

## Editorial of the Fall 2023 issue 49, Vol 15 No 5

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### Editorial by Hannes Stoppel



We are glad to present this issue of the MTRJ. With this MTRJ we are pleased to be able to look at the didactics of mathematics in relation to various areas of Asia. Starting with the eastern part of Turkey, then move through Myanmar and Iran to South Korea and Indonesia, one takes a tour across Asia, what we do with the reports in this issue. Math education of Indonesia is particularly well represented with seven articles. The broad internationality of mathematics education is evident in the participation of Norway and the USA in a study.

MTRJ presents analyses of various types. Quantitative data collections are involved; mixed methods have been used several times to take a look at both a qualitative and a quantitative study. These different types of data collection and data analysis provide a broad view of mathematics education across Asia.

Since Corona, the demand for the use of digital media has been increasing. Several articles report on experiences with the use of digital media and provide impulses for their use.

The technical content of the studies is also evident. It ranges from elementary school over middle school to high school. In addition to examining subject didactics, metacognitive considerations are also considered. One study also looks at connections between cultural and skilled components of education. The studies presented in this issue are very close to practice up to the presentation of finished teaching contents.

Mathematics not only determines our profession lives but is also one of our hobbies. In order to give our readers the opportunity to practice this hobby, tinkering tasks can be found in “The Problem Corner”. We look forward to receiving your solutions of the exercises. To encourage participation, some elegant solutions of readers will be presented in the next issue.

As became visible above, this issue contains teaching-research articles from the perspective of many countries and fulfils MTRJ's desired requirements to publish creative research in mathematics education. We hope we have aroused your curiosity about the articles and hope you enjoy reading them and gain new insights.

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*Alper Yorulmaz, Emel Çilingir Altiner and Sıtkı Çekirdekci (Turkey) p. 5*

This article describes a study and shows results of a study to determine the number sense performances and metacognitive awareness of primary school fourth grade students and the status of metacognitive awareness in predicting number sense. The study leads to interesting insights about these factors and relationships between them.

### **Ethnomathematics Learning Model Based on Motifs of *Dayak Ngaju* Central Kalimantan**

*Jackson Pasini Mairing, Nimi (Indonesia) p. 30*

The aim of this study is to describe the influence of implementing an ethnomathematics learning model combined with batik/carving/painting motives on students' learning outcomes and responses. It contributes to research on the connection between culture and mathematics, which will be interesting to a wide audience ranging from educators to curriculum developers.

### **Cognitive Map: Diagnosing and Exploring Students' Misconceptions in Algebra**

*Analisa Fitria, Subanji, Susiswo and Hery Susanto (Indonesia) p. 49*

The article describes results of a study diagnosing and exploring grade eight students' misconceptions and mistakes when following the rules of algebra. The study results can help instructors teaching algebra.

### **Stimulation of Cognitive and Psychomotor Capability by Game-Based Learning with Computational Thinking Core**

*Erna Andriyani (Indonesia) p. 76*

The author develops a game for understanding connections between multiplication and addition for game-based learning. This is a practical assignment for students of grade three. Educators who wish to develop similar games may be interested in and inspired by the article, learning how to validate such a game.

### **Comparative Analysis of Students' Argumentation Patterns in the Context of Algebraic Problems**

*Hendra Kartika, Mega Teguh Budiarto, Yusuf Fuad, Lauren Jeneva Clark and Kim Jeonghyeon (Indonesia, USA and South Korea) p. 105*

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### **Development of RME Learning Media Based on Virtual Exhibition to Improve Students' High Order Thinking Skills (HOTS)**

*I Putu Ade Andre Payadnya, I Made Wena I, Putu Suarniti Noviantari, I Made Putra Kurniawan Palgunadi and Ayu Dewi Chandra Pradnyanita (Indonesia) p. 129*

To enhance realistic learning and students' thinking abilities, the authors developed virtual exhibition-based realistic learning media for mathematics education and evaluated its quality in improving students' high-level thinking abilities. The study includes detailed mathematical analysis of the data.

### **Characteristics of Differentiated Mathematical Creative Models in Problem-Solving Activities: Case of Middle School Student**

*Heri Purnomo, Cholis Sa'dijah, Erry Hidayanto, Sisworo, Anita Adinda, Abdul Halim Abdullah (Indonesia and Malaysia) p. 157*

The authors assume that creativity is not a gift for all students. The research reveals differences in middle school students' mathematical creativity models of imitation, modification, combination, and creation.

### **Effects of a Working Memory Training Program of Secondary School Students' Mathematics Achievement**

*Wit Yee Ei and Cherry Zin Oo (Myanmar) p. 177*

The goal of the study described in this article is showing that memory training strategies improve mathematics achievement of secondary school students. The article presents a new approach and contributes to the field.

### **Introducing a Teaching Technique for Reducing Students' Mistakes in Simplifying Algebraic Expressions**

*Abolfazl Rafiepour, Nooshin Faramarzpour and Mohammad Reza Fadaee (Norway & Iran)*  
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This study investigates the effect of the separator lines on the learning of grade 6 students in simplifying algebraic expressions. The study proposes new ideas in connection with teaching technology.

### **The Effect of Attitude Towards School on the Students' Happiness: The Moderating Role of Math Anxiety**

*Ahmet Kesici (Turkey)* p. 209

The aim of this study is to examine the moderating effect of mathematics anxiety on the effect of attitude toward school on student happiness among 8<sup>th</sup> grade students.

### **Content Analysis of Students' Argumentation Based on Mathematical Literacy and Creation Ability**

*Usman Mulbar, Nasrullah Nasrullah and Bustang Bustang (Indonesia)* p. 226

This study investigates the argumentation of junior high school students in building their creative reasoning and the relationship between students' argumentation and their mathematical literacy skills. Students' reasoning is analysed in depth through their statements and conclusions during visual problems-solving activity.

### **The Problem Corner**

*Ivan Retamoso (USA)* p. 239

Here you will find sample solutions to recent problems and new problems to tinker with.

## A Study of Number Sense and Metacognitive Awareness of Primary School Fourth Grade Students

Alper Yorulmaz<sup>1</sup>, Emel Çilingir Altınır<sup>2</sup>, Sıtkı Çekirdekci<sup>3</sup>

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*Abstract: The purpose of the current study is to determine the number sense performances and metacognitive awareness of primary school fourth grade students and the status of metacognitive awareness in predicting number sense. To this end, the study employed the relational survey model, one of the quantitative research methods. In the study, it was revealed that the number sense performances of the primary school fourth grade students were at a medium level and did not vary depending on gender. In addition, it was determined that the students' metacognitive awareness was at a high level and that metacognitive awareness did not vary significantly depending on gender. It was found that there is a positive correlation between the number sense performances and metacognitive awareness of the primary school fourth grade students. The students' metacognitive awareness was found to predict their number sense performances at a low level.*

### INTRODUCTION

What is the purpose of mathematics? Is it to reflect on why and how you perform mathematical operations or to remember and apply the right formulas? Mathematics is actually a subject of critical thinking, problem-solving and creativity, contrary to what is known (a subject of the world of formulas and memorization) (Boaler, 2016). People actually organize their own thoughts and decisions while doing math and employing the necessary skills. This requires metacognitive skills.

Metacognition is thinking about thinking. It also includes self-monitoring and correcting one's own learning processes. In mathematics, teachers can use metacognition to teach students how to handle a complex math problem. Just as there is a connection between numbers and operations, there are similar connections between metacognition and problem-solving (McIntosh, Reys & Reys, 1997). When a student first encounters a problem, the student's success in solving that problem depends on his/her ability to be aware of what decision he/she is making and why he/she is making that decision. According to Polya (2014), through the problem-solving stages (understanding, planning, solving and reflecting), students' metacognitive skills and their

awareness of these skills play an important role while applying and arranging their thoughts and decisions. However, metacognition alone is not enough to ensure success. Metacognition serves learning by enabling the use of certain skills (mental calculation, estimation, planning, evaluation, problem-solving...) (Çekirdekçi, 2015). Of these skills, mental calculation and estimation are also important for the development of number sense (Carroll, 1996). When the problem-solving stages of Polya are considered, we see that both number sense and metacognition skills should be used together in order to progress through the stages. Therefore, while number sense is of great importance in developing metacognitive skills (Lee, Ng & Yeo, 2019), metacognitive skills have a significant impact on the performance related to number sense.

### **Number sense**

Mathematics uses number sense, which is the most important component of mathematics, to train the mind in problem-solving and decision making and to foster logical and systematic thinking. Testolin et al. (2020) stated that there is number sense even in animals and they stated that they are numerically encoded in the brain even when we are looking at non-numerical elements we see around us. In addition, Tosto et al. (2014) stated that number sense is not hereditary and that it is associated with general intelligence, but not with gender. From this point of view, it is thought that number sense is a skill that can be developed later.

There are many definitions of number sense. Yang (2003) defined number sense as the ability to produce useful, flexible and efficient strategies for solving numerical problems. Çekirdekçi (2015) defined number sense as a flexible thinking skill based on intuition and senses related to knowing the properties of numbers and operations, working flexibly with numbers in problem situations, making mental calculations, making predictions, developing effective and usable strategies and judging the appropriateness of the results of the problems. Mohamed and Johnny (2010) also stated that individuals with good number sense tend to exhibit characteristics such as logical approach, planning and control, flexibility and conformity and a sense of reasonableness when making mental calculations. They emphasized that these skills are very important for the individual to overcome the numerical problems they encounter in their daily lives.

Enabling students to develop their number sense is considered an important task in national and international mathematics education (National Council of Teachers of Mathematics [NCTM], 2000; Şengül, 2013) because number sense refers to understanding the student's ability to use numbers, operations and their relationships in daily life situations (Yang & Li, 2013). This skill is used to develop useful, flexible and efficient strategies (including mental computation and estimation) for handling numerical situations (Yang & Huang, 2004).

Based on the common ideas about number sense, it is seen that number sense includes knowing the meanings and values of numbers and using this information based on a successful intuition. It can be argued that instead of memorizing rules, the logic of numbers and operations is understood and plans and strategies are developed accordingly with number sense. For example, we can see

that memorization is useless in the learning area of measurement. It is necessary to make a new measurement for each object for which a new measurement value is requested. But a student with developed number sense is adept at approximating without a standard measuring instrument, through his/her experience and self-developed logic and intuitive frameworks. Of course, the student also uses his/her other metacognitive skills in this process. Therefore, it is thought that students' metacognitive awareness will be important in the emergence of this situation.

### **Metacognitive awareness**

As a result of the search for possible solutions to increase students' mathematics achievement, one of the solutions found by educators is to improve students' metacognitive skills (Montero III & Elipane, 2021) because metacognition involves students being aware of their own strengths and weaknesses in their thinking processes, re-checking their thinking processes and managing and arranging their thinking processes while solving a problem (Alindra, Fauzan & Azmar, 2018). Metacognition, which means thinking about thinking, is often related to many skills connected with thinking and learning, such as critical thinking, reflecting on one's own actions and thoughts, solving problems, making decisions, predicting one's own performance on various tasks and monitoring one's own level of understanding (Baş, Özturan-Sağırılı & Bekdemir, 2016). In the field of mathematics, metacognition is explained as a person's knowledge of his/her cognitive processes and his/her perception of a mathematical problem in relation to the process of planning, monitoring and evaluating problem solutions (Flavell, 1979).

Another component of metacognition needed in problem-solving is metacognitive awareness. Metacognitive awareness is defined as the ability of students to reflect on the known and the unknown, to understand how they learn in the context of learning and to control themselves in learning (Herlanti, 2015; as cited in Alindra, Fauzan & Azmar, 2018). According to Boğar (2018), metacognitive awareness is an effort to acquire and use the metacognitive thinking skills that an individual needs throughout his/her life. Metacognitive awareness also refers to the individual's own knowledge, learning processes, knowledge of cognitive and affective states and conscious control and regulation of knowledge. In short, metacognitive awareness can also be expressed as the awareness of what one knows and what one will do in his/her own learning process.

The motto "Knowing when you know is knowing what to do when you don't know" is very suitable for metacognitive awareness. In this way, the student observes himself/herself in the learning processes and tries to compensate for his/her deficiencies. For example, if a student finds that he or she has difficulty measuring an object with a ruler, he/she may try to solve this problem by first checking whether the ruler is broken, whether the point where he/she places the ruler when measuring is the starting point and whether he/she leaves a gap in-between while advancing the ruler or whether he/she uses a ruler of the right size. If he/she realizes that this approach is not working, he/she decides to try a new approach, meaning that he/she has used the metacognitive awareness process.

Studies show that there is a direct relationship between students' metacognition and their success in mathematics (Mevarech & Kramarski, 2003). Rinne and Mazzocco (2014) stated that there is a strong relationship between metacognitive skills and mental arithmetic in children towards the end of primary school and that early metacognitive monitoring ability predicts improvement in mental arithmetic performance. However, it has not yet been determined whether these associations continue to exist and whether the same associations can be observed in young children when different aspects of executive functions and numerical magnitude are included in the process. At the same time, when the studies on metacognitive awareness are examined (Adhytama, 2014), it is seen that students with good metacognitive awareness have better problem-solving strategies and learning outcomes compared to students with poor metacognitive awareness (as cited in Alindra, Fauzan & Azmar, 2018).

### **The relationship between number sense and metacognition**

While solving problems in their classrooms, teachers use helpful and easy strategies specific to the solution of that problem. Students want to use this strategy, but it is important for them to realize that this strategy does not help when faced with different problems. When students realize that not every problem can be solved with the same strategy, that every problem has its own dynamics, then when they encounter different problems, they can develop and use different strategy stages specific to the problem (Waters & Kunnmann, 2010). This process, which holds the student responsible for his/her own learning path, requires teachers to expose their students to various problems until they develop their metacognitive skills. As students acquire a wider range of skills in metacognition, opportunities will arise to develop, evaluate and apply strategies efficiently.

International research on students' problem-solving skills in the context of mathematics education has shown that children do not perform well in multi-stage tasks and that mathematics teachers have difficulty planning and implementing lessons that develop students' problem-solving skills (Kramarski, 2008). Therefore, it has been stated that the concept of metacognition is useful in improving this situation (Schneider & Artel, 2010) because it has been stated that with the inclusion of metacognition in mathematics, executive control, monitoring and self-regulation skills are also included and thus the likelihood of success in mathematics can be increased (Lester, 1982). Verschaffel (1999) pointed out that metacognition has a special importance in the mathematical problem-solving process. In particular, metacognition has a great impact on the processes of using information, distinguishing between necessary and unnecessary information and estimating (Schneider & Artel, 2010). Vo et al. (2014) revealed that students' metacognitive skills specific to the numerical domain predict their mathematical knowledge, and they suggested that children's metacognition is a cognitive ability of mathematics in children. In addition, Bellon, Fias, and De Smedt (2019) stated that the relationship between metacognition and arithmetic performance is a reciprocal relationship where both arithmetic performance and metacognition affect each other. Given that number sense is a necessary process for understanding and learning mathematics in a meaningful way, it can be said that its interaction with metacognition is inevitable.



The development of metacognitive skills and the awareness that a student should not overly rely on a strategy also affect students' number sense skills (Carr, 2010). At the same time, the importance of number sense has been emphasized in terms of providing children with the experience of expressing their thoughts and enabling them to practice about it so that they can develop their metacognitive skills. It has also been emphasized that when metacognition develops, number sense skills will also develop and that there is an active reciprocal interaction between the two (Lee, Ng & Yeo, 2019). Koenig (2020) drew attention to the relationship between number sense and metacognition, stating that talks on numbers, which ensure that number sense skills are fluent and automatic, also enable children to practice expressing their thoughts in order to improve their metacognitive skills. For the development of number sense skills, it is important to determine how the student thinks about numbers, operations and results, how he/she reflects this and which strategies he/she prefers. Strategies, along with number sense and metacognition, are no longer limited to what the teacher teaches, but develop further as the student discovers his/her own strategies and adapts them into new complex problems (Ilko, 2021). As a result, metacognition and number sense involve higher-order thinking skills. Number sense is a way of thinking that requires skills such as perception, attention, flexible thinking, fluency, automaticity, strategy development while metacognition is a cognition that controls these skills.

### **Current study**

Even before children enter primary school, they develop strong informal number sense through their interactions with games, older children and adults (Woods, Ketterlin Geller, & Basaraba, 2018). Number sense is very important for primary school students. The first reason for this is that number sense is a way of thinking that encourages meaning-making and directs problem-solving in a flexible and efficient way (Dunphy, 2007). The second reason is that number sense plays a key role in primary school students' achievement in mathematics (Jordan, Glutting, & Ramineni, 2010). The third reason is that number sense triggers a meaningful learning process (Yang & Li, 2008). Although it has been stated that there are positive relationships between mathematics achievement and number sense performance, there are very few studies on this subject in Turkey and this area of research has only just begun to attract attention (Bütüner, 2018).

While trying to develop number sense in students, it is thought that metacognitive skills are also needed to enable students to develop a deeper understanding. When the existing research is reviewed, it is seen that both number sense and metacognition are skills that should be acquired by students (McIntosh, Reys & Reys, 1997; Singh, 2009; Yang & Hsu, 2009). However, it is seen that studies conducted to include metacognition and number sense together are limited in number (Ilko, 2021; Çekirdekçi, Şengül & Doğan, 2016). Moreover, although most of the relational studies on the relationship between metacognition and mathematical performance have been conducted on middle school students and adolescents, the study by Carr, Alexander and Folds-Bennett (1994) investigated the importance of metacognitive knowledge for primary school students' mathematical performance (Schneider & Artel, 2010). The current study was planned due to the

inadequacy of studies conducted on primary school students and the need to address the relationship between number sense and metacognitive awareness, which are considered to be important in terms of increasing mathematics achievement. Thus, in this study, it was aimed to examine the relationship between number sense performances and metacognitive awareness levels of primary school fourth grade students. To this end, the problem statement of the study is worded as follows; “Is the metacognitive awareness of primary school fourth grade students a significant predictor of their number sense performance?”. In line with the problem statement of the study, the sub-problems of the study are expressed as follows;

1. What is the number sense performance of primary school fourth grade students?
2. Do primary school fourth grade students’ number sense performances vary significantly by gender?
3. What is the metacognitive awareness level of primary school fourth grade students?
4. Do the metacognitive awareness levels of primary school fourth grade students vary significantly by gender?
5. Is primary school fourth grade students’ metacognitive awareness a significant predictor of their number sense performance?

## METHOD

### Research Method

The model of the current study, which examined the relationship between students’ number sense performance and metacognitive awareness, was determined as the relational survey model. Relational survey models are research models that aim to determine the existence or degree of change between two or more variables (Karasar, 2011). In the relational survey model, the data obtained by using measurement tools are analyzed with some statistical methods and the possible relationship between the variables is expressed numerically (Creswell, 2014). In the current study, the exploratory correlational research design, one of the relational research methods, was used. In correlational designs, the relationship between the variables is determined without interfering with the variables, while in exploratory correlational studies, the relationships between the variables are tried to be explored (Büyüköztürk et al., 2015).

### Participants

The target population is defined by Creswell (2012) as “a sampling framework that includes a group of individuals with some common descriptive characteristics that the researcher can identify and work with”. The target population, which is accepted as the universe of the study, consists of fourth grade primary school students attending the schools in a city centre in the north of Turkey in the 2021-2022 school year. The sample of the study consists of 170 fourth grade students attending three primary schools located in a city centre in the north of Turkey in the 2021-2022 school year. The convenience sampling method was used to determine the schools that constituted

the sample group of the study. Convenience sampling is one of the non-random sampling methods in which the data are obtained from a sample easily accessible by the researcher (Büyüköztürk et al., 2015). The reason why the study was conducted on fourth-grade students is that the scales used in the study were aimed at fourth-grade students, as they were thought to be more competent in explaining the solutions of the questions in the number sense test. Of the students participating in the study, 48.2% (f=82) are males and 51.8% (f=88) are females.

### **Data collection tools**

The data of the study were obtained by using two different data collection tools. These two data collection tools used in the study are the “Number Sense Test” developed by Çekirdekci (2015) and the “Metacognitive Awareness Scale Teacher Form” developed by Esmer and Yorulmaz (2017).

The Number Sense Test is a measurement tool with a Cronbach Alpha reliability value of .72, consisting of a total of 11 questions, nine multiple-choice and two open-ended questions, including numbers and operations. A total of 11 items with a discriminant value in the test are gathered under three components with an eigenvalue greater than 1. These components are named as “Knowing the equivalents of numbers and making quantitative reasoning-inferences”, “Calculating the effects of operations using a reference point” and “Knowing the meaning of numbers and thinking flexibly”. Provided that an explanation is made, the lowest score to be taken from the number sense test is 11 and the highest score is 44. In the current study, the Cronbach Alpha reliability value of the number sense test was calculated to be .87.

Another measurement tool used in the study is the “Metacognitive Awareness Scale Teacher Form” adapted by Esmer and Yorulmaz (2017) from the “Metacognitive Awareness Scale for Children (Form A)” developed by Sperling et al. (2002) and adapted into Turkish by Karakalle and Saraç (2007). The metacognitive awareness scale teacher form was prepared for 3rd and 4th grade students and consists of 12 items. The teacher’s form was created by changing the roots and structures of the questions prepared for the students in the original form, and a three-point Likert type (always, sometimes, never) scale was used for respondents to respond the items. The highest score to be taken from the form is 36 and the lowest score is 12. The content and criterion validity of the single-factor form was established and the Cronbach Alpha reliability value was found to be .94. The Cronbach Alpha reliability value of the form in the current study was calculated to be .89.

### **Data collection**

Before the Number Sense Test was started to be answered, one of the researchers informed the students about the purpose and subject of the study and the identities of the researchers. It was emphasized that the explanation part of the questions in the number sense test should not be left blank. Approximately two class hours were given to the students to answer the test. The application of the test was carried out by one of the researchers under the supervision of the classroom teachers. The Metacognitive Awareness Scale Teacher Form was filled in for each student based on the observations of the classroom teachers. The forms were given to the classroom teachers, and after a week, they were taken back from the classroom teachers. The classroom teachers filled the forms by observing the students during this period and taking into account their previous observations of the students.

### **Data analysis**

While analysing the answers given by the students to the number sense test, it was checked whether each item was answered correctly or incorrectly, and then the explanations for the answers to these items were examined. The answers to the items and explanations made were evaluated and scored considering whether they are correct or false and the solution strategy followed and in this way the number sense score was calculated. The strategies used in the solutions of the questions in the test were arranged by taking the coding in the literature as a criterion. The correct answer obtained by using a number sense-based solution (NSBS) in the solutions of the items in the test was given 4 points, and the correct answer obtained by using the rule-operation-based methods (ROBM) was given 2 points. If number sense was used in the solutions of the items, but a wrong answer was found then 3 points were given and 1 point was given if the wrong answer was obtained by using rule-operation-based solutions. If no explanation was given by the students in the solutions of the items or individual generalizations were made while explaining or misconceptions were detected in the explanations (A.C., B.O.A.), then 0 point was given. The items in the metacognitive awareness scale teacher form were prepared in a three-point Likert scale and responses to the items were scored as follows; “always=3”, “sometimes=2”, “never=1”.

A statistical package program was used in the analysis of the data collected with the measurement tools. Before the data were entered into the program, the scores obtained from the metacognitive awareness scale teacher form were entered into the list where the scores taken from the number sense test and the genders of the participants had already been noted.

Missing value analysis was started by checking whether there were missing or incorrect entries for the data group obtained. In the analysis performed to see the rate of missing data, it was observed that there was no missing data in the data group.

Before proceeding to the analysis of the data, the distributions of the data sets were examined. Whether the data sets showed a normal distribution or not was evaluated with reference to the arithmetic mean, mode, median values, and kurtosis and skewness coefficients. In order for the

data to show a normal distribution, the skewness and kurtosis values must be between -1 and +1 (Hair et al., 2013). As a result of the evaluations, it was observed that the mean, mode and median values of the distributions were close to each other, and the kurtosis and skewness values were found to be in the acceptable range. For this reason, it was decided that the data sets had a normal distribution and parametric tests were used in the analysis of the data.

In the first and third sub-problems of the study, descriptive statistics (frequency, percentage calculation, mean, standard deviation) values were calculated, then the general mean scores were calculated and the students were grouped according to their levels. In the second sub-problem, in which whether the students' number sense performances vary significantly by gender was checked, independent samples t-test was used and in the fourth sub-problem, in which whether the students' metacognitive awareness levels vary significantly by gender was checked, Mann-Whitney U, t-test was used. In order to determine whether there is a significant relationship between the students' number sense performances and their metacognitive awareness levels, the Pearson Moment Product Correlation coefficient was calculated in the fifth sub-problem. Regression analysis was used to determine whether the students' metacognitive awareness was a significant predictor of their number sense performance. In the findings section, solutions based on number sense given by the students to the questions in the number sense test and solution examples based on rule-operation-based methods are also included.

## FINDINGS

In this section, the analysis findings obtained as a result of the statistical processes carried out for each sub-problem of the study will be given. The findings related to the first sub-problem of the study "What is the number sense performance of primary school fourth grade students?" are presented in tables including the number sense strategies used in the solutions of the questions in the number sense test, operation-rule-based calculations used in solving the questions in the number sense test, and the arithmetic mean values obtained from the test. The distribution and percentages of the solution strategies used by the students in the solutions of the questions in the number sense test are shown in Table 1.

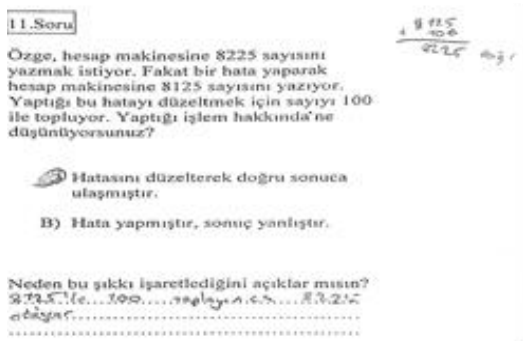
Question Number	Types and Percentages of the Strategies					
	A.C.	NSBS		ROBM		Total (%)
	B.O.A.(%)	Wrong answer (%)	Correct answer (%)	Wrong answer (%)	Correct answer (%)	
1	8.8	50.6	23.5	2.9	14.1	100.0
2	7.6	25.9	35.3	1.2	30.0	100.0
3	0.6	49.4	25.3	17.1	7.6	100.0
4	2.4	21.8	25.9	22.4	27.6	100.0
5	2.4	15.9	76.5	0	5.3	100.0
6	4.7	20.6	66.5	1.2	7.1	100.0
7	1.8	22.4	71.8	0	4.1	100.0
8	13.5	54.1	20.0	4.1	8.2	100.0
9	2.9	18.8	62.4	0	15.9	100.0
10	4.7	27.6	42.4	11.2	14.1	100.0
11	4.1	14.7	40.6	5.3	35.3	100.0

Table 1: Analysis of the strategies used by the primary school fourth graders in the solutions of the questions in the number sense test

When Table 1 is examined, it is seen that in general the students used rule-operation-based methods more and number sense skills less in the solutions of the questions. In the fourth question, the students used rule-operation-based solutions and number sense skills at approximately the same levels. Number sense skills were used the most in the fourth question (50.0%) and the least in the seventh question (4.1%). Examples of the strategies that students used in solving the questions in the number sense test are given in Figure 1.

11<sup>th</sup> question: Özge wants to write the number 8225 into her calculator. But she makes a mistake and writes the number 8125 into the calculator. To correct this mistake she made, she adds the number by 100. What do you think about the operation?

- A) She corrected her mistake and reached the correct result.  
B) She made a mistake, the result is wrong.  
Can you explain why you ticked this option?



1st question: Which of the following is a number smaller than  $\frac{4}{8}$ ? Can you explain how you found the answer?  
*The higher the numerator of the fraction, the smaller it gets*

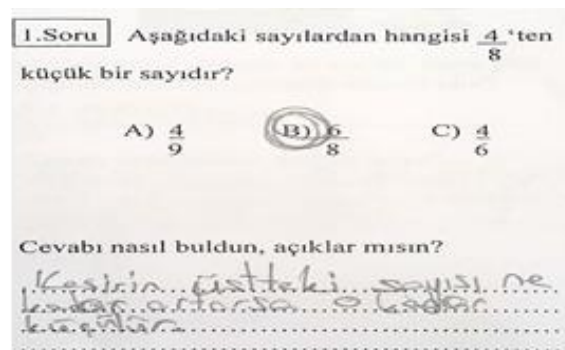
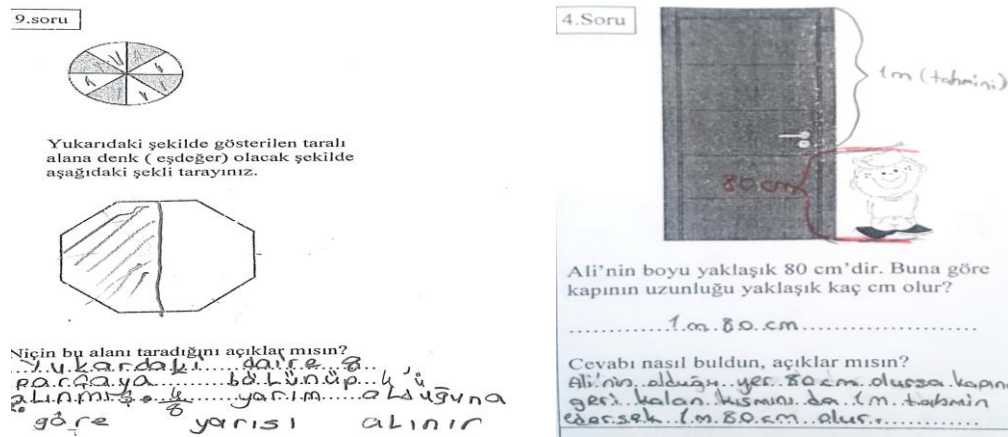


Figure 1: Sample correct and incorrect answers obtained by using the rule-operation-based solution strategies by the students in solving the questions in the number sense test

In the first image given in Figure 1, the eleventh question in the number sense test is shown. In the rule-operation-based correct answer given to this question, the student answered by performing operations that when 100 is added to the given number, the correct result will be obtained. In the first question of the number sense test, which is another example, the student resorted to the rules used to compare fractions. However, here the student misremembered the rule and gave a wrong answer to the question. The student's misremembering of the rule also exemplifies students' misconceptions. The generalization of a feature of the denominator which is used to compare fractions for the numerator can be expressed as the reason for this situation.



**9th question:** Scribble the figure below to be equivalent (equivalent) to the shaded area shown in the figure above. Can you explain why you scribbled this area?

The circle above is divided into eight parts and 4 of them are taken. Since  $4/8$  is half, half is taken

**4th question:** Ali's height is about 80 cm. Accordingly, how many centimetres will the length of the door be approximately?

Can you explain how you found the answer?

If the place where Ali stands is 80 cm, if we estimate the rest of the door as 1 meter, it will be 1 m and 80 cm.

Figure 2: Sample correct and incorrect answers obtained by using number sense-based solution strategies by the students in solving the questions in the number sense test

The student who used the number sense-based solution to solve the ninth question in the number sense test and answered the question correctly realized that the shaded area represented in the image in the question represented  $4/8$  and this value was equal to half. The student divided the blank shape given to him/her into eight equal parts and instead of scribbling the four parts, he/she divided it into two equal parts and took one part. In the fourth question given in Figure 2, another student used the solution based on number sense, but gave an incorrect answer. In order to find the length of the door given in the image, the student made use of the height of Ali standing next to the door and used it as a reference point. As seen in the student's answer, he/she made drawings on the door according to Ali's height, but he/she gave an estimated answer by comparing the remaining part with the reference point. Thus, he/she got the wrong answer.

The arithmetic mean and standard deviation values of the scores obtained from the number sense test by the students participating in the study are given in Table 2.

Students' number sense performances	n	$\bar{X}$	ss
	170	21.75	5.84

Table 2: The arithmetic mean and standard deviation values related to the students' performances in the number sense test

As seen in Table 2, the arithmetic mean of the scores taken by the primary school fourth grade students from the test is  $\bar{X} = 21.75$ . The ratio of the mean obtained to the highest score to be taken from the test corresponds to 49.43%. The arithmetic mean of the students participating in the study is approximately the half value of the highest score to be taken from the test. According to this mean, it can be said that the number sense performances of the students are at the "medium" level. In Figure 3, the categorical distributions of the number sense performances of the students participating in the study are given.

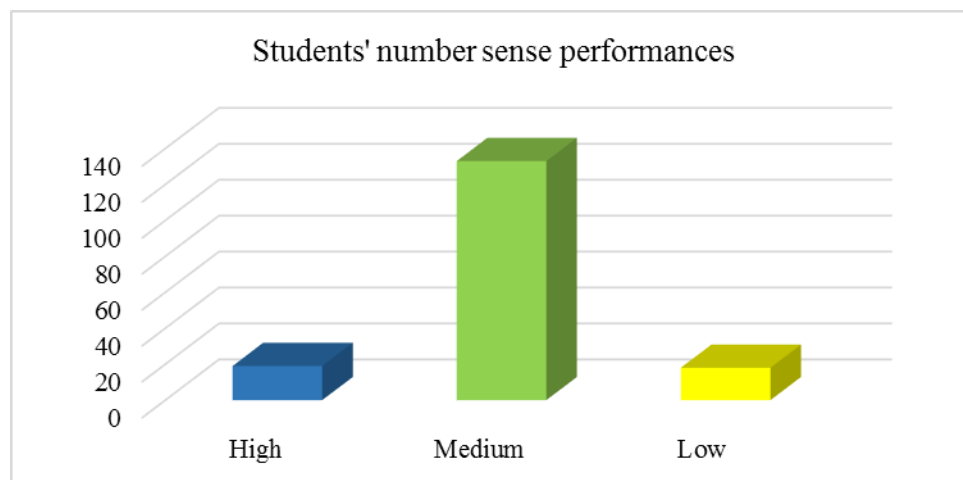


Figure 3: Categorical distributions of the students' number sense performances

When Figure 3 is examined, it is seen that 11.17% of the students ( $f=19$ ) performed at a "high" level according to the scores they got from the 11-question number sense test, while 78.23% ( $f=133$ ) performed at a "medium" level and 10.58% ( $f=18$ ) at a "low" level.

