GeoGebra Integration in High School Mathematics: An Experiential Exploration on Concepts of Circle

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Abstract: This paper is an outcome of using GeoGebra in Nepal's grade ten math curriculum (CDC, 2015). Based on a teaching experiment as one of the research methodologies (Dahal, 2019; Dahal et al., 2019; Dahal et al., 2022a), this paper provides students with active learning environments and the possibility of integrating GeoGebra as an ICT application by documenting the experiences, assessment procedures, emotions, and behaviors, as well as the learning process of eighteen secondary level students (ten boys and eight girls). In this regard, GeoGebra is a computer and online-based application that teaches geometry, algebra, and statistics. GeoGebra's features could help students visualize abstract geometric concepts quickly, correctly, and effectively (Tamam & Dasari, 2021, Dahal et al., 2022a). Theoretically grounded on social constructivism, this paper informs that GeoGebra helps students recall and understand circle terminology, encourages engaged learning through group work, and promotes meaningful learning, conceptual learning, learner-centered teaching, and student motivation. The benefits of using GeoGebra to teach and learn the circle concepts are demonstrated in the experimental classes and reported in this paper. Students become more active builders of the mathematical knowledge of the circle while teaching/learning the concepts of the circle using GeoGebra. It is a crucial ICT tool for supporting innovative approaches to teaching and learning mathematics in the twenty-first century.

Keywords: GeoGebra, circle, teaching experiment, constructivism, innovate approaches

INTRODUCTION

Mathematics teachers and learners present challenges are visualizing abstract geometrical concepts and ideas quickly, correctly, and effectively (Dahal et al., 2022a). This challenge creates an ample opportunity for mathematics teachers and students to learn mathematics using available ICTs tools. Integrating the technological tools to the mainstream of the mathematics classes is one of the ways to connect the conceptual and procedural understanding in the concepts of the circle (Maharjan et al., 2022). Technology integration is likely to be the relevant and innovative way of interactive teaching and learning (Agyei et al., 2021), as technologies are aligned with daily actions. In Nepal, some mathematics teachers are somewhat limited to the chalk-and-talk method of teaching mathematics. For instance, "technology has become an essential component of sustaining civilization, and its incorporation into education is consequently unavoidable" (Ghory & Ghafory, 2021, p. 168).

Likewise, in the professional career of the first author, he realized that students lack the visualizing abilities to understand the concept of the circle. He provides the necessary materials to help students comprehend the ideas of circles, but he notices that they struggle to use those concepts to solve problems and explain the meaning of such concepts while administering various assessments (e.g., quizzes, tests and projects). This might be because of teaching the concepts of the circle through a traditional method, which shall hinder students from manipulating and conceptualizing the circle's properties and visualizing those concepts with animation features. Battista (1999) found that students faced challenges in studying geometrical concepts and struggled to grasp the concepts and required knowledge because of the traditional ways of teaching and learning. Explanation, fear of punishment, textbook-based instruction, and homework and classwork rituals are all part of the traditional teaching methods. Likewise, the first author's students have always struggled to understand geometrical concepts, especially circles. Hence, we have acknowledged the necessity of incorporating innovative pedagogies by incorporating some form of technology. So, there is a need to modify or challenge the existing traditional teaching practices to meet the needs and desires in the era of industrial revolution 4.0 (Tapscott, 2008; Dahal et al., 2020). In the era of industrial revolution 4.0, there are a lot of different kinds of ICT tools used by students when they interact with each other and with teachers (e.g., flipped classrooms, mobile apps, and clickers devices) (Dahal et al., 2022b). GeoGebra is one of the available online and offline tools for visualizing mathematical concepts or the concepts of the circle.

GeoGebra application has become a part of the curriculum in higher secondary education in many countries (Shrestha, 2017a; Shrestha, 2017b; Dahal et al., 2019). The advantages of using GeoGebra, according to Dikovic (2009), are its user-friendly interface, ability to encourage students in project and discovery learning, ability to help students to enjoy ownership of their own creation, ability to encourage collaborative learning, ability to help in visualization of abstract nature of mathematics and ability to engage students in the active construction of knowledge by manipulating variables in GeoGebra (Dahal et al., 2022a). So, GeoGebra has been used to teach mathematics as a pedagogical tool (Putra et al., 2021). A significantly higher achievement was observed among GeoGebra-taught students compared to the control group. Moreover, experimental group students' perceptions of GeoGebra usage were favorable (Joshi & Singh, 2020). Likewise, students can work freely in GeoGebra software without the help of any teacher, which encourages students to discover learning and take ownership of their creations. In this regard, GeoGebra has grown in popularity as an interactive mathematics learning tool. Students may use this application to draw links between symbolic and visual representations. Likewise, GeoGebra is ideal for applying visualization techniques to mathematical concepts. As it can be widely used in teaching and learning mathematics from the primary stage of schooling to higher levels of education in the maximum field of mathematics, it helps students widely analyze mathematics concepts.

Similarly, it can be used in a dynamic way to learn mathematics. GeoGebra is known to be an open-source dynamic mathematics software for innovative learning and teaching purposes for all levels (Kllogjeri & Kllogjeri, 2014). GeoGebra offers a user-friendly environment with an easy-to-use interface, multilingual languages, commands, and a helpline (Hohenwarter et al., 2020).

