A study on the effectiveness of some cognitive activities in teaching integrals in secondary school

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Abstract: Several studies have pointed out difficulties of various types encountered by students in both secondary and high schools to acquire the exact meaning of the integral and to master the ability to invest it correctly in different domains.

This contribution attempts to explore the effectiveness of some cognitive activities engaged in teaching integrals on the achievement of the objectives targeted in secondary school curriculum. For this purpose, a cognitivist analysis of the program is performed by focusing on the implementation of the changes of frames and the conversion of registers of semiotic representation during the treatment of this notion. To explore the effectiveness of these cognitive activities on learning the integrals, a sample of secondary school students were tested.

It emerges from this study that the quality of learning integrals of a real valued function and the formation of correct conceptions on this notion are highly correlated to the investment of this concept in various frames and different semiotic registers. In fact, many of the students tested were unable to perform certain algebraic tasks or to interpret correctly some integrals in familiar situations.

INTRODUCTION

The emergence of analysis results from the reflection on problems that preoccupied both mathematicians and physicists. The first ones, interested in geometric problems inherited from the Greek era, found difficult to transpose classical methods in geometry to situations newly generated by the algebraic manipulations invented by Descartes and Fermat. Physicists concerned with solving problems mainly related to mechanics, found no longer valid their approaches based on intuitive representations of certain concepts. These efforts led to the definition of new concepts, such as fluxions and fluents for Newton, differentials and integrals for Leibniz. Thus, a new field was forged known by the name of infinitesimal calculus. This new type of calculation represents the ancestor of modern analysis as it is defined in an abundant bibliography, as for example in Chauvat (1997) or Bloch (2000).

In this epistemological dynamic, the notion of integral has played a very important role, contributing to the development of several fields in mathematics and through its applications in several fields that fall under other disciplines. It has allowed the institutionalization and representation of geometric (length, area, etc.) and physical (speed, acceleration, etc.) quantities.

In secondary school (17-18 years old), the curricula give the mathematical field of analysis a large place. In this field, the notion of integral of a function has not ceased to be present as an essential course. The main objective is to provide the students the basic knowledge, on this notion, essential for generalizations or extensions required for higher education. A pragmatic reason behind this curricular choice is that the concept of integrals is very invested as a tool in the study of certain programmed phenomena in physics. This changing of the status in learning of integrals is also essential for good cognitive functioning (Douady, 1986). Indeed, the meaning of any notion can only be acquired through situations that invest it. In conceptual field theory (Vergnaud, 1990), all of these situations represent a main component in the characterization of a concept. But, to get different interpretations of an integral requires several registers of semiotic representations. For Duval (1993), the ability to perform manipulations in the required registers and conversions from one to the other is necessary for efficient learning.

Thus, it seems natural to deduce that the teaching of the integrals must take into account this inherent plurality in frames and in registers of semiotic representations. Indeed, the implementation of this change of frames favors some cognitive conditions necessary for the conceptualization of this mathematical knowledge by placing it in situations where the dialectic tool/object can arises (Douady, 1986) and to diversify its semiotic representations (Duval, 2005).

In the he present work, we are interested by the importance of the implementation of this plurality in teaching of integrals in secondary school, in terms of the formation of correct conceptions on this notion and the development of different mathematical capacities transferable in different situations. Thus, it will be a question of answering the problem of the effectiveness of the cognitive activities involved in teaching integrals in secondary school. Accordingly, the research questions will be formulated as follows:

1. What place do the cognitive activities of changing frames and converting registers of semiotic representation occupy in official orientations and in school textbooks with respect to the notion of integral?
2. What is the impact of the cognitive activities targeted in teaching of the integral on its learning?

Taking into account, the qualitative exploratory nature of the current study (Corbin & Strauss, 2007), a methodological protocol is implemented in accordance with this type. Thus, we will proceed to an analysis of the teaching program of the integrals starting with the official framework texts and then its implementation through the textbooks accredited by the Ministry of National Education. After, a test is administered to a group of students, in order to diagnose the impact of the cognitive activities engaged by the official program on learning the integrals.

