Investigation of Students' Algebraic Conceptual Understanding and the Ability to Solve PISA-Like Mathematics Problems in a Modeling Task

Sri Imelda Edo, Wahyuni Fanggi Tasik

Polytechnic of Agriculture Kupang, Prof. Herman Yohannes St. Lasiana, Kupang, Indonesia

sriimeldaedo@gmail.com; wahyunifanggitasik@gmail.com

Abstract: *Several studies related to mathematics understanding found that many undergraduate students lack some basic knowledge of algebra. They memorized only a few topics, formulas, and algorithms without understanding them conceptually, even though they could manipulate those limited number of points correctly or incorrectly. In comparison, most high-achieving students have incomplete solutions in Modeling Mathematics PISA-like tasks in levels 5 and 6, related to the content of change and relationship. In contrast, students with moderate achievement can solve the problem using instinct, trial and error, and logic. Therefore, this study aims to analyze students' algebraic conceptual understanding related to their modeling competence in solving a mathematical problem that is an adapted PISA task, using qualitative research as an appropriate method. It emphasizes a holistic description of the phenomena studied concerning how students work with algebraic conceptual problems and Mathematics task-like PISA problems. This study involved 244 new vocational college students in 5 study programs. Data collection used in this research is students' worksheets, video recording, and interviewing some students to obtain more profound information about their thinking processes. Furthermore, the data was analyzed by holistic description. Interpretation and conclusion using the definition of equation and algebra expression and indicator of modeling competence, mathematical literacy refers to the proficiency level of PISA question given as a guideline to interpret and make a conclusion. Some discovered strategies for solving the problem and implications regarding their mathematical literacy skills related to these tasks are discussed.*

INTRODUCTION

Algebra is a vital field of learning that plays a significant role in mathematical thinking and language expressed with symbols, tables, words, and graphs (Stacey & MacGregor, 1999). An understanding of algebra can help students to recognize the importance of mathematics. They should comprehend symbols and their manipulations to interpret the letters employed in various algebraic situations, the structural aspects, and the solution (Kieran, 2007; Sukirwan et al., 2018). Algebra is regarded as a gatekeeper course since students are expected to pass before moving to the next level. Wu (2001) argued that qualified teachers are needed to teach algebra successfully. Additionally, Gram and Jacobson (2000) stated that high school mathematics is widely regarded as the “gatekeeper” to teaching algebra, a subject that students in US public schools mostly fail.

Algebra is now a required part of most curricula, including vocational college. Therefore, all the college students included in the agriculture field need an understanding of the concepts and skills in using algebraic operations. Mathematics is not the primary major in the vocational curriculum, but it is one of the important subjects that students in agriculture majors are expected to master. Furthermore, agriculture is related to measurements, estimates, and projections involving algebraic operations, such as modeling growth, food supply models, and fisheries market trends.

The demand for algebra at more levels of education is increasing. Wiki Answers, one of the world’s most important questions and answers websites, outlines some current algebra uses (Gunawardena, 2011). For example, companies use algebra to determine their annual budget, including their annual expenditure. Additionally, various stores use algebra to predict the demand for a particular product and subsequently place their orders. It has individual applications in calculating annual taxable income, bank interest, and installment loans.

Several types of research have proposed that student misconceptions or gaps in conceptual knowledge of Algebra lead to incorrect and clumsy procedures for solving problems (Booth & Koedinger, 2008; Jacobson, 1981; Jupri & Drijvers, 2016; Nathan, 2000; Van Lehn & Johnes, 1993). However, in the domain of algebraic problem solving, one type of prior knowledge that is key to learning is a conceptual understanding of features in the problem, including equals sign, variables, like terms, and negative signs (Jupri & Drijvers, 2016). Conceptual knowledge of these features enables the user to recognize the symbols or perform an operation, as well as comprehend the purpose of the equation and the effect of relocating the feature on the overall problem (Nathan, 2000; Van Lehn & Johnes, 1993). Students should have a firm grasp of the problem’s fundamental elements to comprehend the instructional information completely (Booth & Koedinger, 2008; Jacobson, 1981). They are unlikely to demonstrate significant advances in procedural knowledge without this in-depth, relevant knowledge of problem characteristics. Deep strategy construction relies on the inclusion of sufficient information about the problematic aspects that make them appropriate or inappropriate. Therefore, having a high conceptual understanding may be required to solve equations appropriately. However, for students with an insufficient conceptual understanding of the problem’s characteristics, superficial methods such as the one outlined above are likely to dominate (Booth & Koedinger, 2008).

Incorrect procedures are typical when learning Algebra (Sebrechts et al., 1996), which inhibits accurate solutions. Moreover, many university students in the US also lack some basic understanding of algebra (Booth & Koedinger, 2008; Gunawardena, 2011; Jacobson, 1981). As a result, they commit the same mistakes as their secondary school counterparts. They memorized only a few facts, formulas, and algorithms without understanding the concept. However (Rittle-

Johnson & Star, 2014; Star & Seifert, 2020) argued that a critical learning outcome in problem-solving domains is the development of flexible knowledge, where multiple strategies are learned and applied adaptively to a range of situations. Booth and Koedinger (2008) stated that in algebraic problem solving, one type of prior knowledge is the conceptual understanding of features in the problem, including equal signs, variables, terms, and negative signs. For example, understanding the equals sign has previously been crucial for algebraic problem solutions (Knuth et al., 2006).

