Assessment of Teaching Methods in Mathematical Simplicity and Complexity in Rwandan Schools via Pedagogical Content Knowledge

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Abstract: The teaching practices of mathematics are still little-known in Rwandan schools though the Competence-Based Curriculum (CBC) and many pedagogical documents recommend how to assess, what to assess, and when to assess the effective teaching of mathematics. This study aimed to assess Rwandan mathematics teachers' practices through pedagogical content knowledge (PCK). We sampled 14 mathematics teachers having similar educational backgrounds. Seven of them were sampled from Teacher Training Colleges (TTC), and the other seven were non-TTC teachers (selected from general secondary schools). We adopted a pedagogic approach to analyze the assessment practices and the tasks proposed by teachers to students. We also analyzed 35 items related to pedagogical content knowledge for teaching (PCK) among these two groups of teachers. Although the results about differences between two groups of seven teachers were not considered robust differences, it was still revealed that in all item categories related to PCK, TTC teachers have less performance than non-TTC teachers. The lack of mastery of content and specialized knowledge at the university level was found to cause this. We also found challenges related to teachers' assessment skills, especially mathematical complexity, as indicated by the interview results. We, therefore, recommend that all teachers, especially TTC teachers, be offered training in content knowledge so that they strengthen their teaching practices.

INTRODUCTION

Recently, in Rwanda, the poor assessment of students' learning has become an issue that concerns the various actors in education and the educational institution [1], [2]. This specific interest has been expressed in several learning areas and different grade levels of education. Most importantly, the policy of implementation of the Competence-Based Curriculum (CBC) in Rwandan schools introduced prescriptions on the assessment practices and emphasized the learning goals by insisting on how to assess the knowledge, skills, and progress of students' understanding in their learning areas [3].

The report produced in 2020 by the Project for Supporting Institutionalizing and Improving the Quality of School-Based In-service Teacher Training (SBI) activity (SIIQS) also revealed that the main learning activity in Rwanda schools is based on group work activities in almost all lessons observed [4]. Regardless of subjects and grade levels, teachers provide assessment questions that require recalling definitions when assessing. Teachers also assess students by using closed questions to confirm whether the answers are correct or wrong, which is against the prescriptions identified in education policy based on CBC for teaching. However, this type of assessment cannot reveal whether students master the content or not because it is superficial. Therefore, there is a need to develop open questions to dig into students thinking [5]. This kind of higher level of questions to assess students' understanding is critical and crucial in mathematics education in Rwanda and worldwide [6]–[8].

The findings based on observation by Moh'd et al. (2021) also revealed that pedagogical content knowledge (PCK) in classroom practices is low. Johar et al. (2021) argued that teachers' lack of content knowledge influences teaching strategies and leads to students' misconceptions instead of developing students' conceptual understanding (Putrawangsa & Hasanah, 2021). The assessment practices in mathematics teaching are essential, especially when an assessment is done by considering the aspect of mathematics teaching. The assessment practice is essential and helpful because it identifies teaching and learning situations for the different grade levels [10]. Otherwise, the lack of knowledge in assessment practices may lead teachers to use the same practice to assess the same content in different grades, while the curriculum frameworks indicate the additional knowledge to be focused on the flow of each grade level.

However, based on these issues of pedagogical content knowledge for teaching revealed in literature, the current study envisions analyzing teachers' PCK and mathematical tasks in the assessment of students learning. We analyzed them using a specific tool that integrates different pedagogical work in mathematics and characterizes the level of performance in mathematical simplicity and mathematical complexity. We tried to study and understand how teachers in Rwandan schools conceive their assessments and creations of knowledge for their students but did not report all the results in this article since this is a portion of a Ph.D. project. We have therefore chosen to assess teaching methods in mathematical simplicity and complexity in Rwandan schools via pedagogical content knowledge.

Context of the study and description of the analysis tool

Analysis of assessment tasks with a pedagogic approach implies considering the relationships between teaching, learning, and content, between assessment and the construction of subject content [10], [11]. These relations give us the fundamental impression to study the assessment practices of Rwandan teachers in mathematics from a pedagogic point of view. For the present study, we have chosen to analyze and present the mathematical tasks proposed by teachers in schools and compare teachers' results of pedagogical content knowledge. We present below some theoretical elements relating to the analysis of mathematical tasks proposed in the assessment. We then describe how the tool developed and utilized in previous studies is applied to produce results for the current study in Rwandan schools. Hill et al. (2004) utilized a tool to analyze the proposed assessment tasks according to the pedagogic approach. Specifically, this tool focused on strategies, methods, and various teaching techniques associated with instructions. The tool has been designed to analyze teachers' pedagogical content knowledge for teaching and assessment practices in mathematics in the U.S. However, we believe it remains relevant to use it in Rwanda because it is standard for mathematics education. We would therefore like to assess its use in this context. It integrates these two dimensions (teachers' pedagogical content knowledge for teaching and assessment practices in mathematics) to analyze mathematical tasks. These tasks include knowledge of mathematical simplicity and mathematical complexity.