The second sub-problem of the study is "Do primary school fourth grade students' number sense performances vary significantly by gender?" T-test analysis was used to determine whether the number sense performances vary significantly by gender and the analysis results are given in Table 3.



Gender	n	$\bar{X}$	ss	t	sd	p	Cohens's d
Female	88	21.92	5.78	-.386	168	.70	.05
Male	82	21.57	5.93				

Table 3: Results of the t-test conducted to determine whether the students' number sense performances vary significantly by gender

The table shows the results of the t-test to determine the relationship between the number sense performances of the female and male students in the number sense test. When the results of the analysis were examined, no statistically significant difference was found between the number sense performances of the female and male students [ $t_{(113)} = -.386, p > .05$ ].

In addition, the Cohen's d value calculated to find the effect size of gender on number sense performance was found to be .05. A Cohen's d value of .05 is interpreted as a medium effect size (Büyüköztürk, 2007; p. 48). According to this result, the gender variable has a medium effect on the number sense performance of the primary school 4th grade students. This effect does not create a significant difference between the genders. It can be said that gender explains about 5% of the total variance in students' number sense performance.

Within the context of the third sub-problem of the study, the arithmetic mean and standard deviation values obtained as a result of the analyses conducted to determine the metacognitive awareness levels of the students are given in Table 4.

Students' metacognitive awareness	n	$\bar{X}$	ss
	170	30.81	5.93

Table 4: Arithmetic mean and standard deviation values related to the students' metacognitive awareness

When Table 4 is examined, it is seen that the arithmetic mean of the metacognitive awareness scores of the primary school fourth grade students is  $\bar{X} = 30.81$ . The mean of the metacognitive awareness scores of the primary school fourth grade students is seen to be high. This finding indicates that the primary school fourth grade students have high metacognitive awareness.

In the fourth sub-problem of the study, the state of variation of the students' metacognitive awareness levels depending on the gender variable was examined. In order to determine the analyses to be made, the normality distribution was examined with the Kolmogorov-Smirnov Normality test and the distribution graph was examined. As a result of the analyses, it was observed that the data group did not have a normal distribution in terms of metacognitive awareness levels by gender and the Mann-Whitney U t-test, one of the non-parametric tests, was used to determine

the status of variation depending on the gender variable. The results of the analysis are given in Table 5.

Gender	n	Mean Rank	Sum of Ranks	U	p	Cohens' d
Female	88	85.97	7565.00	3567.00	.897	.009
Male	82	85.00	6970.00			

Table 5: The state of variation in the students' metacognitive awareness levels by gender

As can be seen in Table 5, there is no statistically significant difference between the scores of the female students and male students according to the Mann Whitney U-test results ( $U = 3567.00$ ,  $p < .05$ ).

In the fifth sub-problem of the study, the relationship between the two variables was questioned. Thus, the fifth sub-problem of the study is worded as follows; "Is primary school fourth grade students' metacognitive awareness a significant predictor of their number sense performance?" In order to find an answer to this sub-problem, the normality of the data groups was examined and it was observed that the data showed a normal distribution and the Pearson product-moment correlation coefficient test was performed and the analysis results are presented in Table 6.

		Number sense	Metacognitive awareness
Number sense	r	1	.34**
	p		.00
Metacognitive awareness	r	.34**	1
	p	.00	

\*\*  $p < .01$

Table 6: Correlation values showing the relationship between the students' number sense performance and their metacognitive awareness

The result of the simple linear correlation analysis performed to reveal whether there is a relationship between fourth grade students' number sense performance and their metacognitive awareness is presented in Table 6 and here it is seen that there is a positive and significant correlation between the two variables ( $r = .34$ ,  $p < .01$ ). Based on the relationship between the two variables, it can be stated that as students' metacognitive awareness levels increase, their number sense performance will increase, and as their metacognitive awareness levels decrease, their number sense performance will decrease.

Simple regression analysis was used to reveal whether the metacognitive awareness of the primary school fourth grade students was a significant predictor of their number sense performance. The results of the regression analysis performed to determine the common variance between metacognitive awareness and number sense are given in Table 7.

Variable	B	Standard error	$\beta$	t	p	$R^2$	F
Constant	11.41	2.24	-	5.09	.00	.11	22.07
Metacognitive awareness	.33	.07	.34	4.69	.00		

Table 7: Simple linear regression analysis results related to the prediction of the students' number sense performance

As a result of the regression analysis performed, the explained variance value, which is expressed as the coefficient of determination ( $R^2$ ) and used to interpret how much of the situation that occurs in one variable is explained by the other variable, was found to be .11 ( $F= 22.07$ ,  $p<.05$ ). When the coefficient of determination is considered, it can be said that 11% of the total variance in the number sense performance of the primary school fourth grade students is due to metacognitive awareness. Moreover, according to the t-test results related to the significance of the regression coefficients, metacognitive awareness can be expressed as a significant predictor of number sense performance ( $p<.05$ ).

## DISCUSSION

While 11.17% ( $f=19$ ) of the primary school fourth grade students performed at a high level according to the scores they got from the 11-item number sense test, 78.23% ( $f=133$ ) performed at a medium level and 10.58% ( $f=18$ ) at a low level. In addition, the ratio of the mean obtained to the highest score to be taken from the test corresponds to 49.43%. The arithmetic mean of the students participating in the study is approximately the half value of the highest score to be taken from the test. According to this mean, it can be said that the number sense performances of the students are at the "medium" level. Thus, it can be said that very few of the primary school students have a high level of number sense performance, while the majority of them have a medium and low level of number sense performance. This result is similar to the results of the study conducted by Uluçay (2021) to determine the number sense performances of primary school first grade students. However, the results of the current study are different from the results by Yang and Li (2008), Çekirdekci (2015), Can (2019), Gökçe, Güner and Baştuğ (2022) showing that the number sense performances of primary school students are low. They are also different from the studies conducted at different grade levels and in different countries such as Reys and Yang (1998), Yang and Huang (2004), Yang and Li (2008), Singh (2009), Facun and Nool (2012), Purnomo et al., (2014), Cheung and Yang (2020), Yang and Sianturi (2021) also showing that number sense performances are low. The reasons why different results are obtained in the current study may be that the majority of the primary school students in the study group received pre-school education and that the classroom teachers offered a learning environment that would support the number sense performance since their professional experience was ten years or less. In addition, it was revealed that the primary school fourth grade students mostly used rule-operation-based methods in solving the questions in the number sense test and they used solutions related to number sense

skills less. This result is similar to the results of the studies conducted by Yang (1995), Reys and Yang (1998), Yang (2005), Mohamed and Johnny (2010), Gülbağcı Dede (2015), Çekirdekci, Şengül and Doğan (2020). The reason why the students used more rule-operation-based methods may be that primary school teachers rely more on the activities in the mathematics textbooks during the lesson because, in the study conducted by Cheng and Wang (2012), it was stated that mathematics textbooks have an important place in the development of number sense performance. Another important factor in the emergence of this situation may be the fact that the objectives in the Primary School Mathematics Curriculum include number sense components. In the study conducted by Çekirdekci and Yorulmaz (2021), it was stated that all of the number sense components should be addressed in mathematics curriculums implemented in primary schools in Turkey and that increasing the objectives related to the number sense components in the curriculum would reduce the use of rule-operation-based methods and increase the number sense performance.

It was determined that the number sense performance scores of the primary school fourth grade students who participated in the study did not vary significantly depending on gender. Most of the research results in the literature support this situation (Birgin & Peker, 2022; Can, 2019; Çekirdekci, 2015; Gökçe, Güner & Baştuğ, 2022; Menon, 2004; Reys & Yang, 1998; Yang & Li, 2008). Thus, it can be argued that the gender of primary school fourth grade students does not cause a significant change in their number sense performances, and therefore, gender is not a factor that creates a significant difference. As number sense performance is related to mathematics achievement (Çekirdekci, Şengül & Doğan, 2016; Mohamed & Johnny, 2010; Olkun, Mutlu & Sarı, 2017; Reys & Yang, 1998; Yang, Li & Lin, 2008), it can be argued that male and female students can be equally successful in mathematics (Ministry of National Education [MoNE], 2019).

It was revealed that the mean score of the metacognitive awareness of the primary school fourth grade students is 30.81, and this result indicates that their metacognitive awareness is at a “high” level. This result is similar to the study conducted by Akaydın, Yorulmaz and Çokçalışkan (2020) for primary school third and fourth grade students. In line with this result, it can be said that primary school fourth grade students are adept at controlling and directing their mental processes, determining new strategies and developing self-awareness within the context of metacognitive awareness. Primary school students’ having high metacognitive awareness affects their academic achievement in the teaching process to a great extent (Coutinho, 2007; Landine & Steward, 1998; Young & Fry, 2008). When considered in this context, it can be stated that it is an inevitable fact

that students whose metacognitive awareness is developed exhibit higher performance in primary school lessons.

In the study, the metacognitive awareness scores of the primary school fourth grade students were found to be not varying significantly depending on the gender variable. This result is similar to the results obtained from the studies conducted by Özsoy et al. (2010), Hashempour, Ghonsooly and Ghanizadeh (2015), Jaleel (2016), Akaydin, Yorulmaz and Çokçalışkan (2020). Thus, it can be said that the gender variable does not play an important role in the formation of metacognitive awareness. The reasons why the gender factor does not make a difference in the formation of students' metacognitive awareness may be the fact that primary school programs have a structure that provides gender equality, the learning environments are suitable for gender differences, and the behaviors of primary teachers do not differ according to the gender of students.

It was revealed that there is a low level of positive correlation between the number sense performances and metacognitive awareness of the primary school fourth grade students ( $r=.34$ ,  $p<.01$ ). The use of different strategies in the problem-solving process by primary school teachers has an important place in the development of students' metacognitive skills. Metacognition has an important place in using information, distinguishing necessary and unnecessary information and making predictions in the problem-solving process (Schneider & Artel, 2010). Thus, it can be said that metacognition affects the solution of mathematical problems and is also related to arithmetic performance according to Bellon, Fias and Smedt (2019). It was also stated by Lee, Ng and Yeo (2019) that when metacognition develops, the sense of number will also improve, and that they interact with each other. The close relationship of problem-solving and arithmetic in mathematics with metacognition enables us to express that there is a positive relationship between number sense, which is one of the basic components of mathematics, and metacognition. The result of the study indicates that increasing the metacognitive awareness of primary school students will also increase their number sense performance. In addition, it was revealed that metacognitive awareness has a predictive power of 11% in the formation of number sense performances of primary school fourth grade students. In this context, it can be said that metacognitive awareness significantly predicts the development of number sense performance. It was also stated by Vo et al. (2014) that metacognitive skills specific to the numerical domain predict mathematical knowledge. It was also stated by Carr (2010) that the development of metacognitive skills affects students' number sense performance, which is in line with the result obtained in the current study. Therefore, it can be stated that it is important to develop metacognitive awareness in order to increase the number sense performance. As a result, number sense is a way of thinking that requires skills such as perception, attention, flexible thinking, fluency, automaticity and strategy

development. Since metacognition is a cognition that controls these skills, it affects number sense performance.

Number sense is a type of problem-solving competence related to following rule sets and deciding on the appropriate strategy, especially in problem-solving situations, rather than adhering to the algorithm (Fosnot & Dolk, 2001; Putrawangsa, Evendi, & Hasanah, 2020). In addition, problem-solving approaches such as prediction and mental processing, which contribute to controlling the outcome of the problem, involve higher-level thinking skills because they enable students to develop their own strategies (Carroll, 1996; Tsao, 2004). According to the research findings, doing activities such as making predictions, performing mental operations, and developing strategies about numbers and operations in order to improve students' number sense skills will also indirectly affect metacognitive awareness. In this context, conducting activities within the classroom that promote a sense of numbers will contribute to the development of metacognitive awareness. For example, an individual trying to mentally find the answer to the  $168+75$  operation may use the completion strategy. To do this, the number 25 required to complete 75 to 100 can be reached by separating the number 168 into its parts and obtaining an equivalent representation. He can reconstruct 168 as  $125+43$  and get the correct answer from his mind using  $(125+75) + 43$ . Here, various strategies such as flexible use of numbers, equivalent representation of numbers, reasoning, operational knowledge, and the ability to make evaluations will be included in the process (Çekirdekci, 2023). In order to gain this flexibility about numbers, the relationship between small numbers must be well understood. For example, in order to develop number sense skills, activities related to dot counting, which is the ability to perceive the patterned arrangements of objects that make up the multitude in a very short time and express the number of these objects without counting, can be done in the classroom environment. The number 6 will be perceived in a short time through dot counting, and the number 5 will be seen as  $(2+2+1, 3+2)$  during perception. Thus, the student will be able to use the situation appropriate for the context and develop his own strategy. Number talks can also be included in classroom activities to help students develop strategies. Number talks is a teaching strategy that involves classroom conversations around purpose-built calculation problems.

## CONCLUSION

In this study, it was determined that the number sense performances of the primary school fourth grade students were medium. It can be said that very few of the primary school students have a high level of number sense performance, and the majority of them perform at a low or medium level. In addition, in this study, it was revealed that number sense performance scores did not vary significantly depending on the gender variable. It was determined that the metacognitive awareness of the primary school fourth grade students participating in the study was at a high level and that the metacognitive awareness did not vary significantly by gender. It was revealed that there is a

positive correlation between the number sense performances and metacognitive awareness of the primary school fourth grade students. In addition, metacognitive awareness was found to predict the students' number sense performances at a low level. In light of the results of the current study, it can be suggested to design activities to develop metacognitive awareness and to include these activities in the classroom in the mathematics teaching process.

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## Ethnomathematics Learning Model Based on Motifs of *Dayak Ngaju* Central Kalimantan

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*Abstract: Mathematics education in the Dayak Ngaju, Central Kalimantan community played an essential part in its inherent culture, including batik/carving/painting motifs. The moral messages embedded in these motifs serve as the philosophy of life for the Dayak Ngaju community. This study aimed to describe influence of implementing an ethnomathematics learning model based on these motifs on students' learning outcomes and responses. The research used a mixed-method approach with an explanatory design. The research subjects consist of two 9th-grade classes, which were treated to un-purposed selection out of eleven classes in one of the public junior high schools in Palangka Raya, Central Kalimantan, Indonesia. The instruments included post-test, questionnaire (quantitative data), worksheet and interview guide (qualitative data). The results indicated the implementation of this model influenced the learning outcomes. The students also responded positively to the implementation of the model. They felt challenged to solve problems, answer questions, or complete tasks in the worksheet. Furthermore, students were motivated to develop a positive attitude by creating practical videos showcasing positive attitudes and paintings of the Dayak Ngaju motifs.*

Keywords: Dayak Ngaju motifs, ethnomathematics, learning outcomes, local wisdom, students' worksheet

### INTRODUCTION

Mathematics is a universal language that is merged within cultures. The integration of mathematics and cultures is known as ethnomathematics (Prahmana & Istiandaru, 2021). Research had shown that learning based on ethnomathematics is more readily accepted by students. The condition caused by the culture was an inseparable part of student's everyday life. Additionally, ethnomathematics made learning more engaging by applying it in the real-life context of the local community. Such learning conditions could enhance mathematics learning outcomes

(D'Ambrosio, 1985; Risdiyanti & Prahmana, 2020). Therefore, mathematics education needs to be united by local culture and wisdom.

One of the communities that still practices local culture and wisdom in everyday life is the Dayak Ngaju, one of the largest ethnic groups in Central Kalimantan, Indonesia. For example, in addition to resolving societal issues using the official laws of Indonesia, the customary leader of the Dayak Ngaju community, known as *demang*, also plays a role in their resolution. *Jipen* (fines/penalties), imposed by the *demang*, are applied to any individual who violates customary law. Furthermore, local wisdom such as *pintar tuntang harati* (intelligent and well-behaved), *belum bahadat* (living ethically), *handep* (collaboration), and *isen mulang* (perseverance) are educational norms within the Dayak Ngaju society (Usop et al., 2012).

Several studies aimed to explore ethnomathematics within the culture of the Dayak communities, in general, on the island of Borneo. First, a study focused on the *Dadas Bawo* Dance of the Dayak Ma'ayan tribe in Central Kalimantan. The findings revealed that the hand movements, footwork, and formations in the *Dadas Bawo* Dance incorporated concepts of angles (acute, obtuse, and right angles), parallel lines, geometric shapes (triangles and circles), and geometric transformations (reflection, rotation, and translation) (Mangkin, Agustina, & Huriaty, 2021). Second, ethnomathematics research explored the *nugal* tradition of the Dayak Sebaruk community in Jentawang, Ketungan Hilir, West Kalimantan. The result indicated the tradition of *nugal* involved mathematical concepts such as mathematical logic, sequences and series, geometric transformations (translation and reflection), and distances between points (Dian, 2021). Third, the research focused on the wedding tradition of the Dayak Kanayatn community in the Toho sub-district, Mempawah Regency, West Kalimantan. The findings revealed that the dominant mathematical concept within this tradition was counting. This concept was evident when the ceremony leader recites *asa dwa talu ampat lima anam tujuh* at the beginning of the *nyangahatn* activity. The use of seven as a number represents completeness (Eka, Sugiatno, & Munaldus, 2021).

There was only one study on ethnomathematics regarding the motifs of the Dayak Ngaju in Central Kalimantan. The motifs consisted of *batang garing* (tree of life), *tingang* bird (rhinoceros hornbill), *dandang tingang* (tail feathers of *tingang*), *jata* (dragon), plants, and animals. The research findings showed that the motifs incorporated objects and mathematical concepts. The concept of  $x$  –axis reflection was present in the motifs of *tanduk muang* (deer horns) and *dandang tingang*. The  $y$  –axis reflection was observed in the motifs of *batang garing*, *tingang* bird, *taya* tree, and *tanduk muang*. A  $180^{\circ}$  rotation around the point (0,0) was visible in the motifs of the *taya* tree and *buntut kakupu gajah* (elephant's tail). The translation was present in the *jata* motif. The motifs also included mathematical objects such as hexagons in the motifs of *tanduk muang*, or circles in the *jata* and the *batang garing* motifs (Mairing et al., 2022).

The inserted ethnomathematics to mathematics education can come in two ways. First, ethnomathematics can be a medium for a context for mathematical problems or projects. Second,

ethnomathematics can be beneficial to explore and develop a deeper understanding of mathematical concepts. For example, students can explore the geometric transformations to be present in the motifs using the GeoGebra application. In groups, students can set the motif as a background in GeoGebra. They can then identify four points within the motif, where two points come as the results of a geometric transformation from the other two. Furthermore, students can independently discover the moral values/local wisdom embedded in the motif and then apply the values in everyday life.

Several studies intentionally revealed to apply the learning approaches. First, a study examined the effectiveness of PBL (problem-based learning) in ethnomathematics using the motif of *sasirangan* fabric to improve students' problem-solving abilities. The results showed that students who learned with PBL-ethnomathematics had better problem-solving skills than those with traditional methods (Hidayati & Restapaty, 2019). Second, a study focused on practices of ethnomathematics-based instructional materials for junior high school students. The findings indicated that the instructional materials were suitable for mathematics learning (Gusfitri et al., 2022). Third, a study aimed to enhance creative thinking through ethnomathematics based in Surakarta batik. The results showed that this approach could improve students' creative thinking skills (Faiziyah et al., 2020). Fourth, a study aimed to enhance students' better understanding through ethnomathematics-based learning. The results revealed that students who learned using ethnomathematics had better comprehension than those who did not use ethnomathematics learning (Herawaty et al., 2019).

Searching using Google Scholar over the past ten years shows no studies integrating ethnomathematics in the Dayak Ngaju culture into mathematics education. As the research reveals, ethnomathematics in this context carries moral messages that need to be developed by students in the classroom. The moral messages of the ethnomathematics were embedded in the *batang garing* motif, for instance. The messages show that the universe is God's creation, so living beings should remember the Creator by caring for humanity. Also, it displays that the environment is formed as a unified entity. The motif of the *tingang* bird symbolizes purity, greatness, power, firmness, courage, loyalty, and responsibility. The motif of *dandang tingang* represents humanizing oneself by living with politeness and ethics (*belum bahadat*) towards fellow humans, plants, and animals. The *jata* motif serves as a reminder for humans to continue doing good throughout their lives. The motifs of plants and animals carries the moral value of humans caring for their environment and providing essential benefits in life (Mairing et al., 2022).

Therefore, the researchers intended to implement a learning model that integrated ethnomathematics in the Dayak Ngaju motifs. The implementation aimed to describe the process and learning outcomes of applying the model. Thus, the research questions formulated for this study are "Were the learning outcomes of students who learned using the ethnomathematics models based on the Dayak Ngaju motifs higher than 75 (scale 0-100)?" and "How did students respond to the learning with the model?". The results of this research will be beneficial for teachers



who intend to implement the model in developing higher-order thinking skills and positive attitudes.

## METHOD

### Research Approach and Design

The researchers used a mixed-methods research with an explanatory design. The collection of the data of the research was divided into two stages. The quantitative data were collected first, followed by qualitative data. The emphasis of the research findings was placed on the quantitative data, while the qualitative data was to provide explanations for the quantitative finding presentation. The research process involved several stages: formulating research questions for both quantitative and qualitative data, collecting quantitative data, collecting qualitative data, analyzing quantitative data, analyzing qualitative data, and producing the research report (Lodico et al., 2006).

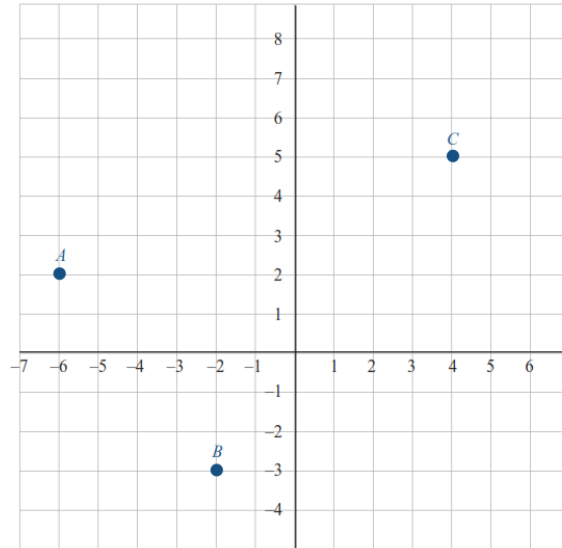
The researchers implemented the ethnomathematics learning model to 9th-grade students in a public junior high school in Palangka Raya Central Kalimantan, Indonesia. The population for this research comprised all 9th-grade students in the selected junior high school. The students in the 9th grade learned the topic of geometric transformations incorporated in the ethnomathematics of the Dayak Ngaju motifs. The population framework consisted of eleven study groups in the 9th grade. In practice, the researchers applied the model to two study groups, which served as the samples for this research. The research sample was randomly selected using clustered random sampling technique, resulting in classes IX-10 and IX-11 as the groups for applying the model. The number of subjects in each class was 29 students.

### Collecting Quantitative Data

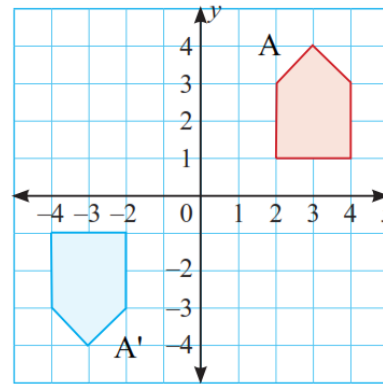
The instruments used to collect the quantitative data were a post-test and a student response questionnaire. The post-test was applied to gain data on students' learning outcomes which later became the answer to the first research question, while the questionnaire was intended to obtain students' responses to the ethnomathematics learning model. The responses from the questionnaire became the answer to the second research question. The indicators and the questions in the questionnaire could be seen in Table 1. The post-test was as follows:

1. The given point  $C(a, b)$  is reflected by line  $x = 2$ . The image is point  $C'(5, 7)$ . Determine the value of  $a + b$ ! Explain your answer!
2. The line  $AB$  is defined by the coordinates of points  $A(-2, 2)$  and  $B(1, 3)$ . After the dilation, the resulting line becomes  $A'B'$ , where the point  $A'$  has coordinates  $(-8, 8)$  and  $B'$  has coordinates  $(4, 12)$ . Determine the scale factor that was used! Explain your answer!

3. A tiger is hunting a deer in the forest. Based on the observation, the deer is located at point  $A$ , and the tiger is at point  $B$ . The deer then moves to point  $C$ .
- Determine the translation that represents the movement of the deer from point  $A$  to point  $C$ . Explain!
  - If the tiger uses the same translation as the deer, will the tiger catch the deer? Explain!
  - Determine the translation that the tiger needs to perform in order to catch the deer! Explain!



4. Shape  $A'$  is the result of a rotation with center  $(0, 0)$  from shape  $A$ .
- Determine the angle of rotation!
  - Prove that the rotation is correct by taking 2 points on shape  $A$  and show that their corresponding shadows on shape  $A'$  are in correct positions.



Questions number 1, 2, 3, and 4 represented the concepts of geometric transformations studied by the students, namely reflection, dilation, translation, and rotation.

Indicators	Number and Statements in the Questionnaire	Questions in the Interview Guide
1. Using the ethnomathematics model could help the students to develop a meaningful understanding of geometric transformations.	1. Using the learning model and the ethnomathematics worksheet has helped me understand the concept of geometric transformations. 6. I can understand the concept of geometric transformations after studying using the ethnomathematics model.	Do the learning model and the ethnomathematics worksheet help you understand the concept of geometric transformations? Why?
2. Using the ethnomathematics model could help	2. Using the model and the ethnomathematics worksheet helped	Do the learning model and the ethnomathematics worksheet help you develop

<b>Indicators</b>	<b>Number and Statements in the Questionnaire</b>	<b>Questions in the Interview Guide</b>
students to develop HOTS (higher-order thinking skills).	me develop good mathematical skills.	higher-order thinking skills? Why?
3. The cultural context and the motifs in the worksheet foster students' motivation to learn and to complete questions, tasks, problems, and projects written in the worksheet.	7. My mathematical abilities improved after studying using the learning model and the ethnomathematics worksheet. 3. The cultural context and the Dayak Ngaju motifs in the ethnomathematics worksheet foster my motivation to learn and complete tasks in the worksheet. 8. I am interested in learning geometric transformations using the ethnomathematics worksheet because it is associated with the culture of the Dayak Ngaju community.	1. Does the cultural context in the model and ethnomathematics worksheet motivate you to learn geometric transformations? Please explain! 2. Which part of the model or the worksheet do you find most interesting? Please explain!
4. Using the learning model encourages students to be actively engaged in their learning.	4. Using the model and the ethnomathematics worksheet encourages me to engage actively and participate in group discussions. 9. I became actively involved in independent and classroom learning by using the model and the ethnomathematics worksheet.	Do using the model and the ethnomathematics worksheet motivate you to actively engage in learning in the classroom? Please explain!
5. Using the ethnomathematics model helps students develop their positive attitudes.	5. Using the model and the ethnomathematics worksheet helps me to appreciate others and my environment. 10. I am motivated to develop positive attitudes after learning using the ethnomathematics worksheet.	Does the moral message in Dayak motifs encourage you to practice it in living your life? Provide an example!
6. The sentences in the worksheet are easy to read, comprehensible, and unambiguous.	11. The sentences in the ethnomathematics worksheet are well-structured and understandable. 16. The sentences in the ethnomathematics worksheet are incomprehensible and confusing.	Are the sentences written in the ethnomathematics worksheet comprehensible? Explain!
7. The mathematical terms, symbols, and formulas are well-written in the worksheet.	12. Terminology, symbols, and mathematical formulas in the ethnomathematics worksheet are well-written.	Are the mathematical symbols in the ethnomathematics worksheet well-written?

Indicators	Number and Statements in the Questionnaire	Questions in the Interview Guide
	17. The ethnomathematics worksheet contains notations and formulas of geometric transformations that are comprehensible.	Can you provide an example?
8. The figures and the motifs in the worksheets are well-depicted.	13. The figures and the motifs in the ethnomathematics worksheet are well-depicted. 18. The ethnomathematics worksheet contains ambiguous and difficult-to-understand figures or motifs.	Are the figures and motifs in the ethnomathematics worksheet clear and visually appealing? Please explain!
9. The provided links in the worksheets are accessible to see the figures and motifs.	14. The links between the figures and the motifs in the ethnomathematics worksheet are easily downloadable. 19. The provided links in the ethnomathematics worksheet can be easy to click to see the figures and motifs.	Why are the images and motifs in the instructional materials not easily accessible using a link? What if we use QR codes instead?
10. The worksheet is applicable for learning activities in groups.	15. The ethnomathematics worksheet is suitable for learning in groups. 20. Completing tasks in the ethnomathematics worksheet becomes enjoyable in a group.	Is the ethnomathematics worksheet suitable for group discussions? Explain!

Table 1: Indicators, Statements in the Questionnaire, and Questions in the Interview Guide

A validity test was conducted by finding the correlation between each question and the total score. The result was a  $p$ -value of less than .05 for each question. Therefore, each question in the questionnaire was valid. Two questions were included for each indicator in the questionnaire to determine reliability. The test result showed a reliability coefficient of  $r = .86$ , indicating high reliability of the questionnaire (Ghufron, 2011). The valid and reliable instrument was filled out by the students through a Google Form at the following link: <https://intip.in/etnomatematika>.

### Collecting Qualitative Data

The collection of qualitative data was conducted in two ways. First, the researchers provided the ethnomathematics worksheet (<https://intip.in/LKPDetnomatematika>) to the students in both classes. The worksheet was completed with guidance for the students to explore the concepts of geometric transformations, routine exercises, mathematical problems, and projects. Additionally, the worksheet included links to learning videos intended to support the guidance. The solutions by the students in the worksheet served as qualitative data used to address the second research question. Second, the researchers conducted interviews using questions in the interview guide listed in Table 1. Some additional questions depended on the student's answers during the interviews. The interviews involved two students representing classes IX-10 and IX-11 to obtain more data.

### Quantitative Data Analysis

The quantitative data in the research consisted of post-test scores and questionnaire scores. Both data were presented in tables or summarized using measures of the central tendency and the data dispersion. Furthermore, the post-test scores were analyzed to conclude the following hypotheses:

$$H_0: \mu = 75$$

$$H_0: \mu > 75$$

where  $\mu$  represented mean of the post-test scores. The result of the hypothesis test was intended to answer the first research question.

### Qualitative Data Analysis

The qualitative data in the research consisted of students' solutions to questions, problems, or projects written in the worksheet, and the interview transcripts. Both qualitative and quantitative data became the answer to the first research question. The transcripts were coded for data triangulation. A conclusion should be supported by two different subject codes (source triangulation) or two different data sources, namely the interview transcripts and the questionnaire responses (method triangulation). Considering both triangulations assured the credibility of the obtained conclusion.

### RESULTS

The implementation of the ethnomathematics learning model on the topic of geometric transformations was applied to five meetings in grade IX. The learning activities using the model in IX-10 and IX-11 were applied by a mathematics teacher. The learning activities consisted of two phases. They were before-class independent learning and face-to-face learning in the classroom. During the independent learning phase, the students engaged in self-directed learning groups using the ethnomathematics worksheet. The worksheet covered the activities to discover geometric transformation concepts, links to YouTube videos, routine exercises, mathematical problems, and projects.

In the face-to-face learning phase, the following activities took place.

- (a) The students began the learning session with a prayer led by one of the students.
- (b) One of the students read aloud the learning objectives stated in the worksheet.
- (c) The teacher motivated the students by showing videos included in the worksheet.
- (d) The teacher facilitated classroom discussions to help students understand concepts that they had not grasped during their independent learning or to address any difficulties encountered in the worksheet.
- (e) The teacher presented problems with cultural or local contexts (ethnomathematics) in the worksheet.
- (f) The students in their groups presented and explained their solutions to the class.

- (g) The teacher facilitated further classroom discussions to deepen students' understanding and encourage them to find alternative answers or solutions.
- (h) The students engaged in reflection and concluded.
- (i) The teacher assigned independent learning tasks to be completed by the students before the next meeting.

One of the advantages emerging in the implementation was the students learning independently in groups to answer questions or complete given tasks in the worksheet to discover specific concepts or formulas. For example, the students learn using YouTube videos to find the results of reflections over the  $y$  –axis from several given points. The results were written and recorded in the worksheet. The students discovered patterns from the previous examples of reflections on three points to find the formula of reflection over the  $y$  –axis (Figure 1).

**1. Pencermian terhadap Sumbu-y**  
 Berdasarkan ciri-ciri sebelumnya, gambar bayangan/hasil pencerminan terhadap sumbu-y (sumbu tegak lurus) menggunakan aplikasi GeoGebra. Ikuti caranya pada video YouTube di link: <https://youtu.be/ivB8OsfVM0>, lalu kalian ulangi lagi pencerminan tersebut menggunakan laptop atau smartphone masing-masing. Catat hasilnya pada tabel berikut.

Titik	Titik Hasil Pencermian (Titik Bayangan)
A(3,5)	A'(-3, 5)
B(2,2)	B'(-2, 2)
C(7,3)	C'(-7, 3)

Keterangan: titik bayangan biasanya disimbolkan menggunakan tanda "aksen", sebagai contoh A'.

Misalkan diberikan titik  $P(x,y)$ , maka titik hasil pencerminan terhadap sumbu-y (jawab berdasarkan pola tiga jawaban sebelumnya pada tabel di atas):

$$P(x,y) \xrightarrow{\text{pencermian terhadap sb-y}} P'(-x, y)$$

**1. Reflection over the-y-axis**

Based on the previous characteristic, draw the resulting image from reflection over the axis (the vertical axis) using GeoGebra. Follow the means on the YouTube video at the link: <https://youtu.be/ivB8OsfVM0>, then you repeat the reflection using your laptop or smartphone. Write the results in the following table.

