

Computational, Logical, Argumentative, and Representational Thinking in the United Arab Emirates Schools: Fifth Grade Students' Skills in Mathematical Problem Solving

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Abstract: *The purpose of this study was to analyze the problem-solving techniques that students in a fifth-grade classroom applied while solving mathematical word problems. Fifth-grade students in a private school with Ministry of Education curricula in Al Ain, Abu Dhabi, were given a set of 15-word problems to solve with detailed justifications. The questions were based on TIMSS past exams with some modifications to fit the local context. Analysis from the study generated five themes that students applied in problem-solving: Logical Thinking Skills, Computational Skills, Problem-solving Strategies, Use of Justifications, and Use of Representations. Pedagogical and curricular implications have been discussed.*

Keywords: *Problem-solving, mathematics learning, gender differences in math, UAE.*

INTRODUCTION

Problem-solving in mathematics is an essential skill for assessing students' understanding, interpreting, and applying mathematical concepts (Carden & Cline, 2015). Researchers in Mathematics Education have emphasized students' problem-solving at the school level (Amado, Carreira & Jones, 2018). However, more needs to be done in problem-solving studies within the United Arab Emirates (UAE), school contexts. There are several methods and approaches to problem-solving (Gourdeau, 2019). Nonetheless, students still struggle when it comes to applying these approaches to their own problem-solving. Still, students are weak in solving mathematical problems, realistic problems, numerical operations, and justifying their answers (Suharta, 2016). Mathematical problem solving is an essential component of, and not separate from, mathematical learning (Nurkaeti, 2018). Problem-solving has been considered a high-level cognitive skill that helps teachers raise expectations for their students' thinking (McCormick, 2016). Although young children have different abilities in using problem-solving strategies, teachers and researchers still need to learn more about the affective and cognitive factors that may contribute to understanding

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variations in problem-solving approaches and their impacts on students' learning of mathematics (Ramirez, Chang, Maloney, Levine & Beilock, 2016).

Although teaching mathematics through problem-solving is considered a challenge for many mathematics teachers, it can foster the students' learning with a more profound understanding and proficiency in using the skills in various situations (Stein, Grover, & Henning, 1996). To use problem-solving as a teaching strategy, many procedures should be considered, such as the teacher's role in selecting the problem-solving strategy, collaborating, and adopting a problem-solving-based curriculum (Cai, 2003a). Hiebert et al. (1996) argued that the curriculum and instructions should be reformed by allowing the students to problematize the subject. They build their proposal based on John Dewey's idea of reflective inquiry, illustrating that through problem-solving, the students will understand and develop their thinking skills (Hiebert et al., 1996). The aim of reforming curriculums based on problem-solving is not to frustrate students and make mathematics difficult, but to help students learn mathematics through problem-solving activities (Battista, 1999; Brown, 2001). We need to encourage the students to think, wonder, and inquire about why things are the way they are and find a solution to this situation. We need students to elicit their curiosity and sense-making skills (Hiebert et al., 1996). Although there has been much research around problem-solving in the last 30-40 years, many questions need to be answered, such as what kinds of instructional activities should be used in teaching through problems (Lester & Cai, 2016). We are concerned about the difficulties that students face in solving problems, and we investigate the reasons behind these obstacles.

Students' average performance in mathematics problem-solving in PISA 2012 was 411 for the UAE students, with a global rank of 40th among 44 participating countries (United Arab Emirates Ministry of Education, 2013). There has been a concern among parents, teachers, school administrators, and the UAE government that "students fall way short of their global counterparts when it comes to maths problem-solving" (Dhal, 2014, April 09th). The government and school leaders have enhanced their efforts to improve the situations in the mathematics classrooms and promote students' computational and problem-solving skills (Organization for Economic Co-operation and Development [OECD], 2020). The United Arab Emirates Government has been considering students' mathematics problem-solving as one of the core concerns. Therefore, it has been an element of close attention in the school inspectors' guidelines and school inspection framework (United Arab Emirates Ministry of Education, 2021). In Abu Dhabi, UAE, several school systems have emphasized mathematics problem-solving as one of the key skills in their curriculum, for example British Schools, International Baccalaureate Schools, American Curriculum Schools, and Indian Schools (Abu Dhabi Department of Education and Knowledge, 2019).

The project was conducted to explore how students in the UAE's local context solve mathematical word problems. Problem-solving in mathematics classrooms is an issue worldwide, and not enough research has been conducted in the UAE context. The study focused on analyzing students'

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problem-solving responses and then extracting possible themes from the students' methodologies in their problem solving, whether intuitively or by design. The research questions for this study were: How do fifth grade students solve real life mathematical problems? Moreover, what are the strategies that the fifth students in the UAE schools use to solve a mathematics problem?

THEORETICAL FRAMEWORK

Mathematics is always considered a difficult subject, although it is found in all other subjects and student's daily lives (Surya, Putri, & Mukhtar, 2017). Problem-solving could be one of the teaching strategies and hence improve the quality of teaching mathematics (Pehkonen, Naveri, & Laine, 2013). The importance of problem-solving is that students can deepen their understanding of mathematical concepts by using them in real-life problems (Beigie, 2008). Problem-solving is a strategy that all students are expected to use in their daily lives and future professions. Therefore, it is crucial to use the problem-solving technique in education to help students manage their lives and solve problems. There are many models for problem-solving, such as the Polya model, the Schoenfeld model, and the Verschaffel model.

Problem-Solving Model: George Polya

Polya's book, *How to Solve It*, has inspired several other research studies and conversations on problem-solving (Polya, 2004). It has even affected problem-solving in non-mathematical contexts. His four steps of problem-solving by design speak to both students and educators and often appear in one form or another in research papers (Kusumadewi & Retnawati, 2020). These four steps are:

- Understanding the Problem: By looking at all aspects of the problem regarding known and unknown variables, there are conditions. Illustrating the problem is a part of understanding it as well.
- Devising a Plan: By trying to form connections between the known and unknown, prior knowledge also plays a role in finding connections between possible past problems and the current problem at hand.
- Carrying out the Plan: Once a plan is devised, carrying it out follows it by checking every step of the plan along the way and double-checking that everything is correct.
- Looking Back: Checking whether the plan worked from different perspectives and whether there was a potentially different plan as well.

Polya et al.'s analysis of problem-solving approaches based on mental agility offers a door into problem-solving techniques as well (Liljedahl et al., 2016, p. 4–5). These techniques are:

- Reduction: Problem solvers intuitively reduce a problem to its basics in a logical manner by visualizing forms of representation for the problem.

- Reversibility: Another successful approach to problem-solving seeks to backtrack trains of thought. Working in reverse as a problem-solving approach is another problem-solving strategy.
- Minding of aspects: Thinking of a problem by keeping a couple of details in mind at the same time is another successful approach used by intuitively good problem solvers.
- Change of aspects: This approach helps good problem solvers avoid getting a mental block by being flexible enough to change their minds and look at the problem from a different perspective when needed.
- Transferring: By recognizing a pattern, successful problem solvers transfer it consciously or subconsciously when solving other problems. Good problem solvers also generalize or transfer their procedures to a different context.

(Liljedahl et al., 2016, p. 4 – 5)

These approaches by successful problem solvers are sometimes inaccessible to intuitive problem solvers who solve the problems but cannot explain their exact train of thought when solving a problem correctly.