Mathematical simplicity: understanding of the task

The language level of the statement, nature and amount of information processed by the student, and an example are considered to determine the level of mathematical simplicity of the task. The example below was taken from a proposed assessment practice by teachers for senior four (S4) students. Instructions for this example were not clear, and as a result, it led to a less explicit understanding of the task for the student. An example was "*find the solution of* $2x^2 = 6$". Even if this algebraic equation problem was not clear in instruction for being answered, the task of finding the solutions to this example is explicit. Without any further indication, the student must understand that any quadratic equation produces none, one, or two real solutions. To this end, in mathematical simplicity, students provide shorter proofs or more straightforward calculations. Thus, some students may find one solution instead of two or obtain the solutions without showing work. When the teacher uses this kind of mathematical simplicity in mathematics teaching, Hill et al. (2004) acknowledged that a teacher teaches common content knowledge.

Mathematical complexity: understanding of the task

In this paper, mathematical complexity refers to the various works conducted in mathematics pedagogics in the different domains concerned by the assessment practices to determine the mathematical complexity tasks of students' mathematical learning [13]. For example, the task of *comparing fraction and decimal numbers* can be more or less complex by playing on different pedagogic variables (size, presentation of numbers, and presence of zeros). Here is an illustration of the different levels of complexity of the task taken from different assessments collected during our study. We referred to the mathematical content in the book of mathematics in senior one (S1) (see REB, 2020, p. 54 and Ndyabasa et al., 2016).

Level 1: With $\frac{1}{10}$ and 0.1, the fraction number making up this decimal number, is the same. The student simply applies the comparison rule studied in class.

Level 2: With $\frac{1}{100}$ and 0.01, the zeros presented in both fraction and decimal numbers increase the complexity of the task, even though a student has certainly already performed this type of comparison.

Level 3: With $\frac{1}{1000}$ and $0.01 - 0.009$; these fraction and decimal numbers are not presented similarly. The student must re-compose the second decimal numbers before the comparison can be performed. The comparison task is therefore made to be more complex here.

The mathematical complexity is not intended to discuss the relevance of students' knowledge or the difficulty of carrying it out but to determine complexity levels that allow this knowledge to be considered in the task analysis. Our definition of complexity is inspired by Kontorovich et al. (2012), and we consider the cognitive of mathematical complexity defined by some other authors [16]. This cognitive complexity level determines a demonstration and the availability of the knowledge that students mobilize when they carry out a mathematical task.

We illustrate another example through the mathematical tasks of dashing an area that corresponds to the fraction to indicate this complexity. The example is "*Shade the area that corresponds to the fraction of* $\frac{1}{4}$ *in Figure A and Figure B, and then do the same to shade the area corresponds to a fraction of* $\frac{5}{10}$ *in Figure C*" (see Figure 1).

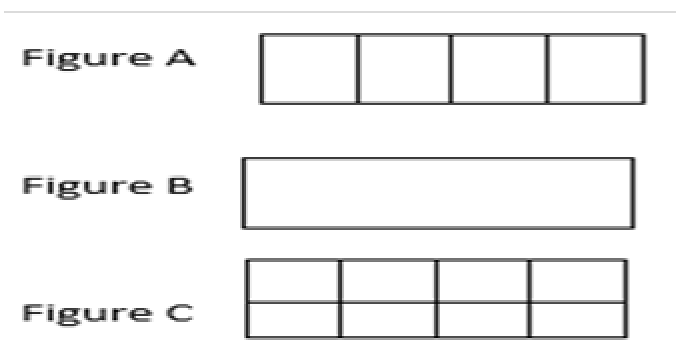


Figure 1: An assigned task for the S1 student

This task in figure A is a low level of mathematical complexity since the students simply apply their knowledge to perform a highly usual task. Figure B allows this task to be done easily because the student has already performed this type of task before in figure A. This time, the student must divide the square into four equal parts. Regarding figure C, students are faced with a highly complex task since they must first realize that the fraction of $\frac{5}{10}$ is equal to the fraction of $\frac{1}{2}$ that he was able to shade in the four rectangles corresponding to half of the square. The students are not guided to perform this task and have never likely performed the task before.

These examples are illustrative of the different levels of complexity as we define it. At the same time, they illustrate the different levels of complexity in different learning areas. Mathematical simplicity and complexity differ in assessment practices context. A task designed for mathematical simplicity requires solving the algebraic equation, and it does not include the context. The task does not include any situation or assessment context except to represent the answers, which are commonly found in the Rwandan curriculum or textbooks, especially in lower-level grades. The task in mathematical simplicity is arguable because the absence of the context in assessment practice limits the opportunity to understand the concept in depth. Contrariwise, the mathematical complexity-designed task incorporates a written teaching scenario. The scenario guides teachers in considering whether students can solve mathematical problems. Clement (1982) advised that teachers must already be aware of misconceptions and confusions students may hold. Based on the above context of teaching and assessment practices, either mathematical simplicity or complexity, the current study is aimed to answer two questions.