Points	The Image Points of Reflection over the y-axis
A(3,5)	A'(-3,5)
B(2,2)	B'(-2,2)
C(7,3)	C'(-7,3)

Note: the image points are usually symbolized using ' sign, for example A'.

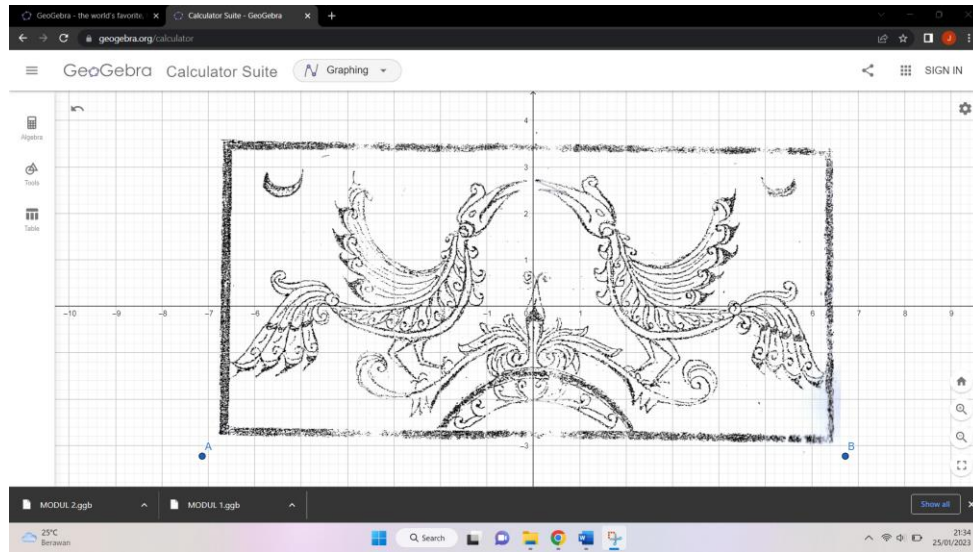
Let given point  $P(x,y)$ , then the image point of reflection over the  $y$ -axis is (use pattern in the answers in the table)

$$P(x,y) \xrightarrow{\text{reflection over the y-axis}} P'(-x, y)$$

(Translated into English)

Figure 1: Students' Answer on the Worksheet to Find the  $y$  –Axis Reflection Formula

The discovered formula was applied to find geometric transformation concepts in ethnomathematics in the Dayak Ngaju motif (Figure 2).



6. Apakah lukisan motif tersebut memuat **refleksi**? .....**YA**.....  
Jika ya, buktikan. Cara pembuktiannya dapat dipelajari melalui video YouTube di link: <https://youtu.be/pRhZ0vhW07I>. Tentukan dua titik lainnya menggunakan GeoGebra untuk memperkuat bukti bahwa refleksi ada pada motif burung *tingang*. Tuliskan buktinya pada tabel berikut.

Titik	Jenis Refleksi	Titik Bayangannya
$(-1,4; 1,9)$	Refleksi terhadap sumbu-y	$(1,4; 1,9)$
$(-1,5; 1,4)$	Sumbu-y	$(1,5; 1,4)$
$(-7; 8)$	Sumbu-y	$(7; 8)$

6. Does the *tingang* bird painting contain reflection? **Yes**  
If yes, prove it. You can learn how to prove it through a YouTube video at the link: <https://youtu.be/pRhZ0vhW07I>. Determine two other points using GeoGebra to strengthen the evidence that the reflection is in the painting. Write the evidence in the following table.

Points	Type of Reflection	The Image Points
$(-1,4; 1,9)$	reflection over the y-axis	$(1,4; 1,9)$
$(-1,5; 1,4)$	y-axis	$(1,5; 1,4)$
$(-7; 8)$	y-axis	$(7; 8)$

(Translated into English)

Figure 2: Students' Solution on the Worksheet to Find Geometric Transformation Concepts in *Tingang* Bird Motif (Salilah, 1984)



Figure 3: Examples of *Dayak Ngaju* Motif Paintings was Made by the Students

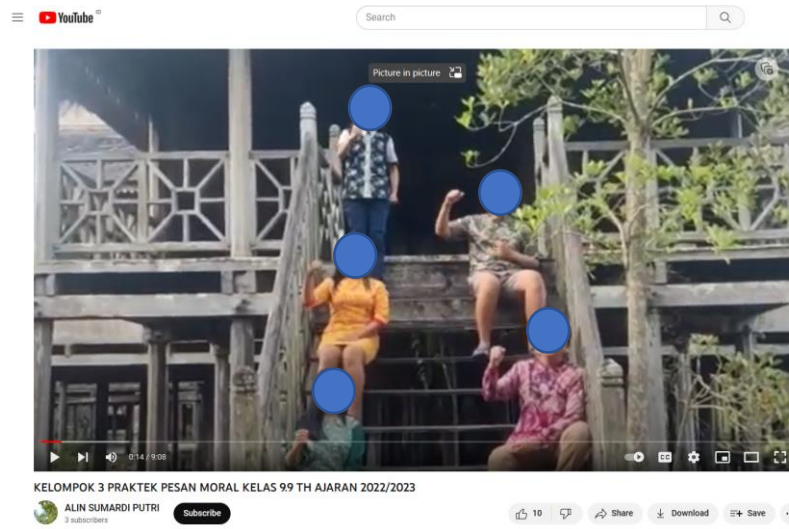


Figure 4: Video of Moral Messages Practice in *Dayak Ngaju* Motifs

Furthermore, the students worked in groups to complete a project. The project was the students creating a drawing of the Dayak Ngaju motif accompanied by the moral message conveyed by the specific motif, and making a practical video based on the message. Examples of the drawings created by the students are presented in Figure 3. One of the sample videos is available to download from the link: <https://youtu.be/ythCAVfB820> (Figure 4).

The students in both classes completed the post-test conducted after the implementation of the ethnomathematics learning model using the Dayak Ngaju motifs. The result showed the overall average score was 84.61 (scale of 0-100). Furthermore, 75% of the students' scores were equal to or above 78.57 (Q1), while the remaining scores ranged from 64.29 (minimum) to 78.57 (Table 2). An example of a student's answers to questions 1-4 is presented in Figure 5.

Classes	N	Mean	StDev	Minimum	Q1	Median	Q3	Maximum
IX-10	29	87,44	7,31	64,29	85,71	92,86	92,86	92,86
IX-11	29	81,77	11,55	64,29	71,43	85,71	92,86	100
All	58	84,61	9,99	64,29	78,57	85,71	92,86	100

Table 2: Summary of Learning Outcomes After Implementing the Model



Jawab: →  
1. Pembahasan  
Rumus yang digunakan untuk pencerminan terhadap sumbu  $x = h$  adalah sebagai berikut.  
 $A(x, y)$  dicerminkan terhadap sumbu  $x = h$  adalah  
 $A'(2h - x, y)$   
keterangan:  
 $A$  : titik A  
 $A'$  : titik A setelah pencerminan  
 $x$  : titik pada sumbu x  
 $y$  : titik pada sumbu y  
 $h$  : bilangan dari sumbu x  
Pada soal diatas titik C dicerminkan terhadap garis  $x = 2$  dan menghasilkan  $C'(5, 7)$ , berarti di ketahui  $h = 2$   
untuk mencari titik C digunakan rumus:  
 $x' = 2h - x$   
 $5 = 2(2) - x$   
 $5 = 4 - x$   
 $x = -1$   
jika dicerminkan terhadap sumbu  $x = 2$  titik y dan y' besarnya sama  
berarti titik C terletak pada  $C(-1, 7)$ , berarti  $a = -1$  dan  $b = 7$   
nilai  $a + b = -1 + 7$   
nilai  $a + b = 6$

Jawaban Nomor 1 dari Siswa SS (Kelas IX-10)

2) Faktor skala pada sumbu  $x = (-8)/(-2) = 4$   
Faktor skala pada sumbu  $y = 8/2 = 4$   
Jadi faktor skala yang digunakan adalah 4.

Jawaban Nomor 2 dari Siswa CJ (Kelas IX-10)

3. A(-6, 2) C(4, 5) B(-2, -3)  
A(-6, 2) → C(4, 5)  
 $-6 + x = 4$      $2 + y = 5$   
 $x = 10$          $y = 3$   
Pasangan bilangan translasi dari A ke C adalah  $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 10 \\ 3 \end{pmatrix}$   
B. Tidak bisa. Harimau tidak dapat menangkap mufa  
B(-2, -3) → B'(8, 0) titik C tidak sama dengan B'  
C. B(-2, -3) → C(4, 5)  
 $-2 + x = 4$      $-3 + y = 5$   
 $x = 6$          $y = 8$   
Pasangan bilangan translasi adalah  $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6 \\ 8 \end{pmatrix}$  agar harimau dapat menangkap mufa  
A. 180° (beratus delapan puluh derajat)

Jawaban Nomor 3 dari Siswa NP (Kelas IX-11)

Answer

The used formula for reflection over line  $x = h$  as follows.

$A(x, y)$  be reflected over line  $x = h$  is  $A'(2h - x, y)$ .

Note:

$A$  : point A

$A'$  : point A after reflection

$x$  : point on  $x$ -axis

$y$  : point on  $y$ -axis

$h$  : number in line  $x$ .

As the point C is reflected over line  $x = 2$  and resulting  $C'(5, 7)$ , it means  $h = 2$ . To find point C', the formula is used:

$$x' = 2h - x$$

$$5 = 2(2) - x$$

$$5 = 4 - x$$

$$x = -1$$

If reflection over line  $x = 2$ , point  $y$  and  $y'$  are the same, so point C is  $(-1, 7)$ . Thus,  $a = -1$  and  $b = 7$ . The value  $a + b = -1 + 7 = 6$ .

The Answer for Number 1 from Student SS (Class IX-10)

2) Scale factor over  $x$ -axis =  $(-8)/(-2) = 4$ .  
Scale factor over  $y$ -axis =  $8/2 = 4$ .  
Thus, used scale factor is 4.

The Answer for Number 2 from Student CJ (Class IX-10)

3)  $A(-6, 2), C(4, 5), B(-2, -3)$

$$\text{a) } A(-6, 2) \xrightarrow{\begin{pmatrix} x \\ y \end{pmatrix}} C(4, 5)$$

$$\begin{aligned} -6 + x &= 4 & 2 + y &= 5 \\ x &= 10 & y &= 3 \end{aligned}$$

Paired number of the translation from A to C is

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 10 \\ 3 \end{pmatrix}.$$

b) No, the tiger can not catch the deer.

$$B(-2, -3) \xrightarrow{\begin{pmatrix} x \\ y \end{pmatrix}} B'(8, 0). \text{ Point C and B' are different.}$$

$$\text{c) } B(-2, -3) \xrightarrow{\begin{pmatrix} x \\ y \end{pmatrix}} C(4, 5)$$

$$\begin{aligned} -2 + x &= 4 & -3 + y &= 5 \\ x &= 6 & y &= 8 \end{aligned}$$

Paired number of the translation is  $\begin{pmatrix} 6 \\ 8 \end{pmatrix}$  for the tiger can catch the deer.

The Answer for Number 3 from Student NP (Class IX-11)

4. A. Sudut =  $180^\circ$   
 B.  $(2,1)$  dirotasi  $R(0, 180^\circ)$  adalah  $(-2, -1)$   
 $(2,3)$  dirotasi  $R(0, 180^\circ)$  adalah  $(-2, -3)$   
 $(3,4)$  dirotasi  $R(0, 180^\circ)$  adalah  $(-3, -4)$   
 $(4,3)$  dirotasi  $R(0, 180^\circ)$  adalah  $(-4, -3)$   
 $(4,1)$  dirotasi  $R(0, 180^\circ)$  adalah  $(-4, -1)$

Jawaban Nomor 4 dari Siswa NM (Kelas IX-11)

- 4) A. The angle =  $180^\circ$ .  
 B.  $(2,1)$  rotated  $R(0, 180^\circ)$  is  $(-2, -1)$   
 $(2,3)$  rotated  $R(0, 180^\circ)$  is  $(-2, -3)$   
 $(3,4)$  rotated  $R(0, 180^\circ)$  is  $(-3, -4)$   
 $(4,3)$  rotated  $R(0, 180^\circ)$  is  $(-4, -3)$   
 $(4,1)$  rotated  $R(0, 180^\circ)$  is  $(-4, -1)$

The Answer for Number 4 from Student NM (Class IX-11)

(Translated into English)

Figure 5: Examples of Students' Solutions to Numbers 1 – 4

The conclusion on the research hypothesis was drawn using the nonparametric Wilcoxon test. This kind of test was selected because the result of the Kolmogorov-Smirnov normality test yielded a  $p$  –value of less than .01, which indicated that the students' outcomes data were not normally distributed with a 95% confidence level. The Wilcoxon test result was a  $p$  –value of 0, meaning it was less than 0.05. The result indicated that the students' outcomes using the ethnomathematics learning model were greater than 75 by a 95% confidence level (Table 3).

Sample	N for Test	Wilcoxon Statistic	P-Value
NILAI	58	1535,00	0,000

Table 3: Wilcoxon Test Result for the Learning Outcomes using the Model

The sample of students' learning outcomes was influenced by the students' positive responses toward the indicators of effectiveness, readability, and practicality of the ethnomathematics model and the worksheet based on the Dayak Ngaju motifs. The average percentage of strongly agree or agree with responses to the indicators were 96.3%, 85.4%, and 95.8%, respectively (Table 4). However, there was one aspect that scored less than 80%, which was the easy access to the figures, the motifs, or the learning videos through the links in the worksheet. The aspect was improved by replacing the link-based accessibility with a QR code (Figure 6).

The positive response was also shown by the interview results of two students, namely AD (a female student from class IX-10) and CH (a male student from class IX-11). Both students responded positively to all the indicators in Table 1 (source triangulation) except for indicator 4. Both students stated that the ethnomathematics learning model could help students understand geometric transformations and develop higher-order thinking skills. Student AD said, "It helps make learning more varied, so it's not boring... we also learn about the connection between culture and mathematics, which makes it more interesting, challenging, and exciting". Student CH also stated, "It helps us learn geometric transformations because the learning incorporates the culture of batik, so we become more aware of the culture in Central Kalimantan... dilation in the batik

pattern requires us to think at a higher level". The interview transcript deals with the questionnaire result (method triangulation).

No	Indicators and Aspects	4 or 3	2 or 1
<b>A Model Effectiveness</b>			
1	Using the ethnomathematics model could help the students develop a meaningful understanding of geometric transformations.	97,9	2,1
2	Using the ethnomathematics model could help students develop HOTS.	91,7	8,3
3	The cultural context and motifs in the worksheet foster students' motivation to learn and to complete questions, tasks, problems, and projects given in the worksheet.	95,8	4,2
4	Using the learning model encourages students to engage in their active learning.	97,9	2,1
5	Using the ethnomathematics model helps students develop positive attitudes.	100,0	0,0
<b>C Model Readability</b>			
6	The sentences in the worksheet are easy to read, understandable, and unambiguous.	81,3	18,8
7	The mathematical terms, symbols, and formulas in the worksheet are well-stated.	91,7	8,3
8	The figures and the motifs in the worksheets are well-depicted.	91,7	8,3
9	The figures and the motifs are accessible through links provided in the worksheets.	77,1	22,9
<b>B Model Practicality</b>			
10	The worksheet is applicable for group learning.	95,8	4,2

Note: 4 = strongly agree, 3 = agree, 2 = disagree, and 1 = strongly disagree

Table 4: Summary of the Students' Responses

6. Apakah lukisan motif tersebut memuat **refleksi**? .....

Jika ya, buktikan. Cara pembuktiannya dapat dipelajari melalui video di QR code di samping. Tentukan dua titik lainnya menggunakan GeoGebra untuk memperkuat bukti bahwa refleksi ada pada motif burung tingang. Tuliskan buktinya pada tabel berikut.



Titik	Jenis Refleksi	Titik Bayangannya
(-1,4; 1,9)	Refleksi terhadap sumbu-y	(1,4; 1,9)
( ... , ... )	...	( ... , ... )
( ... , ... )	...	( ... , ... )

6) Does the *tingang* bird painting contain reflection? .....

If yes, prove it. You can learn how to prove it through the video on the QR code on the side.

Determine two other points using GeoGebra to strengthen the evidence that the reflection is in the painting. Write the evidence in the following table.



Points	Type of Reflection	The Image Points
(-1,4; 1,9)	reflection over the y-axis	(1,4; 1,9)
( ... ; ... )	...	( ... ; ... )
( ... ; ... )	...	( ... ; ... )

(Translated into English)

Figure 6: Improvement Access after Implementation using QR Code

Furthermore, both students stated that the ethnomathematics learning model made students motivated and active in their learning. Student AD stated, "It motivates students to actively engage in learning because it is something new, exciting, and it makes us interested and challenged, which in turn makes us active". Student CH stated, "The ethnomathematics model makes mathematics interesting because it introduces new elements, and it is not merely learning mathematics but also exploring the culture of Central Kalimantan". Moreover, AD mentioned, "The interesting part is when we look for the moral message behind, for example, the *batang garing* motif, we explore its moral message, so we become more aware of this culture. The *batang garing* motif has a deep moral message that symbolizes the different worlds, the upper and lower worlds, which are different, but it is the same still for us as humans". Similarly, CH stated, "The most interesting part for me when I read about the discussion regarding the *tingang* bird, it had certain characteristics that symbolize something".

At indicator 4, there was a difference in responses. Student AD stated that the major constraint was it was difficult to access the provided links in the worksheet, while CH stated that the links were accessible. The links aimed to help students complete the exercises or tasks found in the worksheet. When the researchers was asked about their thoughts if the links could be replaced with QR codes, student AD said, "Yes, it is accessible, there is an application for it." Student CH mentioned, "I have scanned QR codes in the book before". Therefore, both students agreed that using QR codes would be more convenient, as they had experience using QR codes and found them easier to access.

## DISCUSSION

Mathematics is a part of the culture inherent in the community. The integration is called ethnomathematics. A learning model based on ethnomathematics needs to be carefully designed so students can construct understanding meaningfully and develop higher-order thinking skills. Learning experiences or mathematics problems that students are involved with should be more realistic, reflecting everyday life, including the culture embedded in the community (Payadnya et al., 2021). Problems linked to cultural contexts helped students understand the problems better and formed mental images of the problem condition (Suherman & Vidákovich, 2022). This understanding and mental imagery helped students develop plans and implement them to solve the given problems (Ramadhani et al., 2022). The process helped students develop reasoning skills (Nursyahidah & Albab, 2021).

This research showed that the ethnomathematics learning model based on the Dayak Ngaju motifs influenced the learning outcomes. The impact was indicated by the average post-test scores being more than 75 (scale 0-100). The combination of several learning models with ethnomathematics could increase students' ability to solve mathematical problems. The increase occurred through self-reflection on problem-solving plans, monitoring, and evaluation of thinking processes

(Herawaty et al., 2018). Some of these models included contextual learning (Nur et al., 2020), realistic mathematics education (Lubis et al., 2021), inquiry-based learning (Putri & Junaedi, 2022), and problem-based learning (Zaenuri et al., 2020). Learning materials or worksheets based on ethnomathematics could also enhance students' critical thinking and creative thinking (Faiziyah et al., 2020; Imswatama & Lukman, 2018) and students' mathematical literacy (Agusdianita et al., 2021). Furthermore, the probing-prompting learning model based on ethnomathematics could improve students' abilities in mathematical communication and representation (Hartinah et al., 2019; Riwut et al., 2003; Widada et al., 2019).

The influence occurred because the students responded positively to the ethnomathematics learning model. The research showed that students responded to be motivated to engage in active learning because the learning became interesting. The practices of ethnomathematics-learning resources could also encourage students to learn actively (Imswatama & Lukman, 2018). This motivation and active involvement in learning help the students construct a meaningful understanding of mathematical concepts (Herawaty, Sarwoedi, et al., 2019; Herawaty, Widada, et al., 2019). The meaningful understanding became a factor that influences students' ability to solve problems (Mairing, 2018).

## CONCLUSION

Mathematics education is inseparable from the context of everyday life, including the culture inherent in the community. The integration of culture in mathematics education is known as the ethnomathematics learning model. One of the cultures in the Dayak Ngaju community in Central Kalimantan is the batik/carving/painting motifs. The motifs are intertwined with daily life and carry specific moral messages, which serve as the philosophy of life for the Dayak Ngaju community. The Ethnomathematics learning model based on the motifs encouraged the students to actively engage in learning mathematics. The cultural context within the model made the learning process interactive and motivated students to solve questions, tasks, problems, and projects in the worksheet used in the model. Using the model also encouraged the students to acquire the moral messages embedded in the motifs. Therefore, the students responded positively to the implementation of the model. Such a learning environment had an impact on the students' learning outcomes. Moreover, the students produced products such as videos showing they practiced the moral messages (positive attitudes) and some paintings of the Dayak Ngaju motifs.

Expected future research will focus on developing a valid, effective, and practical ethnomathematics model based on the Dayak Ngaju motifs. The model should be applied to a broader school context and involve more participants, including the field operational trials. Additionally, ethnomathematics in the Dayak Ngaju community is not only limited to the motifs but also extends to the traditional dances, the technology of the Dayak Ngaju community, the traditional buildings called *betang*, and the calculation of societal days in the Dayak Ngaju community. The ethnomathematics learning model based on a holistic culture is needed by students to enhance high-order thinking skills and to acquire positive attitudes rooted in the culture (local wisdom).

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## Cognitive Map: Diagnosing and Exploring Students' Misconceptions in Algebra

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*Abstract: This study diagnoses and explores year eight students' misconceptions and mistakes in algebra. An in-depth exploration of students' misconceptions were carried out by conducting interviews based on the students' work as outlined in a cognitive map. Based on the cognitive maps, it can be seen that the students emphasize procedural knowledge rather than a comprehensive understanding of the concept. The students had misconceptions about the variable, which is the core of algebra. The students misinterpreted variables, though they sometimes succeeded in determining variable values from algebraic expressions or equations. The students made mistakes in translating literal sentences into algebraic forms, but they succeeded in completing an algebraic expression. In general, the students were able to operate algebraic forms procedurally. The exploration found that these misconceptions or errors were likely caused by their incomplete knowledge of arithmetic. This problem had an impact when they work on the transition from arithmetic to algebra concepts.*

Keywords: Misconceptions, Cognitive Maps, Algebra

### INTRODUCTION

The Concept image is an individual's overall cognitive structure of a concept. The concept image includes mental image, nature and processes involved in its formation (Tall & Vinner, 1981). Concept images are built over years, originate from individual experiences, and can change when individuals encounter new stimuli. This results in the possibility of incompatibility of students' concept image with the formal definition (Tall & Vinner, 1981). Several studies have shown that individual concept images differ from the formal definition, which is known as a misconception, triggering cognitive conflict (Kang et al., 2010; Ramsburg & Ohlsson, 2016).

Misconceptions are part of the conceptual structure that students have. Holmes et al. (2013) state that misconceptions are part of students' cognitive structures that are not scientifically accurate.

Misconceptions are also described as student conceptions that produce systematic patterns of errors (Smith et al., 1994). When connected with students' prior knowledge, Ojose (2016) argues that misconception is a misunderstanding and misinterpretation caused by students' 'naive theory.' This misunderstanding will cause cognitive conflict in students when faced with the correct concept. Hansen (2020) states that misconceptions indicate a conflict between students' understanding and new mathematical concepts learned (Hansen, 2020). In this study, misconceptions are defined as the incompatibility of students' beliefs with correct scientific concepts.

Misconceptions are different from errors. An error is caused by something that is not patterned and inconsistent. For example, an error happens because of haste and lack of thoroughness (Tooher & Johnson, 2020). However, misconception indicates a person's wrong understanding, and it occurs repeatedly because it is resistant and can produce wrong results (Tooher & Johnson, 2020; Khazanov, 2008). As a result, misconceptions negatively affect learning because they can potentially trigger errors (Zielinski, 2017). Students with misconceptions or insufficient conceptual knowledge perform less optimally in mathematics (Booth & McGinn, 2016). Apart from hindering students' ability to solve mathematical misconceptions, it also hinders students' understanding of new concepts (Stothard, 2021).

According to Irawati et al. (2018), misconceptions can potentially cause errors in forming generalizations about a concept, affecting the learning process in general. Misconceptions among students will be an obstacle to students' success in learning mathematics as a whole, and in the long run, will impact future job opportunities (Ladson-Billings, 1998). Therefore, it is clear that misconceptions negatively influence students' abilities and performance in learning. The results of a literature review conducted by Jamaludin & Maat (2020) on 30 articles that focus on misconceptions show that many students still experience misconceptions in algebra. Students with algebraic misconceptions will likely encounter difficulties using algebra to solve problems (Cline, 2020). Misconceptions about algebra limit students' success in mathematics, thereby limiting their success in school at every level (Zielinski, 2017). Students will also experience difficulties in other branches of mathematics and other related subjects (Efriani et al., 2019; Joanna & Jacquelynn, 2019; Vargová, 2020).

Algebra is one of the main branches of mathematics and has many applications in our daily life. Algebra is closely related to other branches of mathematics, such as probability, geometry, and calculus (Yew et al., 2020). Students are introduced from arithmetic to abstract when learning algebra (Irawati et al., 2018). Algebra uses letters and symbols to represent unknown numbers and describe the mathematical relationship between quantities in formulas and equations (Tooher & Johnson, 2020). Some teachers and students think that algebra is a complicated lesson in mathematics (Das, 2020; Mulungye, 2018; Natalia et al., 2016). One of the factors causing this difficulty is due to the abstract nature of algebra (Rakes & Ronau, 2019). In particular, the abstractness of algebra hinders students from constructing object representations (Kieran, 1992).

Preliminary data shows that students have indications of misconceptions. Students experience confusion in operations involving algebraic forms, for example,  $3x + 4 = 7x$  or  $3x + 2x = 5x^2$ . Students also mistakenly simplify  $\frac{a+b}{a}$  dan  $\frac{a+x}{b+x}$ , into  $b$  dan  $\frac{a}{b}$ . This finding is in accordance with Mulungye (2016), who states that students have misconceptions in operating algebraic forms and mistakenly simplify algebraic expressions. Students also diagnosed misconceptions in the application of the distributive rule. Students work on  $(a + b)^5$  as  $a^5 + b^5$ , and  $3(a + b)^2$  as  $3a^2 + 3b^2$ . When the results of their work regarding the problem were clarified, the students reasoned that they worked based on their intuition (no basis). In Polynomials and Exponents, students also experience misconceptions as they stated that  $y^4 + y^4 = y^8$  and  $\sqrt{x^2 + y^2} = x + y$ . According to Das (2020), most algebraic misconceptions stem from students' misconceptions about arithmetic (Das, 2020).

When and how will someone's misconceptions disappear? This is an important part to be studied. Students with misconceptions tend to continue to experience misconceptions which cause difficulties in learning mathematics (Stothard, 2021). However, misconceptions experienced by students are sometimes not detected by the teacher, so the misconceptions persist for a long time. (Jong et al., 2017; Stothard, 2021). Teachers who are able to diagnose and correct misconceptions can encourage meaningful student learning (Rakes & Ronau, 2019; Vaughn et al., 2020). Therefore, teachers need to detect students' misconceptions to correct and improve them through appropriate learning instructions (Ojose, 2016). Teachers must recognize misconceptions to take preventive and corrective actions for learners (Deringöl, 2019). According to Mulungye et al. (2016), teachers need help identifying misconceptions and knowing the causes of misconceptions in the learning process. If information about student misconceptions is available, it will be easy for teachers to prevent and overcome misconceptions. Ocall (2017) suggests that teachers correct students' misconceptions first before introducing new concepts to students.

This study aims to detect, diagnose and explore students' misconceptions in algebra. Misconceptions in students are often reflected in the results of their work when solving problems (Arnawa, 2019; Russell et al., 2009). Therefore, the thinking students do in solving math problems can be seen from their work in solving problems. The difference between this study and previous research is that the researchers in the current study seek to explore students' algebraic misconceptions by displaying them on a cognitive map.

Cognitive maps are mental images and concepts built to visualize and assimilate information (Sammut-Bonnici & McGee, 2015). Gutiérrez et al. (1991) explained that cognitive maps can be used to examine the uniqueness of one's thinking processes in-depth. Cognitive maps are used to externalize students' thinking, which can help investigate tasks related to students' thinking (Chen et al., 2021). According to Rakes & Ronau (2019), when teachers dig deeper into students' thoughts, they can make interventions that correct misconceptions and strengthen their

understanding of concepts (Rakes & Ronau, 2019). Cognitive maps can be used to explore students' ways of thinking and discover why they experience misconceptions.

This cognitive map can describe causal relationships of various phenomena and concepts and can be modelled (Subanji & Nusantara, 2013). Thus, errors caused by student misconceptions can be corrected with this model. Jacobs (2003) revealed that cognitive maps can show the direction of students' thinking, so that they can be used as a guide for subsequent needs. This study aims to diagnose and explore students' misconceptions and errors in algebra. The exploration was carried out by exploring students' ways of thinking and describing the process of the results of students' work in a cognitive map.

## LITERATUR REVIEW

Over the decades, many research papers about misconceptions and errors in algebra have been published. Many students enter high school with algebraic misconceptions that will limit their mathematics success and future educational attainment (Booth & McGinn, 2016; Zielinski, 2017). It is believed that when students build concepts in learning sometimes they develop concepts that are incomplete, immature, still alternative, and transitional (Mathaba & Bayaga, 2021). The concepts built by these students are entirely correct, partially correct, or entirely inconsistent with scientific concepts. Although students use intuition and a process of trial and error while guessing math results and checking them, they build algebraic concepts independently (Kshetree, 2021). When students carry out algebraic operations, it is said that they carry out thinking processes through mental activity in the student's brain. This process is not only for generating numbers and abstract mathematical concepts but also as an essential skill in quantitative analytical and logical thinking (Faizah et al., 2022).

Learning algebra that lacks meaning usually builds procedural skills without considering conceptual understanding, which can cause errors and misconceptions (Mulungye, 2018). Students are expected to know their knowledge, procedures, and concepts to use the knowledge and apply it in solving problems or assignments. The learner is expected to understand the concept first and then follow the procedure. Learners tend to forget concepts and prioritize procedures. Likewise, it is stated that procedural and conceptual errors are caused by a lack of knowledge or misunderstandings from students about the concept itself (Edogawatte, 2011). Students lacking information or understanding of algebra will result in errors. For example, failure to determine the formula used or error in executing operation signs, which are the basis of algebra, will lead to wrong solutions (Mathaba & Bayaga, 2021).

Several studies have identified a number of misconceptions that students tend to have about algebraic content. The misconceptions experienced by most students relate to equations, negativity, variables, fractions, order of operations, and functions (Booth & McGinn, 2016). Edogawatte (2011) further explores misconceptions by dividing them into three major concepts in algebra: variables, algebraic expressions, and equations. A categorization that places more

emphasis on the process of algebraic operations was identified by Zielienski (2018) and Ojose (2018), namely: negative function input, exponential rules, distribution of negative signs, distribution errors, random cancellation (simplification of algebraic fractions), fractional rules and negative exponential rules.

## COGNITIVE MAP

A person has cognitive maps since he/she is in the womb, which can be seen from the sensorimotor system (Ahmed et al., 2020). The Cognitive Map describes the interconnection between knowledge, problems, procedures, and concepts from the results of one's thinking (Subanji, 2015). Cognitive maps can reveal students' thoughts when exploring a problem (Jonassen, 2003; Toth et al., 2002). Cognitive maps represent complex thinking from various perspectives on science and its context (Chen et al., 2021; Garoui & Jarboui, 2012). Cognitive maps illustrate the formation of learned knowledge, so that a complete schema is formed in a person (Bottini & Doeller, 2020). Therefore, the cognitive map referred to in this study is an image/scheme used to represent individual cognitive structures.

This study uses the cognitive map framework of Peña, et al. (2007). According to Peña, cognitive maps are the result of a cognitive process which is also called cognitive mapping. Cognitive mapping is an internalization-externalization process that represents the qualitative knowledge of an individual and is believed to be of correct value according to the individual's point of view. Thus, this knowledge is unilateral, uncertain, imprecise, unstable, incomplete, and not universal. Basically, a cognitive map is a graph consisting of concepts (C) and causal relationships ( $\rightarrow$ ) which can be seen in Figure 1 (Peña et al., 2007).

Figure 1 explains that, according to Peña, et al., the causal relationship between the two concepts can be direct and indirect. The concept of ( $Ca \rightarrow Cz$ ) is said to be directly causal if there is no other concept between  $Ca$  and  $Cz$  (as shown in Figure 1<sup>a</sup>). However, if there is at least one concept, for example  $Cb$  appearing between the two ( $Ca \rightarrow Cb \rightarrow \dots Cz$ ), then  $Ca$  and  $Cz$  have an indirect causal relationship (Figure 1<sup>b</sup>). The positive or negative sign on the arc indicates the direction of the causal relationship. Positive means that  $Ca$  affects  $Cz$  positively. That is, when  $Ca$  is positive,  $Cz$  will be positive as well. If the sign after the arc is negative ( $Ca \rightarrow - Cz$ ), then the relationship is inverse. This means that if  $Ca$  is negative,  $Cz$  will be positively affected and vice versa.

