Metacognitive Intervention: Can It Solve Suspension of Sense-Making in Integration Problem-Solving?

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Abstract: This study aims to describe metacognitive interventions for students who experience suspension of sense-making when solving integration problems based on the characteristics of students' thinking. This research applied a qualitative research approach and provided essay tests followed by task-based interviews to classify students who experience suspension of sense-making into two categories based on their thinking characteristics namely pure procedural and mixed conceptual-procedural. The metacognitive intervention in this study was carried out by giving metacognitive questions to the participants based on their thinking characteristics. The metacognitive questions given included comprehension, connection, strategy, and reflection questions. The results of this study indicated that mixed conceptual-procedural students were able to raise their sense-making earlier than pure procedural students. It can be concluded that pure procedural students need metacognitive intervention in the form of complete metacognitive intervention while mixed conceptual-procedural students only need partial metacognitive intervention.

Keywords: integration, sense-making, suspension of sense-making, metacognitive, metacognitive intervention

INTRODUCTION

Calculus has been a subject of study in mathematics education since 1980s as conducted by Orton (1983a) & Orton (1983b). Both of his studies focused on how students understood calculus in terms of differentiation and integration. Subsequently, Davis & Vinner (1986) held a research on understanding the concept of limit in calculus and identifying sources of misconceptions. In recent years, research related to calculus in the field of mathematics education includes cognitive processes in learning calculus, barriers to learning calculus, calculus learning practices, and the transition of calculus from high school to university (Bressoud, 2021; Fung & Poon, 2021; Galanti & Miller, 2021; Ghedamsi & Lecorre, 2021; Kashefi et al., 2012; Rasmussen et al., 2014). Calculus has an important role in secondary and higher education as well as various scientific disciplines (Rasmussen et al., 2014; Yoon et al., 2021). Calculus serves as a prerequisite for further studies in areas such as mathematics, computer science, social sciences, biological sciences, physical sciences, education, statistics, engineering, and medicine (Bressoud, 2021; Ghedamsi & Lecorre, 2021). Calculus notions cover real numbers, infinite sequences, functions, continuity, limits, differentials, basic calculus theorems, differential equations, and integrals (Ghedamsi & Lecorre, 2021).

One of the important calculus notions is integration. Integration is defined as a key component of mathematics education in secondary schools and calculus courses in tertiary institutions (Greefrath et al., 2021). Integration are important for understanding various contextual problems, including various contexts in physics and engineering and also important when studying mathematics (Radmehr & Drake, 2017). However, the integration is often considered difficult by most students at both high school and university levels. Furthermore, according to Greefrath et al. (2021), student difficulties do not always arise from a lack of knowledge, but from the activation of less productive cognitive resources. For example, when solving integration problems, especially in area calculations, students often do not involve sense-making. When solving problems, students often do not realize the irregularities that occur, where the area cannot be 0 when the region exists, and the area cannot be negative either. This can be called a suspension of sense-making. Sense-making in mathematics can be defined as developing an understanding of a context, concept, or situation by relating it to existing knowledge (Keazer & Menon, 2016). Furthermore, the suspension of sense-making describes the sense-making disengagement that students do when trying to solve mathematical problems. For example, it occurs when the students ignore realistic considerations in solving mathematical problems (Carotenuto et al., 2021). When sense-making is not involved, the resulting answers will be inaccurate even though they have been carried out with the right procedures. In line with the opinion of Keazer & Menon (2016), sense-making must be inherent in all mathematical activities.

Suspension of sense-making occurs due to several factors. According to Biccard (2018), the suspension of sense-making comes from the disconnection of procedural and conceptual understanding. This is in line with the opinion of Greefrath et al. (2021) that student knowledge is often limited to procedural knowledge because they are good at integrations but have difficulty connecting the different contexts of definite integrations. The terms conceptual and procedural are used to describe knowledge about a network of interrelated relationships between mathematical concepts and knowledge of mathematical symbols, formulas, algorithms, and procedures respectively (Legesse et al., 2020). The procedure is a series of steps, or actions, which are carried out to achieve a goal (Rittle-Johnson et al., 2015). This is in line with Voutsina (2012) that procedural knowledge is defined as knowledge that allows the application of rules, algorithms and procedures to solve problems that are not necessarily meaningful and can be generalized to new contexts and situations. It is the capacity to follow sequential steps to solve mathematical problems or achieve mathematical goals (Hurrell, 2021). The essence of procedural knowledge is that it involves applying sequential action steps and automated techniques to solve problems (Aydın & Özgeldi, 2019). Meanwhile, conceptual knowledge is characterized as a network of connected knowledge, a network in which the connecting links are as prominent as the separate pieces of information. The relationships established by conceptual knowledge serve as guides for understanding problems, and for generating new solution strategies or for adapting existing strategies to solve unfamiliar problems (Aydın & Özgeldi, 2019).

Furthermore, the interactions between procedural and conceptual knowledge can be mediated by metacognitive processes (Braithwaite & Sprague, 2021). Metacognitive is defined as knowledge or cognitive activity that uses cognitive processes as its object (Lingel et al., 2019). Metacognitive involves the ability to actively control various cognitive processes; it is a mental process used to regulate cognitive processes (Radmehr & Drake, 2017). Schoenfeld (2016) states that sense-making requires a metacognitive process. Jivet et al. (2020) then investigated metacognitive influences on sense-making. There are three latent variables for sense-making: transparency of design, reference frames, and support for action. The results of Jivet et al. (2020) research showed that metacognitive influences these three latent sense-making variables. The results of this study are also supported by Franklin et al. (2018) that metacognitive is related to mindset, reflection, sense-making, and the development of personal judgment and is an integral part of student success in learning. Learning that involves metacognitive is seen as able to trigger individuals to carry out a sense-making process (Shilo & Kramarski, 2019).

