Epistemological Obstacles on Limit and Functions Concepts: A Phenomenological Study in Online Learning

Rini Sulastri[1,2], Didi Suryadi[1], Sufyani Prabawanto[1], Endang Cahya[1]

[1]Department of Mathematics Education, Universitas Pendidikan Indonesia, Indonesia,

[2]Department of Mathematics Education, Universitas Serambi Mekkah, Indonesia

rini.sulastri@upi.edu, ddsuryadi1@gmail.com, sufyani@upi.edu, endangcahya@gmail.com

Abstract: Barriers to learning can affect educators and students in achieving learning objectives. One of them is the epistemological obstacle caused by the limited context used when a concept is first learned. The purpose of this study was to explore the epistemological obstacles of mathematics students in the concept of limits and functions during online learning. Data were obtained from 16 first-year students who took a written test on the concept of limit function. Based on the various answers and the presence of errors, nine subjects were selected to be followed by semi-structured interviews. The qualitative research data with this phenomenology approach were analyzed descriptively. The results of the study indicate that there are several concepts of function limits that are misunderstood by subjects who come from learning experiences in high school and also online learning. Epistemological obstacles that occur include an incorrect understanding of the concepts of real and infinite numbers, the value of a function will always be the same as the limit value for the same function, and the use of the substitution method which is generalized to the limit of the function. To overcome these obstacles, the presentation of the material, especially the basic concepts, needs to be done in-depth so that is easy to understand and to minimize the occurrence of misconceptions.

INTRODUCTION

The global pandemic status since the beginning of March 2020 has affected many countries so various strict policies such as social distancing has been implemented. This has resulted in school closures and even a number of schools being completely closed, Indonesia is no exception (Huang, Liu, Tlili, Yang, & Wang, 2020). The government has limited community mobilization by continuing to call for an agenda to work, study, and worship from home. To deal with these conditions, the government has implemented remote learning and teaching programs such as electronic learning (Mailizar, Almanthari, Maulina, & Bruce, 2020). However, not all schools are able to implement e-learning effectively. Lack of experience with e-learning causes difficulties in using online applications (Zaharah, & Kirilova, 2020).

Research conducted (Mailizar, et al., 2020) on the barriers to implementing e-learning during the

COVID-19 pandemic showed that the highest barriers occurred at the student level related to skills and knowledge, motivation, and e-learning infrastructure. The teacher's background has no impact on the level of the barrier. In contrast, a survey conducted (Mushtaha, et al., 2022) shows that the implementation of online learning has a positive impact on aspects of flexibility related to place and time, and aspects of accessibility and effectiveness in the assessment and communication methods used. This shows that the implementation of online learning has a positive and negative impact on its users. In addition, the impact that appears is also related to the learning process carried out.

Several sources of problems in the learning process include the orientation gap between current learning and philosophical orientation; learning flow problems related to didactical design; the problem of organizing didactic situations that are not in accordance with the nature of mathematics and its learning; the problem of conceptual gaps between educators, students, and scientific conceptions; formation of transpositional knowledge (didactic and pedagogical), and various problems related to didactic design (Suryadi, 2019a). Problems related to didactic design are ontological, didactical, and epistemological (Brousseau, 2002; Suryadi, 2019a). The ontological obstacle is related to the gap between the design demands and the child's capacity. Didactical obstacles can arise due to the sequence and stages of the curriculum and its presentation in class. The limited context used in the didactic design can create an epistemological obstacle. In other words, the epistemological obstacle is the gap between the context of the learning experience that has been passed and the demands of linking learning outcomes with various contexts outside that have been experienced (Suryadi, 2019a; 2019b).

Epistemological obstacle can be found in students' errors in answering or responding to questions given (Brousseau, 2002; Cornu, 1991). These errors can be influenced by prior knowledge (Brousseau, 2002). In a study conducted (Moru, 2009), indicators of epistemological barriers were based on students' errors and difficulties in understanding the concept of limits. In this study, epistemological barriers were also traced from students' mistakes in answering several questions about the concept of limits and functions.

One of the gateways to studying advanced science and mathematics is Calculus (Sebsibe & Feza, 2020; Roble, 2017; Sadler & Sonnert, 2016). The concept of limit acts as a central concept in calculus so it must be understood correctly by every student (Zollman, 2014; Artigue 2000; Bezuidenhout, 2001; Cornu, 1991; Oehrtman, 2002; Szydlik, 2000; Williams, 1991; Ervynck, 1981). In addition, limits will also be used in advanced learning (Beynon & Zollman, 2015). Although considered as the basis for understanding calculus, a complete understanding of the concept of limit is still very limited among students (Davis & Vinner, 1986; Tall & Vinner, 1981; Sierpinska, 1987; Cornu, 1981).

Many kinds of research on the limit of functions have been carried out on students, college students, or teachers. The results of research by Arnal-Palacián & Claros-Mellado (2022) show that pre-service teachers use algorithmic procedures in solving limits, and use an intuitive approach to explaining them to students. The results of the research by Fernández-Plaza &

Simpson (2016) show that most students solve limit problems separately, only a few can relate the basic concepts of limits between the limit of a sequence, the limit of a function at a point, and the limit of a function at infinity. However, they often do it in a way that runs counter to formal mathematics. Reed (2018) investigates students' understanding of personal concepts about limits and their relation to formal definitions of limits after students complete introductory calculus lessons. The result shows that students' ability in defining personal concepts related to limits is in a low category. Thus, a conceptual understanding needs to be developed to be able to master the concept of limits.

According to Vinner (1991), a definition has a special role in the technical use of a construct in contrast to the intuitive use and everyday language. In understanding an idea such as the limit of a function, it is necessary to identify it in different representations such as graphic, numeric, algebraic, and verbal. In addition, the coordination of each of the different representation systems is necessary to gain a comprehensive understanding of the idea (Arnal-Palacián & Claros-Mellado, 2022; Duval, 1998). Kidron (2011) conducted a study on the definition of limit which was constructed using a horizontal asymptote definition approach. In this case, a conflict arises between the concept image and the concept definition of the horizontal asymptote. Students consider an asymptote to be a straight line that a curve approaches so that it becomes closer along the line. In addition, the closer it is understood without anything to do with limits.