PISA is the acronym for the 'Programme for International Student Assessment,' the OECD's international program assessing reading, scientific and mathematical literacy (www.oecd.org/pisa). Mathematical literacy is an individual's capacity to formulate, employ, and interpret mathematics in various contexts. It includes mathematical reasoning and using the concepts, procedures, facts, and tools to describe, explain and predict phenomena. It enables individuals to grasp the significance of mathematics in the world and make the sound judgments and choices required of constructive, engaged, and thoughtful citizens (Framework PISA, 2012). This conception supports the importance of students developing a solid understanding of pure mathematics and the benefits of exploring the abstract world. The construct of mathematical literacy, as defined for PISA, emphasizes the need to develop within students the capacity to use the context, and they are expected to have rich experiences within the classrooms to accomplish this.

Mathematical literacy refers to an individual's capacity to *formulate*, *employ*, and *interpret real-world* problems. Stacey (2011) stated the close relationship of this concept to mathematical modeling. However, Edo et al. (2013) found that most high-achieving students have incomplete solutions in Modeling Mathematics Tasks like PISA levels 5 and 6, related to change and relationship content which refer to algebra framework. They cannot solve the non-routine problem correctly since problems cannot be formulated mathematically. In contrast, students with moderate achievement can solve the problem using their "instinct," trial and error," and "logic. Gunawardena (2011) argued that college students pursued similar difficulty in word problems. This is because the frequency of occurrence is very high in all problems, and they should interpret and convert everyday language into algebra. Therefore, this study aimed to examine students' conceptual understanding of solving routine algebraic problems and their ability to solve non-routine problems adapted from PISA tasks.

LITERATURE REVIEW

An equation is a phrase that expresses the equality of two algebraic expressions. For example, in x + 3 = 9, x + 3 is the left side, or left member, and 9 is the right or right member. An Equation may be a true, false or an open sentence such as 2+ 3= 5, 7- 2 = 4, or x + 5 = 9. The number that can replace the variable in an open sentence to make it true is called a root or a solution of the equation (Usiskin, 1999).

Variables have many possible definitions, referents, and symbols (Usiskin, 1999). The first conception considers algebra as generalized arithmetic, and in this conception, a variable is considered as a pattern of generalizing. The second conception suggests that it is a study of procedures for solving certain kinds of problems, and in this conception, a generalization was obtained for a particular question before solving the unknown. Therefore, variables are either unknowns or constants. In the third conception, algebra is the study of relationships among quantities, and these variables tend to vary. The fourth conception accepts algebra as the study of structures, where the variable is little more than an arbitrary symbol. The variable will become an arbitrary object in a structure related to certain properties $2x^2 + ax + 12a^2$. The conception of a variable represented is not the same as any previously discussed notions, and it does not act as an unknown or argument.

The framework of PISA 2012 explains, formulates, employs, and interprets processes related to Mathematical capabilities. For example, formulating situation activities identify the underlying variables and structures in the real-world problem and makes assumptions. Employing mathematical concepts, facts, methods, and reasoning entail tasks such as conceptualizing the problem or interpreting the solution within the original context. Meanwhile, interpreting, applying, and evaluating outcomes include activities to understand the extent and limits of a solution that results from the model employed.

METHOD

This research used the qualitative method to investigate the quality of relationships, activities, situations, or materials. It places a premium on holistic description, that is, on detailing everything that occurs during a particular activity or scenario, rather than comparing the effect of a particular treatment or describing people's attitudes or behaviors.

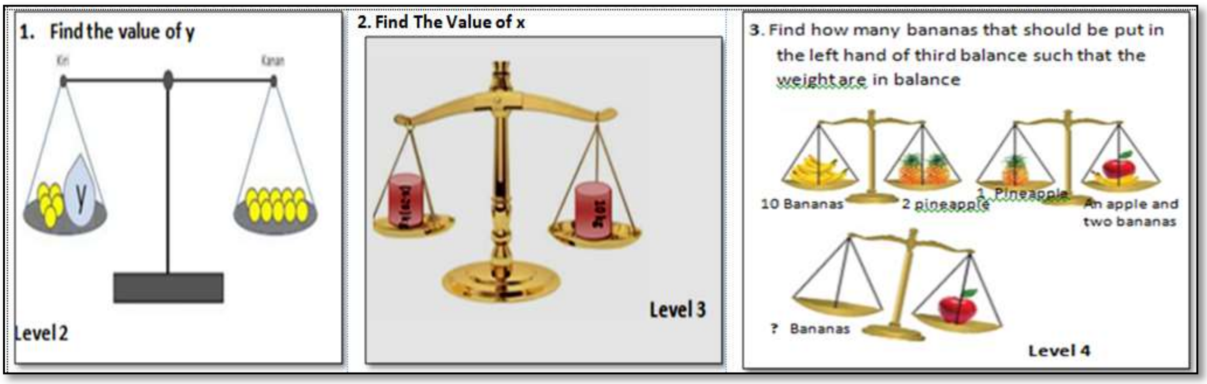
The five steps in qualitative research used according to Fraenkel, Wallen, and Hyun (2014). Identification of the phenomenon to be studied: The quality relationships between students' conceptual understanding about equation and algebra expression and the ability to solve PISA-like mathematics problems in the model task. Therefore, this step involves developing and identifying a valid and reliable mathematical assignment for analyzing the relationship's quality. Identification of the participants in the study: The participants were 244 first-year students from five study programs of a public vocational college, East Nusa Tenggara, Indonesia. Generation of hypotheses: Students with a good conceptual understanding of equation and algebra expression can solve PISA-like mathematics problems related to the model task. Data collection: Using students' worksheets, video recording and interviewing some students to obtain deeper information of their thinking process; data analyzed by holistic descriptive. Interpretation and conclusion: Using the definition of equation and algebra expression and indicator of modeling competence, mathematical literacy refer to the proficiency level of PISA question given as a guideline to interpret and make a conclusion.