The remainder of this paper is organized as follows. We begin by stating a literature overview on works related to teaching and learning integrals followed by a theoretical framework. Then, the methodology framework and the results obtained as well as the ensuing discussion are provided before presenting some conclusions.

LITERATURE REVIEW

In this section, we present a synthesis of some works that have focused on the teaching and learning of the integral. We would like to point out that the researches carried out in connection with this theme are relatively numerous and they concern secondary and higher schools.

In many empirical studies undertook by several authors, for example (Orton, 1983; Thomas & Hong, 1996; Huang, 2012; Hashemi et al., 2014) , it has been inferred that students at school and at university have fundamental difficulties in understanding the concept of integral. As a consequence of his literature review, González-Martin reported in his thesis work (González-martin, 2005) that although it is relatively easy to teach students techniques for calculating derivatives and integrals, there is a great difficulty to bring students truly into the field of analysis and achieving a satisfactory understanding of the concepts and methods of thought that are central in this mathematical area.

This situation of incapacity is expressed by Serhan, in the context of his research on definite integrals, by the fact that students were limited to the procedural knowledge manifested by the ability to calculate a definite integral, which is not the case when it comes to linking this concept with its different representations (Serhan, 2015, p. 15).

In his study, Jones (Jones, 2013, p. 138) considers that the difficulties encountered by students do not necessarily result from a lack of knowledge, but from the activation of cognitive resources that are less productive compared to others. Difficulties of cognitive type have been very recently evoked by Purnomo et al (Purnomo et al., 2022) in their analysis of problem solving process for integral calculus performed on a group of students of Mathematics Education in a university in Indonesia.

Beyond this cognitivist analysis, Belova (2006) refers to the courses given on integral calculus. Qualifying them classical, she considers that they only develop a very limited conceptual understanding of this notion. In this institutional dimension and in order to contribute to the improvement of the teaching of the notion of integral, Luong (2006) conducted a comparative study on the teaching of integrals in high school in France and in Vietnam. Thus, by referring to the anthropological theory of didactics (Chevallard, 1998), the questions of disparities in the two educational systems were studied between the defined integral scholarly knowledge and the knowledge to be taught, prescribed in the official texts, then between this one and what is produced by the teacher following some didactic choices made for teaching and finally between this latter type of knowledge and what has been taught effectively.

In his thesis project, Haddad (2012) studied the difficulties related to the notion of integral encountered by Tunisian students in secondary school and in the first-year university students. This study was conducted with the aim of offering an alternative teaching in secondary school. The author deduced that it is possible to implement a practice that takes into account the links between area, integral and primitive. In 2013, the same author interested by what students newly in university retain from the notion of integral taught in high school. He explored their ability to identify the integral and the area and the modes of validation of the results they produce (Haddad, 2013). His results are aligned with those of Orton (1983) who noted difficulties in articulating the notions of area, integral and primitive, with a group of students asked to calculate, when it is possible and to give a justification otherwise, the area of a part of the plan illustrated by a figure. Almost no student provided justifications for their answers to this question. It was also observed that the symbols used to write and calculate integrals were a source of difficulty.

This lack of in understanding deeply the integral was also noted by Ely (2017) after observing that students were unable to solve some situations slightly modified from those familiar to them. The same author explained this vulnerability by the fact that students have acquired only a procedural knowledge of integration in terms of techniques, without reaching an adequate conceptual knowledge of the inherent structures (Ely, 2017). This was also stated by Akrouti (2019) in his study on students' conceptions of the definite integral when entering university.

According to this concise review, we can conclude that the teaching of integrals in secondary school poses serious problems manifested by a deficiency in terms of acquiring the meaning of this concept, its modes of operation and the tools for validating learners' productions. Consequently, a natural question arises immediately: how can we overcome the issue of apprehension of the meaning and the transferability of the acquired knowledge by the students in integrals in other domains?

The present work represents an attempt to answer this question/issue. The following section offers a theoretical framework that will help us find relevant answers.