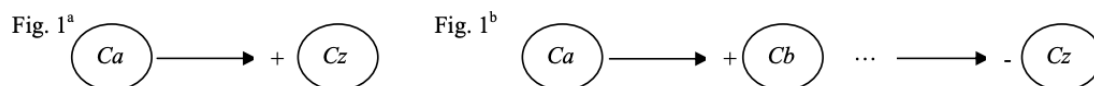


Figure 1: Formal cognitive map model (Peña, et al. (2017))

## METHOD

This research is qualitative. According to Cohen et al. (2018), one of the goals of qualitative research is to describe and explain. Cohen suggests types of research questions in qualitative research, namely, questions relating to: (a) describing a state of affairs, their causes and how this state of affairs is maintained; (b) describing the process of change and its consequences (Cohen et al., 2018). This study will diagnose and explore students' misconceptions. The exploration results will be displayed in the form of a cognitive map to see the occurrence of misconceptions through students' thinking. According to Cangelosi (2013), a combination of tests and interviews can be used to diagnose misconceptions. The diagnostic test can be carefully designed using multiple-choice questions that include a misconception response, a correct response, and a detractor (Russell, O'Dwyer, & Miranda; (2009).

In this study, the data collection was done by giving an algebraic diagnostic test to 68 students of year VIII (13-14 years old) in Banjarmasin. The algebra diagnostic test consists of 24 multiple-choice items adapted from Blessing (2004). The misconception score represents the total number of errors in the student's answers out of the 24 questions. From the results of the diagnostic tests, two students with the highest percentage of misconceptions were selected as research subjects. Furthermore, the research subjects were given an Algebraic four-tier test in the form of questions containing three main categories in Algebra: algebraic expressions, algebraic operations, and models to explore misconceptions from the results of diagnostic identification.

The four-tier diagnostic test is the development of four-level multiple choice questions that are used to see students' understanding of the concept. The first level is a multiple-choice question, and the student must choose an answer, while the second level is students' level of confidence in choosing answers. The third level is the student's reason for answering the question, which is a choice of reasons that have been provided. One open reason is also included. The fourth level is students' level of confidence in choosing reasons (Gurel et al., 2015). The profile of the four-tier-test categories can be seen in Table 1.

## FINDINGS AND DISCUSSION

The results of the diagnostic test showed that 45 out of 68 students had indications of misconceptions with varying scores, ranging from 20.8% to 58.3%. The biggest misconception lies in students' understanding of variables as 41 students experience this misconception. Most students understand the variable as an object. Another misconception is students' understanding of the concept of the equal sign associated with solving algebraic equations. These two concepts are core concepts in early algebra, which makes it easier for students to understand algebra to the next stage (Xie & Cai, 2022).

This study recruited two students with the highest misconception scores as research subjects, namely S1 and S2. Furthermore, S1 and S2 subjects were given a four-tier test to explore their

misconceptions. Based on the four-tier test, it was decided that S1 was included in the MC (misconception) category, and S2 was included in the LK (lack of knowledge) category. Furthermore, in-depth interviews were conducted with S1 and S2 to explore their responses in the diagnostic and the four tier-tests.

Tier 1	Tier 2	Tier 3	Tier 4	Decision
Correct	Sure	Correct	Sure	SC
Correct	Sure	Correct	Not Sure	LK
Correct	Not Sure	Correct	Sure	LK
Correct	Not Sure	Correct	Not Sure	LK
Correct	Sure	Wrong	Sure	Misconception
Correct	Sure	Wrong	Not Sure	LK
Correct	Not Sure	Wrong	Sure	LK
Correct	Not Sure	Wrong	Not Sure	LK
Wrong	Sure	Correct	Sure	LK
Wrong	Sure	Correct	Not Sure	LK
Wrong	Not Sure	Correct	Sure	LK
Wrong	Not Sure	Correct	Not Sure	LK
Wrong	Sure	Wrong	Sure	Misconception
Wrong	Sure	Wrong	Not Sure	LK
Wrong	Not Sure	Wrong	Sure	LK
Wrong	Not Sure	Wrong	Not Sure	LK

Table 1. Four tier-test categories according to Gurel (2015)

### S1 Data Presentation

In the initial identification of the algebraic diagnostic test, S1 achieved a misconception score of 45.8%. Based on the four tier-test, S1 was included in the Misconception (MC) category. S1 is 14 years old and in grade 8 of junior high school. The following is a description of the misconceptions or errors made by S1.

#### 1. Variables representing the number of letters in the expression

S1 was indicated to have a misconception about the concept of variables. S1 assumed that the value of the variable (letter) represents the number of letters in the given algebraic expression. This was obtained from the results of S1 work and interviews.

Q : For  $a + 5$ , what do you think  $a$  denotes?

S1 : 1 (number 1)

Q : Why 1? Not something else, for example, 2 or 3?

S1 : because there is only one  $a$

(The above script was a translation from the Indonesian language. All interview sessions were conducted in the Indonesian language)

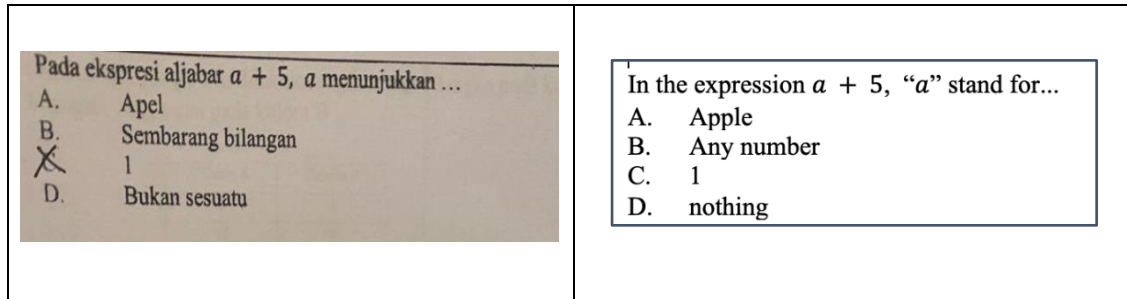


Figure 2: S1's answer sheet

This indication is strengthened by the students' answers on the four-tier test. S1 believes in the wrong concept: that the variable is the number of letters in the algebraic form. Based on this, the researchers wanted to confirm whether S1 can show variables when given any algebraic form.

Q : For  $3a + 8$ , can you tell the variable in the expression?

S1 : "a" Ma'am

Q : Then what is 3 usually called?

S1 : Hmm.. 3 is the coefficient

Q : What about 8? Is it also a coefficient?

S1 : No, because there are no "letters."

Q : Letters? What do you mean? Can you explain?

S1 : There is no x, ma'am, for  $3a$  there is an x, namely a

Q : Well, if 8 is not a coefficient, what is it usually called?

S1 : (long) I don't know ma'am.. just a number.

The results of the interviews show that S1 can identify variables and coefficients, though S1 has a misconception about the definition of a variable.

## 2. Different letters must represent different values

The misconception about the variable concept that S1 has is believing that different letters (variables) cannot represent the same number.



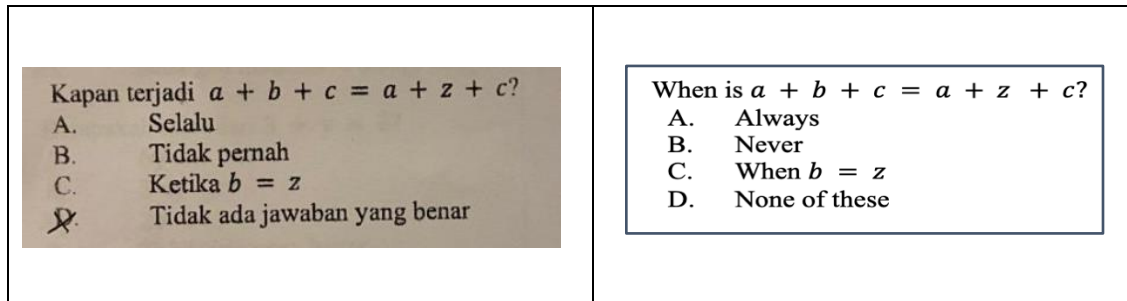


Figure 4. S1's answer sheet

P : in the equation  $a+b+c=a+z+c$ , when do you think the left side is the same as the right side?

S1 : It can't be the same, ma'am.

Q : So you don't think the two equations will ever be the same?

S1 : No

Q : Why?

S1 : Because  $a$  and  $c$  are the same, but  $b$  and  $z$  are different

Q :  $a$  and  $c$  are the same? Does that mean that both sides have the same  $a$  and  $c$ ?

S1 : Yes ma'am, and  $b$  is not the same as  $z$

The statement ' $b$  is not the same as  $z$ ' shows that S1 believes that each different letter must represent a different number. S1 has incomplete knowledge that letters can represent any number, so two different letters can represent the same number. This is because S1 tends to view letters as objects, so when presented with different letters, S1 sees a different object (Yew et al., 2020).

### 3. *Determining variable values from simple algebraic equations*

S1 was able to manipulate the algebraic form of a simple algebraic equation to determine the value of a variable. In item no 2, namely  $a + 5 = 4a$ , S1 procedurally found that the value of  $a$  is  $\frac{5}{3}$ . Therefore, even though S1 has a misunderstanding in interpreting variables, S1 can still determine variable values from a simple equation.

Q : For  $a + 5 = 4a$ , can you find the answer?

S1 : Yes, it's  $\frac{5}{3}$

Q : May I know how?

S1 : I moved  $a$  to the right, so it was  $5 = 3a$ , then  $a = \frac{5}{3}$  was obtained.

Q : Why must it be moved to the right side?

S1 : Usually like that, ma'am. I also don't know why.

By saying 'I moved  $a$  to the right' indicates students' misconceptions about the rules of arithmetic operations (Sarımanoğlu, 2019). As we know, this procedure aims to remove variable  $a$  on the left side of the equation by subtracting it with  $a$ . When this was clarified, S1 was not aware of the concept. S1 has incomplete knowledge of the concept of arithmetic operations, which can potentially cause errors in algebraic operations. According to Booth (2019), the correct answer can be obtained even though students experience misconceptions. Other studies state that students experience pseudo-true thinking processes in solving problems. Pseudo-true occurs when students correctly answer the questions but cannot give reasons for their answers or are wrong in giving reasons (Subanji, 2015). This reinforces the reason why teachers need to diagnose algebraic misconceptions in students because sometimes the misconceptions experienced by students are invisible without us doing a diagnosis (Das, 2020).

#### 4. *Manipulating the algebraic forms presented in sentences*

S1 made an error when asked to manipulate algebraic forms presented in a sentence. This was

<p>Jika 5 ditambahkan ke <math>a + 2</math>, akan menghasilkan...</p> <p>A. <math>a + 7</math></p> <p><del>B. <math>5a + 2</math></del></p> <p>C. <math>7a</math></p> <p>D. Tidak ada yang benar</p>	<p>Add 5 onto <math>a + 2</math> ...</p> <p>A. <math>a + 7</math></p> <p>B. <math>5a + 2</math></p> <p>C. <math>7a</math></p> <p>D. None of these</p>
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because S1 incorrectly interpreted the meaning of the sentence given.

Q : What was your answer to question no 4?

S1 : I answered  $5a + 2$ .

P : How did you get  $5a + 2$ ? Can you explain?

S1 : 5 plus  $a$  plus 2.

Q : Does it mean like this (writing on paper  $(5 + a + 2)$ ).

S1 : No, but  $5 + a$  equals  $5a$ , then plus 2.

The sentence "5 plus  $a$  plus 2" indicates that S1 mistakenly interpreted the question presented in the sentence. S1 interpreted the sentence "if 5 is added to  $a + 2$ " as adding 5 to  $a$ , instead of adding 5 to  $(a+2)$ . Another finding from the interviews is that S1 believes in the wrong rules for adding integers with variables, namely  $5 + a$  becomes  $5a$ . One study stated that this error occurred due to

the duality of the nature of mathematical notation, namely as a process and an object (Joanna & Jacquelynn, 2019). S1 perceives that the answer cannot contain operator symbols. S1 believes the symbol '+' is a command to do something, and the expression that still contains the operation '+' is not a simple form, so it needs to simplify (Ojose, 2016).

In this regard, in the four tier-test, S1 also experienced a conceptual error: adding up  $5x + 3x$  as  $8x^2$ . By believing that  $x$  is the same object, for example, 3 books plus 5 books equal 8 books.

Q : What can you conclude from  $5 + a$ ?

S1 :  $5a$

Q : How about  $5x + 3x$ ?

S1 : Hmm..  $5 + 3$  is 8, so  $5x + 3x$  is  $8x^2$

According to Ojose (2018), S1 applies the wrong rules due to a lack of knowledge. S1 believes in the wrong rule that numbers must be added, and variables must be multiplied to simplify algebraic expressions. This finding is supported by S1's work when asked to solve  $\frac{6}{a} + \frac{6}{b}$ . S1 simplified  $\frac{6}{a} + \frac{6}{b}$  to  $\frac{12}{ab}$ . S1 believes that the quantifier must be added up because it is a number, while the denominator must be multiplied because it is presented in a variable. According to Ojose (2018), this error is related to students' lack of conceptual understanding.

### 5. *The order of the letters in the alphabet determines represented numbers*

Algebraic expressions whose variables are written in the form of letters cause misconceptions among students.

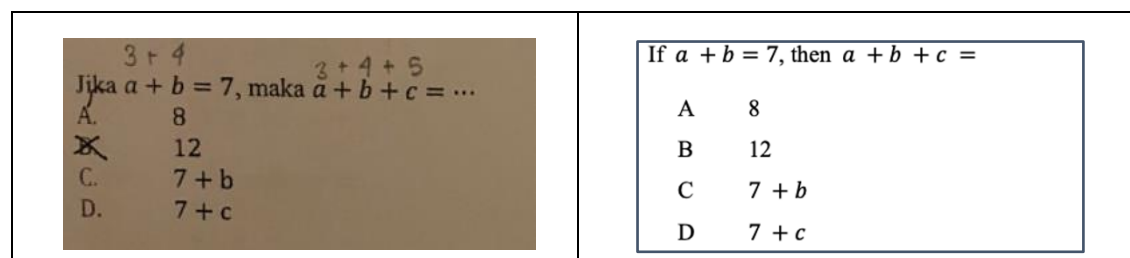


Figure 5: S1's answer

Q : Can you explain why you answered 12 for this question?

S1 : I changed  $a$  to 3,  $b$  to 4, so that  $a + 3$  equals 7

Q : Why not changing  $a$  to 2, and  $b$  to 5?

S1 : If  $a$  is 2, then  $b$  is 3, then the sum will not be 7 ( $a + b = 7$ )

S1 : Because  $c$  will be added later, now  $c$  is 5, so that  $a + b + c = 12$

The sentence " I changed  $a$  to 3,  $b$  to 4, so that  $a + 3$  equals 7" shows that the subject experienced a "trial and error" process in the work. Trial and error were made by selecting numbers based on their intuition. According to Warren (2003), one of the errors that students make is assigning letters a numerical value according to their rank in the alphabet. When students do this, they often assume that variable ' $a$ ' equals 1, variable ' $b$ ' equals 2, and so on. In this case, S1 thought as if there is a correspondence between the order of the linear alphabet and the natural number system, namely  $a = 3$  and  $b = 4$  because the letters  $a$  and  $b$  are sequential in the alphabet. S1 believes that 3 and 4 are the appropriate numbers for the expression.

### 6. Forming algebraic expressions from lateral sentences presented by matching words from "left to right"

In this question, students were asked to form algebraic expressions from the given sentences.

<p>Setengah dari suatu bilangan, dikurangi dari empat hasilnya adalah lima belas. Persamaan yang mewakili pernyataan ini adalah</p> <p>A. <math>4 - \frac{1}{2}x = 15</math>          B. <math>x - \frac{1}{2}x = 15</math>  <input checked="" type="checkbox"/> C. <math>\frac{1}{2}x - 4 = 15</math>          D. <math>\frac{1}{2}x - 4 = 15</math></p>	<p>Half a number, subtracted from four is fifteen. The equation best representing this statement is</p> <p>A. <math>4 - \frac{1}{2}x = 15</math>          B. <math>x - \frac{1}{2}x = 15</math>          C. <math>\frac{1}{2}x - 4 = 15</math>          D. <math>\frac{1}{2}x - 4 = 15</math></p>
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Figure 5. S1's answer

When changing literal sentences to form algebraic expressions, S1 arranged them from "left to right." S1 assumed that the word order in the sentence would map directly to the symbol order that appears in the question. S1 also misinterpreted the sentence "subtracted from four." Russell et al. (2009) refer to this error as a direct translation characterized by a phrase-by-phrase translation of the problem into variables and equations.

### 7. Wrong Interpretation of Sentences

<p>Susan memiliki beberapa kartu gambar. Feby memiliki tiga kali lebih banyak kartu gambar dari Susan. Jika kartu gambar susan dan Feby digabungkan akan berjumlah 36 kartu. Manakah dari persamaan ini yang mewakili koleksi kartu gambar mereka?</p> <p>A. <math>3x = 36</math>  <input checked="" type="checkbox"/> B. <math>x + 3 = 36</math>          C. <math>x + 3x = 36</math>          D. <math>3x + 36 = x</math></p>	<p>Susan has some trading cards. Feby has three times as many trading cards as Susan. They have 36 trading cards in all. Which of these equation represent their trading card collection?</p> <p>A. <math>3x = 36</math>          B. <math>x + 3 = 36</math>          C. <math>x + 3x = 36</math>          D. <math>3x + 36 = x</math></p>
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Figure 6: S1 answer sheet

The question above requires students to read the problem presented in the sentence form and change it to an algebraic equation. S1 mistakenly interpreted the sentence given. The sentence "three times as much" was interpreted by S1 as  $x + 3$ , instead of  $3x$

Q : You chose the answer  $x + 3 = 36$ . May I know what  $x$  here indicates?

S1 :  $x$  is, for example, the card that belongs to Susan

P : Then why did you add  $x$  to 3?

S1 : In this question, it says that there are three times more Feby cards than Susan cards, earlier Susan cards are  $x$ , so 3 was added.

The interview revealed the mistakes/misconceptions experienced by S1, namely the wrong interpretation of the given sentence.

8. *Misconceptions in understanding the distributive nature of negative signs*

S1 succeeded in using the distributive rule for algebraic expressions not containing a negative sign. For  $2 \times (a + b)$ , S1 resolves it into  $2a + 2b$ . An error occurs when a distribution operation contains a negative sign. S1 had a misconception, believing that  $5(p + q) - 3(p + q)$  was  $2p + 8q$ . This was obtained from  $5(p + q) - 3(p + q) = 5p + 5q - 3p + 3q$ . According to Zielienski (2017), this misconception happens because students do not interpret brackets as an entity when dealing with addition or subtraction signs outside the brackets. S1 ignored the consequences of sign rules which can involve positive changes to negative or vice versa (Zielinski, 2017).

9. *Invalid generalization of the distribution rule*

S1 interpreted  $(5 + x)^2$  as  $5^2 + x^2$ . This error develops from over-applying the distributive rule. The distributive property states that  $a(b + c) = ab + ac$ . Subject S1 applied this rule in a new, inappropriate context. According to Chow (2011), this misconception is an invalid distribution. An invalid distribution is also known as an abuse of the distributive nature of algebra (Chow, 2011).

10. *Simplifying superfluous algebraic forms*

For question  $\frac{a+b}{b}$ , the students were asked to simplify the algebraic expression, and S1 simplified  $\frac{a+b}{b}$  as  $a$ .

P : For the expression  $\frac{a+b}{b}$  do you think this can be simplified??

S1 : Yes, Ma'am, ..  $\frac{a+b}{b}$  is the same as  $a$

P : How?

S1 : the letter  $b$  above (the numerator) is crossed out along with the letter  $b$  below (the denominator) (using the rule of cancellation).

There are two mistakes made in S1's answer for this question. First, S1 believes that the variable  $b$  in the numerator and denominator can be removed using the cancellation law. S1 applied the wrong procedure, namely, applying the rules of the cancellation law of multiplication to addition. Second, if cancellation can be made, the variable  $b$  in the numerator and denominator change into 1, so that the algebraic form of  $\frac{a+b}{b}$  can be changed to  $\frac{a+1}{1}$ . According to Mulungye (2016), the main cause of this error was that the subject had an incomplete understanding of arithmetic concepts or failed to transfer arithmetic understanding to an algebraic context.

### 11. Determining the algebraic form of the given geometric pattern

S1 succeeded in determining the number terms of the given geometric pattern. S1 solved the questions that asked to determine the number for the pattern given well. Based on the interview, S1 calculated the number of tiles manually (adding one by one) from pattern 1 to pattern 20. S1 admitted that he did not understand how to generate algebraic expressions to be used as a general formula for a given geometric pattern.

Based on the results of S1's work as a whole, S1's cognitive map is presented in Figure 8 below.



Figure 8: The cognitive map of misconceptions and S1's errors

## S2 Data Presentation

Subject S2 is a year eight student at SMP Banjarmasin, and he/she is currently 14 years old. In the initial identification of the algebraic diagnostic test, S2 experienced a misconception of 58.3%. Based on the results of the four tier-test, S2 is included in the category of students with a *decision lack of knowledge*. Some of the misconceptions experienced by S2 are also misconceptions experienced by S1. The following is an exploration of the misconceptions and mistakes made by S2 that are different from the misconceptions experienced by S2

### 1. A variable represents an object

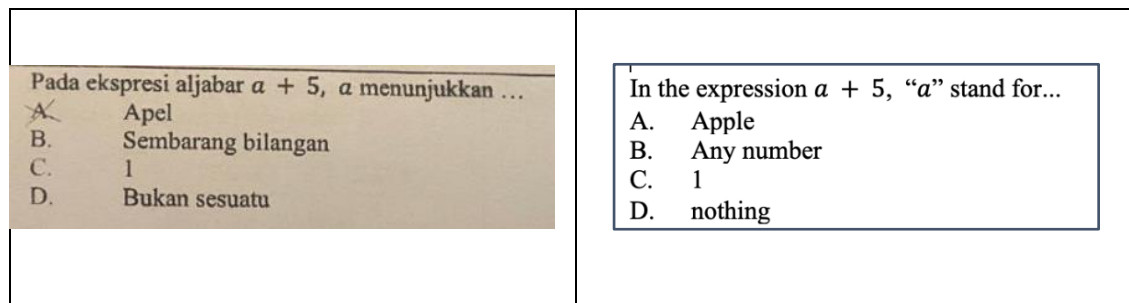


Figure 9: S2's work

S2 interpreted the letter  $a$  in the expression  $a + 5$  as an object, namely an apple. According to Edogawatte (2011), students interpret that there is a close relationship between the letters used as variables and the real-life context. Sometimes students interpret the expression  $8a$  as simple as for 8 apples.

Q : For question  $a + 5$ , you say that  $a$  is an apple?

S2 : Yes, ma'am

Q : Why?

S2 : Because there are no other options that represent letter  $a$ .

The sentence " Because there are no other options that represent letter  $a$ " indicates that S2 relates the letter  $a$  to an object in the real-life context. As in the multiple-choice answers only apple was possible, S2 thought that this option was the answer.

2. The use of rules in manipulating Algebra (operations)

<p>Apa yang dapat kamu simpulkan dari persamaan : <math>a + 5 = 4a</math></p> <p>A. <math>a = \frac{5}{3}</math>  <del>B. <math>4a + 5</math></del>            C. <math>a = \frac{3}{5}</math>            D. <math>a = 0</math></p>	<p>What can you say about 'a' if <math>a + 5 = 4a</math>?</p> <p>A. <math>a = \frac{5}{3}</math>            B. <math>4a + 5</math>            C. <math>a = \frac{3}{5}</math>            D. <math>a = 0</math></p>
<p>Berapakah <math>k</math> jika <math>k - 12 = 4</math>?</p> <p><del>A. -8</del>            B. 3            C. 8            D. 16</p>	<p>What is 'k' if <math>k - 12 = 4</math>?</p> <p>A. -8            B. 3            C. 8            D. 16</p>
<p>Jika <math>5 = 9y</math>, maka <math>y = \dots</math></p> <p>A. <math>5 : 9</math>            B. <math>9 : 5</math>  <del>C. <math>5 - 9</math></del>            D. <math>5 \times 9</math></p>	<p>If <math>5 = 9y</math>, then <math>y =</math></p> <p>A. <math>5 \div 9</math>            B. <math>9 \div 5</math>            C. <math>5 - 9</math>            D. <math>5 \times 9</math></p>

Figure 10 : Student S2's work

S2 made several mistakes in manipulating and performing operations to get variable values from the given algebraic expressions. In-depth interviews were conducted with S2.

Q : In the equation  $a + 5 = 4a$ , can you show which one is the variable?

S2 : Here, Ma'am (pointing to  $4a$ ).

Q : Is  $a$  also a variable (referring to  $a + 5 = 4a$ ).

S2 : .... (pauses) I don't know Ma'am, I don't understand.

Q : Well, for  $4a$  above, what's your reason for saying that  $4a$  is a variable?

S2 : Because it is different from  $a$  and  $5$

Q : Different? Different shape or what?

S2 :  $4a$  is a combination of  $4$  dan  $a$

From the interview, it was revealed that S2 did not recognize variables when presented in an algebraic expression. S2 could not show which one is a variable and which is not a variable. Regarding his work in writing that  $a + 5 = 4a$  was changed to  $4a + 5$ , S2 admitted that he/she did not know the meaning of the problem. S2 assumed that the chosen answer was the most reasonable one among the other choices.



A similar question was given,  $5 = 9y$ , and the students were asked to determine the value of  $y$ . In this section, S2 could understand the intent of the question, which is to determine the  $y$  value of the given equation. S2 confidently determined  $y = 5 - 9$ . Based on the interview, S2 admitted that he/she found the answer by moving number 9 on the right side to the left, so that the sign for number 9 was changed into -9. The misconception that S2 has was that he/she did not consider 9 and  $y$  or  $9y$  as an entity (term). He/she believed that 9 and  $y$  were separate elements, so when 9 was displaced,  $y$  did not follow it. This misconception is consistent with previous research findings that students believe signs, numbers and variables are separate parts (Chow, 2011).

For the next question, in the equation  $k - 12 = 4$ , S2 used his/her understanding of moving segments to eliminate  $k$ . However, the mistake S2 made was not to process the symbols following 12. Therefore, S2 chose the -8 answer with the procedure  $k = 4 - 12 = -8$ . From the work and interviews conducted, S2 experienced misconceptions regarding the rules for conducting operations. According to previous research, this is caused by students' lack of conceptual understanding of operations on arithmetic applied to algebra. (Bush, 2013; Edogawatte, 2011; Mulungye, 2016; Ojose, 2016).

### 3. Words and letters matched from "left to right"

Similar to S1, when changing literal sentences to form algebraic expressions, S2 made the same mistake. S2 composed sentences word for word from "left to right" to form algebraic expressions. S1 assumed that the word order in the problem statement would map directly to the symbol order in the question. S1 also misinterpreted the sentence "subtracted from four."

Russell et al. (2009) refers to this error as a direct translation characterized by a phrase-by-phrase translation of the problem into variables and equations.

<p>Jika <math>n</math> adalah jumlah surat kabar yang dikirimkan Lee setiap hari. Manakah dari ekspresi berikut yang mewakili jumlah total surat kabar yang dikirimkan Lee dalam 5 hari jika pada hari kelima ia mengirimkan tiga surat kabar tambahan?</p> <p>A. <math>5 \times n + 3</math>          B. <math>(5 \times n) + 3</math>          C. <math>(n + 5) + 3</math>  <input checked="" type="checkbox"/> D. <math>3 + (5 - n)</math></p>	<p>If <math>n</math> represent the number of newspaper that Lee delivers each day. Which of the following expressions represents the total number of newspaper that Lee delivers in 5 days if on the fifth day he delivers an additional three newspapers?</p> <p>A. <math>5 \times n + 3</math>          B. <math>(5 \times n) + 3</math>          C. <math>(n + 5) + 3</math>  <input checked="" type="checkbox"/> D. <math>3 + (5 - n)</math></p>
<p>Susan memiliki beberapa kartu gambar. Feby memiliki tiga kali lebih banyak kartu gambar dari Susan. Jika kartu gambar susan dan Feby digabungkan akan berjumlah 36 kartu. Manakah dari persamaan ini yang mewakili koleksi kartu gambar mereka?</p> <p>A. <math>3x = 36</math>          B. <math>x + 3 = 36</math>          C. <math>x + 3x = 36</math>  <input checked="" type="checkbox"/> D. <math>3x + 36 = x</math></p>	<p>Susan has some trading cards. Feby has three times as many trading cards as Susan. They have 36 trading cards in all. Which of these equation represent their trading card collection?</p> <p>A. <math>3x = 36</math>          B. <math>x + 3 = 36</math>          C. <math>x + 3x = 36</math>  <input checked="" type="checkbox"/> D. <math>3x + 36 = x</math></p>

Figure 11: S2's work

S2 made a mistake in making algebraic expressions or equations of the problems presented in sentences.

#### 4. Parenthetical Rules

In this section, S2 did not notice the order in performing operations on algebraic expressions. This sequence must be followed even though the expression does not display parentheses (Zielinski, 2017).

Q : For  $3 + y \times 2$ , did you simplify it to  $6y$ ?

S2 : Hmm yes.

Q : Where does 6 come from?

S2 : I first multiplied  $3 \times 2$  to get 6, then added  $y$  to make  $6y$

Q : Did you mean like this? (writing it down on paper:  $3 \times 2 = 6$ , then  $6 + y = 6y$ )

S2 : Yes, ma'am.

Q : Do you think  $6 + y$  will make  $6y$ , what about  $6 \times y$ ?

S2 :  $6y$  too, ma'am? Well, it should not be the same. It seems I was wrong.

Q : Well, in your opinion, if there is something wrong, what do you think the result of the operation that produces  $6y$ ? Is it  $6 + y$  or  $6 \times y$ ?

S2 : Sorry, ma'am, I still don't really understand.

<p>Berapakah nilai dari <math>3 + y \times 2</math>?</p> <p><input checked="" type="radio"/> A. <math>6y</math></p> <p>B. <math>5xy</math></p> <p>C. <math>2y + 3</math></p> <p>D. Tidak ada yang benar</p>	<p>Find the value of <math>3 + y \times 2</math></p> <p>A. <math>6y</math></p> <p>B. <math>5 \times y</math></p> <p>C. <math>2y + 3</math></p> <p>D. none of these</p>
<p>Bentuk lain dari <math>2 \times (a + b)</math> adalah...</p> <p>A. <math>2a + b</math></p> <p>B. <math>2a + 2b</math></p> <p><input checked="" type="radio"/> C. <math>2ab</math></p> <p>D. Tidak ada yang benar</p>	<p>Another way of writing <math>2 \times (a + b)</math> would be</p> <p>A. <math>2a + b</math></p> <p>B. <math>2a + 2b</math></p> <p>C. <math>2ab</math></p> <p>D. none of these</p>

Figure 12: S2's work

S2 believes that in algebraic expressions he/she is free to carry out operations without being based on sequences or rules in arithmetic. In this case, according to Ojose (2018), S2's work was based more on his/her intuition rather than his/her formal knowledge. Another finding based on the S2's

interview is that there is a misconception about adding up  $6 + y$  to  $6y$ . As discussed in the findings for S1, this misconception occurs because students believe the operator sign should not be in the final answer. The  $+$  sign is an order that students must do so that they work based on their intuition to add  $6 + y$  to  $6y$ . This case also continues when S2 simplified the fraction form, simplifying  $\frac{6}{a} + \frac{6}{b}$  to  $\frac{12}{ab}$ . S2 stated that the results were obtained by adding the same quantifiers and the same denominators. Assuming that  $a + b = ab$  was also incorrect.

In the next question, S2 made the same mistake as he/she added arbitrary numbers with variables,  $2a + b = 2ab$ . Another error found in answer to this question is that S2 did not consider the parentheses contained in algebraic expressions important.

P : For the expression  $2 \times (a + b)$ , you simplified it to  $2ab$ ?

S2 : Yes, ma'am

Q : Okay, can you explain how you got it?

S2 : I multiply 2 by  $a$  then add  $b$

Q : Is it like this?  $2 \times a = 2a$  then  $b$  was added to the result (while writing  $2a + b = 2ab$ )

S2 : Yes ma'am.

Q : Okay, what do you think these brackets mean?

S2 : I don't understand, ma'am (S2 admitted that he/she does not understand the parentheses in the expression).

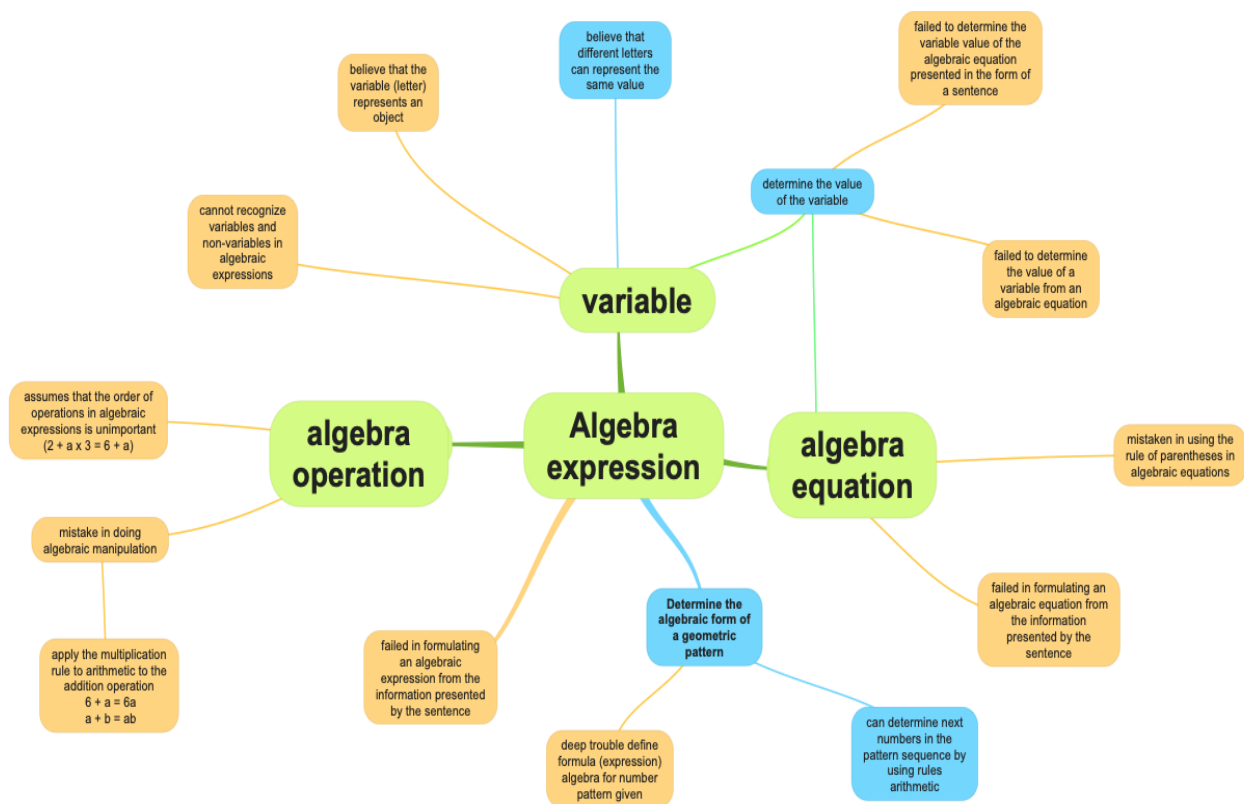
In this regard, the four-tier-test question, S2 also wrote that  $5(p + q)$  is  $5p + q$ . Based on the findings above, it can be stated that S1 has a misconception in or lack of knowledge of using brackets in algebraic expressions. In addition to the above description, S2 also experienced an error in simplifying the algebraic form  $\frac{a+b}{b}$  as  $a$ , generalizing the distributive law  $(5 + x)^2$  as  $5^2 + x^2$ . The interview results stated that S2 worked for no apparent reason (he/she did not understand the procedure he was using), purely using his intuition.