In addition to the above, GeoGebra is a computer and web-based program that helps teachers and students study mathematics, particularly geometry, algebra, and statistics. Numerous features of the GeoGebra software suggest that it could be a valuable tool for assisting students in quickly, accurately, and efficiently visualizing abstract geometrical concepts (Tamam & Dasari, 2021). This plays a vital role in relating mathematics to the student's everyday learning experiences and lives by creating graphs, images, and animations. In Nepal, mathematics' significance is increasing daily in society because of the use of mathematics in various other fields. No doubt, mathematical concepts are indeed attached to every socio-economic activity. The need for mathematics and its application in our day-to-day life has always been an important part of the educational sector. The mathematics teachers of this twenty-first century are supposed to be innovators, creators, and knowledgeable about today's needs in teaching and learning.

Some studies demonstrated that when GeoGebra was applied in teaching mathematics to illustrate mathematical concepts, it helped students visualize and understand concepts through exploration. Further, GeoGebra has a positive impact on students' understanding of geometry. Dogan (2010) claimed that GeoGebra had positively affected students' learning and achievement so that they are motivated toward learning geometry. Blondal et al. (2013) also revealed that students improved their mathematical understanding after using GeoGebra. Students were able to investigate and generate hypotheses, resulting in higher grades. Some of the studies in Nepal have evaluated the impact of GeoGebra in teaching and learning (Shrestha, 2017a; Dahal et al., 2019; Dahal et al., 2022) and showed that secondary-level students demonstrated a better understanding of the concepts while using GeoGebra.

With the support of the second and third authors, this research attempts to help the students while teaching the concepts of circles meaningfully by challenging the traditional teaching methods. This paper aimed to explore the use of GeoGebra in teaching concepts of the circles. Guided by the research question— what role does GeoGebra play in helping students to understand and visualize the concepts of the circle? Starting with the introduction, this paper covers the theoretical framework, method, discussion of findings, and conclusion and way forwards of the study.

THEORETICAL FRAMEWORK

This research is grounded in the theoretical framework of social constructivism. Our study aimed to explore the concepts of the circle using GeoGebra. We implemented teaching experiment methodology (Steffe & Thompson, 2000). This method was chosen to explore the students' firsthand learning and reasoning experiences (Steffe & Ulrich, 2014). Likewise, learning theory of constructivism asserts that rather than passively absorbing information, students actively engage in the process of creating new knowledge. Students build their representations and incorporate new information into their existing knowledge as they experience the world and reflect on their experiences while conceptualizing the concepts of the circle using GeoGebra. Next, social constructivism is an educational philosophy emphasizing learning as a collaborative process. Knowledge develops due to learner's interactions with one another, their culture, and the larger society (Amineh & Asl, 2015).

The teachers must collaboratively guide the learning process to understand students' learning. We are actively involved in regular discussions with students within social constructivism as a theoretical framework. Similarly, we regularly took feedback from the students, motivated them to participate in classroom activities actively, allowed them to operate GeoGebra, and took feedback from witness-researcher about the students' behavioral change and engagements in experiment classes. Within this ethos, the essence of the social constructivism theory is that learning takes place through social interaction. Social constructivism believes social processes are vital for collaborative learning and cognitive development. Vygotsky (1978) believed that classroom community plays an important role in the meaning-making process of the content of any subject matter. It believes that knowledge is socially constructed. “Language, culture, daily activities, material goods, interpersonal engagement, peer interaction, tools, and symbols are all crucial social components in learning” (Dahal et al., 2019, p. 2). Students in the first author's classes are encouraged to participate in peer interaction and conversation. Vygotsky (1978) suggested that cognitive development results from the learners' social interaction.

Group work allows students to explore ideas, beliefs, perceptions, and misunderstandings with their peers and teachers. According to Vygotsky (1978), a difference exists between what a student can do on his own and what the student can do with help from others. Students can do things they would not be able to complete on their own with the support of adults and a greater understanding of others as experts. With the students' pre-existing knowledge of the circle, we incorporated GeoGebra to gain further an in-depth level of understanding of the concepts of the circle. Pictorial images, animations, and adequate illustrations in GeoGebra helped us accomplish the task of making students gain an adequate understanding of the concepts of the circle.

Moreover, social constructivism guided us to make experimental classes collaborative. Our lessons were designed so that students actively discussed the concepts of the circle. We followed the regular schedule of classes and assigned students group assignments to be done at home remotely. Collaboratively they did the assignment at home as well as in the classroom. New forms of ideas emerged through the interaction among the students. So, GeoGebra played the role of catalyst to secure the goals of a social constructivist classroom. Synchronous and asynchronous tools provide a platform for discussion among learners, leading to the social construction of knowledge (Dahal et al., 2020).