THEORETICAL FRAMEWORK

We begin with the point of view of Douady (1986), according to which *"Mathematical knowledge can be effectively constructed by bringing the tool-object dialectic into play within appropriate frames, by the use of problems responding to certain conditions."*

By object, Douady (1986) means *"the cultural object having its place in a larger edifice which is scholarly knowledge at a given moment, socially recognized"*. The word object then refers to the formal representation of a concept, to its cultural aspect. For the same author, a concept is qualified as a tool if *"we focus our interest on the use made of it to solve a problem."* (Douady, 1986).

A frame is *"made of the objects of a branch of mathematics, the relationships between the objects, their various possible formulations, and the mental images associated with these objects and their relationships."* (Douady, 1986).

The word frame is also used with a wide meaning. It can also refer to a field of knowledge that does not belong to mathematics. Changing frames is a way of obtaining various formulations of the same problem, which are not necessarily completely equivalent since each frame involves specific concepts. This diversity gives the possibility of new access to the difficulties encountered and the implementation of tools and techniques that were not essential in the original formulation.

In the teaching-learning process, this change of frames is intentionally implemented on the initiative of the teacher to achieve the tool-object dialectic in order to confer meaning on the mathematical objects targeted. From an epistemological point of view, this change is imposed by the origins of the problems that mathematics tries to solve. The notion of integral is a good illustration of this point. Indeed, the main idea behind its emergence is of geometric origin and the first attempts of calculation emanated from certain intuitive approaches that were common in the study of some phenomena in physics. Consequently, the meaning of this concept and its usefulness can only arise through its investment in situations from different disciplines. In other words, it is the tool status that contributes crucially to the formation of the meaning of a concept.

On the other hand, R. Duval distinguishes mental representations from semiotic representations (Duval, 1995). By the first, he designates all the mental images or conceptions that one may have about an object or a situation. In order to externalize these mental representations, it is necessary to have tools that perform this function. In this context, the same author (Duval, 1993) defines the registers of semiotic representations as being *"...productions constituted by the use of signs belonging to a system of representations which has its own constraints of significance and functioning, the set of these signs is called the register of semiotic representation"*.

This theory underlines the importance of the role played by semiotic representations in the manipulation of abstract objects since they make them visible and accessible. In mathematics, each object has many registers of semiotic representations, each of which provides partial access to the object it represents and allows certain operations to be performed on it. A semiotic system acquires the status of register of semiotic representations if it allows the following three fundamental cognitive activities. First one is the formation of a representation identifiable as a representing of a given register. This formation is subject to some convenient rules, specific to the system used, not only for reasons of communicability, but also for the feasibility of processing by the tools offered by this system. Second one is the transformation of one representation into another without changing the semiotic system. Final activity is the conversion of a representation into another register while retaining all or only part of the content of the initial representation. Conversion is therefore an activity external to the register of the original representation.

Duval (2005) considers that learning in mathematics cannot be dissociated from the activity of recognizing at least two representations of the same object.

The importance of representations in learning mathematics was revealed in many studies. Janvier (1987) considers that the use of representations in mathematical thinking is fundamental. Mainali (2021) recommended that instructional strategies should be focused on incorporating different modes of representation in order to meet the student's preferences for solution strategies.

In the case of the Riemann integral notion, for establishing the properties of integrals and in the case where formal proofs cannot be presented, geometric interpretations by the notion of area prove to be a didactic alternative which has the contribution of bringing the student to perceive the meaning of these properties and to attribute to them a certain legitimacy although it is intuitive. This interpretation is sometimes practiced by illustrations using graphical representations of functions. It is then a transition from the algebraic frame to the geometric and/or graphic frame.

The application of integrals by employing situations from physics or mathematics is another occasion where the change of frames is manifested. This diversification of frames in teaching or learning of integrals implies naturally the use of different registers of semiotic representations. The representation of the integral by the Newton-Leibniz formula is part of the symbolic register but the interpretations of this notion or its applications require representations in other registers. This is the case, among others, in evaluating a geometric or physical quantity or when in illustration by a curve.

It follows then that the conceptualization of the integral is strongly connected to operating this concept in different frames and registers.

METHODOLOGY FRAMEWORK

In this study, we limit ourselves to the case of the Moroccan curriculum. We focus on the teaching program of integrals in secondary school addressed to scientific and technology classes. Therefore, let us begin with an institutional analysis of the integral.

