easy test.

Analysis of Student Answer Patterns on Statistical Reasoning Ability Tests in Cases of Descriptive Statistics in the Context of Malay Ethnomathematics

Further analysis was conducted to see how the pattern of answers of students with high statistical reasoning abilities, namely students with 15MHP code and students with low statistical reasoning abilities, namely students with 20FLP and 22FLP codes based on the level of statistical reasoning ability.

Construct: Describing Data (A1, A2, and A3)

The pattern of student answers with code 15MHP (high statistical reasoning ability) can be seen in Table 5 below:

Table 5.

Student Answer Patterns Code 15MHP (high ability) on A1, A2, A3

<p>a. Berapakah jumlah benang katun, sutera, dan emas-perak yang digunakan dalam penenunan kain songket khas Melayu? katun: 40 sutera: 70 emas-perak: 90</p>	<p>a. How much cotton, silk, gold, and silver thread is needed in the weaving of the traditional Malay Songket based on the graph? Cotton: 40: Silk: 70: Gold-Silver: 90</p>
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<p>b. Berdasarkan grafik di atas, setujuakah kamu jika benang sutera merupakan benang paling sedikit digunakan dibandingkan dengan jenis benang yang lain? Jika tidak setuju, jabarkan alasan kamu!</p> <p><i>tidak setuju... karena di bawah sutera masih ada benang katun yg jumlahnya sedikit.....</i></p>	<p>b. Do you agree, based on the graph above, that silk thread has the fewest uses in the Malay Songket production process compared to other types of thread? I can't entirely agree because the amount of silk threads remains more than the number of cotton threads.</p>
<p>c. Berdasarkan grafik di atas, bandingkan berapa selisih penggunaan benang sutera dan katun, benang sutera dan benang emas-perak serta benang katun dan benang emas-perak dalam proses produksi songket Batubara khas Melayu?</p> <p><i>sutera dan katun : 70 - 40 = 30 katun dan emas perak : 90 - 90 = 0 sutera dan emas perak : 70 - 90 = 20</i></p>	<p>c. What is the difference in the usage of silk and cotton, silk and gold-silver, and cotton and gold-silver threads in the production of a typical Malay Songket, according to the graph above?</p> <p>Silk and Cotton: $70 - 40 = 30$ Silk and Gold-Silver: $90 - 70 = 20$ Cotton and Gold-Silver: $90 - 40 = 50$</p>

Based on students' answers to the 15MHP code on tests A1, A2, and A3 it could be seen that students extract information about the number of cotton, silk, and gold-silver threads well from the graphs presented; is aware of the appropriate information on the graphs presented; can compare the information presented on the graph with the appropriate calculations. The pattern of answers given by students with the 15MHP code is in accordance with level 4 Procedural, namely students can well display, realize, and compare the information presented on the graph. The pattern of the answers was as expected, although the students did not provide a more detailed explanation regarding the answers given. This can be seen in the answers to the A2 test, where 15MHP did not describe the least number of threads according to the graph presented with the aim of supporting disagreement on the statement. 15MHP students have high initial mathematical abilities, so they are able to give correct answers on tests with easy (A1 and A2), and moderate (A3) categories. The student with code 15MHP also has a native Malay ethnic background (not mixed), so that the integrated Malay cultural context (Songket Malay) on the A1, A2, and A3 tests can be easily understood. Different things can be seen from the pattern of students' answers to codes 20FLP and 22FLP which do not present the right information based on the graphs presented. The following is a description of the answer patterns for students with codes 20FLP and 22FLP which are presented in Table 6.

Table 6.

Student Answer Patterns Code 20FLP and 22FLP (low ability) on A1, A2, A3 Tests

Student Answer Patterns Code 20FLP	Student Answer Patterns Code 22FLP
<p>a. Berapakah jumlah benang katun, sutera, dan emas-perak yang digunakan dalam penenunan kain songket khas Melayu?</p> <p>.....20 benang katun benang 50 kain perak.....</p> <div data-bbox="237 596 764 747" style="border: 1px solid black; padding: 5px;"> <p>a. How much cotton, silk, gold, and silver thread is needed in the weaving of the traditional Malay Songket based on the graph? 20 threads dan 50 fabrics</p> </div>	<p>a. Berapakah jumlah benang katun, sutera, dan emas-perak yang digunakan dalam penenunan kain songket khas Melayu?</p> <p>.....Jumlah adalah 90.....</p> <div data-bbox="850 583 1391 720" style="border: 1px solid black; padding: 5px;"> <p>a. How much cotton, silk, gold, and silver thread is needed in the weaving of the traditional Malay Songket based on the graph? The total number of threads is 90.</p> </div>
<p>b. Berdasarkan grafik di atas, setujukah kamu jika benang sutera merupakan benang paling sedikit digunakan dibandingkan dengan jenis benang yang lain? Jika tidak setuju, jabarkan alasan kamu!</p> <p>.....dibuat dengan menggunakan jenis yang dipakai saat songket.....</p> <div data-bbox="237 968 764 1150" style="border: 1px solid black; padding: 5px;"> <p>b. Do you agree, based on the graph above, that silk thread has the fewest uses in the Malay Songket production process compared to other types of thread? The threads are woven using the type used in Songket fabrics.</p> </div>	<p>b. Berdasarkan grafik di atas, setujukah kamu jika benang sutera merupakan benang paling sedikit digunakan dibandingkan dengan jenis benang yang lain? Jika tidak setuju, jabarkan alasan kamu!</p> <p>.....sangat di gunakan untuk kartun.....</p> <div data-bbox="867 989 1375 1157" style="border: 1px solid black; padding: 5px;"> <p>b. Do you agree, based on the graph above, that silk thread has the fewest uses in the Malay Songket production process compared to other types of thread? Cotton is the least commonly used fabric.</p> </div>
<p>c. Berdasarkan grafik di atas, bandingkan berapa selisih penggunaan benang sutera dan katun, benang sutera dan benang emas-perak serta benang katun dan benang emas-perak dalam proses produksi songket Batubara khas Melayu?</p> <p>.....40 adalah penggunaan dengan 20 atas benang perak 40-20=20.....</p> <div data-bbox="237 1377 764 1587" style="border: 1px solid black; padding: 5px;"> <p>c. What is the difference in the usage of silk and cotton, silk and gold-silver, and cotton and gold silver threads in the production of a typical Malay Songket, according to the graph above? 40 threads compared to 70 threads by making a comparison between $70 - 40 = 60$</p> </div>	<p>c. Berdasarkan grafik di atas, bandingkan berapa selisih penggunaan benang sutera dan katun, benang sutera dan benang emas-perak serta benang katun dan benang emas-perak dalam proses produksi songket Batubara khas Melayu?</p> <p>.....Berdasarkan di atas bandingkan 40.....</p> <div data-bbox="867 1388 1375 1587" style="border: 1px solid black; padding: 5px;"> <p>c. What is the difference in the usage of silk and cotton, silk and gold-silver, and cotton and gold silver threads in the production of a typical Malay Songket, according to the graph above?? According to this comparison, the difference is 40.</p> </div>

Based on Table 5 above, it can be seen that the pattern of answers given by the two students (20FLP and 22FLP) has the same pattern. The pattern of answers given does not provide information that matches the graph shown. Students coded 20FLP and 22FLP gave answers that

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were very far from what was expected on the A1, A2, and A3 tests. The tendency of the answer pattern was due to the fact that the two students did not understand what was being asked on the test. Both students were perplexed and found it difficult to discern the correct response. This finding is in accordance with the results of interviews related to the statistical reasoning of the two students.

- R (Researcher)** : *Do you understand the information presented on the graph?*
20FLP : *Honestly, I understand the question (pause).*
22FLP : *I understand the numbers that appear on the graph.*
R (Researcher) : *Why did you answer 20 threads and 50 clothes on the A1 test (To 20FLP students)?*
20FLP : *I was thinking if the A1 test meant subtracting the numbers 70, 90, and 40 on the graph (the face looks confused) and I didn't know that what the A1 test really meant was rewriting the number of each yarn according to the graph.*
R (Researcher) : *How did you get an answer of 90 on the A1 test?*
22FLP : *I was wrong with the test given. I thought that what was being asked was only the number of gold-silver threads, so that's why I wrote 90. I only realized after I got my answers. I panicked because I couldn't see clearly which numbers were on the silk thread, the cotton thread, and the gold-silver thread.*

Based on the results of the interviews above, it is evident that the 20FLP and 22FLP students did not understand well the questions given on the A1, A2, and A3 tests even though the A1 and A2 tests were easy questions. Perplexed and panic are some of the factors that underlie the inappropriateness of the answers given. In addition, the factor of the low initial mathematical ability of the two students could also be one of the factors causing the students' inability to understand and reason about the meaning of the tests given. Initial abilities are needed to support subsequent learning performance (Nogues & Dorneles, 2021). The two students also had inadequate mathematical literacy; thus, they did not interpret the information on the test correctly. Based on this, students with codes 20FLP and 22FLP can only be at level 1 idiosyncratic-statistical reasoning ability.

Construct: Organizing and reducing data (B1, B2, and B3)

The pattern of student answers with code 18MAP (medium ability) on tests B1, B2, and B3 can be seen in Table 7 below.

Table 7.

Student Answer Pattern Code 18MAP (medium ability) on Tests B1, B2, and B3

<p>Berdasarkan pada informasi di atas,</p> <p>a. Kelompokkan data yang diperoleh dalam bentuk tabel dan sesuaikan dengan bulan produksi serta jumlah Okik yang digunakan oleh penenun!</p> <table border="1" data-bbox="358 485 699 947"> <thead> <tr> <th>Bulan Produksi</th> <th>Jumlah Okik</th> </tr> </thead> <tbody> <tr> <td>Januari 2020</td> <td>10 Buah</td> </tr> <tr> <td>Februari 2020</td> <td>10 menjadi 12</td> </tr> <tr> <td>Maret 2020</td> <td>9 Buah</td> </tr> <tr> <td>April 2020</td> <td>9 Buah</td> </tr> <tr> <td>Mei 2020</td> <td>9 Buah</td> </tr> <tr> <td>Juni 2020</td> <td>9 Buah</td> </tr> <tr> <td>Juli 2020</td> <td>14 Buah</td> </tr> <tr> <td>November 2020</td> <td>14 Buah</td> </tr> <tr> <td>Agustus 2020</td> <td>14 Buah</td> </tr> <tr> <td>September 2020</td> <td>14 Buah</td> </tr> <tr> <td>Oktober 2020</td> <td>14 Buah</td> </tr> <tr> <td>Desember 2020</td> <td>17 Buah</td> </tr> </tbody> </table>	Bulan Produksi	Jumlah Okik	Januari 2020	10 Buah	Februari 2020	10 menjadi 12	Maret 2020	9 Buah	April 2020	9 Buah	Mei 2020	9 Buah	Juni 2020	9 Buah	Juli 2020	14 Buah	November 2020	14 Buah	Agustus 2020	14 Buah	September 2020	14 Buah	Oktober 2020	14 Buah	Desember 2020	17 Buah	<p>a. Arrange the tabular data acquired and adjust it to the weavers' production and number of Okik used (based on the information presented related to the weaving culture)!</p> <table border="1" data-bbox="954 520 1377 898"> <thead> <tr> <th>Production Month</th> <th>Total of Okik</th> </tr> </thead> <tbody> <tr> <td>January 2020</td> <td>10 Okik</td> </tr> <tr> <td>February 2020</td> <td>10 becomes 12 Okik</td> </tr> <tr> <td>March 2020</td> <td>9 Okik</td> </tr> <tr> <td>April 2020</td> <td>9 Okik</td> </tr> <tr> <td>May 2020</td> <td>9 Okik</td> </tr> <tr> <td>June 2020</td> <td>9 Okik</td> </tr> <tr> <td>July 2020</td> <td>14 Okik</td> </tr> <tr> <td>August 2020</td> <td>14 Okik</td> </tr> <tr> <td>September 2020</td> <td>14 Okik</td> </tr> <tr> <td>October 2020</td> <td>14 Okik</td> </tr> <tr> <td>November 2020</td> <td>14 Okik</td> </tr> <tr> <td>December 2020</td> <td>17 Okik</td> </tr> </tbody> </table>	Production Month	Total of Okik	January 2020	10 Okik	February 2020	10 becomes 12 Okik	March 2020	9 Okik	April 2020	9 Okik	May 2020	9 Okik	June 2020	9 Okik	July 2020	14 Okik	August 2020	14 Okik	September 2020	14 Okik	October 2020	14 Okik	November 2020	14 Okik	December 2020	17 Okik
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<p>b. Analisislah pada bulan berapa jumlah Okik yang paling banyak digunakan?</p> <p>Bulan Desember mencapai 17 Buah</p>	<p>b. In which month is the highest number of Okik used?</p> <p>In December, the number of Okik reached 17 pieces.</p>																																																				
<p>c. Jika kamu mengurutkan jumlah Okik dari yang paling sedikit hingga paling banyak, tentukan jumlah Okik yang berada pada posisi tengah urutan tersebut!</p> <p>Jumlah = 10 Buah menjadi 2 Buah 10 Jumlah = 9 Buah menjadi 4 Jumlah = 14 Buah menjadi 5 Jumlah = 17 Buah menjadi 1</p>	<p>c. Determine the number of Okik in the middle of the sequence if you order the number of Okik from least to most!</p> <p>The total of Okik = 10 pieces in 2 months The total of Okik = 9 pieces in 4 months The total of Okik = 14 pieces in 5 months The total of Okik = 17 pieces in 1 months</p>																																																				

Based on Table 7 above, it can be seen that 18MAP code students were able to give the correct answer, but still had difficulty communicating the analysis and reasoning process obtained in written form. Seen in the pattern of answers to the B1 test, 18MAP students were able to reduce data from the information presented in tabular form. However, it appears that there is a typo in the February 2020 production month line. 18MAP students have understood that in the February production month, the Okik used by the weavers increased to 12 pieces from the previous 10 pieces. 18MAP students did not reason well with the information they received, but instead wrote

down the information they received in a table, even though the answers they gave were not wrong, only inaccurate and unclear. The same thing is also seen in the pattern of answers to the B3 test, where students are still wrong in understanding what is asked in the test. 18MAP students did not sort the number of Okik used from the smallest number to the largest number so the purpose of the test given was not achieved. The B3 test is intended so that students are able to determine the median value on data that has been sorted from the smallest to the largest data. This is obtained from interviews with student code 18MAP as follows:

- R (Researcher)** : *What do you mean by the answer that in February 2020, the number of Okik used was 10 to 12?*
- 18MAP** : *Oh yes, ma'am, what I mean is that in February 2020, the Okik used by weavers increased, ma'am. In the question, it was written that in February 2020 there were additional weavers, so the Okik used also increased. So, in the February 2020 table, I wrote down Okik which was used from 10 to 12 pieces. That means increased to 12 pieces ma'am.*
- R (Researcher)** : *Okay, how about the answers on the B3 test? What do you mean by the number of 10 pieces in 2 months, 9 pieces in 4 months, and so on? What do you mean that there are 10 Okik in 2 months of production? Please explain again!*
- 18MAP** : *Oh yes ma'am (pause for a moment... continue again). What I understand from the problem is to write down the number of Okik used starting from the least to the most. So, I wrote that there were 10 Okik used in January and February 2020, and soon, ma'am.*
- R (Researcher)** : *Are you sure that's what the B3 test meant?*
- 18MAP** : *I think so ma'am, but it still seems wrong, ma'am (students are not sure about the answer that has been given)*

Based on the pattern of answers and confirmation of the answer process given by students with code 18MAP, it was concluded that students had errors in understanding what was meant by the test given. Understanding the questions or problems given is an important factor in carrying out the data analysis process so that later it can be continued in determining the appropriate statistical calculations. The understanding of the questions given is included in the context of mathematical literacy. Students with low mathematical literacy ability will have an impact on improving other mathematical abilities such as problem-solving, reasoning, and other cognitive abilities. Mathematical literacy ability is proven to provide support for reasoning processes and other cognitive abilities, so that it will help students in improving mathematics learning outcomes (Holenstein et al., 2021). Based on these findings, students with code 18MAP on tests B1, B2 are at level 3 Transitional, where students have been able to reduce data in tabular form even though there are still errors in one of the data provided; able to reduce the data and use it in determining

the largest data. However, on the B3 test, students with code 18 MAP were at level 2 verbal. This is because students are still wrong in determining the most frequently used data according to the information in the ethnomathematics context presented.

Construct: Analyzing and interpreting data (C3 and C4)

The pattern of students' answers with code 15MHP (high statistical reasoning ability) on the C3 and C4 tests can be seen in Table 8 below.

Table 8.

The Pattern of Student Answer Code 15MHP (high ability) on C3 and C4 Tests

<p>c. Berapakah total keuntungan yang diperoleh selama 12 bulan menurut hasil analisis kamu?</p> <p>.....</p> <p>45.7 juta selama 12 bulan</p> <p>.....</p>	<p>c. What is the total profit earned over the course of a year based on the findings of your analysis??</p> <p>45.7 million for 12 months.</p>
<p>d. Coba kamu putuskan, berapa rata-rata keuntungan yang diperoleh dalam penjualan Putu Mayam tersebut?</p> <p>45</p> <p>.....</p> <p>3.833</p> <p>.....</p>	<p>d. Determine the average profit obtained from the sale of Putu Mayam!</p> <p>3.833</p>

Based on Table 8 above, it can be seen that the 15MHP code students did not write down their answers completely and clearly. In the answers to the C3 and C4 tests, students with 15MHP codes were able to analyze the answers correctly but did not provide procedural explanations. The results of the verification and confirmation regarding the answers given by students with code 15MHP through interviews are as follows.

R (Researcher) : How do you get the figure of 45.7 million as a total profit earned for 12 months? Please explain the steps.

15MHP : I got the figure of 45.7 million by adding up all the sales profits from month 1 to month 12.

R (Researcher) : Are you sure 3833 is the correct answer on the C4 test? How did you get that result?

15MHP : At first, I was sure, but after I looked again at the graph, I became unsure of the answer. I have a hard time dividing numbers in decimal form (pause for a moment, then think). So, I think my answer is still not correct, right? (Shows confused face).

R (Researcher)
15MHP

: How did you get that result?

: Initially, I added up all the sales profits from month 1 to month 12. Then, I divided the previous amount by 12 because the question asked was the average profit for 12 months. This is what I doubt, whether the answer is correct or still wrong.

Based on the confirmation of the answer process obtained from students with code 15MHP, students have difficulty when faced with calculating data by hand in the form of decimal numbers. This is in accordance with the findings of Roell (2017) where rational numbers and operations involving fractions and decimals are one of the biggest obstacles for students in learning mathematics, especially statistics. Students will find it difficult to complete decimal number operations without adequate knowledge of the concepts involved. The difficulties experienced by the 15MHP code student ultimately provided further obstacles in the analysis and reasoning process of the results that had been obtained. The results of the 15MHP students' answers on the C4 test were still not correct, because there was an error in placing the position of the decimal number in the division operation (in order to determine the average value). However, 15MHP students can understand the concept of determining the average score on descriptive statistics referring to the results of the interviews obtained. Therefore, students who code 15MHP on the C3 and C4 tests are at level 2 verbally, because students are still not able to carry out the reasoning process correctly and completely. But verbally students are able to understand how to get the right answer (understand the concept of average value). Overall, based on the results of the answers to 22 students, it was found that almost all students had difficulties in completing the C3 and C4 tests, this was also because the two tests were included in the difficult test category (can be seen in Figure 1 Map Wright).

The pattern of students' answers which were analyzed based on the level of initial mathematical ability they had, the applied descriptive statistics learning construct, and the level of statistical reasoning ability, it was found that students with low initial mathematical abilities were still at level 1 idiosyncratic. This is clearly seen from the pattern of answers given and the confirmation of the answers given. Anxiety and students' lack of understanding related to mathematical literacy are the biggest challenges for students with low initial abilities. Meanwhile, students with moderate initial abilities (average) are at level 2 verbal and Level 3 Transitional. Both levels of statistical reasoning ability are dominantly seen in students with moderate initial abilities. This is because students have been able to carry out the process of reduction, analysis, and reasoning on the given problem, but still verbally, and have difficulty communicating it in written form. Students who are able to reach level 4 Procedural statistical reasoning abilities in descriptive statistics learning are students with high initial abilities. This can be seen from the pattern of answers and confirmation of the answers given to indicate that students with high initial abilities are able to perform reduction, analysis, and reasoning on the given problem. Although there are still incomplete and

unclear solutions to problems, they have given the right results. The unique findings are seen specifically in questions C3 and C4 which are difficult questions, where students with high initial abilities also have difficulty in analyzing and reasoning the problems given. Students are still wrong in doing calculations and are not sure about the results and conclusions obtained. In the questions, C3 and C4, students with high initial abilities were still at level 2 verbal. This is because students are able to verbally understand and analyze the problem, but are still wrong in executing the answer process. The results of the analysis of the pattern of answers obtained provide the findings that the level of students' statistical reasoning abilities varies according to the learning construct used. However, the integration of the ethnomathematics context plays an important role for students, both students who have a native Malay ethnic background and a mixed Malay ethnic background, greatly assisting students in understanding the context of the problem presented. This is because students feel close to the problem presented. The ethnomathematics context used also provides a new experience for students in solving math problems more meaningfully (Fouze & Amit, 2018; Jurdak, 2016).

Differential Item Functioning (DIF) based on Gender, Initial Mathematics Ability, and Ethnic Background

The analysis of the pattern of student answers that have been described has answered the first and second research questions. The findings suggest that students' statistical reasoning abilities are closely related to their initial mathematical abilities. The results of the exploration of statistical reasoning abilities show how the level of students' statistical reasoning abilities in each descriptive statistical learning construct is presented in the form of ethnomathematics-based problems (Malay culture). Then, do the results of the exploration of statistical reasoning ability have a bias towards the gender, early mathematics ability, and ethnic background of the students? To answer this research question, we can perform an analysis using Differential Item Functioning (DIF) on the Rasch Model measurement. DIF shows the bias of differences in the answers given by students based on demographic factors, which can be seen by paying attention to the probability value of each item of the question. If the probability value of each item is below 5%, then the item questions that have been completed by students provide a bias and will tend to favor one of the demographic factors (Soeharto & Csapó, 2021). Based on the results of the DIF analysis, it was found that the probability value of all items used in analyzing the pattern of student answers related to the exploration of students' statistical reasoning abilities was not below 5%, which means that the items used did not have a bias towards the pattern of student answers to favor one or the other one demographic factor (gender, early mathematical ability, and ethnic background). The results of a further analysis can be seen in the graphic presented below.

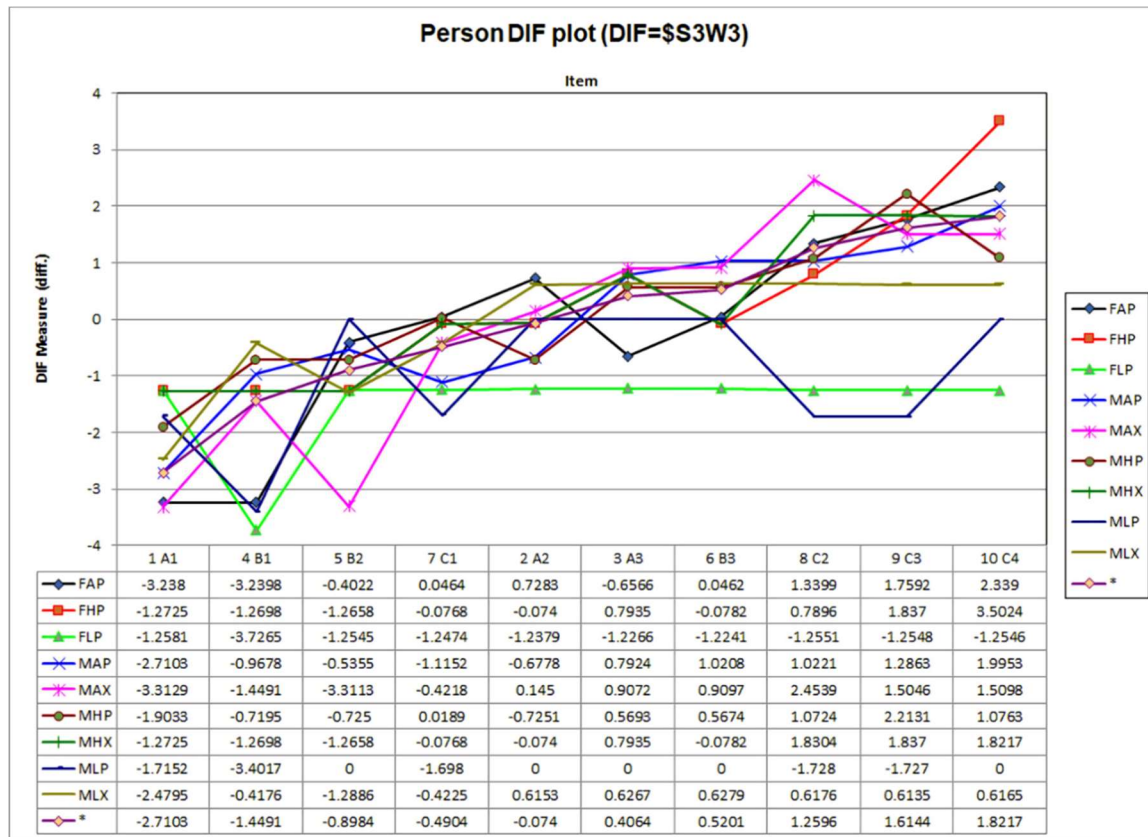


Figure 2.