S2's overall work are presented in S2's cognitive map in Figure 1 below.

S1 and S2's cognitive maps emphasize that the students emphasize procedural knowledge rather than fully understanding the concept. Students with strong conceptual knowledge will be better at solving equations and able to learn new procedures more quickly than students with insufficient conceptual knowledge (Booth & Mc Ginn, 2016). This study shows that S1 and S2 experience misconceptions in the core concept of algebra, namely variables. The basis of algebra is the concept of variables (letters/symbols) and equivalence (Knuth et al., 2005; Weinberg et al., 2016). Referring to Peña et al.'s (2007) cognitive map model, negative direct causal concept refers to

defining variables and determining variable values. S1 and S2 mistakenly interpreted variables, but in some cases, they succeeded in determining variable values from algebraic expressions or equations. The most mistakes made by S1 and S2 are when performing operations in algebra.

The exploration found that this error resulted from their lack of knowledge in the concept of arithmetic operations, so they made mistakes in implementing the concept of algebra. Students who do not understand algebraic concepts, such as expressions, an equal sign, and operation signs, frequently answer math questions using illegal procedures and make mistakes (Edo & Tasik, 2022). This finding is in line with Warren (2003) who asserts that the difficulties experienced by students in algebra stem from inadequate basic knowledge of arithmetic. The potential solution offered is that learning must be meaningful. Students should be encouraged to understand concepts better first so that it is easy to restructure them when accepting new concepts and to master procedural skills (Kshetree, 2020; Mathaba & Bayaga, 2021).



with :

- concept map
- scientific concept
- Misconception/error

Figure 13: S2's Cognitive Map of Misconceptions and Errors

According to Rakes and Ronau (2019), if students are able to connect procedures and concepts, then students' conceptual understanding can produce more robust and consistent procedural skills. Otherwise, students will only use incorrect or incomplete procedures (Skemp 2006). These findings also support the those of several previous studies that misconceptions are often the result of over-generalization or lack of knowledge of previous concepts. Problems arise due to misconceptions or incomplete understanding of concepts or relationships between concepts. In this case, the misconceptions were caused by students' misconceptions of arithmetic and incomplete procedural knowledge of arithmetic. Teachers need to know students' initial knowledge, check it, identify confusion, and then develop new learning ideas (Treagust & Chow, 2013). The negative effects of misconceptions in algebra are emphasized on students' performance and achievement in algebra and other related subjects, which makes detecting these errors an urgent need to overcome them (Yew et al., 2020).

Now, perhaps we can agree that misconceptions in algebra are a problem and can interfere with one's success in mathematics learning. Most likely, instruction in traditional arithmetic and algebra courses is insufficient to address the problem. We attempt to offer instructions that teachers can do in their classrooms to correct students' misconceptions. From the previous discussion, it was found that the instructions carried out were directed at increasing conceptual understanding. The first fundamental thing teachers can do is facilitating students in integrating new information with the knowledge they already know. Teachers must facilitate the transition process from arithmetic to algebra. The results of the restructuring must be made explicit. The teachers can do this by asking students to provide examples and counter-examples of the studied concepts. For example, in the variable concept, when students say that  $a$  refers to an apple, which means that  $2a$  indicates two apples, the teachers can give a counter-example, such as 'what about  $a^2$ ? What happens if the apple is squared? In this way, students are expected to find their own mistakes from what they understand about the concept of variables.

Second, when teaching, teachers must pay attention to the important concepts in algebra and emphasize them. This way will certainly help students build the correct concept. For example, teachers can explain that the equal sign is not only a pointer to the result of a mathematical work (operation = result), but is also an equivalent relationship between two fields separated by an equal sign. Students should also learn that a result of an operation can contain a variable, and it does not have to be a number. This may seem trivial, but it will significantly affect students' understanding of the concept of equality in algebraic equations.

Third, occasionally, teachers can give scaffolding questions during learning. This question is aimed at confirming students' understanding of a concept. Scaffolding questions can also direct students' thinking to the concept to be achieved. For example, when students say that  $\frac{a+b}{b}$  can be simplified to  $b$ , the teacher can ask a scaffolding question, such as, does  $\frac{a \times b}{b}$  also produce  $b$ ?

Students' thinking should be directed to see that it is impossible for  $\frac{a+b}{b}$  and  $\frac{a \times b}{b}$  to produce the same result. That means there is an error made in one of these works.

The essential thing that needs to be considered by teachers to prevent and overcome misconceptions in students is that teachers should not judge algebraic concepts, especially basic algebra, as easy for students. Teachers need to pay more attention to students' procedural abilities when working on questions, especially in a classroom consisting of students from various backgrounds with different experiences. Of course, these factors affect their initial knowledge and the process of restructuring their knowledge during learning. Finally, the points emphasized above are not only recommendations for teachers in teaching but are expected to be a consideration to help teachers provide meaningful learning to avoid and overcome students' misconceptions in algebra.

## CONCLUSION

Students' misconceptions and errors in algebra in the literature review were identified in this study. The cognitive maps drawn from the students' work show that students can recognize algebraic form, and determine variables and non-variables in algebraic expressions/equations. However, they had misconceptions about the definition of the variable itself. The students' cognitive maps also indicate that they had difficulty changing sentences into algebraic expressions/equations but succeeded in determining the variable's value. This finding further confirms that in learning algebra students are likely to master procedural knowledge rather than conceptual one, so they experience misconceptions.

Students' misconceptions in algebra are most likely influenced by their incomplete knowledge of arithmetic, which causes difficulties for students to make the transition from arithmetic to algebra. This study shows that the students' ability to make the transition depends on their ability to understand and use variables to represent unknown entities in mathematical expressions. Their ability to interpret the equal sign as a symbol of mathematical equality between two expressions and solve algebraic problems by representing mathematical ideas expressed in general sentences to mathematical sentences also influence the students' ability to make the transition from arithmetic to algebra.

As mentioned above, teachers can do several things to avoid or overcome misconceptions in students. Teachers should encourage learning to become more meaningful so that students' conceptual understanding is maximized and supports their success in applying procedures in algebra. The teachers also need to facilitate students in integrating new information with prior knowledge they already have by giving examples and non-examples related to certain algebra concepts. Teachers must pay attention to essential algebra concepts and emphasize them when teaching them. Last, the teachers can give scaffolding questions during learning to direct students' thinking.

## LIMITATIONS

There are a number of limitations in this study. First, the researcher did not do thorough observations in the classroom. Second, the intervention was carried out by the researcher by giving scaffolding questions to the subjects. However, it was carried out in a limited time so that it could not lead students' thinking completely, considering that many points from basic algebraic concepts needed to be observed. Last, the researchers did not have the opportunity to explore teachers' opinions further regarding their understanding of misconceptions and didactic actions that have been carried out so far to prevent and overcome misconceptions in their mathematics classes.

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## Stimulation of Cognitive and Psychomotor Capability by Game-Based Learning with Computational Thinking Core

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*Abstract: Cognitive and psychomotor capabilities are two critical interrelated abilities to improve student learning outcomes. Both abilities play a role in understanding new information and developing fine motor skills. Hence, schools train students these two abilities to equip them with basic skills in solving mathematical problems such as basic arithmetic. However, few previous studies have not much discussed the design of learning strategies which successfully integrate these two capabilities. Moreover, these studies only focus on calculations in arithmetic operations, not the conceptual understanding of operations. Therefore, this study aims to describe the development of learning media designs that accommodate student activities through game-based learning to stimulate cognitive and psychomotor capability in conceptual arithmetic operations, especially multiplication. Core computational thinking integrated with interactive game-based learning was used as the learning framework. The research method was called ADDIE comprising five stages of development with data collection techniques of questionnaires, student responses, and tests. Results show that according to experts and students, game-based learning media are valid and practical correspondingly. From the students' responses, it is known that the development of game-based learning can stimulate cognitive and psychomotor capability to solve contextual problems that were previously becoming obstacles for students.*

Keywords: Cognitive, Computational-Thinking, Game-Based Learning, Psychomotor

### INTRODUCTION

Cognitive and psychomotor capabilities are essential components in student learning and development which are also challenges for educators and experts in education. According to Begam and Tholappan (2018), a person's cognitive ability refers to his thinking/mental process continuity. This cognitive process involves acquiring, processing, and applying knowledge, including attention, memory, reasoning, understanding, and problem-solving. On the other hand,

according to Simpson (1972), psychomotor capability refers to students' physical movement skills, including coordination, agility, and fine motor skills, that require practice with measurements based on aspects of speed, accuracy, procedures, and implementation techniques. With this psychomotor capability, students explore phenomena, conceptualize the ideas involved, and apply concepts to new situations (Karplus & Butts, 1977). In the context of learning with abstract objects such as mathematics, the processes contained in this psychomotor ability are interpreted by gaining direct experience and providing opportunities for students to manipulate objects and tools. More clearly, Piaget (1929) claims that students' physical experience in learning mathematics could be obtained by giving students opportunities to explore mathematical concepts through the concrete physical experience before moving on to more abstract representations. These experts generally show that cognitive and psychomotor capabilities play a significant role in learning mathematics, especially in active exploration and reflection activities.

In the midst of this significant role, educators have the main challenge of identifying and overcoming differences in individual cognitive and psychomotor capability. Given the unique differences in the nature and characteristics of these two abilities, educators need to provide personalized instruction and support to meet the learning needs of each student. In addition, educators also need to find strategies to help students develop their cognitive and psychomotor capabilities on an ongoing basis. This requires teachers' focus on creating interesting and challenging learning experiences for students to follow. Teachers need to design lessons that can stimulate these two capabilities, especially in solving real-world problems. Stimulus in problem-solving needs to be accompanied by providing opportunities to train students' physical skills in direct practical activities. Moreover, the corona virus pandemic that hit the world last year has an impact on reducing the level of students' active involvement in learning and their study results (Haryani & Hamidah, 2022; Onyema et al., 2020; Orlov et al., 2021).

A decrease in student learning involvement can also be caused by a lack of their intrinsic or extrinsic motivation (Fatimah & Saptandari, 2022) because there is a reasonably close relationship between the two (Saeed & Zyngier, 2012). This involvement affects students' academic outcomes (Finn & Zimmer, 2012). Hence, an effective learning strategy is needed to increase student learning motivation, such as digital game-based learning, which also functions to encourage students' willingness to learn and self-awareness in both formal and informal learning contexts (Owston, 2009; Yang & Chen, 2010; Yien et al., 2011). This positive implication raises a lot of attention given to the relationship between digital games and the education field (Chiang et al., 2011).

The variety of games implemented in learning makes the development of game application models more adaptive and flexible as learning media based on the material presented (Hays, 2005; Papoutsis & Drigas, 2016). Several studies on multiplication-themed educational games have been carried out, including research on Android-based arithmetic games by Amrizal and Kurniati (2016), mobile educational games for multiplication calculations based on the horizontal method

with Html 5 and Phone Gap by Ricky (2013), and designing learning game application for 3rd-grade math calculation operations using unity by Kristina and Talitha (2021). Those studies are related to developing educational games that contain multiplication calculations through fast multiplication counting activities.

In this study, a game-based multiplication concept learning will be designed with the help of digital game applications of which are accessible on Androids. Students are not only trained to count fast but also understand the concept of multiplication, of which the construction is formed from repeated addition. In learning arithmetic, students need adaptive skills, which involve functional academic skills in the basic operations of addition, subtraction, division, and multiplication (Polspoel et al., 2019). These adaptive skills are needed for everyday life because they involve communication, social life, work, and functional academic skills such as reading, writing, and arithmetic (Ainsworth & Baker, 2004; Hodapp, 2002). Thus, learning arithmetic requires both psychomotor and cognitive abilities, especially if arithmetic problems are in the context of students' everyday problems. However in fact, there are still many students who are afraid of learning mathematics because it is considered difficult and complex (Laurens et al., 2018).

The results of an initial study conducted by the researcher in November 2022 in three elementary schools in the Municipality of Yogyakarta and Bantul Regency show that grade III students still do not understand the concept of multiplication operations and often have difficulty solving contextual problems, especially multiplication abstraction which correlates with modeling problems and the procedure for solving it. Meanwhile, the teacher still uses the rote method to teach multiplication. Students' lack of understanding of multiplication often causes boredom, laziness, and a lack of interest in learning multiplication. This is similar to Thai and Yasin (2016) research on multiplication teaching methods. Therefore, we need alternatives in thinking processes and developing problem-solving strategies, including computational thinking (Wing, 2006).

Along with technological advances in recent years, computational thinking has become an important topic in various fields of life (Lindberg et al., 2019). Various countries have attempted to promote computational thinking education in schools, universities, industries, and government sectors (Lin et al., 2020). A large number of researchers attempt to identify students' computational thinking abilities. For example, research by Yadav et al. (2017) and Denning (2017) which investigated the development of computational thinking-based teaching guidelines for students; research by Shute et al. (2017) regarding the design of a model for assessing student computing learning outcomes; as well as research by Sullivan and Bers (2018) and P´erez-Marín et al. (2020) on computational thinking-based learning performance. These studies are clear evidence that computational thinking has become a basic skill needed in learning in this digital era.

Based on the problems experienced by students in learning the concept of multiplication operations above, the researcher considers it necessary and important to design game-based learning with the help of interactive learning media that can stimulate students' cognitive and psychomotor

capabilities. The developed interactive learning medium is a digital game filled with computational thinking core to help students understand the concept of arithmetic operations, especially multiplication. By integrating computational thinking cores, students learn to understand the concept of multiplication operations by solving contextual problems presented in the digital game.

## LITERATURE REVIEW

Cognitive and psychomotor capabilities have an important role in the formation of adaptive skills, which are the main elements in the elementary school mathematics curriculum (National Math Panel, 2008) and play a fundamental role in solving more complex mathematical problems (Juliana & Hao, 2018; Prendergast et al., 2017). Therefore, teachers need to train these skills to students for mastery of mathematical concept, including the arithmetic concepts. Knowledge of basic arithmetic operation can be achieved if students understand the concept of operations and the links between operations (Rahman et al., 2017).

Cognitive skill is an ability related to a person's thinking activity in receiving, processing, and transmitting the information obtained (Basri, 2018; Darouich et al., 2017). Cognitive capability is often associated with acquiring information for the short term (Darouich et al., 2017), and the long term is seen as an adaptive function of humans to the cultural, social, and emotional environment (Anderson, 1994). In its development, this cognitive ability is significantly influenced by students' thinking activities (Basri, 2018). In mathematics learning, the completion of mathematical tasks is a basis for starting and practicing various students' mathematical thinking activities, including thinking about solving mathematical problems and understanding mathematical content so that the thinking operations that occur become parts of students' conceptual and procedural understanding (Swanson & Williams, 2014). Developing a person's cognitive abilities is directly related to developing psychomotor, social, affective, and adaptive skills.

Psychomotor skill is the ability to perform motor-physical movements related to learning outcomes in cognitive activity (Murrihy et al., 2017). Yet, the link between motor coordination and learning outcomes is largely neglected in the psycho-educational domain. The results of research related to this have been widely published, with the results of a statistically significant relationship between motor difficulties and academic achievements such as language, reading, spelling, and arithmetic (Archibald, L. & Alloway, 2008; Lopes et al., 2013). This shows that cognitive and psychomotor abilities are closely related to achieving meaningful mathematics learning goals. According to Vallori (2014), meaningful learning is signified by some important principles below:

- (a) Open work enables all learners to learn;
- (b) Motivation helps to improve the classroom environment, making learners interested in their tasks;
- (c) Means must be related to the learners' environment;
- (d) Creativity strengthens imagination and intelligence;
- (e) Concept mapping helps learners to link and connect concepts;

(f) Educational curricula must be adapted by considering learners with special needs.

These six important principles show the need for teachers to employ fun learning strategies that can accommodate various characteristics of the student's environment so that students can receive complete information by associating new information with relevant concepts in students' previous cognitive structures. One effective and suitable learning strategy to implement is digital game-based learning (Owston, 2009; Yang & Chen, 2010; Yien et al., 2011). The implication of digital educational games is motivating users in a fun learning atmosphere (Kirriemuir & McFarlane, 2004). These educational games tend to arouse curiosity and challenge users to actively explore games until they feel happy when they can finish the game well so that students are motivated and enjoy learning through the games (Chen et al., 2007; Hong et al., 2009; Moon & Baek, 2009).

Games are designed with various systematic, visual, and kinetic activity loads to stimulate students' skills and awareness of specific knowledge (Besgen et al., 2015; Shuqin, 2012). Various student skills are oriented towards achievement by integrating games into learning, including learning basic mathematics. Through games, students are trained to make decisions by controlling objects in the game for a specific purpose designed in a system or program (Jason in Aprilianti et al., 2013).

Regarding the selection of learning strategies that can increase student learning motivation and develop the skills needed, the education system has integrated Information and Communication Technologies (ICT) especially to improve the quality of student learning in schools (Malik et al., 2017). By utilizing ICT, students can become active learners through dynamic and collaborative learning so that the interactivity and communication of learning increase. In addition, students are stimulated to use their ability to think logically, systematically, and skillfully when making the right decisions facing numerous different possibilities (Munir, 2014).

An alternative learning strategy that follows the achievement of this stimulus and has a wide area of application to solve problems is computational thinking approach (Malik et al., 2017). Furthermore, Malik et al. (2017) explain that in a computational thinking approach, instead of thinking like computers, students think about computing which includes the ability to: (1) formulate problems in the form of computational problems; and (2) develop a good computational solution in the form of an algorithm or explain why no suitable solution is found. According to Ioannidou (2011), the computational thinking approach contains the cores of (1) decomposition, or the ability to break down complex tasks into smaller, more detailed tasks; (2) pattern recognition, or the ability to recognize general similarities or differences which will later help in making predictions; (3) generalization of patterns and abstractions, or the ability to filter information to solve problems; (4) algorithm, or the ability to arrange steps to solve a problem; and (5) debugging, or checking and re-checking every step of problem-solving to ensure the process is correct. If it is incorrect, then exploring why the appropriate solution is not found is necessary.



## METHOD

This study aims to describe the development of instructional media designs that accommodate student activities through game-based learning to stimulate cognitive and psychomotor capability in conceptual arithmetic operations. The research subjects were ten third-grade students from an elementary school in the District of Yogyakarta for the small class trial and 123 third-grade students from three elementary schools, two from Bantul Regency and one from the District of Yogyakarta, for the large class trial. A total of four classes were involved in the large class trial with the latter school consisting of two study groups. Data collection techniques consisted of test and non-test while data analyses included both quantitative and qualitative. Quantitative data analysis was carried out by calculating the mean score of students' tests, validation questionnaire, and student response. The mean score from the last two instruments were converted into the product validity and practicality category by referring to the criteria guidelines on five Likert scales: 'Not Good' for 1; 'Less Good' for 2; 'Good Enough' for 3; 'Good' for 4; and 'Very Good' for 5 (Widoyoko, 2018). The product was said to reach validity and practicality standards if it reached at least "Good" or score 4.

The procedure in this research consisted of an analysis stage, a design stage, a development stage, an implementation stage, and an evaluation stage, or latter abbreviated as ADDIE (Branch, 2009; Sugiyono, 2019). The researcher chose ADDIE model to develop learning media in digital games because it facilitated the construction of students' knowledge and skills in instructional guided learning plans. In addition, this model was also devoted to solving problems related to gaps due to students' lack of knowledge and skills. Furthermore, the ADDIE model contained generative processes by applying concepts and theories to a particular context.

At the analysis stage, the researcher identified the probable causes of incongruity/differences between learning outcomes and theories, concepts, or other learning problems in the multiplication concept material. Identification is based on experiences, preferences, abilities, and student motivation during learning. In addition, the researcher also identified the resources needed during the development process, including the curriculum, the concept of multiplication, the learning models or methods used, teaching materials, facilities, learning environment, technology, and the characteristics of the students involved during the development process. To determine the characteristics of the students at the research site, the researcher conducted written tests at three elementary schools to measure students' understanding of multiplication concepts presented through contextual problems.

At the design stage, the researcher designed a prospective product based on the analysis results of the previous stage and began by selecting the digital games with core computational thinking to stimulate cognitive and psychomotor capability. Next, the researcher prepared the initial design of the media by making a representation of the interactions between the system and its environment in the form of a diagram until it produces a product blueprint. At this stage, the researcher also formulated specific, measurable, applicable, and realistic learning objectives based on appropriate learning strategies.

At the development stage, the researcher developed an initial prototype according to the initial blueprint, including developing test instruments, validation questionnaires and student responses. Likewise, the initial prototype of the media was also validated by experts in the field of learning media and mathematics learning materials. Furthermore, the researcher revised the prototype by accommodating the validators' suggestion, from six validators in total, so the digital game was declared valid and ready to be implemented in both small and large classes.

At the implementation stage, the researcher formulated concrete steps to implement the previously designed learning system. The researcher initially tested the game product in a small trial to 10 third-grade students at an elementary school in the District of Yogyakarta. The try-out was carried out in three meetings, and then the researcher gave a response questionnaire to students as product users. The student responses served as inputs to revise the product. The researcher tested the product again on third-grade students from four classes at three elementary schools. Of the three elementary schools, two were located in Bantul Regency, and one was from Yogyakarta Municipality with two study group classes. To find out the responses of the large trial classes, the researcher distributed response questionnaires of which the results were used as a basis to determine the practicality of the game products.

At the evaluation stage, the researcher continuously evaluated and revised to reach final product. Evaluation was carried out through qualitative and quantitative data analysis. Based on the need analysis results at the first development stage, the researcher analyzed the quantitative data from the results of the test to find out the characteristics of the students regarding their understanding of the multiplication concept in contextual problems. The validation results at the development stage and field trials at the implementation stage were also analyzed. Qualitative data from input, suggestion, and expert criticism were interpreted as a basis for gradual revisions. Furthermore, a quantitative analysis of the validation and student response questionnaires was carried out to assess validity and practicality of the media. All stages of this evaluation were aimed for the feasibility of the final product in terms of content, design, and user-friendliness.

## RESULTS

### 1. Analysis Stage

At this stage, the researcher identified the corresponding curriculum as a guideline for developing digital games. Next, the researcher assessed the learning materials related to the concept of multiplication. In this case, the researcher interviewed a mathematics teacher about the implementation of the school curriculum in teaching multiplication. It was revealed that the teacher taught multiplication only limited to calculating two or more integers with time allocation of  $4 \times 45$  minutes per week distributed in two meetings. To check student understanding, the teacher added one meeting in the form of written test. The limited time allocation turned out to cause problems for students, in which students only memorized multiplication and were oriented towards counting skills only.

The interview also addressed the teacher's teaching method in the expository form. The teacher conveyed the multiplication of two or more numbers directly while teaching the meaning of multiplication as a number multiplied according to the multiplier number. Students who found it difficult to accept the abstraction of the multiplication meaning would eventually choose to memorize the multiplication of integer numbers within the range of 1-10. This was the consequence of the difficulties they experienced in interpreting the teacher's explanation which tended to lead only to calculating numbers. Subsequently, students looked less enthusiastic about participating in learning and were unable to solve contextual problems given by the teacher.

In addition to the learning method, the interview discussed the teacher's teaching materials as well. Books, which are universally textual, served as the main materials. This dissuaded students' interest in learning, which during the Coronavirus pandemic, was often carried out online and utilized more digital learning resources than ever. Therefore, we need interactive digital learning media that can accommodate their learning needs through new post-pandemic habits. Moreover, the initial test result regarding students' understanding of multiplication concept in contextual problems shows that it falls within low category, with a mean score below 50 out of 100. The test result can be seen in Figure 1 below.

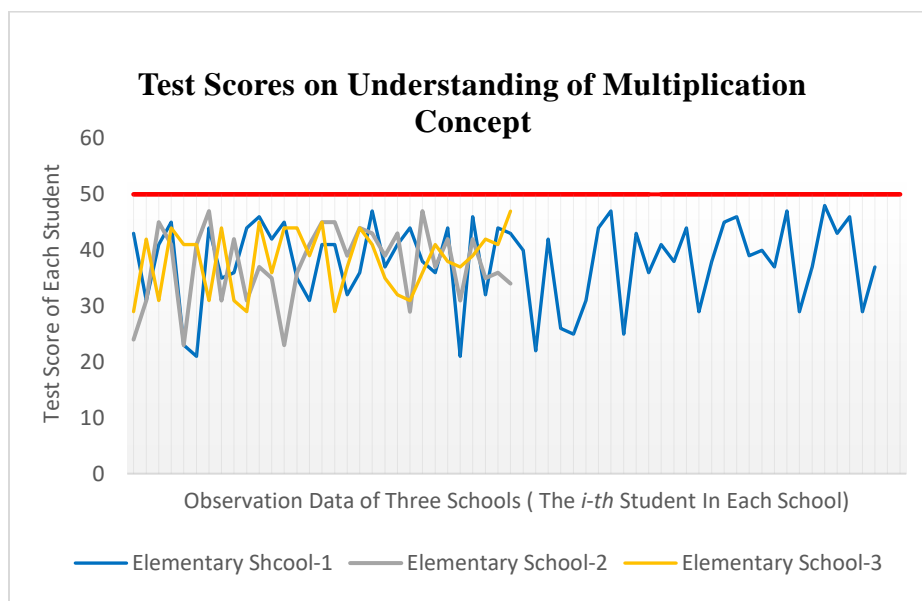


Figure 1. Level of Students' Understanding of Multiplication Concept

From Figure 1 above, we can conclude that students' cognitive and psychomotor capabilities are less than half the maximum score or less than 50. So, on this basis the researcher intends to develop a digital game with core computational thinking to support multiplication concept learning. With this game, it is hoped that students can understand the abstraction of multiplication concepts by solving contextual problems that were previously difficult for students to solve.

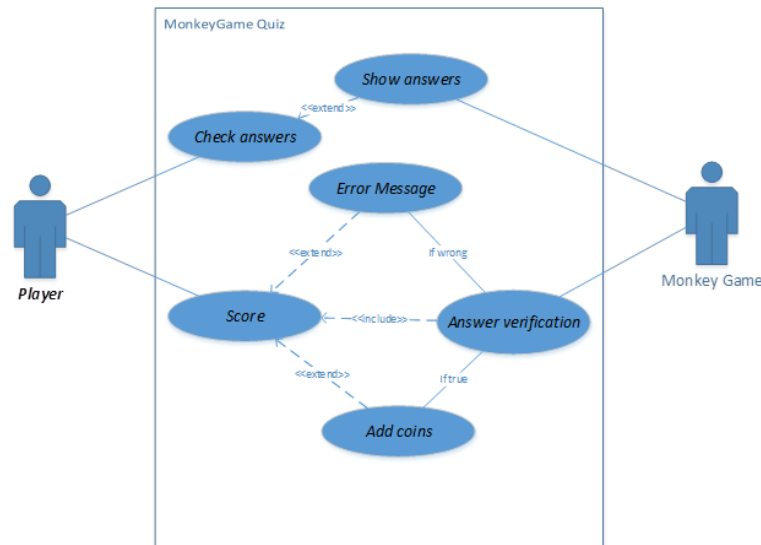
## 2. Design Stage

At this stage, the researcher designed a digital game based on the need analysis result at the previous stage. The researcher then chose digital game media with core computational thinking to stimulate students' cognitive and psychomotor capabilities. Furthermore, at the game design stage, the researcher began creating use case diagrams that describe or represent the interactions between the system and its environment, as shown in Figure 2 (a)-(b) below.

The researcher utilized the use case diagram above to define the functional modeling and operational system requirements by determining the scene method used to build the system from the results of the previous application analysis. At this stage, the researcher decided the name for the game; Monkey Game Arithmetic-CT. In this digital game, there is a monkey character. Monkey is selected since it is a fable main character that sticks in the memories of many children in Indonesia. Children often hear it from their parents during their golden age in reading and listening activities. The fable's monkey character represents an agile animal with a lot of senses and likes to eat fruits, especially bananas and apples. Then the researcher formulated the digital game concept according to the achievement orientation of students' cognitive and psychomotor capability, which were leveled from 1 to 3 on the menu as presented in Figure 3 (a)-(b) below.



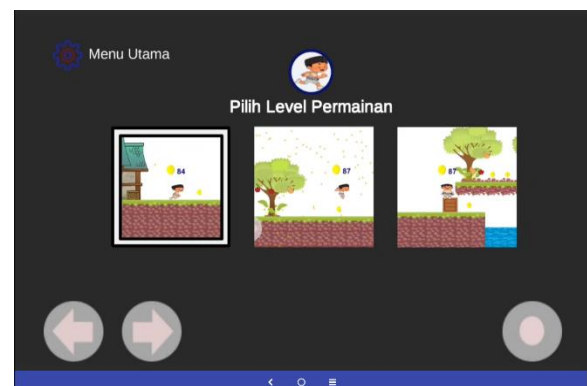
(a) Main Use Case Diagram



(b) Part Diagram of Main Use Case Containing Checking Answers and Feedback  
Figure 2. Digital Game Use Case Diagram



(a) Game Main Menu



(b) First to Third Level Game

First column translation:





Main Menu  
User Manual  
Picture Hint  
Choose a Level  
Back  
About  
Quit



Second column translation:

Choose a Game Level




Figure 3. Game Leveling on the Main Menu

The game level represents the complexity of the contextual problems as shown in Figure 4 (a)-(c) below.

Ada 2  sedang makan buah , tiap  menghabiskan 4 buah .




Berapa buah  yang dihabiskan oleh kedua  tersebut?

Nyatakan dalam bentuk penjumlahan

 +   =  

\*Isikan dengan banyaknya buah yang dimakan oleh satu monyet    \*Isikan dengan banyaknya buah yang dimakan oleh satu monyet





Nyatakan dalam bentuk perkalian



 x   =  

\*Isikan dengan banyaknya monyet    \*Isikan dengan banyaknya buah yang dimakan oleh satu monyet





31 Cek Jawaban Skor

(a) First Level

Ada 3  sedang makan buah , tiap  menghabiskan 9 buah .




Berapa buah  yang dihabiskan oleh ketiga  tersebut?

Nyatakan dalam bentuk penjumlahan

 +   +   =  

\*Isikan dengan banyaknya buah yang dimakan oleh satu monyet    \*Isikan dengan banyaknya buah yang dimakan oleh satu monyet    \*Isikan dengan banyaknya buah yang dimakan oleh satu monyet

Nyatakan dalam bentuk perkalian







 x   =  

\*Isikan dengan banyaknya monyet    \*Isikan dengan banyaknya buah yang dimakan oleh satu monyet




31 Cek Jawaban Skor

(b) Second Level

Translation Column:




There are two  eating , and each  eats four . How many  do the two  eat?

Express in addition statement







 +   =  

Fill in the number of fruits eaten by one monkey      Fill in the number of fruits eaten by one monkey





Express in multiplication statement

 x   =  

Fill in the number of monkeys      Fill in the number of fruits eaten by one monkey




There are three  eating , and each  eats nine . How many  do the three  eat?

Express in addition statement

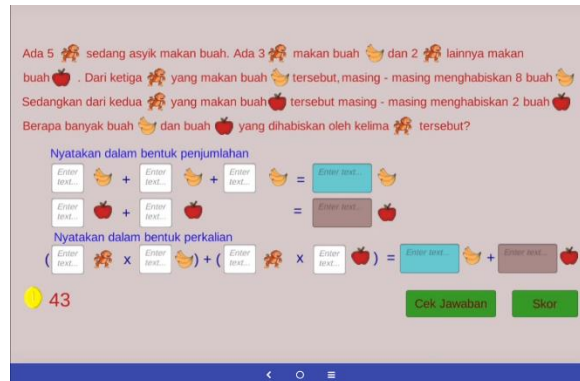
 +   +   =  

Fill in the number of fruits eaten by one monkey      Fill in the number of fruits eaten by one monkey      Fill in the number of fruits eaten by one monkey

Express in multiplication statement

 x   =  

Fill in the number of monkeys      Fill in the number of fruits eaten by one monkey



(c) Third Level

Translation Column:

*There are five 🍌 eating fruits; three 🍌 eating 🍌 and two 🍌 eating 🍌. Of the three 🍌 eating the 🍌, each eats eight 🍌. At the same time, the two 🍌 eating the 🍌, each eats two 🍌. How many 🍌 and 🍌 do the five 🍌 eat?*

*Express in addition statement*

🍌 +  🍌 +  🍌 =  🍌

🍌 +  🍌 =  🍌

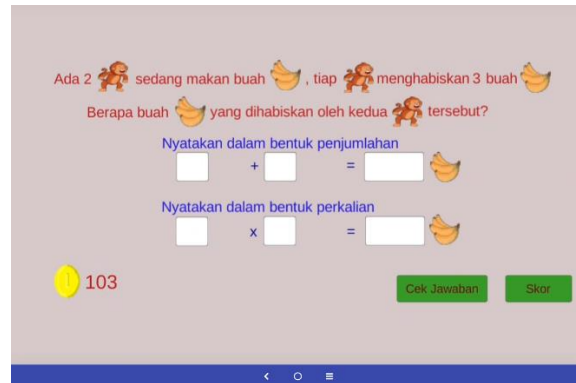
*Express in multiplication statement*

( 🍌 x  🍌) + ( 🍌 x  🍌) =

🍌 +  🍌

Figure 4. Examples of Contextual Problems from First to Third Level

Computational thinking core loads, namely abstraction, algorithm design, pattern recognition, decomposition, and debugging in digital games, are represented in problem-solving activities as presented in Figure 5 below.