Job & Schneider (2014) in their research on the epistemological obstacle in Calculus, used the anthropological approach of Chevallard. The aim is to analyze the history of calculus in building an epistemological model based on pragmatic and define deductive praxeology. More specifically, research on epistemological obstacles to the concept of limit functions was carried out by Moru (2009) with the subjects being first-year mathematics students at the Faculty of Science and Technology. In the results of his research, it was stated that the subject had difficulty in determining or distinguishing between limit values and function values.

Problems that often arise from the concepts of limits and functions relate to whether a function can reach its limits, whether a limit is actually bound, whether a limit is a dynamic process or a static object, and whether a limit is inherently bound to the concept of motion (Williams, 1991). The question "Is a limit reached or not?", according to Moru (2009) as an epistemological obstacle in understanding the concept of limit. According to Tall (1993) some difficulties in understanding the concept of limit include: the use of language and terms such as limits, tends to, approaches, and as small as we please, which has a colloquial meaning contrary to formal concepts; limit processes are not carried out with simple arithmetic or algebra; the process of a variable becoming smaller is often interpreted as a smaller variable quantity; the idea of N getting bigger, implicitly indicates the conception of infinity; confusion about whether a limit is really achievable; and confusion about the part from finite to infinity raises questions about what happens to infinity.

The difficulty in understanding the concept of limit begins with the many informal ideas of the word limit that students already have (Barnes, 1991). Some students understand some aspects of formal mathematical concepts but this can distort their definitions thereby creating a conceptual barrier that makes learning about limits more difficult (Cavanagh, 1996). Students' difficulties regarding the concept of limits are also due to the prerequisite materials that must be mastered such as real numbers, functions, and very small and large numbers (Parameswaran, 2007; Sierpinska, 1987). In addition, the factors that may contribute to students' difficulties regarding limits relate to the predominance of the dynamic and procedural aspects of limits in textbooks and calculus teaching, and students' attitudes towards mathematics (Bezuidenhout, 2001; Parameswaran, 2007; Williams, 1991).

According to Denbel (2014) students have limited conceptions of limit and continuity due to the way the concepts are introduced to students, and the existence of limited concepts related to pre-calculus concepts. Furthermore, the concept of limit is often disputed with a number of questions such as whether a function can reach its limit, whether a limit is actually a limit, whether a limit is a dynamic process or a static object, and whether a limit is inherently related to the concept of motion (Tall, 1993; Williams, 1991).

The results showed that the subject viewed the limit of the function as an unreachable function (Sulastri, et al., 2021). Similarly, research conducted by Denbel (2014) shows that students see limits as unreachable, approximate, limits, dynamic processes, and not as static objects, and are impressed that a function will always have a limit at a point. More deeply students' misunderstandings in understanding limits relate to (a) the relationship between continuous functions and limits, where students think that a function must be defined at a point to have a limit at that point, (b) a function that is not determined at a certain point has no limit value. Students think that when a function has a limit, it must be continuous at that point. (c) The limit is equal to the value of the function at a point, where the limit can be found by the substitution method, and if we get a division of zero by zero, the result is zero. In this case, most students know that any number divided by zero results in undefined.

As for what distinguishes this study from previous research on epistemological obstacles in the concept of limit functions, the focus of the material on the concept of limits in this study is related to the value of the function limit and the value of the function as well as the relationship between these values. In addition, the subjects involved are first-year mathematics students at the Faculty of Mathematics and Teacher Education who have obtained material on function limits in the Differential Calculus course with online learning. This is due to the Covid-19 pandemic, so online learning is applied.

The purpose of this study was to explore the epistemological obstacle experienced by first-year mathematics students in the concept of limit function in online learning. In addition, to reveal the type of error and how to understand the error.

METHOD

This qualitative research uses a phenomenological approach. Epistemologically, the phenomenological approach is based on the paradigm of personal knowledge and subjectivity and emphasizes the importance of personal perspective and interpretation. In other words, phenomenology is concerned with the study of experience from an individual's perspective. The purpose of the phenomenological approach is to clarify specifically and identify and describe certain phenomena through a person's perspective in a situation (Lester, 1999; Creswell, 2012; Freankel, Wallen, & Hyun, 2012). Thus, the purpose of using this approach is to reveal the epistemological obstacles that occur to students during online learning. To reveal this, the researcher analyzed student errors in solving function limit problems and also explored learning experiences on function limits and knowledge relevant to this material.

The subjects in this study were first-year mathematics students from one of the universities in Aceh, Indonesia, totaling sixteen people. They are in a transitional period from secondary education to higher education and have different learning experiences, especially on the concept of limits. At the level of secondary education or equivalent to senior high school, they learn more about the limit of functions procedurally than conceptually because of the demand to pass the final exam. Meanwhile, at the higher education level, which is equivalent to college, the limit of functions is studied in the differential calculus course in the first semester. The learning process in this first year is through online learning due to the COVID-19 pandemic since early 2020.

The data collection instruments in this study were written tests and interview guidelines conducted through the Zoom Meeting and Google Meet applications. This test instrument was designed by the researcher and was adapted from several studies (Moru, 2006; Jordaan, 2005; Denbel, 2014) and refers to the didactic design (Suryadi, 2013) and the didactic situation (Brousseau, 2002). Furthermore, the instrument was validated by mathematicians to get the concept of a limit function that is appropriate and in accordance with scientific concepts. In addition, this test item was tested on high school students (secondary education level with an age range of 16-18 years according to government regulations through the national education system) and also mathematics college students who have studied differential calculus. This is done to determine the readability and accuracy of the questions to be tested on college students. In this study, the test material on the concept of the limit of a function is limited to the relationship between a function and the limit of a function. Two questions or cases in the written test (See Figure 1), the first case is about determining and comparing two functions and their relation to limits, and the second case is about the concept of the relationship between functions and limit functions.