Person DIF on Student Answer Patterns Regarding Gender, Early Mathematics Ability, and Ethnic Background

Based on Figure 2 above, the DIF analysis confirmed that students with female gender, low early math abilities, and ethnic Malay backgrounds had a fixed pattern of answers and did not experience significant changes. This can be seen in the DIF Measure that students have with the FLP code where 9 out of 10 items get scores that are not much different. The results of the DIF analysis concluded that although students have supportive demographic factors, such as gender, high early math abilities, and ethnic backgrounds that are in accordance with the given ethnomathematics-based problems, they do not provide benefits for students in improving learning outcomes, especially those closely related to improving students' statistical reasoning ability. However, it cannot be omitted that students' initial mathematical abilities also have their own role to support the development of other mathematical abilities. This condition is possible, and it can be seen that students with code 15MHP who have high initial mathematical abilities have good answer patterns and are at level 4 procedural on item questions with the construct of analyzing and interpreting

data. On the other hand, students with codes 20FLP and 22FLP who have low initial mathematical abilities and low scorers have a poor answer pattern and are at level 1 idiosyncratic on all items about statistical reasoning abilities. Wyse & Mapuranga (2009) also reported that bias on test items may occur based on demographic factors of research respondents. Based on this, the findings obtained in this study can be re-analyzed to see how big the implications are between other demographic factors by giving different treatments.

CONCLUSIONS

Students' statistical reasoning abilities can be explored well if students and teachers know the level of statistical reasoning abilities they have. The level of statistical reasoning ability can be analyzed by providing a test instrument that is developed based on contextual problems such as culture and traditions that are close to students' daily lives. The integration of the ethnomathematics context in the statistical reasoning ability test instrument helps students understand the statistical problems presented and makes it easier for students to carry out the process of reducing, interpreting, analyzing, and reasoning data for further conclusions. Exploration of the statistical reasoning ability of high school students by giving problems in the ethnomathematics context of Malay culture resulted in the findings that students have different levels of statistical reasoning ability. However, the findings of this study also concluded that although there is no bias in the pattern of answers given by students to demographic factors (such as gender, early mathematics ability, and ethnic background), the early mathematics ability factor still provides its own support in influencing students' statistical reasoning abilities. This shows that students' statistical reasoning abilities can be further improved by paying attention to students' initial mathematical abilities, as well as using the context of the problems presented.

The Rasch Model analysis provides an important role in checking for possible biases in student response patterns based on demographic factors. Rasch's analysis made it possible to further explore biases on demographic factors other than students' initial mathematical ability, gender, and student background. Other internal factors such as the level of learning motivation, the location of the student's residence area, as well as external factors such as giving different treatment to learning can be explored further using the Rasch Model analysis by providing optimal learning treatment.

In this study, giving problems with ethnomathematics contexts was proven to help students understand the problems presented to assist students in conducting the analysis and reasoning process of the data obtained further. These findings provide recommendations for researchers to integrate cultural contexts and traditions close to students in presenting mathematical problems. This study produces limited findings, where the ethnomathematics context presented uses the

cultural context and traditions of the Malay tribe in general, and the number of research subjects used is also small. Therefore, other researchers can continue similar research by paying attention to the demographic factors used, research subjects, and the existence of learning treatments so that the findings obtained can provide significant results.

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Specialized Content Knowledge of pre-service teachers on the infinite limit of a sequence

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Abstract: This paper analyses how pre-service teachers approach the notion of the infinite limit of a sequence from two perspectives: Specialized Content Knowledge and Advanced Mathematical Thinking. The aim of this study is to identify the difficulties associated with this notion and to classify them. In order to achieve this, an exploratory qualitative approach was applied using a sample of 12 future teachers. Among the results, we can affirm that pre-service teachers mainly use algorithmic procedures to solve tasks in which this type of limit is implicit, although they would consider a resolution that specifically involves the notion with an intuitive approach if they had to explain it to their students.

INTRODUCTION

The limit is one of the main notions of Calculus, since its understanding is linked to most of its content (Morales et al., 2013). Consequently, the analysis of the limit and the difficulties associated with its teaching-learning process constitute lines of research in the area of Didactics of Mathematics. This notion, and in particular, the infinite limit, has been studied by many authors. Its difficulties constitute one of the lines of research in didactics of mathematics in recent decades (Dong-Joong, Sfard and Ferrini-Mundy, 2005, Jutter, 2006, Morales, Reyes and Hernández, 2013, Douglas, 2018). Many authors, however, have studied this notion without differentiating whether the limit was of a sequence or of a function (see Claros et al., 2013). In this paper, as already considered by Arnal-Palacián (2019) in a previous phenomenological study in the sense given by Freudenthal (1983), the learning of the infinite limit of a sequence is addressed in a particular way, since each type of limit has peculiarities that cannot be addressed together.

At the educational level, Galileo's and Cantor's notion of infinity appears very late in the mathematical education of students. Salat (2011) proposes, as early as possible, to get students to discuss infinity and its applications in terms of reflecting on their own mathematical thinking. On the other hand, Miranda et al. (2007) pointed out that students continue to experience difficulties associated with the intuitive and formal nature of the mathematical notion of infinity. Some of these difficulties are: a) failure of the link between geometry and arithmetic (epistemological), b)

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difficulties due to the very nature of the notion (didactic), c) the teaching methods used by teachers (didactic), d) eliminating the problem of infinity by taking as many terms as necessary (cognitive), among others. If we look at research on the limit and infinity from the point of view of the study of teachers, there is much less research on the limit and infinity from the point of view of teachers than that on students, although its content is very similar.

In view of the above, the main objective of this study is to analyze the difficulties that pre-service teachers have when working on tasks involving the infinite limit of a sequence.

The analysis of these difficulties will make it possible, firstly, to point out the difficulties associated with this notion when it is presented by pre-service teachers and, secondly, to classify these difficulties in two areas: when the trainee teacher carries out the proposed task or when the trainee teacher has to explain it to future students. We will go deeper into both aspects in order to anticipate such difficulties in a future didactic sequence that could be useful for the trainee teacher.

THEORETICAL FRAMEWORK

The theoretical framework on which this work is based is the difficulties of the notion of limit and Specialized Content Knowledge. We will also relate the latter to Advanced Mathematical Thinking.

The notion of infinite limit

The definition has a distinctive role in the technical use of a construct in contrast to its intuitive and colloquial uses (Vinner, 1991). In the particular case of the definition of the infinite limit of a sequence, as already observed in Arnal-Palacián (2019), it is placed in textbooks shortly before the introduction of the infinite limit of a function and consecutively to the finite limit of a sequence.

Taking into account the variety of definitions of the infinite limit of a sequence found, Arnal et al. (2017) consulted university professors and secondary education teachers with the aim of selecting a correct definition that is accepted by the mathematical community that teaches at different educational levels. The definition selected was the following:

“Let K be an ordered field, and $\{a_n\}$ a sequence of elements of K . The sequence $\{a_n\}$ has for limit 'more infinite', if for each element H of K , a natural number exists v , such that it is $a_n > H$, for all $n \geq v$.” (Linés, 1983, p.29).

Although it was not part of the consultation described above, for the infinite limit of a sequence we present the definition provided by the same author:

“Let $f: X \rightarrow \mathbb{R}$, with $X \subset \mathbb{R}$ not upper bounded. It is said that f has limit $+\infty$, when it tends to $+\infty$, if for every real number H a real number exists K such that it is $f(x) > H$ for all $x \in X$, that complies $x > K$. It is written $\lim_{x \rightarrow +\infty} f = +\infty$.” (Linés 1983, p.201-202).

Among the notions that appear in the definition of the infinite limit of a sequence, we highlight the following: dependence, sufficiently large, infinite processes, bounding, types of infinity and divergence. All of them should be taken into account in the teaching and learning of the notion of the infinite limit of a sequence, specifically in the design of a didactic sequence that addresses the teaching of the infinite limit of a sequence. See Figure 1.

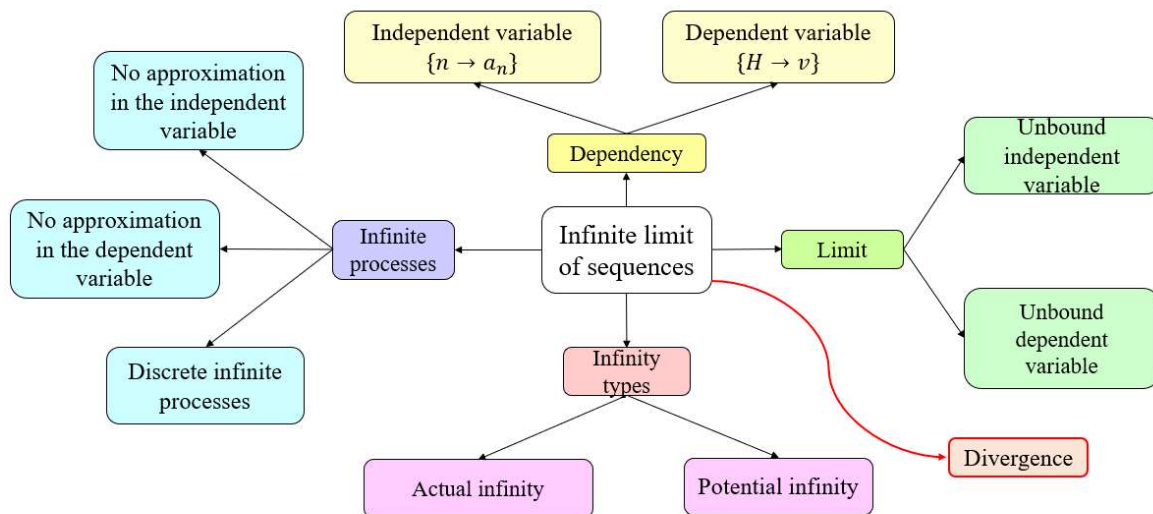


Figure 1. Mathematical notions to be used for the design of a didactic proposal.

Difficulties with the notion of limits

In order to study the notion of the infinite limit of a sequence, it is essential to be aware of the difficulties, obstacles and errors previously studied for the notion of limit. To this end, the works of Duval (1998), Medina and Rojas (2015), Morales et al. (2013), Vrancken et al. (2006), among others, will be taken into consideration.

One way of approaching problems arising from the notion of limit is to use different systems of representation, although there is currently a tendency to teach its calculation from an algorithmic and algebraic approach. Moreover, when the student uses only the algebraic representation, it is practically impossible to detect where the error lies. In many cases, graphical representation is not used because it is not considered as a support for these algebraic processes (Vrancken et al., 2006). In the analysis, a resolution or explanation that considers this algorithmic approach will be treated independently of when the handling of the notion itself is taken into account in a specific way.

In order to begin to understand a notion, as in this case the infinite limit of a sequence, students must identify it in different representations: graphical, numerical, algebraic and verbal. Moreover,

it is necessary to coordinate the different systems of representation in order to obtain a comprehensive understanding of the notion (Duval, 1998).

Along the same lines, Morales et al. (2013) consider that activities related to the notion of the infinite limit using different representations make it possible to identify some difficulties such as: a) justification of the limit, b) application of the conditions of the definition of the notion, c) generalization and d) preference in the use of procedural methods as opposed to conceptual methods.

Regarding infinity, which is closely related to the notion of limit, Medina and Rojas (2015) carried out a review of the obstacles associated with it: non-acceptance of actual infinity and influence of potential infinity (Sierpiska, 1985); separation of the geometric and the numerical, i.e. the continuous from the discrete, since the successful solution of some problems through their geometric interpretation prevents the passage to the notion of numerical limit (Cornu, 1991); generalization of properties from the finite to the infinite, and Leibniz's principle of continuity.

We will take these difficulties into account in the analysis of the pre-service teachers' responses to the proposed task and indicate whether or not they are reproduced in the trainee teachers.

Specialized Content Knowledge

Mathematical as well as psychological and pedagogical content must be integrated into the university training of student teachers (Movshovitz and Hadass, 1990). Furthermore, teacher training, both initial and in-service, is a very important factor in the improvement of mathematics teaching and learning processes. This training must be focused on their professional development and for this it is essential that they put into practice a deep specialized knowledge of the content (Posadas & Godino, 2017).

In recent years, research in mathematics education has been nourished by models of descriptive analysis of the specialized knowledge of teachers (Shulman, 1986), who first included pedagogical content knowledge (PCK) as a differentiating element between the mathematical knowledge that a teacher should have and any other person who performs mathematical tasks. Shulman (1986) described the types of knowledge that a teacher should have in order to be competent in his or her job, initially including knowledge of the content to be taught, knowledge of the pedagogy needed to teach it, and knowledge of the curriculum. In the case of mathematics, Shulman's ideas were adapted by Ball, Thames and Phelps (2008) to give rise to what is known as the Mathematical Knowledge for Teaching (MKT) model. This model is divided into two domains: content knowledge (SMK) and pedagogical content knowledge (PCK). See Figure 2.

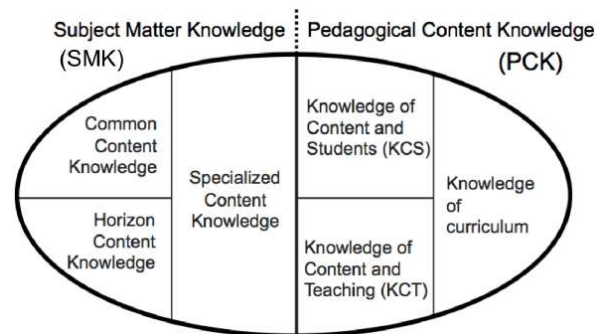


Figure 2: MKT Model (Ball, Thames & Phelps, 2008)

In this study, we will focus on the analysis of the Specialized Content Knowledge (SCK) associated with the infinite limit of a sequence, of the subdomain Subject Content Knowledge (SMK) that considers the mathematical content that a teacher needs from a teaching point of view (Ball et al. 2008). To specify what is and what is not SCK, Ball et al. (2008) list a number of activities that are: a) examining equivalences, b) recognizing what is involved in using a particular representation, c) relating representations to underlying ideas and to other representations, d) choosing and developing useful definitions, e) using mathematical notation and language and critiquing their use, f) connecting a topic being taught to topics from previous or future years, g) presenting mathematical ideas, among others.

SCK is the mathematical knowledge and skill that only teachers need to carry out their work. Mathematical knowledge that is not usually needed for purposes other than teaching.

Furthermore, we relate this mathematical content that a teacher needs to how people who are professionally engaged in mathematics think. The latter is called mathematical thinking and is understood as a spontaneous reflection carried out by mathematicians on the nature of the process of discovery and invention in mathematics, and also on advanced thinking processes, such as abstraction, justification, visualization, estimation or reasoning based on the justification of proposed hypotheses (Cantoral et al., 2000). In mathematical thinking we differentiate between Elementary Mathematical Thinking (EMT) and Advanced Mathematical Thinking (AMT). EMT is characterized by routine tasks in the classroom, using definitions only for the description of already known objects (Calvo, 2001). On the other hand, AMT is characterized by the intervention of the following processes: representation, abstraction, formalization, definition, among others (Garbin, 2015).

The justifications or explanations provided by the pre-service teacher where intuitive approaches are involved will be classified within EMT, while when this is formal it will be classified within AMT (Dreyfus, 1991; Tall, 1991).

METHOD

This study is based on an exploratory qualitative approach. By means of convenience sampling, i.e. the sample is made up of those trainee teachers to whom we have had access, we have analysed 12 students from a Spanish university who are studying for the Master's Degree in Teacher Training. In Spain, this Master's degree is a qualification without it is not possible to teach different subjects in Secondary Education (12-18 years), and its duration is one academic year, 600 hours. Previously, in Spain, students who take it have had to pass a Bachelor's Degree (8 semesters), related to the specialty to be taught. In the case of this study, the previous training of the sample members was as follows: Mathematics (9), Industrial Engineering (2) and Telecommunications Engineering (1). Their training in mathematical notions of calculus during their previous university degree, and in which the notion of limit appeared as it was taken up in the present study, can be found below (Table 1):

Degree	Subjects	Total hours of the subject	Content related to the infinite limit of sequences
Mathematics	Mathematical Analysis I	135	Sequences of real numbers. Convergence. Calculation of limits. Continuity. Limits of functions. Continuous functions. Properties. Weierstrass, Bolzano and Darboux theorems. Classification of discontinuities.
	Mathematical Analysis II	150	Functions of several real variables. Limits and continuity.
Physics	Mathematical Analysis	60	Sequences and limits. Cauchy sequences. Limit of functions. Continuity.
Industrial Engineering	Mathematics	60	Sequences and series of real numbers.
Telecommunications Engineering	Calculus	60	Real functions of a real variable: limits and continuity.

Table 1. University training in calculus.

At the time of the data collection for this study, they attend the last sessions of the subject Design of activities for learning Mathematics. Prior to this, the students have passed the subjects Disciplinary Contents of Mathematics and Curricular and Instructional Design of Mathematics, as well as having completed 100 hours of attendance at an educational centre. The elements of analysis have been the evidence of the work samples of the proposed task (Figure 3).

Study the limit of the sequence a_n defined by $a_n = n^2 - 15$. Justify your answer. Then, indicate the explanation you would give to your students in the classroom.

Figure 3: Proposed task on the limit of a sequence.

For the analysis of the data, an inductive-deductive categorization process was carried out, including the categories expected from the theoretical review, described in the previous section, and also those arising from the analysis of the evidence collected from the trainee teachers' responses, so that all possible situations were included. The two variables taken into account were: task correctness and type of thinking involved. Both variables are used, on the one hand, for the resolution and justification of the resolution and, on the other hand, for the explanation provided to their future students. Therefore, the table we will use in the analysis (see Table 2) will be presented twice in the results section, the first time for the justification of the resolution and the second time for the explanation provided by the trainee teachers to their students.

Correction of the task	Type of mathematical thinking involved
No resolution or explanation included	
Performs incorrectly	EMT Algorithmic Involves the notion in a specific way
	AMT
Performs correctly	EMT Algorithmic Involves the notion in a specific way
	AMT

Table 2: Variables and categories considered for analysis.

- a) In correcting the task we consider:
- Does not include resolution or explanation: those answers in which the teacher does not respond to the proposed task, leaving the answer blank.
 - Performs incorrectly: the trainee teacher makes a mistake when solving the proposed task on the infinite limit of a sequence. This error may be in the use of one of the representation systems, in the understanding of the notion itself or algorithmic procedures, among others.
 - Performs correctly: the trainee teacher correctly solves the proposed task on the notion of the infinite limit of a sequence, both in the algorithmic processes and in the use of the different systems of representation that he/she decides to use. He/she also correctly uses the notation and vocabulary specific to this mathematical notion.
- b) We differentiate the type of thinking developed by the trainee teacher: PME and PMA, described in the previous section, in relation to the type of response provided to each of the two tasks proposed, both for justification and explanation:
- EMT. This category is subdivided into two:
 - Algorithmic: when the pre-service teacher uses properties of the limit that he/she uses routinely.
 - Involves the notion in a specific way: when intuitive approaches are involved, without the use of algorithmic procedures.
 - AMT: The pre-service teacher uses different systems of representation, abstraction, formalization or formal definition of the limit, i.e. when a formal approach is involved.

Afterwards, and given the relevance of the different systems of representation, an account will be made of them, both when they have to justify their answer and in the explanation to their students. This analysis takes into account both the ways of expressing and symbolizing when solving or explaining the limit of a sequence in the proposed task. Among the systems of representation, we consider: verbal, tabular, symbolic and graphic (Janvier, 1987).

In addition to the productions of future teachers, 35 textbooks published in Spain from 1936 to the present day have been considered, from the 1st and 2nd years of baccalaureate (16-18 years), both in Science and Social Sciences. Of these, we will look in depth at the 4 textbooks from 2010 to 2019, i.e. those that could be used by future teachers when mathematics at the educational level for which they are preparing. This analysis will take into account representation systems in which the infinite limit of a sequence is presented: verbal, tabular, graphical and symbolic; its format: definition and example; and the approach: intuitive and formal.

This analysis will serve to compare the way in which future teachers solve the proposed task in relation to the way in which it is presented in textbooks.

RESULTS

Procedures vs. notion involved

Given that the proposed task consists of two parts: the first in which the student would solve the limit, and the second in which he would explain it to his future students, we are going to approach the analysis independently. This analysis will allow us to identify the difficulties presented by the Master's student when solving the task and, on the other hand, to see what misconceptions, or not, the student would transmit when he or she becomes a teacher and has to explain the task to his or her students.

When the pre-service teachers are asked to solve and justify the limit, we obtain the following results (Table 3):

Correction of the task	Type of mathematical thinking involved	Percentage	
No resolution or explanation included		0 %	
Performs incorrectly	EMT	Algorithmic	0 %
		Involves the notion in a specific way	33.33 %
		AMT	
Performs correctly	EMT	Algorithmic	58.33 %
		Involves the notion in a specific way	0 %
		AMT	

Table 3: Results of the answers to solving and justifying the limit of a sequence.

We found that 33.33 % of the pre-service teachers gave incorrect answers. In all cases, these errors occur when the future teacher involves the notion in a specific way from a PME. In all cases,

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elements appear that could appear in the limit of a function, but in no case are they attributed to the notion studied in this paper: the infinite limit of a sequence. Below, we show an example of an answer where the future teacher considers the limit at different points, as if it were a function (Figure 4) and another example where L'Hôpital is applied without indeterminacy and without the appearance of functions (Figure 5). In the latter case, we attribute the error not only to the lack of indeterminacy but also to the confusion between L'Hôpital, used for functions, and the Stolz Criterion, used for sequences.

<p>Para calcular un límite, primero hay que ver para qué valor de n (en qué punto).</p> <p>Por ejemplo si, en este caso, el límite es:</p> <ul style="list-style-type: none"> - cuando n tiende a $0 \Rightarrow -15$ - cuando n tiende a $\infty \Rightarrow \infty$. <p>y segundo sustituir ese valor de n.</p>	<p>To calculate a limit, we must first see for what value of n (at what point).</p> <p>For example, if, in this case, the limit is:</p> <ul style="list-style-type: none"> - when n tends to $0 \rightarrow -15$ - when n tends to $\infty \rightarrow \infty$ <p>And second substitute that value of n.</p>
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Figure 4: Incorrect answer where invalid limits are contemplated and their translation.

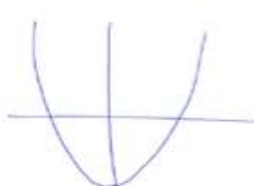
<p>$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} (n^2 - 15)$</p> <p>Si es un número diferente de ∞ decir un número entero y ∞ es el límite y si no decir ∞</p> <p>Si es indeterminación aplicar el L'Hôpital $\rightarrow \frac{(n^2-15)}{1} = 2n - 0$</p> <p>y volver a aplicar el valor de n del límite</p> 	<p>If it is a number different from ∞ it will be an integer and that will be the limit, otherwise, it will give ∞.</p> <p>If it is indeterminacy, I will apply L'Hôpital and I will apply again the value of n of the limit.</p>
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Figure 5: Incorrect answer where the future teacher tries to apply L'Hopital and its translation.

Most of the future teachers are able to solve the limit of the given sequence. Moreover, more than half of them prefer to solve it using algorithmic procedures, without actually using the mathematical notion of the infinite limit of a sequence in a particular way. In these algorithmic processes they employ equivalences, and use the correct mathematical notation and language. See Figure 6 and 7.

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} (n^2 - 15) = \lim_{n \rightarrow \infty} n^2 - \lim_{n \rightarrow \infty} 15 = +\infty - 15 = +\infty$$

Figure 6: Correct answer of a student using an algorithmic procedure in a symbolic way.

El límite en el infinito es infinito puesto que para mirar a que tiende un "polinomio" nos fijamos en el término de mayor grado; en este caso es grado 2 y $\infty^2 = \infty$

The limit at infinity is infinite because to see what a “polynomial” tends to, we look at the term with the highest degree, in this case of degree 2 and $\infty^2 = \infty$.

Figure 7: Correct answer of a student using an algorithmic procedure verbally and its translation.

Only one of the samples showed a development of the AMT. This future teacher not only accounts for the growth of the succession terms, but also relates them to the position they occupy, “As n grows a_n grows over n ” (Figure 8).

$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} n^2 - 15 = +\infty$. Conforme n crece a_n crece por encima de n .

Figure 8: Correct response of a future teacher developing an AMT.

Subsequently, in the second part of the proposed task, when the trainee teachers were asked to explain what their intervention in the classroom would be like, we obtained the following results (Table 4):

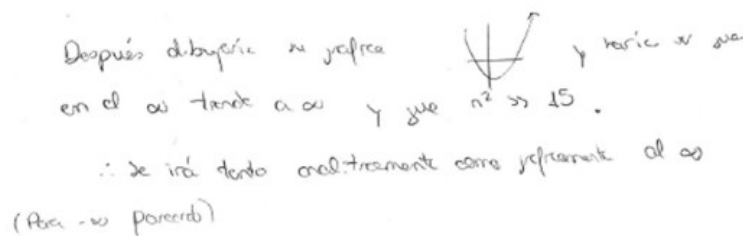
Correction of the task	Type of mathematical thinking involved	Percentage
No resolution or explanation included	No resolution or explanation included	33.33 %
Performs incorrectly	EMT Performs incorrectly	0 %
	EMT Involves the notion in a specific way	16.67 %
	AMT	0 %
Performs correctly	EMT Performs correctly	0 %

	Involves the notion in a specific way	50 %
	AMT	0 %

Table 4: Results of the answers to the explanation of the limit of a sequence.

