Error Analysis of Dyslexic Student's Solution on Fraction Operation Tasks

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Abstract: *How dyslexia students solve number operations is still challenging to unravel. This study aimed at revealing the types of errors conveyed by a dyslexic student in performing fractional operations on mathematical tasks that combined non-verbal text (symbols and pictures) and verbal text. The data were collected using a task-based interview with a 13-year-old dyslexic student that was recorded and focused on the types of errors on fractional operations (addition, subtraction, multiplication, and division). Results depicted that the student overgeneralized whole number operations when adding two fractions with different denominators although she successfully converted the area model of fraction into a correct fraction operation, identified a common denominator but failed to change the fractions into equivalent form when subtracting two mixed fractions, failed to interpret a multiplication word problem into subtraction operation, and applied only part of the "invert and multiply" algorithm on a word problem. The assumption of the phonological disturbances that was found in the student participant's performance was not found consistently in all the given word problem.*

INTRODUCTION

There is a shred of emerging evidence that dyslexia is linked to mathematics difficulties. Although dyslexia is often understood as a reading and writing disorder, studies have reported that dyslexic children and adults are slower and less accurate in remembering arithmetic facts than those non-dyslexic children and adults (Simmons & Singleton, 2006). This is due to the fact that dyslexic children's phonological processing deficits have an adverse effect on the development of arithmetic fact memory (Simmons & Singleton, 2008). Similarly, Simmons and Singleton (2008) report that the main difficulty of dyslexic children is the ability to remember number facts so that they are slow in calculating or verifying sums of numbers. In this case, the memory footprint for an arithmetic question may deteriorate before the answer is calculated. In addition, slow computations can exacerbate this problem as it increases the required times to store the problem in working memory while the answer is computed. In particular, Cornoldi et al (2021) show that a dyslexic student not only has difficulty in reading and writing in terms of alphabetic material, but also numerical material such as symbols.

Nevertheless, some studies indicate insignificant correlation between students' mathematical performance and the symptoms of dyslexia. For example, Simmons' (2002) study showed that there was a statistically significant relationship between non-verbal reasoning ability and place value understanding, but there was no significant relationship between phonological circle function and place value understanding in children aged 7 to 11 years. This finding motivated some researchers to investigate further whether dyslexic weaknesses in processing numbers in mathematical tasks were mainly related to language processing weaknesses (e.g., problems with number facts and exact calculations) or weaknesses in performing mathematical processes, such as comparing a quantity and estimating the results of calculations. This simple question remains a source of controversy. Simmons and Singleton (2009) found that dyslexic children have slower and less accurate memory of numerical facts than those non-dyslexic children, but it has an undisturbed understanding of place value. In addition, Simmon and Singleton (2008) concluded that the existence of the dyslexic group's arithmetical weakness could not be attributed to their dyslexic difficulties or due to their weaker intellectual abilities. More specifically, they added that the aspects of mathematics that are less dependent on verbal codes (e.g., estimation, subitizing) are not impaired. This is reinforced by the findings of Träff and Passolunghi (2015) that dyslexic students performed worse than students in the control group on number fact-taking, multi-step arithmetic problem solving, and multi-digit computation. Their scored arithmetical approximations and conceptual understanding such as place value and principles in count operations did not differ from those in the control group.

There are several research on investigating dyslexic students' number processing skills, however, it is still less and underreported. Place value understanding becomes the main factor affecting students' success in giving solutions on number processing tasks. In relation to dyslexic students' performance on place value, there is evidence that dyslexic students are less accurate and slower in multiplying two single-digit numbers in non-verbal tasks (Boets & De Smedt, 2010). Another finding with the non-verbal task is reported by Koerte et al (2016) that there is no significant difference between the group of dyslexic and non-dyslexic students regarding their performance on nonverbal number line tasks, which is still linked to place value understanding.