Research questions

i. *How do mathematics teachers in Rwandan teacher training colleges and non-TTC teachers who teach in secondary schools perform the items related to pedagogical content knowledge for teaching?*

ii. *To what extent do these teachers understand the assessment practices towards students' understanding of mathematical simplicity and complexity in Rwandan schools?*

This study is the first to assess Rwandan teachers in the African context. It informs researchers on how teachers are skilled in a range of PCK-related fields such as common content knowledge (CCK), specialized content knowledge (SCK), knowledge of content and students (KCS), knowledge of content and teaching (KCT), and content knowledge (KC). It gives room to policymakers on which area needs special input and planning.

METHODOLOGY

A mixed-method research design was used for this study [17]. Quantitative data were collected via a Google form, while Qualitative data were collected face to face in the field. Before collecting data, we applied and got ethical clearance from the research and innovation unit at the University of Rwanda College of Education (URCE). This clearance helped us to seek permission to do research in schools.

Participants

We, at the end of April 2020, invited many teachers through phone calls, and we asked them to participate in the study. We set the schedule together with those who agreed. We selected 14 mathematics teachers in total. The seven mathematics teachers were sampled from Teachers Training Colleges (TTC). The other seven teachers were sampled from the regular or general secondary schools (SS)—here named non-TTC—in Rwanda. All these teachers are experienced in teaching mathematics at the secondary school level. Since Rwanda teachers are using the new curriculum, CBC, they also received many different pieces of training in pedagogical content knowledge for teaching.

Data collection and Procedures

In order to analyze the PCK-related items and the assessment practices of mathematics, teachers who agreed to participate were sent a Google form questionnaire called Measures of Teachers' Mathematical Knowledge for Teaching, MTKT, to answer the survey questions. This process lasted three months (from May to July 2020). This questionnaire comprises 35 items related to PCK (see Hill et al., 2004), and some of them were modified—such as Rwandan names to contextualize the situation—and used in some tasks designed to explore more teachers' knowledge of PCK and the personal dimension of the assessment practices.

The authors developed MTKT to shed light on various teacher's knowledge assessments that were debated around the end of the 20th century in the U.S. Although MTKT was designed for elementary school teachers in the United States, our portion target sample also is characterized by teachers who train primary school teachers in Rwanda. Thus, comparing this part with another part of teachers who teach in general secondary schools can depict how delicate it is to train primary school teachers. The tasks covered mathematical topic areas such as geometry (GEO), rational numbers (RAT), number concept and operations (NCOP), pattern function and algebra (PFA), and proportional reasoning (PR). Box 1 explains the item categories, for example, what kind of knowledge can be categorized as specialized content knowledge, etc.

Box 1: Example of the item category

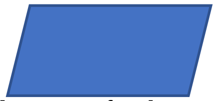
Question-1 depicted from NCOP is an example of **CCK** is about "0 is even", "0 not a number", and "8 is 008." It says: Ms. Diane was working with a new textbook and she noticed that it gave more attention to the number 0 than her old book. She came across a page that asked students to determine if a few statements about 0 were true or false. Interested, she showed them to a colleague who is also a teacher and asked her what she thought. Which statement(s) should the teachers select as being true? (Mark YES, NO, or I'M NOT SURE for each item below.)

	Yes	No	I'm not sure
(a) 0 is an even number	1	2	3
(b) 0 is not really a number. It is a placeholder in writing big numbers	1	2	3
(c) The number 8 can be written as 008	1	2	3

SCK-related question (in FPA) was about explaining reversing inequalities as **question-34** asks: Ms. Alicia was teaching a lesson on solving problems with inequality in them. She assigned the following problem. [$-x < 9$] Marcie solved this problem by reversing the inequality sign when dividing by -1, so that $x > -9$. Another student asked why one reverses the inequality when dividing by a negative number; Ms. Alicia asked the other students to explain. Which student gave the best explanation of why this method works? (Mark ONE answer)

a) Because the opposite of x is less than 9.
b) Because to solve this, you add a positive x to both sides of the inequality.
c) Because $-x < 9$ cannot be graphed on a number line, we divide by the negative sign and reverse the inequality.
d) Because this method is a shortcut for moving both the x and 9 across the inequality. This gives the same answer as Marcie's, but in a different form: $-9 < x$.

Question-18 is related to **GEO** (Geometry), and is an example of **KCS**. It asks: At the close of a lesson on reflection symmetry in polygons, Ms. Mukesha gave her students several problems to do. She collected their answers and read through them after class. For the problem below, several of her students answered that the figure has two lines of symmetry, and several answered that it has four. How many lines of symmetry does this figure have?



Which of the following is the most likely reason for these incorrect answers? (Circle ONE answer.)
a) Students were not taught the definition of reflection symmetry.
b) Students were not taught the definition of a parallelogram.
c) Students confused lines of symmetry with edges of the polygon.
d) Students confused lines of symmetry with rotating half the figure onto the other half.

KCT-related question was depicted from **RAT (**rational numbers) and is presented in **question-28**. Mr. Shadad is using his textbook to plan a lesson on converting fractions to decimals by finding an equivalent fraction. The textbook provides the following two examples: Convert 2/5 to a decimal: 2/5 = 4/10 = 0.4 Convert 23/50 to a decimal 23/50 = 46/100 =0.46.
Mr. Shadad wants to have some other examples ready in case his students need additional practice in using this method. Which of the following lists of examples would be best to use for this purpose? (Circle ONE answer.)
a) 1/4 8/16 8/20 4/5 ½
b) 1/20 7/8 12/15 3/40 5/16
c) 3/4 2/3 7/20 2/7 11/30
d) All of the lists would work equally well.