Institutional place of the integral

According to the framework document for the teaching of mathematics in Morocco (MEN, 2007), the notion of integral (of Riemann) is presented to students in the science and technology classes of secondary school in a chapter entitled integral calculus. The integral of a continuous function is defined by the Newton-Leibniz formula, $\int_a^b f(x)dx = F(b) - F(a)$ where F stands for a primitive of the function f on the interval $[a, b]$. This last concept is a prerequisite for students because it is introduced directly after dealing with the differentiability of a function.

In parallel to these epistemological and didactic choices, the following guidelines are stated:

- to present through examples of some simple functions, the link between the integral of a function over an interval $[a, b]$ and the area of a domain of the plane bounded by the curve of a function f and the lines of equations $x = a$ and $x = b$ and the abscissa axis,
- to restrict evaluating an integral to integration by parts and to the direct use of primitive functions.
- no proof should be presented for the properties of integrals,

- to invest the integrals in situations from physics or mathematics.

The details of the programmed content and the targeted capacities to be built on the learners are presented in Table 1 (MEN, 2007).

Contents of the program	Target capacities
- Integral of a continuous function on a segment. - Properties of the integral: Chasles relation, linearity, integral and order. - The mean value of a function on an interval. - Techniques for calculating integrals. - Calculation of areas and volumes.	- Calculation of the integral of a function by using one of two techniques. - Calculation of the area of a domain of the plane between two curves and two lines parallel to the ordinate axis. - Calculation of the volume of a solid of revolution generated by the rotation of the curve of a function around the abscissa axis.

Table 1: Integral Calculus Teaching Program

Officially, the implementation of this program is manifested by the production of textbooks, published only after an accreditation issued by the Ministry of National Education in accordance with some specific prescriptions drawn up for this purpose.

Methodology

In this subsection, we develop the methodology adopted to provide some answers to our problem and the related questions.

For the first question, which aims to explore the place of the activities of changing frames and converting registers of semiotic representation in official orientations and in school textbooks with respect to the notion of integral, we will analyse the official prescriptions at the light of the theoretical study carried out previously in this paper. It is a question of identifying the frames and the registers of semiotic representation where the activities are supposed to be practiced by the learners as stipulated by the official texts.

We will also undertake an analysis of the activities proposed in the textbooks. The analysis of textbooks is justified by their principal role in the implementation of program prescriptions.

The activities analysed are chosen from mathematics textbooks for science and technology classes, accredited by the Ministry of National Education. There are two textbooks whose data are recorded in the following table 2:

Textbooks	Ministerial accreditation number	ISBN
Fi Rihab Ryadiat	09CB 21207	9954-436-82-0

Al Wadih fi Ryadiat	09CB 21307	9981-30-179-5

Table 2: Identification of textbooks analyzed

In both textbooks, the chapter on integrals is organized as follows: the first part is devoted to preparatory activities for the introduction of new knowledge, the second is dedicated to presenting the main content stipulated by the programs and the last part offers some training activities aiming to consolidate and to invest the new learning.

Our analysis, which focuses on the ten preparatory activities and the 123 training activities proposed in the two textbooks, aims to explore the cognitive activities of frame changes mobilized in the two types of activity. As a result, the preparatory and training activities will be analysed according to the frames and registers of semiotic representations of formulating the activity and those required to produce the expected responses to the instructions.

The analysis will be carried out on the basis of a grid, in Table 3, which gives indicators that allow us to identify in official texts or in the activities of textbooks the frames and registers of semiotic representations involved. The development of this grid was based on the various studies carried out during this research.

	Types of frames or registers	Indicators to identify the frame or the register involved in the activity
Involved frame	Algebraic	The use of algebraic formulas for processing integrals.
	Geometric	The use of geometric concepts to illustrate, interpret or invest integrals.
	Graphic	The use of curves to identify or illustrate an integral.
	Discipline other than mathematics	Physics, biology, …
Involved semiotic register	Graphic register	Formulation of data via curves.
	Algebraic register	The use of function symbols and/or expressions.
	Geometric register	Formulation of data by geometric figures.