While researchers have focused on the number processing skill on whole or natural numbers (e.g. Träff, Desoete, & Passolunghi, 2017; Teixeira & Moura, 2019), research on how dyslexic students performed place value understanding on fraction-related tasks is not reported yet, whereas place value is important as a basis for understanding fraction operations. Place value understanding is important since it can be used to figure out that the numerator and denominator of a fraction were not made up of different groups of place value. Therefore, research on number processing skills that specifically discuss dyslexic students' performance on fraction operation needs to be further studied.

In performing a solution on any fraction-related task, a solver, including a dyslexic student, needs to be aware of the existence of errors when providing the solution. According to Siyepu (2013), an error is an incorrect answer due to planning, where this error is done systematically because it is applied regularly in the same situation as a symptom of the conceptual structure that causes the error. It may be found from students' previous learning, either in mathematics class or from their interactions with the social and physical world (Smith et al., 1993). More specifically, errors in fraction operation have been identified by Brown and Quinn (2006) and organized into six main categories, namely algorithmic applications, applications of basic concept on fraction operation in a word problem, elementary algebraic concept, specific arithmetic skills for algebraic understanding, comprehension of the structure of rational number, and computational fluency. The first two categories become crucial aspects who learn fraction in primary school, need to be proficient as they are frequently found in the students' solution strategies. Regarding algorithmic application as the basic skill on solving fraction operation task, Ashlock (2006) also identified four types of errors, namely incorrectly writing a fraction representing a shaded area of a figure, failing to simplify fraction into the simplest form, incorrectly dealing with numerator and denominator of a fraction when adding or subtraction two fractions. Hwang and Riccomini (2021) also identified the most common errors in students' solutions to the fraction operation task, namely failing to decompose mixed numbers into integers and fractional parts or converting mixed numbers into ordinary fractions when performing addition operations.

This study tried to unpack dyslexic student errors on fractions through a fraction operation task covering both verbal and non-verbal information. Thus, the aim of this study is to analyze the errors of dyslexic student in performing solution strategies on fraction operation tasks covering addition, subtraction, multiplication, or division.

RESEARCH METHOD

Research Design

The present study used a case study research design, which was used to investigate contemporary phenomena in-depth and in the context of the real world (Yin, 2014, p. 237). It was to answer the "how" and "why" questions (Yin, 2014, p. 2), which were relevant to the present study. This study aimed to uncover how a dyslexic student solved problems related to fraction operations by focusing on investigating the types of errors that might occur and investigating why they occurred in a Forum Group Discussion activity with students' parents, mathematics education experts, and outside education experts, ordinary students, and teacher of the participating students. In addition, the researchers had little or no control over the events that occurred during the interview (Yin, 2014).

The student participant, i.e., the dyslexic student, was recruited by means of a letter of consent that the children gave to their parents. At the interview times, she was 13 years old and had normal or corrected-to-normal visual acuity with no hearing loss.

A task-based interview was prepared by writing a semi-structured interview guideline and a set of fraction operation tasks. The set of fraction operation tasks was designed and developed by focusing on the combination of text types, i.e., verbal and non-verbal for every task. A group discussion consisting of ten teachers, the researchers, and an expert in mathematics education was involved in a forum group discussion to review the initial draft of the task. The fraction operation tasks were designed and developed by focusing its feature on four basic fraction operations: addition, subtraction, multiplication, and division. Some of them were in the non-verbal text (symbolic and figural) or verbal text only, while others combined non-verbal and verbal text.

Fraction operation	**Types of text**
Addition	Verbal and non-verbal (figural & symbolic)
Subtraction	Non-verbal (symbolic)
Multiplication	Verbal
Division	Verbal and non-verbal (figural)

Table 1: Feature of task

Table 1 indicates the distribution of tasks regarding the types of text resulted from the revision of the initial draft after the review process. Figure 1 depicts an example of the task of fraction addition and subtraction. The addition task asked student to represent two fraction models as two different fractions and further added those two fractions, while the subtraction task asked the student to subtract two mixed number with different denominators.

Solve the following questions!

$$4\frac{1}{2} - 1\frac{3}{4} = \cdots$$

Write fractions that suit to the following two pictures and solve the questions!

