

Analysis of Problem Solving Process on HOTS Test for Integral Calculus

Eko Andy Purnomo¹, Y.L. Sukestiyarno², Iwan Junaedi³, Arief Agoestanto⁴

¹Doctoral Students of Graduate School, Universitas Negeri Semarang, Indonesia, ²

⁴Departement of Mathematics, Universitas Negeri Semarang, Indonesia

ekoandy@unimus.ac.id, sukestiyarno@mail.unnes.ac.id, iwanjunmat@mail.unnes.ac.id,
arief.mat@mail.unnes.ac.id

Abstract: *Problem-solving is the essence of mathematics and is the main goal in learning mathematics. Many students did not have good problem-solving skills based on the field observations. The problems grew up because the students were not used to solving the problems and the problem-solving stages. They did not include issues with high complexity, such as questions with the High Order Thinking Skills (HOTS) category. The problem-solving steps have been developed, such as; Dewey (1910), Polya (1945), Mason, Burton & Stacey (1982), Schoenfeld (1985), and Wilson et al. (1993). The objective of this current study was to analyze the stages of problem-solving in solving mathematical problems with the High Order Thinking Skills (HOTS) category. The research sample was 57 students of Mathematics Education in one of Private University of Indonesia who took the Integral Calculus subject. This research was a qualitative descriptive study. The data analysis employed an inductive approach where the conclusions were drawn from minor case investigations to provide comprehensive results. The data analysis consisted of reduction data, presentation data, and concluding. There are three results from the current research. First, the students have not implemented problem-solving with problem-solving stages; second, The students fail to solve the problems due to a lack of mathematical literacy skills; last. The incomplete mathematization process causes imperfect problem-solving. Based on the results, the research recommendation is to add stages of problem-solving with two steps: formulating the situation mathematically and understanding mathematical solutions in real life or problems.*

Keywords: *Integral calculus, Mathematical Literacy, Mathematization, Problem Solving, HOTS*

INTRODUCTION

Problems Solving with a high level of difficulty can use the stages of problem-solving. Problem-solving is often used when solving non-routine problems (Temur, 2012), complex problems (Greiff, S & Fischer, 2013) where the problem solver does not know the previous scheme (Schoenfeld, 1992). Problem-solving is used to help problem

solvers learn how to think mathematically (Rott et al., 2021) and systematically (Goulet-Lyle et al., 2020). The problem-solving model should act as a guide to help problem solvers in the thinking process. Problem-solving is the most important skill for students (Damayanti, & Sukestiyarno, 2014; Erlina & Purnomo, 2020; Sulistyarningsih et al., 2021) and is part of the mathematics curriculum in almost all countries, including the United States (Schoenfeld, 2007), Australia (Clarke et al., 2007), Netherlands (Doorman et al., 2007), China (Cai & Nie, 2007), French (Artigue & Houdement, 2007), Hungarian (Szendrei, 2007) and English (Burkhardt & Bell, 2007). In the curriculum, problem-solving is also one of the students' skills (Dagan et al., 2018; Indriyani et al., 2018).

Based on the research, the problem-solving ability of the students was still low (Purnomo & Mawarsari, 2014; Purnomo et al., 2014; Amir, 2015; Wardono et al., 2016); Hidayat & Irawan, 2017; Kusuma et al., 2017; Yeni et al., 2020). Students are still unable to solve Higher Order Thinking Skills (HOTS)-type questions due to a lack of problem-solving skills (Abdullah et al., 2015; Susanto & Retnawati, 2016; Kusuma et al., 2017; Karimah et al., 2018). The difficulties experienced by students when completing mathematics included lack of understanding of the questions and information provided, use of concepts, calculations, inaccuracies (Phonapichat et al., 2014), inability to use correct strategies, and lack of creativity in solving problems (Supandi et al., 2021). It is because the students are not accustomed to solving problems through the stages of problem-solving. As a result, the solution to the problem is not ideal. Based on these studies, it is necessary to improve problem-solving abilities.

In learning mathematics, students learn not only math material but also math skills (Piñeiro et al., 2021; Purnomo et al., 2021), think creatively (Nuha et al., 2018), and learn to face problems (Agoestanto & Sukestiyarno, 2017). Many theories suggest the stages of problem-solving. The first researcher who introduced problem-solving were Dewey (1910), then Polya (1945), Mason, Burton & Stacey (1982), Schoenfeld (1985), and Wilson et al. (1993). Dewey problem-solving consists of five stages: (i) encountering a problem (suggestions), (ii) specifying the nature of the problem (intellectualization), (iii) approaching possible solutions (the guiding idea and hypothesis), (iv) developing logical consequences of the approach (reasoning (in the narrower sense)), and (v) accepting or rejecting the idea by experiments (testing the hypothesis by action) (Dewey,

1910). Polya's most commonly used solution consists of four stages including (i) understanding the problem, (ii) devising a plan, (iii) carrying out the plan, and (iv) looking back (Polya, 1978). Meanwhile, Schoenfeld divides the stages of problem-solving into 5, including (i) analysis, (ii) design, (iii) exploration, (iv) implementation, (v) verification (Schoenfeld, 1992). Based on the analysis, each problem-solving model has its own characteristics.