Research related to the suspension of sense-making has been carried out. Carotenuto et al. (2021), for example, seek to deepen understanding of the suspension of sense-making by conducting empirical and qualitative studies that focus on the effects of variations in the presentation of story questions (text, images, format) on students' approaches to problems. The results of this study depicted that suspension of sense-making is more precisely a phenomenon of activating alternative types of sense-making: various types of sense-making active seem to be strongly influenced by the presentation of word problems. However, this research is still limited to students' numerical answers. Therefore, Carotenuto et al. (2021) suggested further research on the process of suspension of sense-making cognitively, not just on objective reports.

Additionally, Kirkland & McNeil (2021) investigated the suspension of sense-making experienced when working on word problems and examined the design of word problem questions that can trigger students to involve sense-making, reasoning explicitly about the context described in the problem. The results showed that rewriting story problems into "yes/no" questions affected students' problem-solving performance and sense-making. However, further research is needed to determine the mechanisms involved in these effects due to research. It is a quantitative study, but not a qualitative one involving think-aloud as a retrospective follow-up on participants to better understand their thinking during problem-solving.

Furthermore, Bonotto (2003) suggested remedies that can be given to overcome the suspension of sense-making, including (i) replacing word problem-solving with class activities that are more related to realistic conditions that are close to students and consistent with sense-making dispositions; (ii) changing teachers' conceptions, beliefs and attitudes towards mathematics; (iii) making direct efforts to change the socio-math class norms. However, this suggested remedy has not been related to metacognition. Even metacognitive also plays a role in sense-making as explained by Schoenfeld (2016) who concluded that these components are interrelated with one another. This study is also supported by previous research (Jivet et al., 2020; Shilo & Kramarski, 2019).

Therefore, it can be said that so far the students' suspension of sense-making has not been explained as a cognitive process that refers to the findings of previous studies. Previous studies mainly focused on word problems that cause suspension of sense-making. If the suspension of sense-making is described from a cognitive perspective, it will develop basic knowledge about student thinking. This can then be utilized in developing interventions that are based on the characteristics of student thinking. Furthermore, this study aims to explore further the process of metacognitive intervention in students who experience suspension of sense-making in terms of students' thinking characteristics. This research begins by investigating the characteristics of the thinking of students who experience suspension of sense-making in calculus courses, especially in the process of solving integration problems, in determining the area of a region bounded by a curve. This research is expected to expand knowledge related to metacognitive interventions that can be given to students who experience suspension of sense-making according to their thinking characteristics. Calculus learning practitioners can then take advantage of this research to determine appropriate interventions according to the characteristics of students who experience suspension of sense-making.

LITERATURE REVIEW

Metacognitive Intervention

The term “metacognitive” or “thinking about thinking”, refers to a distinct capacity that allows one to think about one’s cognitive processes (Pennequin et al., 2010). Metacognitive theories are those theories of mind that focus on cognitive aspects of the mind (Schraw & Moshman, 1995). This means that metacognitive involves the ability to control various cognitive processes actively. In other words, metacognitive is a mental process used to regulate cognitive processes (Radmehr & Drake, 2017). Metacognitive involves the ability to assess one's knowledge and cognitive abilities and how one monitors and controls their cognition in completing tasks (Bellon et al., 2019). Therefore, metacognitive is essential in mathematics learning activities to help students learn more effectively and efficiently.

However, metacognitive does not always run smoothly without any barriers, students may experience metacognitive failure. Metacognitive failure is related to the response to red flags (Huda et al., 2018). In the metacognitive process “red flag” indicates the need for someone to stop or re-examine the problem-solving process (Goos, 2002). According to (Goos, 2002), there are three times when “red flags” can occur and can identify metacognitive failures, namely: (1) no progress in the process of finding solutions (lack of progress); (2) error detection in the problem-solving process, (3) ambiguous in the final answer (anomalous result). Metacognitive failure occurs when students are unable to detect red flags (blindness), detect the presence of red flags but the actions taken are inappropriate (vandalism), and assumes there are red flags that are not there (mirage) (Rozak et al., 2018).

In mathematics learning activities, metacognitive can be improved by appropriate instruction that facilitate students to reflect on their own thinking (Lai, 2011). Metacognitive interventions can be given so that students could bring up metacognitive processes. One form of metacognitive intervention is in the form of metacognitive questions. Metacognitive questions consist of comprehension, connection, strategic, and reflection questions (Özcan & Erktin, 2015). According to Faradiba et al., (2019), comprehension questions assist students to understand mathematics problems. Connection questions assist students in connecting the given problem to similar or related problems in the past. The strategic questions assist students in determining the best strategy to solve a problem. Finally, the reflection questions direct students to recheck the process of solving problems and their solutions. The grid of metacognitive questions asked in interviews as part of the metacognitive intervention is in Table 1.

Types of Metacognitive Questions		Metacognitive Questions
Comprehension Question	1)	What is the material related to this problem?
	2)	Can you show the area that you are looking for?
Connection Question	1)	Have you ever worked on questions like this before?
	2)	Did you immediately do this problem in the same way as you did before?
	3)	What are the differences between this question and the questions you have done before?
Strategic Question	1)	What strategies can be used to solve the problem?
	2)	Why is this considered the right strategy?
	3)	What is the integration formula?
Reflection Question	1)	Is the process correct?
	2)	Does the solution make sense?
	3)	Is there any other way to solve this problem?

Table 1: The Metacognitive Questions

Suspension of Sense-Making

Sense-making is a means for learning mathematics and is an important goal of learning mathematics (Biccard, 2018; Keazer & Menon, 2016; Palatnik & Koichu, 2017; Sepeng & Sigola, 2013; Weinberg & Thomas, 2018). Sense-making is an essential cognitive process in all mathematical activities (Keazer & Menon, 2016). Sense-making is involved in mathematical activities, including activities of understanding concepts, representations, reasoning, proving, and problem-solving processes (Keazer & Menon, 2016; Mueller et al., 2011; Palatnik & Koichu, 2017; Rau et al., 2012; Smith, 2006). From a problem-solving perspective, sense-making means forming meaning or giving meaning based on experiences that include the context of everyday life as well as concepts and knowledge possessed, which enables a person to recognize how and when to respond to problems appropriately to solve problems effectively (Xiaofang, 2021).