METHOD

We employed a well-known method of research known as the "teaching experiment" (Thompson, 1979; Dahal, 2019; Dahal et al., 2019; Dahal et al., 2022). Likewise, the teaching-research cycle helped us to design, implement, assessment & analysis and refine our plan (Czarnocha et al., 2016). Within teaching experiment and teaching-research cycles, GeoGebra in our study investigated how students conceptualized and visualized the circle's concepts and verifications. The circle concepts taught were chord, diameter, semicircle, segments, sector, central angle, arc, inscribed angle, concentric circles, intersecting circle, con-cyclic points, and cyclic quadrilateral in experimental classes. In addition, a few experimental classes included conceptual and procedural verifications and proofs. Similarly, concepts and verifications such as the inscribed angles of a circle standing on the same arc are equal, the central angle of a circle is double the inscribed angle standing on the same arc, and the inscribed angles standing on the same arc of a circle are equal.

The sum of opposite angles of a cyclic quadrilateral were explored using GeoGebra during the experimental classes. The experimental classes were recorded with the help of the witness researcher. We revisited those recordings while generating the meaning and the themes. Our research was conducted with 18 tenth-grade students where ten were boys and eight were girls. This teaching experiment/cycles encompassed two episodes of fifteen classes in which the concepts of circles were taught intensively. In addition, evaluations were an integral part of the circle concept learning that occurred during experiment classes.

We took both formal and informal field notes throughout experimental classes. Many students engaged in conversations and activities during the intensive instruction. The first author proposed numerous possible explanations on integration of GeoGebra in the concepts of the circle. Likewise, using recorded videos and field notes, potential findings were generated. In addition, the researcher-witness assisted the first author in comprehending the emotions and activities of the students. Being a reflective educator/researcher is also crucial when using the teaching experiment method. We expend more energy as reflective practitioners than teachers (Steffe & Thompson, 2000; Dahal et al., 2022a). These activities, termed "retrospective action", are performed by researchers and educators. The retrospective action allows the researcher to be aware of what occurred in the past or to take a fresh look at what occurred in the past. Reflecting on past teaching experience is one of the most crucial aspects of the teaching experiment method regarding making sense of the collected data. In addition to developing research and research instruments, the other two authors contributed to implementation process, the review of the research process, the completion of the research study, and the writing of the paper.

FINDINGS AND DISCUSSIONS

This section of the research examined the findings and discussed them in-depth. Our study was based on the information we gathered from teaching experiments, likewise, the participants and researcher's reflective journals were also used to generate some findings. The main ideas and findings are put together in different parts of the text using interpretation and analysis. The following six major findings were discussed.

GeoGebra Helps Recall and Understand Terminologies of Circle

As a teacher-researcher, the first author became aware about the diverse group of students' interests, knowledge, and ability to learn mathematics in general and "circle" in particular, which is one of the contents related to mathematics. Even though the circle has been introduced from the primary level and continue till higher grades, we teacher researcher become aware of the students' prior knowledge and experiences, on the concepts of circle and its basic terminologies, and so on, which are required to explore abstract ideas and concept particular in grade ten. It is important to know students' pre-knowledge and experiences not only in teaching circle but also in every topic related to mathematics. Prior knowledge and experiences enable students "understand new concept more quickly, retain their current knowledge more effectively, and transfer their knowledge to new situations more easily" (Klosterman, 2018, p. 12).

Thus, considering the importance of students' prior knowledge and experiences related to circle, the author first recalls the basic terminologies. For this, the first author uses simple question answer method and mathematical quiz incorporating the question from circle from previous grades and the grade ten itself. Such as what is circle? What is chord? How do you differentiate between chord and diameter? What is radius? What is the relation between radius and diameter? If diameter of a circle is 6 cm than what is the radius? How to calculate the area of a circle if the diameter is 10 cm? How do you differentiate between area and circumference of a circle? And so on. In addition, we will use many vocabulary terms when talking about the circle, which is essential to visualize. As shown in figure 1 the first author has demonstrated the basic terminologies of circles through the GeoGebra based on the questions asked them to know their pre-knowledge about the circle. Based on the activities, it has been found that most students can conceptualize and visualize the basic terminologies for the circle that they learned in previous grades. Some students were able to remember the terminologies of the circle because of the activity.