Students' algebraic conceptual understanding was investigated through their ability to solve three problems from junior high school textbooks containing the concept of the equation, variable, equal sign, and operation sign. Meanwhile, students' ability to Solve PISA-Like Mathematics Problems in Modeling Task were analyzed based on the sixth level of modeling Proficiency by (1) Applying given models, (2) recognizing, applying and interpreting basic given models, (3) using a different representational model, (4) Working with explicit models and related constraints with assumptions (5) developing and working with complex models that reflect on modeling processes and outcomes, (6) Conceptualizing and working with models of complex mathematical process and relationships, generalizing and explaining modeling outcomes. Mathematics tasks to investigate students' ability were taken from Pisa-like mathematics task content change and relationship in levels 2, 5, and 6.

RESULTS

The validity, reliability of instruments, and difficulty level were analyzed using the quantitative method. The reliability test showed Cronbach's alpha 0.596, and they were reliable. Furthermore, the difficulty test showed that questions number 4 were easy, while 1,3 and 2 were moderate, and 5 and 6 were difficult.

The first type of problem consists of three mathematics tasks from junior and senior high school textbooks, as shown in Figure 1.



Source: https://www.berpendidikan.com/2016/03/pengertian-dan-contoh-soal-persamaan-linear-satu-variabel-plsv.html	Source: Dris and Tasari (2010)	Source: As'ari et al. (2013)

Figure 1. Linear equation problems to investigate students' conceptual understanding

The student came up with an unexpected outcome when answering question 1, and only 27.66% obtained the correct answers. Meanwhile, 10.64% of students could not construct a mathematical model and gave the wrong answer by guessing. Subsequently, 61.70% made an error in the construct because of misconception or misreading and misinterpreted contextual language in mathematics.

Students' first misconception was assigned labels and arbitrary values, and the variables were misinterpreted as a "label" and as a "thing," as shown in Figure 2.

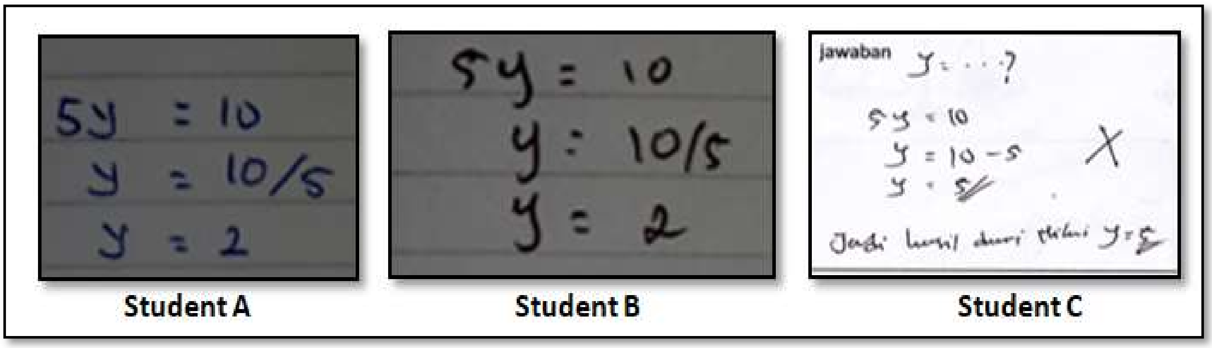


Figure 2. Students' answers in a one-to-one interview for question 1

In this context, students interpreted y as the label of things on the left side of the balance. For example, five oranges and an unknown substance are contained in the sack on the left side of the balance, which is balanced by the weight of ten. Some students translate the context to mathematics expression as 5y = 10. Students A and B have similar answers, but they gave different conceptual understandings of the variable. The answers provided were recorded in the transcript as follow.

Students A.

R: In your opinion, is this an easy, moderate, or difficult problem?

S: I felt that it is an easy question.

R: Please, can you elaborate on your response?

S: There is a balance, and the things on the left side are five oranges with a sack that contain unknown things. As y was used to signify the unknown, the mathematics equation for the items on the left side of balance was 5y.

R: Are you confident, 5y? Would you mind checking your response against the context? Take care to consider the context.

S: While looking back to the context, there are five oranges and a sack; when the sack is denoted as y, the expression should be 5y.

R: Ok. Let me know the meaning of 5y in your opinion.

S: Five oranges with unknown variables.

R: What is the answer to 3x4?

S: 3x4 = 12

R: Was 3 x 4 calculated?

S: (did not respond)

R: Do you know that the concept of multiple comes from addition?

S: Yes, I know

R: how do you express 4+4+4 in multiple forms

S: 3 x 4 or 4 x 3

R: 3 x 4 have a different meaning than 4 x 3; which one is chosen?

S: (Students looked confused)

R: You add 4 three times. How is it converted to multiple concepts?

S: Four appeared three times, and the answer is 3 x 4.

R: What about 5y?

S: five and y

R: Is 5y = 5 (y) and 5 x y

S: All the variables that refer to the symbol of 5 times y are remembered.

R: The prior example 3 x 4= 4+4+4, and 5y = ………

S: y +y +y +y +y

R: How many y is on the left side of the balance?

S: I did not know because the sack was closed.

R: In your opinion, y as a variable refers to an unknown thing. In this context, y refers to…

S: Oooo…. orange (not sure) … a sack, maybe orange

R: Orange, sack, or others? Please make sure your answer is correct

S: sack because letter y is on a sack.

R: thank you for your time, and this will be discussed later when the algebra topics are considered.

Student B

R: In your opinion, is this an easy, moderate, or difficult Problem?

S: It is one of the easy questions.

R: Can you please explain your answer, especially for the equation 5y = 10?

