Investigation of Secondary Students' Epistemological Obstacles in the Inequality Concept

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Abstract: An inequality concept has an important role; even in advanced mathematics, inequality assists in analysis and proof. However, students' understanding of inequality is not satisfying. The fact shows that students experience difficulties and errors in solving inequality problems. These difficulties and errors are not intentional or do not result from students' carelessness in carrying out solutions or students' ignorance (misconceptions). Instead, these difficulties and errors are caused by epistemological obstacles. Therefore, this study explores the epistemological obstacles students face in the inequality concept by analyzing errors found in solving inequality problems. The qualitative research design with a phenomenological approach was chosen to achieve the research objectives by involving 29 eleventh-grade secondary students. The researchers employed a test on the inequality concept to explore students' epistemological obstacles, which consisted of three problems. A one-to-one unstructured interview was also conducted to investigate students' ways of thinking and understanding based on their answers. Furthermore, the data were analyzed using an inductive approach, combining systematic data management methods through reduction, organization, and connection. The data obtained are then presented in the form of narratives and figures. The results showed that the inequality rules, the absence of semantic and symbolic meanings of inequalities, interpreting solutions, and generalizations in the inequality rules become sources of students' errors in solving inequality problems. Thus, we found epistemological obstacles in the inequality concept based on these four types of errors. The obstacles are indicated by students' limitations in understanding and interpreting inequality signs as they solve inequality problems.

INTRODUCTION

Inequality is an expression showing that two quantities are not equal (Frempong, 2012; Gellert, Kustner, & Hellwich, 1975; Gustafson & Frisk, 2008; Postelnicu & Coatu, 1980). In algebra, expressions of two unequal quantities are connected by symbols (Davies & Peck, 1855). Inequality is a scientific discipline with a highly crucial role, contributing to mathematical discoveries from Classical Greek Geometry to Modern Calculus (Fink, 2000). Inequality is also considered a subject that is difficult to define precisely, in which some parts belong to 'algebra' and the others to 'analysis' (Hardy, Littlewood, & Pólya, 1934). It implies that inequality has an important position at a higher mathematics level. Inequality assists in analysis and proof.

However, several previous studies have shown that students and undergraduates still encounter difficulty understanding or applying inequalities. Taqiyuddin et al. (2017) study revealed that students have difficulty solving problems in linear inequalities, e.g., when students are given a linear inequality $9x + 1 > 9x - 2$. While solving the linear inequality, students tend to incorrectly use algebraic operations, such as in $9x + 1 > 9x - 2$, $9x - 9x > -2 - 1$, and $x > -3$. A similar result is also found by Botty et al. (2015), reporting that students have difficulty solving linear inequality problems, e.g., when students draw a graph of linear inequality and identify the area represented by the given inequality. The results obtained an average of 22%, indicating that the test is challenging for students.

In addition, another fact demonstrates that difficulties in solving inequalities are experienced by not only secondary students, but also undergraduate students (Rowntree, 2007), specifically in the following four areas: a) inequalities as equations (Blanco & Garrote, 2007; Vaiyavutjamai & Clements, 2006), b) limited understanding of the terms "greater than" and "less than" and the appropriate relational symbols (Warren, 2006), c) difficulty in connecting and using different problem-solving techniques (Blanco & Garrote, 2007; Tsamir & Almog, 2001), and d) interpreting solutions (Tsamir & Bazzini, 2004). Besides, other studies (Bicer, Capraro, & Capraro, 2014; Blanco & Garrote, 2007; Booth, McGinn, Barbieri, & Young, 2017; Ellerton & Clements, 2011; El-Shara' & Al-Abed, 2010) found that students made basic arithmetic errors due to insufficient knowledge about inequality rules which tend to alter the inequality sign, such as when dividing the inequality by a negative sign.

Several studies have been conducted related to the difficulties faced by students in solving the concept of inequality. However, the studies mentioned above only focused on the difficulties and errors faced by students in solving the concept of inequalities. And how is the impact of applying a model or learning method in learning the concept of inequalities. In this study, we carried out an update by looking at and exploring students' experiences in understanding and interpreting the concept of inequalities by investigating the epistemological obstacles experienced by students in the concept of inequalities.

Concerning this phenomenon, we used the approach of epistemological obstacle analysis and investigation to student difficulties inherent in structuralist thinking. The concept of epistemological obstacles was first introduced by Bachelard (1938), which appeared in his philosophy of science work. Bachelard (1938) explained that "The problem of scientific knowledge must be posed in terms of obstacles [...] we will illustrate sources of standstill and even regression in the very act of knowing, and this is where we will discern causes of inertia that we will call epistemological obstacles." Gutting (1989) points out that the center of Bachelard's philosophy of science is his model of scientific change, which is built around four epistemological categories: ruptures, obstacles, profiles, and acts. Bachelard employs the concept of an epistemological break in contexts. He indicates that, in the first term, scientific knowledge separates from common sense and even the contradiction, and in the second term, ruptures also occur between scientific conceptual elaborations. The end of ruptures, in turn, suggests that there is an obstacle that must be destroyed. Bachelard thus introduced the notion of an epistemological obstacle, understood it as any concept of the method that prevents an epistemological rupture. The idea of an epistemological profile consists of an analysis that reveals the degree to which the individual's understanding of a concept involves elements from various stages of its historical development. Finally, the concept of an epistemological act counterbalances the obstacle and refers to the leaps that the scientific genius introduces in scientific development.