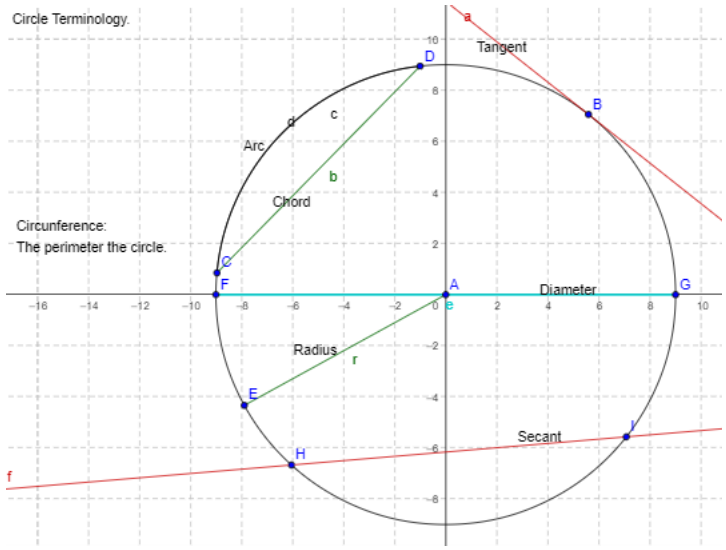

Figure1: Terminologies of the Circles

The activity was helpful for students who had missed a few circle classes in a previous grade. For instance, one of the boy students shared “*sir, this activity has helped me to visualize the concepts of the circle when radius is changed*”. As the students did not get an opportunity to learn and understand the basic terminologies of circle through GeoGebra in previous grades, they found the very first day class of circle with recalling the experiences using GeoGebra was very effective. During the time of the reflection one of the average boy students reflected that “*this was my very first day class of circle using GeoGebra. Even though it is the introductory class of circle, I enjoyed throughout the discussion and activity. I got an opportunity to explore basic concepts related to circle and visualized that through GeoGebra. I hope to explore and solve more abstract problems related to circle through GeoGebra in our upcoming class”*. Such kind of the reflection of the students encouraged the first author to conduct and to experiment the problems and theorems related to circle using GeoGebra.

GeoGebra Encourages Engaged Learning through Group Works

It could be one of the days of October 2018, the first author started the second day of the experiment class with great interest to demonstrate the concepts—chord, diameter, semicircle, segments, sector, central angle, arc, inscribed, angle, concentric circles, intersecting circle, con-cyclic points, and cyclic quadrilateral of the circles. These concepts were recalled during the first day of circle class in grade ten. The first author has spent more than one and a half years teaching this group of students. In this class, first author attempts to teach the students by integrating innovative pedagogies with the help of GeoGebra application as done in day one. In contrary, the first author was quite nervous about the students' reaction and behaviors when he presented the contents of mathematics of grade ten in a very different way. The first author was preparing for the session with great anxiety and curiosity. For instances, on particular class, the first author has to manage many new teaching materials such as a projector and a laptop.

For that day, the first author's planning was to make students visualize in group activities splitting the whole class in five different groups based on the concepts, namely, central and inscribed angle, cyclic quadrilateral; some experimentally verification that equal arcs of a circle subtend equal angles at the center (to mention).

With the help of witness-researcher, as shown in figure 2, the first author demonstrates the concepts of the circle, central angle, and its corresponding arcs with the following steps as students were already habitual for handling the GeoGebra application:

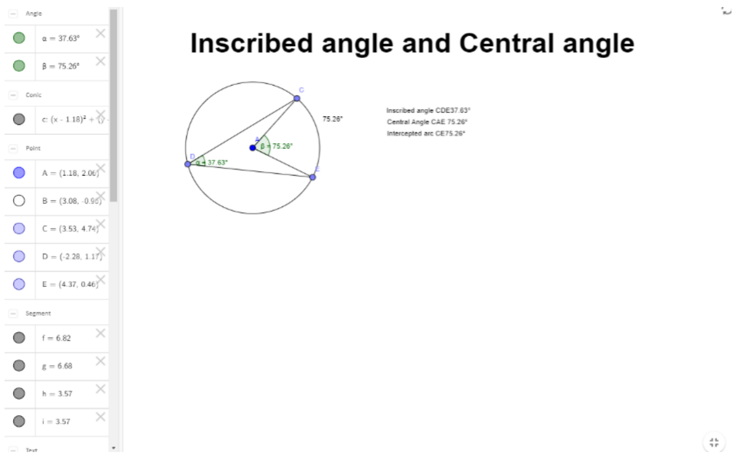

Figure 2: Screenshot of experimental verification

Step 1: Open GeoGebra and hide the axes.

Step 2: Create two circles' radii more than 3 cm with center O.

Step 3: Choose the point C on circumference of both the circles.

Step 4: Create a line between points A & C and B & C.

Step 5: Measure angles ACB and Ref. angle AOB. What do you notice about their measures?

Step 7: Create an arc a, between points A and B in both the circles. What do you notice?

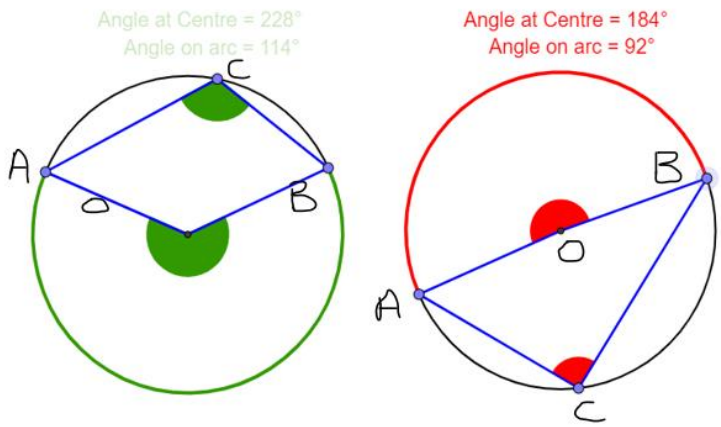

Figure 3: Screenshot of experimental verification

After demonstrating the concepts and responding the concerns raised by the students associated to center angle and inscribed angle standing on same arc. As shown in the figure 3 of two different figures, the first author divided the students into five groups each containing five students and gave them A4 papers. The first author assigned each group the following tasks.

Task I

- Draw a circle of radius 5 cm
- Draw two equal angles at the center
- Measure corresponding arcs of each central angle
- Draw your result.