A lack of ability to solve mathematical issues will impact the development of essential mathematical skills required by the students (Oktaviyanthi & Agus, 2019). Problem-solving can run well when a person has experience solving problems (Greiff, S & Fischer, 2013) and does not experience cognitive barriers (Antonijević, 2016). One of the abilities that support the problem-solving process includes good mathematical literacy skills. Mathematical literacy implies a foundation of knowledge and competence and the confidence to apply knowledge to the practical world. People with mathematical literacy skills can estimate, interpret data, solve everyday problems, reason in numerical, graphic, and geometric situations, and communicate using mathematics (Ojose, 2011). In solving problems, mathematical literacy is a very important part.

In addition to mathematical literacy, other things that need to be considered in problem-solving include mathematical modeling (Klymchuk, 2015). Mathematical literacy deals with real problems hoping that problem-solvers must 'solve' real-world issues that require the skills and competencies they have acquired. A fundamental role in the process is referred to as mathematization. The mathematization process develops concepts and ideas starting from the real world and ultimately reflecting the results obtained in mathematics back to the real world (Lange, 2006).

The problem-solving process in the field looks different from the existing stages. The solved problems contained errors in complicated cycles, and the issues were solved not following the sequence of the previous steps (Rott et al., 2021). This actual process was not considered during the last problem-solving model. Based on this, it is necessary to improve the stages of problem-solving so that it is easier for students to solve these problems.

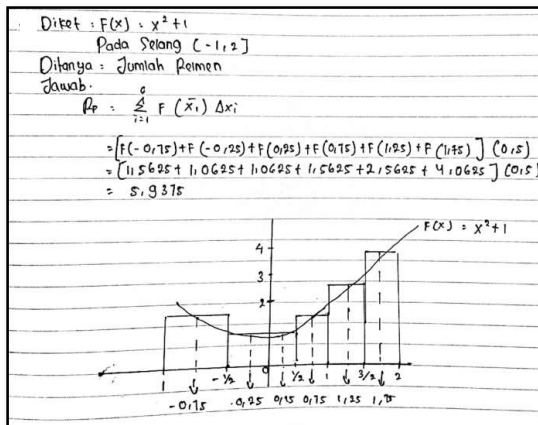
METHODOLOGY

This research is a qualitative descriptive study that describes the process and results of the problem-solving stages. Integral Calculus was taught to 57 students at a private university in Indonesia who were all undergraduates in Mathematics Education. The examination of the problem-solving findings of 57 students yielded six answer categories, with two answers to each question divided. Data was gathered through evaluation tests, observations, and in-depth interviews as part of the triangulation method. A three-tiered evaluation test was devised: simple, medium, and complicated.

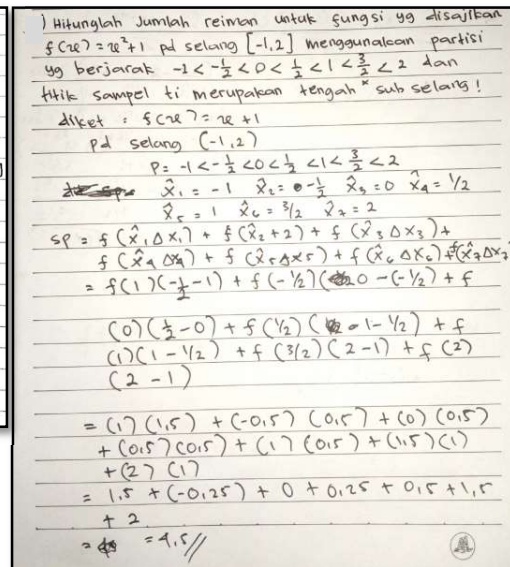
Triangulation was used as a data-gathering method, which involved conducting experiments, observations, and in-depth interviews to gather information (Sukestiyarno, 2020). The data analysis used an inductive approach where conclusions are drawn from small case investigations in detail to provide a big picture (Sukestiyarno, 2020). The data analysis consisted of data reduction, data presentation, and concluding. The data reduction was made by coding the student answers. The coding is used to facilitate the tracking of important data about the exposure of existing data. After the data is reduced, the next step is to present the data by verifying the data with in-depth interviews. The last step is to make conclusions from the data in the field.

RESEARCH FINDINGS

In this current study, the students were asked to work on three questions, and the results were analyzed as follows. The first problem is the calculation of the Riemann sum for the function represented by $f(x) = x^2 + 1$ in the interval $[-1,2]$ using equidistant partitions $-1 < -\frac{1}{2} < 0 < \frac{1}{2} < 1 < \frac{3}{2} < 2$ and the sample point t_i is the middle of the sub-interval! The results of student work can be seen in the image below.