It is known that $f(x) = \frac{x^2 - 9}{x-3}$ and *g(x) = x + 3.* Answer the following questions and their reasons!
Is the function f(x) the same as the function g(x)? Draw the graph!
Find $\lim_{x\to 3} f(x)$ and $\lim_{x\to 3} g(x)$!
What conclusions can be drawn from solution (b)?
Answer the following questions with examples and explanations!
If $\lim_{x\to a} f(x) = L$, is $f(a) = L$?
If $f(a) = L$ and $\lim_{x\to a} f(x)$ exist, is $\lim_{x\to a} f(x) = L$?

Figure 1: Two questions given on a written test to research subjects

Based on various written test answers, nine students were selected (symbolized by M1, M2, …, M9) to take part in an online semi-structured interview. This interview aims to confirm the test results and also to find out their experience in studying limits in the course of differential calculus.

The research data, namely written answers and interview transcripts from the recordings were analyzed descriptively. In addition, the coding of each of these data related to the concept of limits and the relationship between limits and functions is carried out. For the same context, they will be grouped under one code so that several different categories are obtained for the overall written answers and interview transcripts. Furthermore, an analysis of these categories is carried out related to the epistemological obstacles that arise and also based on the learning experience during online learning.

RESULTS

In this section, we will discuss the results of the subjects' answers through written answers and the results of interviews for the two questions or cases of limit functions.

First case: Define and compare two functions and their relation to limit

For the first question of the first point (a), most of the subjects concluded that the two functions f(x) and g(x) are the same functions. From the solution process, it can be seen that the function f(x) is translated by factoring and then dividing regardless of the conditions that result in x+3, which is the same as the function g(x). Thus, the graphs of the two functions are depicted the same (see Figure 2). Most of the subjects did not specifically state what happens at x=3 for the function f(x), only a straight line that passes through x=3 in the graph. From the way of solving the problems and the graphs described, it can be seen that the subject did not understand the concept of functions, especially rational functions correctly, thus ignoring the special rules of these functions.

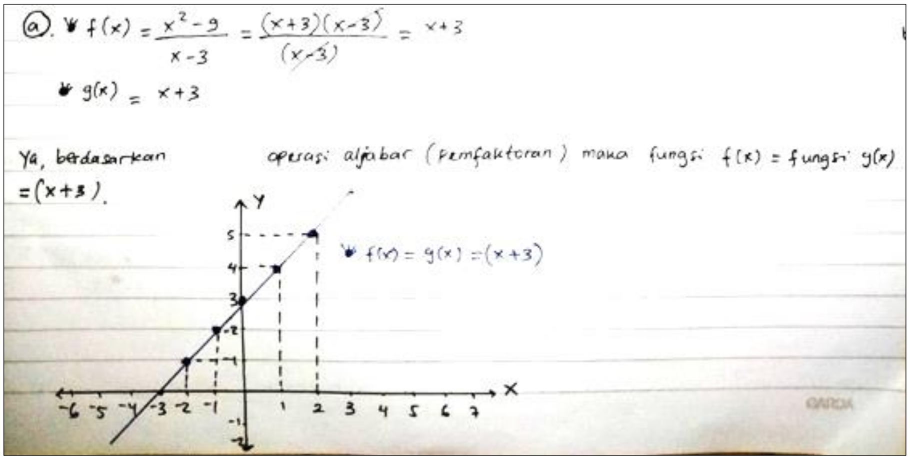

Figure 2: M1 represents f(x) = g(x) so the graph is unified ignoring the special rules of the function f(x)

Only one subject (M9) answered differently that the function f(x) is not the same as g(x). The subject takes any positive and negative x values and x = 0. This also applies to any value of y. Before describing the graph of the function f(x), the subject looks for the turning point with x=1 so that a graph like the one shown in Figure 3. This erroneous understanding of rational functions causes errors in designing the graph of the function. Subjects do not pay attention to the form of rational functions properly, so they assume the function is a quadratic function. The transcript of the interview between the researcher (R) and the subject (M9) is as follows.

R : *Is the function f(x) the same as g(x)?*
M9: *not the same, if the first is a quadratic function, the second is a linear function*
R: *what about the results?*
M9: *for the result, if the linear function directly finds the x value, but for the quadratic function, we have to make a test point first and then we will produce two x values, can it be x twin roots or x is different*
R: *How to find the value of the function f(x)?*
M9: *x squared is simplified one by one, it's still (x^2 – 9) means that it is first translated to its simplest form to be (x-3)(x+3) equalized (x-3).*
R: *What to do next?*
M9: *(x-3) is the same as (x-3) the denominator can be beheaded.*
R: *Why can be beheaded?*
M9: *Because they are the same. The value is 1. So, the result is (x+3)*
R: *Then, what is the conclusion?*
M9: *It's the same, ma'am, with the first function.*
R: *Why is it the same?*
M9: *Because the numbers are the same as if they can be divided like that, for example like 9 and 3, make them still in one, like multiples are the same, multiples of 3. So, when there is a number that can be divided the result is not 1, so the remainder is (x+3).*

From the interviews conducted, M9's answer changed which stated that the two functions f(x) and g(x) were the same. In the answer sheet, the graphs made are also different, the graph of the function f(x) is in the form of a parabola while the function g(x) is a linear line. Finally, M9 mentions that there is an error in understanding the two functions. Since the two functions are the same, the graph is the same. When asked again whether the function f(x) is equal to g(x), M9 states:

> M9: *It is the same, because after the function f(x) will produce a linear function. But not all. Why can the function f(x) in this number be the same as the function g(x) because the number in the value of the function f(x) is equally divisible by multiples, which is a multiple of 3. So, when we simplify the form it produces a linear function so that it is equal to the function g(x) but when the value of the function f(x) is different, meaning not in multiples, it might not be the same function value as g(x).*

Although M9 understands the concept of quadratic functions in describing graphs, in this case, M9 does not understand it in depth. The function f(x) is at first glance a quadratic function. Since this is a rational function, the denominator must also be considered. In solving the function f(x) will produce a linear function which means the graph is a straight line.