Problem-Solving Model: Schoenfeld (2013)

Alan Schoenfeld was impressed by Polya's problem-solving model. For Schoenfeld, "the problem-solving process is ultimately a dialogue between the problem solver's prior knowledge, his or her attempts, and his or her thoughts along the way." He also states that problem-solving is the ability to think mathematically (Garcia, Boom, Kroesbergen, Nunez & Rodriguez, 2019; Liljedahl et al., 2016, p. 14). Schoenfeld's problem-solving model initially included five episodes (Karatas, Soyak & Alp, 2018)—Reading, Analyzing, Exploring, Planning/Implementing, and Verifying. Schoenfeld also adds another dimension to the heuristics of problem-solving by including metacognition and regulation of thoughts. While the concept of metacognition was extensively used in education in the 1980s, it has since been reconceptualized to differentiate it from the regulation of thoughts. By thinking about what they are doing, students are engaged in metacognition, and by controlling these thoughts by posing questions and alternative solutions, they are regulating their cognition and thinking. Both of these serve to help students and they become effective problem solvers (Schoenfeld, 2013). Rott (2012) discussed the basis of the Schoenfeld empirical framework. He illustrated that to theoretical model of the problem-solving process is based on Pólya's list of questions and guidelines. Rott (2012) made a correlation between Polya's steps and Schoenfeld's episodes which is illustrated in Figure 1 below:

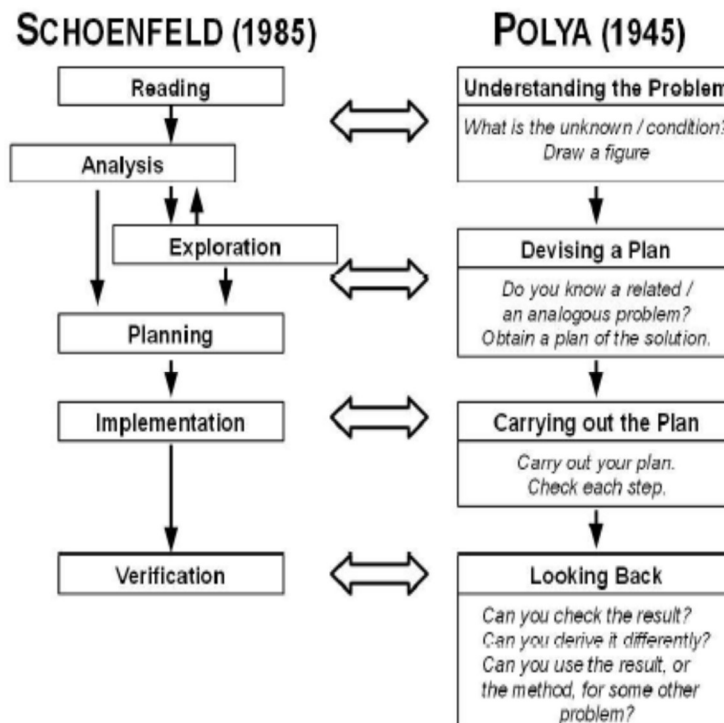


Figure 1: Correlation between Polya's steps and Schoenfeld's episodes

Problem-Solving Model: Verschaffel et al. (1999)

Verschaffel et al. (1999) proposed a recursive problem-solving model that includes five elements: building a representation, deciding the solution method, carrying out the solution, interpreting the solution, and evaluating the outcome. As Verschaffel et al. (1999) suggested, the first step is to involve learners in drawing a picture to represent the problem context. Then, the learner makes a scheme of the variables in a table or a list to distinguish the necessary information in the problem and uses his or her prior knowledge to create a viable representation. After the representation phase, the learner decides the solution approach or method based on his or her prior scheme or new scheme from the representation with a flow chart. He or she may use an initial guess and check approach to reach the most viable route to the solution with the help of a pattern in the given problem context. The learner may simplify the solution process with the necessary formula, tools, or models. The third step is about executing the problem-solving action as planned and getting a viable solution. The learner interprets the solution or outcome in the problem context and evaluates whether the solution is feasible. This way, Verschaffel et al.'s (1999) model is somewhat similar to Polya's problem solving (Garcia, Boom, Kroesbergen, Nunez & Rodriguez, 2019). These three models, as discussed above, served as a basis for our study to analyze and interpret students' problem-solving approaches to the selected problems.

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We are looking at students' problem-solving skills within five different domains: logical thinking, computational skills, justification of answers, problem-solving strategies, and use of representations. Logical thinking ability has been an important cognitive ability to be developed in students while teaching mathematics (Sezen & Bulbul, 2011). Likewise, computational skills provide students ability to carry on mathematical problems in an efficient and fast way by applying appropriate strategy (Borkulo et al., 2020). Mathematics problem-solving also involves reasoning in mathematical context integrating mathematics to the real-life situations. This kind of skills can promote students' ability to justify their reason for particular problem-solving approach opening the possibility to view alternative ways to reach the solution (Glass & Maher, 2004). Such skills can be further strengthened and promoted with the applications of representations of problems and solution models that students construct (Gagatsis & Elia, 2004).

METHODOLOGY

This study aimed to analyze the mathematical problem-solving skills of grade 5 students in Al Ain, Abu Dhabi. A segment of an international test, TIMSS 2015, with a mathematics section was chosen with some modifications to adapt it to the culture of the United Arab Emirates. The test was then translated into Arabic, as the students' mother tongue is Arabic; the sample questions from grade-4 TIMSS were deemed acceptable for the grade 5 students as the test was administered at the start of their new academic year.

Participants

We gained access to a private school in Al Ain through a gatekeeper, one of the researchers. The school is a private MOE system school, with grades from k-12. Students study in co-ed classes up to grade 3, and then boys and girls are taught separately. Participants in this study come from two sections of grade 5: the boys' section (15 participants) and the girls' section (12 participants). After initial consent from the school, the test was administered to the students who attended face-to-face learning. Because of COVID -19 restrictions, the test was presented to the students on the smart screen in their classrooms. In contrast, the students answered on their sheets of paper to minimize any interactions and to maintain social distancing in the classroom. All students in both classrooms were given the same conditions, and the first researcher then gathered the answer sheets from the school.

Materials

The test was obtained from the TIMSS official website ([source: https://www.edinformatics.com/timss/pop1/mpop1.htm?submit3243=Grade+3%2C4+Math+Test](https://www.edinformatics.com/timss/pop1/mpop1.htm?submit3243=Grade+3%2C4+Math+Test)) and was modified to include 15 questions rather than the whole test. This was done since the test was to be conducted in one 45-minute period. The 15 questions chosen were further modified to localize them by changing names when applicable. Although the TIMSS test is for grade 4 students, it was decided to use the same test for grade 5 students. The justification for this was that students had just come from their summer vacation, and before that, they had studied their first

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semester of online learning (spring 2020). The test was administered on September 22nd, 2020, during the start of their fall 2020 semester. The researchers wanted to investigate the students' problem-solving skills regarding word problems and justifying answers. The test was translated to Arabic since the students' first language is Arabic, and having the test in their mother tongue would give them a better opportunity to provide justification for their answers. The medium of teaching mathematics at that school was also in Arabic, hence the justification for translating the test.

Following that, the specification table, Table 1, used for the test, was used to identify the main areas of possible analysis, which are: (1) Operations that include the basic four operations, addition, subtraction, multiplication, and division; (2) Fractions; (3) Geometry; (4) Measurement; and finally (5) Probability and Statistics. Students' answers in both the boys' and girls' sections were analyzed across these areas/strands of mathematics. Below is the specification table, showing the strands of mathematics as observed in the test, with the questions' placement and their weights (Table 1), (see Appendix (1)). The weight was converted a percentage which was calculated based on the number of questions from the overall questions (for example: the questions that were related to the operations had eight questions, therefore $8/15 \times 100\% = 53.3\%$).

