Concept of Polygon: Case Study of Elementary Students' Difficulties

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Abstract: This study aims to determine students' difficulties in understanding the concept of polygons in elementary schools. This research is qualitative research with a case study method. The subjects of this study were 30 grade 5 elementary school students from two different schools in the city of Bandung. The instruments used are test and non-test. The technique of tests is asking some questions about polygons, while the non-test is in the form of interviews. The data collected were analyzed using the three stages of the Miles and Huberman model, including reduction, presentation, and conclusion. The findings in this study indicate that students have difficulty understanding the concept of polygons, namely difficulties in identifying polygons properties, polygons rules and regulations, and determining polygons' names. One of the things that teachers can do to follow up on the problems from these findings is to design learning based on didactic situations that are appropriate and according to the problem faced by students.

INTRODUCTION

Geometry is an important material widely used in various disciplines (Loc et al., 2017). By studying geometry, students can solve various problems in everyday life from different perspectives, build relationships and use geometric representations to simplify abstract concepts (Biber et al., 2018; Filiz & Gür, 2021; Jones, 2002; Sopany & Rahayu, 2019). One of the goals of learning geometry for students is to have basic 21st-century skills, namely reasoning, problem-solving, and critical thinking skills (Erşen et al., 2021; Herbst et al., 2017). Geometry lessons have been taught from kindergarten to university. Geometry learning is very well taught early because students consistently interpret geometric shapes based on how they move their bodies (Douglas H. Clements et al., 2004). Thus, an understanding of the basic concepts of geometry has been instilled in students from an early age (France, 2004; Hallowell et al., 2015). Learning geometry starts at an early age at the kindergarten level, where students perceive the differences between geometric shapes by observing the objects they see in the environment and trying to find aspects of the similarities between them. However, as they age, they continue to study geometry at a higher level from a view of induction and deduction. Students can

experience various errors and misunderstandings in understanding geometric material during this process.

Geometry concepts must be taught hierarchically and sequentially to avoid mistakes and misunderstandings (Filiz & Gür, 2021). Because if students cannot understand the concepts of geometries well at the elementary level, there are critical problems in studying geometry at the next level. Geometry is one of the most challenging materials for students to master (Aksu, 2013; Anggraini et al., 2021; Fajari, 2020; Gal & Linchevski, 2010; Rogers, 1995). That is shown by Indonesian students' achievements in geometry from elementary school until universities are still low (Alawiyah et al., 2018; Hartono, 2020; Puspasari et al., 2015; Sholihah & Afriansyah, 2017). Geometry is considered a complex material to understand because geometry's characteristics require visual abilities or imagination and high analytical skills to understand unreal objects.

In contrast, elementary school students are at the concrete operational stage and must use concrete objects to understand something (Fajari, 2020). One of the geometry materials taught in elementary schools is polygons. Polygon material is one of the basic materials in geometry. One of the topics that must be understood in this material is the context of elements and properties of polygons, so if students still experience limitations in understanding this material, it will be an obstacle to students when understanding and using concepts to solve other geometric problems. Several studies on understanding the concept of polygons reveal the fact that the level of students' geometric perception is not at the expected level as in the quadrilateral material, which is considered one of the most problematic materials for students (Ayvaz et al., 2017; Bernabeu et al., 2021; Biber). et al., 2018; Fernigil L. Colicol et.al., 2017). Among the problems often encountered are students having difficulty describing a shape based on its characteristics (Hidayat, 2019). That also impacts naming polygons, so students do not realize the hierarchical relationship between plane shapes (Fujita, 2012). In addition, students also have problems defining plane shapes (Fujita, 2012; Fujita & Jones, 2007).

Bernabeu et al. (2021) also researched the concept of polygons and the relationship between polygons in third-grade elementary school students. Focus on introducing polygons, the relationship between polygons, and giving reasons to state examples of polygons and non-examples of polygons. As for this study, the researcher analyzed the students' difficulties in understanding the polygon concept, which focused on understanding the concepts of polygons, regular and irregular polygons, and naming polygons in elementary schools. It also addresses the following research questions: What kinds are students' difficulties in mathematics who have learned polygons in geometry material? The hope is that this research can be a reference for teachers and other researchers in designing learning by minimizing the various problems in studying geometry, especially polygon material.