Translation Column:

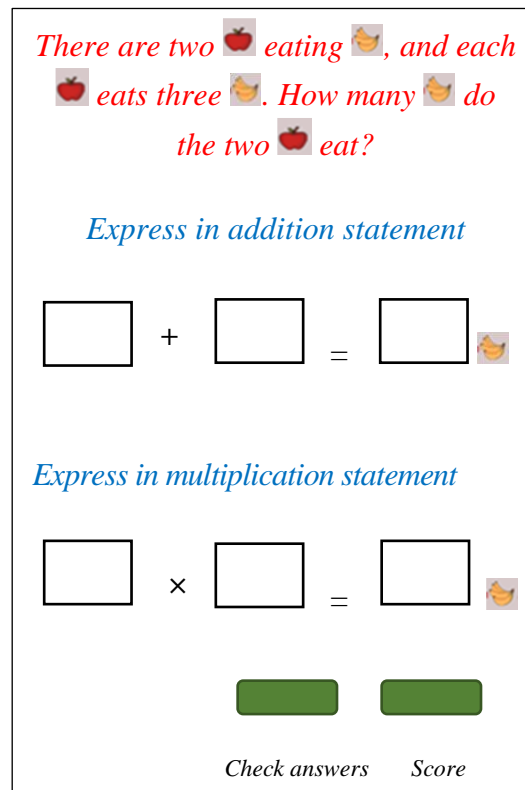


Figure 5. The Abstraction Core of Computational Thinking

Figure 5 shows that students must generalize and identify common cores by accommodating specific details and necessary patterns and ignoring unrelated data to solve the problem. In this game, students must be able to sort out the number of both the monkeys and fruits to fill in each answer box in the addition and multiplication statements. In the game's display, there are not fruit



picture or instruction hints to fill the answer box. So, students have to filter the details of the data as their abstraction entities. The core loads of the algorithm design are presented in problem-solving steps, as shown in Figure 6 below.



Translation Column:

*There are two 🍌 eating 🍌, and each 🍌 eats six 🍌. How many 🍌 do the two 🍌 eat?*

*Express in addition statement*

+  =  🍌

*Fill in the number of fruits eaten by one monkey*      *Fill in the number of fruits eaten by one monkey*

*Express in multiplication statement*

×  =  🍌

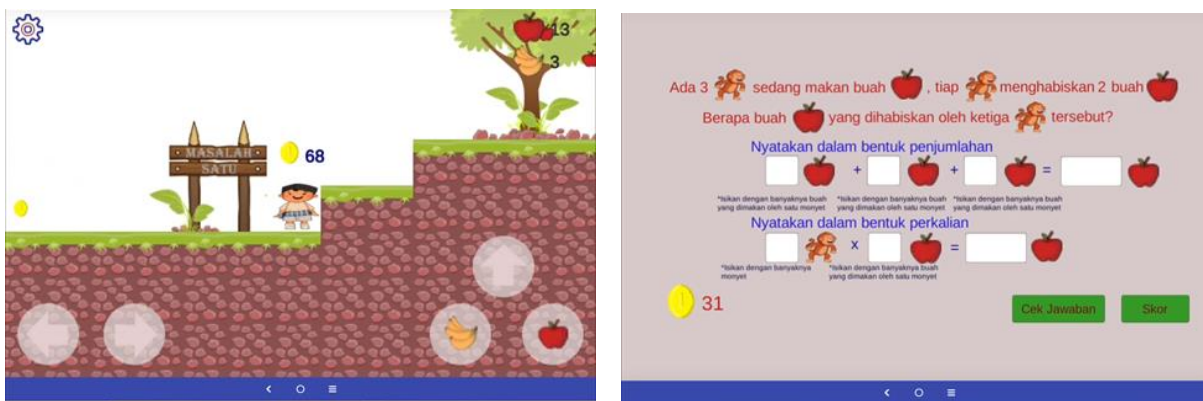
*Fill in the number of monkeys*      *Fill in the number of fruits eaten by one monkey*

Figure 6. The Algorithm Design Core of Computational Thinking

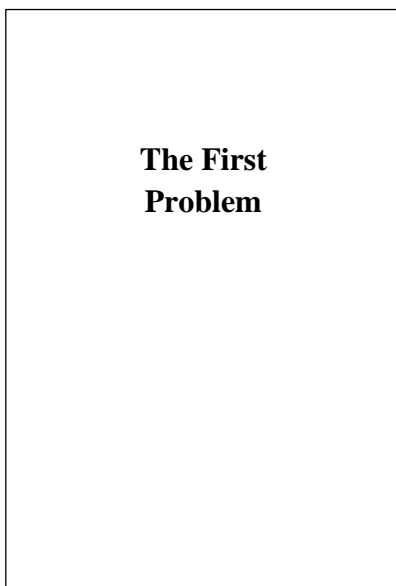
Figure 6 shows that students must develop logical and systematic problem-solving instructions to solve problems. In the first step of this game, students must fill in the number of fruits eaten by one monkey in each statement box of the addition model. In the second step, students must fill in the number of monkeys in the first statement box of the multiplication model. Afterwards, students must fill in the number of fruits each monkey eats in the second statement box of the multiplication model. In the final step, students check their answers by clicking the 'check answers' button, followed by pushing the 'score' button to check the achievement of the problem-solving score.

The core load of pattern recognition is presented in the problem-solving steps, as shown in Figure 7 (a)-(b) below.



(a) The First Problem and Its Content on The Digital Game

Translation Column:



*There are three eating, and each eats two. How many do the three eat?*

*Express in addition statement*

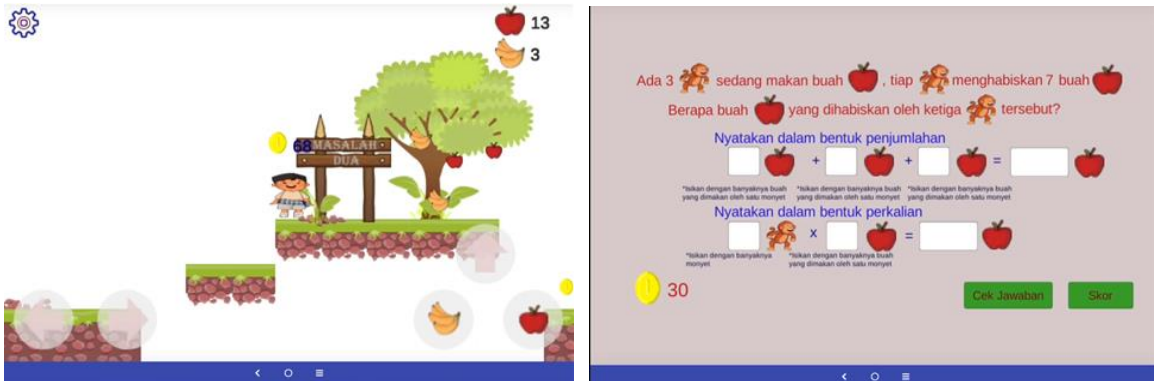
+  +  =

*Fill in the number of fruits eaten by one monkey*      *Fill in the number of fruits eaten by one monkey*      *Fill in the number of fruits eaten by one monkey*

*Express in multiplication statement*

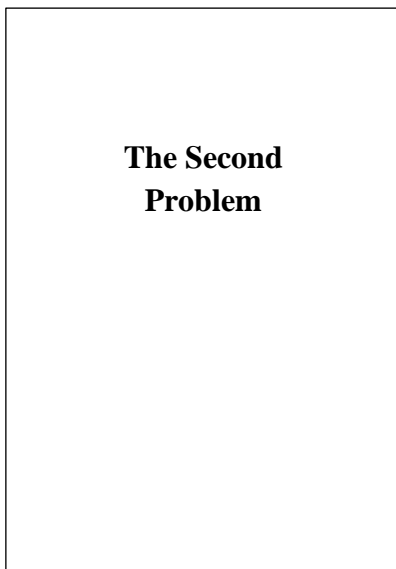
×  =

*Fill in the number of monkeys*      *Fill in the number of fruits eaten by one monkey*



(b) The Second Problem and Its Content on The Digital Game

Translation Column:



*There are three 🍎 eating 🍎, and each 🍎 eats seven 🍎. How many 🍎 do the three 🍎 eat?*

*Express in addition statement*

🍎 +  🍎 +  🍎 =  🍎

*Fill in the number of fruits eaten by one monkey*      *Fill in the number of fruits eaten by one monkey*      *Fill in the number of fruits eaten by one monkey*

*Express in multiplication statement*

🍎 ×  🍎 =  🍎

*Fill in the number of monkeys*      *Fill in the number of fruits eaten by one monkey*

Figure 7. The Pattern Recognition Core of Computational Thinking

Figure 7 (a)-(b) shows that students must be able to see similarities or differences in patterns and methods in the data that will be used in predicting and presenting data to classify problems and provide appropriate solutions. This pattern recognition uses previous experience and prior knowledge as the basis for logical thinking. Then, from this logic, students get new experience and knowledge to solve various identical problems according to patterns they already know. In this game, students must be able to see the data pattern and the regularity of solving the first problem,

as shown in Figure 6 (a) above. The first problem presents a data pattern about three monkeys eating apples, and each monkey eats two. Students are asked to determine how many apples the three monkeys eat.

The second problem is presented using an identical multiplication problem. It shows the existence of three monkeys eating apples, and each monkey eats seven. Students are asked to determine how many apples the three monkeys ate. To solve the first problem, students design a solving algorithm that begins with applying the concept of repeated addition. Then, they continue with the solution step by applying the multiplication concept related to the previous repeated addition concept. Using the patterns and regularities of the data in the first problem, students design solutions to the second problem by predicting the same steps for solving the multiplication problems.

The core load of decomposition is presented in the problem-solving steps, as shown in Figure 8 below. Figure 8 shows that students must be able to break down complex data, problems, or processes into smaller and simpler parts. So, if there is a complex problem, it can be more easily solved by breaking it down. In this case, students must be able to separate the number of bananas and apples eaten by the monkeys.

Ada 5 🐒 sedang asyik makan buah. Ada 3 🐒 makan buah 🍌 dan 2 🐒 lainnya makan buah 🍏. Dari ketiga 🐒 yang makan buah 🍌 tersebut, masing - masing menghabiskan 8 buah 🍌. Sedangkan dari kedua 🐒 yang makan buah 🍏 tersebut masing - masing menghabiskan 2 buah 🍏. Berapa banyak buah 🍌 dan buah 🍏 yang dihabiskan oleh kelima 🐒 tersebut?

Nyatakan dalam bentuk penjumlahan

$$8 \text{ 🍌} + 8 \text{ 🍌} + 8 \text{ 🍌} = 24 \text{ 🍌}$$

$$2 \text{ 🍏} + 2 \text{ 🍏} = 4 \text{ 🍏}$$

Nyatakan dalam bentuk perkalian

$$(3 \text{ 🐒} \times 8 \text{ 🍌}) + (2 \text{ 🐒} \times 2 \text{ 🍏}) = 24 \text{ 🍌} + 4 \text{ 🍏}$$

🟡 28

Cek Jawaban Skor

Translation Column:

*There are five 🍎 eating fruits. There are three 🍎 eating 🍌 and two 🍎 eating 🍌. Of the three 🍌 eating the 🍌, each eats eight 🍌. At the same time, the two 🍌 eating the 🍌, each eats two 🍌. How many 🍌 and 🍌 do the five 🍌 eat?*

*Express in addition statement*

$$\boxed{8} \text{ 🍌} + \boxed{8} \text{ 🍌} + \boxed{8} \text{ 🍌} = \boxed{24} \text{ 🍌}$$

$$\boxed{2} \text{ 🍌} + \boxed{2} \text{ 🍌} = \boxed{4} \text{ 🍌}$$

*Express in multiplication statement*

$$(\boxed{3} \text{ 🍌} \times \boxed{8} \text{ 🍌}) + (\boxed{2} \text{ 🍌} \times \boxed{2} \text{ 🍌}) = \boxed{24} \text{ 🍌} + \boxed{4} \text{ 🍌}$$

Figure 8. The Decomposition Core of Computational Thinking

The core load of debugging is presented in the problem-solving steps, as shown in Figure 9 below.

Ada 2 🐒 sedang makan buah 🍌, tiap 🐒 menghabiskan 4 buah 🍌  
Berapa buah 🍌 yang dihabiskan oleh kedua 🐒 tersebut?

Nyatakan dalam bentuk penjumlahan

$$\boxed{4} \text{ 🍌} + \boxed{4} \text{ 🍌} = \boxed{8} \text{ 🍌}$$

Nyatakan dalam bentuk perkalian

$$\boxed{4} \text{ 🍌} \times \boxed{2} \text{ 🍌} = \boxed{8} \text{ 🍌}$$

25

(a) Problem-solving

Ada 2 🐒 sedang makan buah 🍌, tiap 🐒 menghabiskan 4 buah 🍌  
Berapa buah 🍌 yang dihabiskan oleh kedua 🐒 tersebut?

**Jawaban anda belum tepat.**  
Dalam bentuk perkalian, anda terbalik mengisi jumlah monyet dan jumlah buah yang dimakan oleh monyet.

20

(b) Result of Rechecking Process

There are two 🍌 eating 🍌, and each 🍌 eats four 🍌. How many 🍌 do the two 🍌 eat?

Express in addition statement

4

🍌

+

4

🍌

=

8

🍌

Fill in the number of fruits eaten by one monkey

Fill in the number of fruits eaten by one monkey

Express in multiplication statement

4

🍌

×

2

🍌

=

8


🍌

Fill in the number of monkeys

Fill in the number of fruits eaten by one monkey

Check answers
Score

Your answer is incorrect.




In the multiplication statement, you reversely fill in the number of monkeys and fruits.

Okay

(c) Result of Error Correction from Rechecking Process

Translation Column:

Your answer is correct.



Hooray! You get ten gold coins.

Okay

Figure 9. The Debugging Core of Computational Thinking

In Figure 9, it is shown that students must carry out an inspection or process of rechecking each step of problem-solving to ensure the accuracy. If students make mistakes while solving the

problem, the system will provide feedback in the form of notifications, as shown in Figure 9 (b). The picture states that the multiplication form is not accurate as the boxes for the number of fruits and monkeys are switched. Students are also given a chance to improve their problem-solving and recheck their answers. If the answer is correct, the system will pop up a notification saying that ten gold coins are obtained as a prize.

### 3. Development Stage

This stage began with an instrument feasibility assessment in the form of a product validation questionnaire regarding media and materials and student response questionnaire. The validity test was carried out using expert judgment or reviewing the grid, especially the instrument suitability with the research objectives and questions. Based on the results of expert judgment, the three instruments were declared valid. Furthermore, the material substance and media design of the product were assessed by three validators. The material substance validation is presented in Figure 10 and Table 1 below.

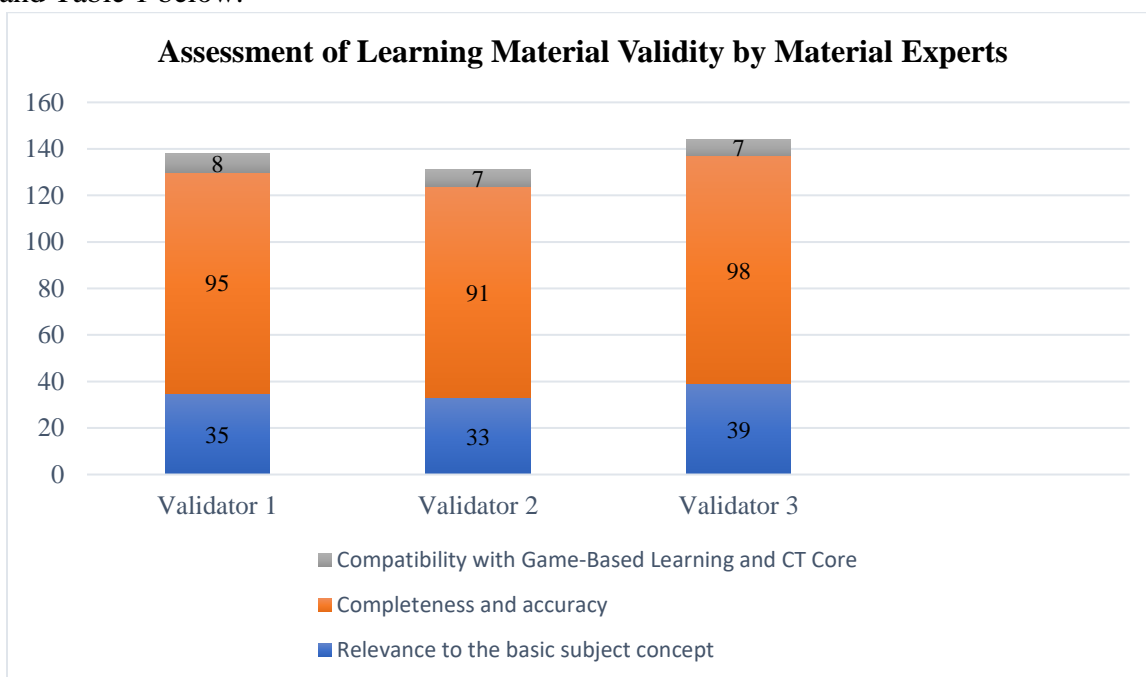


Figure 10. Description of the Assessment Result by Material Experts

Component	Validator-1	Validator-2	Validator-3
Mean	<b>138</b>	<b>135</b>	<b>144</b>
Mean total		<b>139</b>	

Table 1: Average assessment of learning material validity by material experts

Based on the material expert validation, it is known that the mean score of validator 1, 2, and 3 correspondingly are 138, 135 and 144 which fall in “Very Good”, “Good”, and “Very Good” categories. The mean total is 139 belonging to "Very Good" category. Thus, it can be concluded that the material aspect of the learning media is valid. This validity assessment is presented in Figure 11 and Table 2 below.

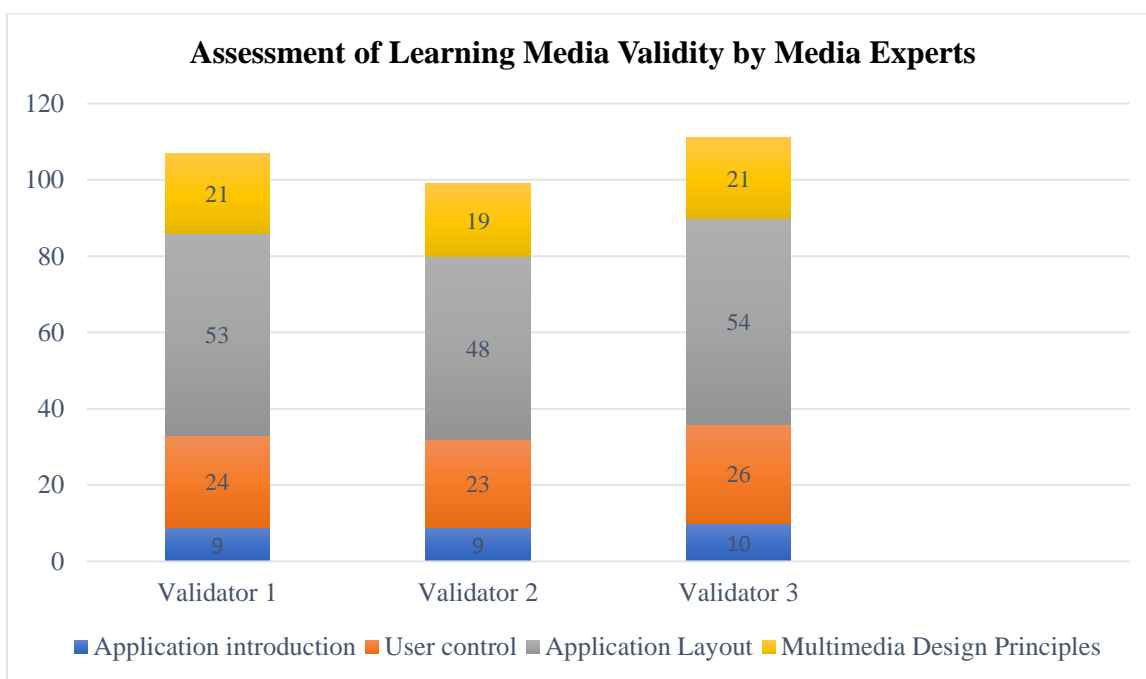


Figure 11. Description of the Assessment Result by Media Experts

Component	Validator-1	Validator-2	Validator-3
Mean	<b>107</b>	<b>99</b>	<b>111</b>
Mean total	<b>105.67</b>		

Table 2: Average assessment of learning media validity by media experts

The means obtained from the three validators are 107 (Very Good), 99 (Good), and 111 (Very Good) with the overall mean score of 105.67 (Very Good). Thus, it can be concluded that the media aspect of the learning media is valid. Conclusively, the interactive digital game based on computational thinking developed in this study has achieved the validity criteria of a product development in the aspects of material and media.



#### 4. Implementation Stage

At the implementation stage, the researcher conducted a small class trial from which was found that the product being developed reached "Very Good" criteria. Next, the researcher conducted a large class trial of four classes of students from three elementary schools. The result of student responses in large classes was 81.9 in "Very Good" criteria. Therefore, we can conclude that students' assessments of the digital game developed in this study reach the practicality principle. 98 out of 123 students showed a very good impression regarding their experience in playing the game.

#### 5. Evaluation Stage

At this stage, the researcher conducted a continuous evaluation which began with evaluating the test result of students' understanding of the multiplication concept in contextual problems. The mean score of the test was less than half of the maximum score, or more precisely less than 50. The evaluation was also carried out on the validation result at the development stage. The researcher made several revisions to the game, especially in the illustration of contextual problems and the appearance of game characters from a media perspective. Regarding the material perspective, the researcher also revised the legibility of contextual problems. According to students' assessment on the use of the game, researchers did not need to make revisions because more than 90% of students was very appreciative and enthusiastic to welcome the user-friendly game.

### DISCUSSION

The use of the digital game integrated into mathematics learning generates a positive impact from students as users. In this game-based multiplication learning, the teacher uses a digital game specially designed to assist students' understanding of multiplication concepts by presenting contextual problems. This educational digital game is called Arithmetic-CT Monkey Game. This game gives students a fun and attractive learning experience with structured game content. Monkey Game Arithmetic-CT trains students' adaptation skills to solve various problems with varied difficulty levels. Even to win the game, students must use their creativity in passing challenges or solving contextual problems. While using this game, students get feedback from the system when they access the answer-checking feature. In this case, students can improve their answers because the game is designed with specific tasks to guide students 'learning by gaming'. Various supporting features are also designed to attract students' attention to this game with a storyline of everyday life. So emotionally, students also encounter meaningful experiences.

Game-based learning supported by the Arithmetic-CT Monkey Game application on the Android platform contains twelve characteristics of digital learning, some of which are enjoyment and fun, rule, control, challenge, stimulant censor, interaction, setting, realism, and victory condition, as stated by Prensky (2003). This learning also refers to two things- education and gameplay-, as well as achieving learning goals and a means of entertainment (Lin et al., 2020). Furthermore, Lin et

al. (2020) state that the use of digital games in learning is designed by integrating the system into the experience of playing games. Because of this, a content design model and game features are often adapted to the behavioral habits of its users, such as rule, target, imagination, mystery, sensory stimulation, and control abilities (Garris et al., 2002). This underlies the conduct of several studies on computer use by children under seven years of age, which is considered to reduce children's important developmental tasks in terms of social and intellectual as well as other types of learning (Healy, 2000).

On the other hand, this game-based learning also loads core computational thinking. There are three levels with different levels of difficulty, namely level one for simple contextual problems that contain one particular variable and involve integers 1-5 as the numbers to be operated on; level two for simple contextual problems that contain one particular variable and involve integers less than ten; and level three for complex contextual problems that contain more than one variable. These problems are posed to assess students' cognitive abilities in sorting concepts into several components (the concept of addition and the concept of multiplication), then linking them together to understand the concept as a whole (the concept of multiplication is constructed from a repeated addition). In this case, core computational thinking abstraction is significant in determining students' success in solving contextual problems in games through analytical activities. The development of logical and systematic problem-solving instructions and the process of rechecking the correctness of each problem-solving step are indicators of students' cognitive capability achievement. Students' cognitive achievement is an implication of the ease of operation, and the continuous interaction between students and games during learning will build students' thinking habits while playing.

The contextual problems of multiplication that students must solve at each level are also used to assess students' psychomotor capability, which can be seen from their attitude or manipulation in the problem-solving process. Students must link various skills based on similarities or differences in patterns to predict or produce appropriate solutions just like core computational thinking pattern recognition. Students should know how to break down complex data or problems in order to effectively solve them, such as decomposition in the core of computational thinking. It is also an indicator of psychomotor abilities that students can achieve. Thus, the experience of 'learning by doing' is obtained by students when playing games, affecting their behavior and psychomotor capability when solving problems. Because with learning game-based mathematics, students transfer the abstractness of mental objects in their cognition into external representations or behaviors that can be observed so that their computational thinking skills increase. This is in line with the research result from Andriyani and Maulana (2019), which shows that a good learning experience is needed to acquire mathematical knowledge with abstract and hierarchical objects. With digital games in learning, students look enthusiastic about using interactive technology. Because technology reduces the abstractness of learning concepts, the students understand a learning situation more quickly (Buliali et al., 2022; Panthi et al., 2021).

## CONCLUSION

The current interactive digital game as a support device for game-based learning can be said to meet the validity principle based on the results of the product feasibility test as indicated by the fulfillment of the "Very Good" category in the aspect of material and media. The practicality of learning media is also indicated by the achievement of "Very Good" criteria regarding student responses. Hence, digital games have effectively addressed contextual multiplication problems that previously posed challenges to students due to their limited grasp of multiplication concept. However, the researcher has not measured the overall effectiveness of digital games in game-based learning, so this possibility opens up as material for further research. Digital games can be an alternative to support students' cognitive achievement by facilitating the translation of abstract images of multiplication concepts and training students' psychomotor capability in solving multiplication contextual problems. By incorporating core computational thinking content, digital games have demonstrated their capability to facilitate cognitive development tasks and enhance psychomotor skills, exemplified by progressive level advancements. As a result, students become accustomed to a "learning by gaming" approach. Furthermore, students' feedback has provided evidence that the core of computational thinking heightens their enthusiasm for learning multiplication. With this core loads, students feel assisted in determining the optimal solution strategy through problem formulation activities and appropriate information processing.

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## Appendix

### ASSESSMENT SHEET OF THE LEARNING MATERIAL VALIDITY BY MATERIAL EXPERTS

#### A. PURPOSE

To assess the validity of the Arithmetic-CT Monkey Game of interactive learning media for application compatibility with Game-Based Learning and CT core quality, completeness, accuracy, and relevance to the basic subject concept by material experts.

#### B. INSTRUCTIONS

1. To Mr./Miss, please assess by giving a tick (√) in the column that has been provided that is appropriate with the following assessment criteria:
  - 1: Not Good
  - 2: Less Good
  - 3: Good Enough
  - 4: Good
  - 5: Very Good
2. To Mr./Miss, please advise on improvement by writing in the comment line suggestions that have been provided.

No	Assessment Criteria	Score				
		1	2	3	4	5
<b>Compatibility with Game-Based Learning and CT Core</b>						
1	The game content present problems whose solutions contain the abstraction core of computational thinking					
2	The game content present problems whose solutions contain the algorithm design of computational thinking					
3	The game content present problems whose solutions contain the pattern recognition of computational thinking					
4	The game content present problems whose solutions contain the decomposition of computational thinking					
5	The game content present problems whose solutions contain the debugging of computational thinking					
6	Games contain special learning that helps to solve problems related to the concept of multiplication.					
<b>Completeness and accuracy</b>						
7	Coherent in the preparation of material from simple concepts to more complex concepts					
8	Diversity in giving examples related to the concept of multiplication					

9	The accuracy of the problem given with the concept of multiplication					
10	Correctness of the problem-solving feedback					
11	Readability and clarity of information contained in-game issues					
<b>Relevance to the basic subject concept</b>						
12	Suitability of the material with the core competencies and basic competencies in the referenced curriculum					
13	The usefulness of games as learning media needed by students and facilitates the achievement of learning objectives					
14	Conformity of material with the truth of its substance					
15	Examples of clarity in illustrating the abstract concept of multiplication					
16	Coverage (breadth/depth) of the material					
17	Factual material and material actualization					
18	Appropriateness of the language used with the level of the cognitive and intellectual development of students					
19	Interactivity between students and games that attract student learning motivation					
20	Instruction traction					

Comment and Suggestion:

.....  
 .....  
 .....

**C. CONCLUSION**

In terms of material aspects, the Arithmetic-CT Monkey Game of interactive learning media states:

1. Worth
2. Worth using after revision
3. Not worth

Please give a circle sign of the choice of numbers provided as the assessment result.

....., ..... 2022  
 Validator,

.....



## Comparative Analysis of Students' Argumentation Patterns in the Context of Algebraic Problems

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*Abstract: The objective of this study was to evaluate and characterize the argumentation patterns used by seventh-grade students in the context of algebraic addition and subtraction problems. A qualitative case study was conducted using the Claim-Evidence-Reasoning (CER) model (McNeill & Krajcik, 2008) and argument quality framework. To describe the arguments put forth by the participants, a sample of ten students with high mathematics proficiency and ten students with low mathematics proficiency were selected from the target population of junior high schools in a large region of West Java, Indonesia. Data collection was carried out using written argumentation frame for algebraic operations (AFAO). The results revealed that, although none of the students employed the  $C_H+E_H+R_H$  pattern (which represents the highest quality of arguments), students with high mathematics proficiency levels exhibited a greater prevalence of these argumentation patterns compared to those with low mathematics proficiency levels. The findings have implications as a valuable resource for teachers in monitoring the advancement of their students and preventing or alleviating diverse difficulties or inaccuracies that they may face. Based on the findings, a specific design for a classroom teaching activity is proposed.*

Keywords: Achievement Gaps, Argumentation Patterns, Algebraic Expressions, Mathematics Proficiency, Mathematical Argumentation

### INTRODUCTION

Argumentation is an essential skill for junior high school students (Knudsen et al., 2014; Ayalon & Even, 2016; Campbell et al., 2019) to develop, construct, and communicate their mathematical knowledge (Stylianides, 2018). In addition, through the process of argumentation, students'

reasoning skills, communication, social behavior, and information-gathering abilities also improve, and it can foster students' conceptual understanding and critical thinking (Nussbaum, 2011). The process of argumentation also serves as a foundation for changing individuals' viewpoints, as it enables the development and restructuring of ideas through analytical thinking, leading to the acquisition of knowledge, and at the core of this process is the notion of cognitive transformation, which entails a modification in cognitive frameworks (Chadha & Van Vechten, 2017).

Furthermore, developing students' mathematical argumentation skills has become a focus of attention in curricula in various countries, and cultivating this skill has become a primary goal in education (Schwarz, 2009; Kollar et al., 2014; Fukawa-Connelly & Silverman, 2015). For example, in the United States, the Common Core State Standards for Mathematical Practice (CCSSMP) includes standards for argumentation. The CCSSMP requires students to "construct viable arguments and critique the reasoning of others" (National Governors Association Center, [NGAC], 2010). This approach encompasses the comprehension and application of assumptions, the development of logical arguments, and the formulation and examination of conjectures in a systematic and rational manner (Lesseig et al., 2019). Principles and Standards for School Mathematics states that "instructional programs should enable students to develop and evaluate mathematical arguments and proofs" (NCTM, 2000).

Although the concept of argumentation is not explicitly stated in the current *Indonesian Mathematics Curriculum* (IMC; Kemendikbud, 2018) for junior high school (Years 7-9), some key elements in the standards define the issue. In the current IMC states that the learning process is developed on the principle of active student learning through activities such as observing, questioning, analyzing, and communicating, as well as strengthening critical learning patterns. Therefore, there is a relationship between the demands of the Indonesian Mathematics Curriculum for junior high school and argumentation skills, specifically in the aspect of communication.

Even though argumentation patterns are useful for identifying and evaluating argument structures and for discovering and producing complete and strong arguments (Macagno & Walton, 2015). Argumentation patterns are also useful for assessing the quality of argumentation (Zalska & Tumova, 2015). From this perspective, students' argumentation skills can be enhanced by teaching them how to construct better arguments, which are complete in all their components (including, besides data and claims, other elements that are usually missing, such as backing or warrant). Argumentation patterns are also crucial for junior high school students to evaluate their own or others' arguments. Their arguments must be sufficiently evidence-based to persuade others, and their reasons must be clear to be evaluated by others (Campbell et al., 2019).

In light of many junior high school students in the United States believe that arguments are contrary to mathematical standards when they take math classes (Forman et al., 1998). Students may think that arguments are not necessary because each problem requires a specific strategy of correct

solutions (which the teacher can or should provide). This is certainly a problem, contrary to the importance of the mathematics proficiency to argue for students. As a result, students in secondary schools often have difficulty in the process of mathematical argumentation (Schwaighofer et al., 2017). One of them is that secondary school students cannot use justification and reason to support their allegations (Kuhn & Moore, 2015; Mayweg-Paus & Macagno, 2016).