The same prospective teachers who performed poorly on the first proposed task, i.e. solving and justifying the limit of a sequence, did not include the explanation they would provide in the classroom of the notion involved.

On two of the occasions when prospective teachers had correctly justified the limit in the first part of the task, using an algorithmic procedure and without mentioning the notion they were using, they provided a wrong explanation in the second part of the task. This corresponds to 16.67 %. Given the presentation format of this second part of the task, which allowed for a more detailed explanation, these prospective teachers used the wrong graphical representation system. The notion represented was a function, which they had not previously used and which was not involved in the task. Moreover, they proposed to perform the limit for $n \rightarrow -\infty$, when this cannot be done for a sequence, but for a function. See Figure 9.



Después dibujaría su gráfica y haría su sea en el ∞ tendiendo a ∞ y que $n^2 \gg 15$.
 \therefore se irá dando analíticamente como se representa al ∞
 (Para $-\infty$ parecería)

Then, I would draw its graph and show that at ∞ and that $n^2 \gg 15$.
 It will go both analytically and graphically to ∞ . (For similar $-\infty$)

Figure 9: Incorrect answer of a future teacher in the explanation of the notion of the limit of a sequence and its translation.

On the other hand, half of the future teachers analyzed used a development of the EMT in their explanation, involving the notion in a specific and correct way. Most of them are those who, in the resolution and justification, used a development of the EMT involving algorithmic procedure. We can observe very brief explanations, as well as a very intuitive view of the limit (Figure 10). We also find explanations where simpler examples are presented, such as $a_n = n^2$ in the tabular representation system (Figure 11).

este número va creciendo según ~~los~~ términos de
la sucesión son mayores ~~por~~ por ello
tiende a infinito.

This number grows as the terms of
the sequence gets larger and larger, so it
tends to infinity.

Figure 10: Example of a response with an intuitive view of the limit and its translation.

n	a_n
1	1
2	4
4	16
\vdots	\vdots

Figure 11: Example of a response using simpler examples.

Representation systems used

Given the relevance of the different systems of representation, which we have already mentioned, we have collected the ones used by trainee teachers by combining their complete response to the two proposed tasks. See Table 5.

	Verbal	Graphic	Tabular	Symbolic
Does not use representation	8.33 %	75.00 %	75.00 %	25.00 %
Uses an incorrect representation	33.33 %	25.00 %	0.00 %	25.00 %
Uses a correct representation	58.33 %	0.00 %	25.00 %	50.00 %

Table 5: Representation systems involved.

The representation system that was most frequently used correctly was verbal (58.33 %), where the future teachers, in addition to verbalizing the algorithmic procedure for the second part of the task, also used it correctly (58.33 %), “subtracting 15 is irrelevant against a power of n^2 when $n \rightarrow \infty$ ” (Figure 12) that they have followed in the first part of the task, specifically involve the notion with which they are working, “the limit tends to $+\infty$ because the numbers are getting bigger and bigger” (Figure 13).

EL RESMAR 15 ES IRRELEVANTE FRENTE
A LA POTENCIA DE n^2 CUANDO $n \rightarrow \infty$

Figure 12: Use of the verbal representation system as an explanation of algorithmic procedure.

el limite tiende a ∞ porque los números sin cada vez
mayores.

Figure 13: Use of the verbal representation system involving the notion of infinite limit.

At the other end of the spectrum is the graphic representation system, which, in addition to not being used by 75% of future teachers, those who do use it present it incorrectly, treating the notion as if it were a function instead of a succession (Figure 14).

Después dibujé la gráfica y hacia ∞ sea
en el ∞ tiende a ∞ y que $n^2 \gg 15$.




Figure 14: Use of the graphical representation system.

The same percentage (75 %) of trainee teachers declines the use of the tabular representation system, although in this case, those who decide to use it use it correctly. See Figure 15.

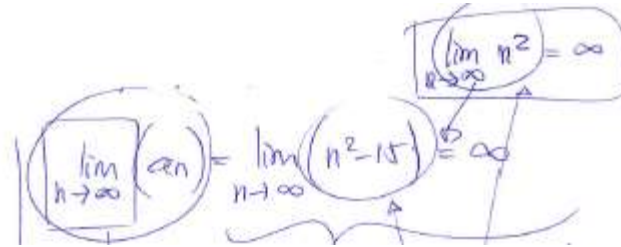
n	1	3	5	límite $\rightarrow \infty$
n^2	1	9	25	∞

Figure 15: Use of the tabular representation system in the second part of the task.

The fourth of the representation systems, the symbolic one, used correctly by 50% of the future teachers, was used to a greater extent in the first part of the task, while in the second part it was relegated to very few cases. In both cases for the use of algorithmic procedures. See Figure 16 and Figure 17.

$$\begin{aligned} \text{Si } n \rightarrow \infty & \quad \lim_{n \rightarrow \infty} a_n = +\infty \\ & \quad \lim_{n \rightarrow -\infty} a_n = +\infty \end{aligned}$$

Figure 16: Incorrect answer using the symbolic representation system.



$$\lim_{n \rightarrow \infty} (a_n) = \lim_{n \rightarrow \infty} (n^2 - 15) = \lim_{n \rightarrow \infty} n^2 = \infty$$

Figure 17: Correct answer using the symbolic representation system.

Textbook analysis

Given that the same number of books was not available for each of the decades, the average number per textbook was taken as a reference for each of the systems of representation: verbal (v), tabular (t), graphic (g) and symbolic (s), and formats: definition (d) and example (e). Also, on the one hand, fragments were collected that have to do with an intuitive approach, linked to the EMT; and on the other hand, fragments with a formal approach, linked to the AMT.

The historical evolution of the plus and minus infinite limit of a sequence is shown below (Figure 18).

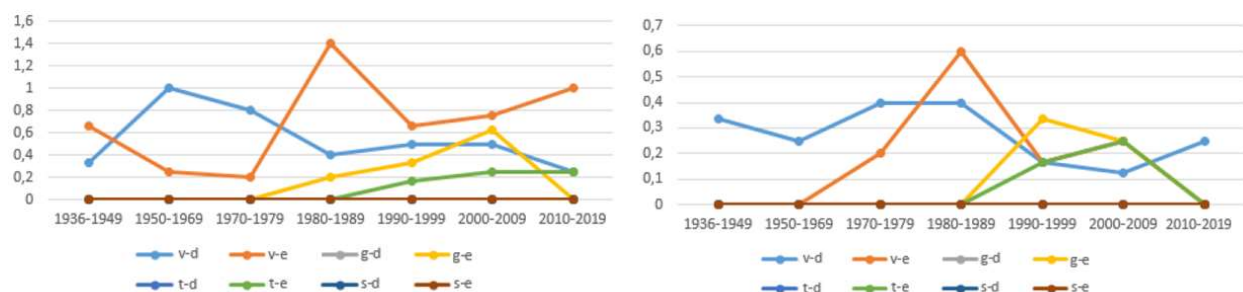


Figure 18. Historical evolution with an intuitive approach. More infinite (left) and less infinite (right).

Although the most infinite and least infinite limit have historically had a similar behaviour, it is important to highlight the presence of the most infinite limit higher than that of the least infinite limit, doubling its frequency in some instances.

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The following image shows some of these fragments analysed (Figure 19 and Figure 20). In the first of them, we observe how the textbook presents an example of the most infinite limit of a sequence in the verbal representation system. While in the second one we observe the intuitive approach in the verbal representation system.

Veamos qué sucede con la sucesión $b_n = \frac{n^2}{n+1}$:

n	1	10	100	1000	10000	100000	1000000
a_n	0,5	9,09	99	999	9999	99999	999999

En esta sucesión los términos no se aproximan a ningún número, en cambio a medida que n aumenta los términos se hacen cada vez mayores.

Let's see what happens for the sequence $b_n = \frac{n^2}{n+1}$.

In this sequence the terms do not approach any number, but as n increases the terms become larger and larger.

Figure 19. Fragment with intuitive approach (t-e) (Valverde et al., 2015)

Por otro lado, la sucesión $b_n = 2n$ verifica que sus términos se hacen cada vez mayores: es decir, *tienden a $+\infty$* : $b_1 = 2, b_2 = 4, b_3 = 6, b_4 = 8, b_5 = 10\dots$

Se dice que el límite de la sucesión es $+\infty$: $\lim_{n \rightarrow \infty} b_n = +\infty$

On the other hand, the sequence $b_n = 2n$ verifies that its terms become greater and greater; that is, they tend to $+\infty$. $b_1 = 2, b_2 = 4, b_3 = 6, b_4 = 8, b_5 = 10\dots$

It is said that the limit of the sequence is $+\infty$: $\lim_{n \rightarrow \infty} b_n = +\infty$.

Figure 20. Fragment with intuitive approach (v-e) (Alcaide et al., 2016)

In a similar way, we present the infinite limit of a sequence from a formal approach. Given the process of generalisation and abstraction that the student has to carry out, on this occasion the difference between the plus and minus infinite limit has not been established. See Figure 21.

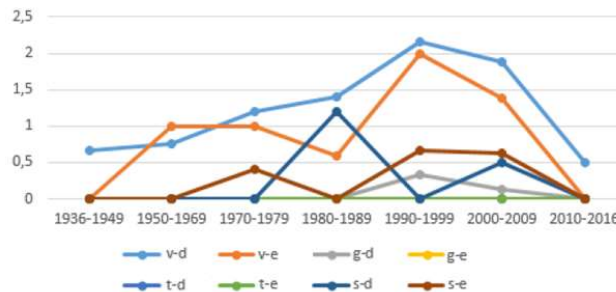


Figure 21. Historical evolution with a formal approach

As can be seen, the formal approach to the notion of limit has been decreasing its presence in Spanish textbooks in recent years. A sample of one of the last fragments found is the following (Figure 22) in the verbal representation system and definition format.

De forma teórica: $\lim_{n \rightarrow \infty} a_n = +\infty$ si para cualquier número, k , podemos encontrar un número natural h tal que:
Si $n > h$, se cumple que $a_n > k$.

Theoretically: $\lim_{n \rightarrow \infty} a_n = +\infty$ if for any number, k , we can find a natural number h such that:
If $n > k$, then $a_n > k$.

Figure 22. Fragment with formal approach (v-d) (Escoredo et al., 2009).

Analysis of future teachers vs. textbooks

The results obtained from the analysis of the answers given by the future teachers are related to those presented in the study of the textbooks we have carried out. In fact, prospective teachers use the verbal representation system more frequently and with a lower percentage of error. This corresponds to the fact that the verbal representation system is the most frequent in recent years in the textbooks when the infinite limit of a sequence has to be presented.

With respect to the graphical representation system, its frequency has declined in recent years and this could explain some of the errors that have arisen when future teachers use this system in their answers to justify their answers or to calculate a limit. To this difficulty we can also add the variety

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of limits of functions presented and the corresponding graphical representations of them, a fact which may lead us to think that there is a limit when n tends to $-\infty$ or when n tends to 0, as some future teachers point out in their answers. We attribute some of these difficulties or errors to what Fernández-Plaza and Simpson (2016) point out, where the symbols used for sequences and functions are almost the same, but connections must be established in the classroom that relate both concepts.

Furthermore, we can also attribute the development of the EMT in textbooks, to the detriment of the AMT, due to the boom that the intuitive approach to the notion studied seems to be experiencing in textbooks. This fact has meant that certain notions are not treated with the rigour and therefore a real understanding of them is not produced.

CONCLUSIONS

In the present study it has been possible to analyze the difficulties existing in pre-service teachers when performing tasks in which the infinite limit of a sequence is involved, both for their resolution and for their future explanation in the classroom. Given the professional development focus of the task, they have needed to put into practice specialized knowledge of the content (Posadas & Godino, 2017). In addition, we consider that this task has been appropriate for developing MKT, as it has allowed the use of multiple representations and has given the pre-service teachers the opportunity to propose mathematical practices for teaching the notion of the infinite limit of a sequence. In particular, and in relation to the SCK (Ball et al. 2008), during its resolution, we have been able to observe how future teachers use equivalences, notation and mathematical language to approach it. On the other hand, we note that they used different systems of representation, but they did not relate the underlying ideas, nor did they connect the notion with topics from previous and future years.

On the other hand, and in relation to the type of Mathematical Thinking, in the first part of the task in which the students had to solve the proposed limit, the correct answers are presented with a development of the EMT from an algorithmic approach and, on one occasion, with a development of the AMT. Incorrect answers are presented when students, from a EMT development, involve the notion of the infinite limit of a sequence. Likewise, we perceive a change in how future teachers will approach the explanation of the limit. They discard the use of the AMT, despite the fact that their future students are at an age when it can begin to be introduced, and when they develop the EMT it is always by involving the notion, without using algorithmic procedures and from a more intuitive point of view. We can therefore affirm that future teachers do not involve the processes of abstraction, formalization and definition (Garbin, 2015), which are characteristic of AMT, and opt for routine tasks in the classroom, which are characteristic of AMT (Calvo, 2001).

Despite Duval's (1998) indications that in order to understand a notion, different representations must be provided, not many future teachers have resorted to showing the notion of the infinite limit

of a sequence in more than one system of representation. Moreover, in no case was coordination between them shown, which would provide a comprehensive understanding of the notion. On the other hand, as stated by Vrancken et al. (2006), future teachers are in favour of the current tendency to consider an algorithmic approach to solving a given limit. This has hardly made it possible to detect errors in the first part of the task. It was in the second part when, by using conceptual methods and different systems of representation (Morales et al. 2013), such as the graph, more difficulties were detected.

In view of the above, we can affirm that future teachers do not have an adequate knowledge of the notion of the infinite limit of a sequence and, consequently, this will undoubtedly be transmitted to their students.

As a future perspective, we consider the creation of a didactic sequence composed of different examples and definitions, developing both the EMT and the AMT, which will help to overcome the difficulties encountered.

Acknowledgements

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The Problem Corner

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The Purpose of **The Problem Corner** is to give Students and Instructors working independently or together a chance to step out of their “comfort zone” and solve challenging problems. Rather than in the solutions alone, we are interested in methods, strategies, and original ideas following the path toward figuring out the final solutions. We also encourage our Readers to propose new problems. To submit a solution, type it in Microsoft Word, using math type or equation editor, however PDF files are also acceptable. Email your solution as an attachment to The Problem Corner Editor iretamoso@bmcc.cuny.edu stating your name, institutional affiliation, city, state, and country. Solutions to posted problem must contain detailed explanation of how the problem was solved. The best solution will be published in a future issue of MTRJ, and correct solutions will be given recognition. To propose a problem, type it in Microsoft Word, using math type or equation editor, email your proposed problem as an attachment to The Problem Corner Editor iretamoso@bmcc.cuny.edu stating your name, institutional affiliation, city, state, and country.

Hello Problem Solvers, solutions to **Problem 2** were submitted, and I am glad to report that they were correct and interesting. The solutions followed different approaches, which I hope will enrich and enhance the mathematical knowledge of our community.

Solutions to **Problem** from a Previous Issue

Interesting “Largest path” Problem

Proposed by Ivan Retamoso

Problem 2

The distance between Paul’s home and his School is 2 miles. One day after his school day is over, Paul decides to walk back home by taking 2 straight paths perpendicular to each other,

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assume the territory where Paul's home and his school are located allows him to do it, any way he wishes as long as the 2 straight paths are perpendicular to each other.

- What is the total length of the largest path Paul can take to go back home from his school?
- Give a compass and straightedge construction of the path you found in part a) starting from the distance between Paul's home and his School which is 2 miles, using any scale to represent a mile.

Solution 1

by Aradhana Kumari, Borough of Manhattan Community College, USA.

This solution uses Calculus via Differentiation, the length of the path was found in general and by setting its derivative equal to zero the maximization of the length of the path was accomplished, this is followed by a step-by-step construction of the optimal path using only straight edge and compass.

Solution:

Part a)

As per question we have

Total length $L = \text{length SA} + \text{length AH}$

$$= 2 \cos \theta + 2 \sin \theta, 0 \leq \theta \leq 90^\circ$$

$$L' = -2 \sin \theta + 2 \cos \theta$$

To find which angle will maximize the total length consider we have to

Consider the equation $L' = 0$

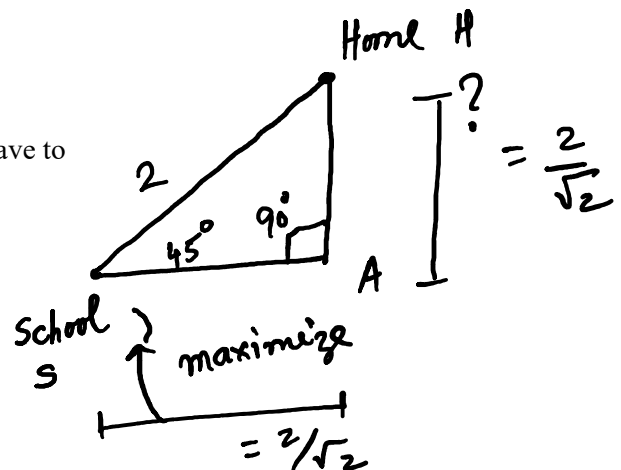
$$\text{We have } -2 \sin \theta + 2 \cos \theta = 0$$

$$-2 \sin \theta = -2 \cos \theta$$

$$\tan \theta = 1$$

$$\theta = 45^\circ$$

Therefore, the total length of the largest path is



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?

$$2 \cos 45^\circ + 2 \sin 45^\circ = \frac{2}{\sqrt{2}} + \frac{2}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}$$

Part b)

Step 1) Draw a line segment of length 4 unit. Call the end points as S (School) and P

Step 2) We will draw the perpendicular bisector of segment SP. Keep the compass at S and draw an arc above and an arc below as shown below.

Step 3) Next move the compass at P and draw an arc above and below as shown below.

Step 4) Join K and L as shown. Segment KL intersect segment SP at the point H at 90° .

Step 5) We will draw the angle bisector of right angle KHS. We keep the compass at H (Home) and draw an arc on the segment HS. Let say this arc intersect the segment SH at point F

Step 6) We keep the compass H and draw an arc on the segment HK. the arc interests the segment HK at point E.

Step 7) We keep the compass at E and draw the arc as shown.

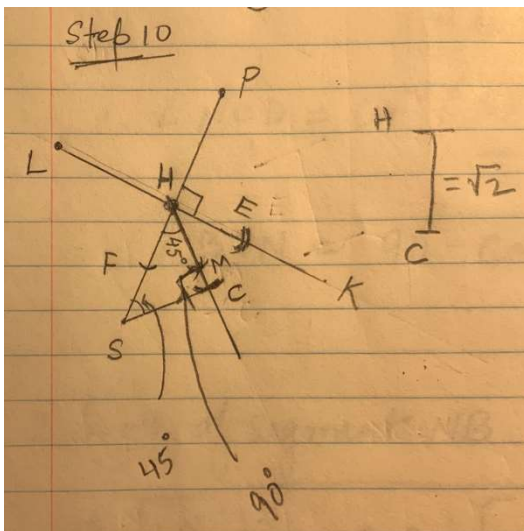
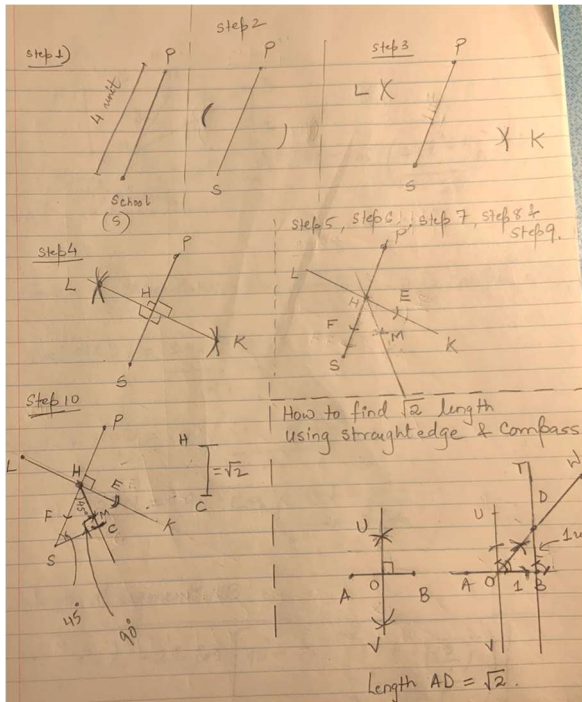
Step 8) We keep the compass at F and draw an arc as shown. These two are met at point M.

Step 9) We join the point M with H. Angle MHS is 45° .

How to find length $\sqrt{2}$ using compass and straightedge

Consider a line segment AB of length 2 and follow the step 1 through 4 as above to find the perpendicular bisector of this line segment AB. Next, we follow steps like 5,6,7 8 and 9 to bisect angle UOB. We draw a line parallel to line UV passing though B. Let line BT and OW intersect at point D. By construction length BD is 1 unit. Consider the right triangle OBD the length of segment OD is $\sqrt{2}$.

Step 10) Using compass measure segment OD. We put the compass at H and draw an arc of length OD. This arc intersects the line HM at C. Hence, we have length HC is $\sqrt{2}$. We draw a perpendicular line passing through the point S and line HM. Length SC is $\sqrt{2}$.



Solution 2

by Jayendra Jha, Arihant Public School, India and Sankalp Savaran, Shiv jyoti Senior Secondary School, India.

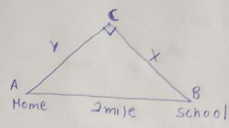
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This solution, interestingly, does not use Calculus, instead the Solvers converted the objective function into a form that can be easily maximized using facts from Trigonometry, amazing!

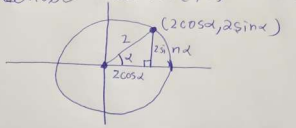
Part a)

Solution :->



Let A, B be location of Home, school which is 2 mile apart.
Paul have to travel distance BC and AC which are perpendicular to each other.
So $x^2 + y^2 = 4$. ——— (1)

Here equation (1) is equation of circle whose center (0,0) radius is 2.



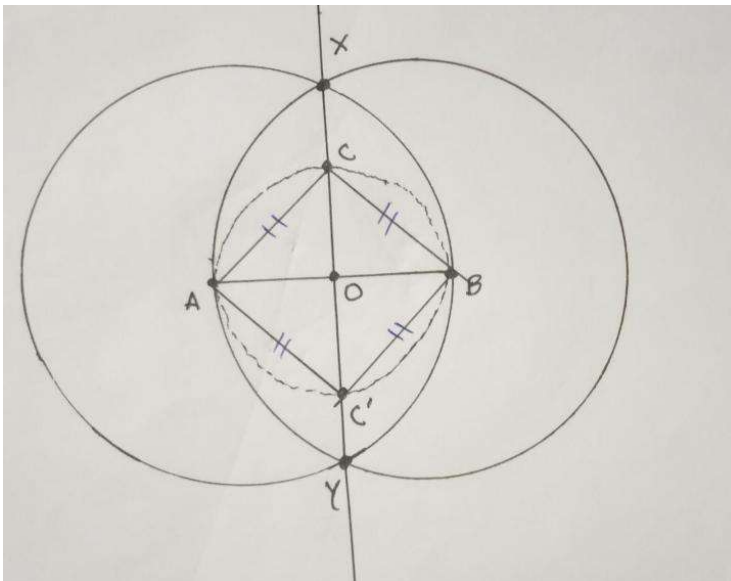
and question is asking for maximum value of

$$\begin{aligned} X+y &= 2\cos\alpha + 2\sin\alpha \\ &= 2(\cos\alpha + \sin\alpha) \\ &= 2\sqrt{2}\left(\frac{\cos\alpha}{\sqrt{2}} + \frac{\sin\alpha}{\sqrt{2}}\right) \\ &= 2\sqrt{2}\left(\sin\frac{\pi}{4}\cos\alpha + \cos\frac{\pi}{4}\sin\alpha\right) \\ &= 2\sqrt{2}\sin\left(\frac{\pi}{4}+\alpha\right) \end{aligned}$$

Max^m value of $\sin\theta$ when $\theta = \frac{\pi}{4} + \alpha$ is +1
So max^m value is $= 2\sqrt{2}$

Part b)

In above part (a), the value of α at which $\sin(\frac{\pi}{4} + \alpha)$ get maximum value is $\frac{\pi}{4}$.
 So value of $X = 2\cos\alpha = 2\cos\frac{\pi}{4} = \sqrt{2}$ and
 similarly $Y = 2\sin\alpha = 2\sin\frac{\pi}{4} = \sqrt{2}$ which
 makes Right isosceles triangle.
 Here is the steps and figure to done
 construction by straight edge and
 compass.



Step (i) Let $AB = 2$ miles. Take a compass and make A as center and B as radius and draw circle T_1 .

Step (ii) Make another circle T_2 by considering B as center and A as radius.