Gambar 1A



Picture 1B

Picture 1. Students' Performance on Number 1

Based on Figure 1a above, it can be analyzed as follows. Student A1 has formulated the situation mathematically by illustrating the problem in a picture. Students have determined the known information by writing what is known, namely the function $f(x) = x^2 + 1$ and the interval $[-1, 2]$. In writing, the elements that are known to be incomplete, such as partition: $-1 < -\frac{1}{2} < 0 < \frac{1}{2} < 1 < \frac{3}{2} < 2$ and the sample point t_i It is the middle of the sub-interval. Students can determine the unknown information, namely the middle of the sub-interval, but it is not written in the answer. Students find the relationship between the data and the strange, but the answer is not reported. The solution plan is not written down, but the answer is. The student does not respond to the query. Since students did not complete the sixth step in problem-solving In-depth interviews were done

The interview results explicate that student A1 draws a graph to see more fully what is known in the problem. Students answered questions using pictures. Then, student A1 did not write the solution plan because he was not instructed to write the solution design in the answer. Student A1 only has one solution design. Student A1 implements the solution plan but does not check every step. It is because the work takes a lot of time.

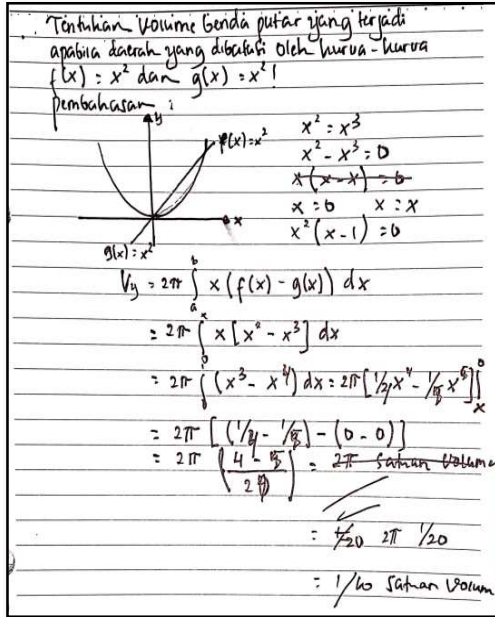
Student A1 contains answers to solving questions. At the end of the solution, student A1 did not respond according to the question.

Students are good at answering questions, but there are stages of problem-solving that have not been implemented, namely in the sixth stage. Student A1 is right by first describing the situation in the form of pictures so that it helps students in answering questions. Based on this, it is necessary to emphasize one more step in the problem-solving process, namely "understanding mathematical solutions in real life or problems." The results of the second student work can be seen in the image below.

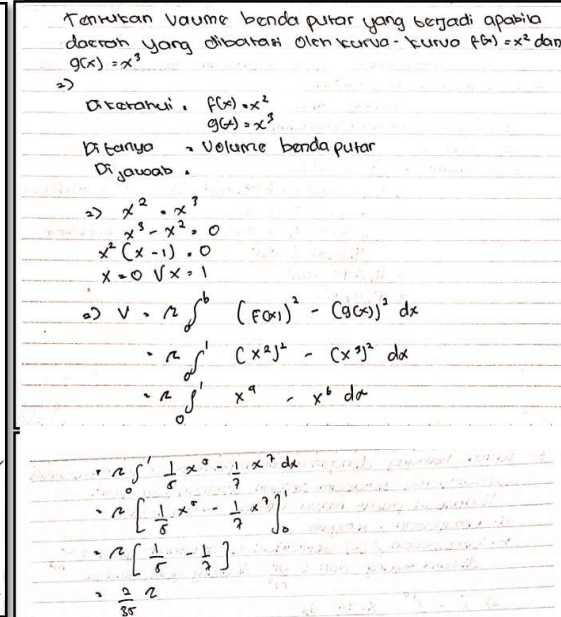
In Figure 1b, it can be analyzed that B1 students do not formulate the situation mathematically. The student has determined the known information by writing what is known, namely the function $f(x) = x^2 + 1$ and interval $[-1,2]$ and partition: $-1 < -\frac{1}{2} < 0 < \frac{1}{2} < 1 < \frac{3}{2} < 2$. Students have also determined the sample points, namely $x_1 = -1, x_2 = -\frac{1}{2}, x_3 = 0, x_4 = -\frac{1}{2}, x_5 = 1, x_6 = -\frac{3}{2}, x_7 = 2$. Students cannot determine unknown information. Namely, the sample point t_i is the middle of the sub-interval. It is what causes students to misperceive the middle point, but students answer the endpoint of the interval. It resulted in the student's answer being wrong. Students did not find a relationship between the unknown data. The solution plan was not written down, but the students immediately wrote down the answers. Student B1 implements the solution plan, but the answer is not quite right. Student B1 does not re-check the answers that have been written. The last step, "understanding mathematical solutions in real life or problems," is not implemented.

The B1 student's error by not formulating the situation in mathematical form resulted in the student being wrong in determining the midpoint. There is still confusion between the middle and the end of the hose. Student B1 knows that the sample point is the end of the interval based on the interviews. It is due to the inability of students to analyze what is known. It is due to the low ability of students in mathematical literacy. It can help students in mathematical literacy by "formulating the situation mathematically." The other solving stages were also not implemented properly, so the students' answers were wrong.