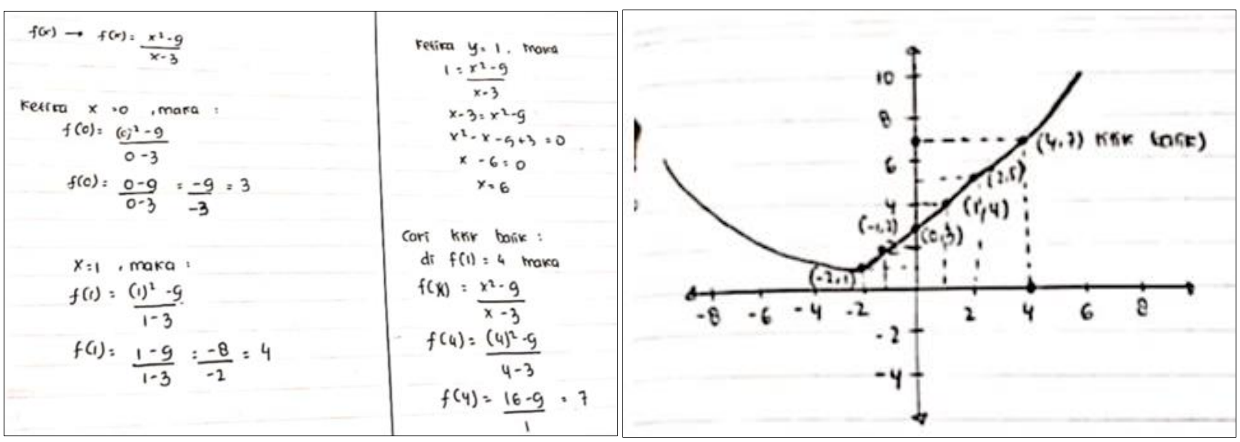

Figure 3: M9's answer to the function f(x)≠g(x) and the graph of f(x)

In determining the limit value of the functions f(x) and g(x), almost all subjects can determine their value in the same way as in the first point (1-a). Where describes $\lim_{x\to 3}\frac{x^2-9}{x-3} = \lim_{x\to 3}(x+3) = 6$ which is obtained from the substitution x=3. Thus, most conclude that the limit f(x) is the same as the limit g(x) because they are both equal to 6 as x approaches 3, but without paying attention to the special rules of the function f(x). In this case, the concept of limit is not properly understood by the subject. They understand the concept of limit is the same as the concept of a function, directly substituting a value of x into the function. Though this concept is clearly different even though it will get the same value for the function and also the known limit.

There is one subject (M2) who gives reasons for elaborating by factoring in solving the function f(x). The reason is that if the function f(x) is substituted directly for x=3 it will produce $\frac{0}{0}$, so to avoid this, the function in the numerator must be factored into $(x^2 - 9) = (x-3)(x+3)$. This solution is actually not wrong, it's just that the subject does not understand further the rules that must be met for a rational function. When obtaining the result in the form $\frac{0}{0}$, the subject states it as an indeterminate form. That is, the subject understands that dividing zero by zero will result in indeterminacy, but the subject does not relate the results obtained with the concept of limits. Subjects do factor to avoid indeterminate results. Although using other ways to get the limit value, the concept of the limit remains that x approaches 3, not x equals 3. This shows that the limit of the function still has a value but is not continuous when x is equal to three.

In giving conclusions about the limit functions of f(x) and g(x), most of the subjects stated that the two limit functions are the same because they have the same limit value. The variety of subject answers can be seen in Table 1.

Answer category	Answer explanation
The limit value of the function is the same as the value of the function	The limit value of the function f(x) is the same as the limit value of the function g(x)
	f(x) = g(x), the limit solution f(x) is different from g(x) but has the same value
	g(x)=x+3 is a factor or simple form of $\frac{x^2-9}{x-3}$
The limit value of the function is not the same as the value of the function	the function f(x) is not the same as g(x) because the limit values are different, f(x) ≠ 3 and g(x) ≠ -3
	The limit f(x) is the limit of the polynomial, while the limit g(x) is the algebraic limit

Table 1: Various conclusions for the answer to the first question

Based on Table 1, it can be seen that most of the subjects stated that the two limit functions f(x) and g(x) were the same for various reasons. For example, based on the process of solving the function f(x) by factoring which produces the same function as g(x), namely (x+3). Thus, the values of the functions f(x) and g(x) will be the same for all x. In this case, the subject does not understand the concept of function properly, especially rational functions. A rational function will have a function value when it fulfills one of the conditions, namely that the denominator cannot be equal to zero. That is, for x=3 does not satisfy the function $f(x) = \frac{x^2-9}{x-3}$ because it produces a zero divisor. So, it must be a function $f(x) = \frac{x^2-9}{x-3}$, with the condition x≠3.

There were two subjects who answered that the value of the functions f(x) and g(x) was the same but with different explanations. In the function $f(x) = \frac{x^2-9}{x-3}$ where x=3 is not met or f(x) is not defined at x=3, otherwise the function g(x) is defined at x=3. Another subject (M2) stated that *"Although the function f(x) = g(x) but the solution to the limit of the function f(x) is different from the function g(x), it will have the same result"*. In this case, the subject only sees the process of solving the function f(x) by factoring. Where the result of factoring is a function of g(x). In other words, the subject is not careful about the concept of a defined or undefined function at a certain point.