Brousseau (2002) offers the concept of epistemological obstacles as the meaning of knowledge (instead of lack of knowledge) that has been considered effective previously, even in certain contexts, which at some point begins to produce answers that are judged to be wrong or inadequate and raise contradictions. Moreover, epistemological obstacles are resistant and appear sporadically even after being overcome; dealing with them requires deeper knowledge which generalizes the known context and requires students to be aware of the obstacles explicitly (Brousseau, 2002). According to Brousseau (2002), such thinking can be applied to analyze the historical origin of knowledge or teaching or spontaneous cognitive development of students' understanding. The search for epistemological obstacles is conducted using two approaches: first, according to Bachelard, historical research by adopting an epistemological perspective, and second, tracing repeated errors in learning mathematical concepts. The two approaches are interrelated: historical-epistemological developments can assist in identifying possibilities of hidden models and suggest the construction of appropriate learning situations to overcome the obstacles found. Besides, students' difficulties and repeated errors indicate an epistemological obstacle. Relevant to this, Brousseau (2002) has proposed methods to find out these epistemological obstacles, including (1) finding repeated errors and asserting the errors are part of knowledge, not ignorance; (2) investigating obstacles in the history of mathematics; and (3) comparing obstacles with history and determining their epistemological characters. Finding epistemological obstacles to a mathematical content can be carried out through historical analysis and analysis of students' ways of understanding as the epistemological obstacles are not related to the way or method the teacher uses in learning; they are results of the nature and characteristics of the mathematical concept instead (Cornu, 2002).

Additionally, epistemological obstacles can be identified by noticing the tendency to generalize certain understandings to all situations. Sierpinska (1987) explained that the duality of epistemological obstacles provides another clue; if the presence of epistemological obstacles in students is associated with beliefs, overcoming these obstacles does not mean replacing their existing beliefs with the opposite ones. It will double the obstacles. Instead, students have to rise from what they believe to analyze, from the outside, ways they use to solve problems, formulate the hypotheses they have understood, and become aware of possible rival hypotheses.

The epistemological obstacle analysis in this study is conducted to explore the obstacles faced by secondary students in understanding and interpreting the inequality concept by exploring their experiences. This analysis is essential since it is a starting point for teachers to develop teaching strategies (Hausberger, 2017). This epistemological obstacle analysis is supported by a "didactic situation theory" (Brousseau, 2002). According to this theory, knowledge construction refers to the implication of interaction between students and problem situations (broader: milieu); that is, dialectical interaction in which students use prior knowledge to revise, modify, complement, or reject constructs of new knowledge. Students' knowledge is attained by adapting their ways of thinking to an environment. In other words, students' errors emerge due to the adaptation process of ways of thinking and understanding in the learning environment (Brousseau, 2002). Therefore, this current study investigates the epistemological obstacles students encounter in the inequality concept through error analysis on the given problems. In addition to that, the researchers analyzed ways of thinking and understanding of each students' errors. An epistemological obstacle is a well-constructed piece of knowledge that is so practical and valuable that it forces the problem-solver to employ the approach it proposes. We took advantage of the chance to explore the concept of epistemic obstacles more thoroughly in order to find a productive approach for the teachers by offering a variety of obstacles and taking into account their relative difficulties. In order for educational practitioners to use this study's findings as a guide in predicting learning obstacles for students in the idea of the inequalities concept. Additionally, it can be utilized as a guide or source of information when proposing the best design in light of current scientific knowledge of the inequalities concept.

RESEARCH METHODOLOGY

This study aims to investigate the epistemological obstacles in the concept of inequalities of secondary students, by exploring their experiences after receiving/through learning in the concept of inequality. Thus, a qualitative research design with a phenomenological approach was chosen. This approach is defined as a qualitative method focusing on understanding and interpreting human life experience as a topic according to its framework, relating to meaning, and how it is obtained from experience (Grbich, 2007; Langdridge, 2007; Suryadi, 2019). The phenomenon being studied was the epistemological obstacle of secondary students in the inequality concept. Further, the phenomenological facts were linked with normal interpretative and relevant theory (pragmatic interpretation). The research techniques and study layout that make up a phenomenological research study's methodology are described below.

1. *Participant selection*

Selecting research participants who have substantial and meaningful experiences with the issue being examined is a requirement for researching the essence of lived experience (Polkinghorne, 1989). Composition and sample size of the study are additional factors to be taken into account when choosing participants. Because the goal of phenomenological research is to collect descriptions of experience rather than generalizable conclusions, participants' representativeness of the general population is not a priority (Cilesiz, 2011; Seidman, 2006). The subjects might be chosen using the purposive sampling technique. This sort of sample is nonprobability (Alhazmi & Kaufmann, 2022). Due to the in-depth nature of the study, sample sizes for phenomenological research are typically not huge; while recommendations for sample size vary, a sample of 3–10 people are typically seen to be suitable (Creswell, 1998; Polkinghorne, 1989).

2. Data collection through phenomenological

In general, phenomenological research, data consist of descriptions of life experiences, which can be collected through interviews, observations, or written self-descriptions (van Manen, 1997). In this study, data were collected through giving tests related to the concept of inequalities and followed by interviews to explore experiences formed from the process of learning the concept of inequalities and how students interpret it.

3. Phenomenal data analysis

The goal of phenomenological research is to identify and dissect the structures, logic, and connections that exist within the experience being studied. The central phase of phenomenological research is data analysis. Data analysis was carried out by adopting a phenomenological approach developed by Hycner (1985) and modified by (Groenewald, 2004), which acknowledges the researcher's interpretive involvement with the data.

4. Validity considerations for phenomenological research

In qualitative research, the term "validity" typically refers to a study's rigor to ensure that the findings are the product of the proper application of methodologies and that the research delivers relevant information based on its epistemology (Guba & Lincoln, 1982; Lincoln, 1995; Merriam, 1995).

