Teachers' Skills for Attending, Interpreting, and Responding to Students' Mathematical Creative Thinking

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Abstract: *This study aimed to explore the mathematics teachers' skills in attending, interpreting, and responding to students' mathematical creative thinking. The data of this study comprised of the teachers' skills in attending, interpreting, and responding to students' mathematical creative thinking gained from observing a recorded video of the teachers' teaching enactment. The data were collected inductively with open coding to examine classroom teaching. Findings suggest that the mathematics teachers raise the attending skills in two categories: activities directly related to students' mathematical creative thinking and activities that support students' mathematical creative thinking. They interpret mathematical understanding in various ways: excluding the justification of right or wrong answers; focusing on right or wrong; focusing on the only correct solution; focusing on deficiencies in the student's working process; focusing on how they wrote and drew on grid paper and anything else indirectly connected to mathematical thinking; stating that working on the problem is easy; comparing the thinking process of the teachers and students. The mathematics teachers responded by giving comments or questions about their students' knowledge, idea, procedure, or mathematical thinking and based on mathematical creative thinking; giving general comments or questions; giving comments or questions to trigger students to share their opinions; asking other students to comment or ask questions about certain students' thinking ideas; asking other students to explain certain students' thoughts; giving comments or questions about students' mathematical creative thinking/open-ended problem/critical thinking.*

INTRODUCTION

The skills of attending, interpreting, and responding to students' thoughts have been the focus of educational research. Those three skills are vital learning components to support students' learning so that the effort to comprehend and develop them becomes one of the focuses of mathematics learning (Luna & Selmer, 2021). Those essential skills need to be developed during teacher training since they affect the effectiveness-based learning and improve students' mathematics competency (Sánchez-Matamoros et al., 2019). Those skills are described as practices with challenging development, yet they could be learned (Tyminski et al., 2021), could be developed from time to time (Jacobs et al., 2010; van Es & Sherin, 2008), and one of the essential components of teaching mastery and learning quality (M. Y. Lee, 2020). Those skills are the tools to assess someone's teaching practice and improve learning (Barnhart & van Es, 2015).

Some experts proposed the connections between the skills of attending, interpreting, and responding to teachers' competencies. Teachers need to attend to specific mathematics ideas on students' papers and produce logical feedback to interpret students' thoughts used later for responding (Krupa et al., 2017). Most teachers showed evidence of attending to the students' thoughts, but they showed a fewer evidence of interpreting students understanding as well, much less evidence of how to respond to students' thoughts based on their understanding (Larochelle et al., 2019). Responding skills seem to be the toughest skills to develop (Barnhart & van Es, 2015; Jacobs et al., 2010; Tyminski et al., 2014). In other words, teachers tend to attend and interpret their students' mathematical thinking instead of responding to them (Land et al., 2019).

Many studies of attending, interpreting, responding to students' mathematical thinking skills have been widely carried out. Research related to these three skills by mathematics teachers in various fields of mathematics studies has also been widely carried out (such as Jacobs et al., 2010; Kiliç & Masal, 2019; Nagle et al., 2020; Sánchez-Matamoros et al., 2019; Walkoe, 2014). Research by Jacobs et al. (2010) focused on integer operations. Previous research by Walkoe (2014) revealed that using video clubs helps teachers be more consistent in following the substance of students' algebraic thinking and reasoning about students' algebraic thinking. Research by Sánchez-Matamoros et al. (2019) used derived material and the result was that students connected the rate of change to the slope of the line and the instantaneous rate of change to the slope of the tangent. Research by Kiliç and Masal (2019) used algebraic material. Research by Nagle et al. (2020) employed statistical material and the results are teacher interpretations that are often evaluative and tend to describe student processes. Research on the three skills is reviewed from a variety of strategies (such as Araujo et al., 2015; Kristinsdóttir et al., 2020; Krupa et al., 2017; Nagle et al., 2020; Roller, 2016; Sánchez-Matamoros et al., 2019). Research by Sánchez-Matamoros et al. (2019) used students' written answers to explore the relationship between the three skills to prospective high school mathematics teachers on students' mathematical understanding. Research by Krupa et al. (2017) designed a curriculum module consisting of pre-and post-assessment, reading, class discussions, structured interviews with high school students, and written reflection to develop pre-service teacher attention in students' mathematical thinking. Research on the three skills involved both teachers and prospective secondary school mathematics teachers (such as Baldinger, 2020; Dyer & Sherin, 2016; Krupa et al., 2017; Larochelle et al., 2019; Nickerson et al., 2017; Roller, 2016; Sánchez-Matamoros et al., 2019; Simpson & Haltiwanger, 2017; Styers et al., 2020; Wallin & Amador, 2019). Research by Dyer and Sherin (2016) identified three types of instructional reasoning about the interpretation of students' thinking used by teachers: (a) making connections between certain moments of student thinking, (b) considering the relationship between students' mathematical thinking and the structure of mathematical tasks, and (c) develop students' thinking tests.

Many researchers define creative thinking from various points of view. Creative thinking is the ability to generate novel ideas or solutions in a problem-solving process (Hadar & Tirosh, 2019), as a mental activity that is used to construct an idea or notion of the "new" (Siswono, 2014), as the ability to generate new ideas or solutions and select unique or the most useful idea or solution to develop or apply in action (Tran et al., 2017). Mathematical creative thinking is the competence to engage productively in the learning, evaluation, and improvement of ideas that can result in original, practical solutions (Suherman & Vidákovich, 2022). The researchers use different indicators in their creative thinking research. Research conducted by (Leikin & Lev, 2013), (Elgrably & Leikin, 2021), and (Levenson, 2022) uses indicators of fluency, flexibility, and originality. The researchers (Sahliawati & Nurlaelah, 2020) used concepts of fluency, flexibility, elaboration, and originality.

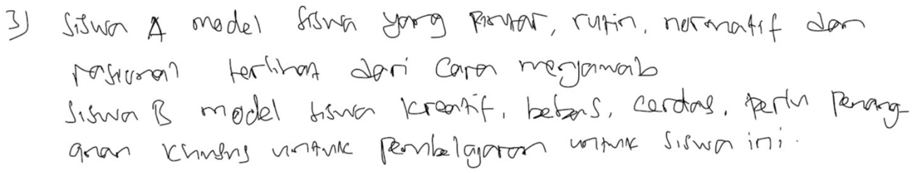
In this research, mathematical creative thinking is defined as the ability to generate ideas in solving mathematical problems with indicators of fluency, flexibility, and originality. In this research, fluency in mathematics is a person's skill produces many mathematically correct answers that is not duplicated, and can generate many meaningful ideas/possibilities/approaches in solving problems. Flexibility is a person's ability to change focus, use different thinking strategies, use various representations, or relate different mathematical topics and is measured based on the classification of student completion in categories and then the number of categories with correct answers is calculated or measured through the number of different methods carried out in solving problems. Originality is a person's ability to produce problem-solving using insights that are new to him.