Table 3 : Analysis grid of activities according to frames and registers

To answer the second question, a test targeting different abilities related to integral calculus is administered (in French) during the 2021-2022 school year, to a group of students of scientific classes. The test (Appendix) consists of seven questions is analysed as presented in Table 4.

Questions	Abilities assessed
Q 1.	The use of the value of the integral $\int_a^b f(x)\,dx$ to estimate the increase of a primitive of a function f on the closed interval $[a, b]$.
Q 2.	Investment of the independence of the value of the integral of the chosen primitive function.
Q 3.	Determination of the sign of an integral.
Q 4.	Mastering of the status of the independent variable $(x, z, \ldots)$ in computing an integral.
Q 5.	Representation of an area by an integral.
Q 6.	Representation of the volume of a solid by an integral.
Q 7.	Investment of the integral for the estimation of a quantity in physics.

Table 4 : Analysis of the administered test

Before the administration of the test and for validation purposes, four mathematics teachers working in secondary school and three researchers in mathematics education were consulted on the test items. After some minor modifications, the test was administered during the first two weeks of May 2022 to students in the aimed classes, who had taken advantage of lessons on the knowledge involved in the test.

Given the constraints imposed by the COVID 19 pandemic, the test was only distributed in 5 secondary schools of the Regional Academy of Education and Training of Rabat-Sale-Kenitra. The total number of participants is 75.

RESULTS

Analysis of the official program

Taking into account the epistemological and didactic choices of the curriculum (MEN, 2007), we deduce the results cited below in connection with the activities of changing frames and registers of semiotic representations.

The naming of the chapter by integral calculus refers, intentionally or not, rather to a field of mathematics where calculations dominate with specific tools and techniques than to a new concept which has its own meaning and which serves in particular to model some geometric or physical objects. Moreover, the change of frames is restricted to calculation of area and volume and in very particular cases.

Regarding the choice to introduce the integral by the Newton-Leibniz formula $\int_a^b f(x)dx = F(b) - F(a)$, which comes under the algebraic frame, we have the following:

- Rather, it attributes legitimacy to the presence of the concept of primitive function in the program without showing the student concretely the need to learn the new concept.

- From a didactic point of view, although this choice makes possible to produce proofs for certain properties accessible by the student in secondary school, it is not invested to avoid excessive recourse to intuition in validating knowledge.

The definition represents the integral of a continuous function f on the interval $[a, b]$ by the symbol $\int_a^b f(x)dx$ bound by an equality relation to the notation $[F(x)]_a^b$, then substituted immediately by the quantity $F(b) - F(a)$, with F is a primitive function of f on $[a, b]$. This generates a real confusion on the exact object referred to in the definition: is it the integral concept, its representation by $\int_a^b f(x)dx$ or the symbol $[F(x)]_a^b$?

We would also like to mention that the official prescriptions (MEN, 2007) do not present any indication on the cognitive importance of the change of frames or registers in the practice of teaching the integral.

Analysis of textbook activities

Using the indicators explained in the analysis grid cited in Table 3, we classified the preparatory and training activities according to the frames or registers of semiotic representations implemented in the formulation of the activity or required to respond to instructions. The results obtained are registered in table 5.

	Frame or register used for formulation of the activity		Frame or register necessary for the formulation of the requested response	
	Preparatory activities	Training activities	Preparatory activities	Training activities
Algebraic	8	118	9	116
Geometric	4	4	0	3
Graphic	3	8	1	5
Discipline other than mathematics	0	6	0	5
Graphic register	4	8	1	5
Algebraic register	18	123	19	116
Geometric register	3	4	0	3

Table 5 : Classification of textbook activities according to frames and registers

Test' results

It is important to state at the beginning that reliability of the test was performed. Using the XLSTAT 2022 to compute Cronbach's α, we found that $\alpha = 0.851$. It is considered that the reliability is acceptable when α is equal or higher than 0.7 (Taber, 2018).

In table 6 below, we present the number of students who succeeded in producing answers and the numbers of correct productions. We point out that for each question, we have considered the answer to be correct when the student manages to produce the object requested in the instruction (value, sign, expression, etc.) and by explaining the approach he has used.