However, during the problem-solving process, the student's ability to consider real-world information might not be applied, and students tend to ignore this information (Fitzpatrick et al., 2020). Several studies have also described "seemingly absurd things that students do at all levels when they try to solve math problems". An example is when the students respond with numerical answers to nonsensical problems or when they ignore realistic considerations in school math problem-solving (Carotenuto et al., 2021). Schoenfeld (1991) then introduced the phrase "suspension of sense-making" to describe students' disengagement with mathematics.

Palm (2008) explained that the phenomenon of suspension of sense-making occurs when students face a problem, they immediately work on the problem in a stereotyped way, without paying attention to the reality of the 'real' situation described in the task. As a result, the solution they find does not match, and in some cases, it even becomes absurd when it is linked to the 'real' situation. This also happens with students when working on problems. Students have a tendency not to use their real-world knowledge properly and ignore that their solutions must make sense in the 'real' situation. The tendency to provide such 'unrealistic' solutions seems strong and not easily overcome by mere hinting.

METHOD

This study applied a qualitative approach because it aimed to describe metacognitive interventions for students who experience "suspension of sense-making". This type of research can be categorized as exploratory descriptive because it describes the results of exploration related to metacognitive interventions given to students who experience suspension of sense-making when application problems based on the characteristics of students' thinking. The research obtained verbal data in the form of students' expressions when they were solving problems. This research was carried out at the Mathematics Education Study Program, in one of the universities in East Java, Indonesia. The participants were students who experienced a "suspension of sense-making".

To begin the study, researchers concerned with learning of integral calculus by employing 47 students in total as the participants. The learning process was carried out in six meetings, and all activities in each meeting were observed and recorded. The learning process implemented is summarized in the following Table 2.

Meeting 1	Meeting 2	Meeting 3	Meeting 4	Meeting 5	Meeting 6
Discussing the Concept of a Definite Integral with the Concept of Riemann Sums	Discuss the definition and nature of definite integrals	Discussing the Fundamental Theorem of Calculus	Discuss Integration Techniques	Discusses the Mean Value Theorem for Integration and Symmetry	Discuss specific integration applications

Table 2: Learning Activities Implemented

The method used in learning is the IMPROVE self-questioning method because it is one of the well-known methods for growing students' metacognition and improving their mathematical problem-solving skills (Shilo & Kramarski, 2019). This method aims to instil key aspects of sense-making in problem-solving by using general questions directed at understanding, strategy, connection, and reflection across the three metacognitive skills. The IMPROVE method has been trusted in metacognitive research on students at various levels of education as well as in teacher professional development (Shilo & Kramarski, 2019). In every learning process, students were trained to always involve metacognitive.

After finishing the learning on integration, students were then given integration problems. After that, an observation was conducted to see whether the phenomenon of "suspension of sense-making" appeared in the student's answers. The problems given were the following.

(1) Determine the area of the region bounded by $f(x) = \sin x\,, 0 \leq x \leq 2\pi$ and the x −axis

(2) Determine the area of the region bounded by $f(x) = x^2 - 4x, -1 \leq x \leq 2$ and the x −axis

When students were asked to solve the problems, there were 16 students out of 47 students immediately applied definite integration concepts for granted without further activation of cognitive resources. Therefore, the final result obtained was less precise. Furthermore, task-based interviews were conducted with students who indicated they were experiencing suspension of sense-making to find out the students' thinking processes when working on the problem and ensure that the errors on the answer sheets occurred due to suspension of sense-making. Based on the task-based interviews conducted to the sixteen participants related to the error occurred due to suspension of sense-making, two characters of the participants were then found, namely pure procedural and mixed conceptual-procedural students.

From sixteen participants who experienced a suspension of sense-making, eleventh of them were identified to be pure-procedural students. Meanwhile, five students were categorized as mixed conceptual-procedural students. All sixteen participants were then explored in-depth interview in which metacognitive questions were asked as the metacognitive intervention. The interview guide was adapted from Faradiba et al. (2019) which can be seen in Table 1.

RESULTS

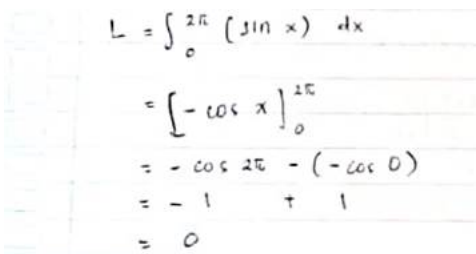
In this study, metacognitive interventions were given in the form of asking metacognitive questions to students who experienced suspension of sense-making. The metacognitive intervention was intended to help students involve their metacognition in problem-solving. Furthermore, this study explored the thinking characteristics of students who experience suspension of sense-making based on the types of conceptual and procedural knowledge involved in the problem-solving process. Next, we described the metacognitive intervention process based on the students' thinking characteristics in the problem-solving process, namely pure procedural and mixed conceptual-procedural. The metacognitive intervention was given during the task-based interview.