Specification Table		
Strand	Question Number	Question Weight
Operations	Addition: 5	53.3%
	Subtraction: 1, 2, 12	
	Multiplication: 13, 15	
	Divison: 4, 7	
Fractions	3, 10	13.3%
Geometry	6 (perimeter)	6.6%
Measurement	8, 9	13.3%
Statistics & Probability	11, 14	13.3%

Table 1. Specification the problems used in the study

Quality Criteria (Validity and Reliability)

When we started to plan for this project, we considered the validity and quality of this study (Creswell, 2016). We tried to answer the question: Is the account valid? The validity comes from the fact that we chose word problems from a previous TIMSS exam for grade four students. The questions were selected to cover the basic strands: operations, fractions, geometry, measurement, statistics, and probability (National Governors Association for Best Practices & Council of Chief State School Officers, 2010; NCTM, 2000). The selected problems in the study were examined to

be modified to fit into the local context of the UAE schools and translated into Arabic for students (see Appendices I and II). The questions were moderated, and language structure was examined to see if fifth graders might understand the questions. The students were not informed before the test about the study. Parents' consent was sought to conduct the research, telling them that their children's identities would not be used in the study. They were also informed about the purpose and benefit of the study in terms of its contribution to knowledge in the field of teaching and learning mathematics. Consent from the school and mathematics teachers was also sought to implement the test in the classroom. The teachers displayed the questions on the smartboard for the students. The students answered the questions on a blank sheet of paper that they brought from home. A strict safety protocol against COVID-19 was followed as per the school rules and government policies, not to distribute any sheets and pencils in the classroom. After completing the test, the students left their answer sheets on their tables, and the teachers later collected them and handed them over to the researchers in a sealed envelope.

Data Analysis

The participants were studying at a private school in Al Ain city following the Ministry of Education curriculum. We completed the requested procedures for the consent form. The test consisted of 15 questions, and the duration of the test was 45 minutes. Because they were studying this way, 12 female and 15 male students sat separately for the exam. The test was conducted at the school, but due to COVID-19 and the conditions of social distancing and general health rules, we presented the test on the screen, and the students used their papers to answer the questions. We analyzed the answers of the students depending on five criteria:

- (1) Logical Thinking: The student's ability to think logically is constructed as s/he acts in his or her environment and tries to make sense of his or her world. When solving a problem, the student tries to relate the problem to his previous knowledge (Micklo, 1995).
- (2) Computational Skills: This is the ability of the students to use basic operations to solve problems. Computational skills are considered a necessary key to solving problems (Hickendorff, 2013).
- (3) Justification of Answers: The National Council of Teachers of Mathematics argues that reasoning skills are vital in studying mathematics. Students should justify their answers and solution steps as well as make and assess mathematical conjectures. Mathematical justification is part of communication skills (Cai, 2003).
- (4) Problem Solving Strategies: Polya's strategy is a famous strategy used in mathematics curriculums to help students deal with mathematical problems. It includes four steps; understanding the problem, planning, carrying out the plan, and looking back (Nurkaeti, 2018).
- (5) Use of Representations: such as diagrams, tables, and pictures to assist the student in reasoning and solving the problem.

To maintain anonymity, we numbered the participants as follows-- F1, F2... F12 for the girls and M1, M2... M15 for the boys. This study's findings helped us learn more about how students think, their mathematical thinking, and their weak and strong points in mathematics. This will help us in

many aspects-- we can judge the strategies of teaching, the mathematics curriculums, and the possible weak points, and how to strengthen them.

Two researchers first graded the test and tabulated the test results by identifying students who had either *answered correctly and provided justification*, *answered correctly but did not provide justification*, *answered partly correctly*; and lastly, *did not answer correctly or did not answer the question*.

RESULTS

Table 2 (analysis of questions) shows students' answers to the 15-question test. Code names were given to students to maintain anonymity. The researchers made a note on three occasions: (//) students correctly solving the problem and providing justification; (/) students partially solving the problem correctly with or without justification; (*) students incorrectly solving the problems.

	M 1	M 2	M 3	M 4	M 5	M 6	M 7	M 8	M 9	M 10	M 11	M 12	M 13	M 14	M 15	F 1	F 2	F 3	F 4	F 5	F 6	F 7	F 8	F 9	F 10	F 11	F 12
1	//	/	*	*	*	*	//	*	*	*	*	*	*	*	/	*	*	//	/	//	/	//	*	/	*	//	//
2	//	*	*	//	*	//	*	//	//	//	*	/	//	//	//	*	*	/	/	//	/	/	*	*	/	/	//
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10	//	//	//	/	//	/	/	//	/	//	/	//	//	//	/	/	*	//	/	//	*	//	*	*	//	//	//
11	//	//	/	//	*	//	//	//	//	//	*	//	//	//	//	/	/	*	/	//	*	//	*	//	//	/	//
12	/	/	*	/	*	*	/	/	*	/	*	/	/	/	/	*	//	/	*	//	//	//	*	*	/	//	*
13	/	//	/	*	/	//	//	//	//	//	/	//	//	//	/	*	*	*	*	*	//	//	*	*	/	//	//
14	/	*	*	*	*	*	*	*	*	*	*	*	*	*	*	/	//	//	*	//	*	//	*	//	*	//	//
15	*	//	*	*	//	*	//	/	//	//	*	/	//	//	*	/	*	*	*	*	*	*	*	*	*	*	*

Table Key: //: has answered correctly and provided justification. /: has partly answered. *: has not answered/ incorrect.

Table 2: Analysis of students' solutions based on operations of addition, subtraction, multiplication, and division

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Table 3 (below) presents the distribution of students' solutions question-by-question. The letter B represented the boys and G represented the girls participants. The test results have been recorded in terms of answered correctly, partial answer and answered incorrectly in four areas of number operations.

<i>Operations</i>									
<i>Addition</i>									
	Question 5								
	B	G	Total	B	G	Total	B	G	Total
<i>Answered Correctly</i>	1	0	1						
<i>Partial Answer</i>	13	5	18						
<i>Answered Incorrectly</i>	1	7	8						
<i>Subtraction</i>									
	Question 1			Question 2			Question 12		
	B	G	Total	B	G	Total	B	G	Total
<i>Answered Correctly</i>	2	5	7	9	2	11	0	5	5
<i>Partial Answer</i>	2	3	5	1	6	7	10	2	12
<i>Answered Incorrectly</i>	11	4	15	5	4	9	5	5	10
<i>Multiplication</i>									
	Question 13			Question 15					
	B	G	Total	B	G	Total	B	G	Total
<i>Answered Correctly</i>	9	4	17	7	0	7			
<i>Partial Answer</i>	5	1	1	2	1	3			
<i>Answered Incorrectly</i>	1	7	9	6	11	17			
<i>Division</i>									
	Question 4			Question 7					
	B	G	Total	B	G	Total	B	G	Total
<i>Answered Correctly</i>	8	0	8	1	2	3			
<i>Partial Answer</i>	4	3	7	13	2	15			
<i>Answered Incorrectly</i>	3	9	12	1	8	9			

Table 3: Analysis of students' solutions based on operations of addition, subtraction, multiplication, and division

1. Number and Operations

(a) Addition

For the question on addition operation, question 5, all male students, except M1, answered the first part of the question and missed the second part, finding the difference between Ahmed and Aysha, which indicates that the boys do not read the question well and do not follow the Polya's model in solving problems. Seven girls did not answer the question, while the rest of the girls answered it partially correctly. This referred to the same reason as the boys that the girls also ignored the second part of the question.

(b) Subtraction

For the first question on subtraction (operations), question 1, the male students seemed confused, and they might have thought that they needed to find $900 - x = 300$, answers of 8 boys were 600. This misunderstanding did not happen with the girls. We can say that the students, mainly males, did not read the question well, and then they did not know what exactly was required. Some students missed the subtraction skill. We can see this in question 2; M12 could write the sentence, but he could not subtract. Question 12 states that Maha placed a box on a shelf with a length of 96.4 cm., while the box has a length of 33.2 cm., and whether there is space for a second box on the shelf, and what would be the maximum length of said box. While none of the boys answered

this question correctly with justification, 5 of the girls did. Ten boys provided partly correct answers, and five responded incorrectly. Two of them answered partially correctly from the girls' side, and five responded incorrectly (Figures 1-4).