Question	Q1	Q2	Q3	Q4	Q5	Q6	Q7
Number of students who provided an answer	53	63	66	37	35	19	19
Number of correct answers	28	17	12	5	17	2	9

Table 6 : Number of correct answers per question

In addition to these quantitative results, the Table 7 presents some errors committed by the students in each of the questions of the test.

Question	Errors observed in the responses
Q 1.	- Since $\int_1^3 f(x)dx = 7$ then $f(x) = 7$. So, $F(x) = 7x$ and $F(3) - F(1) = 14$
Q 2.	- $G(3) - G(2) = 1/3$. - $G(2) - G(3) = 2x - 3x = -x$. - $G(2) - G(3) = G(-1)$.
Q 3.	- J is negative because $(x - 3)$ is negative and ln (x) is positive on $[1; 2]$. - The sign of J is the one of $(x - 3)$.
Q 4.	- The response is yes without any proof. - No relation between the two integrals. - $\int_a^b f(x)dx = [F(x)]_a^b$ and $\int_b^a f(z)dz = [F(z)]_b^a$ but no relation was provided. - $\int_b^a f(x)dx = -\int_b^a f(z)dz$.
Q 5.	- $\int_0^1 f(x)dy$. - $A = \frac{1}{b-a} \int_0^1 f(x)dx$.
Q 6.	- $\mathrm{V} = \int_4^0 \pi(f(x))^2$. - $\mathrm{V} = \int_1^7 \pi(f(x))^2$. - $\mathrm{V} = \int_{-2}^2 y - f(x)$.
Q 7.	- The average speed is $\int_1^7 \mathrm{v}(t)dt$. - The average speed is $\frac{\mathrm{V}(7)-\mathrm{V}(1)}{7-1}$.

Table 7: A list of some errors made by students

DISCUSSION

The analysis of the official program on the teaching of integrals allows us to identify several results. First, the choice to present the integral of a function by the Newton-Leibniz formula prompts us to wonder if the learner acquires the meaning of the integral or if he appropriates the formula for his calculation. Theoretically, the answer is clear. In fact, only problem situations that highlight both the usefulness and the usability of a mathematical object are effective in acquiring its meaning. On the other hand, presenting the concept of integral of a function by the Newton-Leibniz formula is likely to create in the student an ambiguity in the defined object. In other words, it is unclear whether the expression $F(b) - F(a)$ represents for him a definition of the symbol $\int_a^b f(x)dx$ or of the concept integral of the function f on the interval $[a, b]$. It seems then that the course on integrals represents a continuation on the course on primitives where this last notion is invested.

In the programmed content and concerning the cognitive activity involved, there is a dominance of calculus which also extends to situations of application of integrals. Indeed, the official prescriptions aim at the numerical estimation of the surface and the volume by a mechanical restitution of certain formulas, and not to make use of the concept of integral like a tool of modelling situations resulting from the geometry or the physics or possibly another domain.

In connection with the knowledge targeted in the program, it seems important to point out that the question of integrability is not mentioned at all. Learners are completely exempt from asking themselves this natural and fundamental question in mathematics related to the existence of the object studied. In principle, this lack of awareness to wonder about the existence of the integral is unexplained since it is attached to the concept of primitive function which has continuity as unique sufficient condition presented in the program for its existence. Faced with such a didactic choice, the following important question arises: can the student recognize whether a function is integrable from its graphical representation? This question will be put into perspective for a future work.

Concerning the interpretation of integrals, the interest is particularly focused on area and volume. This questions the fact of restricting, in the mathematics program, only to the geometric frame knowing that physics offers a whole variety of situations likely to improve the perception of the notion of integrals.

The analysis of preparatory activities has clearly shown that they are mainly formulated in the algebraic frame and the use of another frame does not appear to be an objective in itself, to promote the formation of another mental or even semiotic representation of the integral. In addition, the change of frames is only carried out in one direction. Indeed, the unique frame required to produce the answers to the instructions of the activities is the algebraic one. It is also clear when we see that no situation from another disciplinary field is suggested among the preparatory activities. We can therefore conclude that the frames used in these activities do not take into account the issue of diversifying the cognitive activities of the student.