Creative thinking is closely related to mathematical thinking. Creative thinking is a subcomponent of mathematical thinking (Kattou et al., 2013)**.** On the other hand, Schoevers et al. (2020) noted that general creativity and mathematical ability could predict mathematical creativity better than general and mathematical creativity. Teaching creative mathematics has become something needed (Luria et al., 2017). Teachers cannot teach creativity and the more teachers teach, the less opportunities students have for creative thinking (Baker et al., 2020). Some studies on mathematics teachers' competency related to mathematical creative thinking are conducted by Levenson (2013), Luria et al. (2017), and Levenson (2021). When deciding the tasks to promote students' mathematical creative thinking, the teachers' considerations should be based on values. The practical value of mathematical creative thinking is various ways to solve mathematics problems (E. S. Levenson, 2021). Teachers could implement a strategy to promote students' creative thinking in the classroom and develop equity principles. The principles include presenting open-ended problems, modeling and encouraging risk-taking, discussions, debating mathematical concepts, concept-based learning, divergent thinking strategies, and incorporating cultural awareness and creativity into curricula and classroom environments (Luria et al., 2017).

The research examines mathematical creative thinking in terms of various points of view. Creative thinking is an educational goal (for example Hadar & Tirosh, 2019; Pendidikan et al., 2018; Tabach & Friedlander, 2017). Based on the Indonesian Ministry of Education and Culture Regulation No 36 of 2018 cited from Pendidikan et al. (2018), Indonesia also lists "creative" as the goal of the 2013 Curriculum. Creative thinking research is competency and stimulus for teachers and prospective teachers (for example, Ayele, 2016; Kaiser et al., 2015; K. H. Lee, 2017; Sánchez et al., 2021; Siswono, 2015). Research by Sánchez et al. (2021) investigated the development of creativity in mathematics classes for pre-service teachers of a secondary school teaching master's program, who were not trained on how to develop creativity. Research on creative thinking as a stimulus to student creativity (for example Bicer et al., 2020; Elgrably & Leikin, 2021; Kurniasih et al., 2020; Molad et al., 2020; Sánchez et al., 2021) as well as research on a person's typology is said to be creative thinking (eg. Aljarrah, 2020; Lassig, 2020). Research by Kurniasih et al. (2020) found that problem posing, asking questions, and using songs were used by 5th-grade elementary school teachers in mathematics lessons to facilitate students' mathematical thinking. Research by Lassig (2020) revealed that there are three types of creativity, namely creative personal expression, boundary-pushing, and task achievement.

A preliminary study for the present research involving junior high school mathematics teachers in Central Java was carried out on May 31, 2021, using PISA questions which were used to measure mathematical creative thinking and the answers of 2 8th grade students. PISA questions, for example in PISA 2012 were used to measure students' creative problem solving abilities (Yang & Fan, 2019) and PISA questions could be used to measure students' creative knowledge in everyday life (Komatsu & Rappleye, 2021). Five teachers were asked to answer questions with open responses related to the skills of attending, interpreting, and responding to students' creative thinking based on PISA questions and 2 students' answers.

Based on the teachers' response to question 2 from the preliminary study with the question "explain in detail, according to you, what each child did in response to the problem", it was known that they attended students' thinking by explaining their activities and their thinking processes (one teacher), writing down the ideas the students chose to solve the problems (three teachers), explaining students' activities when solving the problems (one teacher). Question 3 was related to how they interpreted students' thinking. The results were that the teachers interpreted students' thinking by knowing their deficiency during the process of doing the problems (one teacher), interpreting their thinking process (three teachers), and interpreting their personalities (one teacher). Question 4 related to how teachers responded to students' thinking. They brought up new tasks asking students what steps they took to solve the problems (three teachers) and created new assignments that helped students evaluate their thinking process (two teachers). One of the teachers' responses to answer question 3 shown in Figure 1.



3) Student A is a student who is smart, orderly, normative, and rational as seen on the answers. Student B is a student who is creative, free, smart, and need personal touch.

Figure 1: The teacher's response to question 3

The teacher argued that student B was creative, free, intelligent, and needed personal touch (see Figure 1). This indicated that the teacher interpreted creative thinking by stating that student B was creative. In other words, teachers interpret the personalities of student B. However, the teacher did not provide more explanation to support the statement.

During preliminary research, the teachers demonstrated the activities of attending and interpreting but their comments were not directed to students' mathematical creative thinking. While the problems asked students to conduct creative thinking. One teacher responded by commenting that student B was creative, but it was not supported by any evidence (see Figure 1). Hence, further research is needed to study the teachers' attending, interpreting, and responding skills toward students' creative thinking in learning mathematics in class.

The research objective is to explore the teachers' attending, interpreting, and responding skills to students' mathematical creative thinking. The results of this study are expected to be the first step for further research involving training to identify the characteristics of attending, interpreting, and responding to students' mathematical creative thinking in the setting of teacher professional development. It is supported by Yaakob et al. (2020) that effective teacher professional development is carried out in the form of training.

RESEARCH METHODS

This study applied a qualitative approach with a grounded theory research design. The subjects were three junior high school teachers from different cities in Central Java Province, Indonesia. One teacher had taught for more than 20 years, coded with P1. Another teacher who had taught for 10 to 20 years are coded with P2. Next, a junior teacher who had taught for less than five years is coded with P3. The three junior high school mathematics teachers stated that they were willing to participate in this research. The main requirement for teacher involvement is a teacher who has experience in teaching mathematical creative thinking. P1 is an administrator of the Association of Mathematics Teachers in Semarang, Central Java, and is actively involved in research activities on creative thinking with the first author of this article. P2 is the coach of the student mathematics Olympiad at the school where she teaches. P2 is used to invite Olympiad fostered students to think creatively in solving Olympic mathematics problems. P3 is a teacher who has taught mathematics for less than 5 years. He is a graduate of the mathematics education study program at one of the universities in Central Java and his final thesis has the theme of mathematical creative thinking.

The data is the description of the practice of attending, interpreting, and responding to students' creative thinking carried out by the three teachers The three teachers carried out mathematics learning in 3 meetings each and the lessons were recorded on video. P2 and P3 taught function and linear equations, while P1 taught the Pythagorean Theorem.

Data analysis in this research was carried out following Miles et al.'s (2014) framework. First, researchers critically watched the recorded video and transcribed it into written texts. Next, the researcher reduced the data by choosing information related to the teachers' activities of attending, interpreting, and responding to students' creative thinking. The activities of the teacher attending, interpreting, and responding to students' creative thinking are grouped. The researcher did the coding by compiling an inductive code based on the data that appeared in the learning by each teacher as a category and subcategory of each competency. The learning carried out by teachers involves the interaction of individual students with teachers, groups of students with teachers, and all students with teachers. So, the general coding for students' interaction with the teacher was A1, students in groups with the teacher was A2, and whole students with the teacher was A3. The skills of attending, interpreting, and responding are symbolized by the letters A, I, and R, respectively. The description of the categories and subcategories of each inductively acquired skill is presented in Table 1.