In mathematics education documents, as well as in the mathematical research, arguments are considered vital to mathematics education, but little attention is paid to the pattern of arguments in secondary mathematics classrooms. The objective of this study was to bridge this gap in the literature on mathematical arguments by evaluating and characterizing the written algebraic arguments of students. More specifically, a research question has been raised: How are the patterns structured in students' argumentation during this task?

### THEORETICAL FRAMEWORK

In 1958, Stephen Edelston Toulmin introduced a model that represented the "layout of arguments" (van Eemeren et al., 1996) in his book *The Uses of Argument*. This model has been used in numerous textbooks on argumentation for the analysis, evaluation, and construction of arguments. Toulmin's perspective on argumentation has also had a significant impact on a more theoretical level. In a more practical sense, Toulmin's model is frequently used to analyze argumentation (Metaxas et al., 2016; Doğan & Yıldırım Sır, 2022), often used in studies of written or verbal mathematical argumentation (Zambak & Magiera, 2020). Toulmin's model of argumentation has proven to be effective in analyzing argumentation skills and can also be used as a learning approach for students to construct arguments (Jonassen & Kim, 2009; Metaxas, Potari, & Zachariades, 2016).

In mathematics education research, Krummheuer was the first to apply Toulmin's argumentation scheme (Inglis et al., 2007; Moutsios-Rentzos et al., 2019). Specifically, Krummheuer (1995) only selected three core parts of the argumentation in his work, namely data, claim, and warrant (Conner, et al., 2014). Krummheuer believed that the other three components, namely backing, rebuttal, and qualifier, were less relevant to apply in the context of mathematics. Moreover, reducing the complexity of the Toulmin model is highly useful for students at the school level who may have difficulty applying the scheme in full to identify argumentation components (Kollar et al., 2007).

Some other researchers followed Krummheuer's lead by using only the elements of the core argument (Conner et al., 2014). Based on previous research, the study focuses on three main components of arguments, namely data, claims, and warrants. One model adapted to the Toulmin model is the CER model.

The CER (Claim-Evidence-Reasoning) model was derived from the more complex Toulmin argument model, adapted for use in science education (McNeill & Krajcik, 2008). Fielding-Wells (2016) applied the CER model to mathematical argumentation in primary school students in

Australia, while Graham & Lesseig (2018) did so for high school students in America. Zambak & Magiera (2020) used the model to evaluate the mathematical argumentation abilities of prospective primary and secondary school teachers. They simplified the Toulmin model's three main components for analyzing written argumentation abilities into evidence, reason, and claim, as illustrated in Figure 1.

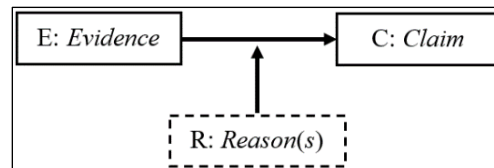


Figure 1: Model for Analyzing Written Argumentation (Zambak & Magiera, 2020)

Claim refers to a student's statement about problem-solving. Evidence is the information collected and used by students to support the truth of the claim. Meanwhile, reason is defined as the justification presented by students to reduce uncertainty and to express a comprehensive solution to all potential issues (Zambak & Magiera, 2020).

Moreover, Krummheuer (1995) asserted that a claim refers to a deduction that can be articulated either prior to or subsequent to the presentation of data (evidence). The relationship between the claim and the evidence can be identified by using terms such as "therefore" or "because." When the evidence is presented first, the reasoning is acknowledged as "evidence, so claim." Conversely, when the claim is presented first, the progression of the argumentation is described as "claim because evidence". Additionally, the term "since" can aid in delineating the correlation between data, claims, and warrants (reasoning). Consequently, the expressions "claim because of data, since reasoning" or "data becomes claim, since reasoning" can portray the progression of an argument (Nordin & Boistrup, 2018).

Within the field of mathematics education, the term "argumentation" can refer to two distinct concepts. Firstly, it may refer to mathematical arguments put forth by both students and teachers within a classroom setting. Secondly, it may refer to arguments made by researchers in mathematics education, pertaining to the nature of mathematical learning and the effectiveness of teaching mathematics in different contexts. According to Sriraman and Umland (2020), mathematical argumentation within the classroom involves presenting a logical sequence of reasoning intended to demonstrate the validity of a mathematical outcome. In the realm of mathematics education research, numerous scholars have emphasized the value of integrating argumentation-based activities in the classroom, as a means of promoting students' comprehension of mathematical concepts and their ability to reason mathematically (Erkek & Bostan, 2019).

Mathematical argumentation is a particular type of conversation characterized by justification, association, and the use of ideas (Ibrahim & Justi, 2016; Uygun & Guner, 2019). This discourse is

aimed at determining the truth of mathematical statements (Knudsen et al., 2014; Rumsey & Langrall, 2016). It can be defined as a series of statements and reasons aimed at demonstrating the validity of a claim (Cardetti, & LeMay, 2018).

Mathematical argumentation involves a range of activities such as conjecturing, testing examples, thought experiments, representing mathematical ideas, taking other perspectives, analyzing, and revising (Staples & Newton, 2016). It requires students to respond to claims made by others with their own arguments and counterarguments, construct explanations, ask questions, and potentially refute others' arguments. Mathematical argumentation is the process of constructing arguments to demonstrate or explain the truth of mathematical statements or solutions to mathematical problems. Since it is a social activity, it is important for researchers to observe and analyze what is happening so that they can understand the nature of students' argumentation.

## METHOD

The present study used a design of qualitative case studies to examine the achievements and skills of seventh grade students in cognitive tasks integrated in the argumentation of selected public schools. To this end, the data collection was carried out using a written argumentation frame for algebraic operations (AFAO). To analyze the data, a specific benchmark is designed to assess content and competence using rating scales and descriptions and existing tools are adapted to measure the learning performance and skill of learners on the task.

### Participants and Context

The determination of research participants in this study consists of four stages, namely: 1) selecting a sample from a selected school, 2) administering a mathematics proficiency test, 3) assessing students' mathematics test responses, and 4) determining participants based on differences in mathematics proficiency. The first stage involves selecting a sample. Six classes of seventh-grade students were purposively selected from a public school in a West Java province, Indonesia, out of a total of 12 classes surveyed. The sample was selected through systematic random sampling, using the numerical or alphabetical order of class naming as a basis. For instance, classes A to F were included in the sample.

The second stage is to administer a Mathematics Proficiency Test (MPT) to the selected classes of students. The MPT consists of 23 multiple-choice questions developed by the researcher and validated. This activity was carried out on February 6, 2023, during the first learning session. During the test administration, each class was supervised by one teacher. The participants in the Mathematics Proficiency Test were 225 students, consisting of 90 male students (40%) and 135 female students (60%). The students' ages ranged from 12-14 years (Mean age = 13.08 years, Standard Deviation = 0.55).

The third stage involved scoring students' MPT responses. The scores of the students were then ranked from highest to lowest. Finally, 10 students with the highest MPT scores and 10 students

with the lowest MPT scores were selected. The results of the student selection are presented in Table 1.

High Group		N	10	Low Group		N	10
Rank	ID	Class	Score	Rank	ID	Class	Score
1	Student 1	E	22	225	Student 225	B	2
2	Student 2	F	22	224	Student 224	A	3
3	Student 3	F	21	223	Student 223	E	4
4	Student 4	F	21	222	Student 222	A	4
5	Student 5	A	20	221	Student 221	E	6
6	Student 6	A	20	220	Student 220	E	6
7	Student 7	B	20	219	Student 219	C	6
8	Student 8	D	20	218	Student 218	B	6
9	Student 9	F	20	217	Student 217	F	7
10	Student 10	A	19	216	Student 216	F	7

Table 1: Research participants

### Data Collection Process

Data were collected by the first author in February 2023. Other teachers were also present to assist with classroom management during data collection. The test was written in Indonesian and was conducted individually in a quiet classroom.

The data in this study were obtained using the Argumentation Frame in Algebraic Operations (AFAO), as presented in Appendix 1. The AFAO was developed based on a question construction framework that was predetermined by the researchers and validated by three expert judges. The CVI scores for the instrument's sufficiency, clarity, coherence, and relevance dimensions ranged from 0.90 to 1.00, indicating excellent content validity (Polit & Beck, 2006). The study utilized the revised Bloom taxonomy cognitive level for evaluating (Anderson & Krathwohl, 2001), which included indicators to verify solutions to algebraic addition and subtraction problems.

The twenty selected participants were provided with AFAO-1, AFAO-2, and AFAO-3 sheets to be completed within a maximum time of 40 minutes. AFAO-2 and AFAO-3 had the same type and content as AFAO-1, with the only difference being the numerical values presented in the questions. AFAO-2 and AFAO-3 were used as triangulation to verify the credibility of the students' answers. The research question was answered using the AFAO instrument (Figure 2). In summary, AFAO is a written survey tool that has been specifically designed to assess and elicit the ideas of learners that are embedded within argumentation.

Ani merupakan siswa kelas VII di salah satu SMP di Kota Bandung. Pada suatu ulangan matematika, Ani diberi soal $(5a - 7b) + (13a + 8b)$ .	Ani is a student in the seventh grade attending one of the junior high schools in the city of Bandung. While taking a mathematics examination, Ani was presented with a mathematical problem in the form of $(5a - 7b) + (13a + 8b)$ .
Proses penyelesaian soal yang dikerjakan oleh Ani adalah sebagai berikut: Langkah-1 $(5a - 7b) + (13a + 8b)$ Langkah-2 $5a - 7b + 13a + 8b$ Langkah-3 $-2a + 21b$	The solution process utilized by Ani is outlined as follows: Step-1: $(5a - 7b) + (13a + 8b)$ Step-2: $5a - 7b + 13a + 8b$ Step-3: $-2a + 21b$
Periksa langkah demi langkah proses penyelesaian soal yang dilakukan Ani. Benar atau salahkah pekerjaan Ani? Jelaskan mengapa!	Please verify, in a step-by-step manner, the problem-solving process executed by Ani. Determine whether Ani's work correct or incorrect? Explain why!

Figure 2: The argumentation task frame (AFAO)

The responses in the AFAO instrument were independently coded by the first author and reviewed by the second and third authors. Moreover, efforts were made to establish a coherent link between the task and the inferences drawn from the participants' responses. To ensure the credibility of the data, this study employed the within-method triangulation (Denzin & Lincoln, 2017). The procedure involved comparing the data obtained from AFAO-1 with that of AFAO-2. If there was consistency between the two sets of data, the first data was considered valid, credible, and suitable for analyzing the research student's argumentation process. However, if inconsistency was observed, it was compared with the data obtained from AFAO-3. This process could continue until consistent data was obtained to be used in the data analysis to reveal the pattern of the research student's argumentation.

Based on data obtained from the work of high-mathematical students in AFAO-1, AFAO-2 and AFAO-3, nine ( $n = 9$ ) data were considered reliable. The only data point of student 50 was deemed unreliable (Figure 3). A non-credible data point was considered invalid. Thus, the nine credible data points obtained from the work of AFAO-1 or AFAO-2 are valid and can be used for analysis.

Furthermore, in the data obtained from the low math ability students' work, there was one data point from student 219 that was deemed not credible as it did not follow AFAO-2 and AFAO-3, thus its data credibility could not be tested. Therefore, one non-credible data point was considered invalid. Therefore, the nine credible data points obtained from the work in AFOA-1 or AFOA-2 were valid and could be used for analysis.

1. Salah karena adanya perpindahan tempat Jadi $5a - 7b + 13a + 8b$ $= -8a + 12b$ AFHA-1	"Wrong due to the displacement of location. Therefore, $5a - 7b + 13a + 8b = -8a + 12b$ ."
1. Benar, karena perpindahannya yang sangat tepat AFHA-2	"Correct due to the very precise displacement."
1. Benar, $-2a + 17b$ itu hitungan yang baik AFHA-3	"Correct $-2a + 17b$ is a good calculation."

Figure 3: Example of non-credible data (Student 50)

### Data Analysis Process

We conducted a four-step analysis of the data. In the first step, we identified the arguments that the student understood when examining the algebraic addition and subtraction problem-solving. To accomplish this process, the researchers calculated the percentage of students who answered "incorrectly" to the question "Check step-by-step the problem-solving process done by Ani. Is Ani's work correct or incorrect?". This step was critical to verify if the question was comprehensible to the students. The second step was to identify the arguments that the student used to justify (evidence and reasoning) when examining the problem-solving. To accomplish this process, the researchers calculated the percentage of students who provided justifications for the prompt, "Explain why!".

The third step was to identify patterns in the responses of the students to the algebraic operations problems based on the dimensions of structure, content, and recipient-orientation (Meyer & Schnell, 2020). In terms of the dimension of structure, this research utilized the CER (Claim, Evidence, Reasoning) model. Using the CER model, the researchers focused on identifying the claims made by each student about the process of problem-solving. The researchers also looked for evidence and reasoning provided by the students to support each claim. Furthermore, the researchers created argument maps by summarizing all the arguments formulated by the students. In each map, the researchers explained all the relationships between the evidence, reasoning, and claims included by the student in their solution narrative.

Component of Argument	Quality	Description	Code
Claim	Low	Not making any claims, or making inaccurate or false claims.	C <sub>L</sub>
	Moderate	Making claims that are accurate but incomplete	C <sub>M</sub>
	High	Making a claim that is both accurate and complete.	C <sub>H</sub>
Evidence	Low	Not providing evidence, or only providing inappropriate evidence (evidence that does not support the claim).	E <sub>L</sub>
	Moderate	Providing precise but insufficient evidence to support a claim. This may include some incongruous pieces of evidence.	E <sub>M</sub>
	High	Providing precise and sufficient evidence to support a claim.	E <sub>H</sub>
Reasoning	Low	Failure to provide reasoning, or only providing reasoning that does not connect evidence to claims.	R <sub>L</sub>
	Moderate	Providing reasoning that connects claims and evidence, reiterating evidence and/or incorporating some, but insufficient scientific principles.	R <sub>M</sub>
	High	Providing reasoning that connects evidence with claims, encompassing appropriate and adequate scientific principles.	R <sub>H</sub>

Table 2: Guidelines for assessing argument quality (McNeill & Krajcik, 2008)

In addition, Meyer & Schnell (2020) stated that several questions can be asked to identify the structure of arguments. For instance, does the claim answer the question? Is there evidence to



support the claim? Is there reasoning to explain how the evidence supports the claim? and so on. The content dimension is related to the use of mathematical rules included in the argument structure, such as whether the mathematical rules used are correct. Meanwhile, the recipient-orientation dimension is related to the language aspect, such as whether the presented argument can be understood as a whole. The final step was to identify the quality of each student's argument components. The researchers adapted the argument assessment guidelines from McNeill & Krajcik (2008) and as shown in Table 2.

## RESULTS AND DISCUSSION

The students' response to the question "*Check step-by-step the problem-solving process carried out by Ani. Is Ani's work correct or incorrect?*" is presented in Table 3.

Group	Percentage of claims	
	"Incorrect"	"Correct"
High ( $n = 9$ )	88,9%	11,1%
Low ( $n = 9$ )	11,1%	88,9%

Table 3: Percentage of responses based on mathematics proficiency levels

As shown in Table 3, from the data of nine high-level mathematics proficiency students that were deemed valid and credible, eight students (88.9%) claimed "*Incorrect*". Only one student (11.1%) claimed "*Correct*". This indicates that the majority of students understood the presented questions and problems. However, these results were in contrast to those of students with low-level mathematics proficiency. From the data of nine students deemed valid and credible, only one student (11.1%) claimed "*Incorrect*". Eight students (88.9%) claimed "*Correct*". This suggests that the majority of students with low-level mathematics proficiency struggled to understand the presented questions and problems.

The justification for the "*Explain why!*" question based on the obtained data showed that all students with high-level mathematics proficiency ( $n = 9$ ) provided evidence and reasoning. In contrast, among students with low-level mathematics proficiency ( $n = 9$ ), only three students provided evidence, and six students provided reasoning. Additionally, the distribution of the quality of claims, reasoning, and evidence on the first problem was also identified, as presented in Table 4.

Component of Argument	Quality	High Group Frequency ( $n = 9$ )	Low Group Frequency ( $n = 9$ )
<i>Claim</i>	Low	1 (11,1%)	8 (88,9%)
	Moderate	5 (66,7%)	1 (11,1%)
	High	3 (33,3%)	0 (0,0%)
<i>Evidence</i>	Low	1 (11,1%)	8 (88,9%)
	Moderate	7 (77,8%)	1 (11,1%)
	High	1 (11,1%)	0 (0,0%)
<i>Reasoning</i>	Low	4 (44,4%)	8 (88,9%)
	Moderate	0 (0,0%)	1 (11,1%)
	High	5 (55,6%)	0 (0,0%)

Table 4: The distribution of argumentation component levels in the first problem

As presented in Table 4, most high-proficiency mathematics students have moderate levels of claim quality (55.6%) and evidence quality (77.8%), with only few students having low (11.1%) or high (11.1%) quality. Meanwhile, the majority of students have high quality reasoning (55.6%), and 44.4% are categorized as having low quality. No students have moderate quality reasoning. These findings indicate that the majority of high-proficiency mathematics students possess good skills in constructing mathematical arguments.

Furthermore, students with low mathematics proficiency mostly have low quality claims, evidence, and reasoning. Only one student (11.1%) had moderate quality claims, evidence, and reasoning. These results indicate that the majority of students with low mathematics proficiency struggle with constructing mathematical arguments.

Based on the quality of arguments presented in Table 4, this study identifies six patterns of students' mathematical argumentation in solving algebraic addition and subtraction problems as follows: 1)  $C_L+E_L+R_L$ . Low claim, low evidence, and low reasoning, 2)  $C_M+E_M+R_L$ . Moderate claim, moderate evidence, and low reasoning, 3)  $C_M+E_M+R_H$ . Moderate claim, moderate evidence, and high reasoning, 4)  $C_H+E_M+R_L$ . High claim, moderate evidence, and low reasoning, 5)  $C_H+E_M+R_H$ . High claim, moderate evidence, and high reasoning, and 6)  $C_H+E_H+R_L$ . High claim, high evidence, and low reasoning. The distribution of argumentation patterns constructed by students with high mathematics proficiency level (HMP) and low mathematics proficiency level (LMP) is presented in Figure 4.

As shown in Figure 4, the distribution of argumentation patterns is highest among students with high mathematics proficiency levels (6 patterns), while only 2 patterns are found in students with low mathematics proficiency levels. At the HMP level, the majority of students ( $f=4$ , 44.5%)

exhibit the argumentation pattern of  $C_M+E_M+R_H$ , while only one student each ( $f=1$ , 11.1%) exhibit the patterns of  $C_L+E_L+R_L$ ,  $C_M+E_M+R_L$ ,  $C_H+E_M+R_L$ ,  $C_H+E_M+R_H$ , and  $C_H+E_H+R_L$ . At the LMP level, the majority of students ( $f=8$ , 88.9%) exhibit the argumentation pattern of  $C_L+E_L+R_L$ . Only one student (11.1%) exhibits the pattern of  $C_M+E_M+R_H$ . Although there is no high-quality argumentation ( $C_H+E_H+R_H$  pattern), the HMP level provides better arguments than the LMP level.

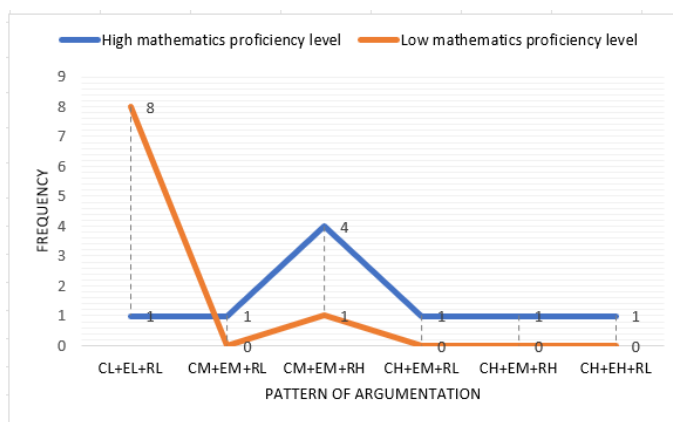


Figure 4: Distribution of argumentation patterns

## Argumentation Patterns of Students with High Mathematics Proficiency Level

### $C_L+E_L+R_L$ Pattern

Figure 6 below presents the results of the work of student 6 who has high mathematics proficiency using the  $C_L+E_L+R_L$  argumentation pattern.

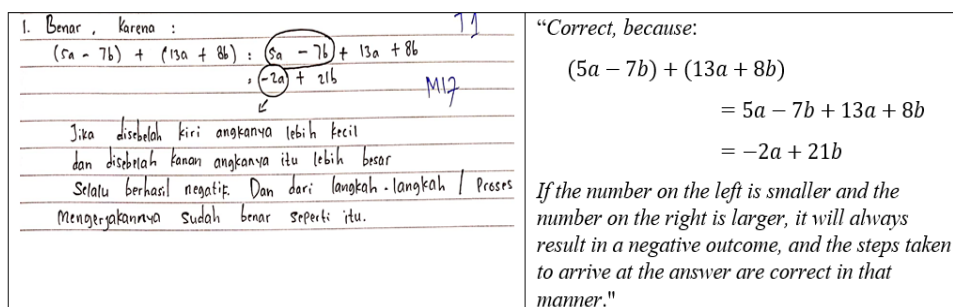


Figure 5: Student 6 Response to problem

As shown in Figure 6, the student made a false claim by stating that Ani's problem-solving process was correct. This resulted in the student providing incorrect or irrelevant evidence and reasoning. The student provided reasoning using the rule of subtracting numbers (incorrect concept) which led to step  $5a - 7b = -2a$  being considered correct. The student did not understand the presented problem. The student also did not understand the rules of subtraction or addition in algebraic forms, that subtraction or addition of algebraic forms can only be done if the terms are of the same type.

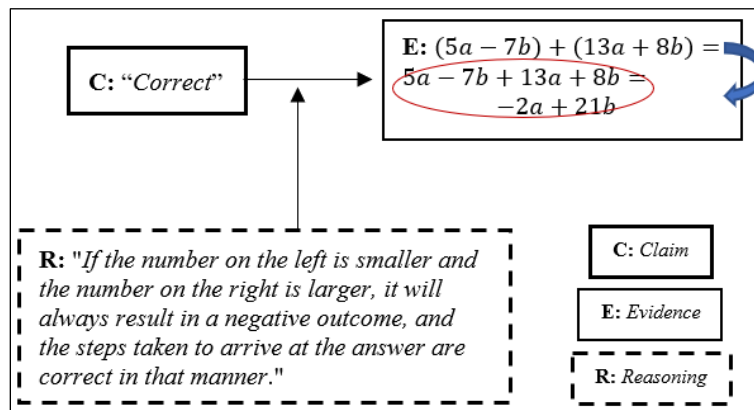


Figure 6: Student 6 Argument map on problem

### CM+EM+RL Pattern

The result of the work of student 9, who has high mathematics proficiency with the  $C_M+E_M+R_L$  argumentation pattern, is presented in Figure 7.

<p>1. Salah.</p> <p>karena, jawaban yang benar adalah :</p> <p>Langkah -1 : <math>(5a - 7b) + (13a + 8b)</math></p> <p>Langkah -2 : <math>(5a \times 13a + 5a \times 8b) + (-7b \times 13a + -7b \times 8b)</math></p> <p>Langkah -3 : <math>65a^2 + 40ab + -91ab + -56b^2</math> (jumlahkan bilangan yang 1 suku)</p> <p>Langkah -4 : <math>65a^2 - 51ab - 56b^2</math> (hasil)</p>	<p>"Incorrect.</p> <p>Because, the correct answer is:</p> <p>Step-1: <math>(5a - 7b) + (13a + 8b)</math></p> <p>Step-2: <math>(5a \times 13a + 5a \times 8b) + (-7b \times 13a + -7b \times 8b)</math></p> <p>Step-3: <math>65a^2 + 40ab + -91ab + -56b^2</math> ("Calculate the sum of a single term.")</p> <p>Step-4: <math>65a^2 - 51ab - 56b^2</math> ("result")</p>
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Figure 7: Student 9 Response to problem

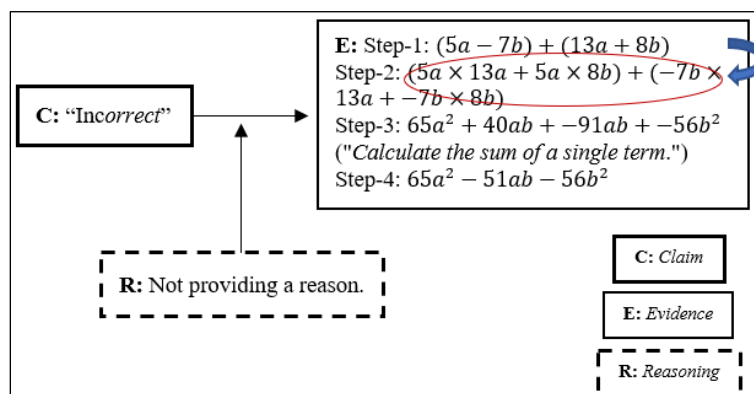


Figure 8: Student 9 Argument map on problem

As shown in Figure 8, the student made a true claim by stating that the solving process performed by Ani was incorrect. However, the student did not provide a reason. Although the student provided evidence, there was evidence that did not match. In the first step, the student provided appropriate evidence by rewriting the problem that needed to be solved. However, in the second step, the student solved the problem using the wrong method  $((5a - 7b) + (13a + 8b) = (5a \times 13a + 5a \times 8b) + (-7b \times 13a + -7b \times 8b))$ . The student used the wrong concept by applying the distributive law  $a \times (b + c) = a \times b + a \times c$  to solve the problem. The student also did not pay attention to the rule of arithmetic operations in the brackets. These results indicate that the student understands the problem, but does not provide a reason. Although providing evidence, there is an inappropriate process.

### C<sub>M</sub>+E<sub>M</sub>+R<sub>H</sub> Pattern

Figure 9 shows an example of the work results of students 1 and 5 who have high mathematics proficiency with the argumentation pattern of C<sub>M</sub>+E<sub>M</sub>+R<sub>H</sub>.

<p>1. Salah, karena Ani tidak menyamakan variabel terlebih dahulu (jika ada dua variabel berbeda maka tidak bisa dijumlahkan atau dikurangkan)</p> $1_1 = (5a - 7b) + (13a + 8b)$ $1_2 = 5a - 13a + 7b - 8b$ $1_3 = -8a + 7b - 8b$ $1_4 = -8a - 1b$ <p>(Student 5)</p>	<p><i>"Incorrect, because Ani did not first make the variables equal (if the variables are different, they cannot be added or subtracted)."</i></p> $(5a - 7b) + (13a + 8b) =$ $5a - 13a + 7b - 8b =$ $-8a + 7b - 8b = -8a - 1b$ <p>(Student 5)</p>
<p>1. salah, karena jika variabel nya berbeda maka tidak bisa di jumlah atau dikurangkan :</p> $\rightarrow (5a - 7b) + (13a + 8b)$ $\rightarrow (5a - 13a) + (7b + 8b)$ $\rightarrow -8a + 15b$ <p>jadi jawaban yang benar adalah <math>-8a + 15b</math></p> <p>(Student 1)</p>	<p><i>"Incorrect, because if the variables are different, they cannot be added or subtracted."</i></p> $(5a - 7b) + (13a + 8b) =$ $(5a - 13a) + (7b + 8b) =$ $-8a + 15b$ <p>(Student 1)</p>

Figure 9: Example responses to problem (Student 1 and 5)

As shown in Figure 10, the student made a true claim by stating that the presented problem-solving process was incorrect. To support this claim, the student provided reasoning that the error in the problem-solving process occurred due to operating (adding or subtracting) different variables. Based on this reasoning, the student presented evidence in the form of problem-solving steps that they believed to be correct. In the process, the initial step taken was correct by grouping like terms. However, in the step  $(5a - 7b) + (13a + 8b) = 5a - 13a + 7b - 8b$ , there was an incorrect process by changing the operation sign (from + to - or vice versa). This result shows that the student was unable to order the algebraic operation steps involving remove brackets when grouping like terms.

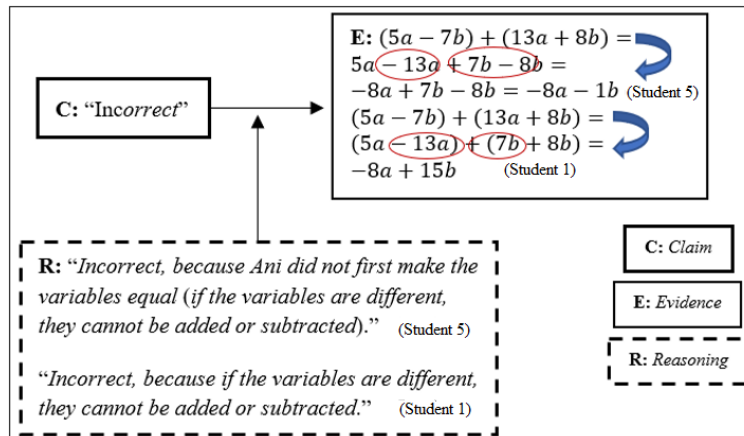
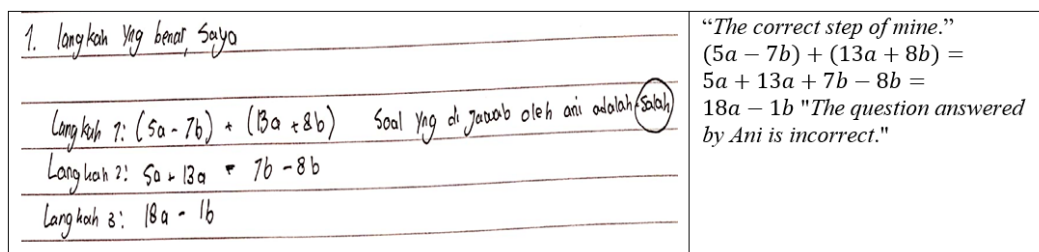


Figure 10: Argument map on problem (Student 1 and 5)

### CH+EM+RL Pattern

The results of the work of student 4, who possesses high mathematical abilities and employs the CH+EM+RL argumentation pattern, can be found in Figure 11.



The figure shows handwritten work on the left and a typed claim on the right. The handwritten work includes:
 

- 1. langkah yg benar, sayo
- Langkah 1:  $(5a - 7b) + (13a + 8b)$  Soal yang di jawab oleh ani adalah (Salah)
- Langkah 2:  $5a + 13a + 7b - 8b$
- Langkah 3:  $18a - 1b$

 The typed claim on the right reads: "The correct step of mine." followed by the correct expansion:  $(5a - 7b) + (13a + 8b) = 5a + 13a + 7b - 8b = 18a - 1b$ . It concludes with: "The question answered by Ani is incorrect."

Figure 11: Student 4 response to problem

As shown in Figure 12, the student made a complete claim that "Ani's answer is incorrect". However, the student did not provide reasoning related to the concept used in the presented problem-solving process. Meanwhile, the student presented evidence to support the claim. In the process, the evidence presented was appropriate, but there was an incorrect problem-solving step. In the step  $(5a - 7b) + (13a + 8b) = 5a + 13a + 7b - 8b$ , there was a change in the operation sign in the term  $8b$  (from  $+8b$  to  $-8b$ ). This result indicates that the student had difficulty in ordering the operation steps involving remove brackets.

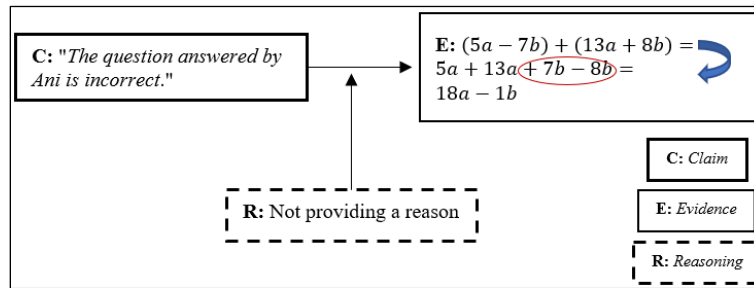


Figure 12: Student 4 argument map on problem

### $C_H + E_M + R_H$ Pattern

Figure 13 presents the results of the work of student 8, who possesses high mathematical abilities and employs the  $C_H + E_M + R_H$  argumentation pattern.

<p>1.). jawabannya adalah Salah.          Karena : Ani di beri Soal <math>(5a - 7b) + (13a + 8b)</math>.          Saat ani menjawab Soal tersebut, ia tidak menyamakan variabelnya.          Seharusnya,          Langkah - 1 <math>(5a - 7b) + (13a + 8b)</math>          Langkah - 2 <math>(5a - 13a) + (7b + 8b)</math>          Langkah - 3 <math>= -8a + 15b</math></p>	<p>"The answer provided by Ani is incorrect."          "Due to receiving a problem <math>(5a - 7b) + (13a + 8b)</math>, Ani failed to equate the variables when answering the problem. The correct approach should have been to do so."          Step-1: <math>(5a - 7b) + (13a + 8b)</math>          Step-2: <math>(5a - 13a) + (7b + 8b)</math>          Step-3: <math>-8a + 15b</math></p>
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Figure 13: Student 8 response to the problem

Figure 14 depicts that the student made a claim that the given answer is incorrect due to the use of different variables in the operations. Moreover, the student presented evidence in the form of an alternative problem-solving approach. In step  $(5a - 7b) + (13a + 8b) = (5a - 13a) + (7b + 8b)$ , the student performed the required steps by grouping and operating with similar terms. However, there were modifications made in the operation signs, specifically in the terms  $13a$  (changed from  $+13a$  to  $-13a$ ) and  $7b$  (changed from  $-7b$  to  $+7b$ ). Consequently, the final result of the problem-solving process was incorrect.