S: The balance context showed that the variables on both sides are the same since an equal sign was used. The left side of the balance has five oranges, and a sack contains an unknown number of oranges. Let orange be denoted as y; then five oranges should be 5y. Therefore, the equation; 5y=10 is formed. The fixed value of y is 2 or y =2

R: Orange was denoted as y, then five oranges should be 5y" do you mean that y refers to orange?

S: yes, of course, y refers to orange for the question asked for the value of y.

R: what is the meaning of y =2? Can you interpret the result?

S: y =2 means 2 oranges in the sack.

R: Does it mean the total number of oranges on the left and right sides of the balance are 7 and 10?

S: No since the oranges in both sides have to be the same.

R: please look back to the balance! There are 10 oranges on the right side of the balance. The left side has five oranges and a sack containing an unknown number of oranges. How many oranges are in the sack equal 10 on the left side?

S: It is easy, add 5 oranges to make the numbers on the left side of the balance as many as on the right side.

R: What is the value of y?

S: y is equal to five

R: How is the result interpreted?

S: The numbers of oranges in the sack are five.

R: y refers to orange, the number of oranges, and the number of oranges in the sack.

S: y refers to the number of oranges in the sack.

R: This will be discussed during the algebra topic.

Students seem to lack understanding of the variable's concept and operation sign from the transcription. Instead, they bring prior knowledge and solve equations using routine algorithms without understanding. Meanwhile, there is no clear understanding of the function of the feature in the equation and how changing the location can affect the overall problem.

The second type of misconception was miscellaneous forms of an incorrect answer. Figure 3 illustrates students' responses to this type of inaccuracy.

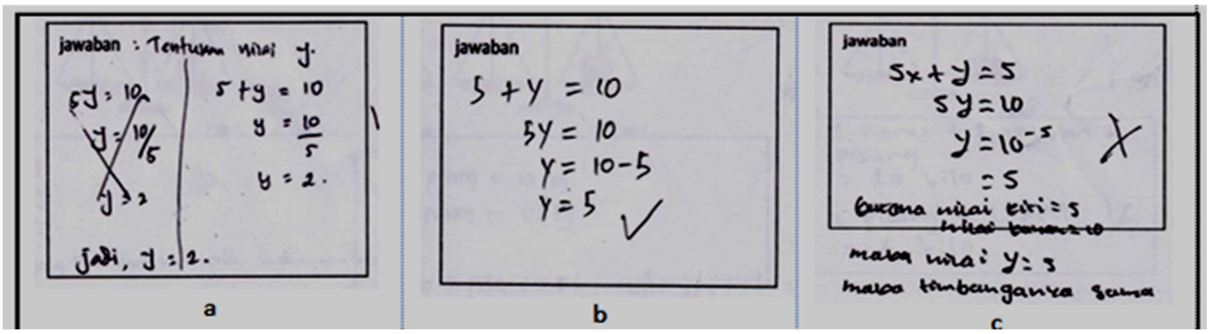


Figure 3. Procedural errors students made for question1

There were different answers for the same question, significantly simplifying algebraic expressions, where incorrect rules were applied. Figures 3a and 3b showed that the problems were correctly formulated. However, they performed the wrong operation to simplify the equation. The student in Figure 3a separated the variable on the left side of the equal sign to move the constant to the right side. Then, it was divided on the right side of the equal sign to the constant on the left side. The algorithm's final steps are always reached by dividing the number on the right side by the left side. In contrast, Figure 3b does not have enough knowledge to operate algebra expressions and simplify the equation; hence, a double error was committed without understanding. The value of y was not substituted to the original equation to evaluate the equality of the expressions. Figures 3c showed that students found difficulties formulating the problem in the mathematics form. They had known that they should add five oranges on the left side of the balance to make it balance. However, they did not know how to communicate their thinking mathematically. Memorization algorithms or procedures were conducted to simplify algebra expressions to find the unknown value. They do not entirely understand the concept of equality and equation.

Students felt that question 2 was more accessible than 1, but some made the same error. For example, 65.95% of solved question 2 correctly, 21.28% cannot construct a mathematical model and gave the wrong answer by guessing, and 12.77% gave miscellaneous forms of incorrect answers. Students with problems performing algebra operations consistently made the same type of errors. For question 2, various incorrect solutions were provided, as shown in Figure 4.

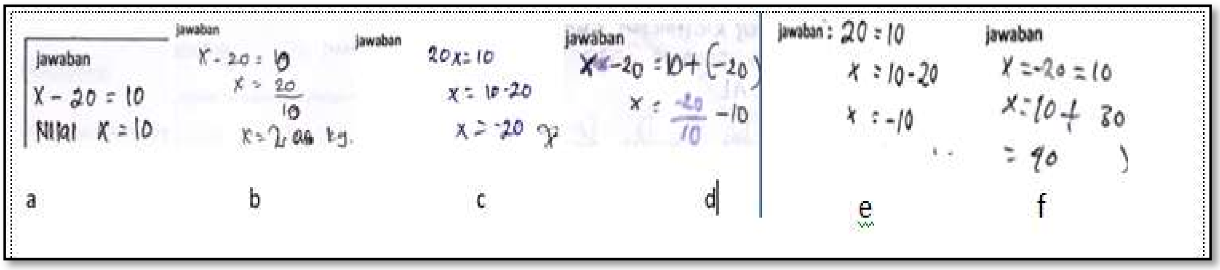


Figure 4. Students' miscellaneous forms of incorrect answers students made to question 2

Students have the same errors and misconceptions in solving questions 1 and 2. This fact is relevant to (Wu's 2001) statement that algebra is regarded as a gatekeeper course. Those who successfully pass through will be promoted to the next level. (Jacobson, 1981) also stated that High school mathematics is widely regarded as the" gatekeeper" to college. Students' answers in Figures 4e and 4f showed that they did not understand the concept of the equal sign.