The second problem is to determine the volume of a rotating object that occurs when the area bounded by the curves $f(x) = x^2$ and $g(x) = x^3$. The results of student work can be seen in the image below.



Picture 2a



Picture 2b

Picture 2. The Students' Performance on Number 2

Based on Figure 2a, it can be concluded as follows. By drawing a representation of the problem, student A2 has expressed the situation quantitatively. Student A2 has created a graph to represent the already known data. Anonymous data, namely the point of intersection between two charts, can be selected by students. $f(x) = x^2$ and $g(x) = x^3$. Students find the relationship between data and the unknown. Namely, the intersection point is used as the upper and lower limits. The student did not write the solution plan. Students do not carry out the last stage in problem-solving, namely "understanding mathematical solutions in problems or real life."

Based on the results of the interviews, it was concluded that the students were less precise in presenting the picture in question. Students draw a graph of the function $f(x) = x^2$ as a straight line. The graph $f(x) = x^2$ should be a curved curve. It is due to the lack of good literacy at the "mathematically formulating the situation" stage. The

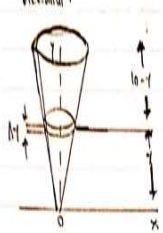
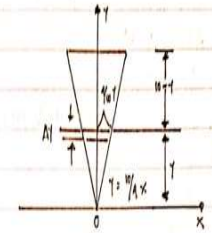
solution plan is not written down but directly written the answer. Students carry out the solution plan and check it at every step. The student does not return the response according to the question. The results of the second student work are as follows.

Figure 2b Student B2 does not formulate the situation mathematically. It does not illustrate the problem in the form of a picture. The student has determined the known information by determining $f(x) = x^2$ and $g(x) = x^3$. Students can determine the unknown information, namely the intersection point between the graphs $f(x) = x^2$ and $g(x) = x^3$. Students find the relationship between known and unknown data; namely, the intersection point is the upper and lower limits. The solution plan is not written down but directly writes the answer and the solution plan is not right. The student implements the solution plan but does not check every step. The student does not return the response according to the question.

Based on the interviews, it was concluded that students were wrong in determining the completion of the volume of a rotating object. Students should distinguish between finding volume using the disc, ring, or cylinder method. It is due to the lack of good literacy at the stage of "mathematically formulating the situation." The solution plan is not written down, but the students already know the steps. The solution strategy is incorrect because the students follow the solution strategy but do not double-check each step. The pupil does not respond to the query. The results for the third question are as follows.

Diket: tinggi tangki = 10 kaki
Jari-jari = 4 kaki
Ditanya: a. Tentukan gaya yang diperlukan
a. Merivati tepi atas tangki
b. Mencapai 10 kaki di atas puncak tangki
Jawab:
Sebuah gambar ~~tanpa~~ ~~ada~~
Perhatikan sebuah kerucut memiliki jari-jari $\frac{4}{10}y$, maka V kompartan
 $\Delta V = \pi \left(\frac{4}{10}y\right)^2 \Delta y$ dan beratnya (δ gaya berat) $F = \delta \pi \left(\frac{4}{10}y\right)^2 \Delta y$
dengan $\delta = 62,4$ (kepadatan air)
 \Rightarrow Gaya yang diperlukan adalah $10-y$. Jadi kerja Δw adalah
 $\Delta W = \delta \pi \left(\frac{4}{10}y\right)^2 \Delta y (10-y)$
Meneranya
 $w = \delta \pi \frac{4}{25} \int_0^{10} (10y^2 - y^3) dy = \delta \pi \frac{4}{25} \left[\frac{10y^3}{3} - \frac{1}{4}y^4 \right]_0^{10} \approx 26,130$
Jadi kerja untuk memompa air sampai ke tepi tangki adalah
 $26,130$ kaki pond.
 \Rightarrow Sama dalam diatas, Sebarang air dalam kerucut harus diangkat
 $20-y$, maka
 $w = \delta \pi \int_0^{10} \left(\frac{4}{10}y\right)^2 (20-y) dy = \delta \pi \frac{4}{25} \int_0^{10} (20y^2 - y^3) dy$
 $dy = \delta \pi \frac{4}{25} \left[\frac{20y^3}{3} - \frac{1}{4}y^4 \right]_0^{10} \approx 130,69$
Jadi kerja untuk memompa air sehingga ke tepi tangki adalah
 $130,690$ kaki pond.