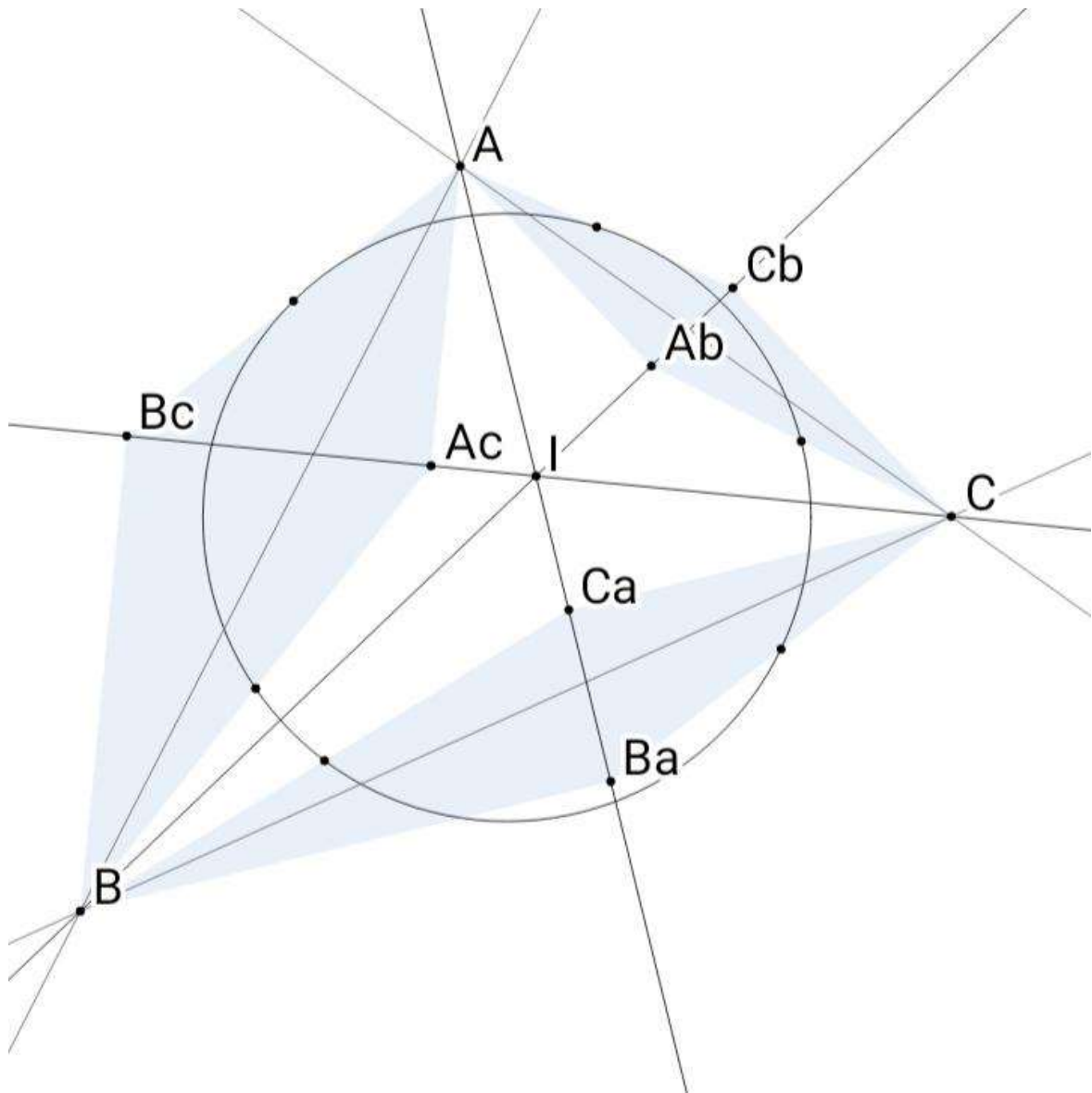
Step (iii) Circle T_1 intersect circle T_2 at two points X and Y .

Step (iv) Pass a line through XY which intersects AB at O by using straightedge.

Step (v) Make a circle T_3 whose center is O and radius is OA .

Step (vi) Circle T_3 cut ~~the circle~~ ^{Line} XY at two points C' and C'' .

Hence \rightarrow Path ACB and $AC'B$ becomes two possible ways.



In addition to having solved problem 2, Jayendra Jha and Sankalp Savaran sent me a conjecture they discovered which I wanted to share with all of you for the sake of Mathematical Research.

CONJECTURE: Let I be in the center of $\triangle ABC$ and Let Ab, Ac be orthogonal projection from A on Line BI and CI and similarly define $\{Ba, Bc, Ca, Cb\}$ cyclically then the Circumcentre of $(\triangle AAcBc); (\triangle AAbCb); (\triangle BAcBc); (\triangle BCaBa); (\triangle CBaCa); (\triangle CCbAb)$ Lies on Same Circle as shown in Given Figure.

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References:

[1] Van LamoenCircle <https://mathworld.wolfram.com/vanLamoenCircle.html>

[2] Discussion of above conjectured with geometers on Euclid messenger group:
<https://groups.io/g/euclid/topic/88234292#3766>

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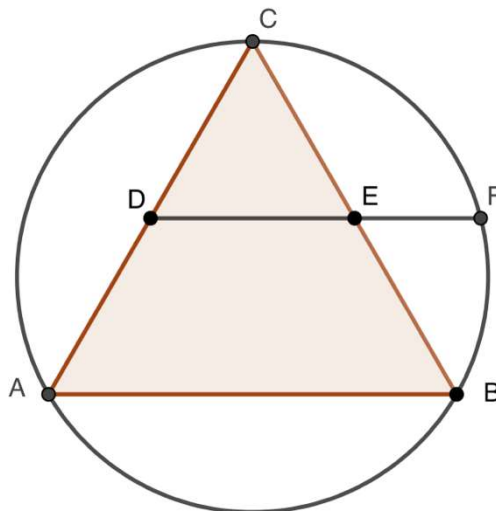
archanajha112018@gmail.com

Dear Problem Solvers,

I really hope you enjoyed solving Problem 2, below is the next problem, proposed by Aradhana Kumari, Borough of Manhattan Community College, USA.

Problem 3

Triangle ABC is an equilateral triangle inscribed in a circle. D and E are the mid points of sides AC and BC respectively. Find the ratio $\frac{\text{length DF}}{\text{length DE}}$



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Analysis of Problem Solving Process on HOTS Test for Integral Calculus

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Abstract: *Problem-solving is the essence of mathematics and is the main goal in learning mathematics. Many students did not have good problem-solving skills based on the field observations. The problems grew up because the students were not used to solving the problems and the problem-solving stages. They did not include issues with high complexity, such as questions with the High Order Thinking Skills (HOTS) category. The problem-solving steps have been developed, such as; Dewey (1910), Polya (1945), Mason, Burton & Stacey (1982), Schoenfeld (1985), and Wilson et al. (1993). The objective of this current study was to analyze the stages of problem-solving in solving mathematical problems with the High Order Thinking Skills (HOTS) category. The research sample was 57 students of Mathematics Education in one of Private University of Indonesia who took the Integral Calculus subject. This research was a qualitative descriptive study. The data analysis employed an inductive approach where the conclusions were drawn from minor case investigations to provide comprehensive results. The data analysis consisted of reduction data, presentation data, and concluding. There are three results from the current research. First, the students have not implemented problem-solving with problem-solving stages; second, The students fail to solve the problems due to a lack of mathematical literacy skills; last. The incomplete mathematization process causes imperfect problem-solving. Based on the results, the research recommendation is to add stages of problem-solving with two steps: formulating the situation mathematically and understanding mathematical solutions in real life or problems.*

Keywords: *Integral calculus, Mathematical Literacy, Mathematization, Problem Solving, HOTS*

INTRODUCTION

Problems Solving with a high level of difficulty can use the stages of problem-solving. Problem-solving is often used when solving non-routine problems (Temur, 2012), complex problems (Greiff, S & Fischer, 2013) where the problem solver does not know the previous scheme (Schoenfeld, 1992). Problem-solving is used to help problem

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solvers learn how to think mathematically (Rott et al., 2021) and systematically (Goulet-Lyle et al., 2020). The problem-solving model should act as a guide to help problem solvers in the thinking process. Problem-solving is the most important skill for students (Damayanti, & Sukestiyarno, 2014; Erlina & Purnomo, 2020; Sulistyarningsih et al., 2021) and is part of the mathematics curriculum in almost all countries, including the United States (Schoenfeld, 2007), Australia (Clarke et al., 2007), Netherlands (Doorman et al., 2007), China (Cai & Nie, 2007), French (Artigue & Houdement, 2007), Hungarian (Szendrei, 2007) and English (Burkhardt & Bell, 2007). In the curriculum, problem-solving is also one of the students' skills (Dagan et al., 2018; Indriyani et al., 2018).

Based on the research, the problem-solving ability of the students was still low (Purnomo & Mawarsari, 2014; Purnomo et al., 2014; Amir, 2015; Wardono et al., 2016); Hidayat & Irawan, 2017; Kusuma et al., 2017; Yeni et al., 2020). Students are still unable to solve Higher Order Thinking Skills (HOTS)-type questions due to a lack of problem-solving skills (Abdullah et al., 2015; Susanto & Retnawati, 2016; Kusuma et al., 2017; Karimah et al., 2018). The difficulties experienced by students when completing mathematics included lack of understanding of the questions and information provided, use of concepts, calculations, inaccuracies (Phonapichat et al., 2014), inability to use correct strategies, and lack of creativity in solving problems (Supandi et al., 2021). It is because the students are not accustomed to solving problems through the stages of problem-solving. As a result, the solution to the problem is not ideal. Based on these studies, it is necessary to improve problem-solving abilities.

In learning mathematics, students learn not only math material but also math skills (Piñeiro et al., 2021; Purnomo et al., 2021), think creatively (Nuha et al., 2018), and learn to face problems (Agoestanto & Sukestiyarno, 2017). Many theories suggest the stages of problem-solving. The first researcher who introduced problem-solving were Dewey (1910), then Polya (1945), Mason, Burton & Stacey (1982), Schoenfeld (1985), and Wilson et al. (1993). Dewey problem-solving consists of five stages: (i) encountering a problem (suggestions), (ii) specifying the nature of the problem (intellectualization), (iii) approaching possible solutions (the guiding idea and hypothesis), (iv) developing logical consequences of the approach (reasoning (in the narrower sense)), and (v) accepting or rejecting the idea by experiments (testing the hypothesis by action) (Dewey,

1910). Polya's most commonly used solution consists of four stages including (i) understanding the problem, (ii) devising a plan, (iii) carrying out the plan, and (iv) looking back (Polya, 1978). Meanwhile, Schoenfeld divides the stages of problem-solving into 5, including (i) analysis, (ii) design, (iii) exploration, (iv) implementation, (v) verification (Schoenfeld, 1992). Based on the analysis, each problem-solving model has its own characteristics.

A lack of ability to solve mathematical issues will impact the development of essential mathematical skills required by the students (Oktaviyanthi & Agus, 2019). Problem-solving can run well when a person has experience solving problems (Greiff, S & Fischer, 2013) and does not experience cognitive barriers (Antonijević, 2016). One of the abilities that support the problem-solving process includes good mathematical literacy skills. Mathematical literacy implies a foundation of knowledge and competence and the confidence to apply knowledge to the practical world. People with mathematical literacy skills can estimate, interpret data, solve everyday problems, reason in numerical, graphic, and geometric situations, and communicate using mathematics (Ojose, 2011). In solving problems, mathematical literacy is a very important part.

In addition to mathematical literacy, other things that need to be considered in problem-solving include mathematical modeling (Klymchuk, 2015). Mathematical literacy deals with real problems hoping that problem-solvers must 'solve' real-world issues that require the skills and competencies they have acquired. A fundamental role in the process is referred to as mathematization. The mathematization process develops concepts and ideas starting from the real world and ultimately reflecting the results obtained in mathematics back to the real world (Lange, 2006).

The problem-solving process in the field looks different from the existing stages. The solved problems contained errors in complicated cycles, and the issues were solved not following the sequence of the previous steps (Rott et al., 2021). This actual process was not considered during the last problem-solving model. Based on this, it is necessary to improve the stages of problem-solving so that it is easier for students to solve these problems.

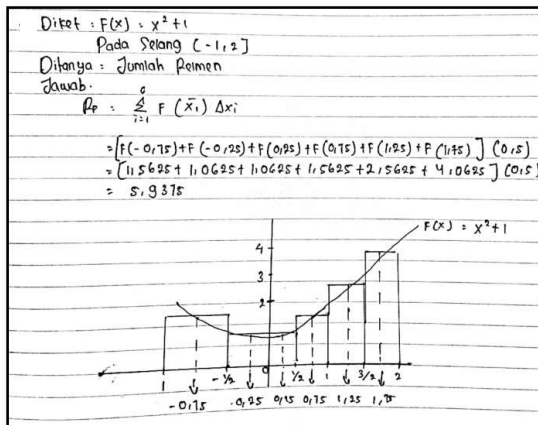
METHODOLOGY

This research is a qualitative descriptive study that describes the process and results of the problem-solving stages. Integral Calculus was taught to 57 students at a private university in Indonesia who were all undergraduates in Mathematics Education. The examination of the problem-solving findings of 57 students yielded six answer categories, with two answers to each question divided. Data was gathered through evaluation tests, observations, and in-depth interviews as part of the triangulation method. A three-tiered evaluation test was devised: simple, medium, and complicated.

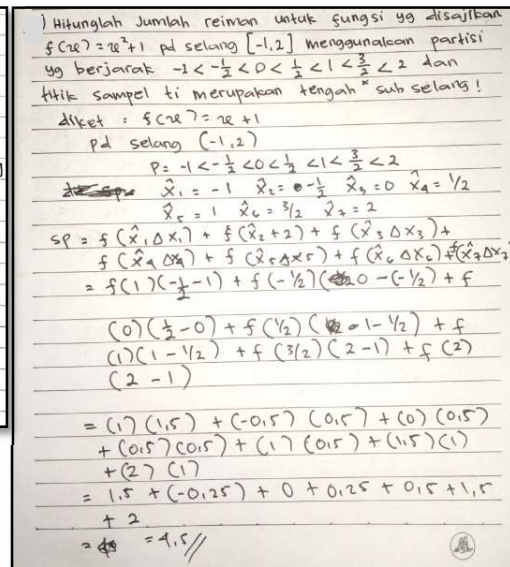
Triangulation was used as a data-gathering method, which involved conducting experiments, observations, and in-depth interviews to gather information (Sukestiyarno, 2020). The data analysis used an inductive approach where conclusions are drawn from small case investigations in detail to provide a big picture (Sukestiyarno, 2020). The data analysis consisted of data reduction, data presentation, and concluding. The data reduction was made by coding the student answers. The coding is used to facilitate the tracking of important data about the exposure of existing data. After the data is reduced, the next step is to present the data by verifying the data with in-depth interviews. The last step is to make conclusions from the data in the field.

RESEARCH FINDINGS

In this current study, the students were asked to work on three questions, and the results were analyzed as follows. The first problem is the calculation of the Riemann sum for the function represented by $f(x) = x^2 + 1$ in the interval $[-1,2]$ using equidistant partitions $-1 < -\frac{1}{2} < 0 < \frac{1}{2} < 1 < \frac{3}{2} < 2$ and the sample point t_i is the middle of the sub-interval! The results of student work can be seen in the image below.



Gambar 1A



Picture 1B

Picture 1. Students' Performance on Number 1

Based on Figure 1a above, it can be analyzed as follows. Student A1 has formulated the situation mathematically by illustrating the problem in a picture. Students have determined the known information by writing what is known, namely the function $f(x) = x^2 + 1$ and the interval $[-1, 2]$. In writing, the elements that are known to be incomplete, such as partition: $-1 < -\frac{1}{2} < 0 < \frac{1}{2} < 1 < \frac{3}{2} < 2$ and the sample point t_i It is the middle of the sub-interval. Students can determine the unknown information, namely the middle of the sub-interval, but it is not written in the answer. Students find the relationship between the data and the strange, but the answer is not reported. The solution plan is not written down, but the answer is. The student does not respond to the query. Since students did not complete the sixth step in problem-solving In-depth interviews were done

The interview results explicate that student A1 draws a graph to see more fully what is known in the problem. Students answered questions using pictures. Then, student A1 did not write the solution plan because he was not instructed to write the solution design in the answer. Student A1 only has one solution design. Student A1 implements the solution plan but does not check every step. It is because the work takes a lot of time.

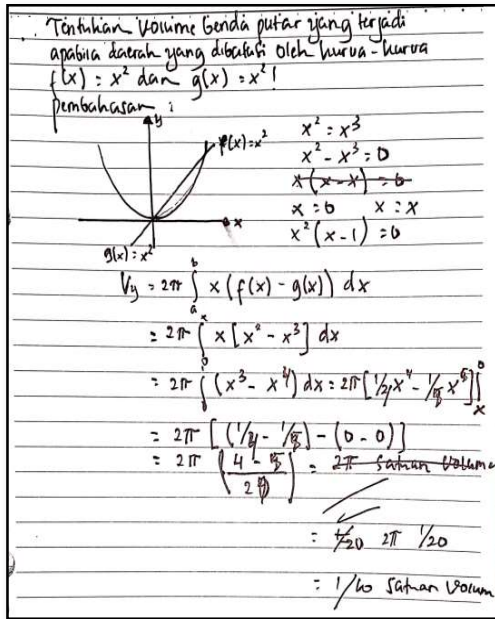
Student A1 contains answers to solving questions. At the end of the solution, student A1 did not respond according to the question.

Students are good at answering questions, but there are stages of problem-solving that have not been implemented, namely in the sixth stage. Student A1 is right by first describing the situation in the form of pictures so that it helps students in answering questions. Based on this, it is necessary to emphasize one more step in the problem-solving process, namely "understanding mathematical solutions in real life or problems." The results of the second student work can be seen in the image below.

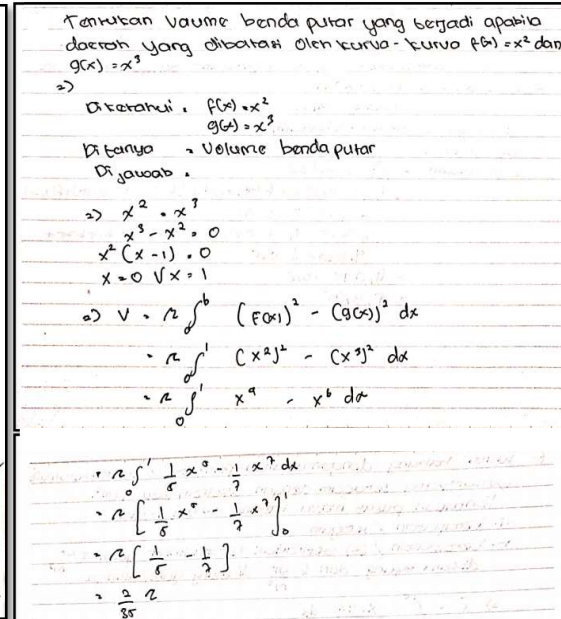
In Figure 1b, it can be analyzed that B1 students do not formulate the situation mathematically. The student has determined the known information by writing what is known, namely the function $f(x) = x^2 + 1$ and interval $[-1,2]$ and partition: $-1 < -\frac{1}{2} < 0 < \frac{1}{2} < 1 < \frac{3}{2} < 2$. Students have also determined the sample points, namely $x_1 = -1, x_2 = -\frac{1}{2}, x_3 = 0, x_4 = \frac{1}{2}, x_5 = 1, x_6 = \frac{3}{2}, x_7 = 2$. Students cannot determine unknown information. Namely, the sample point t_i is the middle of the sub-interval. It is what causes students to misperceive the middle point, but students answer the endpoint of the interval. It resulted in the student's answer being wrong. Students did not find a relationship between the unknown data. The solution plan was not written down, but the students immediately wrote down the answers. Student B1 implements the solution plan, but the answer is not quite right. Student B1 does not re-check the answers that have been written. The last step, "understanding mathematical solutions in real life or problems," is not implemented.

The B1 student's error by not formulating the situation in mathematical form resulted in the student being wrong in determining the midpoint. There is still confusion between the middle and the end of the hose. Student B1 knows that the sample point is the end of the interval based on the interviews. It is due to the inability of students to analyze what is known. It is due to the low ability of students in mathematical literacy. It can help students in mathematical literacy by "formulating the situation mathematically." The other solving stages were also not implemented properly, so the students' answers were wrong.

The second problem is to determine the volume of a rotating object that occurs when the area bounded by the curves $f(x) = x^2$ and $g(x) = x^3$. The results of student work can be seen in the image below.



Picture 2a



Picture 2b

Picture 2. The Students' Performance on Number 2

Based on Figure 2a, it can be concluded as follows. By drawing a representation of the problem, student A2 has expressed the situation quantitatively. Student A2 has created a graph to represent the already known data. Anonymous data, namely the point of intersection between two charts, can be selected by students. $f(x) = x^2$ and $g(x) = x^3$. Students find the relationship between data and the unknown. Namely, the intersection point is used as the upper and lower limits. The student did not write the solution plan. Students do not carry out the last stage in problem-solving, namely "understanding mathematical solutions in problems or real life."

Based on the results of the interviews, it was concluded that the students were less precise in presenting the picture in question. Students draw a graph of the function $f(x) = x^2$ as a straight line. The graph $f(x) = x^2$ should be a curved curve. It is due to the lack of good literacy at the "mathematically formulating the situation" stage. The

solution plan is not written down but directly written the answer. Students carry out the solution plan and check it at every step. The student does not return the response according to the question. The results of the second student work are as follows.

Figure 2b Student B2 does not formulate the situation mathematically. It does not illustrate the problem in the form of a picture. The student has determined the known information by determining $f(x) = x^2$ and $g(x) = x^3$. Students can determine the unknown information, namely the intersection point between the graphs $f(x) = x^2$ and $g(x) = x^3$. Students find the relationship between known and unknown data; namely, the intersection point is the upper and lower limits. The solution plan is not written down but directly writes the answer and the solution plan is not right. The student implements the solution plan but does not check every step. The student does not return the response according to the question.

Based on the interviews, it was concluded that students were wrong in determining the completion of the volume of a rotating object. Students should distinguish between finding volume using the disc, ring, or cylinder method. It is due to the lack of good literacy at the stage of "mathematically formulating the situation." The solution plan is not written down, but the students already know the steps. The solution strategy is incorrect because the students follow the solution strategy but do not double-check each step. The pupil does not respond to the query. The results for the third question are as follows.

Diket: tinggi tangki = 10 kaki
Jari-jari = 4 kaki

Ditanya: a. Tentukan gaya yang diperlukan
a. Merivati tepi atas tangki
b. Mencapai 10 kaki di atas puncak tangki

Jawab:

Sebuah tangki kerucut memiliki jari-jari 4 y, maka V kompartan
Perhatikan sebuah kerucut memiliki jari-jari $\frac{4}{10}y$, maka V kompartan
 $\Delta V = \pi \left(\frac{4}{10}y\right)^2 \Delta y$ dan beratnya (gaya berat) $F = \delta \pi \left(\frac{4}{10}y\right)^2 \Delta y$
dengan $\delta = 62,4$ (kepadatan air)

\Rightarrow Gaya yang diperlukan adalah $10 - y$. Jadi kerja Δw adalah
 $\Delta W = \delta \pi \left(\frac{4}{10}y\right)^2 \Delta y (10 - y)$

Integrasinya
 $w = \delta \pi \frac{4}{25} \int_0^{10} (10y^2 - y^3) dy = \delta \pi \frac{4}{25} \left[\frac{10y^3}{3} - \frac{1}{4}y^4 \right]_0^{10} \approx 26,130$

Jadi kerja untuk memompa air sampai ke tepi tangki adalah
26,130 kaki pond.

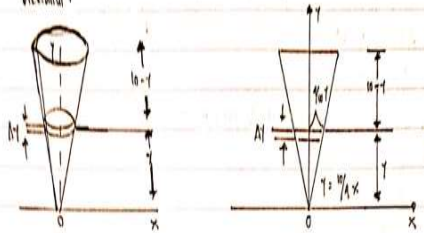
\Rightarrow Sama dalam diatas, Sebarang air dalam kerucut harus diangkat
 $20 - y$, maka
 $w = \delta \pi \int_0^{10} \left(\frac{4}{10}y\right)^2 (20 - y) dy = \delta \pi \frac{4}{25} \int_0^{10} (20y^2 - y^3) dy$
 $dy = \delta \pi \frac{4}{25} \left[\frac{20y^3}{3} - \frac{1}{4}y^4 \right]_0^{10} \approx 130,69$

Jadi kerja untuk memompa air sehingga ke tepi tangki adalah
130,690 kaki pond.

Picture 3a

Tangki kerucut tersebut terisi penuh dengan air. Alesa tangki tangkanya 10 kaki
dan jari-jari tangkanya 4 kaki. Tentukan gaya yang diperlukan untuk memompa air!
Persewaan A. Merivati tepi atas tangki

Ditanyakan:



Ditanya: Merivati tepi atas tangki!

Jawab: $F = \int_0^{10} \delta \pi \left(\frac{4y}{10}\right)^2 (10 - y) dy = \delta \pi \frac{4}{25} \int_0^{10} (10y^2 - y^3) dy$
 $= \frac{4\pi(62,4)}{25} \left[\frac{10y^3}{3} - \frac{y^4}{4} \right]_0^{10} = 26,130$ kaki pond

Ditanyakan: B. Mencapai 10 kaki Puncak Tangki

Ditanyakan: Gerak sama dengan A

Ditanya: 10 kaki Puncak Tangki!