Metacognitive Intervention in Pure Procedural

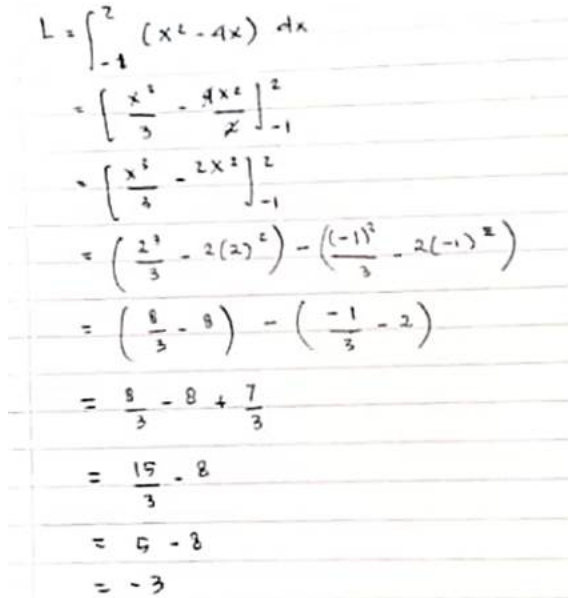
After being given two integration problems, pure procedural students immediately solved the problems using the usual procedures by applying the integration formula. However, they applied this formula without involving the meaning of the procedure carried out and identified it as experiencing suspension of sense-making. It is because they did not realize the awkwardness that the area cannot be 0 in problem 1 and is unlikely to be negative in problem 2 (Figure 1). Then a metacognitive intervention was carried out on pure procedural students. Initially, a metacognitive intervention was given in problem 1 with a description of the problem, namely: Determine the area of the region bounded by

$$f(x) = \sin x,\ 0 \leq x \leq 2\pi \quad (1)$$

and the x-axis. As shown in Figure 1a, when the students worked on problem 1, they answered that the area of the region is 0. This is seen as unreasonable because the area exists, so the area cannot be 0. One of the pure procedural student's initial answers to problem 1 can be seen in Figure 1a.



(a)



(b)

Figure 1: (a) Student's Answers to Problem 1 (b) Student's Answers to Problem 2

Pure procedural students worked on Problem 1 by applying the procedure purely without paying attention to the concept. Even though, pure procedural students had a good understanding of the problems given. This can be seen from the comprehension questions described as follows (P is one of the researchers and S1 is one of the pure procedural students).

P : What material is related to this problem?
S1 : Regarding the application of integrations in area calculations ma'am
P : Try to draw and indicate the area you want to find the area of
S1 :

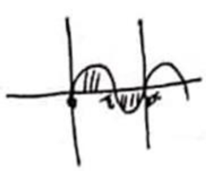


Figure 2: Image of the area sought by S1 (one of the pure procedural students)

Based on the interview with all pure procedural students in the comprehension question section, it is known that students have known how to draw the area they want to find even though they did not describe it during the problem-solving process. This shows that the students have a good understanding of the problem to be worked on.

Next, the metacognitive intervention was continued by providing connection questions. The example of the interview is as follows.

P : *Have you ever done something like this before?*
S1 : *Yes, ma'am.*
P : *Is there any difference between this problem and the problem you have done before?*
S1 : *Yes, at least the curve and the boundaries, ma'am.*
P : *If you look again at the area, you just drew, were there no special conditions that distinguished this problem from the other questions you used to work on before?*
S1 : *(trying to think)*
Erm... What is it, ma'am? Nothing seems to be

Based on the interview, pure procedural students were still not aware of the special conditions that exist in the problem where there should be different treatment between the areas above the x-axis and below the x-axis. Therefore, the metacognitive intervention was continued by providing strategic questions. The example of the interview excerpt with one of the pure procedural students is as follows.

P : *Tell me about the strategy you used to solve the problem.*
S1 : *Yes, ma'am, using integration... all that's left is to enter the graph function formula and its boundaries*
P : *Okay, now try to remember your answer again (while pointing at the student's answer) and tell me yesterday how come you answered like that. What's the story?*
S1 : *All that remains is to substitute the curve formula f(x) = sin x and the boundaries, Ma'am (while reading the answers), and the result are 0. (Answers read by students can be seen in Figure 1a)*
P : *When you found the answer, did you check again or not?*
S1 : *No, Ma'am.*
P : *Why not check again*
S1 : *Yes, because I think it is correct Ma'am. There's nothing strange about my answer, I did it smoothly too.*
P : *Oh, so there are no obstacles in solving this problem?*
S1 : *No, Ma'am.*

At this strategic question stage, the students were asked about the strategy used, possible strategic choices, and how the strategy was implemented and monitored. From the strategy questions asked, it is known that students directly applied the curve and boundary formulas that they previously knew in integration to determine the area of the area. The students were not aware of the red flag in the form of an anomaly. This indicates a blindness type of metacognitive failure. The last question for Problem 1 is related to the reflection question. Meanwhile, the following is a sample excerpt from an interview related to the reflection question.

P : *If you look back now, is there anything strange about your answer?*

S1 : *(Checking the answer again) It's correct ma'am. Isn't that right,* Ma'am?

$$\int \sin x \, dx = -\cos x \quad (2)$$

P : *Yes, nothing else is strange, right?*
S1 : *No mom*
P : *Try to look again at the graph that you drew at the beginning. Is it possible that the area is 0?*
S1 : *Oh yes, it can't be 0, ma'am. This is the area.*
P : *What's wrong with it?*
S1 : *(Rethinking the answer)*
S1 : *Oh yes, ma'am, there are curves above the x-axis and below the x-axis. So it seems that you have to calculate it separately, ma'am.*

Based on the interview at the reflection question stage, it is known that the students reflected and re-checked their answers because they were asked by the researcher as part of the intervention. The results of the reflection show that the students initially experienced blindness-type of metacognitive failure because they were not aware of any irregularities in the obtained results. However, after being given metacognitive questions, they could raise metacognitive awareness, engage sense-making, and be aware of existing irregularities. Finally, they could find the right answer by changing the strategy, as shown in Figure 3.