Figure 1: Example of addition and subtraction task

The interview activity was conducted through an online platform synchronously and had been recorded for about an hour. During the interview, the student participant explained how to answer the given question with the guidance of the interviewer. Through the interview, the interviewer got the student participant's thinking process in the fraction operation task.

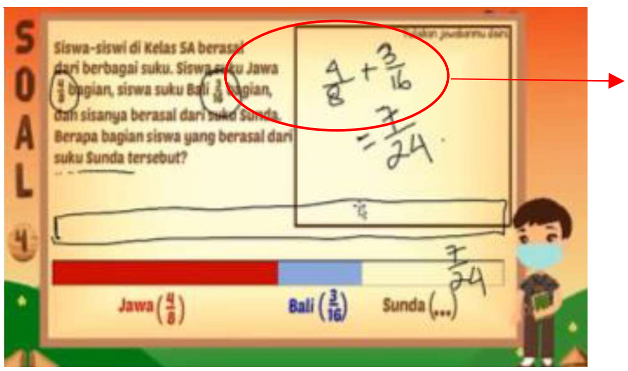
Data Analysis

Data analysis was conducted through a Focus Group Discussion (FGD) of several experts in East Java, Indonesia. In general, the FGD aimed to explore the level of consensus of the participants on the interpretation of the work carried out by students. Technically, the researchers presented the recorded video of interviewing the student participant and showed some student participant's responses on the task. On the other hand, the FGD was also used to collect student participant's opinions, ideas, and beliefs of the FGD participants on topics related to how normal students compared with dyslexic students in terms of solving fraction operation problems, how the symptoms of dyslexia on the mathematical ability of dyslexic children, and issues related to relevant to the research discussion.

RESULTS AND DISCUSSION

Errors in Addition and Subtraction

The feature of the task for addition of two fractions was verbal and non-verbal (figural & symbolic). A word problem, "The students in class 5A come from various ethnic groups, $\frac{4}{8}$ of Javanese students, $\frac{3}{16}$ of Balinese students, and the rests are Sundanese. How many students are Sundanese?" did not lead the participant to do subtraction, for example, by subtracting 1 as whole by the sum of $\frac{4}{8}$ and $\frac{3}{16}$. Instead, she added the two numbers (symbolic code) emerging in the written text without any further consideration of the contextual meaning of the word problem. She wrote the sum of $\frac{4}{8}$ and $\frac{3}{16}$ was $\frac{7}{24}$. This could be considered as overgeneralized whole number operations in which the numerator as well as the denominator in two fractions were added. When the interviewer asked her, "Do you use this bar to guide you understand the problem?", she said," Yes. I did". However, she could not explain how she used the bar as part of her solution by concerning the size of the bar representing $\frac{4}{8}$ and $\frac{3}{16}$. Therefore, the students solved the problem without understanding the problem as a whole.

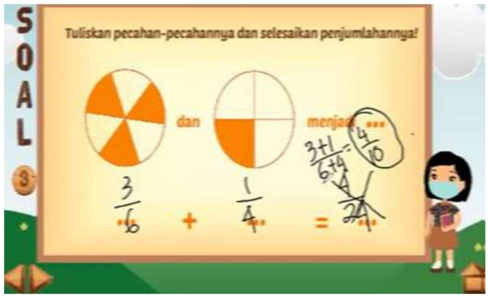


(Overgeneralized whole number operations)

Students in grade 5A came from various ethnic. $\frac{4}{8}$ of them are Javanese, $\frac{3}{16}$ of them are Balinese, and the rest are Sundanese. How many students are Sundanese?

Figure 2: Participant's solution on fraction addition task

This finding was consistent with her performance on another addition/subtraction task, namely adding two fractions by writing the fraction of two area models (see Figure 3). Moreover, the student participant solved the problem without understanding the problem as a whole. She immediately added up the numbers that appeared in the problem. While, she successfully represented the area circle model with correct fraction, she failed to add up the two fractions by adding the numerator and denominator without changing to the same denominator first.