Jawab: Air di kerucut diangkat $20 - y$
 $F = \delta \pi \int_0^{10} \left(\frac{4y}{10}\right)^2 (20 - y) dy = \delta \pi \frac{4}{25} \int_0^{10} (20y^2 - y^3) dy$
 $= \frac{4\pi(62,4)}{25} \left[\frac{20y^3}{3} - \frac{y^4}{4} \right]_0^{10} \approx 130,690$ feet-pond

Picture 3b

Picture 3. The Students' Performance on Number 3

Based on Figure 3a, it can be seen that A3 students do not formulate in a mathematical situation. Student A3 has determined the known information by writing down what is known, namely the height and radius of the tank. Students also write down the questions asked in the questions. A3 students can determine the unknown information, namely the radius of a circle with a thickness y at the height of Δy , which is $\frac{4y}{10}$. Students find the relationship between the data and the unknown, namely the volume of the disc, namely $\Delta v = \pi \left(\frac{4y}{10}\right)^2 \Delta y$ and its weight (gravity) $\delta \pi \left(\frac{4y}{10}\right)^2 \Delta y$ with $\delta = 62,4$ (density of water). The solution plan is not written on the answer sheet but directly reports the answer. Execute the solution plan and check every step. There are two steps: looking for work for

a water pump a). past the top edge of the tank, and b). reach 10 feet above the top of the foot. The required force is $10-y$. so work $\Delta w = \delta\pi\left(\frac{4y}{10}\right)^2 \Delta y 10 - y$. So, the work to pump water up to the edge of the tank is 26,138 pound-feet. Same with the previous point problem, now the water in the cone must be lifted $20 - y$ then. So, the work to pump the water up to the edge of the tank is 130,69 pound-feet. A3 students have carried out the last stage in problem-solving.

Based on the interviews, A3 students have answered correctly, but there are stages of problem-solving that have not been carried out. A3 students do not formulate in a mathematical situation lacking in detail in presenting known things, for example, by giving pictures. The other solving steps have been carried out well by A3 students.

Figure 3b for B3 students is almost the same as A3 students. A graphic depicts the discrepancy between B3 students' work and their peers in the section that they are very familiar with each other. It means that B3 students have formulated a mathematical situation through a presentation with images. This step will make it clearer to write down what is known in the problem, look for what is not known in the problem, and answer questions at each step. But the answers of B3 students are incomplete because they do not return the solutions according to the questions. B3 students do not carry out this last step. Based on interviews with B3 students, the answers were not returned to the questions because the students thought that the problem solving was finished when they got the answers. It is because students do not know the problem-solving process well. One solution to complete students' responses by adding one problem-solving stage is "adding the mathematization process, namely understanding mathematical solutions in problems or real life."

The design of the new problem-solving stages could be applied to know the advantages and disadvantages if implemented in the field. Therefore, based on aforementioned findings, a new problem-solving stage design was made into six stages of problem-solving as shown in Table 1.

Stages	Stages of problem-solving	Description
Stage 1	Formulate the situation mathematically	Students change problems in mathematical situations
Stage 2	Understand the problem	Students identify things that are known in the problem
Stage 3	Plan problem solving	Students plan problem solving to be carried out
Stage 4	Carry out problem-solving	Students carry out problem-solving by the problem-solving plan
Stage 5	Review the results of problem-solving	Students check the correctness of the answers that have been made
Stage 6	Understand mathematical solutions in real life or problems	Students relate the answers they found into real life.

Table 1. New problem-solving stage design

RESEARCH DISCUSSION

This current study aims to analyze the problem-solving process in the HOTS category questions in universities. This research focuses on how the stages of problem-solving can help students solve problems. The analysis results will see the weaknesses and strengths of the existing problem-solving steps. Based on this analysis, a new solution stage will optimize the process and problem-solving results. The results of the student problem-solving process can be concluded that students have difficulties at the beginning of the problem-solving process. For example, in question no. 1, student B1 could not determine the midpoint, in question no. 2, student A2 was less precise in drawing the graph $f(x) = x^2$. The lack of mathematical literacy skills causes this student's difficulty to improve literacy skills in problem-solving. It is necessary to have stages of formulating the situation mathematically. When students can do this stage, they will be able to describe what is known in the problem to look for things that are not known in the issue. Students have difficulty dealing with non-routine problem situations (Temur, 2012) and lack problem-solving experience (Greiff, S & Fischer, 2013), so it is necessary to increase mathematical literacy. One way to improve mathematical literacy is to model

mathematically (Klymchuk, 2015). Through this activity, students will know more deeply about analyzing what is known and looking for things that are not known in the problem.

The next student error is at the end of the problem-solving stage. The student does not return the answer according to the question. The mathematization process is not done well, namely understanding mathematical solutions in real life. This process is a refinement of the mathematization and modeling activities in real situations. Modeling in real problems can help in the problem-solving process (Stender & Kaiser, 2015; Purnomo et al., 2020) and increase the creativity of the problem-solving process (Schindler & Lilienthal, 2020). Mathematical modeling is considered central to an important element in problem-solving (Carotenuto et al., 2021). When students are not used to solving the problems, the problem-solving steps are not being by the stages (Nurkaeti, 2018), and the results are not optimal (Greiff, S & Fischer, 2013). The problem-solving process contains errors and does not follow a predetermined order, as in the normative model (Rott et al., 2021). So, it is necessary to improve the problem-solving stage to make it easier for students to understand.

The results showed that 1). Students have not implemented problem-solving with problem-solving stages; 2). Students cannot solve problems due to a lack of mathematical literacy skills; 3). Students have not carried out a complete mathematization process. Based on this analysis, it is necessary to add additional stages in the problem-solving process, namely 1). steps added at the beginning before problem-solving by formulating the situation mathematically. The goal is to improve mathematical literacy skills. Students will use this ability to carry out the next stage's problem-solving process. 2). adding the last step by inserting the mathematization process, namely understanding mathematical solutions in real life or problems. The goal is to become reflective problem solvers who can become good (Evans, 2015) by returning answers according to the existing issues. By adding these two stages, the problem-solving stage will become complete. The result is that the problem-solving process will run well, and students' problem-solving abilities will increase.

CONCLUSION

The results showed that 1). The students have not implemented problem-solving with problem-solving stages; 2). The students cannot solve problems due to a lack of mathematical literacy skills; 3). The students have not carried out a complete mathematization process. Based on these conclusions, the recommendation from this research is to add stages of problem-solving. Steps are added at the beginning before problem-solving by formulating the situation mathematically and at the ending problem-solving process by inserting the mathematization process, namely, understanding mathematical solutions in real life.

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Computational, Logical, Argumentative, and Representational Thinking in the United Arab Emirates Schools: Fifth Grade Students' Skills in Mathematical Problem Solving

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Abstract: *The purpose of this study was to analyze the problem-solving techniques that students in a fifth-grade classroom applied while solving mathematical word problems. Fifth-grade students in a private school with Ministry of Education curricula in Al Ain, Abu Dhabi, were given a set of 15-word problems to solve with detailed justifications. The questions were based on TIMSS past exams with some modifications to fit the local context. Analysis from the study generated five themes that students applied in problem-solving: Logical Thinking Skills, Computational Skills, Problem-solving Strategies, Use of Justifications, and Use of Representations. Pedagogical and curricular implications have been discussed.*

Keywords: *Problem-solving, mathematics learning, gender differences in math, UAE.*

INTRODUCTION

Problem-solving in mathematics is an essential skill for assessing students' understanding, interpreting, and applying mathematical concepts (Carden & Cline, 2015). Researchers in Mathematics Education have emphasized students' problem-solving at the school level (Amado, Carreira & Jones, 2018). However, more needs to be done in problem-solving studies within the United Arab Emirates (UAE), school contexts. There are several methods and approaches to problem-solving (Gourdeau, 2019). Nonetheless, students still struggle when it comes to applying these approaches to their own problem-solving. Still, students are weak in solving mathematical problems, realistic problems, numerical operations, and justifying their answers (Suharta, 2016). Mathematical problem solving is an essential component of, and not separate from, mathematical learning (Nurkaeti, 2018). Problem-solving has been considered a high-level cognitive skill that helps teachers raise expectations for their students' thinking (McCormick, 2016). Although young children have different abilities in using problem-solving strategies, teachers and researchers still need to learn more about the affective and cognitive factors that may contribute to understanding

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variations in problem-solving approaches and their impacts on students' learning of mathematics (Ramirez, Chang, Maloney, Levine & Beilock, 2016).

Although teaching mathematics through problem-solving is considered a challenge for many mathematics teachers, it can foster the students' learning with a more profound understanding and proficiency in using the skills in various situations (Stein, Grover, & Henningen, 1996). To use problem-solving as a teaching strategy, many procedures should be considered, such as the teacher's role in selecting the problem-solving strategy, collaborating, and adopting a problem-solving-based curriculum (Cai, 2003a). Hiebert et al. (1996) argued that the curriculum and instructions should be reformed by allowing the students to problematize the subject. They build their proposal based on John Dewey's idea of reflective inquiry, illustrating that through problem-solving, the students will understand and develop their thinking skills (Hiebert et al., 1996). The aim of reforming curriculums based on problem-solving is not to frustrate students and make mathematics difficult, but to help students learn mathematics through problem-solving activities (Battista, 1999; Brown, 2001). We need to encourage the students to think, wonder, and inquire about why things are the way they are and find a solution to this situation. We need students to elicit their curiosity and sense-making skills (Hiebert et al., 1996). Although there has been much research around problem-solving in the last 30-40 years, many questions need to be answered, such as what kinds of instructional activities should be used in teaching through problems (Lester & Cai, 2016). We are concerned about the difficulties that students face in solving problems, and we investigate the reasons behind these obstacles.

Students' average performance in mathematics problem-solving in PISA 2012 was 411 for the UAE students, with a global rank of 40th among 44 participating countries (United Arab Emirates Ministry of Education, 2013). There has been a concern among parents, teachers, school administrators, and the UAE government that "students fall way short of their global counterparts when it comes to maths problem-solving" (Dhal, 2014, April 09th). The government and school leaders have enhanced their efforts to improve the situations in the mathematics classrooms and promote students' computational and problem-solving skills (Organization for Economic Co-operation and Development [OECD], 2020). The United Arab Emirates Government has been considering students' mathematics problem-solving as one of the core concerns. Therefore, it has been an element of close attention in the school inspectors' guidelines and school inspection framework (United Arab Emirates Ministry of Education, 2021). In Abu Dhabi, UAE, several school systems have emphasized mathematics problem-solving as one of the key skills in their curriculum, for example British Schools, International Baccalaureate Schools, American Curriculum Schools, and Indian Schools (Abu Dhabi Department of Education and Knowledge, 2019).

The project was conducted to explore how students in the UAE's local context solve mathematical word problems. Problem-solving in mathematics classrooms is an issue worldwide, and not enough research has been conducted in the UAE context. The study focused on analyzing students'

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problem-solving responses and then extracting possible themes from the students' methodologies in their problem solving, whether intuitively or by design. The research questions for this study were: How do fifth grade students solve real life mathematical problems? Moreover, what are the strategies that the fifth students in the UAE schools use to solve a mathematics problem?

THEORETICAL FRAMEWORK

Mathematics is always considered a difficult subject, although it is found in all other subjects and student's daily lives (Surya, Putri, & Mukhtar, 2017). Problem-solving could be one of the teaching strategies and hence improve the quality of teaching mathematics (Pehkonen, Naveri, & Laine, 2013). The importance of problem-solving is that students can deepen their understanding of mathematical concepts by using them in real-life problems (Beigie, 2008). Problem-solving is a strategy that all students are expected to use in their daily lives and future professions. Therefore, it is crucial to use the problem-solving technique in education to help students manage their lives and solve problems. There are many models for problem-solving, such as the Polya model, the Schoenfeld model, and the Verschaffel model.

Problem-Solving Model: George Polya

Polya's book, *How to Solve It*, has inspired several other research studies and conversations on problem-solving (Polya, 2004). It has even affected problem-solving in non-mathematical contexts. His four steps of problem-solving by design speak to both students and educators and often appear in one form or another in research papers (Kusumadewi & Retnawati, 2020). These four steps are:

- Understanding the Problem: By looking at all aspects of the problem regarding known and unknown variables, there are conditions. Illustrating the problem is a part of understanding it as well.
- Devising a Plan: By trying to form connections between the known and unknown, prior knowledge also plays a role in finding connections between possible past problems and the current problem at hand.
- Carrying out the Plan: Once a plan is devised, carrying it out follows it by checking every step of the plan along the way and double-checking that everything is correct.
- Looking Back: Checking whether the plan worked from different perspectives and whether there was a potentially different plan as well.

Polya et al.'s analysis of problem-solving approaches based on mental agility offers a door into problem-solving techniques as well (Liljedahl et al., 2016, p. 4–5). These techniques are:

- Reduction: Problem solvers intuitively reduce a problem to its basics in a logical manner by visualizing forms of representation for the problem.

- Reversibility: Another successful approach to problem-solving seeks to backtrack trains of thought. Working in reverse as a problem-solving approach is another problem-solving strategy.
- Minding of aspects: Thinking of a problem by keeping a couple of details in mind at the same time is another successful approach used by intuitively good problem solvers.
- Change of aspects: This approach helps good problem solvers avoid getting a mental block by being flexible enough to change their minds and look at the problem from a different perspective when needed.
- Transferring: By recognizing a pattern, successful problem solvers transfer it consciously or subconsciously when solving other problems. Good problem solvers also generalize or transfer their procedures to a different context.

(Liljedahl et al., 2016, p. 4 – 5)

These approaches by successful problem solvers are sometimes inaccessible to intuitive problem solvers who solve the problems but cannot explain their exact train of thought when solving a problem correctly.

Problem-Solving Model: Schoenfeld (2013)

Alan Schoenfeld was impressed by Polya's problem-solving model. For Schoenfeld, "the problem-solving process is ultimately a dialogue between the problem solver's prior knowledge, his or her attempts, and his or her thoughts along the way." He also states that problem-solving is the ability to think mathematically (Garcia, Boom, Kroesbergen, Nunez & Rodriguez, 2019; Liljedahl et al., 2016, p. 14). Schoenfeld's problem-solving model initially included five episodes (Karatas, Soyak & Alp, 2018)—Reading, Analyzing, Exploring, Planning/Implementing, and Verifying. Schoenfeld also adds another dimension to the heuristics of problem-solving by including metacognition and regulation of thoughts. While the concept of metacognition was extensively used in education in the 1980s, it has since been reconceptualized to differentiate it from the regulation of thoughts. By thinking about what they are doing, students are engaged in metacognition, and by controlling these thoughts by posing questions and alternative solutions, they are regulating their cognition and thinking. Both of these serve to help students and they become effective problem solvers (Schoenfeld, 2013). Rott (2012) discussed the basis of the Schoenfeld empirical framework. He illustrated that to theoretical model of the problem-solving process is based on Pólya's list of questions and guidelines. Rott (2012) made a correlation between Polya's steps and Schoenfeld's episodes which is illustrated in Figure 1 below:

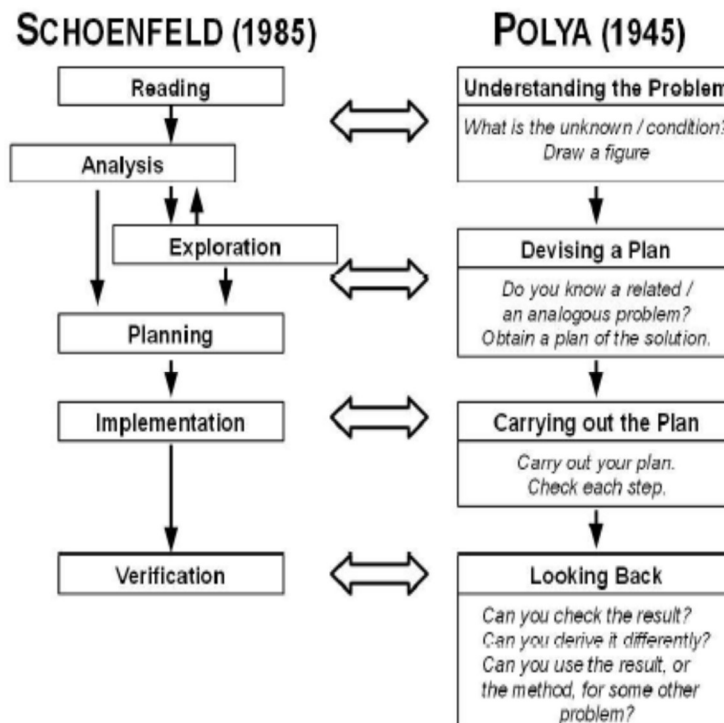


Figure 1: Correlation between Polya's steps and Schoenfeld's episodes

Problem-Solving Model: Verschaffel et al. (1999)

Verschaffel et al. (1999) proposed a recursive problem-solving model that includes five elements: building a representation, deciding the solution method, carrying out the solution, interpreting the solution, and evaluating the outcome. As Verschaffel et al. (1999) suggested, the first step is to involve learners in drawing a picture to represent the problem context. Then, the learner makes a scheme of the variables in a table or a list to distinguish the necessary information in the problem and uses his or her prior knowledge to create a viable representation. After the representation phase, the learner decides the solution approach or method based on his or her prior scheme or new scheme from the representation with a flow chart. He or she may use an initial guess and check approach to reach the most viable route to the solution with the help of a pattern in the given problem context. The learner may simplify the solution process with the necessary formula, tools, or models. The third step is about executing the problem-solving action as planned and getting a viable solution. The learner interprets the solution or outcome in the problem context and evaluates whether the solution is feasible. This way, Verschaffel et al.'s (1999) model is somewhat similar to Polya's problem solving (Garcia, Boom, Kroesbergen, Nunez & Rodriguez, 2019). These three models, as discussed above, served as a basis for our study to analyze and interpret students' problem-solving approaches to the selected problems.

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We are looking at students' problem-solving skills within five different domains: logical thinking, computational skills, justification of answers, problem-solving strategies, and use of representations. Logical thinking ability has been an important cognitive ability to be developed in students while teaching mathematics (Sezen & Bulbul, 2011). Likewise, computational skills provide students ability to carry on mathematical problems in an efficient and fast way by applying appropriate strategy (Borkulo et al., 2020). Mathematics problem-solving also involves reasoning in mathematical context integrating mathematics to the real-life situations. This kind of skills can promote students' ability to justify their reason for particular problem-solving approach opening the possibility to view alternative ways to reach the solution (Glass & Maher, 2004). Such skills can be further strengthened and promoted with the applications of representations of problems and solution models that students construct (Gagatsis & Elia, 2004).

METHODOLOGY

This study aimed to analyze the mathematical problem-solving skills of grade 5 students in Al Ain, Abu Dhabi. A segment of an international test, TIMSS 2015, with a mathematics section was chosen with some modifications to adapt it to the culture of the United Arab Emirates. The test was then translated into Arabic, as the students' mother tongue is Arabic; the sample questions from grade-4 TIMSS were deemed acceptable for the grade 5 students as the test was administered at the start of their new academic year.

Participants

We gained access to a private school in Al Ain through a gatekeeper, one of the researchers. The school is a private MOE system school, with grades from k-12. Students study in co-ed classes up to grade 3, and then boys and girls are taught separately. Participants in this study come from two sections of grade 5: the boys' section (15 participants) and the girls' section (12 participants). After initial consent from the school, the test was administered to the students who attended face-to-face learning. Because of COVID -19 restrictions, the test was presented to the students on the smart screen in their classrooms. In contrast, the students answered on their sheets of paper to minimize any interactions and to maintain social distancing in the classroom. All students in both classrooms were given the same conditions, and the first researcher then gathered the answer sheets from the school.

Materials

The test was obtained from the TIMSS official website ([source: https://www.edinformatics.com/timss/pop1/mpop1.htm?submit3243=Grade+3%2C4+Math+Test](https://www.edinformatics.com/timss/pop1/mpop1.htm?submit3243=Grade+3%2C4+Math+Test)) and was modified to include 15 questions rather than the whole test. This was done since the test was to be conducted in one 45-minute period. The 15 questions chosen were further modified to localize them by changing names when applicable. Although the TIMSS test is for grade 4 students, it was decided to use the same test for grade 5 students. The justification for this was that students had just come from their summer vacation, and before that, they had studied their first

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semester of online learning (spring 2020). The test was administered on September 22nd, 2020, during the start of their fall 2020 semester. The researchers wanted to investigate the students' problem-solving skills regarding word problems and justifying answers. The test was translated to Arabic since the students' first language is Arabic, and having the test in their mother tongue would give them a better opportunity to provide justification for their answers. The medium of teaching mathematics at that school was also in Arabic, hence the justification for translating the test.

Following that, the specification table, Table 1, used for the test, was used to identify the main areas of possible analysis, which are: (1) Operations that include the basic four operations, addition, subtraction, multiplication, and division; (2) Fractions; (3) Geometry; (4) Measurement; and finally (5) Probability and Statistics. Students' answers in both the boys' and girls' sections were analyzed across these areas/strands of mathematics. Below is the specification table, showing the strands of mathematics as observed in the test, with the questions' placement and their weights (Table 1), (see Appendix (1)). The weight was converted a percentage which was calculated based on the number of questions from the overall questions (for example: the questions that were related to the operations had eight questions, therefore $8/15 \times 100\% = 53.3\%$).

Specification Table		
Strand	Question Number	Question Weight
Operations	Addition: 5	53.3%
	Subtraction: 1, 2, 12	
	Multiplication: 13, 15	
	Divison: 4, 7	
Fractions	3, 10	13.3%
Geometry	6 (perimeter)	6.6%
Measurement	8, 9	13.3%
Statistics & Probability	11, 14	13.3%

Table 1. Specification the problems used in the study

Quality Criteria (Validity and Reliability)

When we started to plan for this project, we considered the validity and quality of this study (Creswell, 2016). We tried to answer the question: Is the account valid? The validity comes from the fact that we chose word problems from a previous TIMSS exam for grade four students. The questions were selected to cover the basic strands: operations, fractions, geometry, measurement, statistics, and probability (National Governors Association for Best Practices & Council of Chief State School Officers, 2010; NCTM, 2000). The selected problems in the study were examined to

be modified to fit into the local context of the UAE schools and translated into Arabic for students (see Appendices I and II). The questions were moderated, and language structure was examined to see if fifth graders might understand the questions. The students were not informed before the test about the study. Parents' consent was sought to conduct the research, telling them that their children's identities would not be used in the study. They were also informed about the purpose and benefit of the study in terms of its contribution to knowledge in the field of teaching and learning mathematics. Consent from the school and mathematics teachers was also sought to implement the test in the classroom. The teachers displayed the questions on the smartboard for the students. The students answered the questions on a blank sheet of paper that they brought from home. A strict safety protocol against COVID-19 was followed as per the school rules and government policies, not to distribute any sheets and pencils in the classroom. After completing the test, the students left their answer sheets on their tables, and the teachers later collected them and handed them over to the researchers in a sealed envelope.

Data Analysis

The participants were studying at a private school in Al Ain city following the Ministry of Education curriculum. We completed the requested procedures for the consent form. The test consisted of 15 questions, and the duration of the test was 45 minutes. Because they were studying this way, 12 female and 15 male students sat separately for the exam. The test was conducted at the school, but due to COVID-19 and the conditions of social distancing and general health rules, we presented the test on the screen, and the students used their papers to answer the questions. We analyzed the answers of the students depending on five criteria:

- (1) Logical Thinking: The student's ability to think logically is constructed as s/he acts in his or her environment and tries to make sense of his or her world. When solving a problem, the student tries to relate the problem to his previous knowledge (Micklo, 1995).
- (2) Computational Skills: This is the ability of the students to use basic operations to solve problems. Computational skills are considered a necessary key to solving problems (Hickendorff, 2013).
- (3) Justification of Answers: The National Council of Teachers of Mathematics argues that reasoning skills are vital in studying mathematics. Students should justify their answers and solution steps as well as make and assess mathematical conjectures. Mathematical justification is part of communication skills (Cai, 2003).
- (4) Problem Solving Strategies: Polya's strategy is a famous strategy used in mathematics curriculums to help students deal with mathematical problems. It includes four steps; understanding the problem, planning, carrying out the plan, and looking back (Nurkaeti, 2018).
- (5) Use of Representations: such as diagrams, tables, and pictures to assist the student in reasoning and solving the problem.

To maintain anonymity, we numbered the participants as follows-- F1, F2... F12 for the girls and M1, M2... M15 for the boys. This study's findings helped us learn more about how students think, their mathematical thinking, and their weak and strong points in mathematics. This will help us in

many aspects-- we can judge the strategies of teaching, the mathematics curriculums, and the possible weak points, and how to strengthen them.

Two researchers first graded the test and tabulated the test results by identifying students who had either *answered correctly and provided justification*, *answered correctly but did not provide justification*, *answered partly correctly*; and lastly, *did not answer correctly or did not answer the question*.

RESULTS

Table 2 (analysis of questions) shows students' answers to the 15-question test. Code names were given to students to maintain anonymity. The researchers made a note on three occasions: (//) students correctly solving the problem and providing justification; (/) students partially solving the problem correctly with or without justification; (*) students incorrectly solving the problems.

