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A Review of Math Girls 3: Gödel's Incompleteness Theorem by Hiroshi Yuki and How It Can Be Used to Teach Introductory Proofs

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Abstract: Math Girls 3:Gödel's Incompleteness Theorem is a student led adventure through advanced mathematics topics, proof writing, and finally the proof of the title theorem itself. We offer a review of the book from the perspective of both the casual read and its potential for use in an undergraduate Introduction to Proofs course. We discuss both how a teacher trying to connect disparate topics in an Introduction to Proofs course may consider using the book, as well as the academic and emotional guidance it can give a student working through understanding proof writing for the first time.

Introduction

Hiroshi Yuki has branded a new type of mathematics novel where the author teaches mathematics to the reader through interactions between three female students and one male narrator. His first two Math Girls books on infinite series (Yuki, 2011) and Fermat's last theorem (Yuki, 2012) were so well received when translated into English from Japanese by Bento Books that he created three additional books more accessible to a younger audience on integers, equations and graphs (Yuki, 2014), and trigonometry (Yuki, 2014). We consider his third Math Girls book on Gödel's Incompleteness Theorem in this review and are excited that the remaining Math Girls books on Randomized Algorithms, Galois Theory, and the Poincare Conjecture are now available in their English translations as well. Many mathematical novels as well as nonfiction mathematical stories exist (Doxiadis 2000, Jackson 2000) weaving the stories of mathematicians, their struggles and obsessions, as well as descriptions of famous problems together. Few strive to teach the reader mathematics from the ground up in a story telling fashion as Yuki does. We describe the book and some specific topics it covers, then discuss potential impacts on a student struggling in mathematics and conclude with how the book could be used in an Introduction to Proofs course.

Book Topics Description

Math Girls 3 starts with entertaining riddles and takes the reader into the world of math from a student perspective. While constructing a compelling narrative of the narrator's emotional turmoil during the school year with his study group (Yuri, Tetra, and Miruka), Yuki is able to teach Gödel's Incompleteness Theorem and the necessary preliminary mathematical topics. The story starts with the narrator and his cousin, Yuri, walking through logic problems together and slowly builds an understanding of surrounding topics until the reader is ready to grasp Gödel's Incompleteness Theorems. While the book is centered around Gödel's Incompleteness Theorems, it is not necessary for the reader to fully devote themselves to learning Gödel's Incompleteness Theorem to be able to gain meaningful insight from Hiroshi Yuki's, "Math Girls 3: Gödel's Incompleteness Theorem." Advanced concepts covered in this book include Peano's axioms (Chapter Two), set theory (Chapter Three), limits (Chapter Four), formal systems (Chapter Five), proof theory (Chapter Five), epsilon-delta proofs (Chapter Six), diagonalization (Chapter Seven), and trigonometry (Chapter Nine).

Use in an Introduction to Proofs Course

When constructing an Introduction to Proofs course for undergraduate students, there are varied, seemingly disconnected topics that must be covered before being able to prove anything. Set theory must be known to show that the set of two things are equal and notation that is used in proofs must be known to be able to read proofs. Jumping from topic to topic before really getting into understanding what constitutes a proof can make students struggle to find connections between the concepts, which may cause them to feel developing an expertise in proof writing is unattainable. While unlikely to be sufficient as a stand-alone text for an Introduction to Proofs course, the book can serve as a narrative for students to read throughout the semester and a more standard textbook may be used to go into more depth to each topic. Students primarily use textbooks to look for examples of how to do specific types of problems (Benesh, 2006) and often skip reading the narrative, if it is present, on how concepts connect to each other. Math Girls 3 is less dense than a traditional textbook and can serve to motivate students on why they are learning a particular concept or proof technique (Hembree, 1990). Additionally, few math and science books adequately represent female students (Becker 2021), which Math Girls 3 refreshingly flips. Storybooks have been shown to have a positive impact on mathematical vocabulary and problem-solving skills in elementary age students (Hassinger-Das 2015, Wangid 2021) but is seldom used at higher educational levels. Furthermore, an Introduction to Proofs course is a significant transition for mathematics students between the problems-based courses such as Calculus and Differential Equations to proof based courses such as Analysis and Abstract Algebra. This transition can cause students to become disheartened about mathematics due to their struggles to understand elements of proof. Math Girls 3 stresses the importance of learning how to embrace the struggle of learning new topics and the reader will find themselves fitting in with the characters

that each serve a niche in the classroom. For those who are able to grasp concepts quickly, dialogue in the study group will give different perspectives of the same topic, providing a deeper understanding of the topic. For those who feel disparaged because they need to work harder to grasp the same concepts, the narrative provides constant reassurance of the hard work required to understand material. With each topic that is taught, Yuki provides a character who fits different types of students in the classroom, producing a moment of clarification to ensure a secure understanding is being built.

Yuki is able to build empathy with the reader by illustrating the anguish that all mathematicians at all levels have felt when it seems that the problem at hand is too overbearing to be able to conquer. The book provides opportunities to gain new perspectives from “peers" when a student is unable to articulate their struggles or feelings of inadequacy to others. Students may find themselves becoming study partners with the narrator, Tetra, Miruka, and Yuri with well-constructed dialogue that allows students a window to what a constructive partnership with classroom peers can be.

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