5. Ethical considerations in phenomenological research

Participants' privacy and confidentiality must be protected in these situations since failing to do so could harm their reputation or have other negative effects. By using pseudonyms in place of identifiable information such as references to names and localities and by allowing participants to read the final report and point out any weaknesses or objectionable portrayals, it is possible to protect participants' privacy. Another important ethical factor in phenomenological research is reciprocity. According to calls for reciprocity, both the researcher and the subject of the research should gain from the act of doing it. Researchers are requested to share some of the perks associated with their privileged and intellectual positions in exchange for the opportunity to share sensitive details about their lives, even though there is no financial compensation for doing so (Lincoln, 1995).

The subjects involved in this study were 29 eleventh-grade secondary students in Medan City. The test was administered to all students, but from the test results, the subject was then reduced to nine students, and in the final stage, it was reduced again to three students. According to Miles and Huberman (1994), data reduction refers to the process of selecting, focusing, simplifying, abstracting, and changing data that appears in field notes or written transcriptions. Data not only needs to be summarized for easy management but also needs to be modified so that it can be understood concerning the problem being addressed. In other words, at this data reduction stage, there will also be a process of coding, summarizing, and also partitioning or creating parts. In addition, data reduction can also be interpreted as a form of analysis that sharpens, classifies, and directs research objectives. The data for this article was derived from students' learning experience in the inequality concept.

Data were collected through tests and interviews designed to explore students' experiences and how these experiences influence students' perceptions of the inequality concept. The test given to the students was related to the inequality concept. Also, a one-to-one interview was conducted after the researchers analyzed the test results to ascertain the experiences and obstacles encountered by the students. In detail, the test is presented in Table 1.

Problem Type	Problems
1	Please solve the linear inequalities below and present the solutions using interval notations: a. $3(2x-9)<9$ b. $-4(3x+2)\leq 16$.
2	Please use the inequality notation to describe the expression "there exist all values of x within an interval $(-3,5]$".
3	Please explain all values of x with the distance of 4 from number 5. Sketch this expression in a number line, state it using an inequality, and find the solutions.

Table 1: Solving Inequality Problems

All data in this study were transcribed, and a pseudonym was given to each participant. The data were analyzed using an inductive approach, which combines systematic data management methods through reduction, organization, and connection (Dey, 1993; LeCompte, 2000), and the data obtained are then presented in the form of narratives and images. Overall, data analysis was carried out by adopting the phenomenological approach developed by Hycner (1985) and modified by (Groenewald, 2004), which acknowledges the researcher's interpretive involvement with the data. In this study, data analysis was carried out through five steps that have been simplified by Groenewald, including 1) Bracketing and phenomenological reduction; 2) Delineating units of meaning; 3) Clustering of units of meaning to form themes; 4) Summarising each interview, validating it and where necessary modifying it; and 5) Extracting general and unique themes from all the interviews and making a composite summary. As previously stated in the introduction section, epistemological obstacles can be identified by analyzing students' errors in solving inequality problems. In addition to that, this study analyzes other phenomena that appear in students' answers. A theme was made based on patterns and types of errors found to classify student errors. The researchers also reviewed the results of previous studies as our assumptions about ways of thinking and understanding aspects of each error type. After the classification process and descriptive analysis of students' errors, a one-to-one task-based interview was conducted. This interview aims to strengthen the descriptive aspects of ways of thinking and understanding behind the errors obtained.

RESULTS

The results are presented in three stages. First, it presents categories of student errors based on themes formed by the patterns and types of errors. Second, besides the categories of errors, patterns, and types of errors, students' way of thinking and understanding behind each underlying error is also analyzed. Third, the epistemological obstacles to the inequality concept are based on students' ways of thinking and understanding. Based on the analysis of student errors in solving the concept of inequality, the discussions with lecturers in Mathematics Education, and previous research, several themes of the source of student errors have emerged, namely: (a) inequalities rules (Bicer et al., 2014), (b) the absence of semantic and symbolic meanings of inequalities (Blanco & Garrote, 2007), (c) interpreting solutions, and (d) generalizations in inequality rules. Descriptions of student errors in the task of inequality concept are summarized in Table 2.

Types of errors	Descriptions of errors	Ways of Thinking	Ways of Understanding
Inequalities rules	Students do not understand the rules/reasons for changing the direction of inequalities when multiplying or dividing inequalities by negative numbers.	Multiplication and division by negative numbers do not change the sign of inequality.	The result of multiplying or dividing an inequality by a negative number does not affect the inequality sign.
The absence of semantic and symbolic meanings of inequalities	Students do not understand or misunderstand notation in inequality	• Students understand the form of inequality notation "≤", "≥" is equal to equality sign "<", ">"	There is no difference in the semantic and symbolic meaning of the inequality notation.

		• There is no difference between interval and closed notation in expressing in the form of inequalities.	
Interpreting solution	Students cannot write their solutions in interval notation through the solution of inequalities and think that only one value makes the inequality correct.	The final result of the operation of solving an inequality is the inequality solution, for example $3(2x-9)<9$, $6x-27<9$, $6x<9+27$, $6x<36$, $x<6$ For the students, $x<6$ is the solution for $3(2x-9)<9$	Student assume that the solution set cannot be written in an interval or a finite set; students also, write closed intervals like $[-1,\infty)$
Generalization in the rule of inequalities	Students use the absolute value inequality rule in solving linear inequalities.	Students generalize the inequality process used in absolute value inequalities in solving linear inequalities.	Students generalize the understanding of absolute value inequality to linear inequality.

Table 2: Descriptions of student errors

The main reason why the students in this study committed basic arithmetic errors was that they did not have sufficient knowledge about the rules of inequality that change the direction of inequalities when multiplying or dividing the inequalities by negative numbers. It is seen in the results of students' work of task 1 in Figure 1.