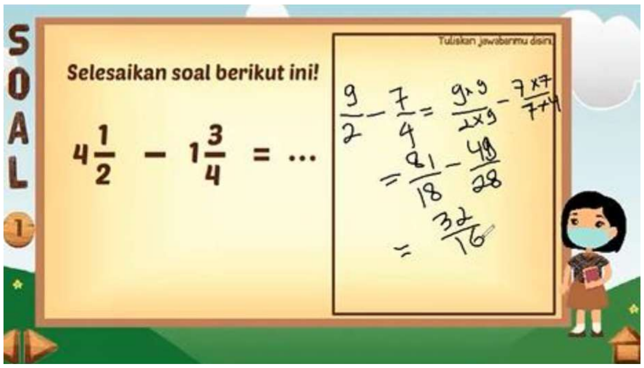


Write fractions that suit to the following two pictures and solve the questions!

Figure 3. Participant's solution on determining and adding fraction

As with most children, the participant overgeneralized operations on fractions such as the fact that the numerator was operated alone with the numerator, and the denominator was also operated independently with the denominator although she successfully converted the shaded figures into appropriate fractions. In this case, no clue indicating the existence of part of the task dealing with the written text as the phonological structure that makes her difficult to understand. From a cognitive perspective, it was possible to process fractions either componential — as two separate integers (3 and 16) or holistically as one (rational) number with one overall magnitude (i.e., the numeric value 3/16). This distinction between component and holistic processing was useful for understanding why people had difficulty in solving fractional problems: many of the errors that students made in fractional problems appeared to be due to their dependence on component processing in problems that required holistic processing.

The student was given word problems, as well as direct computation such as "solve the following problem; $4\frac{1}{2} - 1\frac{3}{4}$". In this problem, the student participant was asked to determine the result of subtracting two mixed fractions. To solve the problem, the student participant converted mixed fractions into common fractions correctly, then she got $\frac{9}{2}$ and $\frac{7}{4}$. In the subtraction operation, the student realized that the denominators were different so it took another step before subtracting. But when the student tried to convert to the same denominator, the student failed to convert it into an equivalent form. She multiplied the first fraction by $\frac{9}{9}$ and the second fractions by $\frac{7}{4}$, so the operation became $\frac{81}{18} - \frac{49}{28}$. Then, she subtracted the numerator and denominator separately without concerning that the denominator was still different. By subtracting the numerator, the student participant got 32. But she incorrectly subtracted the denominator (see Figure 4). She subtracted the smaller number from the larger number but forgot to put negative sign in the result and got the wrong answer. Thus, a hypothesis was rising due to the student participant's attempt to extend the subtraction algorithm for natural numbers and to apply it directly to fractions. In this regard, similar findings of Brown and Quinn (2006) reveal that students in their study subtracted the numerators and subtracted the denominators. Interestingly, the algorithm that the student participant applied was also incorrect. While the subtraction of the numerators was correct, that of the denominator was incorrect. Apparently, she subtracted 18 from 28 leading to the result of 16, which was in this case she also made a slip by writing it as 16 instead of 10. There was a hypothesis that she applied the commutative property of subtraction on a natural number, which was wrong. This needed further clarification.



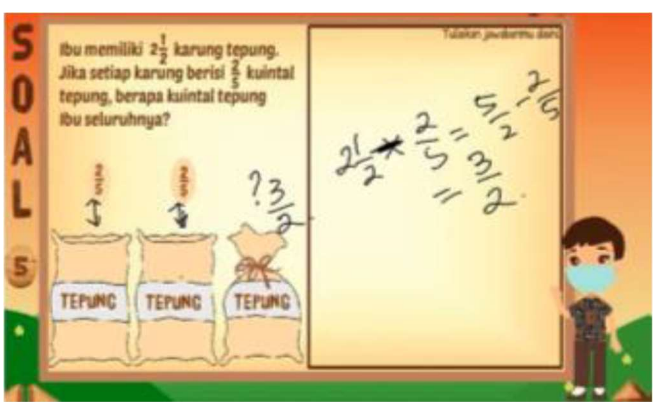
Solve the following problem!