Subject M15 stated that the function f(x) is different from g(x) but the explanation is wrong. Where the subject stated that "*the function f(x) is not the same as g(x) because the limit values are different, f(x)= 3 and g(x)= -3"*. This is not correct because of the concept of a function where all the x domains of real numbers will satisfy the function g(x). In addition, in this first point, what is being asked is the similarity of the two functions, not related to the limit of the function. In other words, the subject does not understand the context of the problem correctly. After being investigated, it turns out that the subject's answers cover all the questions in the second case. In this case, the subject combines the answers to the three questions relating to functions and function limits. The written answers can be seen in Figure 4.

From Figure 4, it can be seen that the subject understands the concept of the limit of a rational function by mentioning the condition for the limit of its function which is not defined at x=3. This is because when x = 3 then the denominator returns zero. On the other hand, the subject made a mistake in substituting the value of x=-3 in the limit of the function g(x) which resulted in the wrong graph of the function being described. In other words, the subject's understanding of how to make graphics still needs attention. Thus, it can be stated that this subject is less thorough in understanding the context of the problem and also the concepts of limits and functions. In addition, there is a subject (M9) who does not directly state the conclusion to the two known limit functions f(x) and g(x) but only provides an explanation. In this case, the subject only concludes with the definition of the type of limit function for the functions f(x) and g(x) along with how to solve it. The subject can distinguish between the two limit functions, but this answer is not what is expected from the conclusion of the process of solving the limit functions of f(x) and g(x).

"Limit f(x) is a polynomial limit, where the solution is to simplify the form of the function by dividing the largest power, after being simple, substitute the value of x into the equation of the function f(x). While the limit of the function g(x) is an algebraic limit where the only way to solve it is to substitute/replace the value of x into the equation of the function. If the substitution method doesn't work, then use the factoring method or multiply by common roots".

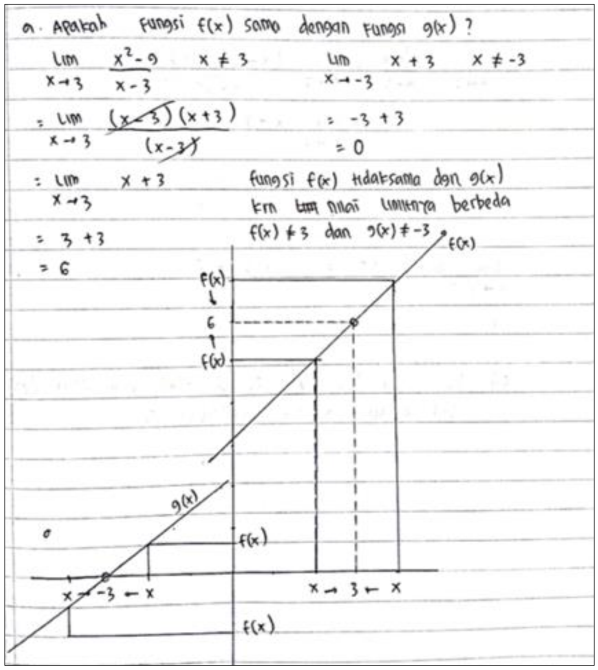

Figure 4: The answer of the subject who does not understand the context of the question and the explanation is wrong

In addition, another subject mentioned a way of solving a function that is to "*calculate the limit value x→3 with the denominator ≠a. If* $\lim_{x \to a} X$ *then the result is a. Using the factoring method if* $\lim_{x \to a} \frac{x}{x-a}, x - a \neq 0$". Based on the subject's answer, information was obtained that the subject had not correctly understood the concept of function for all domain points. This resulted in an understanding of the concept of limit is wrong.

In solving the function and the limit of this function, it is necessary to pay attention to the known form of the function. For the function f(x) there is a condition that must be met, namely x≠3 because for x=3 it will produce $\frac{0}{0}$, which means the function f(x) is not defined at x=3. In contrast to the function g(x) which satisfies all x real numbers. That is, the two functions are only the same for all domains f(x) i.e. real numbers except when x=3. In terms of limits, for the limit functions f(x) and g(x) have the same value when x→3 is close to 6.

Second case: Concept of the relationship between function and limit of a function

Based on all of the subject's answers to these questions, three categories of answers can be grouped, namely statements that are true, statements that are not necessarily true, and statements that are different. In the first question, there are 50% of the subjects answered correctly that if the limit value of a function is L then the function value is also L, and six people answered not necessarily correct because it depends on the known function, while the other subjects answered differently for the known limit and function values. For the second question, most of the subjects (81%) answered correctly for the known statement that if the function value is L and the limit value exists, then the limit value is the same as the function value for the same function.

In answering the first case related to the relationship between the function's limit value and the function value, *"If* $\lim_{x \to a} f(x) = L$*, then f(a) = L"*, most of the subjects stated it was true if the function's limit value was L then the value of the function will also be L. This value is obtained from the process of substituting the value of an into the function f(x) (see Figure 5). In this case, the subject did not understand the concept of function and function limit and their relation correctly. In other words, the subject's understanding of the types of functions is limited to simple functions. Where each domain that is substituted into the function easily produces a function value. The concept of the limit of a function is considered the same as the concept of a function. To get the limit value, they do it by substituting a value that is approximated without meaning to be approximated. As with functions, taking any number of domain members is then substituted into the known function. Whereas the concept of limit is very different from the concept of function. The following is a transcript of the interview between the researcher and M1 related to understanding the concept of function and function limit.