Question-18 is related to **PR (**proportional reasoning)**,** and it is an example of **KC**. It asks: Mr. Mutabazi's students were comparing different rectangles and decided to find the ratio of height to width. They wondered, though, if it would matter whether they measured the rectangles using inches or measured the rectangles using centimeters. As the class discussed the issue, Mr. Mutabazi decided to give them other examples to consider. For each situation below, decide whether it is an example for which different ways of measuring produce the same ratio or a different ratio. (Circle PRODUCES SAME RATIO, PRODUCES DIFFERENT RATIO, or I'M NOT SURE for each.)

	Produces the same ratio	Produces different ratio	I'm not sure
a) The ratio of two people's heights, measured in (1) feet or (2) meters.	1	2	3
b) The noontime temperatures yesterday and today, measured in (1) Fahrenheit or (2) Centigrade.			
c) The speeds of two airplanes, measured in (1) feet per second or (2) miles per hour.			
d) The growths of two bank accounts, measured in (1) annual percentage increase or (2) end-of-year balance minus beginning-of-year balance.			

Apart from the MTKT, researchers provided a set of two tasks (one for solving a quadratic equation and another for solving a word problem) for qualitative data collection. We used both MTKT to generate quantitative data and follow-up them with two tasks to generate qualitative data. This helped us to triangulate our results. After submitting their responses, we selected four teachers (two from TTC and two from non-TTC) for an interview discussion in June 2021. They were selected from the most that had misconceptions while solving two tasks provided. These teachers were visited by researchers, and a semi-directive interview was conducted. By interview discussion or semi-directive interview, we mean that researchers sat together with each of the selected participants, presented them with tasks, asked him/her to perform, and discussed how the teacher performed the task.

Items Selection and Data analysis

The items for use were selected based on the related topic areas that are generally found crossing grade levels in the content of Rwandan textbooks and competence-based curricula. These topic areas include geometry (GEO), rational numbers (RAT), number concept and operations (NCOP), pattern function and algebra (PFA), and proportional reasoning (PR). Table 1 represents the item category.

Item category	GEO	RAT	NCOP	PFA	PR
Common content knowledge (CCK)			2		
Specialized content knowledge (SCK)		1	6	4	
Knowledge of content and students (KCS)	1		9		1
Knowledge of content and teaching (KCT)	4	1	4		
Content knowledge (KC)			1		1
Total	5	2	22	4	2

Table 1: Number of items category selected from MKT

Based on this procedure, we could identify questions that could be asked during the interview to help us understand the performance of the 35 items and tasks given to participating teachers. Each interview was transcribed, and the teachers' answers were recorded. We did this because the teacher's answers allowed us to identify elements of practice that we felt were essential. Thus, interpretive and descriptive statistics were used to deliver results.

To have an image of how the item category and assigned tasks are performed, the item difficulties were calculated to clear up whether we measured similar constructs of two groups of teachers. All teachers' responses were entered into SPSS, version 25. Thus, we estimated the point biserial correlation to rate the number of right and wrong answers that teachers gave on the items and the total scores that the teachers received when summing up the scores across the items. The answer of each teacher was recorded for each question. Firstly, the number of teachers in each group who performed well in each item category was computed. Secondly, the average scores for each teacher along each MKT test item were computed. Then, multivariate analysis of MANOVA was estimated to compare multivariate sample means teachers' performance in the item category between the non-TTC and TTC teachers. The separate ANOVA was also estimated to find out whether there is a significant difference between dependent variables (mean scores) on the item categories. Since it is not convincible to report statistical differences between such small groups of 7 each, we discussed the differences on a qualitative basis. We also analyzed the teacher's understanding of the assigned tasks related to the item categories through the interview. Therefore, the conclusion was drawn based on the results obtained from the analysis.

RESULTS

Regarding the first question of the study *[How do mathematics teachers in Rwandan teacher training colleges and non-TTC teachers who teach in secondary schools perform the items related to pedagogical content knowledge for teaching?],* we found that the overall performance of the item category was less performed by TTC teachers (M=2.69, Std=1.737) comparing to the non-TTC (SS) teachers (M=3.41, Std=2.091). With the descriptive statistics, the performance of the item category through the mean (M) scores and standard deviation (Std) is indicated in Table 2. Note that the average reflects on the number of teachers (in each group, there are seven teachers). The most performed skill by both groups of teachers was common content knowledge (CCK), although teachers teaching in general secondary schools outperformed (M=5.25 out of 7 teachers, Std=1.708) those teaching in primary teacher training colleges (M=3.50, Std=1.914). Similarly, the least performed skill was content knowledge (CK) by both groups, and non-TTC teachers outperformed (M=2.50, Std=1.773) TTC teachers (M=1.88, Std=1.727) too.