In contrast, 91 students, or 39%, can correctly model problems in question 3. It was argued that this problem is like a two-linear equation system unless presented in the balancing context. Almost every student who fails to solve question 1 successfully solves question 3.

The second type of problem was PISA-Like Mathematical tasks taken from the contest of mathematics literacy questions in 2011 published by the Journal on Mathematics Education. There were 3 PISA-Like Mathematical tasks, to examine students' modeling proficiency, as shown in Figures 5 and 7

The growth of coral reef length can be predicted used the following function

$$y = 0,75\,x - 0,5.$$

y: the length of coral reef (mm)
x: the height of coral reef (mm)

Question 4

Find the length of coral reef if it has 40 mm in height.

Source: Contest Literacy of mathematics 2011 (Translate in English)

Figure 5. Question 4 (mathematics task like PISA level 2)

Question 4 was easy, and 72.34% solved this problem correctly. However, several students were put equal signs improperly. An example of errors and misconceptions in solving question 4 is shown in Figure 6.

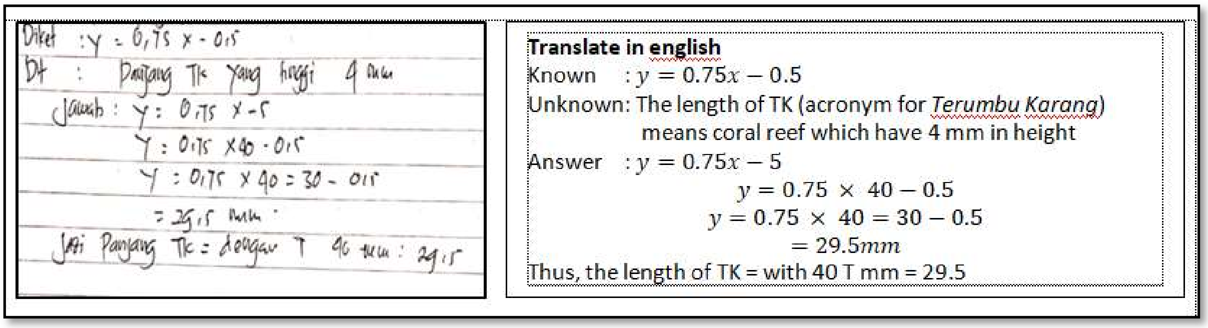
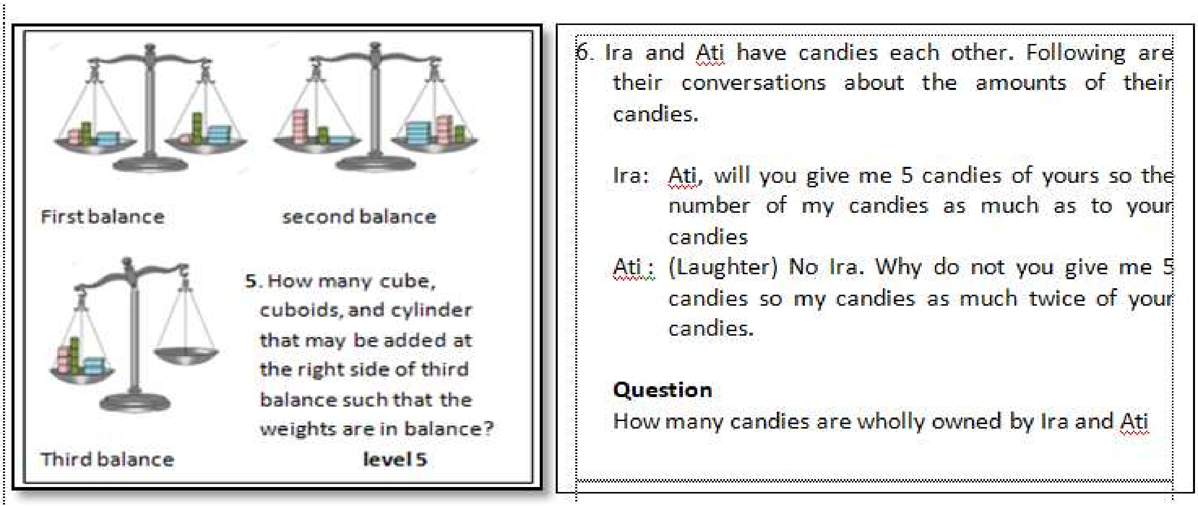


Figure 6. Students lack understanding of the equal sign concept to question number 2

The student's answer showed in Figure 6, "…, y = 0,75 x 40 = 30 − 0.5, … ", where x was substituted with 40, and the result of 0.5 was subtracted from 0.75 (40). The lack of equal sign and operation sign concepts was inferred from these answers, and an equal sign was used to separate some calculation steps. In addition, the students did not understand the expressions $0{,}75 \times 40 \neq 30 - 0{,}5$.

The second and third mathematics tasks like PISA were in questions 5 and 6, as shown in Figure 7.



Source: Edo, Putri, and Hartono (2013)

Figure 7. Questions 5 and 6 (mathematics tasks like PISA levels 5 and 6)

In addition, 46.81% solved the task correctly, 14.89% did not answer the question, and 21.28% could not construct the mathematics model since wrong answers were provided by guessing.

However, 17.02% construct mathematics models correctly but cannot continue finding the value of each variable to answer the question, as seen in Figure 8.