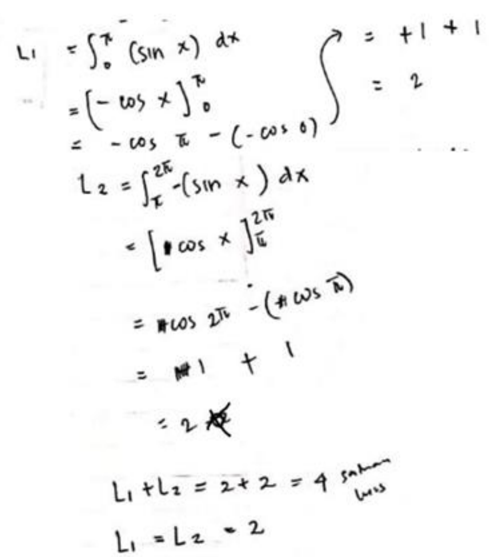


Figure 3: Student's Answer to Problem 1 after Metacognitive Intervention

In Problem 1, the pure procedural students already had a good understanding of the problem, as can be seen from their answers at the comprehension question stage. At the connection stage, the students were not yet aware of the direction of the questions in the connection stage so at the strategic question stage they were still firm on the answer. At the reflection stage, the students could finally raise their awareness, engage sense-making, and realize there were irregularities in the results obtained. After realizing the mistake, the students could make corrections properly even though they experienced confusion because they returned 0 results on L_2. When they got a value of 0, they could get involved in sense-making and realized that the area couldn't be 0, and found something wrong with the positive and negative values in the area of calculation.

Next, in Problem 2, Determine the area of the region bounded by

$$f(x) = x^2 - 4x, -1 \leq x \leq 2 \quad (3)$$

and the x −axes. Pure procedural students answered Problem 2 as can be seen in Figure 1b. The students also had a good understanding of Problem 2. They were able to indicate the area to be searched for when asked even though initially it was not written on the answer sheet as shown in the sample in Figure 4.

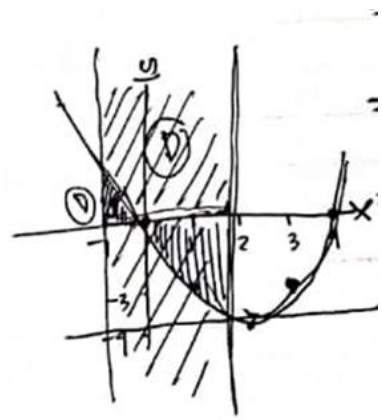


Figure 4: The Results of the S1 Graphic on Problem 2

Based on the results of the interview in the comprehension question stage, it is known that students had a fairly good understanding of Problem 2. They could draw graphs and show the area to be searched for. Furthermore, at the connection question stage, the researcher related it to Problem 1. It can be seen from the sample interview excerpt with S1 as follows.

P : *If you look at the graphics in Problem 2, what do you think is similar to problem 1 that we have just discussed?*

S1 : *This is the same as a curve that lies above the x-axis and below the x-axis. Now, if you look again, the area under the x-axis is bigger, ma'am. This is probably why my answer ended up being negative. If the number 1 was the area above and below the x-axis is the same, then the result is 0.*

P : *Well, that's right. Is it possible that the area is negative?*

S1 : *No mom*

Based on the connection questions, the students looked back at their answers to problem 1, rethought their answers to problem 2, and began to realize the awkwardness of that area was negative. Therefore, it can be said that students' metacognitive awareness appears at this stage spontaneously.

Because students were already aware of the awkwardness that area could not be negative at the connection question stage, then at the strategy question stage the researcher only confirmed a more appropriate strategy to be able to solve Problem 2. An example of the students' answers is in Figure 5. The figure describes S1's answer to Problem 2 after realizing the awkwardness of that area was impossible negative.

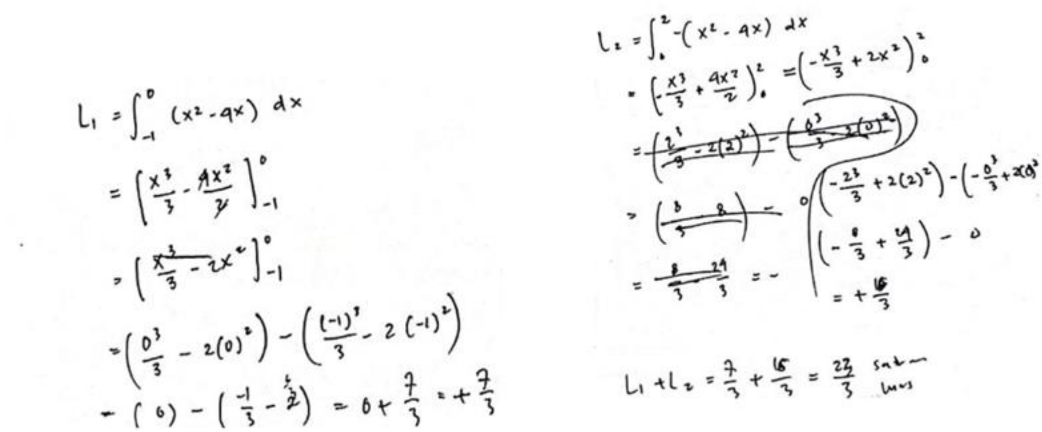


Figure 5: Improvement of S1 Answers on Problem 2 after Metacognitive Intervention

In the process of solving Problem 2, pure procedural students had also started to involve their sense of making and were aware of red flags. They revised their answers that were considered odd and, in the end, they were able to solve Problem 2 properly by finding the right answer.

Finally, after being given a reflection question, pure procedural students could realize that before solving integration problems, they need to know the curve image and the region to be searched for its area first. They cannot directly substitute curve and boundary formulas. Even though the results look right, and it looks like there are no significant obstacles, the results will be awkward.