M1: *In my opinion, it depends on the conditions. Sometimes there is a limit value but the limit value is not the same as the function value. So, it could be that the limit value of a function has a limit value and there is a limit value that is not the same as the function value. Now for my explanation, I give an example where the limit value exists and has the same function value as the limit value. So that when the limit value of a function exists, it is the same as the function value, which is equal to L. ... And it will have the same value when the a value is substituted into the function.*

R: *Means for the first case, is the statement true?*

M1: *Yes, because I gave an example where the limit value and function are the same. The limit value of a function, if it has a limit value, the limit value is the same as the function value, then it will be the same as the function value.*

R: *So, is this a conclusion for all or just a special case?*

M1: *Especially for this question, ma'am.*

R: *So, what is the conclusion for the statement if the limit is L, is the function value also the same as L?*

M1: *In my opinion yes, ma'am, when the limit value is L then the function value will also be L.*

Based on the answers during the interview, M1 added that the written answer was a true statement with the examples given. In this case, M1 has not been consistent in providing explanations regarding the concept of limits and functions. Even though at the beginning of the conversation, M1 has shown the correct concept that the statement can be true or false depending on the function given. However, at the end when asked again, M1 states that the limit value will always be the same as the function value.

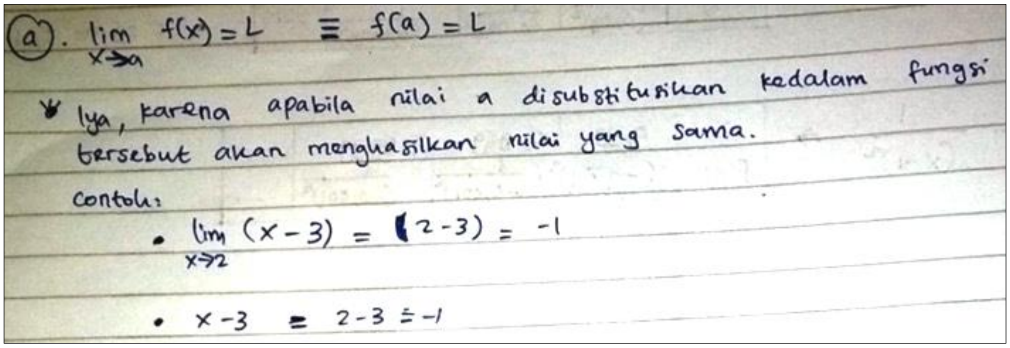

Figure 5: M1 answer for function value equal to limit

Another subject also stated the same for the statement that the value of the limit is the same as the value of the function with the reason "*when the limit is L, the left and right limits are defined in L, then the function will also be worth L for the same point (x=a)*". One of the subjects gave an explanation related to continuity that "*lim f(x) = L, f(a) = L then lim f(x) = f(a) =L means that the function f is defined in a and the limit value is the same as the function value then f(x) is continuous at a. Must meet continuous conditions, (1) defined in a or f(a) exists, (2) lim left = lim right or limit value exists, (3) f(a) = limit value*". From this explanation, it can be seen that the subject understands the concept of continuous limit but is not yet firm for the answer in this first case.

On the other hand, there is a subject (M2) that states that the value of the function will be equal to the value of the limit of the function depending on the known form of the function, but this does not apply to all functions. In the answer, it is not necessarily written that f(a) = L because in $\lim_{x \to a} f(x)$ may be worth $\frac{0}{0}$ so it needs to be changed. The subject mentions an example, namely $f(x) = \frac{x^2-9}{x-3}$. In this case, the subject understands the concept of the limit of a function but is less precise in the concept of a function for a known domain, for example when x=3. In solving the limit case $f(x) = \frac{x^2-9}{x-3}$ by factoring $(x^2 - 9) = (x - 3)(x + 3)$ to get $\lim_{x \to 3} \frac{x^2-9}{x-3} = \lim_{x \to 3} \frac{(x-3)(x+3)}{x-3} = \lim_{x \to 3}(x + 3) = 6$. On the other hand, for the value of the function f(x) when x=3, the subject only substitutes the value of x=3 into the function without factoring and providing an explanation for the value of the function obtained is $\frac{0}{0}$. This is not correct because the function f(x) can also be factored in to get the value of the function. During the interview, M2 re-explained the example given where the example function can also be changed or factored in to avoid the $\frac{0}{0}$ form. M2 states that the function is x≠3 because when x=3 the function has no value. In this case, f(x) is defined for all values of x except when x=3.

Several other subjects also stated that if the value of a limit function is L, then the function value is not necessarily L as well. M4 only includes one simple example, namely $\lim_{x \to 2} f(x) = 5$ and $f(2) = 5$ without knowing the form of the function with the result being a number. From this explanation, it can be seen that the subject understands the concept of the limit of the function where the value of the limit of the function will not be right at that point but approaches it from both sides, namely the left side and the right side. In the written answer, the subject does not explain the concept of function. Similarly, M7 mentions that there are two possibilities for the known statement. The statement will apply to function values equal to the limit, and also applies to function values different from the limit. The subject explained that "*there is a graph of a continuous function where the limit value and the function value are the same. There is a graph of a discontinuous function where the limit value and the function value are not the same*". In this case, M7 adds examples for both statements accompanied by graphs (see Figure 6). This shows that M7 understands the concept of limit especially in distinguishing between continuous and discontinuous limits. Although the example given is very simple without any explanation of the form of the function given.

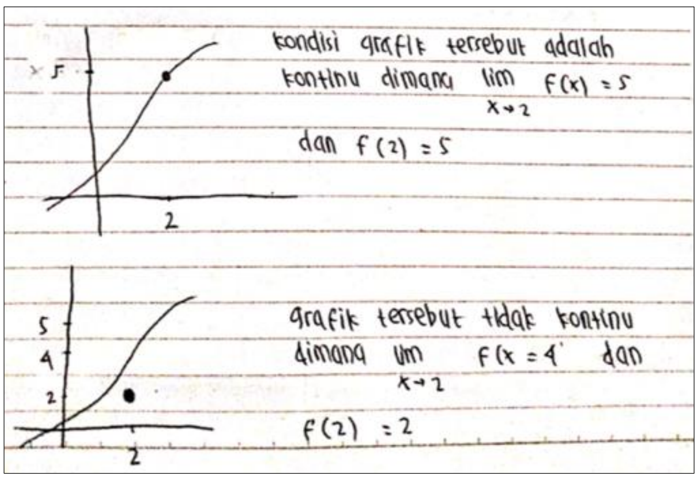

Figure 6: M7 gives an example of a function value equal to limit and vice versa

There are subjects who answer that the limit value is different from the function value. According to M9, the limit of a function is not the same as a function because "$\lim_{x \to a} f(x) = L$ *is a limit function where when a function x or f(x) where x→a is defined in L, while f(a) = L is a function (a) which has a value of L*". The subject provides an example of a simple function by way of substitution. This shows that the subject does not fully understand the concept of limits and functions and is limited to simple functions.