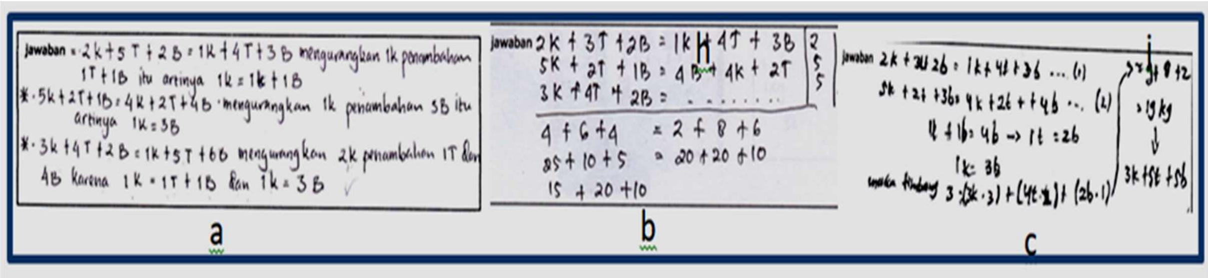


Figure 8. Students' incomplete answers for question number 5

The answers in Figure 8a showed that students can model the problem correctly and simplify it to the simplest expression. However, the value of *B* was not substituted to find *k* and *t*. The students can formulate the problem mathematically but fail to simplify the unstructured linear equations system. The interview section reported that the linear equation has three variables and two equations. Therefore, the equation system cannot be solved simultaneously to obtain the value of each variable. In performing the algorithm, different challenges were encountered. The last response showed that the students gave the correct answer for the weight of the things on the left side of the balance, but an incorrect final answer was provided.

In contrast, students who failed to solve question number 2, displayed in Figure 2a, can solve this Mathematics task like PISA level 5, as shown in Figure 9.

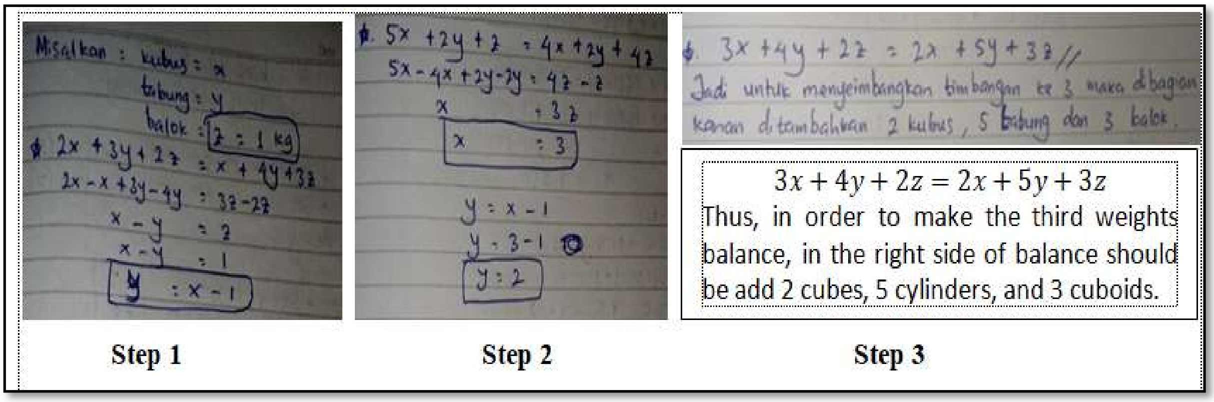


Figure 9. The correct answer for question 1

The real-world contexts are translated to mathematics language by denoting *kubus* (cube) as (x), *tabung* (cylinders) as y, and *balok* (cuboids) as z. Subsequently, each mathematics model was simplified, and the value of z was substituted with the simplest equation to obtain x and y.

Additionally, no students can correctly solve problems in level 6 since 63.83% did not answer, 21.28% guessed, and 14.89% made errors in modeling the problem. Students failed to transfer everyday language to mathematics because they misread and misinterpreted the problems, as shown in Figure 10.

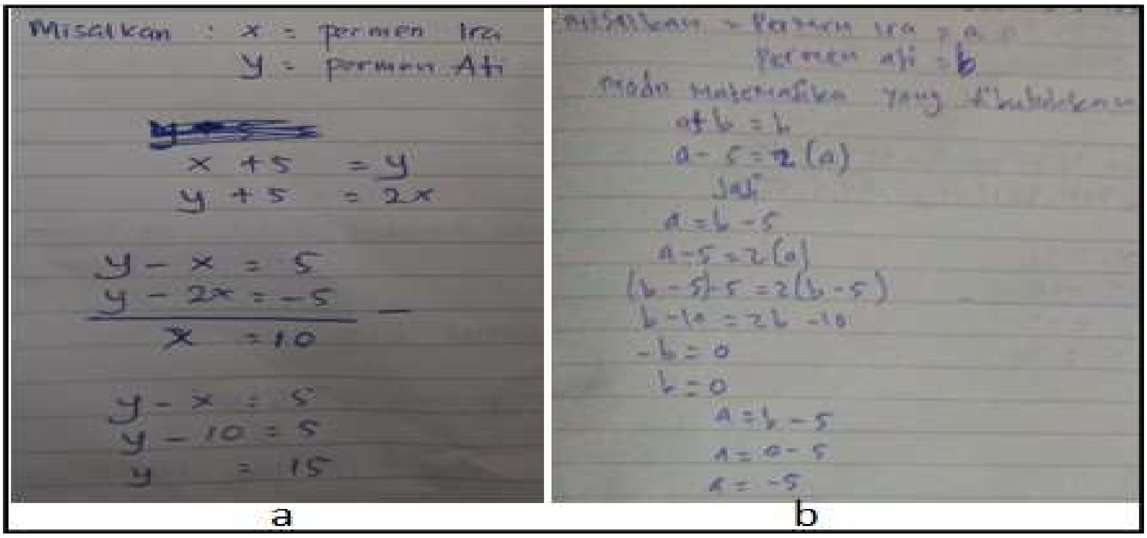


Figure10. Students misreading or misinterpreting problem number 6

This study showed that students with a weak understanding of algebra concepts found difficulty solving routine mathematics tasks like PISA. However, some modeled a mathematics task like PISA and failed to solve the equation due to a lack of procedural knowledge and skill in manipulating and simplifying algebra expressions. As a result, they made some errors and misconceptions.