Noticing Component (Code)	Category (Code)	Subcategory (Code)
Attending (A)	Activities that are directly related to students' mathematical creative thinking (X)	Attending the process/result of students' creative thinking (X1)
		Asking for an explanation about their thinking steps (X2)
		Detailing or not detailing students' thinking strategies (X3)
		Asking for justification for their reasoning (by giving guided questions, giving hints/keywords, bringing up sentence

		phrases, asking students using "How and Why") (X4)
		Reminding the concepts and principles they had learned or the relevant ones (X5)
		Emphasizing what they had done (X6)
	Activities that support students' mathematical creative thinking (Y)	Attending their articulation, how students draw, and gesture when students explain their thinking (Y1)
		Allowing their mathematical reasoning development (provide the widest opportunity for students to explore mathematical ideas, provide opportunities for students to state true or false the results of thinking students or other groups of students, say that students can get various answers) (Y2)
		Asking them if they had finished the tasks (Y3)
Interpreting (I)	excluding justification of right or wrong answers (B)	-
	Focusing on the correctness or incorrectness of a solution (showing which one was correct or incorrect, putting a checkmark on the right solution, using words to interpret correct or incorrect solution implicitly, or showing a smiley face) (C)	-
	focusing on the only correct solution (D)	-
	focusing on the deficiency of students' working process (E)	Adding important information missing (E1)
		Asking for the clarification of students' statements (E2)
		Giving questions or comments about students' reasoning to check whether the student's answer is correct or not (E3)
		Pointing out students' mistakes related to the procedures of doing the problems (E4)
		Asking if their students were aware of their mistakes made in the process (E5)

	focusing on how they wrote and drew on grid paper and anything else indirectly connected to mathematical thinking (F)	-
	stating that working on the problem is easy (G)	-
	comparing the thinking process of the teachers and students (H)	-
Responding (R)	giving comments or questions about their students' knowledge, idea, procedure, or mathematical thinking and based on mathematical creative thinking (K)	Giving comments/questions to examine students' thinking (such as using why and how questions) (K1)
		Giving questions/guided comments to help students think creatively about mathematics (K2)
		Giving follow-up questions to confirm students' mathematical reasoning (K3)
		Giving comments/questions about students' thinking mistakes (K4)
		Giving comments about the relevant concepts/principles/calculations (K5)
	giving general comments or questions (such as any question, do you understand, can you do it, and what is the conclusion) (L)	-
	giving comments or questions to trigger students to share their opinions (M)	-
	asks other students to comment or ask questions about certain students' thinking ideas (N)	-
	asking other students to explain certain students' thoughts (O)	-
	giving comments or questions about students' mathematical creative thinking/open-ended problem/critical thinking (P)	-

Table 1: Categories and subcategories of attending, interpreting, and responding to students' creative thinking raised by the three teachers and obtained inductively

RESULTS AND DISCUSSION

P1 delivered learning material about the Pythagorean Theorem. As shown in Figure 2 below, he applied various attending, interpreting, and responding patterns.

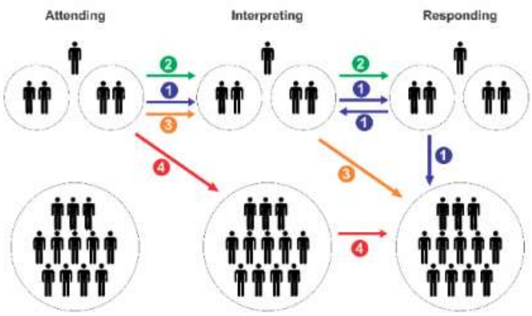
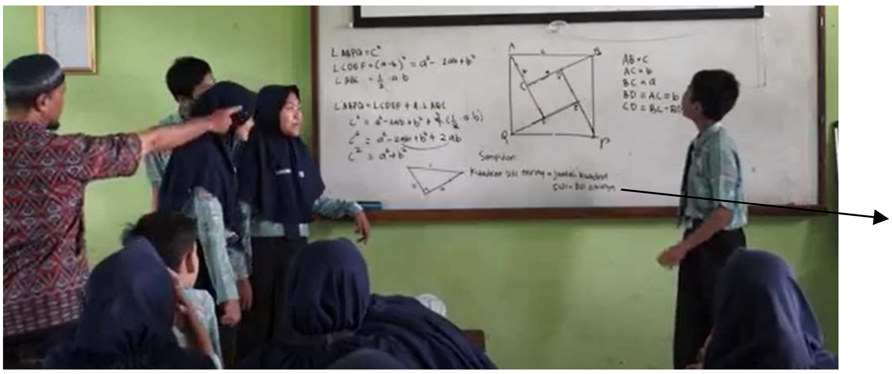


Figure 2: P1's Patterns

P1 performed four patterns (see Figure 2). Pattern 1 was AA2-IA2-RA2-IA2-RA2-RA3 as the dominant pattern done by him. On pattern 1, the interpreting skills were followed by repeatedly responding to the interaction between a group of students with P1 until they understood. This process was ended with the teacher classically responding to students. Pattern 2: AA2-IA2-RA2. Pattern 3: AA2-IA2-RA3. Pattern 4: AA2-IA3-RA3.

In learning meeting 1 with the material on Proving the Pythagorean Theorem, 4 kinds of mathematical problems are provided to prove the Pythagorean Theorem. Every 2 groups of students get 1 task to prove the Pythagorean Theorem. However, in presentations by group representatives and classical discussions, only 2 ways of Proving the Pythagorean Theorem are discussed in class. The results of the work of proving the Pythagorean Theorem by one group of students are presented in Figure 3 below.



Conclusion: Square of hypotenuse = sum of the square of the other sides

Figure 3: The results of the work of proving the Pythagorean Theorem

Based on Figure 3, P1 performs the skills of attending, interpreting, and responding to students' thinking focus on the interaction of a group of students with the teacher by the pattern 2: AA2-IA2-RA2. In attending, P1 performs the activities coded by AA2, the category for attending coded by X and Y. The subcategories of X that are done by P1 were X2, X3, and X4. The subcategory of Y is done by P1 only Y1. P1 asked one of the groups to write down the results of their work on the blackboard as shown in Figure 3 above. After finishing the work, P1 asked one of the group representatives to explain the results of their work. P1 applies the skill of attending, coded X2 by saying, "Now you read it. Explain this one. (pointing to the results of student work)". The following is an excerpt from the conversation between P1 and student representatives in a group (PS).