The dominance of the algebraic frame in this type of activity, essentially intended for the formation of mental representations of integrals among students, has another disadvantage for their semiotic practice. The only register involved is the algebraic one, manifested by the use of symbols and expressions since they are necessary for the calculation. This fits well with the algebraic frame used. In fact, the formulation of the activities or the expected answers only requires the exercise of computational tasks. Thus, no diversification in terms of semiotic representations is required in these activities.

The dominance of the algebraic frame extends widely to training activities. Basically, these activities aim to make learners acquire automatism in the use of techniques or procedures and to implement the tool aspect of newly learned mathematical objects. The achievement of these two cognitive objectives can only be the result of students' work in situations emanating from various frames and registers where the activities of change are practiced in all directions. However, the analysis of the items of this last type of activity reveals that apart from the mechanical use of algebraic properties of integrals, no result is the fruit of a specific thought to the concept of integral. In fact, the calculation of integrals, especially when it comes to calculating area or volume, is dominated by the determination of primitive functions and the techniques behind them. Questions on bounding integrals are solvable by the properties of the order in the set of real numbers or numerical functions.

From the analysis of the prescribed program and of the preparation and training activities proposed in the textbooks, it appears that access to the meaning of the integral and its properties is hampered by the abuse of computational tasks which do not allow to vary the cognitive activity required of the student and to develop his capacities in mathematical languages.

The analysis of the students' answers to the test reveals a clear deficiency in terms of restitution of knowledge. The number of correct answers provided for questions Q1 and Q2 related to the relationship between the integral and the notion of primitive functions is too unsatisfactory since the program is quite articulated on this notion. Furthermore, errors revealed in Table 7 concerning Q1 and Q2 show that some students confuse the integral of a function and its primitives.

The inability of the majority of students tested to answer correctly to question Q3 although it only requires the execution of a two-step procedure, determination of the sign of the integrand function and comparison of the bounds of the integral, is completely understandable since the program and the activities that implement it are too centered on the mechanical calculation of the value of an integral by the Newton-Leibniz formula or by integration by parts while the properties of the new notion are little discussed if we dare not say marginalized. Beyond the fact that some students consider ln x to be positive accordingly to the sign of the variable x, they do not take the order of the bounds into account to determine the sign of the integral.

According to the low number of students who answered the question Q4 and the errors made in this question, we can deduce that the symbolic representation of the integral is not mastered.

For the investment of the integrals in the situations Q5 and Q6 resulting from the geometry, incapacities are well observed. The transition from the geometric frame to the algebraic one then poses difficulties for the students. This last statement is clearly observed from the obtained results about question Q5 where evaluating an area is targeted. The answers to question Q6 presented in Table 7 show also that the students use the integral although it is not necessary.

Finally, from question Q7 where it is proposed to evaluate a mean value in a contextualized situation, it turns out that the students did not master one of the very interesting interpretations of the integral.

Following the results of our analysis and the ensuing discussion, some conclusions emerge. They will be the focus of the following last section.

CONCLUSION

This study starts from the point of view according to which the access of a learner to the meaning of mathematical concepts necessarily passes through their treatment in several frames put into play intentionally. This condition aims at the relevant formation of mental representations on a given concept to facilitate its treatment in different frames. This position motivated us to question the effectiveness of the cognitive activities involved in the teaching of integrals in secondary school on the learning and the investment of this notion.

The choice of the notion of integrals was first motivated by its importance in mathematics and in other disciplines. After their emergence to solve classical problems of geometry or physics, they quickly used other concepts, such as limits and functions, to gain new momentum in the field of mathematics, and then by its place in secondary and higher education mathematics curricula.

To determine what place occupies the cognitive activities of change of frames and conversion of registers of semiotic representation in the official orientations and in the textbooks relatively to the concept of integral, an analysis of the texts of framing in vigour has been achieved. Such an analysis, supported by our bibliographical review, allows us to conclude that there is a total domination of the computational processing of integrals which in turn calls on semiotic representations mainly in the algebraic register. The same conclusion was deduced from the analysis of textbooks. In the preparatory activities proposed therein, it has been revealed that the computational activity takes precedence. And as a result, the algebraic frame is the most favored to bring into play since the manipulations required for the determination of the primitive functions are algebraic. It follows that the choice of preparatory activities is completely subject to the introduction of the integral by the Newton-Leibniz formula. It is a restrictive choice, in the sense that it does not provide a set of frames and registers of varied semiotic representations.