For the statement, if $\lim_{x \to a} f(x) = L$, then it is not necessarily the value of *f(a)=L*. It is based on the known form of the function. The limit value of the function will be the same as the function value if the known function does not require certain conditions as in the rational function. Rational functions must be considered in the denominator, if the denominator returns zero then the function is not defined at the specified point.

For the second statement, it is known that the value of the function *f(a)=L* and $\lim_{x \to a} f(x)$ exists, then $\lim_{x \to a} f(x) = L$. Most of the subjects stated that the statement was true based on the examples included with the substitution method. When searching through interviews, M1 stated that the answer was different from the written answer. According to M1 the limit value and function value will be the same or different depending on the given function. The transcript of the interview with M1 is as follows.

M1: *For this second condition, it depends on the function. Sometimes there is a function value when we substitute it, it is not the same as the existing limit value. That is, there is a function that does not have a limit value but has a function value*. (Next, the researcher asked M1 to write down an example in a notebook, as shown in Figure 7).

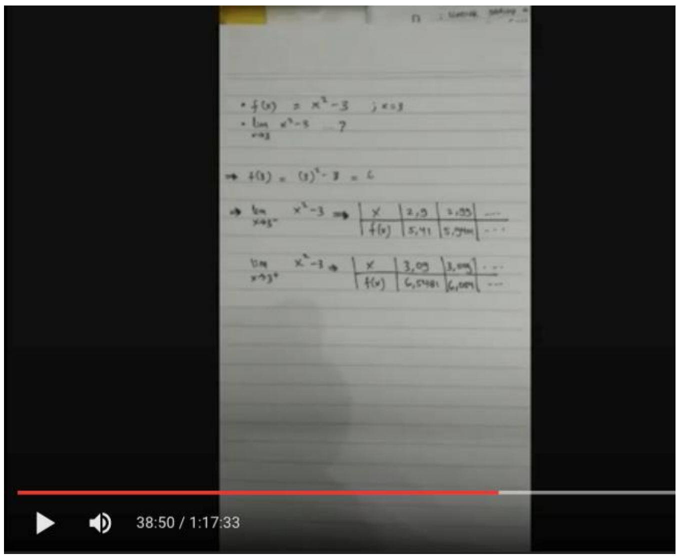

Figure 7: M1 gives an example of a function value not equal to the limit value

One of the subjects gave a reason with the answer "*yes, it means a continuous function where the function value and the limit value are the same*". Several other subjects answered the statement correctly by giving examples of rational functions so that the solution was by factoring. On the other hand, there is a subject (M9) who states that the statement is not the same as the reason "*the difference between f(a) – L with* $\lim_{x \to a} f(x)$ *or* $f(a) = L \neq \lim_{x \to a} f(x)$". From the results of the interview, M9 did not understand the context of the question because it had never been studied. M9 stated that there was no relationship between limits and functions. According to him, the function was correct. maps only one pair of domains to codomains while limits cannot be explained. In everyday life, limits are seen as limits such as debit card limits, while in the context of mathematics, limits are something that is close.

Based on the answers and explanations of the subjects regarding the relationship between functions and function limits, most of the subjects did not understand the concept properly. Especially when faced with cases like this that require analysis of the concepts of limits and functions. Even in this case, the subject is still limited in providing examples for each statement. The included examples are still in the form of simple algebraic functions. Even though there are still many forms of functions and function limits that can justify the statement or cancel it.

From the two statements, the same conclusion is obtained that if it is known that $\lim_{x \to a} f(x) = L$, then the value of f(a) is not necessarily the same as L. On the other hand, if we know the value of the function of $f(a) = L$ and $\lim_{x \to a} f(x)$ exists, then $\lim_{x \to a} f(x)$ is also not necessarily equal to L. Based on this, it can be said that the value of the function and the limit value of the function can be the same or different values. This is because the value of the function and also the value of the limit of the function depend on the known function.

DISCUSSION AND CONCLUSIONS

In building the concept of limits, there is at least three knowledge that must be possessed, namely actions, processes, and objects (Cottrill, et al., 1996). The development stages include actions internalized into processes and processes encapsulated into objects. This is investigated in the learning process using computers in studying the concept of limits. On the other hand, Moru (2009) investigated the development of the limit concept without using a computer. The results show that the subject understands whether or not there is a limit to the indeterminate, $\frac{0}{0}$ for functions represented algebraically. On the other hand, in this study, the subject erred in the concept of a limit which resulted in the division of a number n (where n≠0) by zero. Subject assumes that the division results in undefined also applies to limits. This concept only applies to division and differs in the limit case (Sulastri, et al., 2022). For geometric functions, the subject denies that there is a limit to undefined functions. According to the subject the limit value can only be determined for algebraic functions. In addition, the subject is confused about the concept of the limit value and function value (Moru, 2009). Likewise, with the results of this study where the subject also has not been able to understand the concept of limits and functions correctly.