Metacognitive Intervention on Mixed Conceptual-Procedural

After being given two integration problems, mixed conceptual-procedural students were known to have drawn graphs and understand the purpose of drawing these graphs. Even though they understand the meaning of drawing graphs when solving integration problems, it turns out that drawing graphs are also a routine procedure that students usually do. So, in this study students with these characteristics are hereinafter referred to as mixed conceptual-procedural students. Mixed conceptual-procedural students are identified as experiencing suspension of sense-making because they did not realize the awkwardness that the breadth could not be 0 in Problem 1 and could not be negative in problem 2. Then a metacognitive intervention was carried out on mixed conceptual-procedural students. First, the process of metacognitive intervention in Problem 1 will be described. Figure 6 presents the example of mixed conceptual-procedural students' answers to Problem 1.

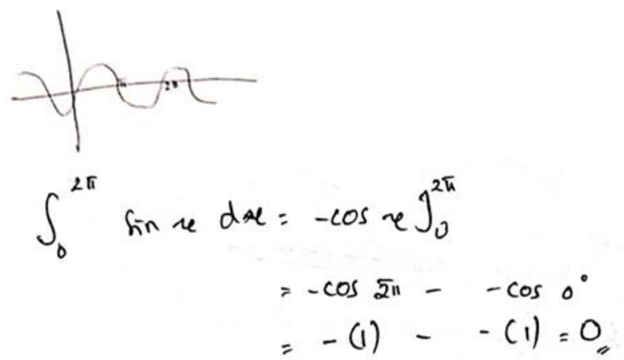


Figure 6: The Answer of Students with Mixed Conceptual-procedural to Problem 1

In contrast to pure procedural students who immediately solved problems by applying certain integration concepts, without drawing a graph first, mixed conceptual-procedural students drew their graphs even though they did not indicate the area to be searched for in the image. However, it is known through the comprehension questions that S2 (one of the mixed conceptual-procedural students) had already understood the given problem and could indicate the area to be searched for.

P : *What material is related to this problem?*
S2 : *Regarding the application of integration ma'am*
P : *How do you know that this problem is integration related?*
S2 : *These are like questions that are usually done in the Calculus course, Ma'am. For questions like this, use the integration.*
P : *Okay, here you have drawn the curve. Can you show the area to find the area?*
S2 : *This or that?*

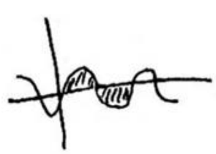


Figure 7: The figure of the area that the mixed conceptual-procedural students are looking for

Based on the interview in the comprehension question section, it is known that students knew the area they wanted to explore even though they did not describe it when solving problems as it is shown by the sample in Figure 7. This shows that they had a good understanding of the problem to be worked on.

Next, the researcher continued the metacognitive intervention by providing connection questions as follows.

P : *Have you ever done something like this before?*

S2 : *Yes ma'am, yesterday in integral calculus class I often worked on problems like this.*

P : *How to solve this kind of problem?*

S2 : *Yes, first draw it ma'am, then when you already know which area we use the integration, we substitute the boundary and curve formula.*

P : *Oh, I see. Then why do we need to draw the curve first before working on it?*

S2 : *Just so you know the area, ma'am, and so you know the boundaries.*

P : *OK about the boundaries. How is that a way of knowing the boundaries?*

S2 : *Yes, seen from the curve ma'am. Sometimes the areas are separated, so to calculate the area you have to divide each area and then add them up. But in question 1, it immediately connects. So yes, the boundaries goes straight from 0 to* 2π

P : *Previously you said that sometimes there are separate areas, what do you mean?*

S2 : *Yes, for example, there is an intersection of 2 curves, ma'am. It's an area that intersects, we don't know where. Sometimes connected, sometimes disconnected.*

P : *Okay, if we look again at the curve image in problem 1. Can't this be said to be separate too? There's one above the x-axis and one below the x-axis. How's that?*

S2 : *(thinking back on the answer while looking at the pictures and making doodles)*

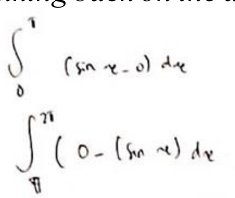


Figure 8: Doodles made by mixed conceptual-procedural students

The one below the x-axis should be negative, right, ma'am?

P : *Naah… That's right, where did you get it?*

S2 : *(explains Figure 8)*
Here, ma'am, I think of this x-axis as f(x) = 0, and then I use the concept of the area bounded by two curves

Based on the interview, mixed conceptual-procedural students assumed that the area to be determined was continuous, so there was no need to divide it per region. They did not realize that there are areas located above and below the x-axis, which should require different treatment when calculating their area. Here the students were again identified as experiencing suspension of sense-making because they did not realize that the areas that were located above the x-axis and below the x-axis should also be said to be separate and to calculate the area, they could not directly apply integration with a boundary of 0 to 2π.

However, when the questions were continued, it was seen that students began to raise their awareness, namely realizing that the area under the x-axis should be negative as shown in Figure 8. Mixed conceptual-procedural students assumed that the x-axis is equal to $f(x) = 0$ and applied the concept of the area between the two curves. Here it can be seen that they did not just think procedurally but started associating with concepts. Therefore, they already realized that the areas located above and below the x-axis need different treatment.

Furthermore, because the students had raised awareness, the researcher only confirmed the strategy used at the strategic question stage. They realized that the answer could not be 0 and that there was an error in the initial strategy used. Mixed conceptual-procedural students immediately tried to implement a new strategy, namely by calculating the area of each area that lies above and below the x-axis

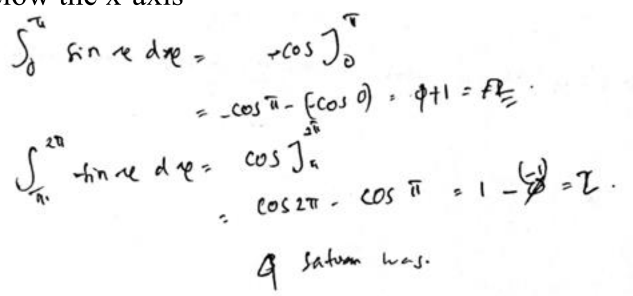


Figure 9: Answers to Problem 1 by S2 after the Metacognitive Intervention

Based on the example of the student's answers in Figure 9, it is known that students reflected and re-checked their answers when they found results that were deemed inappropriate and, in the end, get the right solution.