Task II

- Draw circle of radius 5 cm
- Draw two equal arcs
- Measure corresponding angles at center subtended by each equal arcs
- Draw your result.

After the allocated time was over, the researcher asked each group to share their results. To surprise, the first author found that each team concluded the correct result: in a circle, equal central angles subtend equal arcs and equal arcs of a circle subtend equal central angles.

First author has started teaching decade before obtaining a post graduate diploma (PGD) in education. First author, as a teacher, used to think that curriculum is merely a collection of content material to be delivered to the students within the academic year. Students' interest and motivation are often ignored in his class. While teaching mathematics by integrating GeoGebra and collaborative teaching methods, we found students engaged in doing mathematics and deriving results from different activities rather than listening to talk and staring at the white board. They could share their ideas among their peers and work in a miniature knowledge society. We realized that students learn many life skills form the group works. The following figure 4 demonstrates the students' skills as the form of engaged learning of the concepts of the circle.

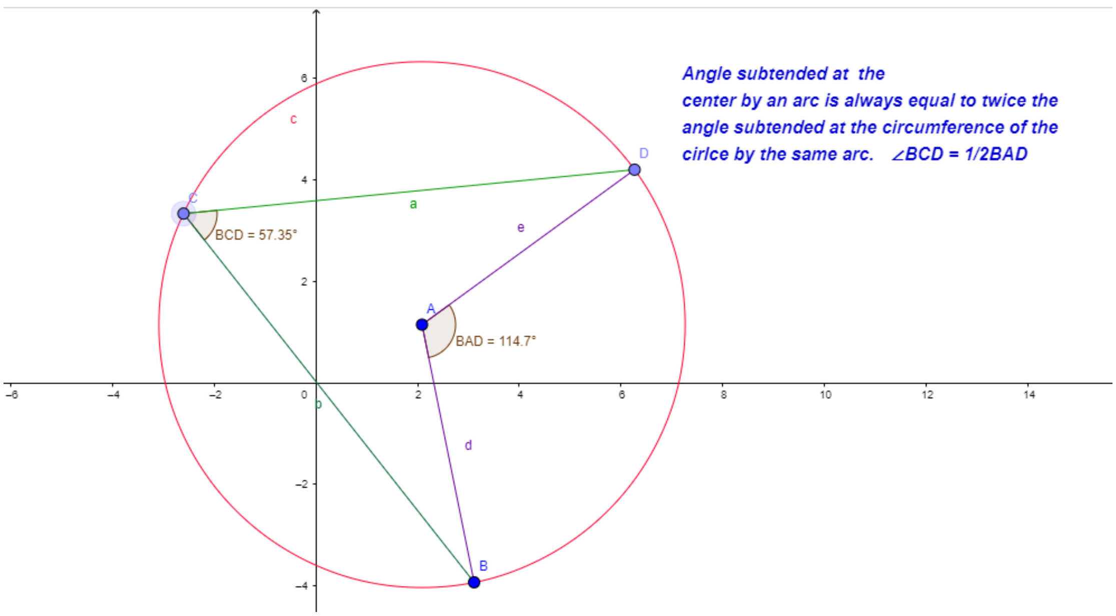

Figure 4: Screenshot of students' creation

The creative task done by students shown in figure 4, as a teacher we may not be able to solve other problems in our profession, but at least we can change our teaching by incorporating cooperative learning. We know that loss of students' interest is one of the major causes of students' failure. Among the strategies, integrating the GeoGebra while teaching general mathematics and circle is particularly likely to motivate the students to learn mathematics in unique ways.

In collaborative ICT class, first author, performed as a mediator. He mediates learning through dialogue and collaboration. We managed the proper environment and physical structure within the classroom. The availability of all the teaching/learning materials is the most important factor for the students to collaborate, and we had to prepare a lot in advance. We managed the structures so that the students may easily interact and perform.

In applying collaborative learning methods with GeoGebra, we encountered a number of obstacles, including the pace at which students followed the first author's instructions, distractions from peers, and a lack of participation on the discussion are among others. Most parents have a traditional learning approach in mathematics, so they expect their children to be educated similarly. They think that their children are deprived of learning mathematics from the teacher and their friends. They believe there should be competition among the students, not cooperation while learning mathematics (Panthi & Belbase, 2017). Students sometimes become arrogant and do not follow the instructions given to them. Some students are passive learners by nature, and some are noisy. Beside all these obstacles, it is worthy to adopt a collaborative method of learning mathematics integrated with technology.

GeoGebra Promotes Meaningful Learning

First author was comfortable compared to the previous day of day four. He did not have to run for materials and arrange rooms for the session. He planned to visualize central angle, inscribed angle, major segment, and minor segment with the help of GeoGebra in the experimental class as in a form revision where some of the concepts were illustrated in the previous class. Major task for the session was to verify experimentally, angles in the same segment of a circle are equal.

As planned by first author, we were waiting for students of grade ten in ICT room with the required teaching aid. Most of the students came in time except a few. Taking the situation easily, we started the sharing the task that was plan for day five with the help of GeoGebra.