DISCUSSION

Students' first type of error was assigned labels, arbitrary values, or verbs for variables and constants. This error contains several subcategories, and students tend to misinterpret a variable as a "label" and as a "thing" rather than a number. Misinterpreting letters as labels is a fundamental misconception that will lead to many other errors in algebra, and college students pursued similar interpretations of variables (Gunawardena, 2011; Widodo et al., 2018). Additionally, different interpretations of letters in different contexts may cause students to mix up and misinterpret the use of variables. Capraro and Joffrion (2006) stated that the variable is liable to change, especially suddenly and unpredictably. However, restricted solution sets are always provided when students encounter variables in algebraic situations. They are introduced as specific references to the value of a particular variable name. According to mathematical literacy, as defined for PISA, students can formulate situations because of misunderstanding the variable concept. It means that algebraic conceptual understanding supports the individual capacity to formulate situations mathematically.

Furthermore, students' second type of misconception applied many illegal procedures in manipulating algebraic expression and equations, as seen in Figures 3a, 3b, 3c, 4a, 4b, 4c, 4d and 4f. Students should have a firm grasp of algebra's structure and characteristics to comprehend algebraic expressions. This is consistent with Van Lehn and Jone's (1993) study, where student misconceptions or gaps in conceptual knowledge of Algebra lead to incorrect and buggy procedures for solving problems. Capraro and Joffrion (2006) also stated that the procedural approach of translating from mathematical words to symbolic representations did not help students succeed on the items that required skills. Therefore, teachers should prepare students not to carry out algebraic procedures but to solve problems and represent situations. Procedures are almost meaningless without conceptual understanding. Connections make mathematics meaningful, memorable, and powerful. In conclusion, students fail to employ mathematical concepts, facts, procedures, and reasoning because they tend to manipulate algebraic expressions by memorizing given algorithms without deep conceptual understanding.

The third type of error was a misunderstanding of the algebra expression concept. Students' answers in Figures 4e and 4f showed that they struggled with algebra expression and equal sign concepts. This fact is in line with the Knuth et al. (2018) study, where one of the most common misconceptions in understanding equations is the significance of the equal sign (=). Students forget that the equal sign means "operations equal answer" and are usually presented with the material in an "operations on the left-hand side of the equal sign" manner. As a result, they did not solve the problem by understanding the concept of the equation but used wrong rules that were persistently fixed in their minds. Another misconception was that they put an equal sign to separate some calculation steps. They were using the equal sign as a step marker and also violated the equivalence property by equalizing statements that were not equal to each other, as displayed in Figure 6a. Knuth et al. (2006) stated that students often think of the equals sign as an indicator of the result of operations being performed or the answer to the problem rather than the equivalence of two phrases. It means that students fail to employ mathematical concepts, facts, procedures, and reasoning because they lack an understanding of equal sign concepts.

The fourth type of error and misconception was manipulating and simplifying non-routine algebra expressions as shown in Figure 8.Booth and Koedinger (2008) said that using these incorrect strategies may persist because many of the procedure's students attempt to use will lead to a successful solution. Unfortunately, without adequate knowledge of the problem features, students cannot distinguish between the situations in which the strategy will work and the ones where it is not applicable. It means that some students can formulate the problem correctly, but they fail to employ the process because they cannot simplify unstructured linear equation system.

The fifth type of error was translating a real-world problem from natural to algebraic languages, as displayed in Figures 3a, 3b, 3c, 4a, 4b, 4c,10a, and 10b. Furthermore, the error was caused by misreading and misinterpreting problems. The frequency of occurrence is very high in all types of problems because they must interpret and convert everyday language into algebra. This is in line with Gunawardena (2011), where students need to do more than the other three conceptual areas of variables, expressions, and equations. This is because a word problem may contain concepts related to one or more of the above three areas.

CONCLUSION

This study discovered that students encountered difficulties solving high-level mathematics tasks such as PISA, committed errors, and formed misconceptions due to limited comprehension of algebra's structural properties. Students that lack a conceptual grasp of algebra, such as expressions, equal sign concepts, and operation sign concepts, frequently answer mathematics problems using memorized procedures. They used many illegal procedures and made errors in manipulating algebra for easy or complex problems. Therefore, the struggle to solve PISA-Like mathematics problems for levels 5 and 6 was high. Teachers are recommended to facilitate the teaching and learning process with activities that can explore students' basic skills and encourage them to construct their understanding of algebra concepts before solving complex problems. According to Usiskin (1999), students should be introduced to the fundamentals of algebra and develop the context, not as meaningless symbols. Furthermore, they should involve all other mathematics as motivation for solving the algebra and as avenues for application. The most complex ideas should be broken down into subtopics instead of learning in one year. Some students with the capacity to formulate complicated mathematical problems cannot solve non-routine linear equations. Furthermore, those that can correctly solve PISA-Like mathematics problems for level 5 failed to solve conceptual questions because of the misconception of variable

ACKNOWLEDGMENTS

The authors are grateful to the Polytechnic of Agriculture Kupang (Politeknik Pertanian Negeri Kupang) for funding and providing the opportunity and facility to conduct this study.

REFERENCES

[1] As'ari, A. R., et al. (2013). *BukuMatematika SMP Kelas 8 Semester 2 Kurikulum 2013*. Jakarta: Pusat Kurikulum dan PerbukuanKemendikbud.