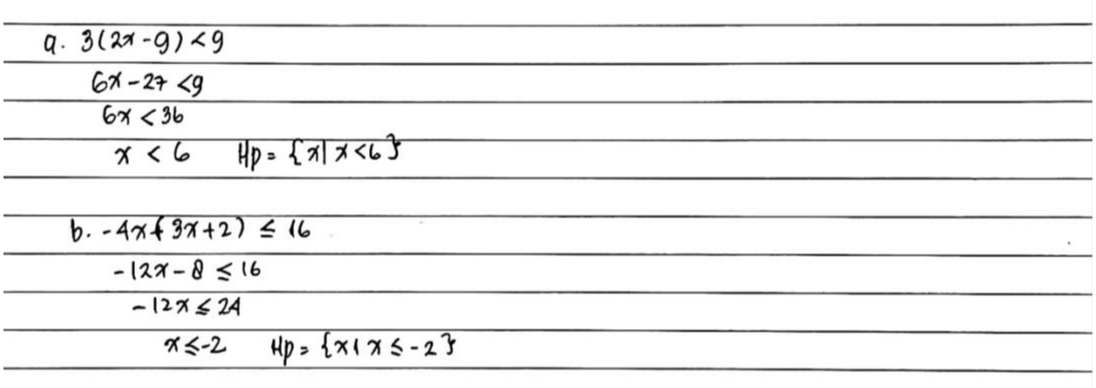


Figure 1: Example of Student Errors in Inequality Rules

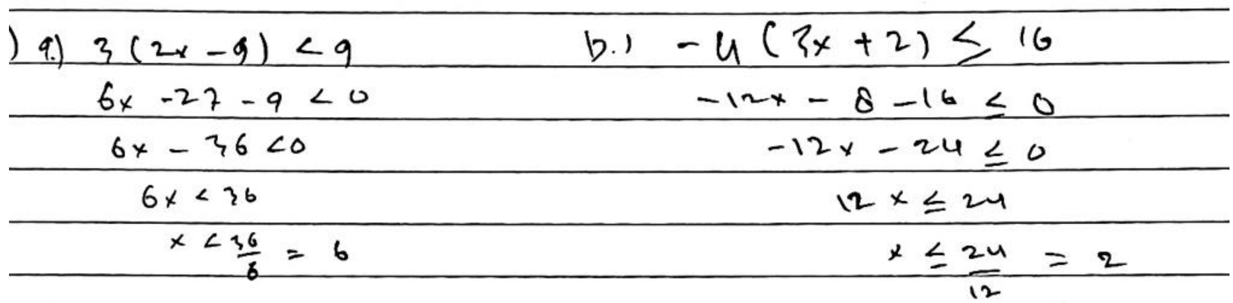
Figure 1 shows that operationally students have solved the inequality of $-4(3x+2) \leq 16$ correctly. Yet, due to the lack of understanding of the inequality rules, students did not change the direction of the inequality when multiplying or dividing by negative numbers. Thus, this student error influenced the solution set of the inequality. The teacher needs to explain and discuss the reasons why we change the direction of the inequality when multiplying or dividing the inequality by negative to overcome this difficulty. Understanding the logic of inequality instead of only memorizing the rules should allow students to understand the inequality concept.

The student errors in understanding the inequality concept were also caused by the students' way of thinking that inequality is equal to equality. It is found based on interviews between researchers and respondents, as summarized below.

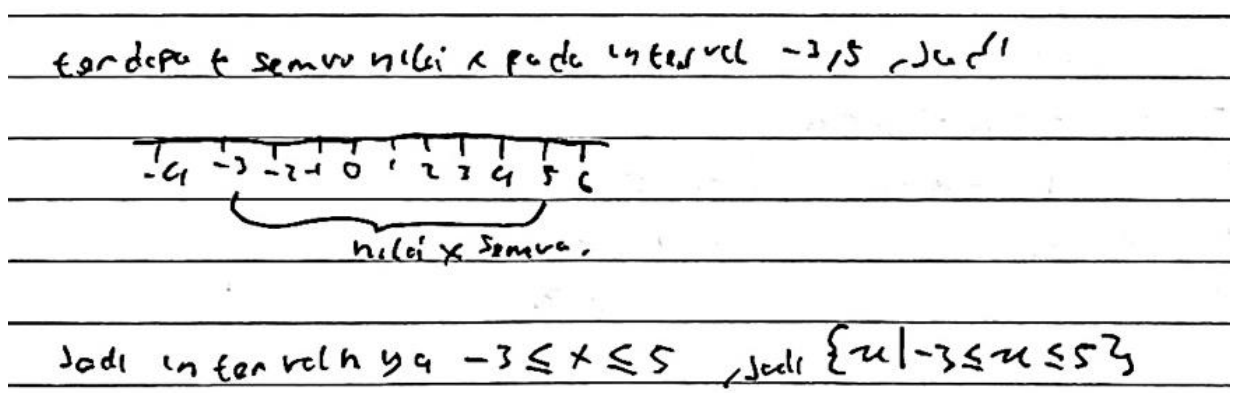
Researcher : What do you think is inequality?
Respondent: Statements compare two or more variables
Researcher : What does that meaning of comparing?
Respondent: Greater than, less than
Researcher : Ok, let's pay attention to Task 1; try to read the two forms of inequality.
Respondent: a) three times $2x$, minus nine is less than nine; b) negative four times $3x$ plus two is less than or equal to sixteen.
Researcher : Ok, let's see your work of part b!
Respondent: Ok
Researcher : Do you think that your solution is correct?
Respondent: Yes, the form of $-4(3x+2) \leq 16$, is simplified to $-12x-8 \leq 16$. Then, 8 is moved to the opposite side because it equals 16. Then, we get $-12x \leq 24$, and the result is $x \leq -2$, so the solution set is $\{x|x \leq -2\}$
Researcher : Try to recheck whether the solution is correct!
Respondent: Hmm, it seems correct because it is the same as when we solve linear equations
Researcher : Ok, I see! What do you think happens when an inequality is multiplied or divided by a negative number?