$$4\frac{1}{2} - 1\frac{3}{4} = \cdots$$

Figure 4. Participant's solution on mixed fractions operation

Errors in Multiplication and Division

The tasks of multiplication and division of fractions were given in a word problem. "A mom has $2\frac{1}{2}$ sacks of flour. If each sack contains $\frac{2}{5}$ quintals of flour. How many quintals of flour does mom have in total?". She admitted that she did not really understand the whole problem. This was indicated in the Figure 5. When the interviewer asked her, "What do you think about this problem?", she said "It is difficult". Afterwards, interviewer re-explained the task and the student participant conveyed $2\frac{1}{2} - \frac{2}{5}$ as the solution.

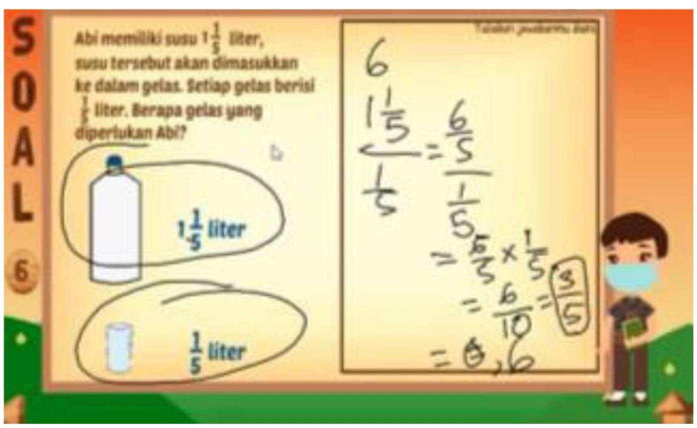


Mother has 2 ½ sack of flour. If every sack contains 2/5 quintals of flour, how many quintals of flour does mother have?

Figure 5. Participant's solution on multiplication of fractions

Figure 5 shows that this problem was supposed to be a multiplication operation problem, but the student participant failed to understand that the solution required the use of multiplication. Instead, the student participant solved this problem using the subtraction operation. In this subtraction operation, the student added the denominator and numerator without converting it to a form with the same denominator. In addition, the student performed a subtraction operation by subtracting a large number by a small number regardless the location of the number. Thus, again, she tried applying commutative rule of natural number operation incorrectly as found in her work on the subtraction task (Figure 4). She also got the wrong answer for 2 – 5, wrote 2 as the solution that led multiple errors, did not understand the operation required, and had not internalized the activity applied.

On the other hand, the student participant succeeded understanding the meaning of a word problem, "Abi has $1\frac{1}{5}$ liter of milk. The milk will be poured in some glasses. Each glass contains $\frac{1}{5}$ liter of milk. How many glasses does Abi need?". This problem was a division operation problem and the student participant was able to take the first step as indicated in the Figure 6.



Abi has $1\frac{1}{5}$ liter of milk. The milk will be poured in some glasses. Each glass contains $\frac{1}{5}$ liter of milk. How many glasses does Abi need?

Figure 6. Participant's solution on division of fractions

According to Figure 6, the first step was right. The interviewer asked, "which part of the question that helped you solving the problem?" and the student participant answered, "the picture, because there is a big bottle and a small glass". Then the interviewer asked, "what about if there is no picture? Will you understand the problem easily?", she said "if there is no picture, I will try to use my imagination". It indicated that dyslexic students often used picture to understand something and had difficulty in understanding a word problem to make any abstraction.

Based on the division of fractions' rules that we needed to convert it into multiplication, the student participant succeeded to convert it into multiplication. However, student participant forgot to invert the second fraction. It should be $\frac{5}{1}$ but she writes $\frac{1}{5}$. In the multiplication operation, the numerator was multiplied correctly, but the student treated 5×5 as $5 + 5$ in the denominator. This finding was quite interesting since most errors within a multiplication problem found when a solver should keep the denominator the same before multiplying corresponding numerators and denominators. According to Yin (2014), this finding was related to the fact that students might believe that if denominators were equal. They should keep the denominator in the solution, otherwise, the denominators should be combined using the operation provided. There was a student's belief that if the size of the denominator of a fraction was the same, then the denominator must still appear in the final answer regardless of the fraction operation used such as multiplication.