	TTC		**Non-TTC**	
Item Category	Mean of teachers	Std.	Mean	Std.
CCK	3.50	1.915	5.25	1.708
SCK	2.33	2.140	2.75	2.364
KCS	3.47	0.964	4.05	1.615
KCT	2.33	1.225	3.78	1.922
CK	1.88	1.727	2.50	1.773
Total	2.69	1.735	3.41	2.091

Table 2: Descriptive statistics of mean and Std. of teachers in two groups who performed for each item category

Since our sample was small, we also chose to analyze the individual performance in each item category. The results showed that almost all non-TTC teachers performed better in the CCK item category with the total mean (M = 86%) except for one teacher (non-TTC4) who performed 33% in this item category. On the other hand, except for one teacher (TTC7) who performed CCK 100%, TTC teachers have performed this item category at the average mean of (M = 57%). Generally, non-TTC teachers performed in all item categories at the average mean (M = 57%), while TTC teachers performed at the average mean of (M = 40%). Table 3 presents the individual performance of each teacher across each item category (skills).

CODES	CCK	SCK	KCS	KCT	CK	Mean
TTC1	33	33	53	56	38	42
TTC2	67	33	32	33	38	40
TTC3	67	25	37	0	25	31
TTC4	33	42	42	44	25	37
TTC5	33	38	74	44	38	45
TTC6	67	29	63	44	25	46
TTC7	100	25	47	11	0	37
Mean	57	32	50	33	27	40
Non-TTC1	100	38	16	44	25	45
Non-TTC2	100	50	74	44	38	61
Non-TTC3	100	38	79	67	38	64
Non-TTC4	33	29	63	44	25	39
Non-TTC5	67	50	68	67	50	60
Non-TTC6	100	33	47	56	38	55
Non-TTC7	100	33	58	56	38	57
Mean	86	39	58	54	36	54

Table 3: Descriptive statistics (%) of the individual performance of each teacher in each item category

The teacher who performed well in specialized content knowledge (SCK) was found among SS (non-TTC) teachers (SS-2 and SS-5) and got 50% mean scores. In the knowledge of content and students (KCS), all teachers are known as the average was 58% among SS teachers and 50% among TTC teachers. This was similar to knowledge of content and teaching (KCT); however, content knowledge displayed the lowest scores for all teachers in both groups. Only one teacher (SS-5) could get 50%, while others got below the average of the total score (50%).

We also estimated the MANOVA to determine whether there are significant differences between two groups of teachers (TTC and Non-TTC or SS) on the item categories. Table 4 presents the results through the Tests of Within-Subjects Effects.

Source	Dependent Var	Type III Sum of Sq	df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	TTC	23.805a	4	5.951	2.116	0.09	0.125
	SS	39.685b	4	9.921	2.483	0.053	0.144
Intercept	TTC	314.717	1	314.717	111.894	0.000	0.655
	ss	578.908	1	578.908	144.879	0.000	0.711
group	TTC	23.805	4	5.951	2.116	0.090	0.125
	SS	39.685	4	9.921	2.483	0.053	0.144
Error	TTC	165.945	59	2.813			
	SS	235.753	59	3.996			
Total	TTC	652.000	64				
	SS	1018.000	64				
Corrected Total	TTC	189.75	63				
	SS	275.437	63				

a R Squared = .125 (Adjusted R Squared = .066)
b R Squared = .144 (Adjusted R Squared = .086)

Table 4: Analysis of ANOVA of dependent variables on item categories

A separated ANOVA was also conducted for dependent variables, with each ANOVA evaluated at α level of $.05$. There was no significant difference between TTC and SS (non-TTC) on item categories $F(4,59) = 2.116, p = .090$, partial $\eta^2 = .125$, with an estimated marginal mean for TTC (M = 3.500, SD =.839) for CCK; (M = 2.333, SD =.342) for SCK; (M = 3.474, SD =.385) for KCS; (M = 2.333, SD =.559) scoring less than SS (M = 5.250, SD =.999) for CCK; (M = 2.750, SD =.408) for SCK; (M = 4.053, SD =.459) for KCS; (M = 3.778, SD =.666) for KCT; and (M = 2.500, SD =.707) for CK. Therefore, Figure 2 presents the marginal mean of MANOVA between two groups on each item category.