Respondent: The result is negative
Researcher : Then, what about the inequality sign?
Respondent: It remains the same

The interview indicates that students believe that multiplying or dividing an inequality by negative numbers does not affect the sign of the inequality. Students' errors in understanding the rules of inequality are also related to the absence of the semantic and symbolic meaning of the inequality, as indicated by the student's assumption that inequality is equal to equality. Tamba and Saragih (2020) stated that students see signs as having no semantic meaning other than connecting two members of the inequality. They solve inequalities by replacing the sign "=" with the inequality sign "<", ">", "≤", or "≥". The absence of semantic and symbolic meaning of inequality is also indicated by student errors in understanding the notations in the concept of inequality, as shown in Figure 2.



(a)



(b)

Figure 2: (a) and (b) Examples of the Absence of Semantic and Symbolic Meaning Errors

Figure 2(a) shows that students tend to replace the inequality notation "<", "≤" with "=", likewise in understanding the interval notation in writing the set of solutions or vice versa in changing the form of interval notation to the form of inequalities. On the other hand, Figure 2(b) indicates that students interpret the interval notation representing the inequality notation "≤", "≥" as equal to "<", ">" sign. It is also confirmed by the results of interviews as follows.

Researcher : Look at Task 1; what should you do?

Respondent: Determining the set of solutions in interval notation of the two forms of inequality

Researcher : Ok, what is your solution?

Respondent: For part (a) it is obtained $x < 6$, part (b) is $x \geq -2$

Researcher : Then, what about the set of solutions in interval notation?

Respondent: a is $(-\infty, 6)$, and b is $(-2, \infty)$

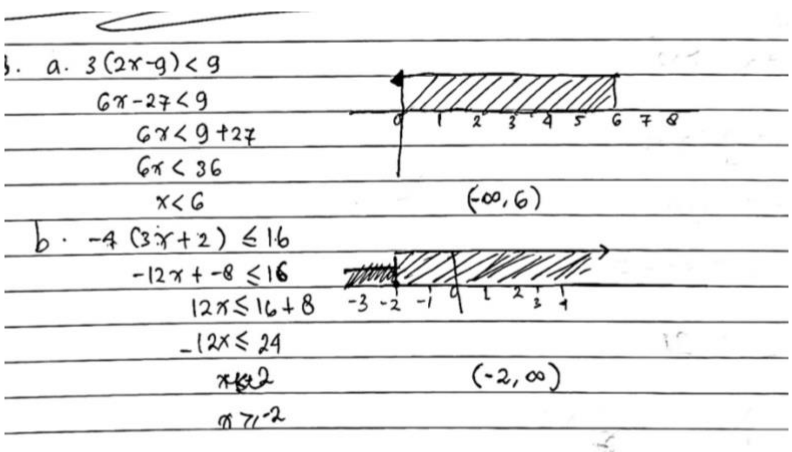


Figure 3: Student's Work in Interpreting Solutions

Researcher : Please recheck; is it correct?

Respondent: Yes

Researcher : In your opinion, what is the set of solutions in interval notation?

Respondent: Writing the set of solutions in brackets

Researcher : I see, then what do a $(-\infty, 6)$, and b $(-2, \infty)$ mean?

Respondent: It means that the value of x is between what I wrote

Researcher : Then, there is no difference between the sign of "<" and "≥", isn't it?

Respondent: Hmm, yes.

The interview excerpt describes the absence of semantic and symbolic meaning of the inequality so that it is one of the sources of student errors in interpreting the solution. Most students view that the set of solutions cannot be written in intervals or finite sets; also, writing closed intervals like $[-1, \infty)$. The absence of semantic and symbolic meanings of inequalities also causes students to make errors and even be unable to change the context of the problem into a mathematical model or form of inequality because they do not understand well the expressions that can represent the inequality notation "<", ">", "≤", and "≥". As seen in Task 3, students were asked to explain all the values of x with the distance of 4 from number 5. Students must at least be able to write the form of the inequality to get the correct solution. However, most students could not do it, as seen in Figure 4.

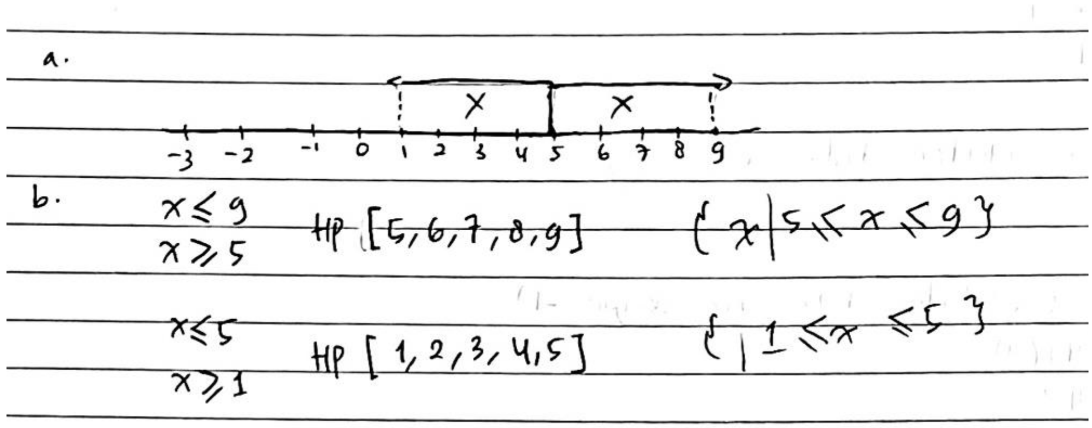


Figure 4: Students' Limitations in Changing the Context of Problems to Inequality

Based on students' understanding, "$x \leq 9$", "$x \geq 5$" and "$x \leq 5$", "$x \geq 1$" are correct forms of inequality to explain all values of x with the distance of 4 from number 5. Another type of student error in completing Task 1 is the generalization of inequality rules. Students tend to think that the rules of the inequality concepts apply to all forms of inequality, as illustrated in Figure 5.