METHOD

This research uses a qualitative approach with a case study method. According to (Gall et al., 2009), the case study method is a method used to explain certain phenomena, whether in the form of processes, individuals, programs, and so on. Thus, the case study method can be used as an appropriate method to explore students' difficulties in understanding the concept of polygons in elementary schools.

The subjects in this study were fifth-grade elementary school students from as many as 30 students (aged 10 – 11 years) from two different schools in the city area. The details are as follows: 16 State Elementary School students in large schools and 14 State Elementary School students in small schools. The selection of student groups was because the polygon material had been given in the previous class. The instruments used include test and non-test instruments. The test instrument is in the form of questions related to the polygon concept, and non-test instrument is in the form of in-depth interviews to strengthen the data obtained.

The researcher traced the students' difficulties from the aspect of understanding the polygon concept by giving tests to 30 fifth-grade students who had studied polygon material. A total of 9 questions have been given to students related to the concept of polygons. These questions are designed to identify difficulties in students' understanding of polygon material related to understanding the concept of polygons. The questions given consisted of one question about recognizing polygonal shapes, one question about regular and irregular polygon shapes, and seven questions about names of plane shapes. The instruments given to students have gone through a qualitative validation process in the realm of material, construction, and language involving two mathematics education experts and two elementary school teachers. After the validation process, the instrument was declared suitable for use according to research needs.

Question Number	Criteria	Category
1	Have more than three sides	Understanding polygons
	Has an angle of more than three	
	Closed curve	
	All sides are line segments	
2	All sides are the same length	Understanding regular and irregular polygons
	All angles are the same	
3	Saying the name of the wake correctly and correctly	Mention the names of plane shapes/polygons
	Name the shapes based on the number of sides, such as triangles, rectangles, etc.	

Table 1: Criteria for polygon questions

Miles and Huberman models are used in analyzing the data of this study. These stages consist of data reduction, data presentation, and conclusion drawing (Miles, M. B. & Huberman, 1994). At

the data reduction stage, the researcher recorded student responses in answering questions related to polygons material. Furthermore, the data is presented starting by grouping the types of student responses based on their level of difficulty. In the last step, the researcher drew conclusions after going through the process of analyzing the difficulties experienced by students with polygonal material.

RESULTS

The analysis was carried out based on the component aspects of students' understanding of the characteristics of polygons. Next is to determine the percentage of students who have difficulty understanding polygons. The following shows the percentage of students who have difficulty answering the questions related to the polygon material.

Number of Questions	Percentage of correct answers (%)	Percentage of incorrect answers (%)
1	50	50
2	30	70
3.a.	76.7	23.3
3. b.	33.3	66.7
3. c.	40	60
3. d.	3.3	96.7
3. e.	26.7	73.3
3. f.	30	70
3. g.	13.3	86.7

Table 2: Percentage of student answers

Based on the table above, the details of the findings in this study consist of understanding the concept of polygons, the concepts of regular polygons and irregular polygons, and determining polygon names.