Picture 3a

Tangki kerucut tersebut tergenangi sepenuhnya dengan air. Alesa kerugi tangkinya 10 kaki dan jari-jari tangkinya diam 4 kaki. Tentukan gaya yang diperlukan untuk memompa air!
Penyelesaian: a. Merivati tepi atas tangki
Meneranya:


Ditanya: Merivati tepi atas tangki!
Jawab: $F = \int_0^{10} \delta \pi \left(\frac{4y}{10}\right)^2 (10-y) dy = \delta \pi \frac{4}{25} \int_0^{10} (10y^2 - y^3) dy$
 $= \frac{4\pi(62,4)}{25} \left[\frac{10y^3}{3} - \frac{y^4}{4} \right]_0^{10} = 26,130$ feet-pond
Penyelesaian: b. Mencapai 10 kaki Puncak Tangki
Diketahui: Gambar sama dengan A
Ditanya: 10 kaki Puncak Tangki!
Jawab: Air di kerucut diangkat $20-y$
 $F = \delta \pi \int_0^{10} \left(\frac{4y}{10}\right)^2 (20-y) dy = \delta \pi \frac{4}{25} \int_0^{10} (20y^2 - y^3) dy$
 $= \frac{4\pi(62,4)}{25} \left[\frac{20y^3}{3} - \frac{y^4}{4} \right]_0^{10} \approx 130,690$ feet-pond

Picture 3b

Picture 3. The Students' Performance on Number 3

Based on Figure 3a, it can be seen that A3 students do not formulate in a mathematical situation. Student A3 has determined the known information by writing down what is known, namely the height and radius of the tank. Students also write down the questions asked in the questions. A3 students can determine the unknown information, namely the radius of a circle with a thickness y at the height of Δy , which is $\frac{4y}{10}$. Students find the relationship between the data and the unknown, namely the volume of the disc, namely $\Delta v = \pi \left(\frac{4y}{10}\right)^2 \Delta y$ and its weight (gravity) $\delta \pi \left(\frac{4y}{10}\right)^2 \Delta y$ with $\delta = 62,4$ (density of water). The solution plan is not written on the answer sheet but directly reports the answer. Execute the solution plan and check every step. There are two steps: looking for work for

a water pump a). past the top edge of the tank, and b). reach 10 feet above the top of the foot. The required force is $10-y$. so work $\Delta w = \delta\pi\left(\frac{4y}{10}\right)^2 \Delta y 10 - y$. So, the work to pump water up to the edge of the tank is 26,138 pound-feet. Same with the previous point problem, now the water in the cone must be lifted $20 - y$ then. So, the work to pump the water up to the edge of the tank is 130,69 pound-feet. A3 students have carried out the last stage in problem-solving.

Based on the interviews, A3 students have answered correctly, but there are stages of problem-solving that have not been carried out. A3 students do not formulate in a mathematical situation lacking in detail in presenting known things, for example, by giving pictures. The other solving steps have been carried out well by A3 students.

Figure 3b for B3 students is almost the same as A3 students. A graphic depicts the discrepancy between B3 students' work and their peers in the section that they are very familiar with each other. It means that B3 students have formulated a mathematical situation through a presentation with images. This step will make it clearer to write down what is known in the problem, look for what is not known in the problem, and answer questions at each step. But the answers of B3 students are incomplete because they do not return the solutions according to the questions. B3 students do not carry out this last step. Based on interviews with B3 students, the answers were not returned to the questions because the students thought that the problem solving was finished when they got the answers. It is because students do not know the problem-solving process well. One solution to complete students' responses by adding one problem-solving stage is "adding the mathematization process, namely understanding mathematical solutions in problems or real life."

The design of the new problem-solving stages could be applied to know the advantages and disadvantages if implemented in the field. Therefore, based on aforementioned findings, a new problem-solving stage design was made into six stages of problem-solving as shown in Table 1.

Stages	Stages of problem-solving	Description
Stage 1	Formulate the situation mathematically	Students change problems in mathematical situations
Stage 2	Understand the problem	Students identify things that are known in the problem
Stage 3	Plan problem solving	Students plan problem solving to be carried out
Stage 4	Carry out problem-solving	Students carry out problem-solving by the problem-solving plan
Stage 5	Review the results of problem-solving	Students check the correctness of the answers that have been made
Stage 6	Understand mathematical solutions in real life or problems	Students relate the answers they found into real life.

Table 1. New problem-solving stage design

RESEARCH DISCUSSION

This current study aims to analyze the problem-solving process in the HOTS category questions in universities. This research focuses on how the stages of problem-solving can help students solve problems. The analysis results will see the weaknesses and strengths of the existing problem-solving steps. Based on this analysis, a new solution stage will optimize the process and problem-solving results. The results of the student problem-solving process can be concluded that students have difficulties at the beginning of the problem-solving process. For example, in question no. 1, student B1 could not determine the midpoint, in question no. 2, student A2 was less precise in drawing the graph $f(x) = x^2$. The lack of mathematical literacy skills causes this student's difficulty to improve literacy skills in problem-solving. It is necessary to have stages of formulating the situation mathematically. When students can do this stage, they will be able to describe what is known in the problem to look for things that are not known in the issue. Students have difficulty dealing with non-routine problem situations (Temur, 2012) and lack problem-solving experience (Greiff, S & Fischer, 2013), so it is necessary to increase mathematical literacy. One way to improve mathematical literacy is to model

mathematically (Klymchuk, 2015). Through this activity, students will know more deeply about analyzing what is known and looking for things that are not known in the problem.