In Problem 1, mixed conceptual-procedural students already had a good understanding of the problem, as seen from the students' answers at the comprehension question stage. At the connection stage, they could also answer questions well, they understood the importance of drawing curves before determining the area. They had also begun to realize that there were special conditions in Problem 1 at the connection question stage. Furthermore, at the strategic question stage, the students mixed conceptual-procedural started implementing a new strategy, having realized the need for a different treatment between the areas located above and below the x-axis. In the process of work, they always reflected when they found inappropriate results. In the end, the students could revise correctly and find the right results.

Next, in Problem 2, figure 10 shows the initial answer to Problem 2 of one of the mixed conceptual-procedural students.

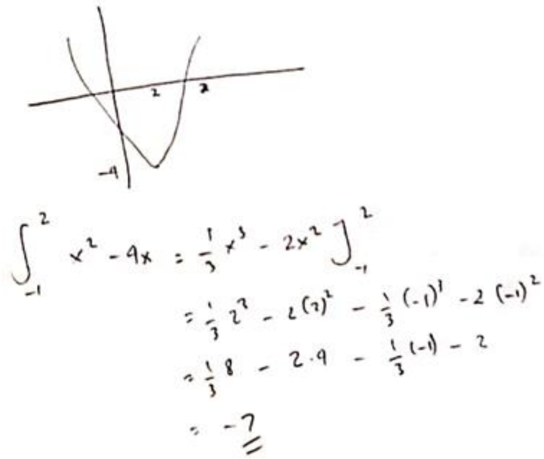


Figure 10: The Answer to Problem 2 by the student with mixed conceptual-procedural

Before being asked further regarding the answer to Problem 2, the mixed conceptual-procedural students immediately realized that area could not be negative. This is because mixed conceptual-procedural students had received metacognitive intervention in Problem 1, so students' metacognitive spontaneously appeared in Question 2. Mixed conceptual-procedural students could bring up metacognitive awareness spontaneously and were able to realize the irregularities that existed even though they had not been given metacognitive questions related to Problem 2. After carrying out the process of re-checking the answers, S2, for example, realized that there was something wrong with the graphic image and the integration formula. Following are the improvements made by mixed conceptual-procedural students on Problem 2.

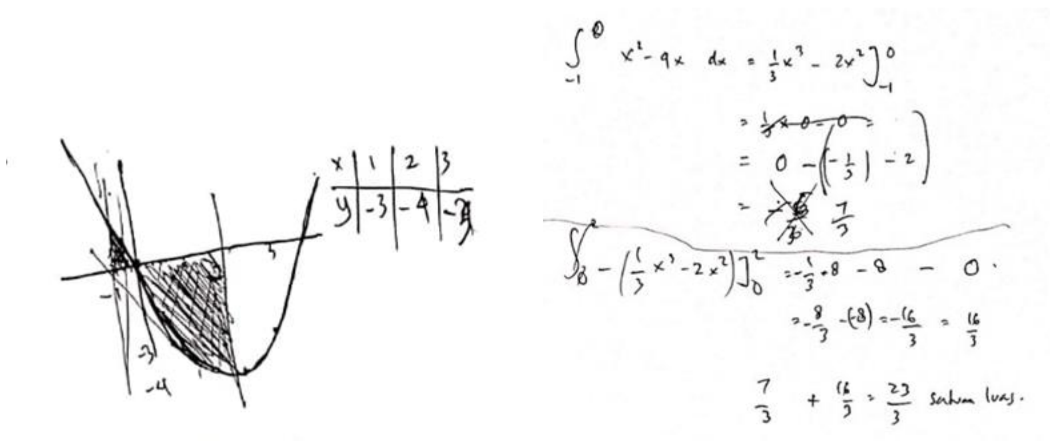


Figure 11: Improvements of mixed conceptual-procedural student answers to Problem 2

The research results presented can be summarized in the following table.

Thinking Characteristics	Types of Suspension of Sense-making	Metacognitive Intervention
Pure Procedural	Do not realize the awkwardness that the area cannot be 0 or negative	Comprehension, Connection, Strategic, Reflection Question
Mixed Conceptual-Procedural	Do not realize that the graphs above and below the x-axis require different treatment	Comprehension, Connection Question
	Do not realize the awkwardness that the area cannot be 0 or negative	Comprehension, Connection Question

Table 3: Metacognitive Interventions based on Thinking Characteristics

DISCUSSION

Pure Procedural Students

Pure procedural students initially experienced suspension of sense-making when solving Problems 1 and 2. This happened because of three factors. (1) Pure procedural students did not activate cognitive resources in the form of sense-making during the problem-solving process. This is in conjunction with Greefrath et al. (2021) that errors do not always arise from a lack of knowledge but from the activation of less productive cognitive resources. (2) Pure procedural students are identified as experiencing metacognitive failure of the metacognitive blindness type because they are not aware of the red flag in the form of an anomaly (Goos, 2002). (3) Pure procedural students do not connect conceptual and procedural understanding, so they experience suspension of sense-making according to opinion (Biccard, 2018).

Furthermore, the metacognitive interventions given to pure procedural students are summarized in Table 2. In the end, after being given metacognitive interventions in the form of metacognitive questions: comprehension questions, connection questions, strategic questions, and reflection questions. Pure procedural students can raise metacognitive awareness at the reflection question stage. Meanwhile, metacognitive awareness is a process of using reflective thinking in developing one's awareness of personal knowledge, tasks, and strategies in a context (Kesici et al., 2011). Pure procedural students could overcome the problem of suspension of sense-making experienced and be able to find solutions to Problems 1 and 2 appropriately. Based on the results of the metacognitive intervention given in the form of metacognitive questions, pure procedural students were able to raise metacognitive awareness and realize existing irregularities. In this case, it can be noted that pure procedural students need complete metacognitive intervention.