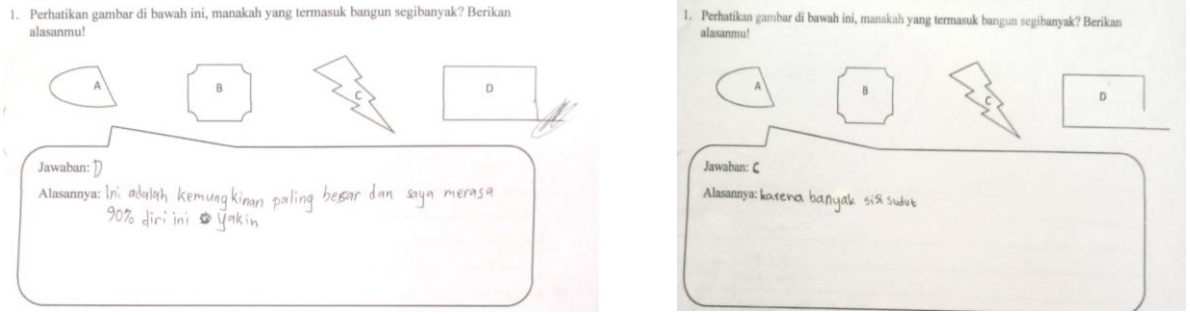
Difficulty in the Concept of Polygons

Students' understanding of the polygon concept has shown in question Number 1. Based on the percentage of students' answers, some students can show or determine the polygon shape from several other shapes that are not polygonal. However, out of 50% of the students, not all could mention the right reasons for determining polygons. The following are some of the reasons given by students for the correct answer choices.

Question Number 1 Correct answer choice C	Reason:
S1	because it has 7 polygons
S2	because it has many angles
S3	has seven sides
S4	because it has many sides
S5	because it has more angles
S6	because it has 11 polygons
S7	because it looks much and looks good.
S8	because it has 11 facets
S9	because it is different from the others
S10	The C-sided shape has more angles than the other side shape.
S11	because the arrangement is more numerous than the others
S12	reasons because they are different from each other.
S13	because the C has more facets than the others.
S14	because it has more squares.
S15	because the highest number is C.

Table 3: The reasons given by students for the correct answer choice for question number 1

Based on the table above, it can be seen that there are still many students who cannot mention all the characteristics of polygons. In this question, students do not understand the nature of polygons, so most students choose the correct answer but give the wrong reason. The reasons given by students only meet one of the characteristics or properties of polygonal shapes such as the nature of having many sides and many angles. As for the nature that the polygon must be a closed curve, none of the students mentioned it. So, for the choice of a shape in the form of an open curve, students take the initiative to draw a line or close the shape. An example of a student's answer can be seen in the following picture:



(a) (b)

Figure 1: (a) and (b) Example of student answers for question number 1

Subject 1 chose the shape "D" as shown in Figure 1(a) as a polygon by connecting the broken lines. The reasons given are:

Researcher: what about question number 1? Can you answer it?

Subject 1: yes Miss.

Researcher: what is asked in this question?

Subject 1: about polygons, Miss, asked to choose among the existing shapes, Miss.

Researcher: what answer did you choose?

Subject 1: "D" Miss.

Researcher: why is the shape said to be polygonal?

Subject 1: because this shape (while showing the picture on the question sheet) is neater and has many sides, Miss

Students assume that polygons are only quadrilaterals that are neat and orderly in shape. For students who choose the correct answer, namely "C" (as shown in Figure 1(b)). The responses given are:

Researcher: do you have difficulty answering question number one?

Subject 2: mmmm…, (while smiling)

Researcher: what answer did you choose?

Subject 2: "C" Miss

Researcher: why did you choose C?

Subject 2: because it has many sides, Miss…

Researcher: do you have any other reason?

Subject 2: no Miss

That shows the students still have limited knowledge about the polygon concept even though there are 50% of students chose the correct answer. However, the reason is not right, so the student assumes that the polygon shape is a shape that only has many sides without paying attention to the properties involved else. Students have difficulty determining the properties of polygons.

Difficulty in the Concept of Regular Polygons

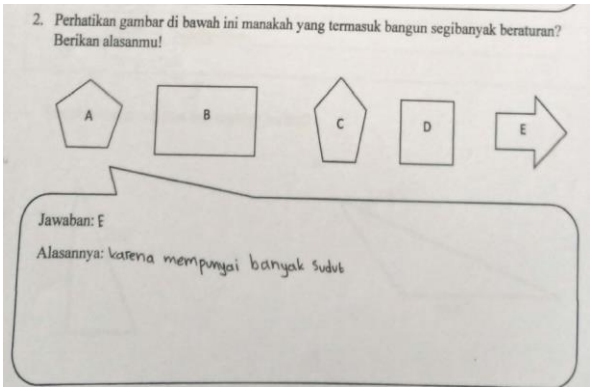
Regular polygons are part of the polygon material that students can understand after students can determine and distinguish polygon and non-polygonal shapes. To find out students'

understanding of the regular polygon can be seen in the student's ability to answer question number 2. The percentage of students who can answer the question by determining the regular polygon from several polygon shapes given is only 30% of 30 students. From this percentage, not all students gave the right and correct reasons for determining the answer. The following are some of the reasons given by students regarding choosing the correct answer:

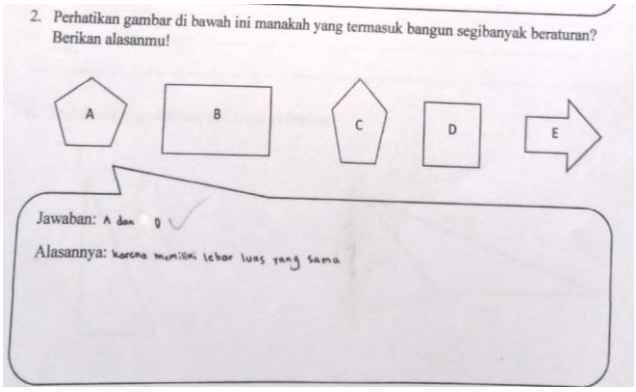
Number of Questions	Answer Choices and Student Reasons
2	Only choosing A, the reason being because it has 5 sides Only choosing A, the reason being because there are more sides. Only choose D, the reason is because it looks neat. Only choose D, the reason is because it has many polygons and is not too small. Choose A and D, the reason is Because they have the same width. Only choose A, the reason is Because the plane A has many sides but is not regular like E Only chose A, the reason is because it has five sides and is regular Only chose A, the reason is Because A has many regular sides Just chose A, the reason is Because it is neater and has many

Table 4: The reasons were given by students for the correct answer choice for question number 2

Based on the table above, students chose the correct answer but could not give a good reason to state that the shape was a regular polygon. In this question about regular polygons, most students (46.7%) stated the E shape as a polygon for various reasons. One of the reasons expressed by the students was because the shape E has many angles, this shows that students think that a regular polygon is only a shape that has many angles when compared to other shapes. The examples of student answers are presented as follows:



(a)



(b)

Figure 2: (a) and (b) Example of student answers for question number 2

Difficulty in Determining the Name of the Polygonal Shape

Students' ability to determine polygons' names is still having difficulties, as found in this study. The plane shapes that are presented to be named are rectangle, square, right triangle, rhombus, parallelogram, kite, and isosceles trapezoid. The results obtained are as follows:

Number of questions	Student Answers
3. a	Rectangle, rectangle.
3. b	Square, regular square, rectangular shape, square, square, rectangle, square, block, rectangle, square.
3. c	Equilateral triangle, oblique triangle, flat shape triangle, isosceles triangle, long triangle, triangle, right triangle, triangle, *bermuda* triangle (names given by students based on their own words), irregular triangle, inverted triangle, square, acute angle triangle, rectangle.
3. d	Square, quadrangle, quadrilateral, square 5, pentagon, quadrangle, cone, rhombus, rhombus, parallelogram, square kite, *lontong* (rhombus-shaped food name), dubus (The name given by the student is based on his own words because it sounds almost the same as a cube in Indonesian), kite quadrilateral, square triangle, square crystal.
3. e	Parallelogram, slanted square, oblique rectangle, oblique rectangle, parallelogram, irregular rectangle, parallelogram, slanted rectangle, rhombus, oblique rectangle, cube, flat shape, square side, square, oblique facet, facet, facet 4.
3. f	Kite shape, flat shape down, square, kite square, long cone, kite drawing, kite, kite, parallelogram, rectangular kites, kites, kites, rectangles, flat triangles, kites, quadrilaterals, crystal terms, kite squares, triangles, rectangles.
3. g	Trapezoid, square box, rectangular shape, flat shape, hollow square, quadrilateral, radial symmetrical/parallel square, quadrangle, inclined square, pentagon, oblique, square, square, steel front cage, square facet, triangle, square, square, square.