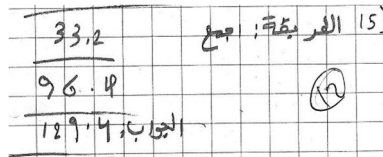


Figure 2: F8's Answer & Justification

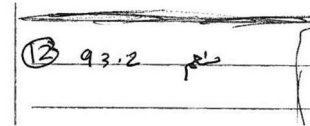


Figure 3: F10's Answer & Justification (Q12)

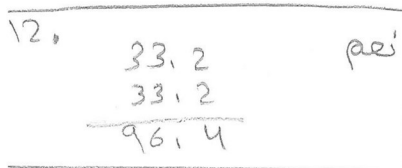


Figure 4: M14's Answer & Justification (Question 12)

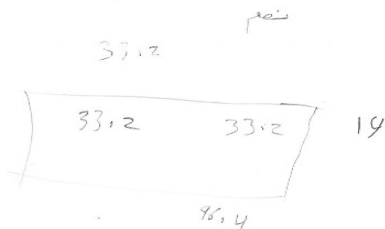


Figure 5: M10's Answer & Justification (Question 12)

Further analysis of the incorrect answers highlights computational mistakes, such as subtracting 3.2 instead of 33.2, adding 33.2 twice, and writing it as 96.4 instead of 66.4, which were common errors among the incorrect answers. Interestingly, using representation to show the solution was only used by one student (M10).

(c) Multiplication

Another question, Q13, states that "A teacher manages to correct ten student tests in half an hour, how many would he be able to correct in an hour and a half?". The boys' answers showed nine correct answers with justification, five partially correct answers, and one incorrect answer. In comparison, the girls' responses showed four correct answers with justification, one partially correct answer, and seven wrong answers. The correct answers either showed the repeated addition of 10 (3 times) or multiplied 10 by 3.

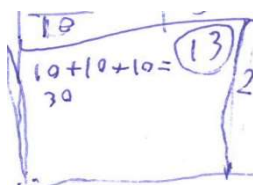


Figure 7: M2's Answer & Justification (Question 13)

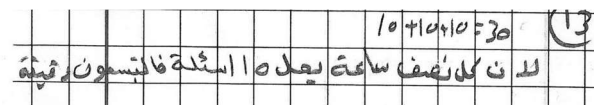
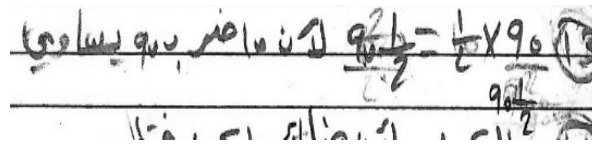


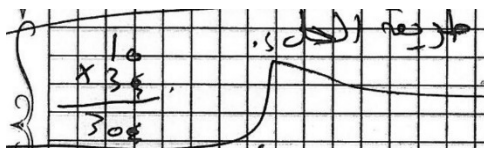
Figure 6: F6's Answer & Justification (Question 13)



Handwritten student work for Figure 8. The top part shows a math problem in Arabic: "ساعتان ونصف يساوي 90 دقيقة" (Two and a half hours is 90 minutes). Below this, there is a calculation: $90 - 10 = 80$. The number 13 is circled next to the calculation.

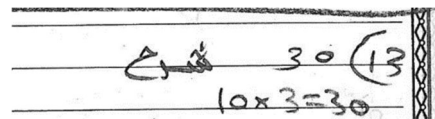
Figure 8: F3's Answer & Justification (Question 13)

F3's answer shows that the student understands that an hour and a half is 90 minutes; however, it also indicates that the student is not able to figure out how to solve the problem, so there is a lack of problem-solving skills and use of the appropriate operation for the word problem. Like many others, two students, M2 and F6, solved the problem by repeated addition, and F6 added that the hour and a half is 90 minutes, hence adding 10 three times. It is noteworthy that F11 was among the few students in both classes who answered the question using multiplication. Three students used multiplication to answer the question in the girls' class, and one answered with repeated addition. The boys' answers were obtained through repeated addition, as none tried to use multiplication. While the correct answers show a good understanding of time and computations, more practice with logical thinking problems is needed. The incorrect answers showed the importance of highlighting appropriate operations (some students subtracted $90-10$; while others added $10 + 30$) to be used, emphasizing students' understanding of time.



Handwritten student work for Figure 10. It shows a multiplication problem on a grid: $10 \times 3 = 30$. There is also some Arabic text written to the right of the grid.

Figure 10: F11's Answer & Justification



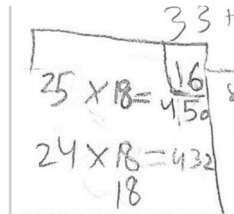
Handwritten student work for Figure 9. It shows a multiplication problem: $10 \times 3 = 30$. The number 30 is circled, and the number 13 is written next to it.

Figure 9: F7's Answer & Justification (Question 13)

In this group, Q15 poses the following problem: "25 X 18 is more than 24 X 18. How much more?" At the face value, students would need to perform the operations and then possibly subtract. However, if students solve this problem logically, it is assumed they would not need to perform any operations, as 25 follows 24 and will be logically higher by 18.

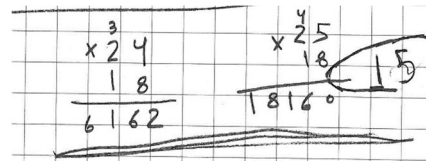
Upon analysis of the students' answers, it was found that seven boys answered correctly, and none of the girls answered correctly.

What is noticeable about the boys' and girls' answers is that none of the boys' solutions show the multiplication steps, which means they used a calculator or performed the task on some scrap paper. All the correct answers obtained from the boys showed the first step of multiplication and the second step of subtraction. When it comes to the solutions from the girls, all the girls attempted to solve the multiplication and then performed subtraction. However, their multiplication skills are



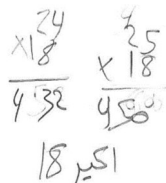
Handwritten student work for Figure 12. It shows two multiplication problems: $25 \times 18 = 450$ and $24 \times 18 = 432$. There are some additional markings like "33+" and "18" written below the second problem.

Figure 12: M4's Answer & Justification (Question 15)



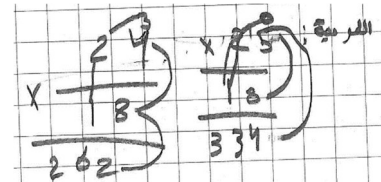
Handwritten student work for Figure 11 on grid paper. It shows two multiplication problems: $24 \times 18 = 6162$ and $25 \times 18 = 18160$. There are some corrections and markings, including a circled "15" in the second problem.

Figure 11: F9's Answer & Justification (Question 15)



Handwritten student work for Figure 14 showing two division problems: $24 \div 18 = 1$ and $25 \div 18 = 1$. The answer "1" is written below each problem.

Figure 14: M10's Answer & Justification (Question 15)



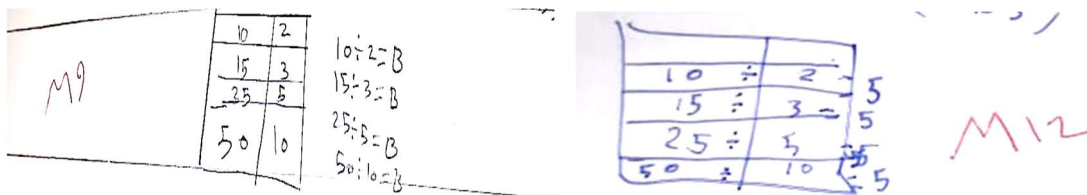
Handwritten student work for Figure 13 on grid paper. It shows two multiplication problems: $24 \times 18 = 202$ and $25 \times 18 = 334$. There are some markings and corrections, including a circled "18" in the second problem.

Figure 13: F8's Answer & Justification (Question 15)

lacking, and they often face difficulty when multiplying by the second digit. It is noteworthy that none of the students tried to solve this problem by using logical thinking (bigger by 18), and all used the two steps of first multiplication and then subtraction. This highlights the need to expose them to logical word problems and further cement their multiplication skills.

(d) Division

Although the answer of student M9 for Q4, Figure 14 (a), is wrong for the first question for the division problem, but we need to stop at his way of thinking; he divided each number in column A by its corresponding number in column B and wrote the answer "B" for all the statements. We can say that his thinking is logical, but he missed the division skill. Several other students, such as M12, F4, F5, F7, F11, and F12, thought the same way but found the answer.