[2] Booth, J. L., & Koedinger, K. R. (2008). Key misconceptions in algebraic problem solving.*Proceedings of the Annual Meeting of the Cognitive Science Society*, *30*(30), 571-576. California: the University of California. https://escholarship.org/uc/item/5n28t12n

[3] Capraro, M. M., &Joffrion, H. (2006). Algebraic equations: Can middle-school students meaningfully translate from words to mathematical symbols? *Reading Psychology*, *27*(2–3), 147–164. https://doi.org/10.1080/02702710600642467

[4] De Lange, J. (2006). *Mathematical literacy for living from OECD-PISA perspective. Tsukuba Journal of Educational Study in Mathematics,* *25*, 13-35. http://www.human.tsukuba.ac.jp/~mathedu/2503

[5] Dris, J., &Tasari, J. (2010). *MatematikaJilid 1 untuk SMP dan MTs Kelas VII*. Jakarta: Pusat Kurikulum dan PerbukuanKemendikbud.

[6] Edo, S. I., Hartono, Y., & Putri, R. I. I. (2013). Investigating secondary school students' difficulties in modeling problems PISA-model level 5 and 6. *Journal on Mathematics Education*, *4*(1), 41–58. https://doi.org/10.22342/jme.4.1.561.41-58

[7] Fraenkel, J. R., Wallen, N. E., & Hyun, H. H. (2014). *How to Design and Evaluate Research in Education*. New York: McGraw-Hill Education

[8] Gunawardena, E. (2011). Secondary school students‟ misconceptions in algebra. *Doctoral Thesis*. Ontario Institute for Studies in Education, University of Toronto.

[9] Jacobson, A. S., & Ackerman, P. D. (1981). Cost Reduction Projections for Active Solar Systems. *Proceedings of the Annual Meeting - American Section of the International Solar Energy Society*, *4*(2), 1291–1295.

[10] Jacobson, K. G. (1999). Central tensions: A critical framework for examining high school mathematics and mathematics education. *Doctoral Dissertation*. The Claremont Graduate University and San Diego State University.

[11] Jupri, A., &Drijvers, P. (2016). Student difficulties in mathematizing word problems in Algebra. *Eurasia Journal of Mathematics, Science and Technology Education*, *12*(9), 2481-2502. https://doi.org/10.12973/eurasia.2016.1299a

[12] Kieran, C. (2007). Developing algebraic reasoning: The role of sequenced tasks and teacher questions from the primary to the early secondary school levels. *Quadrante*, *16*(1), 5–26. https://doi.org/10.48489/quadrante.22814

[13] Knuth, E. J., Stephens, A. C., McNeil, N. M., &Alibali, M. W. (2006). Does understanding the equal sign matter? Evidence from solving equations. *Journal for Research in Mathematics Education*, *37*(4), 297–312. https://doi.org/10.2307/30034852

[14] McNeil, N. M., Weinberg, A., Hattikudur, S., Stephens, A. C., Asquith, P., Knuth, E. J., &Alibali, M. W. (2010). A is for apple: Mnemonic symbols hinder the interpretation of algebraic expressions. *Journal of Educational Psychology*, *102*(3), 625-634. https://doi.org/10.1037/a0019105

[15] Nathan, M. J., & Koedinger, K. R. (2000). Teachers' and researchers' beliefs about the development of algebraic reasoning. *Journal for Research in Mathematics Education, 31*(2), 168-190. https://doi.org/10.2307/749750

[16] Rittle-Johnson, B., & Star, J. R. (2007). Does comparing solution methods facilitate conceptual and procedural knowledge? An experimental study on learning to solve equations. *Journal of Educational Psychology, 99*(3), 561-574. https://doi.org/10.1037/0022-0663.99.3.561

[17] Sebrechts, M., Enright, M., Bennett, R. E., & Martin, K. (1996). Using algebra word problems to assess quantitative ability: Attributes, strategies, and errors. *Cognition and Instruction, 14*(3), 285-343. https://doi.org/10.1207/s1532690xci1403_2

[18] Stacey, K. (2011). The PISA View of Mathematical Literacy in Indonesia. *Journal on Mathematics Education, 2*(2), 95–126. https://doi.org/10.22342/jme.2.2.746.95-126

[19] Stacey, K., & MacGregor, M. (1999). Implications for mathematics education policy of research on algebra learning. *Australian Journal of Education*, *43*(1), 58–71. https://doi.org/10.1177/000494419904300105

[20] Star, J. R., & Seifert, C. (2006). The development of flexibility in equation solving. *Contemporary Educational Psychology, 31*(3), 280-300. https://doi.org/10.1016/j.cedpsych.2005.08.001

[21] Sukirwan, Darhim, Herman, T., &Prahmana, R. C. I. (2018). The students' mathematical argumentation in geometry. *Journal of Physics: Conference Series, 943*(1), 012026. https://dx.doi.org/10.1088/1742-6596/943/1/012026

[22] Van Lehn, K., &Johnes, M. R. (1993). What mediates the self-explanation effect? Knowledge gaps, schemas or analogies? *Proceedings of the Fifteenth Annual Conference of the Cognitive Science Society*, pp. 1034–1039. http://www.public.asu.edu/~kvanlehn/distrib/CSC93-abstract.html%5Cnhttp://alumni.media.mit.edu/~bsmith/courses/mas964/readings/CSC93.pdf

[23] Widodo, S. A., Prahmana, R. C. I., Purnami, A. S., &Turmudi. (2018). Teaching Materials of Algebraic Equation. *Journal of Physics: Conference Series, 943*(1), 012017. https://dx.doi.org/10.1088/1742-6596/943/1/012017

[24] Wu, H. (2001). How to prepare students for algebra. *American Educator, 25*(2), 10-17. https://math.berkeley.edu/~wu/wu2001.pdf