P1: Come on now you read. Explain this one.
PS: Square…(silent)
P1: Explain this. Come on, explain. The ABPQ square is formed by? (points to the image as shown in Figure 3)
PS: ABPQ square is formed…
P1: formed from…
PS: The ABPQ square is formed from…(silent)
P1: How many wakes did it come from? (points to picture)
PS: Five
P1: Explain. There. Explain there (ask students to explain to their friends). If you look at the picture…
PS: ABPQ consists of 5 shapes
P1: That is?
PS: Triangle ABC
P1: How much?
PS: As much as 4
P1: Yes go on. Continue
PS: Square CDEF as much as 1.

It appears in the conversation above, the student was asked to prove the Pythagorean Theorem using some of the rectangle and right triangles used. P1 asks for X4 in reaching conclusions by providing guided questions (e.g., How many shapes are there?, How many?). P1 also provides hints/hints/keywords (ABPQ square formed by ?). P1 brings up sentence phrases (for example, when you look at a picture). For the Y activity, P1 asks students to point to pictures when explaining. This means P1 attending by coded Y1. In another conversation, it was also seen that the P1 activity asked students to point to the picture when explaining. The following is an excerpt of the conversation (the P1's statement in bold).

P1: **Show. Show with your hands**
PS: side AB equals c, side AC equals b, side BC equals a (while pointing to the picture)
P1: Yes. Continue. BD?
PS: BD equals AC
P1: Yes go on
PS: CD equals BC minus BD
P1: **Where's BD? (ask students to point to the picture to show the position of BD). The result (pointing to the calculation result) is?**
PS: CD equals BC minus BD equals a minus b (while pointing to the work result)

In interpreting, P1 performs the activities coded by IA2. The categories for interpreting were coded by B, C, and E. The subcategory for E that is done by P1 was E2. P1 interprets students' understanding with code C by speaking true or false and uses words/sentence phrases as implicit interpretations of true or false answers. When the student's explanation is correct, P1 says "Yes, that's right". The P1's statement means to say the correct answer. P1 also uses the word as an implicit interpretation of the correct answer by saying "Yes go on", as seen in the two conversations above. Based on the student's writing, P1 asked twice why there were $-2ab$ and $2ab$ in one equation that disappeared in the next solution step. Group representatives always answer because the types of variables are the same. This student's answer is wrong. However, the teacher did not state the answer was right or wrong. This shows that P1 interprets with code B. The interpretation of code E is carried out with subcategories E2. The conclusion written by a group of students is "*Square of hypotenuse = sum of squares of other sides*". P1 asks "*Where does the square of the hypotenuse come from? What is the square of the hypotenuse?, Yes, where did that come from? There is the sentence square of the hypotenuse*".

In responding, P1 performs the activities coded by RA2. The categories for responding were coded by K, L, M, N, O, and P. The subcategories of K that are done by P1 were K1, K2, and K3. For the response activity in code K, P1 asks questions to explore students' thinking (using Why?), coded by K1. When the group representative said that the area of ABPQ was equal to c squared, P1 asked the students "*Why is c squared?*". On another occasion, after the group representative explained the results of their work, the teacher asked the students "*Let the children see this. How can it be missing -2ab, 2ab? Why is it suddenly like this? Please explain.*" (points to $-2ab, 2ab$, and $c^2 = a^2 + b^2$). Here's the conversation, P1 responding by "Why".

P1: So that we find that c squared is equal to a squared plus b squared. Now let's take it to the triangle ABC earlier. We bring c here (pointing to side AB). What is a in a right triangle?
PS: hypotenuse
Q1: Why does it say hypotenuse? It's not tilted though? (Points to an image)
PS: Because it is in front of a right angle.

P1 also responds with code L (say hello, got it?). P1 responds with code K2, continued with K3, and ends with K2. This fact can be seen in the following conversation between P1 and student representatives in the group.

P1: (takes a marker and writes on the whiteboard). The variable type is the same. How much is $3a$ plus $5a$?
PS: $8a$
P1: How come it doesn't disappear? It says the type of variable is the same
PS: since it's a plus, it's a minus in the sum (pointing to $-2ab$ and $2ab$)
P1: So what is the total result?
PS: zero

Pattern 4: AA2-IA3-RA3 appears in the lesson by P1 at meeting 2 with the material determining the type of a triangle based on the length of its sides and applying the Pythagorean Theorem. Each group of students is given an investigation sheet to determine the type of triangle based on the existing side length measurements, measure the angle of the triangle using an arc, and compare the squares of the length of the side of the triangle to conclude whether the triangle is acute, right or obtuse. The following Figure 4 is the result of the work of one group of students according to the problem in attachment 2 which is written on the whiteboard.

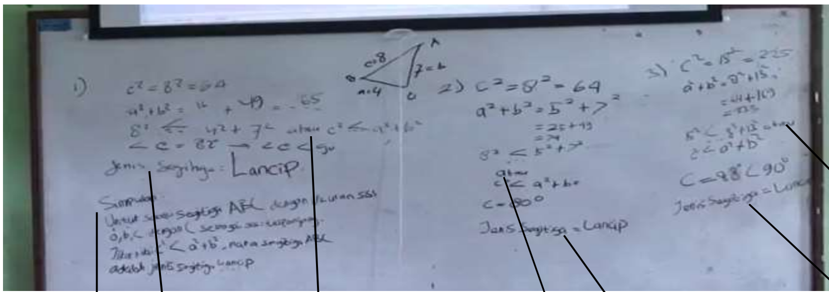


Figure 4: one of the results of the work of a group of students meeting 2 in class P1

Or

Type of triangle: acute

Type of triangle: acute Or Or Type of triangle: acute

Conclusion: For a triangle ABC with sides a, b, c (with c as the longest side). If the value $c^2 < a^2 + b^2$ then triangle ABC is an acute triangle

The activity of attending in pattern 4, P1 the interaction of a group of students, coded AA2, in pattern 4 was carried out in category X, subcategory X1, and category Y, subcategory Y2. P1 knows that students have difficulty writing problems on the whiteboard so P1 assists by writing down some information that students must write on the whiteboard (see Figure 4, work 1). As long as the group representatives write the results of their work on the whiteboard, P1 attended to the results of their work. This fact shows that P1 performs AA2 category X subcategory X1. Then P1 asked another group to state whether the work in Figure 4 was true or false and the group working on the same problem stated that the work was correct. This fact shows that P1 performs AA2 in category Y and subcategory Y2.

The activity of interpreting in pattern 4 is done by P1 in code IA3. P1 interprets students' thinking classically in category E, subcategory E1. P1 asked all students to add important information that should exist but have not been written by students, namely $a, b,$ and c as the sides of the triangle and c as the longest side of the triangle.