Table 5: Student's answer to question number 3

Students still have difficulty in determining the name of the plane shape given to the problem, both in mentioning a special name or the name of the shape based on its many sides. Based on the student's answers above, some students mention the name of the plane shape that relates to the objects around them, such as the rice cake to build a rhombus. This is because of the rice cake (ketupat) that they often encounter in their daily lives. And some students mention kites as “crystal facets” because the kite pictures given are associated with crystal shapes in two dimensions. The following is an example of the answers given by students:

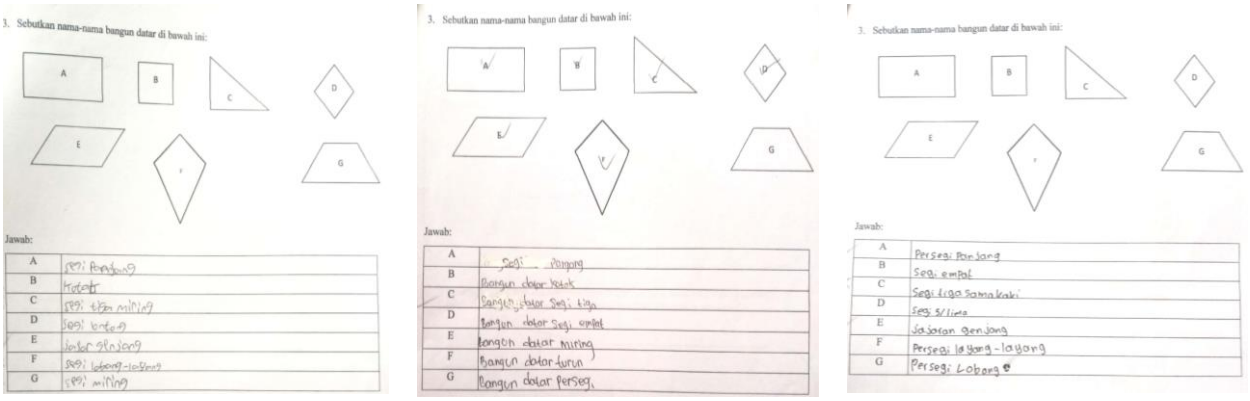


Figure 3: Example of student answers for question number 3

DISCUSSION

This study investigates students' conceptual understanding of polygons. In particular, students understand the concepts of polygons, regular and irregular polygons, as well as the names of plane shapes that have been taught in elementary school. The findings of this study indicate that some students (50%) succeeded in determining the polygon shape but could not give the right reasons for the choice answers that they are chosen. This study shows that students do not understand the characteristics of polygons. Misunderstanding students happen because some students lack a basic conceptual understanding of geometry (Chiphambo & Feza, 2020). In addition, students have several misconceptions and lack background knowledge and reasoning in studying geometry material (Özerem, 2012).

The concept of plane shapes, especially the concept of polygons, is one of the concepts that must be taught to students so that it is easy to understand the following material in learning mathematics, especially geometry material. Because when students do not understand the characteristics in terms of many, students will have difficulty in solving problems related to plane shapes, such as determining the area or circumference of a given polygon (Sholihah & Afriansyah, 2017). The difficulty of students in using concepts is the inability of students to express the meaning of terms that represent the concept of polygons and the inability of students to remember a condition that is sufficient for an object to be expressed in terms that represent the concept of polygons (Fauzi & Arisetyawan, 2020).

In addition to the concept of a polygon related to the properties of the polygon itself, students still have difficulty distinguishing the shape of a regular polygon and an irregular polygon. This situation is in line with the expression (Fitri & Lena, 2021) that there are several obstacles for students in studying polygons, including difficulties in understanding the types of plane shapes, sorting out the properties of regular and irregular polygons, and difficulties in determining names based on their properties. The concept of geometry is complex for students. That is because conceptual development in geometry involves several skills and mental constructions that build

on each other in complicated ways (Walcott et al., 2009). If students do not understand the names of the given shapes, students will experience problems in classifying plane shapes. As revealed in research (Hallowell et al., 2015; Žilková et al., 2015), the students have difficulty identifying the properties of plane shapes correctly, and the effects are students having difficulty in classifying geometry shapes. Fujita et al. (2019) suggest that a systematic stage is needed in defining and classifying geometric shapes. That is because students' interpretation of the meaning of mathematics is subjective and temporary but, in the process, becomes more subtle and objective.