	M 1	M 2	M 3	M 4	M 5	M 6	M 7	M 8	M 9	M 10	M 11	M 12	M 13	M 14	M 15	F 1	F 2	F 3	F 4	F 5	F 6	F 7	F 8	F 9	F 10	F 11	F 12
1	//	/	*	*	*	*	//	*	*	*	*	*	*	*	/	*	*	//	/	//	/	//	*	/	*	//	//
2	//	*	*	//	*	//	*	//	//	//	*	/	//	//	//	*	*	/	/	//	/	/	*	*	/	/	//
3	//	//	//	/	//	//	//	/	*	//	*	//	//	//	*	*	*	/	*	*	*	/	/	*	*	*	
4	//	//	//	/	*	/	/	//	*	//	//	*	//	//	/	/	*	//	*	*	*	*	/	/	/	*	
5	//	/	/	/	/	/	/	/	/	/	*	/	/	/	/	*	*	*	*	*	/	/	/	*	/	*	/
6	//	/	/	//	/	//	//	//	/	*	*	//	//	/	//	*	*	*	*	*	*	*	*	*	*	//	/
7	//	/	/	/	*	/	/	/	/	/	/	/	/	/	/	*	*	*	*	/	*	*	*	*	//	/	//
8	//	//	/	/	/	//	//	/	/	//	/	//	/	//	/	*	*	//	*	//	/	/	*	/	/	/	/
9	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	//	*	//	//	//	*	*	*	//	//
10	//	//	//	/	//	/	/	//	/	//	/	//	//	//	/	/	*	//	/	//	*	//	*	*	//	//	//
11	//	//	/	//	*	//	//	//	//	//	*	//	//	//	//	/	/	*	/	//	*	//	*	//	//	/	//
12	/	/	*	/	*	*	/	/	*	/	*	/	/	/	/	*	//	/	*	//	//	//	*	*	/	//	*
13	/	//	/	*	/	//	//	//	//	//	/	//	//	//	/	*	*	*	*	*	//	//	*	*	/	//	//
14	/	*	*	*	*	*	*	*	*	*	*	*	*	*	*	/	//	//	*	//	*	//	*	//	*	//	//
15	*	//	*	*	//	*	//	/	//	//	*	/	//	//	*	/	*	*	*	*	*	*	*	*	*	*	*

Table Key: //: has answered correctly and provided justification. /: has partly answered. *: has not answered/ incorrect.

Table 2: Analysis of students' solutions based on operations of addition, subtraction, multiplication, and division

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Table 3 (below) presents the distribution of students' solutions question-by-question. The letter B represented the boys and G represented the girls participants. The test results have been recorded in terms of answered correctly, partial answer and answered incorrectly in four areas of number operations.

<i>Operations</i>									
<i>Addition</i>									
	Question 5								
	B	G	Total	B	G	Total	B	G	Total
<i>Answered Correctly</i>	1	0	1	0	0	0	0	0	0
<i>Partial Answer</i>	13	5	18	5	5	10	5	5	10
<i>Answered Incorrectly</i>	1	7	8	7	7	14	7	7	14
<i>Subtraction</i>									
	Question 1			Question 2			Question 12		
	B	G	Total	B	G	Total	B	G	Total
<i>Answered Correctly</i>	2	5	7	9	2	11	0	5	5
<i>Partial Answer</i>	2	3	5	1	6	7	10	2	12
<i>Answered Incorrectly</i>	11	4	15	5	4	9	5	5	10
<i>Multiplication</i>									
	Question 13			Question 15					
	B	G	Total	B	G	Total	B	G	Total
<i>Answered Correctly</i>	9	4	17	7	0	7	0	0	0
<i>Partial Answer</i>	5	1	6	2	1	3	1	1	2
<i>Answered Incorrectly</i>	1	7	8	6	11	17	0	0	0
<i>Division</i>									
	Question 4			Question 7					
	B	G	Total	B	G	Total	B	G	Total
<i>Answered Correctly</i>	8	0	8	1	2	3	0	0	0
<i>Partial Answer</i>	4	3	7	13	2	15	0	0	0
<i>Answered Incorrectly</i>	3	9	12	1	8	9	0	0	0

Table 3: Analysis of students' solutions based on operations of addition, subtraction, multiplication, and division

1. Number and Operations

(a) Addition

For the question on addition operation, question 5, all male students, except M1, answered the first part of the question and missed the second part, finding the difference between Ahmed and Aysha, which indicates that the boys do not read the question well and do not follow the Polya's model in solving problems. Seven girls did not answer the question, while the rest of the girls answered it partially correctly. This referred to the same reason as the boys that the girls also ignored the second part of the question.

(b) Subtraction

For the first question on subtraction (operations), question 1, the male students seemed confused, and they might have thought that they needed to find $900 - x = 300$, answers of 8 boys were 600. This misunderstanding did not happen with the girls. We can say that the students, mainly males, did not read the question well, and then they did not know what exactly was required. Some students missed the subtraction skill. We can see this in question 2; M12 could write the sentence, but he could not subtract. Question 12 states that Maha placed a box on a shelf with a length of 96.4 cm., while the box has a length of 33.2 cm., and whether there is space for a second box on the shelf, and what would be the maximum length of said box. While none of the boys answered

this question correctly with justification, 5 of the girls did. Ten boys provided partly correct answers, and five responded incorrectly. Two of them answered partially correctly from the girls' side, and five responded incorrectly (Figures 1-4).

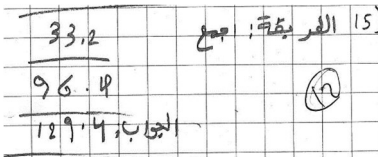


Figure 2: F8's Answer & Justification

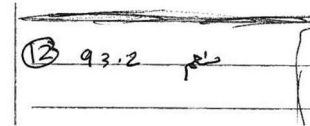


Figure 3: F10's Answer & Justification (Q12)

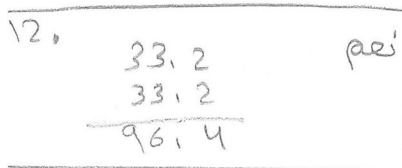


Figure 4: M14's Answer & Justification
(Question 12)

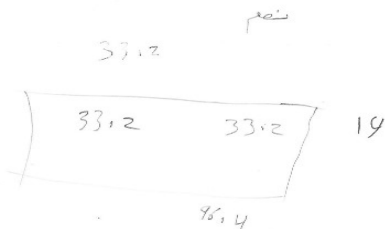


Figure 5: M10's Answer & Justification
(Question 12)

Further analysis of the incorrect answers highlights computational mistakes, such as subtracting 3.2 instead of 33.2, adding 33.2 twice, and writing it as 96.4 instead of 66.4, which were common errors among the incorrect answers. Interestingly, using representation to show the solution was only used by one student (M10).

(c) Multiplication

Another question, Q13, states that "A teacher manages to correct ten student tests in half an hour, how many would he be able to correct in an hour and a half?". The boys' answers showed nine correct answers with justification, five partially correct answers, and one incorrect answer. In comparison, the girls' responses showed four correct answers with justification, one partially correct answer, and seven wrong answers. The correct answers either showed the repeated addition of 10 (3 times) or multiplied 10 by 3.

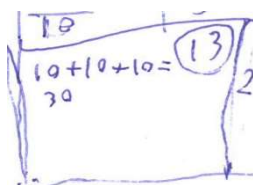


Figure 7: M2's Answer & Justification
(Question 13)

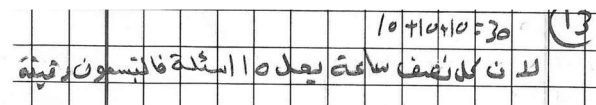


Figure 6: F6's Answer & Justification (Question 13)

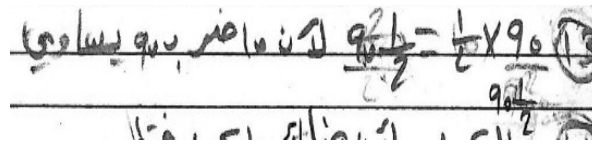


Figure 8: F3's Answer & Justification (Question 13)

F3's answer shows that the student understands that an hour and a half is 90 minutes; however, it also indicates that the student is not able to figure out how to solve the problem, so there is a lack of problem-solving skills and use of the appropriate operation for the word problem. Like many others, two students, M2 and F6, solved the problem by repeated addition, and F6 added that the hour and a half is 90 minutes, hence adding 10 three times. It is noteworthy that F11 was among the few students in both classes who answered the question using multiplication. Three students used multiplication to answer the question in the girls' class, and one answered with repeated addition. The boys' answers were obtained through repeated addition, as none tried to use multiplication. While the correct answers show a good understanding of time and computations, more practice with logical thinking problems is needed. The incorrect answers showed the importance of highlighting appropriate operations (some students subtracted 90-10; while others added 10 + 30) to be used, emphasizing students' understanding of time.

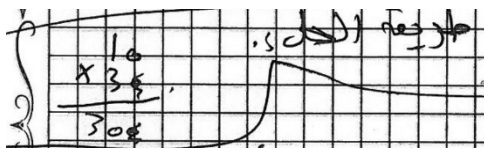


Figure 10: F11's Answer & Justification

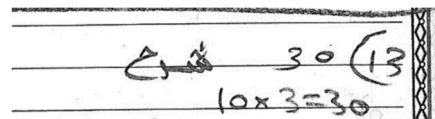
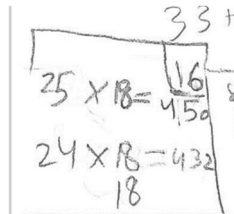


Figure 9: F7's Answer & Justification (Question 13)

In this group, Q15 poses the following problem: "25 X 18 is more than 24 X 18. How much more?" At the face value, students would need to perform the operations and then possibly subtract. However, if students solve this problem logically, it is assumed they would not need to perform any operations, as 25 follows 24 and will be logically higher by 18.

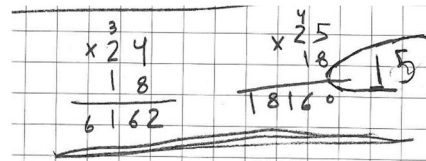
Upon analysis of the students' answers, it was found that seven boys answered correctly, and none of the girls answered correctly.

What is noticeable about the boys' and girls' answers is that none of the boys' solutions show the multiplication steps, which means they used a calculator or performed the task on some scrap paper. All the correct answers obtained from the boys showed the first step of multiplication and the second step of subtraction. When it comes to the solutions from the girls, all the girls attempted to solve the multiplication and then performed subtraction. However, their multiplication skills are



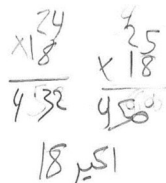
Handwritten work for Figure 12: $25 \times 18 = 450$ and $24 \times 18 = 432$. There is a subtraction of 18 from 450 to get 432. A box is drawn around the calculations with "33+" written above it.

Figure 12: M4's Answer & Justification (Question 15)



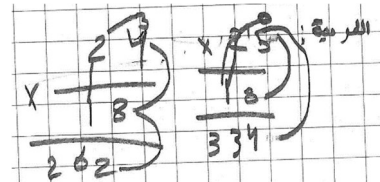
Handwritten work for Figure 11: $24 \times 18 = 6162$ and $25 \times 18 = 18160$. The calculations are written on a grid.

Figure 11: F9's Answer & Justification (Question 15)



Handwritten work for Figure 14: $24 \div 18 = 1$ and $25 \div 18 = 1$. The answer "18" is written below.

Figure 14: M10's Answer & Justification (Question 15)



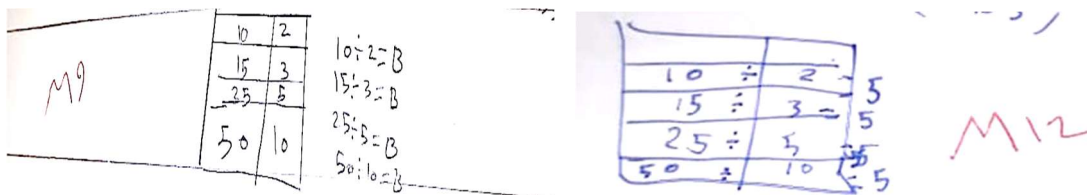
Handwritten work for Figure 13: $24 \times 18 = 202$ and $25 \times 18 = 334$. The calculations are written on a grid.

Figure 13: F8's Answer & Justification (Question 15)

lacking, and they often face difficulty when multiplying by the second digit. It is noteworthy that none of the students tried to solve this problem by using logical thinking (bigger by 18), and all used the two steps of first multiplication and then subtraction. This highlights the need to expose them to logical word problems and further cement their multiplication skills.

(d) Division

Although the answer of student M9 for Q4, Figure 14 (a), is wrong for the first question for the division problem, but we need to stop at his way of thinking; he divided each number in column A by its corresponding number in column B and wrote the answer "B" for all the statements. We can say that his thinking is logical, but he missed the division skill. Several other students, such as M12, F4, F5, F7, F11, and F12, thought the same way but found the answer.



Handwritten work for Figure 15: Student M9's work shows a table with numbers 10, 15, 25, 50 in column A and 2, 3, 5, 10 in column B. Next to it are equations: $10 \div 2 = B$, $15 \div 3 = B$, $25 \div 5 = B$, $50 \div 10 = B$. Student M12's work shows a similar table with equations: $10 \div 2 = 5$, $15 \div 3 = 5$, $25 \div 5 = 5$, $50 \div 10 = 5$.

Figure 15: Students M9's and M12's answers of (Question 4).

For the second question, Q7, in this category (division), most of the male and female students answered the question, but they seemed not to know how to justify their answers.

(2) Fractions

Table 4 (below) presents the distribution of students' solutions question-by-question on the areas of fractions. The letter B represented the boys and G represented the girls participants. The test results have been recorded in terms of answered correctly, partial answer and answered incorrectly in four areas of number operations.

<i>Fractions</i>			
Question 3			
	B	G	Total
Answered Correctly	10	0	10
Partial Answer	2	3	5
Answered Incorrectly	3	9	12
Question 10			
	B	G	Total
Answered Correctly	9	6	15
Partial Answer	6	2	8
Answered Incorrectly	0	4	4

Table 4: Analysis of Questions (Fractions)

For the first question on fractions, Q3, male students could represent the shaded parts in fraction form and compare them to find the equivalent fractions, while none of the girls answered correctly with justification. Question 10 (fraction) provided the students with a rectangle with ten equal parts, two of them were shaded, and eight were not. The students had to choose the best answer that represented the shaded parts; they were given four answers: (a) 2.8 b) 0.5 c) 0.2 and d) 0.02. Almost all the boys answered correctly, with 9 of them justifying an equivalent fraction ($2/10$). Six of the boys chose the correct answer but did not justify their choice. Regarding the girls' responses, 6 of them answered the question correctly with justifications, and 2 of them did not provide any justifications. The four incorrect answers showed a misconception of the relation of parts to the whole.

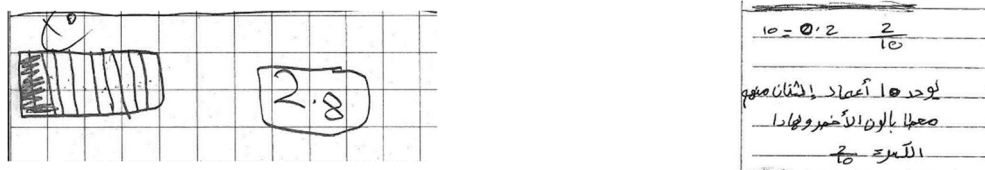


Figure 16: F2's and F10's Answer & Justification

(3) Geometry

Table 5 (below) presents the distribution of students' solutions question-by-question in the area of geometry. The letter B represented the boys and G represented the girls participants. The test results have been recorded in terms of answered correctly, partial answer and answered incorrectly in four areas of number operations.

<i>Geometry</i>			
	Question 6		
	B	G	Total
<i>Answered Correctly</i>	8	1	9
<i>Partial Answer</i>	5	1	6
<i>Answered Incorrectly</i>	2	10	12

Table 5: Analysis of Questions (Geometry)

In the geometry problem, Q6, one of the students, M8, used the trial and error method to solve the problem, finding the missing line segment in a rectangle. He first tried the number 5, then added to see if the sum was not 20, then used the number 6, which gave the correct answer. Another student, M9, drew a rectangle and put a measure of 4 units on the longest side and six units on the shortest, which indicates the missing sense of the number and its values. Most of the girls divided to find the missing side, $20 \div 4 = 5$ (see Figure 16b in the middle).

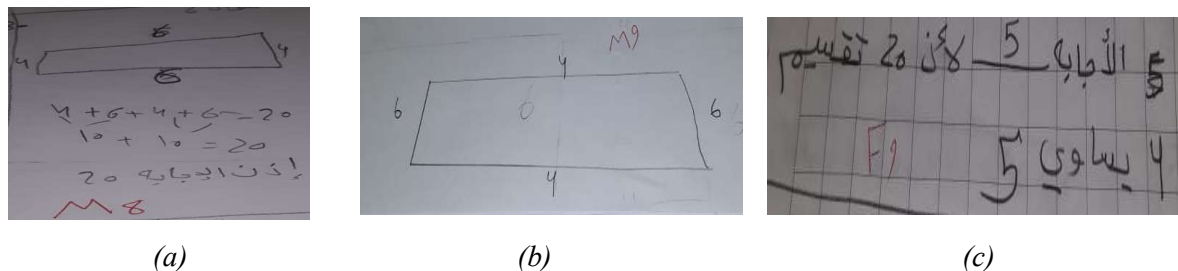


Figure 17: Students M8's, M9's, and F9 answers of (Question 6)

(4) Measurement

Table 6 (below) presents the distribution of students' solutions question-by-question in the areas of measurement. The letter B represented the boys and G represented the girls participants. The test results have been recorded in terms of answered correctly, partial answer and answered incorrectly in four areas of number operations.

Measurement

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	Question 8		
	B	G	Total
Answered Correctly	7	2	9
Partial Answer	8	6	14
Answered Incorrectly	0	4	4
	Question 9		
	B	G	Total
Answered Correctly	0	6	6
Partial Answer	0	0	0
Answered Incorrectly	15	6	21

Table 6: Analysis of Questions (Measurement)

Question 8

For question 8, which asks the students to find the mass of 1,000 clothespins knowing that the mass of each one is 9.2 grams, only two girls could justify their answers, while almost half of the boys could do that. On the other hand, no one of the male students failed in answering this question, while one-third of the students have not responded to the question.

In this group (measurement), the next question, Q9, in this group (measurement), compares the number of footsteps of four students to show who has the biggest footstep. When it comes to gender differences, all the 15 boys answered it incorrectly by commenting that ‘Mohanad,’ who had to take ten footsteps, had the largest size footstep. Whereas the girls’ answers were split in half, where 6 of them answered similarly to the boys, and the rest responded correctly by making the connection that the least number of footsteps meant that person had the largest size footstep, which was identified. Figure 17 shows M14’s answer, “Mohanad has the largest footstep because he has ten footsteps,” indicating that students’ understanding of the largest in size overlaps with the highest number of steps. Almost all students (both boys and girls) who chose “Mohanad” justified it by saying he had the greatest number of steps.

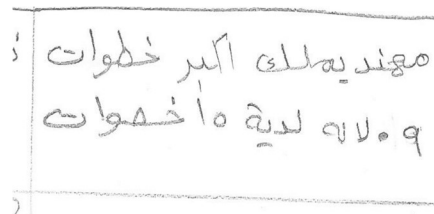


Figure 18: M14's Answer & Justification (Question 9)

Alternatively, we had the answer from student F7, who justified her response by saying, “Reem, seven steps. She has a few footsteps, which means she has a large footstep”. It is also worth

exploring whether the choice of the largest step belonging to a female impacted students' answer choices.

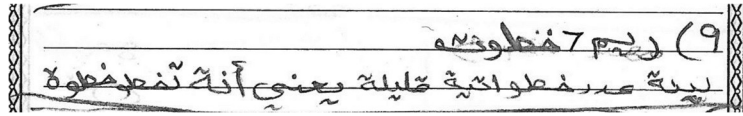


Figure 19: F7's Answer & Justification (Question 9)

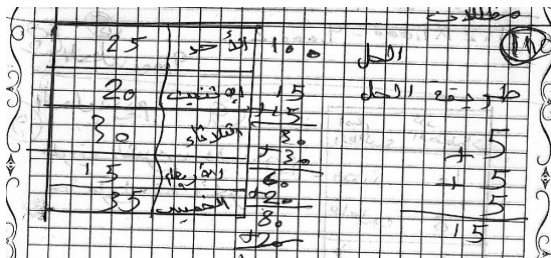
(5) Probability and Statistics.

Probability & Statistics			
Statistics			
	Question 11		
	B	G	Total
Answered Correctly	12	5	17
Partial Answer	1	4	5
Answered Incorrectly	2	3	5
Probability			
	Question 14		
	B	G	Total
Answered Correctly	0	7	7
Partial Answer	1	1	2
Answered Incorrectly	14	4	18

Table 7: Analysis of Questions (Probability & Statistics)

The first question in this group (statistics), Q11, shows a bar graph of the number of milk cartons sold in a school on the five school days and then asks about the total number of milk cartons sold in a week. Most of the boys (12 out of 15) answered correctly and justified having to add a separate number of sales each day to reach the total amount. Five of the 12 girls' answers also showed a similar justification.

One of the female students, F11's answer shows computational errors as she justified it correctly; however, she miscalculated as she added the numbers. While another student, F6's answer, shows



Handwritten calculation: $25 + 20 + 30 = 75$

Figure 21: F11's Justification and Answer (Question 11) Figure 20: M'3 Answer & Justification (Question 11)



that she miscomprehended the question and answered that “Thursday has the highest sales,” this was also present in other students’ answers who gave the number of sales per day but did not add them. The student M3’s answer also showed the addition of the first three days (Sunday – Monday – Tuesday) and then disregarded Wednesday’s and Thursday’s sales.

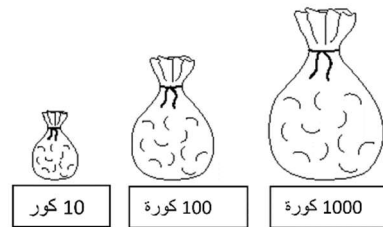


Figure 22: Question 14 diagram

In another question related to probability, Q14 states that there is one red ball in each of the bags that have 10, 100, and 1000 balls, and if a student must pick a ball without looking at any of these bags, which bag will give the highest probability of picking a red ball? Interestingly, 14 of the 15 boys stated that choosing a ball from the largest bag would give them the highest chance of picking the red ball “because it has the most balls.” The choice of justification signifies that the students either did not read the question correctly (there is only one red ball) or assumed that the more balls there were, the higher the chance they had (see figure 22).

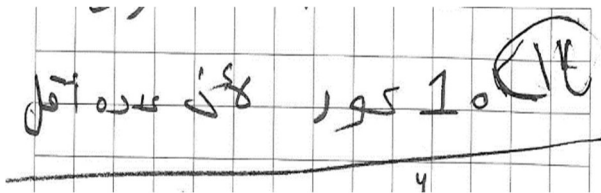


Figure 24: F9's Justification (correct) (Question 14)



Figure 23: M8's Justification (incorrect) (Question 14)

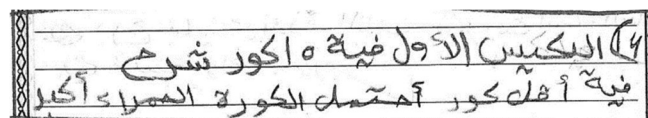


Figure 25: F7's Justification (Correct) (Question 14)

While most of the justifications for the incorrect choices showed that there was a misconception about having a higher chance with a higher number of balls, we can also see examples of the justifications of students F7 and F9 (see figures 23 and 24), who both justified that the smaller number of balls is small, with student F7 adding that the lowest number has the highest probability of picking a red ball.

DISCUSSION

The analysis of the 15 problem-solving questions presented to the fifth-grade students at a UAE Ministry of Education curriculum system school in Al Ain highlighted some key issues that have been discussed based on the five criteria: (1) logical thinking, (2) computational skills, (3) justification of the answers, (4) problem-solving skills and (5) usage of representations such as the use of a diagram, tables, and pictures, to solve.

Logical Thinking

Mathematics teachers should teach students logical thinking in mathematics because it helps them construct mathematical understanding (Weber, 2005). In our study, we conclude that students don't use logical thinking when solving mathematical problems. In question 15 (multiplication of 24 and 25 by 18), no students tried and solved it without computations, which signifies that those students read the question and answered it as a routine or procedural question without really thinking about it. While students are naturally familiar with their timetables (up to 12), they also need to understand the operations and how the increase in numbers is related, such as the algebraic structures $2x$ and $3x$ and their differences. More exercises related to number sense and operations might be helpful to students to train them in logical thinking about this question. This part also relates to algebraic thinking and generalization ability in computational reasoning. Algebraic thinking in solving a problem can help students solve the problem in more than one method or way. Solving a problem using algebraic thinking helps students build algebraic methods of thinking about the problem (Wahyuni, Herman & Fatimah, 2020).

Computational Skills

In problem-solving, computational thinking is an essential part of the solution to the problem. The students need to understand the problem and identify the variables using computational skills to get the solution (Sanford & Naidu, 2017). There were some errors in computational skills, most notable in the multiplication of two digits by two digits (as per the girls' answers that showed the steps). It was noteworthy that none of the girls' section could successfully perform the multiplication of two digits by two digits. The boys' answers did not provide the working out of the solution to gauge their level of multiplication skills.

Some computational errors and subtraction related to hastiness or skill errors, such as when students tried to subtract 33.2 from 96.4 and students either subtracted 3.2 or 63.2 rather than the actual number: 33.2. Computational skills (repeated addition – multiplying two digits by one) were noted as good since most students could perform them flawlessly. Computational skills for adding more than two digits were also performed with minimum mistakes. Computational fluency is one of the skills that help students solve problems, as mastery of these skills enables students and gives them a strong foundation. This is also found in various problem-solving theories and models. "Mastery of computation provides the foundation for more complex mathematics skills, such as problem-solving and algebra" (Nelson & Powell, 2017 p. 3).