The next student error is at the end of the problem-solving stage. The student does not return the answer according to the question. The mathematization process is not done well, namely understanding mathematical solutions in real life. This process is a refinement of the mathematization and modeling activities in real situations. Modeling in real problems can help in the problem-solving process (Stender & Kaiser, 2015; Purnomo et al., 2020) and increase the creativity of the problem-solving process (Schindler & Lilienthal, 2020). Mathematical modeling is considered central to an important element in problem-solving (Carotenuto et al., 2021). When students are not used to solving the problems, the problem-solving steps are not being by the stages (Nurkaeti, 2018), and the results are not optimal (Greiff, S & Fischer, 2013). The problem-solving process contains errors and does not follow a predetermined order, as in the normative model (Rott et al., 2021). So, it is necessary to improve the problem-solving stage to make it easier for students to understand.

The results showed that 1). Students have not implemented problem-solving with problem-solving stages; 2). Students cannot solve problems due to a lack of mathematical literacy skills; 3). Students have not carried out a complete mathematization process. Based on this analysis, it is necessary to add additional stages in the problem-solving process, namely 1). steps added at the beginning before problem-solving by formulating the situation mathematically. The goal is to improve mathematical literacy skills. Students will use this ability to carry out the next stage's problem-solving process. 2). adding the last step by inserting the mathematization process, namely understanding mathematical solutions in real life or problems. The goal is to become reflective problem solvers who can become good (Evans, 2015) by returning answers according to the existing issues. By adding these two stages, the problem-solving stage will become complete. The result is that the problem-solving process will run well, and students' problem-solving abilities will increase.

CONCLUSION

The results showed that 1). The students have not implemented problem-solving with problem-solving stages; 2). The students cannot solve problems due to a lack of mathematical literacy skills; 3). The students have not carried out a complete mathematization process. Based on these conclusions, the recommendation from this research is to add stages of problem-solving. Steps are added at the beginning before problem-solving by formulating the situation mathematically and at the ending problem-solving process by inserting the mathematization process, namely, understanding mathematical solutions in real life.

REFERENCES

- [1.] Abdullah, A. H., Abidin, N. L. Z., & Ali, M. (2015). Analysis of students' errors in solving Higher Order Thinking Skills (HOTS) problems for the topic of fraction. *Asian Social Science*. <https://doi.org/10.5539/ass.v11n21p133>
- [2.] Agoestanto, A, Sukestiyarno, Y. . & R. (2017). Analysis of mathematics critical thinking students in junior high school based on cognitive style. *Journal of Physics: Conference Series*, 824:012052, 755(1). <https://doi.org/10.1088/1742-6596/755/1/011001>
- [3.] Amir, M. F. (2015). Proses Berpikir Kritis Siswa Sekolah Dasar Dalam Memecahkan Masalah Berbentuk Soal Cerita Matematika Berdasarkan Gaya Belajar. *Jurnal Math Educator Nusantara*, 01(02), 159–170. <http://ojs.unpkediri.ac.id/index.php/matematika/article/download/235/150>
- [4.] Antonijević, R. (2016). Cognitive activities in solving mathematical tasks: The role of a cognitive obstacle. *Eurasia Journal of Mathematics, Science and Technology Education*, 12(9), 2503–2515. <https://doi.org/10.12973/eurasia.2016.1306a>
- [5.] Artigue, M., & Houdement, C. (2007). Problem solving in France: Didactic and curricular perspectives. *ZDM - International Journal on Mathematics Education*, 39(5–6), 365–382. <https://doi.org/10.1007/s11858-007-0048-x>
- [6.] Burkhardt, H., & Bell, A. (2007). Problem solving in the United Kingdom. *ZDM - International Journal on Mathematics Education*, 39(5–6), 395–403. <https://doi.org/10.1007/s11858-007-0041-4>
- [7.] Cai, J., & Nie, B. (2007). Problem solving in Chinese mathematics education: Research and practice. *ZDM - International Journal on Mathematics Education*, 39(5–6), 459–473. <https://doi.org/10.1007/s11858-007-0042-3>
- [8.] Carotenuto, G., Di Martino, P., & Lemmi, M. (2021). Students' suspension of sense making in problem solving. *ZDM - Mathematics Education*, 0123456789. <https://doi.org/10.1007/s11858-020-01215-0>
- [9.] Clarke, D., Goos, M., & Morony, W. (2007). Problem solving and Working Mathematically: An Australian perspective. *ZDM - International Journal on Mathematics Education*, 39(5–6), 475–490. <https://doi.org/10.1007/s11858-007->