Handwritten student work for Figure 15. On the left, student M9's work shows a table with two columns: Column A (10, 15, 25, 50) and Column B (2, 3, 5, 10). To the right, the calculations are: $10 \div 2 = B$, $15 \div 3 = B$, $25 \div 5 = B$, and $50 \div 10 = B$. On the right, student M12's work shows a similar table with Column A (10, 15, 25, 50) and Column B (2, 3, 5, 10). To the right, the calculations are: $10 \div 2 = 5$, $15 \div 3 = 5$, $25 \div 5 = 5$, and $50 \div 10 = 5$.

Figure 15: Students M9's and M12's answers of (Question 4).

For the second question, Q7, in this category (division), most of the male and female students answered the question, but they seemed not to know how to justify their answers.

(2) Fractions

Table 4 (below) presents the distribution of students' solutions question-by-question on the areas of fractions. The letter B represented the boys and G represented the girls participants. The test results have been recorded in terms of answered correctly, partial answer and answered incorrectly in four areas of number operations.

<i>Fractions</i>			
Question 3			
	B	G	Total
Answered Correctly	10	0	10
Partial Answer	2	3	5
Answered Incorrectly	3	9	12
Question 10			
	B	G	Total
Answered Correctly	9	6	15
Partial Answer	6	2	8
Answered Incorrectly	0	4	4

Table 4: Analysis of Questions (Fractions)

For the first question on fractions, Q3, male students could represent the shaded parts in fraction form and compare them to find the equivalent fractions, while none of the girls answered correctly with justification. Question 10 (fraction) provided the students with a rectangle with ten equal parts, two of them were shaded, and eight were not. The students had to choose the best answer that represented the shaded parts; they were given four answers: (a) 2.8 b) 0.5 c) 0.2 and d) 0.02. Almost all the boys answered correctly, with 9 of them justifying an equivalent fraction ($2/10$). Six of the boys chose the correct answer but did not justify their choice. Regarding the girls' responses, 6 of them answered the question correctly with justifications, and 2 of them did not provide any justifications. The four incorrect answers showed a misconception of the relation of parts to the whole.

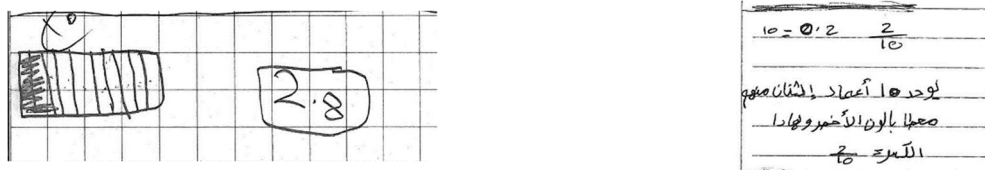


Figure 16: F2's and F10's Answer & Justification

(3) Geometry

Table 5 (below) presents the distribution of students' solutions question-by-question in the area of geometry. The letter B represented the boys and G represented the girls participants. The test results have been recorded in terms of answered correctly, partial answer and answered incorrectly in four areas of number operations.

<i>Geometry</i>			
	Question 6		
	B	G	Total
<i>Answered Correctly</i>	8	1	9
<i>Partial Answer</i>	5	1	6
<i>Answered Incorrectly</i>	2	10	12

Table 5: Analysis of Questions (Geometry)

In the geometry problem, Q6, one of the students, M8, used the trial and error method to solve the problem, finding the missing line segment in a rectangle. He first tried the number 5, then added to see if the sum was not 20, then used the number 6, which gave the correct answer. Another student, M9, drew a rectangle and put a measure of 4 units on the longest side and six units on the shortest, which indicates the missing sense of the number and its values. Most of the girls divided to find the missing side, $20 \div 4 = 5$ (see Figure 16b in the middle).

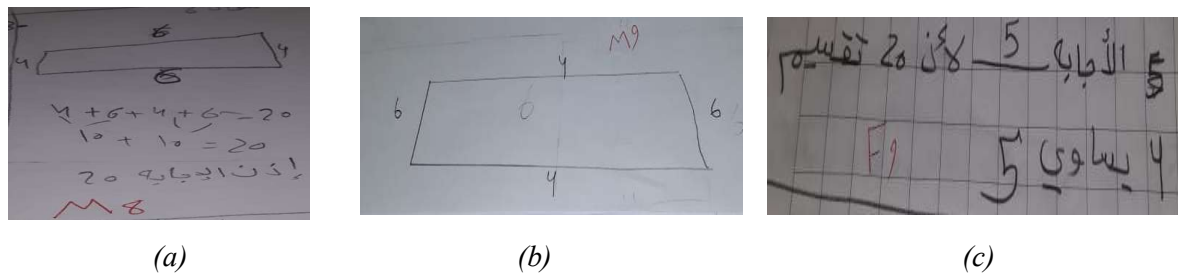


Figure 17: Students M8's, M9's, and F9 answers of (Question 6)

(4) Measurement

Table 6 (below) presents the distribution of students' solutions question-by-question in the areas of measurement. The letter B represented the boys and G represented the girls participants. The test results have been recorded in terms of answered correctly, partial answer and answered incorrectly in four areas of number operations.

Measurement

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	Question 8		
	B	G	Total
Answered Correctly	7	2	9
Partial Answer	8	6	14
Answered Incorrectly	0	4	4
	Question 9		
	B	G	Total
Answered Correctly	0	6	6
Partial Answer	0	0	0
Answered Incorrectly	15	6	21

Table 6: Analysis of Questions (Measurement)

Question 8

For question 8, which asks the students to find the mass of 1,000 clothespins knowing that the mass of each one is 9.2 grams, only two girls could justify their answers, while almost half of the boys could do that. On the other hand, no one of the male students failed in answering this question, while one-third of the students have not responded to the question.

In this group (measurement), the next question, Q9, in this group (measurement), compares the number of footsteps of four students to show who has the biggest footstep. When it comes to gender differences, all the 15 boys answered it incorrectly by commenting that ‘Mohanad,’ who had to take ten footsteps, had the largest size footstep. Whereas the girls’ answers were split in half, where 6 of them answered similarly to the boys, and the rest responded correctly by making the connection that the least number of footsteps meant that person had the largest size footstep, which was identified. Figure 17 shows M14’s answer, “Mohanad has the largest footstep because he has ten footsteps,” indicating that students’ understanding of the largest in size overlaps with the highest number of steps. Almost all students (both boys and girls) who chose “Mohanad” justified it by saying he had the greatest number of steps.

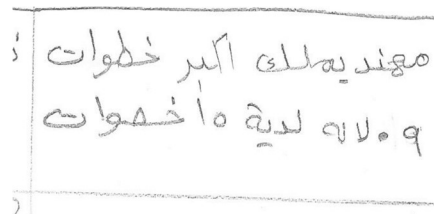


Figure 18: M14's Answer & Justification (Question 9)

Alternatively, we had the answer from student F7, who justified her response by saying, “Reem, seven steps. She has a few footsteps, which means she has a large footstep”. It is also worth

exploring whether the choice of the largest step belonging to a female impacted students' answer choices.

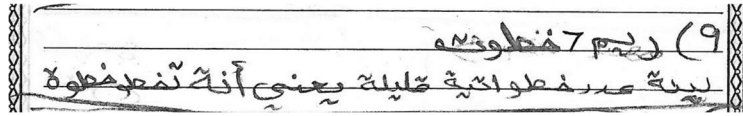


Figure 19: F7's Answer & Justification (Question 9)

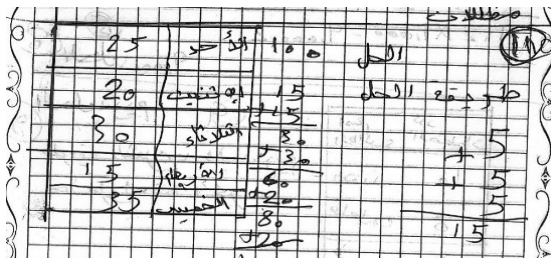
(5) Probability and Statistics.