Problem-Solving Skills

Students showed good problem-solving abilities when solving problems, even when their answers were not 100% correct. Students used many strategies, such as trial and error, explaining alternative solutions (fractions), and breaking down the information to better understand the question. It was noted that sometimes students seemed to read part of the questions, but not all, and this naturally affected the way they went about solving the questions. Problem-solving skills can be re-emphasized in the classroom by teaching these skills and having exploration classes where students try to understand the concepts through problem-solving. Metacognition and regulating their thoughts will also serve to help students stay on the right track and to understand that problem solving is a process that is constantly evolving (Reiss & Torner, 2007).

Justification of Answers

It was noticed that students sometimes found it challenging to justify their answers, and some copied the wording in the questions. Some tried to justify and use as few words as possible to explain their answers and how they got to them. This demonstrates the need for more exercises in which students are asked to justify their answers; this will also help them retrace their steps and think logically about their answers and whether they are appropriate logical answers to the problem. Various problem-solving models recognize the importance of justifying answers and seeking logical estimates of solutions, such as Polya's model and others. Asking problem-solvers to look back and think of other solutions would help ingrain this skill in students (Reiss & Torner, 2007).

Usage of Representations

Usage of representations was not present in students' answers. There was one example in a question related to subtraction (drawing of a shelf with a box on it and the respective lengths of the shelf and box). However, no other occurrences of this question were recorded for other students. In the question related to geometry, a couple of students also used pictures to help them understand the question (the perimeter of a rectangle). However, besides these two occurrences, there was no other usage of representations. This can highlight students not being used to the idea of representing their solutions or their thought process with diagrams, tables, or pictures. This can be emphasized to students during their classwork with problem-solving to reinforce the usage of

representations in problem-solving, especially since this is one of the main elements of various problem-solving models such as Polya's problem-solving model, as well as Schoenfeld's and various others (Liljedahl, Malaspina, Santos-Trigo, Bruder, 2016).

IMPLICATIONS AND LIMITATIONS

Students should be exposed to more word problems and given ample time to digest the information, read the question thoroughly, and devise a possible plan for a solution. Students should be encouraged to show their justifications for the answers and how they arrived at specific solutions. By making it a daily practice in mathematics classrooms in the UAE, students who are not trained in problem-solving will learn to question themselves and their intuition and hone their knowledge and skills in problem-solving. More focus should be given to computational skills in the lower and upper primary grades so students become mathematically fluent. Introducing technology to help students as an alternative to the drill and practice methods could be a way of encouraging students to take ownership of their learning through online practice in mathematics games and then with formative assessments in class in various ways (e.g., the Kahoot program as a tool for quizzes and games can be included).

As we see, problem-solving is the central part of international exams such as PISA and TIMSS (IEA, 2015, OECD, 2018). Educators need to adopt problem-solving as a competency that must be included in mathematics and other subjects. Teachers should adopt one of the problem-solving models, such as the Polya model, Schoenfeld model, or Verschaffel model. This will help the students to organize their thoughts while dealing with a problem. They have to learn how to understand, plan, solve, and check the solution.

The study was specific to a private MOE school in Al Ain with 27 students who attended either the boys' (n=15) or girls' (n=12) sections. The findings cannot be generalized to other schools in the area since the sample for study is small and because other schools can have different types of curricula in several languages. Another limitation of the study is that, because the test was conducted in two classes, issues such as using calculators or other devices to help with computations were not controlled. Hence, it is recommended to conduct studies with the same control situations as much as possible. A third limitation is concerned with the questions that were chosen from the TIMSS exam, which had varying weights amongst the strands and were not balanced between the strands. For example, the weight of the questions in the operations strand is 53.3%, while the weight of the questions in the geometry strand is 6.6%. In the future studies, we have to pay attention to this point.

CONCLUSION

Overall, five themes emerged from this study: logical thinking, computational skills, problem-solving skills, justification of answers, and usage of representations. In general, the students used these themes randomly, i.e., there was no training for the students on how to solve problems using such themes that organized their thinking and hence their strategy. According to the National Council of Teachers of Mathematics (NCTM, 2000), mathematics has five basic abilities: communication, reasoning, problem-solving, mathematical connection, and representation. This shows the strong relationship between problem solving and mathematics (Novitasari, Setianingsih, & Novitasari, 2019).

In conclusion, this study has highlighted a few issues in problem-solving in elementary schools in the UAE context. What has been gleaned from this preliminary study can be further explored to gain deeper insight into problem-solving. Students in the fifth-grade classes in this study have shown great aptitude in justifying their problem-solving strategies and show potential for more significant growth.

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Appendix (1)
Fifth grade problem-solving Questions-English

How Do We Think?

Name: _____

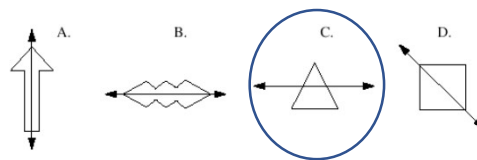
Day & Date: _____

Grade: _____

Dear Students,

Please answer the following questions, providing a short explanation of how you reached your conclusion. You can describe your thought process, or draw a diagram to show how you reached your answer. Below is an example of this:

Example 1: Which of these does not show a line of symmetry? Circle the correct answer and then provide an explanation.



Explanation: If we fold figure C in half, we can see that we have two different shapes, they are not identical, and that shows us that this is not a line of symmetry. However, in figures A, B and D, if we fold the shapes on the lines, we end up with two identical shapes. The two shapes are the same, and that shows us that these lines are lines of symmetry.

- End of Example –

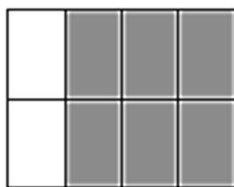
- 1) When you subtract one of the numbers below from 900, the answer is greater than 300. Which number is it?
- A) 823
- B) 712

- C) 667
- D) 579
- 2) Tanya has read the first 78 pages in a book that is 130 pages long. Which number sentence could Tanya use to find the number of pages she must read to finish the book?

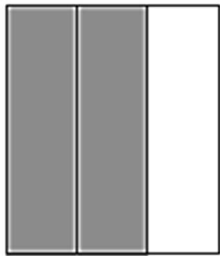
- 3) Each figure represents a fraction. Which two figures represent the same fraction?



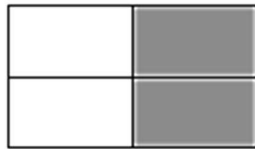
1



2



3



4

- A) 1 and 2 B) 1 and 4 C) 2 and 3 D) 3 and 4

- 4) What do you have to do to each number in column A to get the number next to it in Column B?

Column A	Column B
10	2
15	3
25	5
50	10

- 5) Kyle and Bob are playing a game. The object of the game is to get the highest total of points. This chart shows how many points they scored.

Scorecard

Player	Kyle	Bob
Round 1	125	100
Round 2	125	125
Round 3	150	100
Round 4	50	150

Who won and by how many points?

- 6) A thin wire 20 centimeters long is formed into a rectangle. If the width of this rectangle is 4 centimeters, what is its length?
- 7) There are 54 marbles, and they are put into 6 bags, so that the same number of marbles is in each bag. How many marbles would 2 bags contain?
- 8) The weight (mass) of a clothespin is 9.2 g. Which of these is the best estimate of the total weight (mass) of 1000 clothespins?

Name	Number of Paces
Stephen	10
Erlane	8
Ana	9
Carlos	7

- A) 900 g
 B) 9,000 g
 C) 90,000 g
 D) 900,000 g

9) Four children measured the width of a room by counting how many paces it took them to cross it. The chart shows their measurements.

Who had the longest pace?

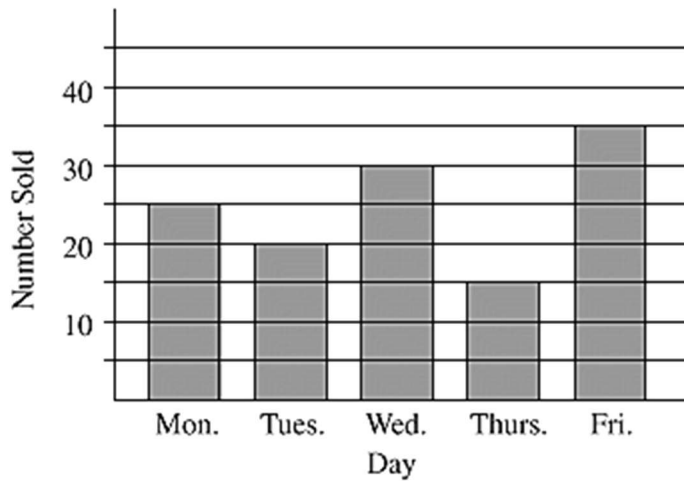
- A) Stephen
 B) Erlane
 C) Ana
 D) Carlos

10) **Which number represents the shaded part of the figure?**



- A) 2.8
 B) 0.5
 C) 0.2
 D) 0.02

11) The graph shows the number of cartons of milk sold each day of a week at a school.

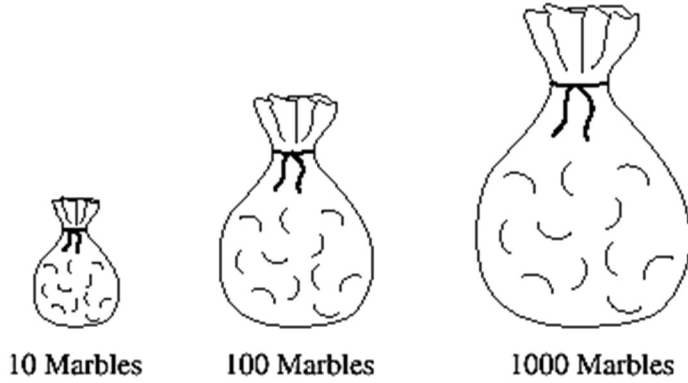


How many cartons of milk did the school sell that week?

12) Julie put a box on a shelf that is 96.4 centimeters long. The box is 33.2 centimeters long. What is the longest box she could put on the rest of the shelf?

13) A teacher marks 10 of her pupils' tests every half hour. It takes her one and one half hours to mark all her pupils' tests. How many pupils are in her class?

14) There is only one red marble in each of these bags?



Without looking in the bag, you are to pick a marble out of one of the bags. Which bag would give you the greatest chance of picking the red marble?

15) 25×18 is more than 24×18 . How much more?

Referred from:

<https://www.edinformatics.com/timss/pop1/mpop1.htm?submit3243=Grade+3%2C4+Math+Test>

Appendix (2) Fifth grade problem-solving Questions-Arabic

كيف أفكر؟

اليوم والتاريخ: _____

اسم الطالب: _____

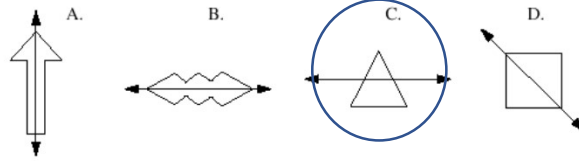
الصف: _____

عزيزي الطالب/ة،
بين يديك اختبار بسيط يعزز من التفكير الرياضي والمنطقي لدى الطالب. هذا مجرد اختبار تشخيصي و لا علاقة له بأية درجات. بعد قراءة الأسئلة والإجابة عنها، اشرح طريقة الحل. بإمكانك وصف طريقة تفكيرك أو رسم نموذج توضيحي يبين طريق توصلك للحل. انظر للمثال 1:

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مثال 1: أي من الأشكال التالية لا يمثل خط تناظر؟ حوِّط على الإجابة الصحيحة و اشرح.



شرح طريقة التفكير:

عند طي الشكل C لا نحصل على جزئين متماثلين، إذا هذا لا يمثل خط التناظر. و لكن عندما نثني الأشكال في A و B و D نحصل على أشكال متناظرة أي مطابقة.

(1) عندما طرحنا عدد ما من 900 كان الناتج أكبر من 300. ما هو هذا العدد؟ اشرح

- (A) 823
(B) 712
(C) 667
(D) 579

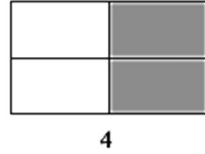
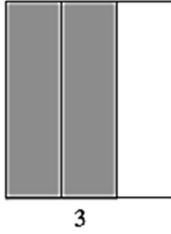
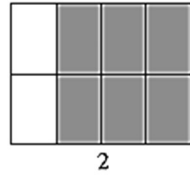
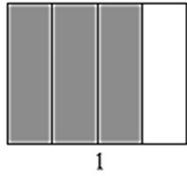
شرح طريقة التفكير:

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(2) قرأت أمنة أول 78 صفحة من كتاب يحوي 130 صفحة. اكتب جملة عددية توضح عدد الصفحات التي يجب أ، تقرأها أمنة لتنتهي الكتاب؟

شرح طريقة التفكير:

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(3) كل شكل مما يلي يمثل كسراً. أي شكلين منهم يمثل نفس الكسر؟ اشرح

(A) 1 و 2

(B) 1 و 4

(C) 2 و 3

(D) 3 و 4

شرح طريقة التفكير:

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Column A	Column B
10	2
15	3
25	5
50	10

(4) ما الذي يجب أن تفعله لكل رقم في المجموعة A لتحصل على الرقم المناظر له في المجموعة B؟

شرح طريقة التفكير:

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(5) يلعب أحمد وعائشة لعبة يكون الفائز فيها من يحصل على أعلى النقاط. يبين الجدول أدناه النقاط التي سجلها كل منهما. من الفائز وبكم نقطة فاز؟

بطاقة تسجيل النقاط		
عائشة	أحمد	اللاعب
100	125	ال الجولة الأولى
125	125	ال الجولة الثانية
100	150	ال الجولة الثالثة
150	50	ال الجولة الرابعة

شرح طريقة التفكير:

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(6) استخدم خالد سلكاً رفيعاً طوله 20 cm لصنع مستطيل عرضه 4 cm. فما هو طول هذا المستطيل؟

شرح طريقة التفكير:

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(7) يوجد مع راشد 54 كرة صغيرة، قام بتوزيعها على 6 أكياس بالتساوي. كم عدد الكرات في كيسين؟

شرح طريقة التفكير:

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(8) وجدت فاطمة أن كتلة مشبك الغسيل تساوي 9.2 g . أي مما يلي يمثل تقريبا كتلة 1000 مشبك غسيل؟ اشرح

900 g (A)

9,000 g (B)

90,000 g (C)

900,000 g (D)

شرح طريقة التفكير:

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(9) قام 4 طلاب بقياس عرض الغرفة باستخدام خطواتهم و يبين الجدول التالي عدد الخطوات التي احتاجوها للقياس. من منهم يملك أكبر خطوة و لماذا؟

عدد الخطوات	اسم الطالب
10	مهند
8	فاطمة
9	فيصل
7	ريم

شرح طريقة التفكير:

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10) ما هو العدد الذي يمثل الجزء المُضَلَّل، اختر الاجابة الصحيحة. كيف عرفت؟

- (a) 2.8
(b) 0.5
(c) 0.2
(d) 0.02

شرح طريقة التفكير:

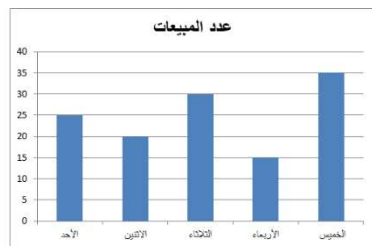
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11) يوضح الرسم البياني عدد مبيعات عبوات الحليب في المدرسة لمدة اسبوع. كم عدد مبيعات الحليب خلال هذا الاسبوع؟

شرح طريقة التفكير:

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12) وضعت مها علبة على رف طوله 96.4 سنتمتر. طول العلبة 33.2 سنتمتر. هل تستطيع مها وضع علبة أخرى على الرف؟ ما أقصى طول للعلبة الثانية؟

شرح طريقة التفكير:

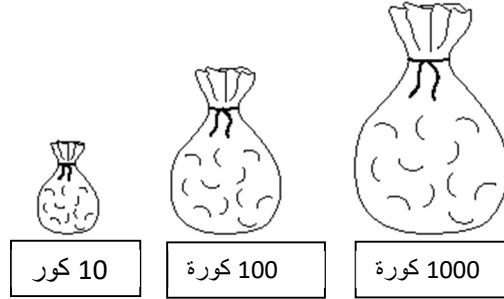
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13) يصحح معلم اختبارات 10 طلاب في نصف ساعة. استغرق ساعة و نصف لالنتهاء من تصحيح جميع اختبارات الصف الخامس. كم طالب في الصف الخامس؟

شرح طريقة التفكير:

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14) في كل من الأكياس الثلاثة التالية كرة حمراء واحدة. عليك سحب كرة من دون النظر في داخل الكيس. أي من الأكياس يعطيك احتمالاً أكبراً بسحب كرة حمراء؟ حوِّط على الكيس و اشرح



شرح طريقة التفكير:

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15) 25 ضرب 18 أكبر من 24 ضرب 18. بكم أكبر؟ _____

شرح طريقة التفكير:

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- انتهت الأسئلة -

Constructing Students' Thinking Process through Assimilation and Accommodation Framework

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rustanto.rahardi.fmipa@um.ac.id

Abstract: Thinking is a tool to construct knowledge in learning mathematics. However, some college students have not been fully aware of the importance of constructing their knowledge. Therefore, this study aims to explore students' thinking processes in completing mathematical proofs through assimilation and accommodation schemes. This research was conducted on students majoring in mathematics from three different universities in East Java as research subjects. The data was collected through a mathematical proof test instrument and interviews which is then qualitatively analyzed. The results of the study show that there were students who completed the test through the assimilation scheme only, and there were students who completed the test using both assimilation and accommodation schemes. Students construct their thinking processes through 5 stages, namely: identifying, determining rules to be used, proving with symbol manipulation, reviewing, and justifying. Students use the five stages of thinking to construct knowledge. However, students who use assimilation schemes made some errors in proving the mathematics problem due to their carelessness in doing the proving with symbol manipulation and reviewing stages.

INTRODUCTION

Thinking process is an important component to know someone's abilities and talents in learning mathematics (Polly et al., 2007; Uyangör, 2019). Thinking can be said as a tool for learning mathematics and a tool to construct one's knowledge (As'ari et al., 2019; Fisher, 2005). Thinking process includes reasoning that occurs through a mental activity in the students' brain. This reasoning can occur when the students are performing algebraic operations, problem solving, decision making, critical thinking, reflective thinking, or analytical thinking. This process is not only to produce abstract mathematical numbers and concepts but also as an important skill in thinking analytically and logically, as well as reasoning quantitatively (Onal et al., 2017).

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Thinking analytically is a highly necessary thinking process used to solve mathematical proof problems. At the university level, these problems are formal which require analytical thinking capability. However, some students tend to complete the mathematical proving problems related to abstract algebra intuitively (Korolova & Zeidmane, 2016). Intuitive mathematical proving is not necessarily wrong but students could possibly use the wrong concept in solving the problems. Some students solved subgroup problems in abstract algebra by using the Lagrange Theorem because they understand only the Lagrange Theorem concept and unfortunately do not understand subgroups concept very well (Leron & Hazzan, 2009; Leron, 2014). Therefore, students who complete an abstract algebra proving by using only the existing knowledge, need to construct their thinking process in order to accept a new knowledge scheme. The new knowledge scheme can be built by assimilation and accommodation. Thus, the question of this research is "how does the students' thinking process in solving the algebra proving problem based on the assimilation and accommodation framework?"

This research focuses on how do the students build their knowledge in solving algebra proofing problems through constructive thinking. Piaget said that the thinking process could be done through a construction process that occurs based on the previous knowledge to gain a new one. This construction could have occurred through five components, namely activating previous knowledge, owning and understanding a new knowledge, using the knowledge, then reflecting (Aseeri, 2020). Construction was the process of student's interaction related to previously owned ideas with new ideas to understand a concept being studied. Construction could be combined with interaction due to the existence of knowledge that were being used to perform a mental activity (Guler & Gurbuz, 2018).

Assimilation and Accommodation Framework

Piaget's theory states that there are two kinds of adaptation process of each individual to their environment; assimilation and accommodation (Kaasila, et al., 2014). Piaget divided the intellectual growth that occurs through one's mental activity into the following six steps: reflexively, obtained through a fundamental adaptation, interest on a new situation, relation to new discoveries, and combining the discoveries in mental activities (Piaget, 1965). A new scheme obtained by the students could be included in the assimilation object by organizing a new definition. The scheme on Piaget's theory contained assimilation and accommodation as a process of knowledge translation. Both were influenced by the development of Piaget's theory in mathematics learning (Ernest, 2003).

Assimilation is a process conducted by students in inserting a new stimulus into the existing scheme. The assimilation was a positive influence of the environment that occurs on one's mental activity. At the time a new object is being assimilated into the existing scheme. While the accommodation is a process of adjusting the schemes conducted by students to build a new scheme based on the existing scheme. Accommodation indicates that the process which is conducted by

the student is influenced by the object being transformed. In other words, assimilation and accommodation could be represented as an interaction between the subject and the object which makes assimilation and accommodation closely related (Zhiqing, 2015). At the time when assimilation is dominated by a new scheme, then the scheme is a part of the accommodation. Therefore, assimilation can occur even though there is no accommodation, but accommodation will not occur without the existence of assimilation. For instance: students who have learnt about addition operation of natural numbers but have never learnt about the addition operation of fractions will solve the mathematics problem as follow $\frac{2}{3} + \frac{1}{2} = \frac{3}{5}$. In this process, students only perform assimilation as they only use the previous knowledge without reconstructing to gain new knowledge about addition operation of fractions. If students are able to operate addition in the form of “ $\frac{2}{3} + \frac{1}{2} = \frac{4+3}{6} = \frac{7}{6}$ ”, these students already performed accommodation as they equalized the denominators into 6 before adding the numerators into $4 + 3 = 7$.

Students can construct their knowledge when doing the assimilation to form a new scheme. Assimilation and accommodation are the adaptation process to the environment based on cognitive structures. While assimilation is the process of interpreting an event by using the existing cognitive structures, accommodation on the other hand is the process of increasing knowledge by modifying the existing knowledge or cognitive structures to gain a new experience (Kaasila et al., 2014; Netti et al., 2016). Therefore, in the process of assimilation, a new stimulus is directly absorbed and integrated into the existing knowledge schemes. Meanwhile the process of accommodation on the existing knowledge structures cannot directly absorb the new stimulus; it needs a phase to integrate the stimulus. The process of assimilation and accommodation can be illustrated into a diagram in order to help us understand the process or procedure of those two adaptation process (Subanji & Nusantara, 2016).

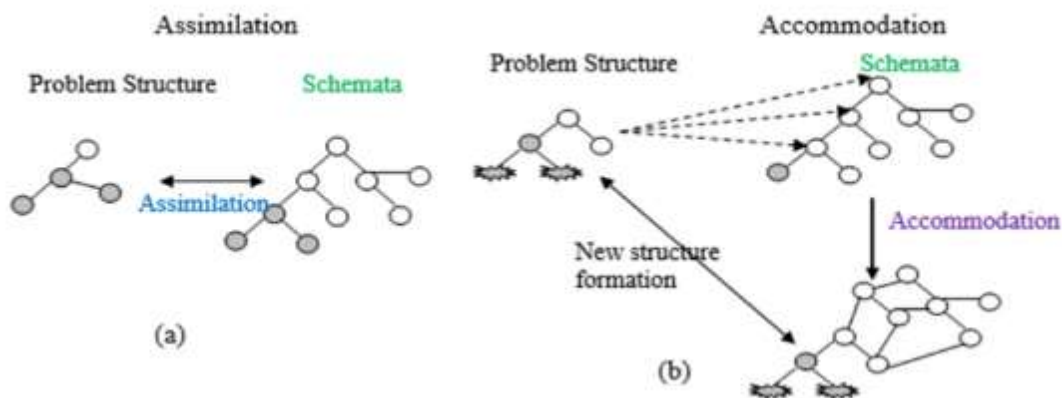


Figure 1: Assimilation and accommodation process

Figure 1 (a), shows that assimilation occurs when the structure of the problem is in accordance with the existing scheme. It will be interpreted directly into the correct way in order to form new

structures. Figure 1 (b), shows that the structure of the thinking scheme does not match with the structure of the problem. The students need to convert the new scheme with the existing schemes in order to create a new thinking structure related to the problem when they constructing correctly. Therefore, the thinking activity through the assimilation and accommodation framework in this research can be seen in Table 1.

Table 1: Thinking process based on assimilation and accomodation schemes.

Thinking Process	Mental Activity
Assimilation	Employing an existing scheme to solve the mathematical proof problem
Accommodation	Employing both the existing scheme and a new scheme in order to solve mathematical proof problems

METHOD

This research is a qualitative research with descriptive explorative. Data for this research is collected through written test and interview. The researcher is the main instrument in collecting and analyzing the data obtained from written test results and interview. This research was conducted to college students in mathematics department from three different universities. The three universities are in Jombang, Mojokerto and Malang city. The subjects were chosen based on the students' abilities in constructing their knowledge through assimilation and accommodation scheme as shown in Table 1. The participant are those mathematics students who have passed the abstract algebra course. From 78 students in three different universities in East Java, Indonesia, 9 students were able to do assimilation without accommodation, and 13 students were able to do assimilation and accommodation. The students who were chosen as the research subjects were those who were able to reveal their thinking process verbally. Table 2 shows the number of students who could construct their idea through thinking process.