The activity of responding to P1 in pattern 4 is carried out in code RA3. P1 responded by giving general questions, category L to all students. P1 asked, "*Do you agree or disagree? Did you understand? There are 3 conclusions, what are the conclusions*?". P1 also responds classically in category K and subcategory K5. P1 writes one of the conclusions and students are classically asked to read the other conclusions together. The following is a conversation between P1 and all students (S).

P1: For a shape (meaning a triangle) whose sides are known, $a, b, and\ c$ with c the longest side. One. If applicable $c^2 < a^2 + b^2$ (read c squared less than a squared plus b squared) or in general language, if the square of the longest side is less than the sum of the squares of the other sides, which triangle is formed?
S: acute triangle (all students answered in unison).
P1: Let's read another conclusion
S: Two. If applicable $c^2 > a^2 + b^2$ (read c squared more than a squared plus b squared) or in general language, if the square of the longest side is more than the sum of the squares of the other sides, the triangle formed is obtuse (all students answered in unison).

During the third meeting, the teacher discussed the kinds of triangles (acute, right, and obtuse) with the Pythagorean Theorem. The teacher gave three problems shown in Figure 5.

Mr. Danar is a mathematics teacher. To teach about two-dimensional figure in the classroom, he would make a teaching instrument of triangle. Based on the previous design, the triangle would have a perimeter of 12 meters.
a. Find out the triangle's sides to make an acute triangle!
b. Find out the triangle's sides to make a right triangle!
c. Find out the triangles' sides to make an obtuse triangle!

Figure 5: Problem 2 in P1's class

A group representative wrote their solution for a problem in Figure 5. The P1's pattern for attending, interpreting, and responding for solution 2a was Pattern 3. Figure 6 below shows the results of the work of one group of students and P1 shows the mathematical principles used.

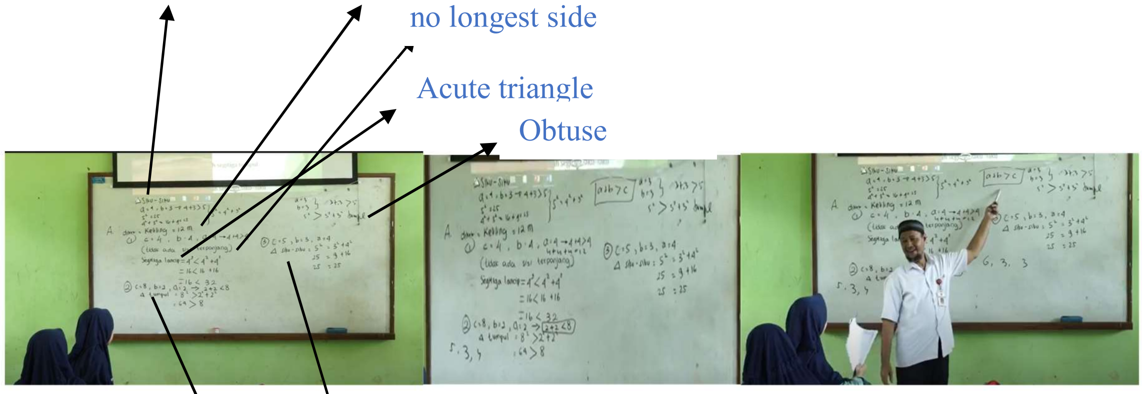


Figure 6: The results of group work and Mr. Rd pointing to the mathematical principles used

The excerpt of the discussion below was between teacher (P1) and student (S)

P1: First, they (he meant the group that wrote the solution) choose $4, 4, 4$ with a requirement to not use the longest side. Yes. Is $4 + 4 + 4 = 12$?
S: Yes
P1: Okay. It means…can I borrow the marker? (The teacher wrote $4 + 4 + 4 = 12$). This is the requirement for the perimeter. It needs the perimeter requirement. The perimeter is 12, then, $4 + 4$ equals what?
S: 8
P1: 8 is more than 4. The requirement of inequality of $a + b > c$ (a plus b greater than c). Right? (Writing $a + b > c$ on the board)
S: Yes.
P1: You need to remember this. You can't do it randomly. You need to use this postulate. You can't choose any random number without meeting this requirement (Pointing to $a + b > c$. Very well. The requirement is to fulfill $4 + 4 > 4$. Right? (Pointing to $4 + 4 > 4$)
S: Correct.

The excerpt shows the attending skills coded AA2 (attending the interaction between students' group and the teacher), category X and subcategory was X3 (in this case not in detail), and X5. P1 reminded triangle inequality and perimeter conditions problem in Figure 5, coded as X5. P1 interpreted the groups' work with the code of IA2, with the category coded was E and the subcategory coded was E1 (the perimeter requirements). Next, P1 responded with the code RA3. The category code was K and the subcategories were K3 and K5. P1 asked if the three measurements $(4, 4, 4)$ fulfill the perimeter requirements, if $4 + 4 > 4$ fulfill the inequality requirement of side lengths to students classically, coded as K3. The P1's responding skills were shown by a classical comment about the relevant concepts of the requirements of perimeter and inequality of a triangle measurement, coded by K5.

Dealing with works on the board in Figure 6 as a solution problem part c in Figure 5, P1 also applied the skills of attending, interpreting, and responding with pattern 1 (AA2-IA2-RA2-IA2-RA2-RA3). P1 used attending skills with the interaction code between a group of students and the teacher. The category of attending was coded by X and the subcategory of attending was X1. Next, the teacher applied interpreting skills of the interaction between a group of students and the teacher (IA2). This category was E and the subcategory E3. P1 asked the groups if $(8, 2, 2)$ fulfill the perimeter and triangle inequality requirements. It turned out that the groups were aware that $(8, 2, 2)$fulfill the requirements of triangle perimeter but did not fulfill the requirement of inequality. P1's response had the interaction code of RA2 with the category coded by K and the subcategory was coded by K4. This interview excerpt shows P1's question, "*Why do you choose?*" The next process was the skills of interpreting coded IA2, and the subcategory E1. That was the information on the instructions written on the discussion paper, the requirements of triangle sides inequality that if the longest side was c, then the formula should be $a + b > c$. The next skills were the responding coded RA2, the category coded by M. The P1 asked other groups about the lengths of triangle sides to make an obtuse triangle. One of them answered $(6, 3, 4)$. The P1 did not say it correctly or wrong, but applied interpreting skills code IA2, the subcategory E3. The P1 asked other groups if $(6, 3, 4)$ fulfilled the triangle inequality and perimeter requirements or not. The following process was the skills of responding coded RA2, the category coded by M. Another group answered $(6, 3, 3)$. The teacher interpreted it with code IA2 and the subcategory E3. The P1 commented that $(6, 3, 3)$ fulfill triangle perimeter requirements and asked if the numbers fulfill the triangle inequality requirements. The processes of IA2 and RA2 were repeated in students' solutions of $(7, 2, 3)$; $(6, 4, 2)$; $(6, 5, 1)$; $(5, 5, 2)$. Finally, the teacher provided a response code R3 (classical interaction between students and teacher) category coded by P. See the discussion excerpt between P1 and a group's representative (KS).