Probability & Statistics			
Statistics			
	Question 11		
	B	G	Total
Answered Correctly	12	5	17
Partial Answer	1	4	5
Answered Incorrectly	2	3	5
Probability			
	Question 14		
	B	G	Total
Answered Correctly	0	7	7
Partial Answer	1	1	2
Answered Incorrectly	14	4	18

Table 7: Analysis of Questions (Probability & Statistics)

The first question in this group (statistics), Q11, shows a bar graph of the number of milk cartons sold in a school on the five school days and then asks about the total number of milk cartons sold in a week. Most of the boys (12 out of 15) answered correctly and justified having to add a separate number of sales each day to reach the total amount. Five of the 12 girls' answers also showed a similar justification.

One of the female students, F11's answer shows computational errors as she justified it correctly; however, she miscalculated as she added the numbers. While another student, F6's answer, shows



Handwritten calculations: $25 + 2 + 30 = 75$ and the number 11.

Figure 21: F11's Justification and Answer (Question 11) Figure 20: M'3 Answer & Justification (Question 11)



that she miscomprehended the question and answered that “Thursday has the highest sales,” this was also present in other students’ answers who gave the number of sales per day but did not add them. The student M3’s answer also showed the addition of the first three days (Sunday – Monday – Tuesday) and then disregarded Wednesday’s and Thursday’s sales.

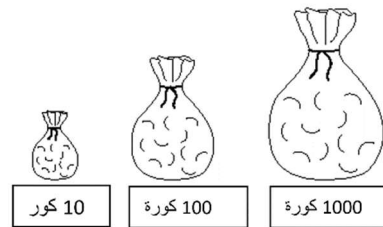


Figure 22: Question 14 diagram

In another question related to probability, Q14 states that there is one red ball in each of the bags that have 10, 100, and 1000 balls, and if a student must pick a ball without looking at any of these bags, which bag will give the highest probability of picking a red ball? Interestingly, 14 of the 15 boys stated that choosing a ball from the largest bag would give them the highest chance of picking the red ball “because it has the most balls.” The choice of justification signifies that the students either did not read the question correctly (there is only one red ball) or assumed that the more balls there were, the higher the chance they had (see figure 22).

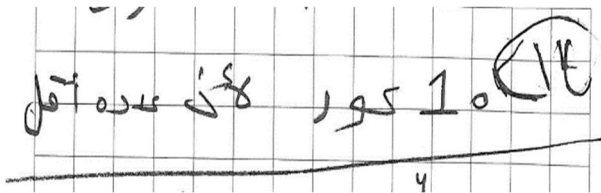


Figure 24: F9's Justification (correct) (Question 14)




Figure 23: M8's Justification (incorrect) (Question 14)

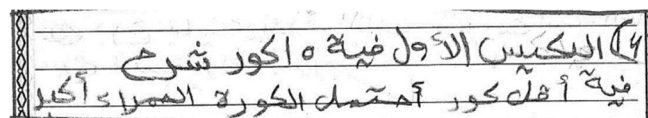


Figure 25: F7's Justification (Correct) (Question 14)

While most of the justifications for the incorrect choices showed that there was a misconception about having a higher chance with a higher number of balls, we can also see examples of the justifications of students F7 and F9 (see figures 23 and 24), who both justified that the smaller number of balls is small, with student F7 adding that the lowest number has the highest probability of picking a red ball.

DISCUSSION

The analysis of the 15 problem-solving questions presented to the fifth-grade students at a UAE Ministry of Education curriculum system school in Al Ain highlighted some key issues that have been discussed based on the five criteria: (1) logical thinking, (2) computational skills, (3) justification of the answers, (4) problem-solving skills and (5) usage of representations such as the use of a diagram, tables, and pictures, to solve.

Logical Thinking

Mathematics teachers should teach students logical thinking in mathematics because it helps them construct mathematical understanding (Weber, 2005). In our study, we conclude that students don't use logical thinking when solving mathematical problems. In question 15 (multiplication of 24 and 25 by 18), no students tried and solved it without computations, which signifies that those students read the question and answered it as a routine or procedural question without really thinking about it. While students are naturally familiar with their timetables (up to 12), they also need to understand the operations and how the increase in numbers is related, such as the algebraic structures $2x$ and $3x$ and their differences. More exercises related to number sense and operations might be helpful to students to train them in logical thinking about this question. This part also relates to algebraic thinking and generalization ability in computational reasoning. Algebraic thinking in solving a problem can help students solve the problem in more than one method or way. Solving a problem using algebraic thinking helps students build algebraic methods of thinking about the problem (Wahyuni, Herman & Fatimah, 2020).

Computational Skills

In problem-solving, computational thinking is an essential part of the solution to the problem. The students need to understand the problem and identify the variables using computational skills to get the solution (Sanford & Naidu, 2017). There were some errors in computational skills, most notable in the multiplication of two digits by two digits (as per the girls' answers that showed the steps). It was noteworthy that none of the girls' section could successfully perform the multiplication of two digits by two digits. The boys' answers did not provide the working out of the solution to gauge their level of multiplication skills.

Some computational errors and subtraction related to hastiness or skill errors, such as when students tried to subtract 33.2 from 96.4 and students either subtracted 3.2 or 63.2 rather than the actual number: 33.2. Computational skills (repeated addition – multiplying two digits by one) were noted as good since most students could perform them flawlessly. Computational skills for adding more than two digits were also performed with minimum mistakes. Computational fluency is one of the skills that help students solve problems, as mastery of these skills enables students and gives them a strong foundation. This is also found in various problem-solving theories and models. "Mastery of computation provides the foundation for more complex mathematics skills, such as problem-solving and algebra" (Nelson & Powell, 2017 p. 3).

Problem-Solving Skills

Students showed good problem-solving abilities when solving problems, even when their answers were not 100% correct. Students used many strategies, such as trial and error, explaining alternative solutions (fractions), and breaking down the information to better understand the question. It was noted that sometimes students seemed to read part of the questions, but not all, and this naturally affected the way they went about solving the questions. Problem-solving skills can be re-emphasized in the classroom by teaching these skills and having exploration classes where students try to understand the concepts through problem-solving. Metacognition and regulating their thoughts will also serve to help students stay on the right track and to understand that problem solving is a process that is constantly evolving (Reiss & Torner, 2007).

Justification of Answers

It was noticed that students sometimes found it challenging to justify their answers, and some copied the wording in the questions. Some tried to justify and use as few words as possible to explain their answers and how they got to them. This demonstrates the need for more exercises in which students are asked to justify their answers; this will also help them retrace their steps and think logically about their answers and whether they are appropriate logical answers to the problem. Various problem-solving models recognize the importance of justifying answers and seeking logical estimates of solutions, such as Polya's model and others. Asking problem-solvers to look back and think of other solutions would help ingrain this skill in students (Reiss & Torner, 2007).

Usage of Representations

Usage of representations was not present in students' answers. There was one example in a question related to subtraction (drawing of a shelf with a box on it and the respective lengths of the shelf and box). However, no other occurrences of this question were recorded for other students. In the question related to geometry, a couple of students also used pictures to help them understand the question (the perimeter of a rectangle). However, besides these two occurrences, there was no other usage of representations. This can highlight students not being used to the idea of representing their solutions or their thought process with diagrams, tables, or pictures. This can be emphasized to students during their classwork with problem-solving to reinforce the usage of

representations in problem-solving, especially since this is one of the main elements of various problem-solving models such as Polya's problem-solving model, as well as Schoenfeld's and various others (Liljedahl, Malaspina, Santos-Trigo, Bruder, 2016).