Table 2: The construction of students' thinking process

University	Number of Students	Assimilation	Assimilation and Accommodation
A	27	2	4
B	31	4	5
C	20	3	4
Total	78	9	13

From Table 2, we can see that 9 students were able to do the thinking process of assimilation and 13 students were able to do the thinking process of assimilation and accommodation. In general, one out of nine students could express their mind verbally in solving problem through assimilation. Two out of 13 students could express their mind in assimilation and accommodation. In short, three students were chosen as the research subjects. Table 3 showed the number of research subjects in this study.

Table 3: Selection of subjects

Thinking Process	Number of Students	Research Subjects
Assimilation	9	1
Assimilation and Accommodation	13	2
Total number	22	3

Table 3 shows that there were 3 students who were selected as research subjects. The 3 subjects are Dwi as subject 1, Alex as subject 2, and Dita as subject 3 (pseudonym). They were selected as research subjects as they were able to do verbal and written communication related to the thinking process that have been conducted in completing the abstract algebra proving test. This is due to the fact that thinking process is a form of communication between individuals and themselves based on cognitive activities they have conducted (Sfard & Kieren, 2001; Sfard, 2012).

The main instrument in this qualitative research is the researcher assisted with research instruments in the form of mathematical proving problem test and interview. The mathematical proving test instrument used in this research was adapted from Hungerford (2000). as follows:

*“Let $p * q = p + q - pq$ with p, q elements of natural numbers in binary operations. Determine whether $p * q = p + q - pq$ is semigroup or not!”*

The proving test consisted of semigroup material in abstract algebra. Semigroup is non-empty set G together with a binary operation $*$ on G that is associative $a(bc) = (ab)c$ for all $a, b, c \in G$ (Hungerford, 2000).

The definition of semigroup:

- A binary operation $*$ on a non-empty set G is a function $\mu: G \times G \rightarrow G$.
- An operation $*$ on a set G is associative if $(a * b) * c = a * (b * c)$ for every $a, b, c \in G$

The data analysis used in this research is a qualitative with the following details:

Data analysis was conducted by observing the results of written tests and semi-structured interviews. In this research, interviews were used as a triangulation to obtain valid data. Creswell (2012) stated that the validity and reliability test of qualitative research can be done through triangulation. The researcher conducted task-based interviews on subjects with the help of a tape recorder and field notes containing important points from the subjects' expressions. The results of the interviews were transcribed exactly to the subjects' answers and expressions and then reduced based on assimilation and accommodation presented in Table 1. The data is presented in matrix form as one of the methods of qualitative research data analysis (Miles et al., 2014). This matrix is a table containing the relationship between variables obtained from the results of written tests and interviews. In this research, the researcher was actively involved in designing research, collecting data, and analyzing data.

RESULT AND DISCUSSION

The results of this research showed that there were three students selected as the research subjects. The selection of three subjects was based on their oral and written communication skills in constructing knowledge to complete a mathematical proving test. The three subjects were able to complete the test by integrating the previously owned knowledge scheme with a new scheme.

Subject 1

Dwi completed the test by assimilation because she performed the procedural proving by using the existing knowledge scheme. She identified the problem by reading the information that would be proven and then she wrote that the natural numbers with binary operations at $p * q = p + q - pq$ is closed. The claim was given spontaneously because she did not think of any element of the natural numbers by symbol of N . Dwi used the knowledge scheme about real numbers to complete the test. She considered that natural numbers were real numbers that are closed to all types of operations of numbers in the form of addition, subtraction, multiplication, and division operation. This can be seen from Figure 2.

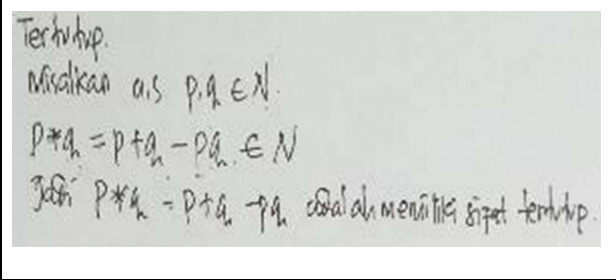
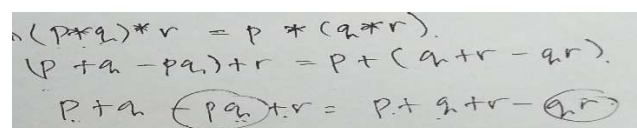
 <p>Teknik Misalkan as $p, q \in N$ $p * q = p + q - pq \in N$ Jadi $p * q = p + q - pq$ adalah memiliki sifat tertutup.</p>	<p>Translation: Closed For example, take any $p, q \in N$ $p * q = p + q - pq \in N$ So, $p * q = p + q - pq$ has a closed nature.</p>
--	---

Figure 2: Answer of subject 1 on the first stage

On the second stage, subject 1 used the associative nature to prove the semigroup. She proved the associative nature by using the existing knowledge scheme about real numbers to prove the semigroup of the natural numbers. She used the symbols $p, q, r \in R$ to prove the associative nature. Dwi performed algebraic operations by manipulating symbols. Firstly, she assumed that on $(p * q) * r = p * (q * r)$ the associative nature was not applicable because the results of algebraic operations between the left-hand and right-hand side of the equation were not the same as in the circle sign in Figure 3.



$(p * q) * r = p * (q * r)$
 $(p + q - pq) + r = p + (q + r - qr)$
 $p + q - (pq) + r = p + q + r - (qr)$

Figure 3: The initial answer of the subject before the construction process

The subject then constructed the knowledge by re-proving to make sure that she got the right answer of associative proofing. She used cancellation characteristics by crossing out the same elements between the right-hand and left-hand side, so that it is obtained $(p * q) * r = p * (q * r)$ as in Figure 4.

<p>Asso siatif. as $p, q, r \in \mathbb{N}$ $(p * q) * r = p * (q * r)$ $(p + q - pq) + r - (p + q - pq)r = p + (q + r - qr) - p(q + r - qr)$ $p + q - pq + r - pr + qr - pqr = p + q + r - qr - pq + pr + pqr$ $p + q - pq + r = p + q + r - pq$ Jadi $(p * q) * r = p * (q * r)$ berlaku sifat asosiatif Jadi $p * q = p + q - pq \in \mathbb{N}$ berlaku semigrup</p>	<p>Translation: Take any $p, q, r \in \mathbb{N}$ $(p * q) * r = p * (q * r)$ $(p + q - pq) + r - (p + q - pq)r$ $= p + (q + r - qr) - p(q + r - qr)$ $p + q - pq + r - pr + qr - pqr$ $= p + q + r - qr - pq + pr + pqr$ $p + q - pq + r = p + q + r - pq$ So, $(p * q) * r = p * (q * r)$ applies semigroup</p>
---	---

Figure 4: The answer of subject 1 on the second stage

The subject's proving process shows that the closed and associative nature were applicable. At first, she assumed that $(p * q) * r$ was not associative because she obtained $(p * q) * r \neq p * (q * r)$. Furthermore, the subject constructed the knowledge that she had in order to obtain $(p * q) * r = p * (q * r)$ as in Figure 4. From the proving result of the closed and associative nature, Dwi concluded that $(p * q) = p + q - pq$ is a semigroup on binary operations of natural numbers. This is indicated from the interview transcript as follows:

R : Why?

D : Because in the beginning I did an algebraic and the result was $(p * q) * r \neq p * (q * r)$. However, after I carefully observed by decomposing it one-by-one, the result showed that $(p * q) * r = p * (q * r)$

Dwi as subject 1 completed the mathematical proving test related to the semigroup by first identifying the problem. Identification of the problem is done spontaneously by mentioning the semigroup conditions in the form of closed and associative nature. Then she gave a claim that $p * q = p + q - pq$ is closed on the binary operation of natural number (\mathbb{N}). Then she proved the associative nature by using the assimilation scheme to obtain $(p * q) * r = p * (q * r)$. She proved it through symbol manipulation in algebraic operations and obtained $(p * q) * r \neq p * (q * r)$. After that she claimed that $p, q \in \mathbb{N}$ with respect to binary operations on natural numbers is not associative. However, Dwi conducted a review on her result by re-checking it again. From

the review, she found that the associative nature that she previously concluded was incorrect. Then she re-constructed her knowledge to perform algebraic operations again and obtained $(p * q) * r = p * (q * r)$. Therefore, Dwi justified that $p * q = p + q - pq$ for all $p, q \in N$ is semigroup of binary operations. The justification was done analytically based on the thinking construction, but the final conclusion that she gave was incorrect. Dwi performed procedural proving as she only explained the proving of semigroup in natural number as in the proving procedure for real number. Although she had written N in her proving, she did not realise that N is a natural number. Thus, she only performed assimilation without accommodation as she did not reconstruct her previous knowledge to conduct the proving of N as natural number.

Subject 2

Alex used his previous knowledge scheme about semigroups proving on real numbers to prove the semigroups on natural numbers. It can be seen from the mental activity performed by Alex in identifying the problems. He mentioned that the semigroup requiring the closed and associative nature. Then he proved and concluded that $p * q = p + q - pq$ for all $p, q \in N$ is not semigroup of binary operations because it did not fulfill the associative nature. The conclusion was correct but the steps taken in reaching the conclusion were not correct. From his proving of the closed nature, an error was seen. The right answer should: $p * q = p + q - pq$ for all $p, q \in N$ is not closed on binary operations.

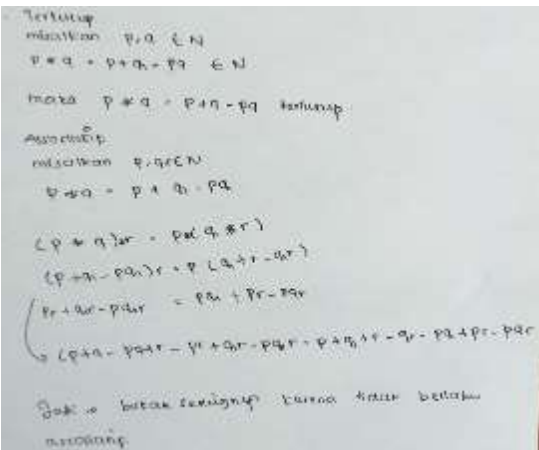
	<p>Translation:</p> <ul style="list-style-type: none"> • Closed <p>For example, $p, q \in N$</p> $p * q = p + q - pq \in N$ <p>Then</p> $p * q = p + q - pq \in N$ <p>Are closed</p> <ul style="list-style-type: none"> • Associative <p>For example, $p, q \in N$</p> $p * q = p + q - pq$ $(p * q) * r = p * (q * r)$ $(p + q - pq)r = p(q + r - qr)$ $pr + qr - pqr = pq + pr - pqr$ $(p + q - pq + r - pr + qr - pqr$ $= p + q + r - qr - pq + pr$ $- pqr$ <p>So, it is not a semigroup because it does not apply associative nature</p>
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Figure 5: The result of subject 2

Figure 5, shows that Alex described his proving through the closed nature and the associative nature was performed spontaneously as he only described it procedurally. The proving result showed that the claim was correct that the problem was not a semigroup, but the steps taken

by Alex to prove it were incorrect. Alex was doubtful about the result of the associative nature proving, so that he constructed the existing knowledge to be re-prove by taking any element of the set of natural numbers in the form of $N = \{1,2,4,5\}$ as in Figure 6.

<p>misalkan. $p=1 \quad q=2$ $p=4 \quad q=5$ $1+2-(1 \times 2)$ $4+5-(4 \times 5)$ $3-2=1$ $9-20=-11$</p> <p>Jadi ditinjau dari pembuktian pada sifat tertutup, $p * q = p + q - pq$ bukan termasuk semigrup karena definisi tersebut hanya berlaku pada angka tertentu saja. misalkan jika dimasukkan pada bilangan asli hasil operasinya bukan termasuk anggota himpunan bilangan asli.</p>	<p>Translation: For example, $p = 1; q = 2$ $p + q - pq = 1 + 2 - (1 \times 2) = 3 - 2 = 1$ For example, $p = 4; q = 5$ $p + q - pq = 4 + 5 - (4 \times 5) = 9 - 20 = -19$ So, it is not closed nature, then $p * q = p + q - pq$ is not a semigroup.</p>
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Figure 6: The results of Alex's thinking construction

Alex did the proving twice by substituted elements of the set of Natural Numbers (N). First, Alex considered $p = 1$ and $q = 2$ to obtain $p + q - pq = 1 + 2 - 2.1 = 1$, because 1 is an element of N as a set of natural numbers, then he claimed that $p * q = p + q - pq$ for all $p, q \in N$ is closed. Second, he did the proving by assuming that $p = 4$ and $q = 5$ and then obtained $p + q - pq = 4 + 5 - 4.5 = -11$. Since the result is -11, he changed his claim into $p * q = p + q - pq$ for all $p, q \in N$ was not semigroup because it is not closed on binary operations in natural number.

*A : Means that for example something like this, $p = 1$ and $q = 2$, then $p + q - pq = 1 + 2 - 2.1 = 1$ is obtained. The result is the natural numbers, ma'am.
 For example, $p = 4$ and $q = 5$ and then $p + q - pq = 4 + 5 - 4.5 = -11$
 Oh, right.... It's not, ma'am.
 So that the closed nature is not applicable, isn't it?*

The thinking process conducted by Alex in completing the test was by identifying the problem first. The information in the problem stated that N is a set of natural numbers, but Alex used a real number scheme to prove it. This is because he only knew the semigroup proving in real numbers in which it can be said that he did an assimilation process. The proving of closed nature was done only by looking at the information in the problem then made a claim that $p * q = p + q - pq \in N$ is closed on binary operations. After that he performed algebraic operations by manipulating symbols to prove the associative nature. From the proving of associative nature, it was obtained that $p * q = p + q - pq \in N$ was not associative on binary operations because $(p * q) * r \neq p * (q * r)$

$(q * r)$. Then Alex made a new claim that $p * q = p + q - pq \in N$ was not semigroup on the binary operations because it did not fulfill the associative nature.

Alex conducted a review on his own result. He checked the correctness of the claim by proving the problem using the elements of natural numbers. Then he obtained the result in the form of a negative number (-11) so that he changed the claim in which $p * q = p + q - pq \in N$ was not closed on binary operations because -11 is not an element of the natural numbers. From the claim, he said that proving associative nature was not needed because the first condition of the semigroup was not fulfilled. After that, he made a justification that $p * q = p + q - pq \in N$ was not semigroup on binary operations.

Based on the result of exploration to subject Alex, it is known that he conducted procedural proving as he used real numbers to proof close property of natural numbers which resulted in incorrect conclusion. However, Alex tried to re-examine the statement in the test and presupposed the element of natural numbers in the form of $N = \{1,2,4,5\}$ to perform the close property proving. Thus, Alex actually performed assimilation but obtained the incorrect conclusion. He then reconstructed his knowledge by presupposing any element of natural numbers so as to say that he performed accommodation.

Subject 3

Dita identified the problem that would be proven in almost the same way as what subject 2 did. First, Dita identified the problem by using semigroup proving on real numbers and integers. Dita claimed that $p * q = p + q - pq \in N$ with $p, q \in N$ is a semigroup on binary operations because it fulfilled the closed nature and the associative nature.

<p>▷ Tertutup $p * q \in \text{bil. asli}$ $p * q = p + q - pq \in \text{bil. asli}$</p> <p>▷ Asosiatif $p, q, r \in \text{bil. asli}$ $(p * q) * r = p * (q * r)$ $\underbrace{(p + q - pq)}_p * \underbrace{r}_q = \underbrace{p}_p * \underbrace{(q + r - qr)}_q$ $p + q - pq + r - (p + q - pq)r = p + q + r - qr - p(q + r - qr)$ $p + q - pq + r - pr - qr + pqr = p + q + r - qr - pr - pr + pqr$ $p + q - pq + r - pr - qr + pqr = p + q - pq + r - pr - qr + pqr$ Karena $(N, *)$ memenuhi sifat asosiatif & tertutup maka $(N, *)$ merupakan semigrup</p>	<p>Translation:</p> <ul style="list-style-type: none"> closed $p * q \in \text{natural numbers}$ $p * q = p + q - pq \in \text{natural numbers}$ Associative $p, q, r \in \text{real numbers}$ $(p * q) * r = p * (q * r)$ $(p + q - pq) * r = p * (q + r - qr)$ $p + q - pq + r - (p + q - pq)r$ $= p + q + r - qr$ $- p(q + r - qr)$ $p + q - pq + r - pr - qr + pqr$ $= p + q + r - qr - pq$ $- pr + pqr$
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	$p + q - pq + r - pr - qr + pqr$ $= p + q - pq + r - pr$ $- qr + pqr$ <p>Because the natural number satisfies the associative and closed nature, so that natural number is a semigroup on the binary operations</p>
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Figure 7: The result of subject 3

From Figure 7, it can be seen that Dita did the closed nature proving only by writing $p * q = p + q - pq \in N$. She only paid attention to the shape of the symbol without paying attention to the element of the set of natural numbers, it is to say that the proving was done spontaneously. Then the associative nature proving was done through manipulation of symbols by assuming that on the left-hand side $p + q - pq = p$; $r = q$ and on the right-hand side $p = p$; $q + r - qr = q$. From this assumption, she performed algebraic operations and obtained the result of $(p * q) * r = p * (q * r)$. Her mistakes in deciphering the associative nature proving resulted in errors in her claim. Dita claimed that $p * q = p + q - pq$ is a semigroup on binary operations of natural numbers. Then she reconstructed her knowledge by saying that the proving of semigroup of natural numbers needed not only algebraic symbols but also needed to be proven by using numbers which were elements of natural numbers. She said that it was based on the thinking process so that it was not written on the answer paper. This was revealed in the interview transcript as follows:

- R : From the claim you have obtained, are you sure that $(p * q) * r = p * (q * r)$ included in semigroups on binary operations of natural numbers?
- Di : Actually, I'm not sure about that ma'am ...
Because the proving of the semigroup on original numbers will be more valid if it is to be done by using the algebraic symbols and also the numbers
- R : What do you mean by that?
- Di : Let me explain this ma'am ... suppose I take $p = 10$ and $q = 12$ so we get $p * q = p + q - pq = 10 + 12 - 10.12 = 32 - 120 = -98$

Dita identified the semigroup problem by assimilation based on the known semigroup definition. She said that the semigroup contained a closed and associative nature. She did the proving of closed nature just by looking at $p * q = p + q - pq \in N$. Then she did the proving of the associative nature by performing algebraic operations through symbol manipulation. Dita said that $p * q = p + q - pq \in N$ is associative because $(p * q) * r = p * (q * r)$. Then she claimed that $p * q = p + q - pq \in N$ is a semigroup on binary operations of natural numbers because it satisfied the closed and associative nature. This claim existed because she performed procedural proving without paying attention to the element of natural numbers.

However, Dita conducted a review of her proving that has been done. She conducted accommodation by reconstructing the existing knowledge schemes. She assumed that the natural numbers are $p = 10$ and $q = 12$ which are then substituted into $p * q = p + q - pq$. He obtained -98 as the result of the substitution while -98 is not a natural number. Therefore, she changed her claim by saying that $p * q = p + q - pq \in N$ was not semigroup on binary operation of natural numbers as it doesn't apply the close property.

Based on the exploration process done to all subjects, it is obtained that the construction of students' thinking in completing mathematical proving tests related to abstract algebra can be simplify as shown in Table 4. The following Table 4 will explain the constructing activities conducted by students based on the assimilation and accommodation framework.

Table 4: The thinking process of the subjects.

Subjects	Schemes	Mental Activities	Students' Construction
Dwi	Assimilation	Performed algebraic operations to get $(p * q) * r \neq p * (q * r)$ so that $p * q = p + q - pq$ with binary operations in natural number is not associative but it is closed.	Reviewed the results of the associative nature proving then performed algebraic operations through symbol manipulation. The review was conducted to change the claims that stated $(p * q) * r \neq p * (q * r)$ into $(p * q) * r = p * (q * r)$. Justified that $p * q = p + q - pq$ is semigroup to respect binary operations of natural numbers.
Alex	Assimilation and accommodation	Identified the problem by using semigroup proving of real numbers based on his previously understood scheme. Then did the proving by manipulating the symbols.	Determined mathematical rules in the form of semigroup definitions in the set of real numbers. He then performed the reconstruction by presupposing the element of natural number in the form of $p=4$ and $q=5$. Gave justification by changing the claim which initially was in the form of $p * q = p + q - pq$ is semigroup to respect with binary operations in natural numbers, and then the claim became $p * q = p + q - pq$ with binary operations in natural numbers is not semigroup because it doesn't fulfill the closed nature.
Dita	Assimilation and accommodation	Identified the problem that would be proven by using previous knowledge related to the proving of semigroups of real numbers and integers.	Gave a statement that to prove the semigroup does not only need the mathematical symbols, but also the numbers which are elements of natural numbers. Then substituted the natural numbers element into $p * q = p + q - pq$.

			Changed the claim with a new claim in the form of $p * q = p + q - pq$ with a binary operation in N is not a semigroup because it didn't meet the requirement of closed nature of the natural numbers.
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Students first constructed their knowledge from assimilation and then performed accommodation. The students' thinking construction began by conducting problem identification to solve the mathematical proving problem. The identification was done as a first step in understanding the problem to be proven (Öztürk & Kaplan, 2019). Then separated the object with its context (Sternberg et al., 2008). In this research, the students tried to understand the problem to be proven by identifying the information presented in the problem. The students mentioned that in the proving of semigroup in a non-empty set G with binary operations, the semigroup requirements in the definition need to be understood first.

The semigroup definition includes the closed and associative nature (Hungerford, 2000). The students mentioned that non-empty set of G could be real numbers or integers. Then the students used the scheme of knowledge about real numbers to prove the semigroup of natural numbers. The proving of closed nature happened quickly by looking at the symbols $p, q \in N$ and $p * q = p + q - pq$ without considering about the members of natural numbers set. The proving that has been done through thinking quickly and automatically is called thinking intuitively (Leron & Hazzan, 2009; Leron, 2014). After that, the students performed algebraic operations by manipulating symbols to prove the associative nature. Symbol manipulation is an activity conducted by students to solve mathematical problems related to algebra (Bleiler et al., 2014). The students performed algebraic operations to prove the associative nature of $(p * q) * r = p * (q * r)$. From the results of the closed and associative nature proving, the students gave a claim that $p * q = p + q - pq \in N$ was a semigroup on binary operations of natural numbers. Claims are statements that are often used in solving mathematical proving problems that need to be verified (Panza, 2014).

Furthermore, the students reviewed or re-checked the claims they made (Mason, 2010). The students constructed their knowledge to check the correctness of the claims (Quansah et al., 2018). The students did the thinking construction by re-considering the statements that would be proven by assimilation and accommodation. The students said that the proving of semigroup of natural numbers was not only by using algebraic symbols but also by using numbers that are elements of a set of natural numbers. Then the students did the proving again by using numbers to get new claims: $p * q = p + q - pq \in N$ was not semigroup because it didn't meet the requirement of closed nature on binary operations of natural numbers. The students can make justification from the proving activity twice. Justification shows the confidence level of the students on the conclusions made based on scheme (Mason, 2010).

The result found that intuitive and analytical thinking are not two separate things because students can construct intuitive and analytical thinking processes using assimilation and then accommodation schemes (Rusou et al., 2013; Iannello & Antonietti, 2008). Some of the students solved the problems of semigroup intuitively because they only used the assimilation scheme to construct their existing knowledge. Whereas the other students who were able to solve the problem intuitively and analytically because they did the process of assimilation and accommodation. At the students accepted the problems, they intuitively solved it based on their existing knowledge, even though the context of the problem was different. Therefore, the students' thinking process in constructing their knowledge to complete the proving of abstract algebra can be described as follows (see Table 5):

Table 5: students' thinking process based on assimilation and accommodation schemes

Steps	Activity
Identifying	Mentioning information in the question or problem
Determining rules	Using definition concept.
Proving with symbol manipulation	Performing algebraic operations to prove
Reviewing	Re-checking the claim that has been made. If they are not sure yet, then the proving activity needs to be done again
Justifying	Make a conclusive conclusion based on the result of the review

Students are able to combine intuitive and analytical thinking to make reasoning in solving mathematical problems (Macchi & Bagassi, 2012). Intuitive and analytical thinking are two different things (Rusou et al., 2013). Intuitive thinking is a model of thinking that occurs quickly, spontaneously, automatically (Leron, 2014). Meanwhile, analytical thinking is a model of thinking which is conducted through a slow process related to mathematical rules. Analytical thinking is related to situations, practices, statements, ideas, theories, and arguments (Thaneerananon et al., 2016). The process of analytical thinking starts from observation, determining the supporting rules, and checking or rejecting intuitive responses (Sternberg et al., 2008). The supporting rules act as a guarantor for the students in giving reason for each step of the mathematical proof (Faizah et al., 2020a).

CONCLUSION

Based on the result of this research, it can be concluded that accommodation happens when students re-construct their knowledge based on the assimilation scheme through 5 steps of thinking process. The five steps are the identification; determining the mathematical rules to be used; carrying out the mathematical proving by means of symbol manipulation, review, and justification.

Therefore, the finding of this research can be used as a tool to develop students' knowledge in solving the mathematical proving problems through assimilation scheme and accommodation scheme to ensure that the proving is not conducted spontaneously. Students should understand the

meaning of each symbol presents in the question to avoid misconception to the result of the proving that have been performed.

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