P1: Now about this problem (Meaning problem part c as shown in Figure 5). $8 + 2 + 2$ equals what?
KS: 12
P1: It meets the requirements of triangle perimeter, doesn't it?
KS: Yes
P1: What about the requirements of the triangle?
KS: No
P1: $2 + 2 < 8$. Is it one of the triangle requirements?
KS: No
P1: Hello…Is it one of the requirements? (Looking at other groups)
KS: No
P1: Why do you choose? (Looking at the group that did the problem)
P1: What? It was … what was the information? It was (The P1 referred to the investigation sheet) information about what you used, right? You used the requirements of triangle sides inequality. If c is the longest side, it should be $a + b$ (a plus b) more than?
KS: c
P1: $a + b > c$. That's your guide. That's the formula. Okay, what about other groups? What about yours?
KS: $(6, 3, 4)$

P1: According to group 4, it should be $(6, 3, 4)$ (Writing 6,3,4 on the board). $6 + 3 + 4$ equals what? (Asking to group 4)
KS: 13
P1: What? 13? What is the perimeter? (Asking other groups)
KS: 12

Research findings on P2's skills in attending, interpreting, and responding to students' mathematical creative thinking. P2 delivered the lesson on the function and equation of the straight line. P2 applied two patterns of skills; pattern 1 was AA2-IA2-RA2, while pattern 2 was AA1-IA1-RA1-RA3, as shown in Figure 7.

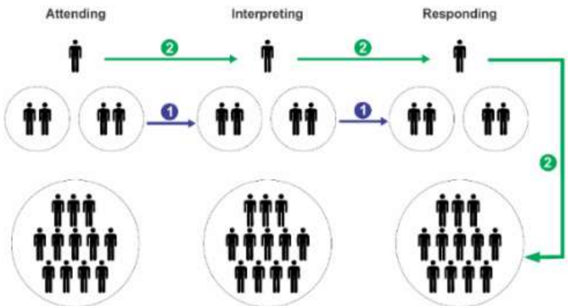


Figure 7: Patterning process of three skills by P2

ht

line. Two problems were given. One of which is shown in Figure 8 below.

1. It is known that point $P = (0,5)$ and $Q = (5,0)$
   a. Draw line g, straight line through point P and Q
   b. Formulate the equation of line *g* through P and Q
   c. Draw other straight lines parallel to line *g* (at least three lines)
   d. Create the formula for straight lines made of question c). Explain

Figure 8: Problems discussed by P2 at class

P2 asked students to work the problems in pairs and went around to see their discussion process. P2 also gave questions to trigger each group to reconstruct the relationship between the line equations, parallel line, gradient of the line, and gradient of parallel lines. The short academic hour made the application of attending, interpreting, and responding to students' creative thinking get started when a group started to finish their task even if it was incomplete. So, those skills were not applied to the finished answer.

During the first meeting, P2 used pattern 1 of AA2-IA2-RA2. The skill of attending with a code of AA2 was applied with category X and subcategory X2. Some pairs responded, "by drawing."

P2 interpreted the statement by saying, "*Yes*," and coded it IA2. This showed that the P2 interpreted the category C. P2 responded with code RA2, category K, and subcategory K2. P2 demonstrated the steps for drawing a straight line through the points of P and Q and called it the line g. When students were drawing it too, P2 attended to her students' working process and result. This was coded AA2, category X, and subcategory X1. The skills of interpreting were coded IA2, the category C. P2 do it often. Next, the responding skills were coded RA2 with the category K and the subcategory K4. P2 commented that some students made mistakes when writing the letter g. Some students wrote it with capital G, but it should be lowercase since it represented a line. The next mistake was that some students drew a line that looked like a line segment, so P2 asked them to draw a long line to show their difference. During the first meeting, P2 frequently applied pattern 1 for questions b, c, and d. The excerpt below was the discussion between the teacher (P2) and some pairs of students (PS) when doing problem 1a in Figure 8.

P2: How? How to draw a line g? The line g must pass through the points P and Q. How?
PS: Drawn
P2: Yes. Stay connected to point P and point Q. Connect point P and point Q to form a straight line. Name the line. Yesterday I had told you how to name lines. Name the line. Maybe at the bottom, maybe at the top. Name the line.
PS: (Drawing a line). We think we made a mistake….
P2: (Attending the drawing process and seeing the result of the line g) That is correct. The letter g should be lowercase. Make it long, don't make it too short, just like that. Line g could be above it or below. (Referring to the position of letter g while pointing to the line they drew)

Students in the group have difficulty solving problem 1 part c in Figure 8. This makes P2 detail what strategies the students should make. P2 attended to category X and subcategory X3. P2 stated "*Take a look first. You have found the equation for the line g. If you have trouble answering question c, what does question c mean? You are asked to draw parallel lines. Parallel lines have the same gradient. Now, the line you will make later the gradient, the slope is the same. You've learned to make gradients. Remember the formula is the value of y per value of* x".

In general, in interpreting the results of student work on all problems in the first meeting, P2 often said that the process or the result of student work was right or wrong, either explicitly or implicitly (saying yes right, yes right, wrong, wrong, okay, yes, okay continue). This shows P2 interprets in category C. P2 also frequently interpreted the interaction between students' groups with the teacher in category F. Some of the P2 statements are written below.

"Your drawing is too small."
"Make the line precise. Draw it long. Don't limit it. Okay, do it again.

Pattern 2 is applied by P2 with the same categories and subcategories as pattern 1. The difference is only in its application in individual student interactions with the teacher.

The research results of P3's skills in attending, interpreting, and responding to students' mathematical creative thinking. P3 delivered the learning material of function and the equation of a straight line. He applied those three skills in two patterns, as seen in Figure 9. Pattern 1 was AA2-IA2-RA2, while pattern 2 was AA2-IA2-RA2-RA3. P3 dominantly applied both patterns.

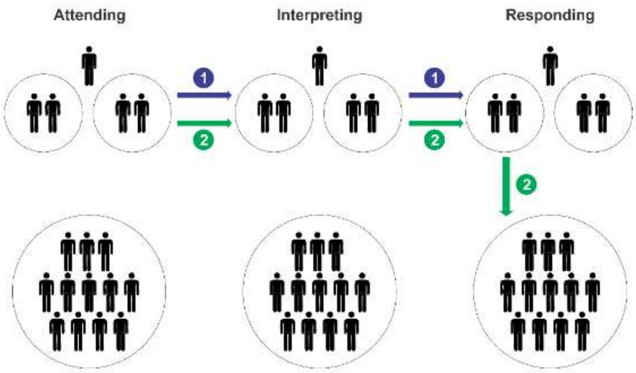


Figure 9: The third pattern of those three skills by P3

In general, P3 conducted the learning process by doing pair discussions as P2 did. When students discussed in pairs, P3 approached each pair. P3 helped them think by asking guided questions and reminding them about their learned concepts. The limited learning time available made the application of attending, interpreting, and responding to students' creative thinking get started when a pair started finishing the problems even if it was incomplete.