IMPLICATIONS AND LIMITATIONS

Students should be exposed to more word problems and given ample time to digest the information, read the question thoroughly, and devise a possible plan for a solution. Students should be encouraged to show their justifications for the answers and how they arrived at specific solutions. By making it a daily practice in mathematics classrooms in the UAE, students who are not trained in problem-solving will learn to question themselves and their intuition and hone their knowledge and skills in problem-solving. More focus should be given to computational skills in the lower and upper primary grades so students become mathematically fluent. Introducing technology to help students as an alternative to the drill and practice methods could be a way of encouraging students to take ownership of their learning through online practice in mathematics games and then with formative assessments in class in various ways (e.g., the Kahoot program as a tool for quizzes and games can be included).

As we see, problem-solving is the central part of international exams such as PISA and TIMSS (IEA, 2015, OECD, 2018). Educators need to adopt problem-solving as a competency that must be included in mathematics and other subjects. Teachers should adopt one of the problem-solving models, such as the Polya model, Schoenfeld model, or Verschaffel model. This will help the students to organize their thoughts while dealing with a problem. They have to learn how to understand, plan, solve, and check the solution.

The study was specific to a private MOE school in Al Ain with 27 students who attended either the boys' (n=15) or girls' (n=12) sections. The findings cannot be generalized to other schools in the area since the sample for study is small and because other schools can have different types of curricula in several languages. Another limitation of the study is that, because the test was conducted in two classes, issues such as using calculators or other devices to help with computations were not controlled. Hence, it is recommended to conduct studies with the same control situations as much as possible. A third limitation is concerned with the questions that were chosen from the TIMSS exam, which had varying weights amongst the strands and were not balanced between the strands. For example, the weight of the questions in the operations strand is 53.3%, while the weight of the questions in the geometry strand is 6.6%. In the future studies, we have to pay attention to this point.

CONCLUSION

Overall, five themes emerged from this study: logical thinking, computational skills, problem-solving skills, justification of answers, and usage of representations. In general, the students used these themes randomly, i.e., there was no training for the students on how to solve problems using such themes that organized their thinking and hence their strategy. According to the National Council of Teachers of Mathematics (NCTM, 2000), mathematics has five basic abilities: communication, reasoning, problem-solving, mathematical connection, and representation. This shows the strong relationship between problem solving and mathematics (Novitasari, Setianingsih, & Novitasari, 2019).

In conclusion, this study has highlighted a few issues in problem-solving in elementary schools in the UAE context. What has been gleaned from this preliminary study can be further explored to gain deeper insight into problem-solving. Students in the fifth-grade classes in this study have shown great aptitude in justifying their problem-solving strategies and show potential for more significant growth.

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Appendix (1)
Fifth grade problem-solving Questions-English

How Do We Think?

Name: _____

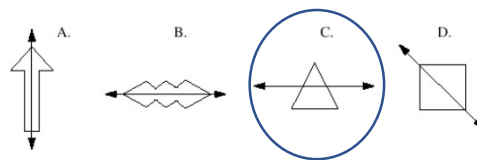
Day & Date: _____

Grade: _____

Dear Students,

Please answer the following questions, providing a short explanation of how you reached your conclusion. You can describe your thought process, or draw a diagram to show how you reached your answer. Below is an example of this:

Example 1: Which of these does not show a line of symmetry? Circle the correct answer and then provide an explanation.



Explanation: If we fold figure C in half, we can see that we have two different shapes, they are not identical, and that shows us that this is not a line of symmetry. However, in figures A, B and D, if we fold the shapes on the lines, we end up with two identical shapes. The two shapes are the same, and that shows us that these lines are lines of symmetry.

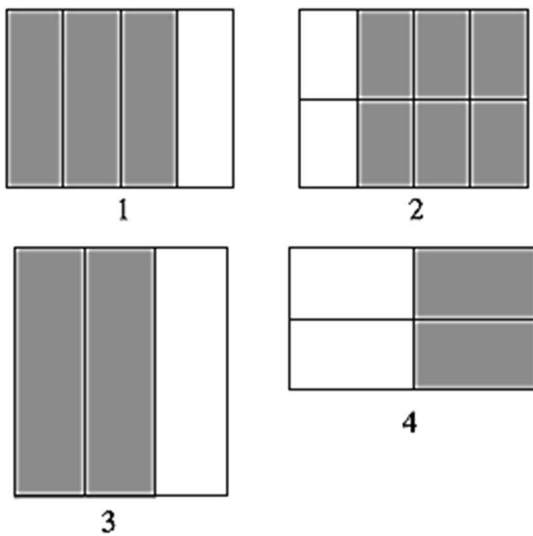
- End of Example –

1) When you subtract one of the numbers below from 900, the answer is greater than 300. Which number is it?

- A) 823
 B) 712

- C) 667
- D) 579
- 2) Tanya has read the first 78 pages in a book that is 130 pages long. Which number sentence could Tanya use to find the number of pages she must read to finish the book?

- 3) Each figure represents a fraction. Which two figures represent the same fraction?



- A) 1 and 2 B) 1 and 4 C) 2 and 3 D) 3 and 4

- 4) What do you have to do to each number in column A to get the number next to it in Column B?

Column A	Column B
10	2
15	3
25	5
50	10

- 5) Kyle and Bob are playing a game. The object of the game is to get the highest total of points. This chart shows how many points they scored.

Scorecard

Player	Kyle	Bob
Round 1	125	100
Round 2	125	125
Round 3	150	100
Round 4	50	150

Who won and by how many points?

- 6) A thin wire 20 centimeters long is formed into a rectangle. If the width of this rectangle is 4 centimeters, what is its length?
- 7) There are 54 marbles, and they are put into 6 bags, so that the same number of marbles is in each bag. How many marbles would 2 bags contain?
- 8) The weight (mass) of a clothespin is 9.2 g. Which of these is the best estimate of the total weight (mass) of 1000 clothespins?

Name	Number of Paces
Stephen	10
Erlane	8
Ana	9
Carlos	7

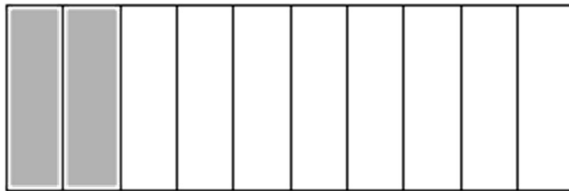
- A) 900 g
 B) 9,000 g
 C) 90,000 g
 D) 900,000 g

9) Four children measured the width of a room by counting how many paces it took them to cross it. The chart shows their measurements.

Who had the longest pace?

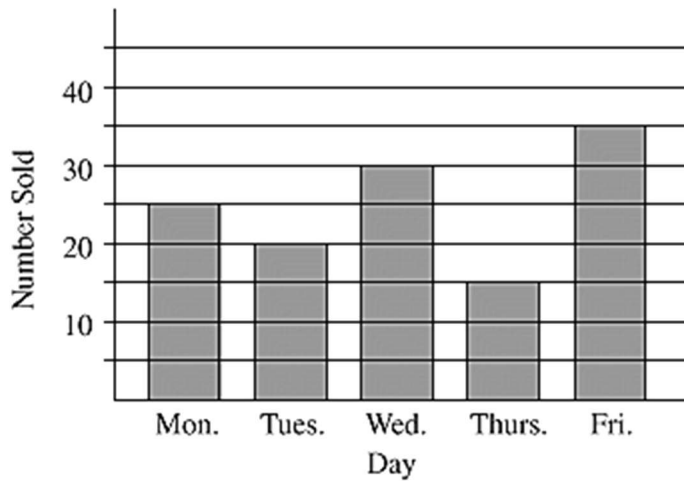
- A) Stephen
 B) Erlane
 C) Ana
 D) Carlos

10) Which number represents the shaded part of the figure?



- A) 2.8
 B) 0.5
 C) 0.2
 D) 0.02

11) The graph shows the number of cartons of milk sold each day of a week at a school.