Based on the student participant's solution on several fraction operation tasks, it showed her inconsistent behavior in solving fraction operations that led to errors in performing fraction operations. Many researchers argued that such errors occurred because of insufficient understanding of fraction concepts (e.g., Newton, 2014). The possible reason that caused the student participant to think inconsistently in solving fraction operation problems was due to the symptoms of dyslexia that made her difficult in understanding a word problem. This was linear with the student participant's statement that she always looked for numbers or pictures that helped her in understanding a word problem and if there was no picture, she tried to imagine the problem visually. On the other hand, after failed to understand the problem, the student participant often failed to do the fraction operation. This was not caused by her inability to do the operation. Similarly, Singleton (2008) stated that dyslexic students were slow in the calculation because they had difficulty remembering number facts.

In regard to phonological processing during her mathematical computation skills, the moment where the student participant failed to convert all the word problem into a precise mathematical procedure indicated that she might find difficulties in phonological processing when interpreting the written text. It could be explained that there was a relationship between reading skills and general computational skills (Newton, 2014; Yang et al. 2021), which explained the possibility that reading and mastery of mathematics, including number processing skill, might influence the growth of phonological processing (Hecht, 2001).

The results of this study also addressed to other unanswered questions to be further studied regarding the cognitive processed performed by a dyscalculic student to solve fraction operation tasks. It was interesting when the student participant tried to focus on the symbolic information (e.g., finding any number within the whole text) instead of the written text information. It challenged to understand whether her preference was due to the symptoms of dyslexia or her weak number processing skills. Thus, her actual cognitive processes needed to be investigated through another method such as eye-tracking. In relation with fraction, for instance, eye-tracking could examine whether an individual could solve a fraction comparison task using componential strategies, which relied on the fraction numerators or denominators, or a combination of both, or holistic strategies that concerned on the magnitude of fraction (Obersteiner & Tumpek, 2016). In normal people, eye-tracking had also been reported as a tool to measure the amount of fixation concentrated on the denominator or numerator of a fraction when comparing fractions or even adding fractions (Huber, et al 2014). Thus, processing the denominator of a fraction tended to require more cognitive effort than processing the numerator of a fraction. How the implications of this finding with the alleged performance of a dyslexic student when comparing or adding two fractions needed to be investigated further.

CONCLUSIONS

The present study shows dyslexic students' difficulty in solving fraction operation tasks. The tasks consist of addition, subtraction, multiplication, and division problem that are presented in a word problem or calculation task. Findings also suggest that a dyslexic student often thinks inconsistently in doing fraction operations, such as generalizing integer operations when adding two fractions with different denominators and applying incomplete parts of the "inversion and multiplication" algorithm. Moreover, she experienced difficulties in understanding a word problem indicated by her failure to translate the textual information into appropriate mathematical symbols and operations. However, when some prompts are given, they seem to understand the meaning of the task more easily. For some cases, they need pictures and numbers to help them understand the problem. It is speculated that the phonological glitches seen in student participants' performances in word tasks are not consistently seen in all given word tasks. This reinforces the previous finding that understanding of values does not seem to rely much on phonological processing, including those related to fractional operations.

Although the findings of this study may add relevant literature towards the insights on how dyslexic students deal with the number operation, a weakness was the limited space and time to work with the student participant due to the challenge of having an interview with her in this Covid-19 pandemic situation that may affect the internal validity of the findings. We need to understand the correct moment of dyslexic students working and explaining their solution. The potential future research in connection with this study finding is the incorporating eye-tracker tools and its developed software to accurately investigate the dyslexic students' cognitive process.

ACKNOWLEDGEMENT

The authors were grateful to DRPM Dikti for giving us Applied Research Grant with grant number 288/SP2H/LT/DRPM/2021. We also thanked SD Laboratorium Unesa, Indonesia, for becoming the research partner and assisting the dyslexic student for data retrieval.

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