After drawing pictures, first author instructs students to do the following activities:

1. Move point D. Measure the central angle, inscribed angle, and the intercepted arc.
2. Move point C. Measure the central angle, inscribed angle, and the intercepted arc.
3. Move point E. Measure the central angle, inscribed angle, and the intercepted arc.
4. What is the relationship between the central angle and the inscribed angle?
5. What is the relationship between the central angle and corresponding arcs?
6. What is the relationship between the inscribed angle and the corresponding arc?

We collected information from all the students, and we found that student's active engagement during the teaching and learning circle using GeoGebra to determine the relationships between the central angle, its opposite arc, and the inscribed angle, its opposite arc, angles in the same segment of a circle are equal produces satisfactory outcomes. For example, one of the female students stated, "*Sir, this is a completely novel and innovative learning experience for me. I can now see the relationships between central angles, their opposite arcs, and inscribed angles and their opposite arcs".* Then we told students to do the other activity in their note copy with the help of a compass, ruler, and pencil. They did the activity enthusiastically and the session was concluded by wrapping up the session with feedback from the students.

GeoGebra Promotes Conceptual Learning

It was first author's regular class and the sixth day of the experimental class in grade ten and his students were eagerly waiting for him. When he entered ICT room, students greeted him with a smile. As a teacher, we have felt that it is very difficult to precede the class if the students show indifference in our presence. His priority is always to maintain harmony and warm relationships with students so that teaching does not remain the only means of livelihood. In the session, as per first author plan was to visualize concyclic points of a circle, cyclic quadrilateral, angles of a cyclic quadrilateral, opposite angles of a cyclic quadrilateral.

Before starting the session, the first author tried to recall the contents discussed in previous classes. First author requested the students to illustrate with the figure of the circles—central angle, inscribed angle, intercepted arc, and their relationship. Most of the students could explain and illustrate the concepts with the figure of the terms mentioned above of a circle. We were very happy at that movement.

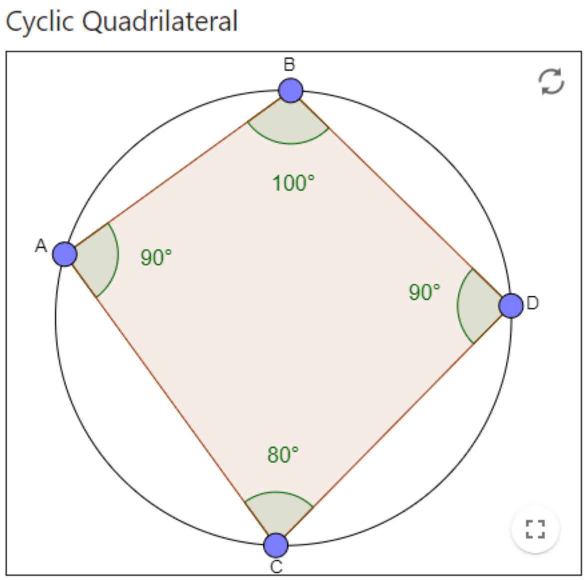

Figure 5: Screenshot of cyclic quadrilateral

First author started the session with a GeoGebra window where a cyclic quadrilateral was drawn as shown in figure 5 alongside. He asked the students to draw two sets of different circles in their notebook. He measured all the angles of the cyclic quadrilateral and told the students to do the same. On the GeoGebra window, we found that the sum of opposite angles of a cyclic quadrilateral is two right angles. First author asked the students to conclude the result. Most of the students got the expected result. Some students could not measure angles with a protractor. We assisted them in measuring the angles. They were happy when they were able to get the desired result. When the students do any activity with satisfaction it gives immense pleasure. Students understood that cyclic quadrilateral, by definition, is any quadrilateral that can be inscribed inside a circle. Also, they learned that the sum of opposite angles of a cyclic quadrilateral is two right angles.

After proving the theorem experimentally in the notebook and on the GeoGebra sheet, we proceed to prove the theorem theoretically. We proved the theorem easily fulfilling the required procedure for the examination. We should always keep in mind that we are teaching students in Nepal, and they are evaluated on three hours written test with specific requirements of a solution. This is an important in the case of teaching secondary level students. We were happy that first author could make the students learn mathematics conceptually.

GeoGebra Promotes Learner-Centered Teaching

We are in the middle of Dashain and Tihar of 2019 AD. Dashain and Tihar are the national festivals of Nepali. All the people seemed to be busy preparing for the festival. They have just enjoyed the festival of joy and are preparing for the festival of light and sweets--Tihar. We were busy collecting data for the research. First author was implementing lesson plans hoping to do the best. Sometimes, we rewarded through students' success and sometimes we have to regret choosing this profession when we see students failing in the subject.

Writing about first author, he has been teaching from a long decade. However, on the experimental class, he taught students to prove theoretically that standing on the same arc, the angle in the center is double that of the angle at the circumference. He proved the theorem that states that the angle in the semicircle is one right angle with the help of GeoGebra as shown in figure 6 below.