Mixed Conceptual-Procedural Students

Like pure procedural students, mixed conceptual-procedural students also did not involve cognitive resources or sense-making in the beginning process of solving a problem. As a result, mixed conceptual-procedural students experienced a suspension of sense-making where students did not realize the awkwardness that the area cannot be 0 in problem 1 or even negative as in problem 2. This is in line with Greefrath et al. (2021) that errors do not always arise from a lack of knowledge but from the activation of less productive cognitive resources.

At first, mixed conceptual-procedural students had drawn graphs and knew the area to be searched for. Graphics are drawn to help define boundaries. This indicates that students have started associating with the concept and not just doing it procedurally. However, at first, the mixed conceptual-procedural students did not realize that a different treatment was needed between graphs located above the x-axis and below the x-axis. Students only focused on finding the overall area without distinguishing the areas below and above the x-axis first. This indicates that students are again experiencing suspension of sense-making because it fails to recognize the existence of different contexts and leads to the need for different treatment between the regions located above and below the x-axis. This is further associated with visual metacognitive concepts.

Visual metacognition is the ability to evaluate one's performance on visual perception tasks (Rahnev, 2021). Visual metacognition is concerned with how people give judgments of confidence in perceptual tasks. According to Sternberg and Sternberg (2012), perception is a set of processes which include recognizing, organizing, and making sense of the sensations we receive from environmental stimuli, which in this case are graphics. Visual metacognitive and awareness are considered to be closely interrelated, with knowledge of the correctness of perceptual choices depending on the level of awareness of the stimulus (Jachs et al., 2015). Visual metacognition is an important skill in our daily lives that enables us, for example, to recognize our poor ability to see in foggy conditions and thus, drive more slowly (Rahnev, 2021). Furthermore, in this study visual metacognitive is associated with how students perceive graphs.

In the visual metacognitive process, failures in visual awareness can occur, such as intentional blindness and change blindness (Ortega et al., 2018). Inattentional blindness is defined as a failure to pay attention to the unexpected but is fully visible when attention is diverted to another aspect of the display being seen (Jensen et al., 2011). The lack of attention of people who experience inattentional blindness can be caused by a demanding main task (Redlich et al., 2021). Change blindness is a staggering failure to detect substantial visual changes. Change blindness, for example, occurs when someone divides attention. When an individual divides attention, the individual is often not aware of some cognitive limitations, such as failure to pay attention to unexpected important changes that occur, and there is an inability to accurately record the events we see (Ortega et al., 2018). Both types of visual metacognitive failure reveal a startling discrepancy between what we believe we see and what we see.

The failure experienced by mixed conceptual-procedural students in viewing graphs was identified as inattentional blindness because these pure procedural students failed to notice that there are areas located below and above the x-axis. Mixed conceptual-procedural students only focus on finding the overall area without first dividing the areas below and above the x-axis.

Then after being given a metacognitive intervention to mixed conceptual-procedural students, they could raise their metacognitive awareness at the connection question stage in problem 1. Students began to realize that there are different contexts and the need for different treatment between the areas located above the x-axis and below the x-axis. After the mixed conceptual-procedural student realizes his mistake, the student can correct his answer and get the right solution from problem 1. Then in Problem 2, the mixed conceptual-procedural student could raise his metacognitive awareness spontaneously. In the end, students with mixed conceptual-procedural could fix the error in Problem 2 and could determine the right solution. The metacognitive interventions provided are summarized in Table 2.

In the end, based on the results of the metacognitive intervention given in the form of metacognitive questions, students only need questions up to the connection question stage. In other words, mixed conceptual-procedural students do not need complete metacognitive questions, or only need partial metacognitive intervention up to the connection question stage.

CONCLUSION

Students often do not involve sense-making when solving problems. This is hereinafter referred to as suspension of sense-making. In this study, students who experienced suspension of sense-making were grouped into pure procedural and mixed conceptual-procedural students. The suspension of sense-making that occurs is caused by several factors: (1) activation of students' cognitive resources is less productive; (2) there is a gap between conceptual and procedural understanding; (3) there is a type of metacognitive failure of metacognitive blindness because they are not aware of the red flag in the form of an anomaly; (4) it is found the presence of inattentional blindness visual metacognitive failure in the process of interpreting graphs.

Furthermore, students who experience suspension of sense-making are given intervention in the form of a metacognitive intervention. Students with pure procedural characteristics can raise metacognitive awareness and engages sense-making at the reflection question stage, so pure procedural students can be said to need complete metacognitive intervention starting from comprehension, connection, and strategic questions to reflection. Meanwhile, students with mixed conceptual-procedural characteristics can raise metacognitive awareness and engages sense-making at the questioning stage connection question or it can be said that it does not require complete intervention, hereinafter referred to as partial metacognitive intervention.

Consequently, metacognitive intervention can solve suspension of sense-making in integration problem-solving. Both pure-procedural and mixed conceptual-procedural students who experiencing a suspension of sense-making in solving integration problem may eventually engage in sense-making after accepting metacognitive intervention. Therefore, metacognitive interventions can accommodate students who experience a suspension of sense making in accordance with their individual thinking characteristics. Metacognitive intervention can activate students’ metacognitive awareness and make students engage sense-making in solving integration problem. Metacognitive intervention can complement the learning strategies used in calculus learning, especially in integration notion.

This research has not yet found subjects with pure conceptual characteristics. Therefore, further research is suggested to be able to reveal whether students who solve problems with pure conceptual knowledge characteristics may also experience suspension of sense-making. It is also recommended for further research to investigate further related to metacognitive intervention in students with pure conceptual characteristics in the problem-solving process concerning the suspension of sense-making.

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