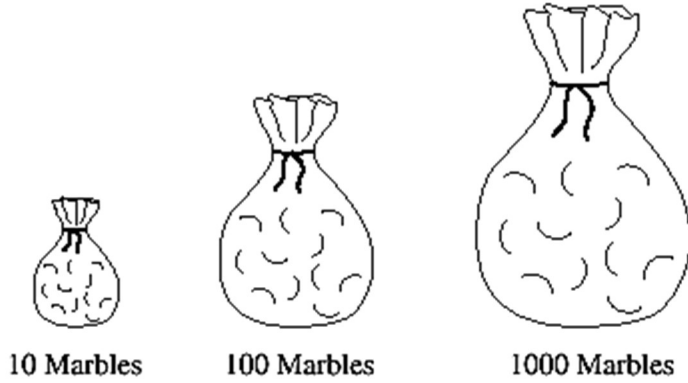


How many cartons of milk did the school sell that week?

12) Julie put a box on a shelf that is 96.4 centimeters long. The box is 33.2 centimeters long. What is the longest box she could put on the rest of the shelf?

13) A teacher marks 10 of her pupils' tests every half hour. It takes her one and one half hours to mark all her pupils' tests. How many pupils are in her class?

14) There is only one red marble in each of these bags?



Without looking in the bag, you are to pick a marble out of one of the bags. Which bag would give you the greatest chance of picking the red marble?

15) 25×18 is more than 24×18 . How much more?

Referred from:

<https://www.edinformatics.com/timss/pop1/mpop1.htm?submit3243=Grade+3%2C4+Math+Test>

Appendix (2) Fifth grade problem-solving Questions-Arabic

كيف أفكر؟

اليوم والتاريخ: _____

اسم الطالب: _____

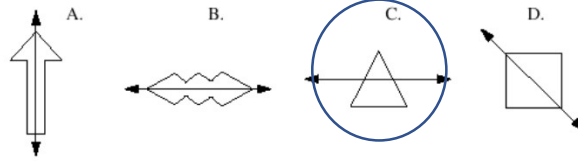
الصف: _____

عزيزي الطالب/ة،
بين يديك اختبار بسيط يعزز من التفكير الرياضي والمنطقي لدى الطالب. هذا مجرد اختبار تشخيصي و لا علاقة له بأية درجات. بعد قراءة الأسئلة والإجابة عنها، اشرح طريقة الحل. بإمكانك وصف طريقة تفكيرك أو رسم نموذج توضيحي يبين طريق توصلك للحل. انظر للمثال 1:

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مثال 1: أي من الأشكال التالية لا يمثل خط تناظر؟ حوِّط على الإجابة الصحيحة و اشرح.



شرح طريقة التفكير:

عند طي الشكل C لا نحصل على جزئين متماثلين، إذا هذا لا يمثل خط التناظر. و لكن عندما نثني الأشكال في A و B و D نحصل على أشكال متناظرة أي مطابقة.

(1) عندما طرحنا عدد ما من 900 كان الناتج أكبر من 300. ما هو هذا العدد؟ اشرح

- (A) 823
(B) 712
(C) 667
(D) 579

شرح طريقة التفكير:

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(2) قرأت أمنة أول 78 صفحة من كتاب يحوي 130 صفحة. اكتب جملة عددية توضح عدد الصفحات التي يجب أ، تقرأها أمنة لتنتهي الكتاب؟

شرح طريقة التفكير:

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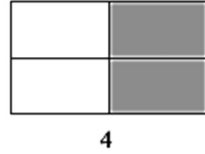
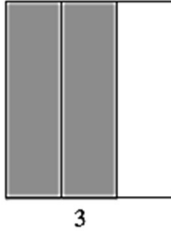
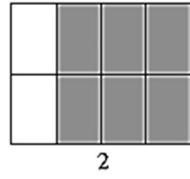
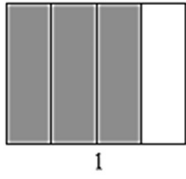
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(3) كل شكل مما يلي يمثل كسراً. أي شكلين منهم يمثل نفس الكسر؟ اشرح

(A) 1 و 2

(B) 1 و 4

(C) 2 و 3

(D) 3 و 4

شرح طريقة التفكير:

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Column A	Column B
10	2
15	3
25	5
50	10

(4) ما الذي يجب أن تفعله لكل رقم في المجموعة A لتحصل على الرقم المناظر له في المجموعة B؟

شرح طريقة التفكير:

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(5) يلعب أحمد وعائشة لعبة يكون الفائز فيها من يحصل على أعلى النقاط. يبين الجدول أدناه النقاط التي سجلها كل منهما. من الفائز وبكم نقطة فاز؟

بطاقة تسجيل النقاط		
عائشة	أحمد	اللاعب
100	125	ال الجولة الأولى
125	125	ال الجولة الثانية
100	150	ال الجولة الثالثة
150	50	ال الجولة الرابعة

شرح طريقة التفكير:

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(6) استخدم خالد سلكاً رفيعاً طوله 20 cm لصنع مستطيل عرضه 4 cm. فما هو طول هذا المستطيل؟

شرح طريقة التفكير:

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(7) يوجد مع راشد 54 كرة صغيرة، قام بتوزيعها على 6 أكياس بالتساوي. كم عدد الكرات في كيسين؟

شرح طريقة التفكير:

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(8) وجدت فاطمة أن كتلة مشبك الغسيل تساوي 9.2 g . أي مما يلي يمثل تقريبا كتلة 1000 مشبك غسيل؟ اشرح

900 g (A)

9,000 g (B)

90,000 g (C)

900,000 g (D)

شرح طريقة التفكير:

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(9) قام 4 طلاب بقياس عرض الغرفة باستخدام خطواتهم و يبين الجدول التالي عدد الخطوات التي احتاجوها للقياس. من منهم يملك أكبر خطوة و لماذا؟

عدد الخطوات	اسم الطالب
10	مهند
8	فاطمة
9	فيصل
7	ريم

شرح طريقة التفكير:

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10) ما هو العدد الذي يمثل الجزء المُضَلَّل، اختر الاجابة الصحيحة. كيف عرفت؟

- (a) 2.8
(b) 0.5
(c) 0.2
(d) 0.02

شرح طريقة التفكير:

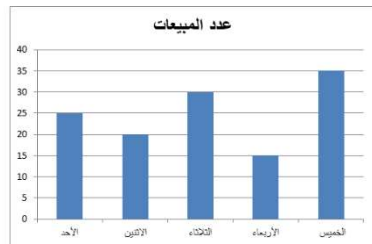
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11) يوضح الرسم البياني عدد مبيعات عبوات الحليب في المدرسة لمدة اسبوع. كم عدد مبيعات الحليب خلال هذا الاسبوع؟

شرح طريقة التفكير:

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12) وضعت مها علبة على رف طوله 96.4 سنتمتر. طول العلبة 33.2 سنتمتر. هل تستطيع مها وضع علبة أخرى على الرف؟ ما أقصى طول للعلبة الثانية؟

شرح طريقة التفكير:

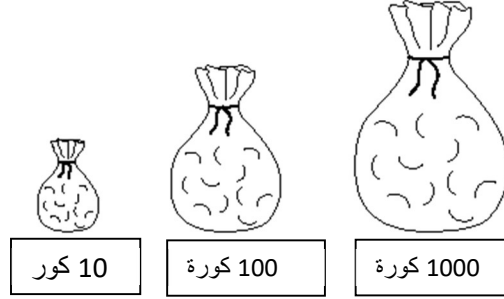
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13) يصحح معلم اختبارات 10 طلاب في نصف ساعة. استغرق ساعة و نصف لالنتهاء من تصحيح جميع اختبارات الصف الخامس. كم طالب في الصف الخامس؟

شرح طريقة التفكير:

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14) في كل من الأكياس الثلاثة التالية كرة حمراء واحدة. عليك سحب كرة من دون النظر في داخل الكيس. أي من الأكياس يعطيك احتمالاً أكبراً بسحب كرة حمراء؟ حوِّط على الكيس و اشرح



شرح طريقة التفكير:

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15) 25 ضرب 18 أكبر من 24 ضرب 18. بكم أكبر؟ _____

شرح طريقة التفكير:

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- انتهت الأسئلة -