The findings in this study also show that students use informal language (own language) in determining the names of geometric shapes. For example, when students state that the rhombus is a *lontong* (food name "ketupat"), this shows that students can integrate real life with geometric shapes. However, the actual concept of students does not understand how the characteristics of the rhombus itself. Likewise, with parallelogram shapes, students use non-mathematical language when mentioning the name of the shape. Students use the term "sloping rectangle or rectangle with beveled edges" to construct a parallelogram (Walcott et al., 2009). Although there are students who understand geometry in formal language, they talk about it informally (Budiarto & Artiono, 2019). In studying geometry, students not only have to understand theorems but also need students' ability to understand terms in geometry so that they do not become obstacles in learning geometric concepts (Chiphambo & Feza, 2020) because in geometry many subjects are interconnected. Therefore, geometry teachers need to investigate the understanding of their students to provide meaningful learning experiences at certain developmental levels. (Feza & Webb, 2005).

Based on Van Hiele's thinking stage, the students in this study had not yet reached the Analysis stage. This problem is because there are still many students who are not able to identify the properties of polygons (Žilková et al., 2015). in addition, several studies revealed that the students' geometric perception level was not at the expected level (Clements & Battista, 1992). Several factors cause students not to be at the right stage in learning geometry, including a lack of understanding of concepts and properties of polygons, a lack of welling understanding of the previous material, and a lack of student skills in using geometric ideas in solving mathematics problems. (Sholihah & Afriansyah, 2017).

Thus, the findings of this study are not in line with the expectations (The National Council of Teachers of Mathematics (NCTM), 2000), which claims that "In Pre-K to grade two all students must recognize, name, construct, draw, compare, and sequence two- and three-dimensional shapes". However, most of the children in this study did not succeed in identifying the names of polygons. Therefore, teaching polygons material is necessary to cultivate basic concepts. Teachers can direct students to gradually understand a concept from polygonal shapes and provide examples and non-examples of polygonal shapes so that students can describe polygons, both regular and irregular polygonal shapes. This problem is in line with what was expressed by (Bernabeu et al., 2021) in teaching the concept of polygons, and it is necessary to define the relationship between perceptual, discursive, operative, and sequential understandings as teaching

objectives that explicitly pay attention to the properties of existing shapes. In a sense, to stimulate understanding of the polygon concept, it is expected to recognize examples and non-examples of polygon and be able to turn non-examples into examples of polygonal shapes by explaining and drawing.

CONCLUSIONS

The conclusion that can be summarized from the discussion above is that students have difficulty understanding the concept of polygons. The students' problem with polygons concepts is that students find it complicated to describe the properties of polygons, so they have difficulty defining the shape of polygons. The impact of students' misunderstanding of the properties of polygons is the difficulty in determining regular and irregular polygons. In addition, the problem that occurs to students is that students give inappropriate reasons to state the nature of the shapes. The next problem is that there are still students who cannot mention plane shapes with the correct names, resulting in students having difficulty learning the following geometric material.

The problems of students are the incompleteness of students understanding of a concept, whereas it becomes a part of learning the following concepts. For example, if students do not know the names of the shapes given by the teacher, this will result in students cannot describe the properties of these shapes. In other words, students do not complete the stage of thinking level one from the level of thinking based on Van Hiele's theory and will have problems continuing to the next level of thinking. Giving a diagnostic test before learning is one of the ways to solve these problems. The diagnostic test is necessary to find students' difficulties regarding the prerequisite material for further instruction. Thus, based on the test results, we can design learning activities based on the students' levels of thinking.

Understanding the concept in the study of geometry is necessary so that it does not become a problem in learning other mathematical materials. Thus, teachers should be able to make learning designs with appropriate didactic situations to make students understand the concepts being taught by paying attention to the problems faced by students. This research is limited to polygon material. The recommendation for further researchers is to conduct similar research on other materials. This research can be a discourse of knowledge that may be useful for future researchers to conduct research with similar themes so that, for example, they can generalize the conclusions of richer research results.

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