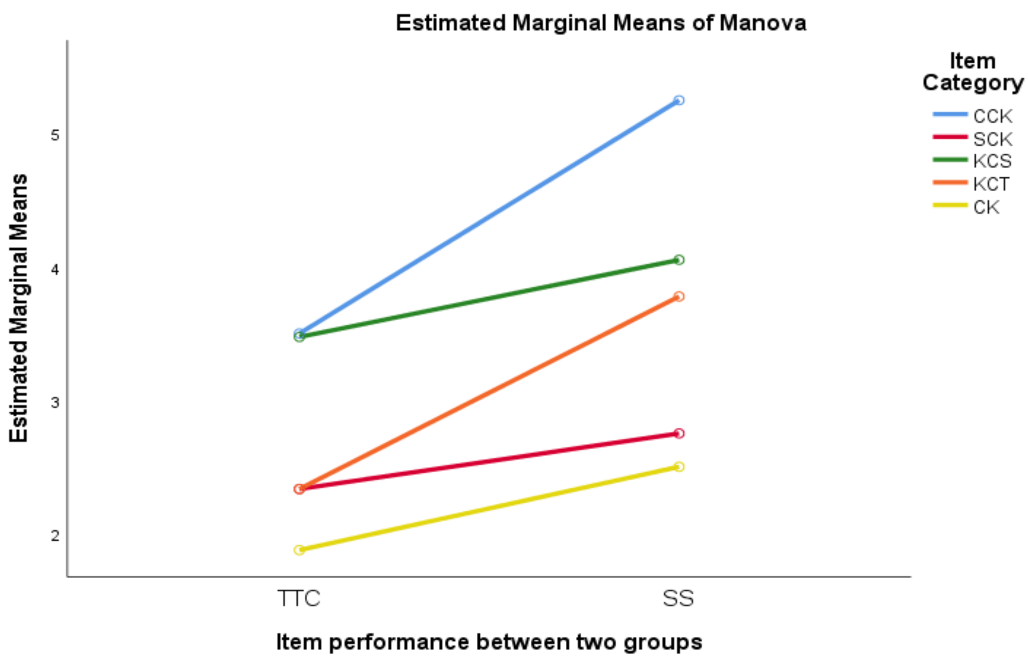


Figure 2: Performance between two groups on each item category

We had five groups in variables: the item categories and dependent variables, in two groups: SS (Non-TTC) and TTC. Within each item category, the SS teacher has performed highly better than the TTC teacher. To find out whether there was a significant difference between dependent variables on the item categories, a multivariate analysis of variance revealed that there was no significant difference between TTC and SS teachers when considered jointly on the variables in item categories, Wilk's $\Lambda = .798, \mathrm{F}(8{,}116) = 1.73, \mathrm{p} = .099,$ partial $\eta^2 = .107$

Regarding the second question *[To what extent do these teachers understand the assessment practices towards students' understanding of mathematical simplicity and complexity in Rwandan schools?]*, we noted that most of the items offered to participating teachers were at a high level of complexity. Though all these items were set to be answered in multiple-choice, teachers who participated in the study revealed weak knowledge of assessment in mathematics teaching, as indicated through the interviews. Though they tried to give explanations, they revealed mistakes in choosing right or wrong answers. This result is reassuring that teachers must understand or know what students have to do in presenting the tasks to assess students' knowledge. Mathematical complexity tasks are often tasks where teachers ask the students to do tasks that are not explicit, and it is up to students to understand and discover ways to perform the tasks. To understand how teachers perceived the designed task from mathematical simplicity and mathematical complexity, we present the following designed task as one of the examples used throughout the interview.

Task 1: Quadratic equation

To solve an equation of $2x^2 = 6$*,? which one of the following will yield the correct answer? (Circle ONE answer.)*

a) Divide both sides by 2*, which gives* $x^2 = 3$*, and divide both sides again by* x *to get* $x = 3$

b) Simplify x^2 *and* x*. Then divide both sides by* 2 *and get the answer of* $x = 3$

c) Take the square root of both sides after diving by 2*.*

d) Use the method of sum and product of roots to find solutions.

To do this task, teachers did not yield good results from it. We found that only seven out of 14 teachers could give the correct answer. Six out of 14 answered that the correct answer is to divide both sides by 2, which gives $\mathrm{x}^2 = 3$, and divide both sides again by x to get $\mathrm{x} = 3$. There was also one out 14 participants (see SS1 (Non-TTC1) in Figure 3) who said that to answer this question correctly, x^2 and x must be simplified first and later, divide both sides by 2 and get the answer of $\mathrm{x} = 3$. Therefore, based on the results of this task, it seems that teachers are not concerned about offering their students tasks that may lead them to think explicitly. This task was also designed based on mathematical complexity in relation to PCK knowledge. In an interview, teachers revealed less knowledge of assessing mathematical complexity.

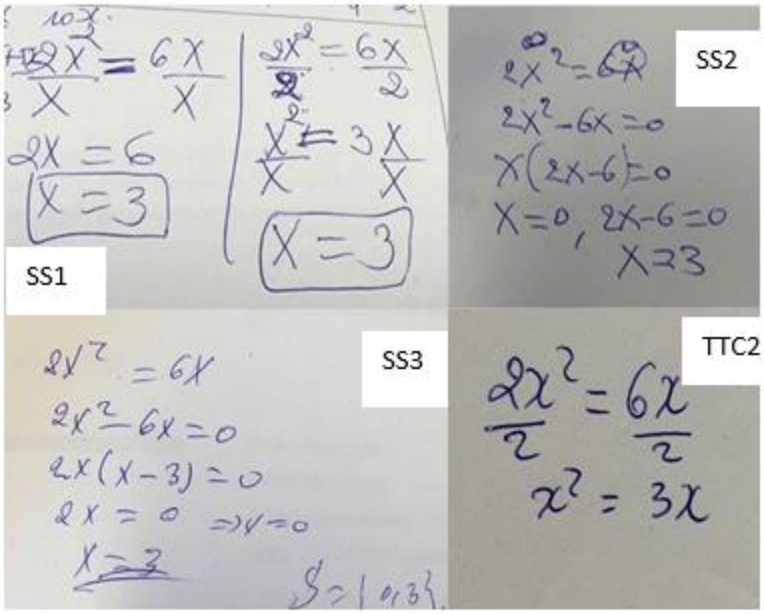


Figure 3: Sample of teachers' performance during quadratic equation solving. Note that SS refers to Non-TTC