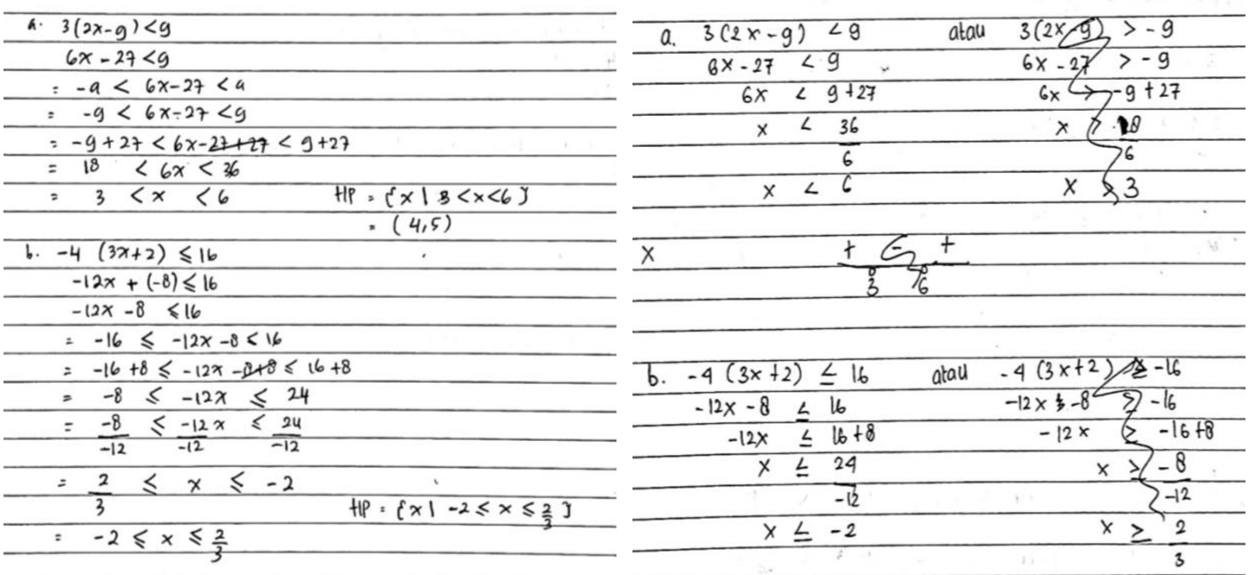


Figure 5: Errors in Generalizing the Inequality Rules

DISCUSSION AND CONCLUSIONS

The students' results showed their difficulty interpreting mathematical concepts and processes related to inequalities. The difficulties and errors found in students did not arise by chance but rather from a stable conceptual framework of students based on their previous knowledge. Radatz (1980) explained that students' errors in mathematics education are not only the result of ignorance, situational accident, insecurity, carelessness, or unique condition, as was assumed in behavioristic educational theory, but also are determined causally, systematically, persistently. It will last for some time unless there is pedagogically action from adults (teachers).

There are some pivotal aspects highlighted in this paper. Many students do not understand the concept of inequality. Most of them have not found a difference between the concept of inequality and equality. Based on students' understanding, the difference is only between the symbols. For example, the symbol "$=$" is for equality, and one of the symbols "$<$", "$>$", "$\leq$", or "$\geq$" is for inequality. These symbols have no semantic meaning for the students because they are used only as a liaison between two inequality members. In understanding the rules of inequality, students tend to view inequality as the same as equality, and for them, this is not an error. This fact is in line with Symonds (1922) that students do not see it as a mistake because they do not understand the symbol's meaning and do not understand its significance.

As an illustration, in solving the inequality of $-4(3x+2) \leq 16$, the students did not make mistake arithmetically since the results given by most of the students were $x \leq -2$ and some even wrote $x = -2$. This mistake is due to the limitations of students in understanding the concept of changing the inequality symbols when an inequality is multiplied or divided by a negative number. As explained earlier that understanding the logic of inequality should allow students to gain a deeper comprehension of the concept of inequality rather than just memorizing the rules. Related to the previous inequality, an understanding of the change in inequality symbol could be given in the following way $-4(3x+2) \leq 16$; $-12x - 8 \leq 16$ (add both side by (8)); $-12x - 8 + 8 \leq 16 + 8$; $-12x \leq 24$ (add both side by $(12x)$); $-12x + 12x \leq 24 + 12x$; $0 \leq 24 + 12x$ (add both side by (-24)); $0 + (-24) \leq 24 + (-24) + 12x$; $-24 \leq 12x$ (multiply both sides by $(\frac{1}{12})$); $-24\left(\frac{1}{12}\right) \leq 12x\left(\frac{1}{12}\right)$; $-2 \leq x$ is equal to $x \geq -2$.

Another method based on Nebesniak (2012) is that besides focusing on procedures and computations, a teacher must include conceptual comprehension related to prior knowledge while encouraging students' understanding and ability to think mathematically. In this case, to consistently solve inequalities correctly, students need to understand the reason behind the rule. For example, Nebesniak mentioned that the teacher could start the lesson with a correct statement $4 < 6$, draw the two numbers on a number line, and discuss what will happen to the statements and graphs if positive numbers are added to both sides of the inequality. The discussion continues by adding the negative numbers on both sides and subtracting the positive and negative numbers.

The final stage is multiplying and dividing by positive and negative numbers into both sides and discussing the inequality symbol.