0045-0

- [10.] Dagan, M., Satianov, P., & Teicher, M. (2018). Creating Use of Different Representations as an Effective Means to Promote Cognitive Interest, Flexibility, Creative Thinking, and deeper understanding in the Teaching of Calculus. *Mathematics Teaching-Research Journal*, 10(3–4), 41–51.
- [11.] Damayanti, & Sukestiyarno, Y. L. (2014). Meningkatkan Karakter dan Pemecahan Masalah Melalui Pendekatan Brain-Based Learning Berbantuan Sirkuit Matematika. *Kreano: Jurnal Matematika Kreatif-Inovatif*, 5(1), 82–90. <https://doi.org/10.15294/kreano.v5i1.3281>
- [12.] Dewey, J. (1910). *How we think*. In D C Heath.
- [13.] Doorman, M., Drijvers, P., Dekker, T., van den Heuvel-Panhuizen, M., de Lange, J., & Wijers, M. (2007). Problem solving as a challenge for mathematics education in The Netherlands. *ZDM - International Journal on Mathematics Education*, 39(5–6), 405–418. <https://doi.org/10.1007/s11858-007-0043-2>
- [14.] Erlina & Purnomo, E. A. (2020). Implementasi Lesson Study Melalui Model Pembelajaran Problem Based Learning Materi Spltv Kelas X Iik. *AlphaMath : Journal of Mathematics Education*, 6(1), 36–45. <https://doi.org/10.30595/alphamath.v6i1.7619>
- [15.] Evans, B. R. (2015). Breakthrough Moments In Problem Solving. *Mathematics Teaching-Research Journal*, 8(1).
- [16.] Goulet-Lyle, M. P., Voyer, D., & Verschaffel, L. (2020). How does imposing a step-by-step solution method impact students' approach to mathematical word problem solving? *ZDM - Mathematics Education*, 52(1), 139–149. <https://doi.org/10.1007/s11858-019-01098-w>
- [17.] Greiff, S & Fischer, A. (2013). Measuring Complex Problem Solving: An educational application of psychological theories. *Journal of Educational Research Online*, 5(1), 38–58. <http://www.j-e-r-o.com/index.php/jero/article/download/338/160>
- [18.] Hidayat, A., & Irawan, I. (2017). Pengembangan LKS Berbasis RME Dengan Pendekatan *Problem Solving* Untuk Memfasilitasi Kemampuan Pemecahan Masalah Matematis Siswa. *Jurnal Cendekia : Jurnal Pendidikan Matematika*, 1(2), 51–63. <https://doi.org/10.31004/cendekia.v1i2.20>
- [19.] Indriyani, F., Nurcahyono, N. A., & Agustiani, N. (2018). Analisis Kemampuan Pemecahan Masalah Siswa Berdasarkan Langkah Ideal Problem Solving. *PYTHAGORAS: Jurnal Program Studi Pendidikan Matematika*, 7(2), 56–67. <https://doi.org/10.33373/pythagoras.v7i2.1296>
- [20.] Karimah, R. K. N., Kusmayadi, T. A., & Pramudya, I. (2018). Analysis of difficulties in mathematics learning on students with guardian personality type in problem-solving HOTS geometry test. *Journal of Physics: Conference Series*. <https://doi.org/10.1088/1742-6596/1008/1/012076>
- [21.] Klymchuk, S. (2015). Solving Application Problems Using Mathematical Modelling Diagrams. *Mathematics Teaching-Research Journal*, 7(3).
- [22.] Kusuma, M. D., Rosidin, U., Abdurrahman, A., & Suyatna, A. (2017). The Development of Higher Order Thinking Skill (Hots) Instrument Assessment In Physics Study. *IOSR Journal of Research & Method in Education (IOSRJRME)*, 7(1), 26–32. <https://doi.org/10.9790/7388-0701052632>