During the second meeting, P3 delivered the material of the equation of a straight line. He evenly distributed three problems to six pairs of students, so each problem was done by two different pairs. Figure 10 below shows problem 3 in the P3 class.

Problem 3
Draw a rectangle $ABCD$ with the length of 5 units and the width of 3 units on a Cartesian Coordinate

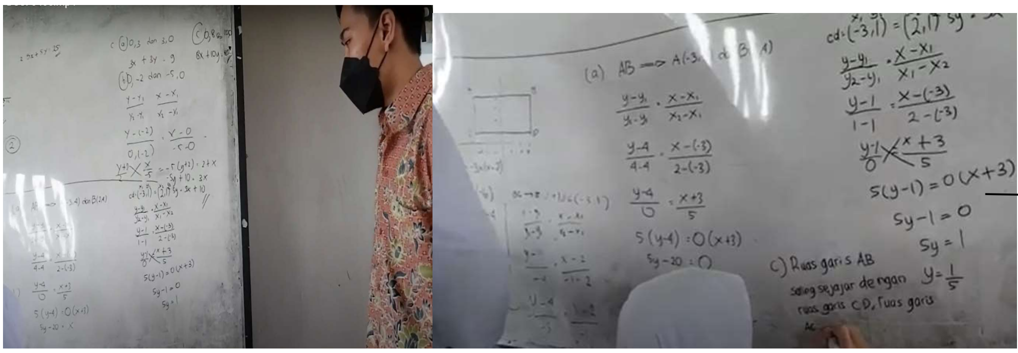
a. Formulate the equation for line segments $AB, BC, CD, dan\ DA$
b. Decide the domain and range for each equation in question a)
c. Describe the connection between each line (For example, based on its gradient)

Figure 10: P3’s mathematics problem

P3 asked one pair with problem 3 in Figure 10 to write down their work on the board. It seemed that P3 applied pattern 2 (AA2-IA2-RA2-RA3) of the process of attending, interpreting, and responding to students' mathematical creative thinking. In the skills of attending, the interaction was between students in group and teacher (AA2), the category X with the subcategory X3 (in this case in detail). The statement of P3 is written below. P3 is also attended by category Y with subcategory Y2 (for the last statement).

> *"This pair draws a rectangle on a Cartesian Coordinate. The length is 5 units, and the width is 3 units. The position is not limited. The first pair draws it like this. So, you can draw it anywhere (Meaning everywhere on the Cartesian Coordinate) as long as the length is 5 units and the width is 3 units…The drawing does not need to be the same. I remind you that the drawing does not need to be the same. The drawing is like this. Then, the equation of each group would be different.*

In Figure 11 below, the student representatives wrote down the process of finding the equation of the line CD through C(-3,1) and D(2,1) by writing $\frac{y-1}{0} = \frac{x+3}{5} \leftrightarrow \frac{-5}{0}$. P3 does not immediately blame the student's work but asks the students to multiply the cross and write it down to be $5(y-1) = 0(x+3)$. Students are then asked to describe it and produce $5y - 1 = 0, 5y = 1$ The job is wrong, it should be $5y - 5 = 0, 5y = 5, y = 1$. P3 does not blame, but allow other students the opportunity to comment. When the student representative looks for the equation of the line AB with A(-3,4) and B(2,4), she wrote $5(y-4) = 0(x+3), 5y - 20 = x$, P3 does not blame or justify but ask the question "0 when multiplied by x how much?". The student realizes her mistakes and justifies the next calculation process. This shows that P3 interprets in category E and subcategory E3. P3 always interprets by giving a checkmark on the right solution. This shows that P3 interpret in category C.



The line segment AB is parallel to the line segment CD, the line segment AC

Figure 11. Written answers of group representatives in class P3

P3 applied the skills of interpreting in the form of interaction code IA2, category C (just as P2), and category E with the subcategory E3.

A pair asked P3 about problem 3 part c in Figure 10. P3 guided them by asking them a question.

P3: Now, look. The line segments AB and AC are what?
KS: Perpendicular
P3: Yes, they are perpendicular. You may choose. AB and CD are parallel. AB and AC are what? You may choose. The c. It's the example. You may take another one.

After the pair understood, one of them wrote on the board. P3 demonstrated the skills of responding to code RA2, category K with the subcategory K2. P3 then applied the skills of responding to the creative thinking coded RA3 by showing various possibilities of rectangle shapes and available solutions based on the rectangles made by students. P3's statement below showed category P, mainly associated with the open-ended problem and creative thinking.

> *This is the solution of group 1. For the other groups, your solution does not need to be the same. Yes. Drawing a rectangle. I will show you some examples. It could be like this. (P3 drew some possible rectangles on the Cartesian Coordinate) You may put your rectangle here, as long as the length is 5 units and the width is 3 units. You may also you're your rectangle here (P3 drew a rectangle) ... The most important thing is that the line segment of AB and CD are parallel. AC and BD are also parallel. AC and AB are perpendicular. CD and BD are also perpendicular too.*

Pattern 1 is applied by P3 with the same categories and subcategories as pattern 2. However, P3 does not respond classically.

The three teachers applied the skills of attending, interpreting, and responding in various ways, as shown in Table 1. The similarity and differences practice of three skills by three teachers shown in Table 2. Based on Table 2, the three teachers applied the skills of attending students' creative thinking in category X and its subcategory was X3. The teachers detailed the thinking strategy their students used with an expectation that they would understand the concept and thinking strategy better to solve the mathematics problems. According to Jacobs et al. (2010), the strategy in detail is essential as it could be a window of children's understanding. In mathematics class, attending students' strategies effectively involved the teachers in tracing all strategies applied by students with a focus on the essential mathematics components Styers et al. (2020). The teachers' attending skills of detailing or not detailing students' creative thinking strategies were used as the fundamental of interpreting students' thinking. This statement is supported by Amador et al. (2016) that itemizing students' thinking strategy is one of the characteristics of advanced skills of attending, and it underlies the teachers to interpret students' mathematical thinking. P1 frequently allowed other groups to assess the correctness or incorrectness of the work of a group written on the board. P3 did the same and also said that students could have various solutions for problem 3 depending on the shapes of the rectangle they drew. P1 and P3 attend by category Y and the subcategories Y2. Their decision not to tell them the correct solution encouraged them to validate their argument and finally led them to be independent and confident learners (Francisco & Maher, 2011).