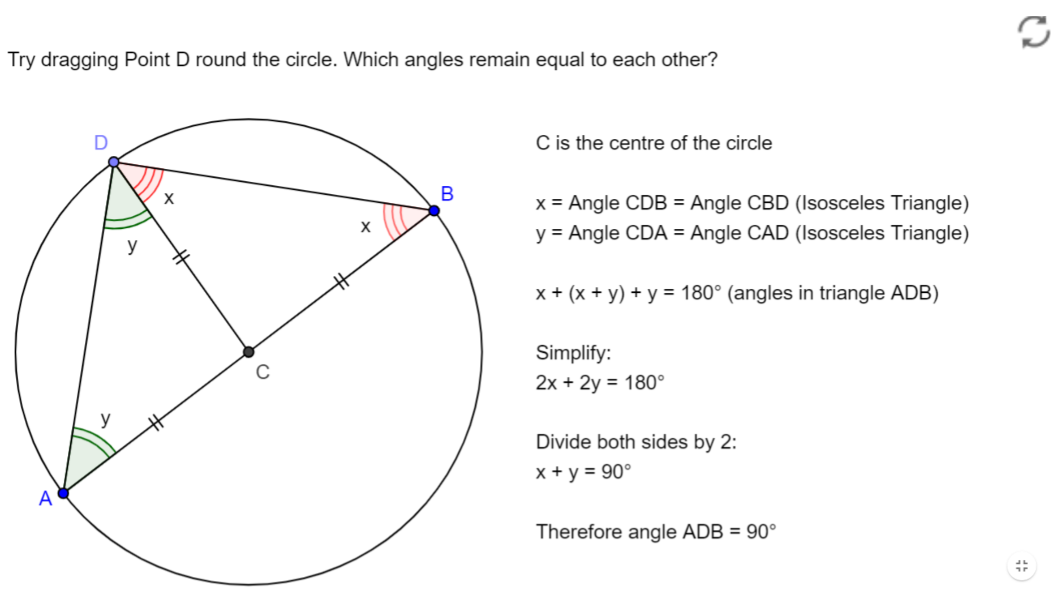

Figure 6: Proof of inscribed angle of the semicircle is one right angle

For instance, examinations in Nepal, most of the students must demonstrate the exact solution that was taught and/or illustrated in the examples. However, we could see the students struggling to maintain the required standard of proving theorems theoretically. This standard includes—proof statements, clear figure, given statements to prove, construction (if any), and a proof table including statements and reasons. We had to help them in proofing as per the standards. Nevertheless, GeoGebra only illustrates the visual form of the concepts.

For the proof of the theorem, first author asked for a volunteer role from one of the students. After all, we believe that learning is an active process. One student was happy to solve the theorem on the whiteboard. She explained the solution to her friends. Everyone understood the theorem. We could see the brightness in students' eyes, which we think is the most important indication of students' understanding of the content. In such a context, the student who solved the problem seemed to be excited and happy. First author asked her, *"How are you feeling now?"* She shared, *"Sir, I generally was not encouraged to involve myself in the classroom activities of GeoGebra. I hope this is true for other friends. Solving problems on the whiteboard and showing my talent were not practiced. This is probably my first time standing in front of my friends and you to solve the problem with the help of GeoGebra. I am feeling honored and responsible, sir"*. Being teachers, we must celebrate the progressive methods of teaching that encourage us to implement learner-centered methodologies in teaching and learning activities. By keeping learners at the center of learning and activities, students should be given opportunities to think, plan, implement, and execute by actively participating in the process. Keeping students engaged in the learning process is challenging, but the tasks with rich learning materials and resources simplify the learning process and help in a learner-centered learning environment. Moreover, in today's digital world, the use of technology and materials developed by using digital technologies seem to engage students effectively. GeoGebra is among those ICT applications for encouraging students' active participation in learning to the circle concepts.

GeoGebra Motivates Students in Learning

GeoGebra helps teachers motivate students in learning mathematics. Those leanings are of the curriculum and/or outside of the curriculum. Role of GeoGebra is very important for students of various backgrounds. The students with less motivation can be attracted towards learning. In one of the experimental classes, we explore how changes in arcs changes the center and inscribed angle. One of the students, motivated by replying that *this is exact learning with visualizing the concepts of arcs*. However, traditional classes only focus on the content of the study. But the ICT integrated classes to some extent motive students for leaning mathematics. Role of GeoGebra is very important for visualizing and animating the concepts of the circles.

Some of the glimpses of the experimental classes are as follows:

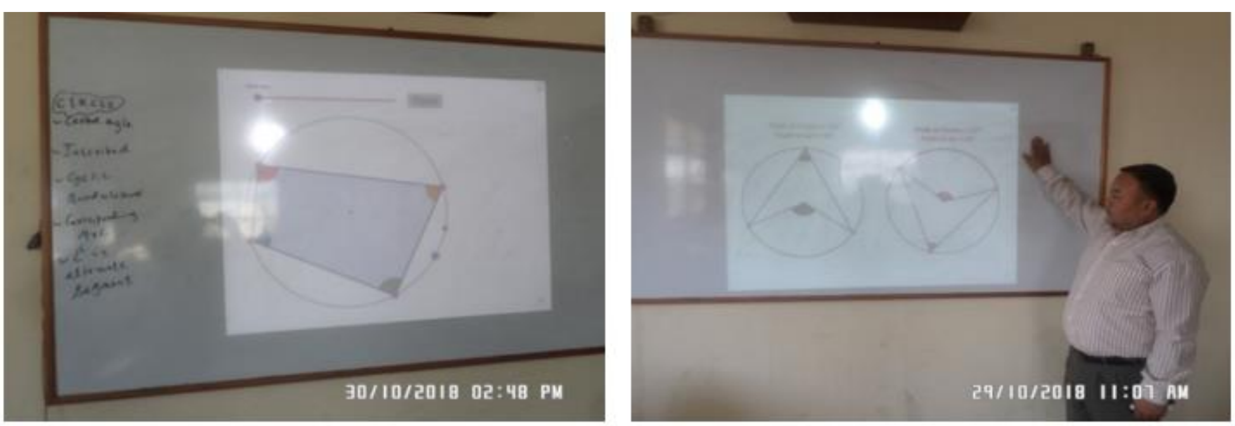

Figure 7: Glimpses of the experimental classes

Above glimpses demonstrate some of the concepts of the circles using GeoGebra that plays an important role for mathematics teaching and learning to visualize the concepts of the circles. The notion of mathematics would not have made sense without integrating GeoGebra in mathematics class, and it is very helpful for a strengthening the concept of mathematics especially the concepts of the circles. GeoGebra excels the learners for deep learning in mathematics, which helps develop logical thoughts and imagination. No doubt, GeoGebra plays an important role in teaching.

It is likely to be claimed that GeoGebra is a powerful visualizing tool that provides students with various experiences demonstrating the essential abilities to select when and how to utilize it in teaching and learning. Near to the final, from experiment classes, we can conclude that GeoGebra plays a vital role in visualizing the concepts of the circle namely, chord, diameter, semicircle, segments, sector, central angle, arc, inscribed, angle, concentric circles, intersecting circle, con-cyclic points, and cyclic quadrilateral as of the motivating ICT tool to name and among other verifications and proofs.

CONCLUSIONS AND WAY FORWARDS

The paper discussed the circle's concepts of chord, diameter, semicircle, segments, sector, central angle, arc, inscribed, concentric circles, intersecting circle, con-cyclic points, and cyclic quadrilateral. In addition, concepts and verifications as inscribed angles of a circle standing on the same arc are equal, the central angle of a circle is double of the inscribed angle standing on the same arc, the inscribed angles standing on the same arc of a circle are equal and the opposite angles of a cyclic quadrilateral are supplementary were explore using the GeoGebra during the experimental classes. The purpose of the experimental classes was to see how GeoGebra could be integrated in mathematics classrooms especially while introducing the concepts of the circle for learning and assessments. In this study, the first author, in collaboration with the second and third authors, utilized a teaching experiment to determine students' mathematical comprehension of circle concepts. The second and third authors also collaborated to determine how students conceptualized changes in the circle concepts using animations features. For the purpose, we employed a teaching experiment approach as the form of teaching research cycle to understand students' engagements during the experimental classes based on the research question, what role does GeoGebra play in helping students to understand and visualize the concepts of the circle?

Similarly, we put our effort into understanding students' conceptual understanding on the concepts of circles of intensive teaching in two episodes of fifteen classes. In Euclidean geometry, a circle is a basic shape. It is the set of all points in a plane that are at a given distance from a given point, the center; alternatively, it is the curve traced out by a point that moves in such a way that its distance from a given point remains constant. On contrary, drawing the various concepts of the circles on the white paper limit the students’ conceptual understanding, but GeoGebra helps to visualize the changes made and its exact figure of the circles and associated concepts. These engagements offer the students for meaningful and conceptual learning of the concepts of the circle.

Further, various individuals have concluded that there should be a variety of approaches to mathematics learning and instruction, including the use of teaching tools proven to increase students' interest in mathematics. This is one of the most significant discoveries to date. Math teachers should have access to this software to offer students a broader perspective on mathematics and to help them develop their critical and creative thinking skills. GeoGebra is one of the mathematical software programs on this list. As a result, expanding the use of ICT applications in mathematics classes could assist teachers and students in contextualizing mathematical concepts.

During the research process, we saw students actively participating in experiment classrooms. Similarly, we found that GeoGebra is user-friendly software. The findings of the study namely, GeoGebra promotes meaningful learning, GeoGebra encourages conceptual learning, GeoGebra promotes learner-centered teaching, and GeoGebra motivates students to learn mathematics concluded that using GeoGebra in the classroom can improve students' conceptual and procedural understanding on the concepts of the circle. Similarly, students' engagements in class and their replies after that revealed that GeoGebra can assist students for visualizing the abstract geometrical concepts of the circles quickly, correctly, and effectively.

Likewise, GeoGebra helps improve students' understanding of the circle concept. Integration of GeoGebra allows students to become active learners in the classroom. It helps teachers to make student-centered mathematics classrooms. It also helps to minimize unnecessary distraction in the classroom. All these benefits of using GeoGebra in mathematics classroom makes mathematics classroom effective and meaningful. So, as a teacher-researcher, we suggest that all the mathematics teachers to use GeoGebra in their classrooms. In this paper, we explored the use of GeoGebra in teaching geometry circles. There are many other mathematics contents where GeoGebra can be used effectively. Many algebraic concepts and transformation can be made visible and easy for students. So, we recommend mathematics teachers explore the possible use of GeoGebra in teaching other concepts on the contents of the school mathematics and beyond.

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