- [23.] Lange, J. de. (2006). Mathematical Literacy For Living From OECD-PISA Perspective. *Tsukuba Journal of Educational Study in Mathematics*, 25(September), 13–35.
- [24.] Nuha, M. A., Waluya, S. B., & Junaedi, I. (2018). Mathematical creative process wallas model in students problem posing with lesson study approach. *International Journal of Instruction*, 11(2), 527–538. <https://doi.org/10.12973/iji.2018.11236a>
- [25.] Nurkaeti, N. (2018). Polya’S Strategy: an Analysis of Mathematical Problem Solving Difficulty in 5Th Grade Elementary School. *EduHumaniora | Jurnal Pendidikan Dasar Kampus Cibiru*, 10(2), 140. <https://doi.org/10.17509/eh.v10i2.10868>
- [26.] Ojose, B. (2011). Mathematics literacy : are we able to put the mathematics we learn into everyday use? *Journal of Mathematics Education*, 4(1), 89–100.
- [27.] Oktaviyanthi, R., & Agus, R. N. (2019). Eksplorasi Kemampuan Pemecahan Masalah Berdasarkan Kategori Proses Literasi Matematis. *Jurnal Pendidikan Matematika*, 13(2), 163–184. <https://doi.org/10.22342/jpm.13.2.7066.163-184>
- [28.] Phonapichat, P., Wongwanich, S., & Sujiva, S. (2014). An Analysis of Elementary School Students’ Difficulties in Mathematical Problem Solving. *Procedia - Social and Behavioral Sciences*, 116(2012), 3169–3174. <https://doi.org/10.1016/j.sbspro.2014.01.728>
- [29.] Piñeiro, J. L., Castro-Rodríguez, E., & Castro, E. (2021). Mathematical problem-solving in two teachers’ knowledge models: A critical analysis. *Mathematics Teaching-Research Journal*, 13(1), 71–93.
- [30.] Polya, G. (1978). How to solve it: a new aspect of mathematical method second edition. In *The Mathematical Gazette* (Vol. 30, p. 181). <http://www.jstor.org/stable/3609122?origin=crossref>
- [31.] Purnomo, E.A. & Mawarsari, V. D. (2014). Kemampuan Pemecahan Masalah Melalui Model Pembelajaran Ideal Problem Solving Berbasis Project Based Learning. *Jkpm*, 1(1), 24–31.
- [32.] Purnomo, E. A., Dalyono, B., & Lestariningsih, E. D. (2021). Developing e-learning media on education statistics subject. *Journal of Physics: Conference Series*, 1918(4). <https://doi.org/10.1088/1742-6596/1918/4/042116>
- [33.] Purnomo, E. A., Winaryati, E., Hidayah, F. F., Utami, T. W., Ifadah, M., & Prasetyo, M. T. (2020). The implementation of Maple software to enhance the ability of students’ spaces in multivariable calculus courses. *Journal of Physics: Conference Series*, 1446(1), 0–6. <https://doi.org/10.1088/1742-6596/1446/1/012053>
- [34.] Purnomo, E. A., Fathurohman, A., & Budiharto. (2014). Keefektifan Model Pembelajaran IDEAL Problem Solving Berbasis Maple Matakuliah Kalkulus II. *Jkpm*, 1(2), 7–11.
- [35.] Rott, B., Specht, B., & Knipping, C. (2021). A descriptive phase model of problem-solving processes. In *ZDM - Mathematics Education*. <https://doi.org/10.1007/s11858-021-01244-3>
- [36.] Schindler, M., & Lilienthal, A. J. (2020). Students’ Creative Process in Mathematics: Insights from Eye-Tracking-Stimulated Recall Interview on Students’ Work on Multiple Solution Tasks. *International Journal of Science and Mathematics Education*, 18(8), 1565–1586. <https://doi.org/10.1007/s10763-019->

10033-0

- [37.] Schoenfeld. (2007). Problem solving in the United States, 1970–2008: research and theory, practice and politics. *Zdm*, 39(5–6), 537–551. <https://doi.org/10.1007/s11858-007-0038-z>
- [38.] Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sensemaking in mathematics. In D. A. Grouws (Ed.). In *Handbook for research on mathematics teaching and learning*. MacMillan.
- [39.] Stender, P., & Kaiser, G. (2015). Scaffolding in complex modelling situations. *ZDM - Mathematics Education*, 47(7), 1255–1267. <https://doi.org/10.1007/s11858-015-0741-0>
- [40.] Sukestiyarno, Y. L. (2020). *Metode Penelitian Pendidikan* (2nd ed.). Unnes Press.
- [41.] Sulistyaningsih, D., Purnomo, E. A., & Purnomo. (2021). Polya's problem solving strategy in trigonometry: An analysis of students' difficulties in problem solving. *Mathematics and Statistics*, 9(2), 127–134. <https://doi.org/10.13189/ms.2021.090206>
- [42.] Supandi, S., Suyitno, H., Sukestiyarno, Y. L., & Dwijanto, D. (2021). Learning barriers and student creativity in solving math problems. *Journal of Physics: Conference Series*, 1918(4), 042088. <https://doi.org/10.1088/1742-6596/1918/4/042088>
- [43.] Susanto, E., & Retnawati, H. (2016). Perangkat pembelajaran matematika bercirikan PBL untuk mengembangkan HOTS siswa SMA. *Jurnal Riset Pendidikan Matematika*. <https://doi.org/10.21831/jrpm.v3i2.10631>
- [44.] Szendrei, J. (2007). When the going gets tough, the tough gets going problem solving in Hungary, 1970-2007: Research and theory, practice and politics. *ZDM - International Journal on Mathematics Education*, 39(5–6), 443–458. <https://doi.org/10.1007/s11858-007-0037-0>
- [45.] Temur, Ö. D. (2012). Analysis of prospective classroom teachers' teaching of mathematical modeling and problem solving. *Eurasia Journal of Mathematics, Science and Technology Education*, 8(2), 83–93. <https://doi.org/10.12973/eurasia.2012.822a>
- [46.] Wardono, Waluya, S. B., Mariani, S., & Candra, S. D. (2016). Mathematics Literacy on Problem Based Learning with Indonesian Realistic Mathematics Education Approach Assisted E-Learning Edmodo. *Journal of Physics: Conference Series*, 693(1). <https://doi.org/10.1088/1742-6596/693/1/012014>
- [47.] Yeni, E. M., Wahyudin, & Herman, T. (2020). Difficulty analysis of elementary school students in mathematical problem solving in solutions. *International Journal of Scientific and Technology Research*, 9(3), 44–47.