In addition, another reason for students' errors is because of the absence of inequality semantic and symbolic meaning. Students often see the symbol only as a link between two inequalities in interpreting the statement. Students mistakenly add or exclude values in the solutions without the inequality semantic and symbolic meaning. For example, in task 1, some students wrote the solution $x \leq -2$. They add -2 to their solution by writing open brackets like $(-2, \infty)$. This fact shows that students do not have an efficient semantic meaning of inequality such as "less than" or "less than and equals to".

To overcome this problem, Rubenstein and Thompson (2001) suggested that some math words need to be emphasized by the teacher to understand the semantic and symbolic meanings. As Usiskin (1996) stated, mathematical symbols are how we write mathematics and communicate mathematical meaning. Tent (2000) also explained that one way that can be done to increase students' semantic and symbolic meaning about inequalities is by reading an inequality in more than one way. For instance, $x < -2$ means x is smaller than -2, x is not greater than -2 or equal to -2, x is neither greater than -2 nor equal to -2.

Students' errors due to the absence of inequality semantic and symbolic meaning are also related to their difficulties in interpreting solutions. They have limitations in interpreting the solution whether to use (1) set notation, (2) number line, or (3) interval notation. As an illustration, in completing task 3, students were required to change the context of the problem into a mathematical model (inequality form) to interpret the solution correctly. To interpret the solution correctly, students must understand that the inequality symbol "less than" has a different meaning from "less than or equal to". Furthermore, dealing with numbers and algebra gives students a semiotic challenge because symbols act as processes and concepts (Tall, 2008).

Mathematical notations or symbols create the basis of mathematical communication; therefore, students must understand them and relate them to meaning. This statement is due to the diversity of symbols and their meanings in different contexts (Mutodi & Mosimege, 2021). Symbol load, unfamiliarity, and greater density, according to the study, may confront students with difficulties when learning mathematics. Extensive research on secondary students' understanding of mathematical symbols revealed that symbols' clarity and abstraction could be a learning obstacle. This study adds to that debate by highlighting the time required to master certain symbols and the lack of appropriate instructional strategies to promote competence with mathematical symbols. Mitigating the obstacles of mathematical symbolism is still a complex topic for teachers to incorporate into their lessons (Bardini & Pierce, 2015).

Another student's error in solving inequalities is the generalization of the inequality rules. Students assume that every rule in the concept of inequality can be used in all forms of inequality. As shown in Figure 5, to solve the inequality "$3(2x-9) < 9$" " students used the inequality rule "if $x \in R$,

$a \in R$, and $a > 0$, then $x < a$, if and only if $-a < x < a$", the inequality "$3(2x-9) < 9$" in task 1 asked students to determine all possible sets of solutions in interval notation so that the statement "$3(2x-9) < 9$" is true. Meanwhile, applying the inequality rule "$-a < x < a$" to the absolute value of inequalities will produce the wrong solution. In the absolute value concept, a number x can be considered its distance from zero on the number line, regardless of its direction.

Overall, at least two research findings are based on the investigation results of students' experiences in solving inequality problems. First, students make mistakes in completing the task of inequalities such as inequalities rules, the absence of inequalities semantic and symbolic meanings, interpreting solutions, and generalizing the inequality rules. Some research results also explained that there are difficulties for students in solving problems related to the concept of inequality (such as Abu Mokh, Othman, & Shahbari (2019); Almog & Ilany (2012); Konnova, Lipagina, Postovalova, Rylov, & Stepanyan (2019); Lo & Hew (2020); Makonye & Shingirayi (2014); Switzer (2014)).

Second, based on the analysis results, it was found that there were epistemological obstacles in the concept of inequality, which are reflected in the limitations of students in understanding and interpreting inequality symbols. The limited meaning of this inequality symbol is a recurring error in solving the inequality concept in this study. For example, students only interpreted the notation "$\leq$" as something smaller than or equal. Thus, students cannot use these inequality notations correctly when different situations arise.

As seen in task 3, there was given a statement, "Explain all values of x with the distance of 4 from number 5". In this context, students faced difficulty in solving it. This difficulty is due to the limitation of students in understanding and interpreting the inequality symbol. The word "within 4" is a keyword that can be represented by the inequality symbol "$\leq$". Since x is with the distance of 4 from number 5, it means that x is less than or equal to 4, so with the distance from x to 5 can be represented as $|x-5|$ then$|x-5| \leq 4$. Furthermore, epistemological obstacles in the concept of inequality are also found in another research, (such as Bicer et al., 2014; Blanco & Garrote, 2007b; Makonye & Shingirayi, 2014; Nyikahadzoyi, Mapuwei, & Chinyoka, 2013; Tamba & Saragih, 2020)

This study reveals the value of novelty related to the concept of inequality, namely the existence of epistemological obstacles that cause students' errors. Students' errors in solving inequality occur because of their limitations in understanding and interpreting inequality symbols. According to the perspective of "epistemological obstacles", one of the most important goals of historical studies is to find problems and systems of constraints (situation fundamentals) that must be analyzed to understand existing knowledge where the findings are related to the solution of these problems. In some languages, the word inequality can assume two different versions. For example, in French, these words are *inégalité* (in Italian: *disuguaglianza*) and *inéquation* (*disequazione*). Concerning these words, the differences will be summarized as follows: an *inéquation* is a mathematical statement of an *inégalité*. Both from a logical point of view and an educational point of view, there is a big difference between inequalities like "$x + 2 < 3$" and inequalities like "$1 + 2 < 5$", the epistemological status is different (Bagni, 2005).