Task 2: Word problem

To illustrate the differentiation of the PCK knowledge relative to the same tasks proposed, it seems relevant to mention the sort of tasks proposed in the interview. The following example was also used to analyze the task related to word problem tasks. Teachers were asked to write an equation from the following statement *"there are 4 times as many chalks as pencils"* using the alphabet C as the number of chalks and P as the number of Pencils. Figure 4 illustrates how the participating teachers wrote an equation from the statement.

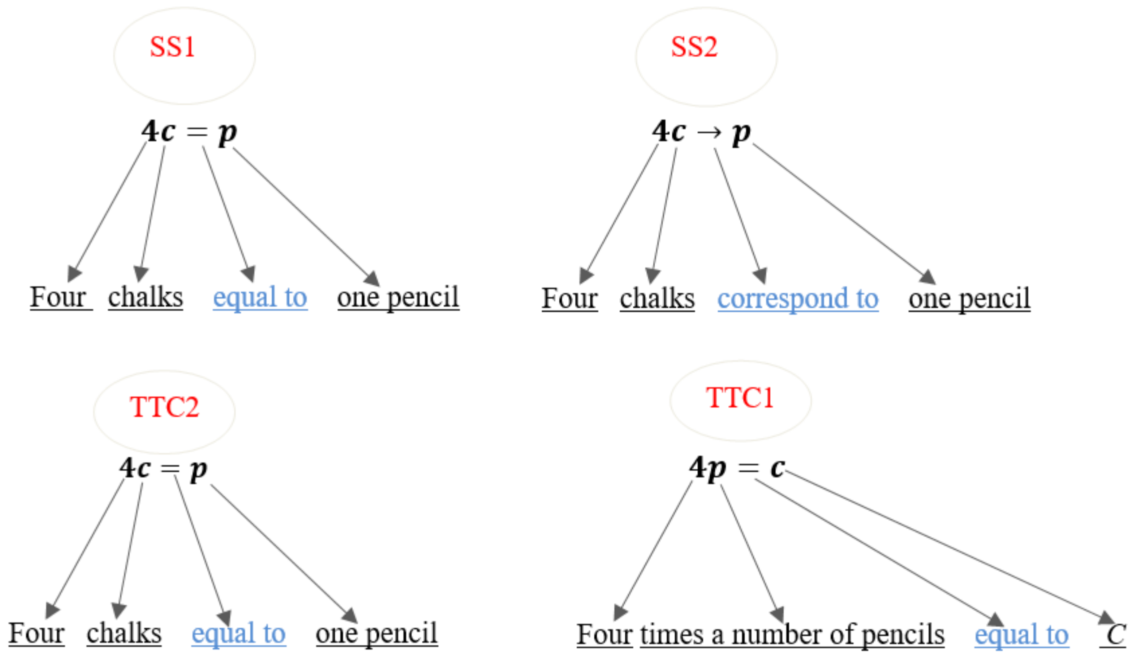


Figure 4: Demonstration of how the interviewed teachers wrote an equation from the statement

Results showed that most participating teachers did not perform this task as it was in a more complex situation that applies to the higher complexity level. We found that teachers were not aware that complexity is not only based on the mathematical knowledge they want to assess but also on methodological skills.

It is true for only equation number 4 (TTC1) that the equation fits the statement. Equation number 2 (SS2) was identified as a function (referring to mapping each element of the domain to exactly one element of the range). This characteristic of equation number 2 allows for one-to-one and many-to-one relationships. However, some participants through interviews have even included one-to-many relations, which are not functions. Through the interview, participating teachers provided examples to disprove the statement by justifying the written answers. They made misconceptions, specifically by using the following logic.

Wrong interpretation. We asked, if there are 100 chalks, how many pencils will be there? In their explanations, many of them said,

"c and p can represent something you can multiply. They wrote an equation as $100c = 25p$ *and said, if there are 100 chalks, there will be 25 pencils. Then to prove this, it is just a matter of simplifying the number here. They point a finger on 100 and 25. So if you divide both sides by 25, then you will get* $4c = p$ *which describes the statement to be correct."*

More than (60%) of participants committed this mistake $4c = p$, and a few of them (14%) made the same mistakes by not identifying an equation as the reversal equation but thinking that it is the function (direct variation).

<u>*Correct interpretation*</u>. On the other hand, the interviewees who gave correct answers also gave relevant explanations for their written work. For example, one teacher explained how he proved the statement as follows.

"The number of chalks is bigger than the number of pencils, right? Then, if there are 100 pencils, the number of chalks will be 100 times four. He wrote an equation, $100c = 400p$*. Therefore,* $C = 4P$*."* We then asked him how he came up to knowing that, and he said, "*to be equal, the number of chalks would be equal to the number of pencils times four.*"

About (84%) of all participating teachers did not provide any relevant explanations about the relationships between the two quantities (C and P). Through the analysis, we found that only 16% of participating teachers could cognitively identify this relationship and explain why the given answer is correct.