A limit is indicated by the symbol →. Most of the subjects understand that in determining the limit value, for example when x→0 it is replaced by x=0 in a known expression. This is one of the sources of confusion and errors experienced by the subject in determining the limit value. The use of this generalized substitution method applies to continuous functions. In addition to this context, it is referred to as an epistemological barrier. According to Tall (1991), generalization is one of the epistemological barriers. Similarly, Bachelard states that several types of epistemological barriers include the tendency to generalize, the barrier caused by natural language, and the tendency to rely on mistaken intuitive experience (Herscovics, 1989). The epistemological aspects that are problematic in the concept of limits occur in the relationship between graphical and arithmetic representations of mathematical content, processes and objects, static and dynamic interpretations, and intuitive ideas and mathematical specifications (Hofe, 2003).

In relation to functions, there are limitations in manipulating algebraic functions (Sebsibe, Dorra, & Beressa, 2019), for example being faced with a division by zero situation which results in no function value. This interpretation is assumed to be the same as the concept of limit (Fischbein, 1999; Moru, 2009). This understanding causes problems in understanding the concepts of limits and functions. A limit is seen as a point that is approached without reaching it, or a point that is approached and reached (Taback, 1975; Cornu, 1991). In this study, the subject did not explicitly state whether the limit was being approached without being reached or when it was being approached.

Based on the process of solving the function limit problem carried out by the subject, most of them understand and do it operationally, namely explaining in the act of calculating. This shows that the limit is associated with calculations, which come from previous learning experiences when high school (Moru, 2009). Learning at school, where the limit of function material is in the second grade even semester. Learning is not optimal because students are focused on being able to solve mathematical problems operationally to face the national exam. In addition, the subject matter of school limits was obtained through online learning due to the COVID-19 pandemic in the first year of the lockdown.

The application of online learning has positive impacts such as flexibility in place and time, accessibility, and effectiveness of assessment and communication methods (Musthaha, et al., 2022). Findings from research by Qutishat, Obeidallah, & Qawasmeh (2022) show the success of implementing online learning with material understood by students, but student interaction during learning is low. Difficulties in online learning can occur if schools do not have experience with electronic learning such as teachers not understanding how to use online applications (Zaharah & Kirilova, 2020). The same difficulty also occurred to the subjects in this study which was their first experience. In addition, the lack of understanding of basic concepts such as the concept of real numbers, inequalities, functions, and others also greatly affects the study of function limits. In the teaching process, the lecturer no longer explains in detail the concept because it is considered to have been studied in basic mathematics courses such as Elementary Algebra. The results of research by Mailizar, et al. (2020) show that the barriers that have the highest impact on electronic learning are at the student level. A teacher needs to ensure an accurate representation of mathematical tools in selecting technology to use in the classroom. Thus, in the online and offline teaching and learning process, special attention must be paid to overcoming the difficulties and misconceptions identified in learning a material (Sebsibe, et al., 2019).

Subjects easily understand a function or limit which is represented in a simple algebraic form such as the examples given for known cases. In determining the value of a function, the subject performs by substitution of some arbitrary real number or value specified. The same way is also done to find the limit value of a function. In this case, the subject generalizes the direct substitution method to find the limit value. This mistake made the concept of function and limit and their relation not understood properly. In other cases, there are subjects who understand that the limit cannot be reached or is close to the specified point (not exactly at that point), compared to the concept of a function that can be substituted right at that point. This is similar to William's (1991) statement that there are several confusing concepts related to limits, including whether a function can reach its limit; whether limit is limit; whether limit is a dynamic process or a static object; and whether limits are inherently tied to the concept of motion. These problems lead to incomplete conceptions of limits (Denbel, 2014). According to Williams (1991), this conception relates to the process of limiting by the mathematical community before Cauchy's formal definition of the definition of limit using epsilon-delta.

In the nineteenth century, Weierstrass wrote a formal definition of limit that requires a high level of logical and syntactic knowledge to understand, namely $\lim_{x \to a} f(x) = L$ where $\forall \epsilon, \exists \delta; 0 < |x - a| < \delta \Rightarrow |f(x) - L| < \epsilon$. Easily understanding this definition can be done with an informal interpretation that is, a limit is a number whose y-value of a function can be as close as desired, by choosing x in the interval x as small as necessary (Sarvetani, 2011).

Several studies that discuss the formal definition of limits (Cornu, 1991; Cottrill et al., 1996; Ervynck, 1981; Fernández, 2004; Tall & Vinner, 1981; Vinner, 1991; Williams, 1991) show that there are various reasons students have difficulty communicating a coherent understanding of the formal definition of limits. One of them is the struggle of students in understanding algebraic notation in the definition of limit ε–δ (Cornu, 1991; Cottrill et al., 1996; Ervynck, 1981; Fernández, 2004). Students' difficulties also occur in understanding what ε and δ represent; the relationship between the variables (and parameters) in the definition; and why |x – c| must be positive, while |f(x) – L| is not (Fernández, 2004). Another difficulty in the formal definition of limits is due to the struggle students have with quantification (Cottrill et al., 1996; Dubinsky, Elterman, & Gong, 1988; Tall & Vinner, 1981).

The findings of Beynon & Zollman (2015) show that many students do not use a formal definition of limit to solve limit problems. In this case, their understanding of the definition of the concept of limit is inconsistent with the definition of the formal concept. Only students who have high abilities have openness to using formal mathematical concepts. The same thing is also obtained from other studies (Tall, 1990) that students have strong procedural knowledge compared to conceptual understanding of a mathematical concept. This causes students not to be able to solve problems by applying stage of problem-solving. Other causes are the lack of mathematical literacy skills and imperfect mathematization processes (Purnomo, et al., 2022).

In the case of the value of a function with a limit value, most of the subjects stated that the value of the function will always be equal to the limit value for a function that is known to be the same. Other errors also occur in the prerequisite material for limit functions such as the concept of real numbers and the concept of infinity. The subject has a limited conception of the basic concept. To overcome these errors, the presentation of the material, especially the basic concepts, needs to be done in-depth so that is easy to understand and to minimize the occurrence of misconceptions.

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