Noticing Component	Similarity	Difference
attending	All teachers, attending in category X. Its subcategory was X3.	P1 and P3, attending by category Y and the subcategories Y2 P1 and P2 attending by category X and the subcategories X1 and X2 P1 attending by category X and the subcategories X4 and X5 P1 attending by category Y and subcategory Y1
interpreting	All teachers interpreting in the category C	P1 and P3, interpreting by category E and subcategory E3 P1, interpreting by category B, and category E and sub-categories E1 and E2 P2, interpreting by category F
responding	All teachers responding in category K with subcategory K2	P1 and P2 responding by category K with subcategory K4 P1 and P3 responding by the category P, mainly associated with the open-ended problem and creative thinking P1 responding by category K with subcategories K1, K3, and K5; and category L, M, N, and O.

Table 2: Comparison of noticing between teachers with different teaching experiences

P2 and P3 often stated the correctness or incorrectness of students' work. P3 also applied checkmarks to point out the correct solution. From the discussion excerpt, it was known that P1 also did it by smiling. It showed that those three teachers interpreted their students' understanding with a category C. The teachers' comment on the correctness of the solution was considered to show the characteristics of students' solution, while the incorrect solution showed students' confusion, wrongdoing, or misunderstanding (Crespo S, 2000).

P1 and P3 interpret by category E and subcategory E3. This demonstrated that P1 and P3 did not quickly claim that their students understood or not, but they tried to explain further the mathematical meaning of solving a mathematics problem. P1 reduced the interpretation of the correctness or incorrectness of their solution and changed the discourse into the activity of finding out the information about the mathematics information and concept his students missed when solving the given problem. Based on Crespo et al. (2000) stated that the teachers' experience in interacting with students helps them change their students' understanding.

P2 discussed with her students by pointing out their mistakes when drawing the line and Cartesian Coordinate. P2 commented on their work directly by telling them to draw long lines to differentiate it from line segment and asking them to draw a large Cartesian Coordinate. P1's questions confirmed students' mistakes, and finally, they became aware of their thinking mistakes. P1 ended the question-and-answer session by saying, "Why did you choose it?". P2 and P1's follow-up action was asking students to revise their mistakes. This illustrated that teachers could apply various ways to point out their students' mistakes and fix them. Those mistakes could be helpful for the teachers. Based on Shaughnessy et al. (2021) argued that students' mistakes help teachers learn their students' thinking patterns and get them to learn the mathematics contents better that could be used to build interactions with students.

The teachers applied the skills of responding during the interaction between students in groups. It was always ended with responding to the classical interaction between students and them. They also demonstrated the skills of responding by supporting students' thinking. The way is the teachers brought up guided questions to help students solve the problems. P1 responded to his students' thinking by widening their ideas. P1 widened his students' thinking by giving a follow-up question, "Is there any side length that makes an obtuse triangle?".

Responding children's mathematical thinking could be done by asking questions designed to support or broaden students' thinking (Jacobs & Ambrose, 2020), redirect (Lineback, 2015), or introduce "further problem" (Jacobs et al., 2010), and pose the problem responsively (Land et al., 2019). The teachers' activities to support students' thinking were to make sure that their students understand the problem, change the mathematical problem to suit students' understanding, explore anything they had done, and remind them to apply other strategies (Jacobs & Ambrose, 2020). Some of the teachers' activities to widen students' thinking were to support the reflection of the strategy they had applied, to find out more strategies and discover the relationship between them, make connections between their thinking with symbolic ideas, and bring up further problems related to the problem they had just solved (Jacobs & Ambrose, 2020).

CONCLUSIONS

The mathematics teachers raise the attending skills in two categories, namely, activities that are directly related to students' mathematical creative thinking and activities that support students' mathematical creative thinking. The subcategories for activities that are directly related to students' mathematical creative thinking are attending the process/result of students' creative thinking; asking for an explanation about their thinking steps; detailing or not detailing students' thinking strategies; asking for justification for their reasoning (by giving guided questions, giving hints/keywords, bringing up sentence phrases, asking students using "How and Why"); reminding the concepts and principles they had learned or the relevant ones; and emphasizing what they had done. The subcategories for activities that support students' mathematical creative thinking are attending their articulation, how students draw, and students gesture when they explain thinking; allowing their mathematical reasoning development (provide the widest opportunity for students to explore mathematical ideas, provide opportunities for students to state true or false the results of thinking students or other groups of students, say that students can get various answers); and asking them if they had finished the tasks.`

The mathematics teachers interpret students mathematical creative thinking in various ways, namely excluding justification of right or wrong answers; focusing on right or wrong; focusing on the only correct solution; focusing on deficiencies in the student's working process; focusing on how they wrote and drew on grid paper and anything else indirectly connected to mathematical thinking; stating that working on the problem is easy; and comparing the thinking process of the teachers and students. The subcategories for focusing on the deficiency of students' working process are adding important information missing; asking for the clarification of students' statements; giving questions or comments about students' reasoning to check whether the student's answer is correct or not; pointing out students' mistakes related to the procedures of doing the problems; and asking if their students were aware of their mistakes made in the process.

The mathematics teachers responded by giving comments or questions about their students' knowledge, idea, procedure, or mathematical thinking and based on mathematical creative thinking; giving general comments or questions (such as any question, do you understand, can you do it, and what is the conclusion); giving comments or questions to trigger students to share their opinions; asks other students to comment or ask questions about certain students' thinking ideas; asking other students to explain certain students' thoughts; giving comments or questions about students' mathematical creative thinking/open-ended problem/critical thinking. The subcategories for giving comments or questions about their students' knowledge, idea, procedure, or mathematical thinking and based on mathematical creative thinking are giving comments/questions to examine students' thinking (such as using why and how questions); giving questions/guided comments to help students think creatively about mathematics; giving follow-up questions to confirm students' mathematical reasoning; giving comments/questions about students' thinking mistakes; and giving comments about the relevant concepts/principles/calculations.

Senior teachers and junior teachers responded with teacher comments/questions related to mathematical creative thinking/open-ended questions. Teacher responses in the form of teacher follow-up questions to confirm students' mathematical reasoning were carried out by senior teachers. Senior teachers and young teachers respond with teacher comments/questions related to students' thinking mistakes. So, there is a need for further research to explore why students make mistakes in mathematics as part of the teacher's skills in interpreting students' mathematical understanding. Teachers in this study did not receive training in the skills of attending, interpreting, and responding to students' mathematical creative thinking. The follow-up is to provide training to mathematics teachers in these three skills as their professional development and to further investigate the implementation of these three skills in the professional development of mathematics teachers. Prospective mathematics teachers can learn these three skills so it is necessary to do research of these three skills by involving prospective mathematics teachers so that they have the initial experience to improve the quality of the implementation of the three skills in their teaching practice in the future.

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