Besides the limitations of students in understanding and interpreting inequality notation, we see that students' knowledge of prerequisites before arriving at the concept of inequality also acts as an epistemological obstacle. As an illustration, students' knowledge of the concept of numbers greatly contributes to understanding the concept of inequality comprehensively. Based on the facts of research findings, students often make mistakes in operating the results of inequalities that are multiplied or divided by negative numbers. This error can also be identified because students forget to change the inequality sign, but not a few of these errors are also because students do not understand the concept of a comprehensive number. The concept of numbers has an important role in helping students to understand more advanced mathematical ideas (Schröder et al., 2022), therefore the concept of numbers and the concepts associated with them are central to learning mathematics starting from elementary school to intermediate (ages 6 to 17 years) (Elias et al., 2020). Rips et al. (2008) explained that the concept of numbers plays an important role in many mathematical activities, for example, counting and arithmetic. In addition to its practical role, the concept of numbers also has a central place in mathematical theory.

Therefore, analyzing epistemological obstacles is not only focused on how the mistakes made by students, it is also necessary to explore how the historical-epistemological development of a concept is. Obstacles are something that cannot be separated from the learning process for students. So, reflection becomes important to overcome these obstacles in changing the learning model as important content in the learning process (Maknun et al., 2022). So that way we can help identify possible hidden models and suggest the construction of appropriate learning situations to overcome the obstacles found. In terms of the overall concept of inequality, and particularly as it pertains to solving algebraic inequalities in which a student must multiply or divide both sides of the inequality by a negative number, we can add a numerical explanation that uses a concrete example to help solidify the concept. For example, we know that $1 < 2$ (1 is to the left of 2 on the number line). If we multiply both sides by "-3," we would get $-3 < -6$ if we forget to "flip" the inequality sign. The issue is that -3 is to the right of -6 on the number line, which means we should now have $-3 > 6$. I've found this example tends to help students conceptualize this idea in a conceptual manner when using only numbers and the idea with the number line (numbers to the right are greater than numbers to the left).

An epistemological profile consists of an analysis that reveals the extent to which an individual's understanding of a concept involves elements from various stages of its historical development (Bachelard, 1938). So based on this it can be understood that in anticipating epistemological obstacles, it is necessary to conduct a thorough search of students' understanding related to the concepts they are learning with previous concepts related to the concept. So that when students are faced with different situations, they no longer have obstacles to handle them. Trouche (2016) explains that what is definitely important for learning mathematics, is the conceptual component of the schema, i.e., operational invariance: concept-in-action and theorem-in-action, i.e., implicit properties, which are not necessarily true but appear as relevant in certain situations domain. For example, when learning to multiply two integers, students usually develop a strong theorem in action as 'the product of two numbers is a number that is greater than the initial two numbers; and the powerful concept-in-action as 'multiplication is the engine for increasing numbers. Such operational invariants are relevant in a particular domain (which is the reason for their rejection) and turn into a bottleneck as soon as the mathematical context exceeds this domain. For example, when a positive integer is multiplied by a negative number, students tend to be confused about whether the product is greater or less than the initial number.

While doing a math task, a student may point out an error. Error is not only the effect of ignorance, uncertainty, and chance, but also the effect of previous knowledge which is interesting and successful, but is now exposed as wrong or irrelevant. Errors of this type are erratic and unpredictable. These errors can be identified by reviewing the results of student work. So, when making mistakes in understanding inequality notation in solving inequalities. Several questions arise (do students not understand the meaning of inequality notation? Do students consider inequality the same as equality? Do students not understand the nature of inequality? Does the error originate from students' prior knowledge?). Of course, students have reasons or ideas that support these answers. Students may not realize that what they are doing is wrong, because it makes sense to them. Errors are not always the effect of ignorance, uncertainty, or chance; they can result from an interesting and successful application of a piece of prior knowledge, but in other contexts exposed as errors or simply not adapted (Brousseau, 2002). In other words, students use concepts in certain contexts and apply them to other contexts (Brousseau, 2002). This is in line with the view of Modestou and Gagatsis (2007) that the obstacles of epistemological origin manifest in mistakes that are not made by chance and can be reproduced and persisted. Epistemological obstacles are characterized by their appearance both in the history of mathematics and in everyday mathematical activity.

The results showed four types of students' errors in solving inequalities: inequalities rules, the absence of inequalities semantic and symbolic meanings, interpreting solutions, and generalizations of the inequality rules. The concept of equality is still a reference for students in solving inequality problems. Then, the limitation of understanding and interpreting inequality symbol is the leading cause of failure to comprehensively understand the concept of inequality. Meaninglessness is also one of the main problems in dealing with inequality. For that reason, it is necessary to consciously pay attention to how the concept of inequalities is introduced to avoid learning inequalities being reduced to mere mechanical tasks. Each solution should enable students to understand the meaning of the process they follow to reach the correct solution of an inequality. Otherwise, the procedures that they learn will be a source of error.

In addition, this study has research limitations. Where this research only focuses on epistemological obstacles, which are one of two types of learning obstacels, namely ontogenic obstacles and didactic obstacles. So that students' obstacles to the concept of inequalities are only seen from the student's point of view and mathematical characteristics (the concept of inequalities) through the exploration of their experiences in learning the concept of inequalities starting from junior high school to high school. So that some of the information needed relating to student knowledge of the concept of inequalities cannot be explored optimally and thoroughly about past involvement with aspects of inequality. With this limitation, it is hoped that it can be improved in further research.

These findings contribute in several ways to our understanding of the epistemological obstacles faced by students in the concept of inequalities and provide a basis for knowing what material most lead to student misunderstandings. Some practical recommendations for educators and further researchers to follow up on these findings are (1) epistemological obstacles are interpreted as knowledge, so that when there are obstacles to students in understanding and interpreting a concept. The teacher should not ignore it, instead the teacher should use it to provide insight for the student to analyze from the outside the way he used to solve the problem to formulate the hypothesis that he has understood so far, and become aware of possible rival hypotheses; (2) providing a comprehensive understanding of the meaning of inequality notation is very important, so that when students are faced with different contexts and situations, they do not have difficulty applying it; (3) students' mastery of prerequisite knowledge really needs to be emphasized to make a better understanding in understanding the concept of inequalities.

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