DISCUSSION

In general, teachers were found to be good at common content knowledge (CCK), knowledge of content and students (KCS), and knowledge of content and teaching (KCT) because of in-service teachers' training offered in 2016 during the implementation of the competence-based curriculum (CBC) by various partners such as Rwanda Basic Education Board (REB) with various partners such as VVOB, British Council, IEE, JICA, SOMA UMENYE, BLF [18] to mention few. Teachers are well trained in different active learning techniques to cater to learners and the classroom atmosphere. However, specialized content knowledge (SCK) and content knowledge (CK) were found problematic among Rwandan teachers. Both TTC and SS teachers did not get half of the total score (50%) on related items. It appears that math education, in general, does not equip students with the "necessary and enough knowledge" in math. It may also be explained by the fact that students are given many subjects to cover; for instance, some students are currently given to complete three subjects (such as Mathematics-Physics-Education) in only three years at URCE.

We found a significant disparity among participating teachers and the complexity of the proposed tasks. However, this led us to question whether teachers are always aware of the complexity of the tasks they give to students in assessment. Even though on the whole, few complex tasks were proposed, we realized, after interviewing teachers, that those who had proposed them were not necessarily aware of their actual level of complexity and that this could impact the results of the weakest students. To illustrate our point, we discuss the points based on the presented results obtained from the analysis. The task concepts used and presented in the section require prerequisite knowledge of algebraic equations. Though it is still a mathematically questionable statement, from the fundamental theorem of a quadratic equation of the form $ax^2 + bx + c = 0$, as a second-order polynomial, it can be guaranteed that it has two solutions in a single variable. Many common mistakes and misconceptions in this task were to think first about simplifications. The respondents did not realize that if we simplify—using techniques such as factorizing—quadratic equations, we get a linear equation that is guaranteed to have one solution, no solution, or infinitely many solutions but not two solutions. Another thing that is realized from some participants is that, for example, one teacher, through the interview, identified all steps by finding solutions to the quadratic equation, but he selected the wrong answer.

To estimate this level of complexity, we also discussed the concepts of fraction and decimal numbers prepared by teachers. The given fraction and decimal numbers, as the example, were not pre-sentenced in the same way as the others. However, Level 3 makes the comparison task more complex. Definitely, that unusual comparison of the fraction and decimal numbers requires the student to put the proposed decimal numbers in the same "format" before applying any comparison rule. However, without being indicated to the student and being alerted to this fact of higher levels of complexity, the concept of the fraction and decimal presented was in mathematical simplicity.

Therefore, through the interview, we sought to determine whether the interviewed teacher had deliberately chosen to make these tasks more complex by questioning him about level 3. His response was: "*I did not realize this problem. It is just a typo*." This answer is doubly problematic. This teacher was not aware of the complexity generated by the unusual presentation of these decimal numbers of level 3. However, as [12] advised, the choices teachers make in the presentation of the example or exercise generated are decisive for the success or failure of students. Teachers' PCK knowledge is essential to ensure the validity of students' success or failure in his/her learning. If teachers cannot determine the complexity of their students' evaluative tasks, they cannot even design valid assessments. In fact, there should be a link between PCK and knowledge of students and mathematics because a teacher who has a strong knowledge of students' learning should also have a basic knowledge of the mathematics they study (Hill et al., 2004). Thus, PCK is basically not enough for math teachers. They need content knowledge too.

CONCLUSION

Our pedagogical approach allowed us to take a detailed look at the mathematics assessment practices of a sample of 14 mathematics teachers, considering more precisely the nature and complexity of the tasks proposed in assessment practices. For triangulation purposes, we used quantitative and qualitative data collection. Thirty-five tasks (as a sample of these tasks was presented in box 1) adapted from Hill et al. (2004) generated quantitative data. Based on these results, we formulated two tasks (one for solving a quadratic equation and another for solving a word problem) and generated qualitative data. We realized that the responses to assessed tasks proposed by these teachers were of a low level of complexity. Considering the PCK knowledge and the level of assessment practices identified by the participating teachers, we question the overall validity of the assessment practices, even if an individual analysis would be worthwhile to affirm this with more certainty. By this, we mean that the assessments proposed in mathematics by the teachers in our sample do not allow us to determine what their students know. Moreover, passing low complexity tasks does not provide information on the level of resistance or confrontation of students' knowledge and skills to more complex tasks. We believe assessing teachers reflects the students' learning and acquiring knowledge and skills. The results obtained from the study are unique but not conclusive, as our approach was also unique. Based on the results, it seems that from the pedagogical freedom granted to Rwandan teachers, they lack specialized content and content knowledge training. In fact, majoring in a specific and specializing in one subject is needed among higher teacher training institutions.

Therefore, we recommend that URCE and other teacher training colleges cater to the researchers in this field should use a combination of theories and make interventions to investigate why TTC teachers perform less than non-TTC teachers while all teachers have similar attributes. An increase in teachers' samples is needed for researchers to compute inferential statistics and figure out the real difference between these teachers.

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