The result is then clear, these activities do not contribute to build an exact conceptions on the integrals. This dysfunction is accentuated by the proposals for training activities that aim in principle, among others, to reinvest knowledge in other contexts so that it acquires a cultural status. From the analysis of this type of activity, it follows that the most dominant ability is the determination of the numerical value of the integral. This statement is also true for the calculation of areas or volumes where the only task assigned to the student is the use of formulas, simple to reproduce, for domains in the plan or in space described in the same way in all activities.

Faced with this assessment and as an answer to the second question, it is difficult to confirm that the proposed training activities contribute to master the sense of the integral or to develop the capacity to invest knowledge on this notion in other disciplines. The results deduced from the test administered confirmed this. Indeed, apparent difficulties in applying integrals in the estimation of certain quantities were revealed.

Overall, after this analysis which focused on the three levels of the curriculum, the design, the implementation and what is supposed to be learned, it turns out that the lack of intentional engagement in the teaching of the integrals of the situations which impose a diversification of frames for processing integrals and of registers to represent it semiotically presents didactic defects manifested by the abusive recourse to computational tasks. This weighs cognitively, access to the meaning of the integral of a function and the acquisition of the ability to use it is to be considered with delicacy.

At the end of this work, we would like to point out that during this work, other questions arose. We have not been interested in them because they go beyond the objectives of this article. But, they have generated great motivation for future work articulated on the following two main questions:

- What is the impact of teaching the definite integral centered on a graphical approach? In particular, can the learner recognize the properties of a definite integral from its representative curve?
- What place occupies the validation and how it is performed in the students' productions about integrals?

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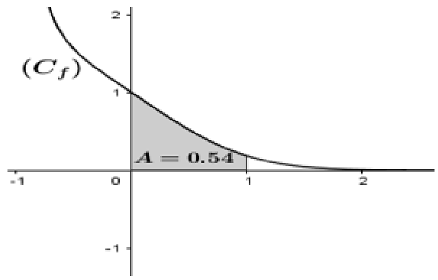
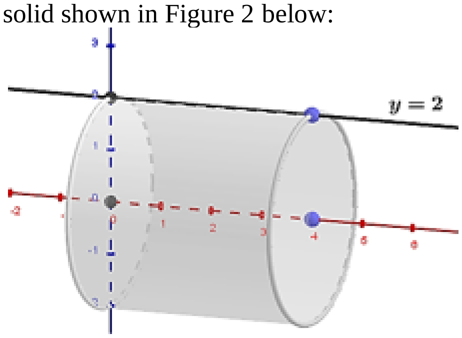
Appendix: Test

Q 1.	Let f be a continuous real function on $[1,3]$ such that $\int_1^3 f(x)\,dx = 7$. F denotes a primitive function of f on $[1,3]$. Evaluate $F(3) - F(1)$.
Q 2.	Let f be a continuous function defined on [2;3] such that $\int_2^3 f(t)\,dt = 1/3$ and G a primitive function of f on $[2;3]$. Give the exact value of $G(2) - G(3)$.
Q 3.	What is the sign of the integral $J = \int_2^1 (x-3)\ln(x)\,dx$? Justify the answer.
Q 4.	f is a continuous function defined on $[a,b]$. Is there any relationship between the following two integrals: $\int_a^b f(x)dx$ et $\int_b^a f(z)dz$? If yes, precise it?
Q 5.	Let f be the function represented by the curve (C_f) in an orthonormal frame $(o;\vec{\imath};\vec{\jmath})$. A is the area of the domain of the plane colored in gray (Figure 1). Express by an integral the area A and give the value of this integral.  Figure 1
Q 6.	Calculate the volume of the solid shown in Figure 2 below:  Figure 2
Q 7.	Give the value of the average speed between the instants $t_1 = 1$ and $t_2 = 7$ of a moving body whose instantaneous speed is $v(t) = 4t + 3$.