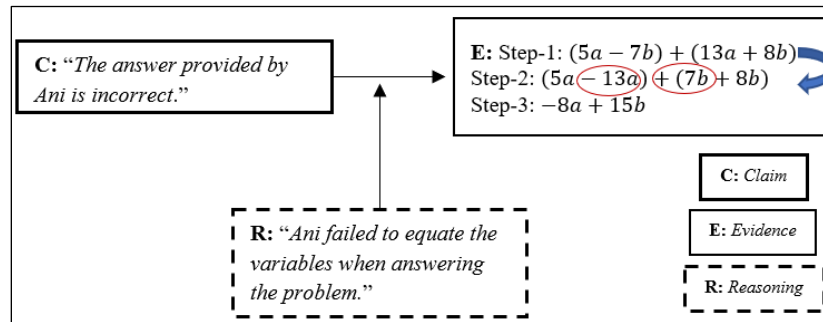


Figure 14: Student 8 argument map on problem

### **C<sub>H</sub>+E<sub>H</sub>+R<sub>L</sub> Pattern**

The findings of the performance of student 3, who exhibits a high level of mathematical proficiency with the argumentation pattern of C<sub>H</sub>+E<sub>H</sub>+R<sub>L</sub>, are displayed in Figure 15.

1. Proses yang benar	proses yang salah yang di jawab oleh Ani.	"The incorrect process that was answered by Ani."
Langkah-1 $(5a-7b) + (13a+8b)$	Langkah-1 $(5a+7b) + (13a+8b)$	"The correct process,"
Langkah-2 $5a+13a-7b+8b$	Langkah-2 $5a-7b+13a+8b$	Step-1 $(5a-7b) + (13a+8b)$
Langkah-3 $18a+1b$	Langkah-3 $-2a+21b$	Step-2 $5a+13a-7b+8b$
		Step-3 $18a+1b$

Figure 15: Student 3 response to the problem

Figure 16 illustrates that the subject made a distinct claim by stating that "Ani's answer process is incorrect." However, the subject did not provide reasoning pertaining to the location of the error in the presented problem-solving steps. Meanwhile, the subject put forth alternative evidence of a proper problem-solving approach that is adequate to substantiate the claim.

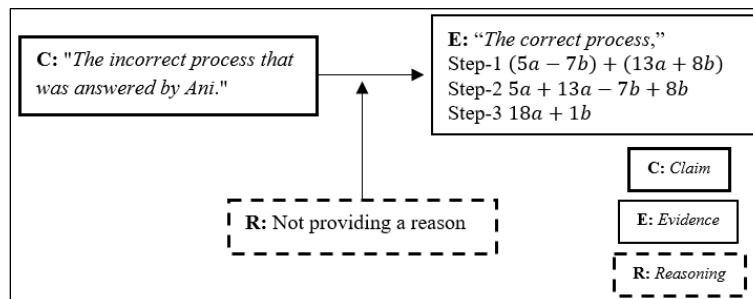


Figure 16: Student 3 argument map on problem



## Argumentation Patterns of Students with Low Mathematics Proficiency Level

### CL+EL+RL Pattern

The outcomes of the student's responses based on this pattern comprise two types, namely: 1) simply rewriting the problem, and 2) providing arguments, but with inappropriate claim, reasoning, and evidence. Specifically, Figure 17 below presents an example of the student's response.

$\begin{aligned} & (5a-7b) + (13a+8b) \\ & + 5a - 7b + 13a + 8b \\ & - 2a + 21b = 67 \end{aligned}$ <p>Periksa jawaban, demi langkah, PROSES PEMECAHAN SOAL yang dilakukan Ani (Student 224)</p>	$\begin{aligned} & (5a - 7b) + (13a + 8b) \\ & 5a - 7b + 13a + 8b \\ & - 2a + 21b = 67 \end{aligned}$ <p>"Please examine step by step the problem-solving process carried out by Ani." (Student 224)</p>
<p>I benar, dikarenakan Ani menghitung soal nya. perjelasannya: <math>(5a-7b) + (13a+8b)</math> Ani: <math>5a-7b + 13a+8b</math> <math>-2a+21b</math> Jadi benar Ani: <math>-2a+21b</math> (Student 223)</p>	<p>"Correct, because Ani calculated the problem. Explanation: <math>(5a - 7b) + (13a + 8b)</math> <math>5a - 7b + 13a + 8b</math> <math>- 2a + 21b</math>. Therefore, the result is Ani's expression: <math>-2a+21b</math>." (Student 223)</p>
<p>1. Ani jawabanja benar <math>= (5a-7b) + (13a+8b)</math> <math>- 2a - 7b + 13a + 8b</math> <math>- 2a + 4b</math> (Student 221)</p>	<p>"The answer provided by Ani is correct." <math>(5a - 7b) + (13a + 8b)</math> <math>5a - 7b + 13a + 8b</math> <math>- 2a + 21b</math>. (Student 221)</p>

Figure 17: Example of student's response to problem (Student 221, 223 and 224)

As delineated in Figure 18, within the CL+EL+RL argumentation pattern, an LMP student exists who abstains from expressing a viewpoint on the accuracy of the presented problem-solving steps. Additionally, the student refrains from presenting any reasoning or supporting evidence. Rather, the student merely reiterates the problem statement that was initially provided. Moreover, some students make erroneous claims by asserting the veracity of "Ani's answer". Consequently, the student proffers arguments that lack a cogent connection between the evidence and the claim. Furthermore, the student exclusively presents evidence by reciting the problem-solving process that was originally furnished in the query.

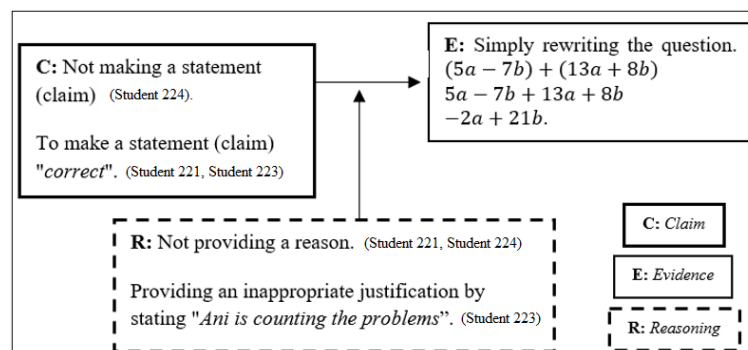


Figure 18: Argumentation map of student's response to the problem (Student 221, 223 and 224)

### $C_M+E_M+R_H$ Pattern

The outcomes of the task completed by the student 217 who possesses low mathematical proficiency and employed the  $C_M+E_M+R_H$  argumentation pattern, are displayed in Figure 19.

Soal: Ani diberi Soal $(5a - 7b) + (13a + 8b)$ . 1) Jawab: $(5a - 7b) + (13a + 8b)$ . $5a - 7b + 13a + 8b$ $= 8a + 15b$ Salah karena ani tidak menyamakan Variabel.	Ani was given a task $(5a - 7b) + (13a + 8b)$ . Respond: $(5a - 7b) + (13a + 8b)$ $5a - 7b + 13a + 8b$ $= 8a + 15b$ . "Incorrect due to Ani's failure to equate the variables".
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Figure 19: Student 217 Response to the problem

As indicated in Figure 20, the student posits that the presented steps for problem-solving are erroneous. Additionally, the student cites the reason for this fallacy as the failure to perform operations using identical variables. Nonetheless, the solution proffered by the student does not correspond with the aforementioned rationale. The student offers an alternative solution; however, it entails performing operations on disparate variables.

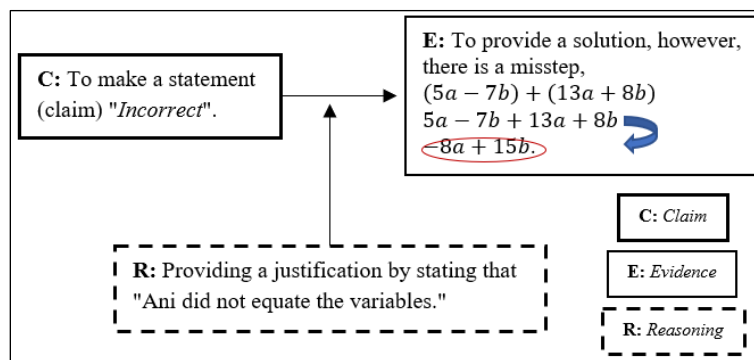


Figure 20: Student 217 Argument map on problem

## CONCLUSIONS

The study examined the abilities of high and low proficiency mathematics students in constructing mathematical arguments while solving algebraic addition and subtraction problems. The research concludes that students with advanced mathematics skills are more effective at constructing persuasive arguments than those with lower proficiency. However, even among high proficiency students, limitations in their ability to argue effectively were observed. In accordance with the findings of Farra et al. (2022), students occasionally encountered difficulties in providing justifications for their responses, and a few resorted to reproducing the phrasing used in the questions.

The study identified six distinct patterns of mathematical argumentation that students utilized when solving algebraic addition and subtraction problems. Although none of the students employed the  $C_H+E_H+R_H$  pattern (the highest quality of arguments), Students with high

mathematics proficiency levels were found to exhibit a greater prevalence of these argumentation patterns than those with low mathematics proficiency levels. Students with high proficiency tended to rely on the  $C_M+E_M+R_H$ ,  $C_H+E_M+R_H$ , and  $C_H+E_M+R_L$  argumentation patterns. Nevertheless, even those students who utilized intricate argumentation patterns were not invariably capable of presenting thorough and precise arguments. In certain instances, they demonstrated inadequate understanding of algebraic concepts, such as the associative and distributive law, or made errors when solving problems, such as incorrect ordering of the algebraic operation steps involving remove brackets. Ojo (2022) stated that the utility and significance of algebra are commonly perceived to derive from its concepts and mode of reasoning.

We argue that the highest quality of arguments does not relate to particular mathematics proficiency levels. This implies that while argumentation patterns can serve as an advantageous instrument for students to enhance their mathematical argumentation and problem-solving proficiencies, they also require a profound understanding of algebraic concepts and procedures. In Addition, through comprehension of students' argumentation patterns, teachers can anticipate the possible procedures that students may undertake while solving a mathematical problem. Consequently, teachers can observe students' progress and preclude or mitigate various challenges or inaccuracies encountered by students.

Teachers should use effective teaching strategies that encourage their students to effectively communicate their mathematical knowledge using various methods, including verbal and non-verbal as well as written argumentation. For low-proficiency mathematics students, interventions that focus on improving their argumentation skills may be particularly beneficial. In this regard, teachers may need individualized support and guidance to help these students develop mathematical argumentation skills. Thus, it is essential to devise an instruction methodology that facilitates the adaptation of students to addressing supplementary algebraic problems. These problems require the use of argumentation patterns and algebraic concepts, while encouraging students to recognize the arguments they use in their explanations and identify any challenges they face during the activity.

## RECOMMENDATIONS

Based on the findings of the study, a recommendation is put forth for a specific design of a classroom teaching activity comprising of seven sequential steps. First, introduce the topics of algebraic addition and subtraction problems and explain the importance of constructing persuasive arguments when solving them. Then, give a written argumentation frame for algebraic operations (AFAO) and ask students to solve them individually or in pairs. Subsequently, students should share solutions and arguments with the class and encourage each other to constructively criticize each other's arguments. Explain the six different mathematical argument patterns identified in the study to the students, including the  $C_H+E_H+R_H$  pattern, which represents the highest quality of arguments. Provide examples of each argumentation pattern and ask students to identify the

pattern(s) they used in their argumentation. Provide feedback on the quality of the students' arguments by focusing on their use of argumentation patterns and their understanding of algebraic concepts and procedures. Lastly, encourage students to reflect on their own learning and identify areas where further practice and support are needed.

Throughout the activity, it is important to emphasize that the pattern of arguments does not relate to particular mathematics proficiency levels. Instead, it requires a profound understanding of algebraic concepts and procedures. By using argumentation patterns as an instrument to enhance their mathematical argumentation and problem-solving proficiencies, students can develop a deeper understanding of algebraic concepts and procedures, and improve their ability to construct persuasive arguments when solving algebraic problems.

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## Appendix 1

### Argumentation Frame in Algebraic Operations (AFAO)

The questionnaire consists of two sections. The first is intended to simplify analysis. In the second section, you must use your previous knowledge of the algebraic addition and subtraction axioms. This questionnaire is not part of your regular algebra activity, so it does not affect your results. Your name is not linked to your responses.

#### SECTION A: Demographic information

Personal Particulars	
Name:	Name of the School:
Gender:	Class:
Date of Birth:	

#### SECTION B: Algebra Task

##### Instructions

- *This questionnaire will not affect your grades. Please do not spend a lot of time on a single statement – your first thoughts are usually the best.*
- *Write your responses on the spaces provided after the statement. Please respond to every statement – it's important that you respond to each statement honestly.*
- *All the information will be used for research purposes only. Your responses will be treated confidentially. Your responses will not reveal any information that could identify you.*
- *This survey should take you about 40 minutes to complete.*

Ani is a student in the seventh grade attending one of the junior high schools in the city of Bandung. While taking a mathematics examination, Ani was presented with a mathematical problem in the form of  $(5a - 7b) + (13a + 8b)$ .

The solution process utilized by Ani is outlined as follows:

Step-1:  $(5a - 7b) + (13a + 8b)$

Step-2:  $5a - 7b + 13a + 8b$

Step-3:  $-2a + 21b$

Please verify, in a step-by-step manner, the problem-solving process executed by Ani. Determine whether Ani's work correct or incorrect? Explain why!



## Development of RME Learning Media Based on Virtual Exhibition to Improve Students' High Order Thinking Skills (HOTS)

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*Abstract: Students' higher-order thinking skills (HOTS) in Indonesia are still low and have not experienced significant development. To enhance HOTS, modern technology-based learning media are needed to support realistic mathematics learning. Innovative learning media can enhance realistic learning and students' thinking abilities. Therefore, the authors aim to develop virtual exhibition-based Realistic Mathematics Education (RME) learning media and evaluate its quality in improving students' high-level thinking abilities. The results of this research are significant in bridging the gap by providing flexible and easily accessible alternative learning materials, addressing the shortage of interactive technology-based mathematics learning media for boosting students' HOTS. This research was conducted at Widiatmika Middle School, with classes 8A and 8B participating. The authors employed the Borg and Gall Research and Development (R&D) method to develop the learning media. Several data collection instruments were used, including interviews, observations, questionnaires, validation sheets, and tests. Data analysis was carried out to determine whether the product met the criteria for validity, effectiveness, and practicality. The validation results indicated that the virtual exhibition-based RME media achieved an ideal total average percentage of 87.27% in media expert validation and 91% in material expert validation. Thus, it can be concluded that the virtual exhibition-based RME media developed falls within the 'very good' criteria for use in mathematics learning. The students' HOTS test results in the Limited Test showed a completion rate of 60%, which increased to 79% in the Field Test. The average student opinion score regarding the quality of virtual exhibition-based RME media is 83%. The application of RME learning combined with technology has a positive impact on enhancing students' HOTS. RME learning using virtual exhibition media provides students with a modern, effective, and enjoyable approach to learning mathematics.*

## INTRODUCTION

Mathematics is crucial for the development of a nation's human resources (HR). Higher-order thinking skills (HOTS) are essential for successfully understanding and enhancing mathematical abilities, as well as comprises a series of learning competencies that progressively evolve into a specialized skill that students must master in today's education (Arifin & Retnawati, 2017). HOTS is indispensable for students because the real-world problems they encounter are often complex, unstructured, novel, and demand more advanced thinking skills than mere application of previously acquired knowledge (Riadi & Retnawati, 2014).

The importance of students' HOTS is inversely proportional to the fact that students' mathematical thinking skills are low in Indonesia (Mandini & Hartono, 2018). Data from the Trends International Mathematics and Science Study (TIMSS) in 2015 showed that Indonesia's ability in mathematics was ranked 48 out of 50 countries where Indonesian students lack high-level mathematical thinking skills (Mullis et al., 2015). In addition, Tanudjaya and Doorman (2020) stated that the low achievement of Indonesian students in HOTS has not improved over the years. Furthermore, the low HOTS of students cause problems in ongoing mathematics learning and does not support meaningful mathematics learning, especially its application in real life (Pangestika & Cahyaningsih, 2022). Because of this, the problem of the lack of HOTS students must be addressed using mathematics learning that emphasizes real world contexts.

Realistic Mathematics Education (RME) is a learning approach that prioritizes real-world contexts and was developed in 1971 by a group of mathematicians at the Freudenthal Institute, Utrecht University in the Netherlands (Heuvel et al., 2014). This approach offers a potential solution to address students' lack of Higher-Order Thinking Skills (HOTS). According to RME, the mathematics classroom isn't merely a place for transferring mathematical knowledge from teacher to student. Instead, it serves as an environment where students can rediscover mathematical ideas and concepts by engaging with authentic problems. In the context of realistic mathematics learning, students start with real problems before entering a more formal setting. With guidance from their teacher, they're encouraged to reconstruct their understanding of mathematical concepts and then apply these concepts to everyday problems or other fields. RME offers students the opportunity to think critically and creatively when analyzing mathematical concepts within real-world contexts. Therefore, the development of Higher-Order Thinking Skills (HOTS) in students is highly relevant to RME, as this approach provides students with the space to explore mathematical concepts comprehensively and utilize their thinking skills to solve contextual problems (Nurmudi, 2020).

The deficiency in students' mathematical thinking skills can also stem from the absence of interactive learning tools and media that can stimulate their learning motivation, thereby hindering the development of their thinking skills (Milovanović et al., 2013). Research conducted by Milovanović et al. demonstrates that multimedia can facilitate students' comprehension of learning

materials and their ability to apply knowledge to mathematical problems and exercises (Chen et al., 2023). Another study conducted by Nusir et al. focused on the impact of using multimedia in mathematics education (Nusir et al., 2013). Their findings indicate that interactive multimedia, incorporating images and animations within educational games, proves highly effective in motivating young children to engage in learning and enhancing their mathematical skills.

In the era of educational disruption, where learning increasingly occurs online or in hybrid formats and heavily relies on technology, it is imperative that learning materials are rooted in interactive technology and harness the potential of the internet and cyberspace (Coman et al., 2020). One such technology that can be effectively employed in education is virtual exhibitions. A virtual exhibition is a digital collection encompassing images, sounds, manuscripts, documents, and other content related to history, science, and culture, accessible electronically (Astita et al., 2015). For instance, research conducted by Li (2023) highlights that virtual exhibitions can serve as interactive learning tools, offering students an enjoyable learning experience. Through virtual exhibitions, students can gain an immersive learning experience akin to navigating a virtual gallery brimming with knowledge. These virtual exhibitions facilitate interactive and meaningful learning, ultimately enhancing students' comprehension and higher-order thinking skills (Darwis & Hardiansyah, 2023). Moreover, the design of virtual exhibition-based learning is relatively straightforward, and platforms such as ArtSteps provide comprehensive services for creating virtual exhibitions.

The potential offered by utilizing virtual exhibitions in the context of realistic mathematics learning is exceptionally promising. This research bridges a crucial gap by presenting an alternative, flexible, and easily accessible learning medium, addressing the limitations of interactive technology-based mathematics learning resources. The design employed incorporates the principles of Realistic Mathematics Education (RME), with an emphasis on activities that foster connections between mathematical concepts and real-world applications, as well as the process of uncovering these concepts and solving contextual problems. This fusion of technology and realistic learning approaches presents students with a captivating and enjoyable means of learning mathematics while optimizing their high-level thinking abilities in every learning activity. Consequently, this research is dedicated to the development of virtual exhibition-based RME learning resources, with the overarching goal of enhancing students' Higher-Order Thinking Skills (HOTS). The study aims to create and assess the quality of virtual exhibition-based RME learning materials to elevate students' cognitive abilities.

## LITERATURE REVIEW

### Learning Media

Reiser and Dempsey define learning media as physical equipment used to present learning to students (Reiser & Dempsey, 2012). This definition highlights that any physical equipment, whether it's textbooks, visual aids, audio materials, computers, or other resources, falls under the category of learning media. Learning media encompasses all physical tools and materials utilized

by instructors, lecturers, teachers, tutors, or other educators to facilitate learning and achieve learning objectives.

The learning media in question spans traditional formats like chalk, handouts, diagrams, slides, overheads, real objects, video recordings, and films, as well as advanced media such as computers, DVDs, CD-ROMs, the Internet, and interactive video conferencing (Romadoni & Rudhito, 2016). Learning media's development is of paramount importance because it can enhance the quality of education, align with the evolving paradigms of education, meet market demands, and align with the global vision of inclusive education. Learning media is an integral part of the learning process, serving as an indispensable tool that accelerates comprehension, enhances quality, and lays a solid foundation for learning.

### **Realistic Mathematics Education (RME)**

In realistic mathematics learning, the educational process commences with something real or closely related to students' real-life experiences. "Realistic" in this context doesn't necessarily mean it must be tangible or exist in reality; it can also pertain to concepts that can be imagined by students and are relevant to the topics under study (Febrian, & Perdana, 2017). Such imaginings can be harnessed within the learning process, fostering meaningful engagement for students.

Soedjadi (2014) proposed that Realistic Mathematics Education (RME) represents an innovative approach to mathematics education that aligns with constructivist theory. Within RME, there is a heightened focus on tapping into the potential inherent in each child or student. A teacher's belief in this potential significantly influences how they manage mathematics instruction and shapes students' comfort in engaging in activities commensurate with their abilities. These dynamics have a profound impact on the teaching culture and the learning culture. This innovation in mathematics learning not only transforms the conceptual landscape of mathematical materials and their interrelationships but also, and no less importantly, shifts the educational culture toward a more dynamic approach, all while staying within the boundaries of social ethics. Under this innovative framework, students are encouraged to express their opinions, accept differing viewpoints, and appreciate the importance of negotiation in life. Meanwhile, the role of the teacher must evolve from one of traditional "teaching" to that of a facilitator.

### **Virtual Exhibitions**

A virtual exhibition is a collection of digital replicas of real events or objects developed using multimedia and virtual reality tools to create a simulated environment on a computer, delivered via the web (Khoon Ramaiah, 2008). This technology aims to provide users with the same satisfaction as if they were physically present at the event or interacting with the objects. Typically, virtual exhibitions involve simulating real environments using virtual reality tools, which can be more complex, expensive, and time-consuming to develop compared to simple online exhibits.

Exhibitions can be categorized based on several factors, including: 1) type (aesthetic and reconstructive); 2) goals (fundraising, appreciation, and festivals); 3) the number of participants (single, joint, and group); 4) space (formal, informal, real, and illusory); 5) interests (economic, educational, political, and cultural); and 6) tempo (permanent, incidental, and periodic) (Khairunnisa et al., 2021). Therefore, when classifying a virtual exhibition, it falls into the category of an illusory space due to its existence in the virtual realm, represented as a computer-generated simulation.

### **High Order Thinking Skills (HOTS)**

High-order thinking skills, often referred to as higher-order thinking skills, encompass problem-solving, creative thinking, critical thinking, argumentation, and decision-making abilities. According to Newman and Wehlage, the cultivation of high-order thinking skills empowers students to discern ideas clearly, engage in effective argumentation, tackle problems adeptly, construct coherent explanations, formulate hypotheses, and gain a deeper understanding of complex concepts (Widodo & Kadarwati, 2013).

Vui further explains that high-order thinking skills manifest when an individual links new information to existing knowledge stored in their memory, thereby connecting, reorganizing, and expanding upon this information to accomplish a goal or find solutions to challenging problems (Kurniati et al., 2016). The primary aim of high-order thinking skills is to enhance students' cognitive abilities, especially in critical thinking when processing various types of information, employing creative problem-solving using their acquired knowledge, and making decisions in complex situations (Saputra, 2016).

## **METHODS**

### **Place and Time of Research**

The location of this research is at Widiatmika in the the 2022/2023 academic year. The subject of this research is students in 8<sup>th</sup> grade of 8A and 8B.

### **Research Design and Procedure**

This research is a type of research and development or Research and Development (R&D) that aims to produce certain products, and test the effectiveness of these products (Sugiyono, 2015). The method used by the authors in developing learning media is the R&D method modified from Borg and Gall which includes (Borg & Gall, 1983):

#### 1) Potentials and Problems

Before developing learning media, a needs analysis was carried out in the form of a preliminary study at Widiatmika Middle School which was carried out in the form of interviews conducted by researchers with one of the mathematics teachers at Widiatmika Middle School on August 1, 2022. Researchers found problems with students' HOTS

abilities and the unavailability of media. technology-based interactive learning suitable for RME learning.

2) Collecting Data

The next step is to collect references for the development of media-based learning media obtained from relevant sources, namely various articles and other references obtained from the internet. Researchers also collect sources from valid websites and YouTube to learn tutorials for preparing virtual exhibition-based media.

3) Product Design

Planning to make the initial product is to collect materials that will be done by searching through the internet. Simultaneously, the preparation of material taken from the main material, such as books, journals, theses, and others is also carried out. In designing RME media based on virtual exhibitions, the things to do are: determine the template, fill in the template with the exhibition design, determine the material, determine the placement of the material, as well as fill in the virtual exhibition with prepared materials, re-check.

4) Design Validation

Design validation is an assessment process to measure the effectiveness of a product design rationally not based on field facts. Professional experts are presented in the assessment of new products created. At this stage, material and media experts validate the initial design of learning media which will later obtain appropriate learning media for use. Design validation is carried out in two parts. First, the material expert test aims to examine aspects of the feasibility of the content presented in the form of the suitability of the material with the curriculum (content standards), language feasibility, and presentation. Two material experts consisting of lecturers and mathematics teachers who are experts in the material given. second, the media expert test aims to determine the accuracy of the minimum standards that examine aspects of visual communication, language feasibility, and software engineering contained in learning media. Two experts in the field of educational technology were presented to assess the products that have been made.

5) Design Revisions

After the product design has been validated by material and media experts, criticism and suggestions will be obtained so that developers know the drawbacks of the learning media that have been made.

6) Product Trial

The finished product is then tested in learning activities. The product trial was carried out using a small group evaluation involving 10 students of class 8A. An of Widiatmika Middle School, and a field test involving 42 students of class 8B of Widiatmika Middle School.

7) Product Revisions

From the results that have been tested on the product, if the product is not perfect, then revisions and improvements are made to the learning media made so that it can produce a final product that is ready to be used in schools.

## Population and Sample

The population of this study is grade 8th Widiatmika Middle School. This study had 2 trials, namely a limited trial using a sample of 10 students from class 8A, and a field trial using 42 students of class 8B. The sample selection used a purposive random sampling technique where the sample was determined with certain considerations. In this case, the consideration is to select students and classes that are representative and consist of students with various levels of ability.

## Data Collection Techniques

This research uses several data collection instruments, namely:

### 1) Interviews

Interviews were conducted to find out the responses, comments, and suggestions of eighth-grade mathematicians. The interview method was chosen because the researcher could be closer to the informant so that the information obtained was more in-depth. The interviews were conducted before the manufacture of learning media to obtain information about problems in learning mathematics and knowing the responses of the media for learning mathematics.

### 2) Observations

This observation was carried out by researchers regarding the learning process carried out and knowing the teaching materials used, seeing the characteristics of students or student behavior patterns during the learning process. In this study, the type of non-participant researcher observation was used, that is, the only came to the place where the activity of the person being observed, but the researcher was not involved as a researcher and as an independent observer in the activity.

### 3) Questionnaires

Questionnaires are used to collect data about the accuracy of the components of learning media, the accuracy of the design or design of learning media, and the accuracy of the content of learning media. In this development research, the questionnaire was presented by researchers using google forms (Google Forms) to obtain data from all students regarding the accuracy and attractiveness of e-mail.

### 4) Validation Sheet

The validation sheet for learning media devices is used to obtain information about the quality of learning media based on the assessment of the expert validator. The information obtained through this instrument is used as a consideration in revising the learning media that has been developed to produce a valid final product.

### 5) Test

This test is carried out on students before and after using the media that has been developed. The test results are used to determine the effectiveness of the media. The test used in this

study is a description test consisting of HOTS questions on circle material. This test consists of 5 questions that are given to students at the end of the lesson.

### **Product Trial Techniques**

Product trials are considered necessary because the products produced are really of high quality, effective, and on target. In addition, product trials are a condition that must be met by a researcher who conducts a development research model. Product trials are carried out after the resulting product is declared valid by validators. Product trials conducted should pay attention to the following: 1) trial design and 2) test subjects.

#### **1. Trial Design**

In this study, three types of trials were carried out, namely validity, effectiveness, and practicality. For the data obtained to be more accurate for further improvement and development research. The three types of tests can be described as follows:

##### **a. Validity Test**

The validity test is a trial carried out to obtain data in the form of validity assessments and suggestions on the level of validity of the product being developed, which in this case is a learning medium. The validators referred to in this development research are media experts as well as material experts.

##### **b. Effectiveness test**

Effectiveness testing is carried out by piloting the product-to-product users. Product trials were carried out in small groups of students who had never studied the material used. The purpose of this test is to assess the effectiveness of the product in the learning process for the improvement of HOTS students.

##### **c. Practicality test**

A practicality test is a trial carried out to obtain data at the level of ease of a product being developed. To find out the level of practicality of the product in this development research can be obtained using a student response questionnaire.

#### **2. Test Subjects**

The test subjects used in this development research are expert groups and target users of the product. In this case, the expert group in question is a validator consisting of expert lecturers at the Mathematics Education Study Program at the FKIP Universitas Mahasaraswati Denpasar and mathematics teachers at Widiatmika Middle School. The target product users in this study are class VIII students of Widiatmika Middle School.



### Data Analysis Techniques

Data analysis is carried out to find out an overview of the quality of the results of the learning media products developed, through quantitative data that has been obtained so that it will be known whether the products developed, namely virtual exhibition-based RME learning media, have met the criteria for validity, effectiveness, and practicality. The data analysis techniques obtained from each instrument are described as follows.

#### a. Quality Criteria

Research data in the form of qualitative data is converted into quantitative data by determining the average value. After that, the value is converted into a qualitative value that reflects the quality of the media according to the category of ideal assessment criteria as follows.

No	Quantitative Score Range	Qualitative Category
1	$\bar{X}_i + 1,5 SB_1 < \bar{X}$	Very good
2	$\bar{X}_i + 0,5 SB_1 < \bar{X} \leq \bar{X}_i + 1,5 SB_i$	Well
3	$\bar{X}_i - 0,5 SB_1 < \bar{X} \leq \bar{X}_i + 0,5 SB_i$	Enough
4	$\bar{X}_i - 1,5 SB_1 < \bar{X} \leq \bar{X}_i - 0,5 SB_i$	Not enough
5	$\bar{X} < \bar{X}_i - 1,5 SB_1$	Very less

Table 1. Ideal Assessment Criteria

Information:

$\bar{X}_i$  : ideal average that can be found using the formula

$$\bar{X}_i = \frac{1}{2} \times (\text{ideal maximum score} + \text{ideal minimum score})$$

$SB_i$  : The ideal standard deviation that can be found by the formula

$$SB_i = \frac{1}{6} \times (\text{ideal maximum score} + \text{ideal minimum score})$$

Ideal maximum score =  $\sum$  item criteria  $\times$  the highest score

$$\text{Ideal minimum score} = \sum \text{item criteria} \times \text{lowest score}$$

The ideal percentage of Student Activity Sheets (P), namely:

$$\bar{P} = \frac{\text{Assessment result score}}{\text{Ideal maximum score}} \times 100\%$$

## b. Media Assessment by Experts

As for the results of the assessment regarding the quality of the media in the form of qualitative data, the data is then converted into quantitative data with the following conversion guidelines:

Information	Score
Very Less (SK)	1
Less (K)	2
Enough (C)	3
Good (B)	4
Very Good (SB)	5

Table 2. Guidelines for Converting Qualitative Data by Experts

After obtaining data regarding the quality of medias in quantitative form, then the data is processed to determine the quality of medias per aspect and as follows:

### 1) Material Expert

After obtaining data regarding the quality of medias in quantitative form, then the data is processed to determine the quality of medias per aspect. Calculations of material experts on the Aspects of Concept Truth, Aspects of Concept Depth, Aspects of Breadth of Concepts, Aspects of Implementability, and Aspects of Linguistics are as follows:

$$\bar{p} = \frac{\text{Assessment result score}}{\text{Ideal maximum score}} \times 100\%$$

Information:

$$\begin{aligned} \text{Ideal maximum score} &= \frac{\sum \text{item criteria} \times \text{the highest score}}{\text{number of indicators for each aspect}} \\ &= \frac{\text{Total score of each aspect}}{\text{number of indicators for each aspect}} \end{aligned}$$

For material expert assessment as a whole use the following formula:

$$\bar{p} = \frac{\text{Assessment result score}}{\text{Ideal maximum score}} \times 100\%$$

Information:

$$\begin{aligned} \text{Ideal maximum score} &= \sum \text{item criteria} \times \text{the highest score} \\ \text{Assessment result score} &= \frac{\text{Jumlah skor keseluruhan aspek}}{\text{number of indicators for all aspects}} \end{aligned}$$

## 2) Media Expert

After obtaining data regarding the quality of medias in quantitative form, then the data is processed to determine the quality of medias for the Media Anatomy Aspect, Image Quality Aspect, and Full View Aspect, as follows:

$$\bar{p} = \frac{\text{Assessment result score}}{\text{Ideal maximum score}} \times 100\%$$

Information:

$$\text{Ideal maximum score} = \sum \text{item criteria} \times \text{the highest score}$$

$$\text{Assessment result score} = \frac{\text{Total score of media anatomy aspect}}{\text{number of indicators for each aspect}}$$

For aspects of media experts as a whole use the following formula:

$$\bar{p} = \frac{\text{Assessment result score}}{\text{Ideal maximum score}} \times 100\%$$

Information:

$$\text{Ideal maximum score} = \sum \text{item criteria} \times \text{the highest score}$$

$$\text{Assessment result score} = \frac{\text{Overall view aspect score sum}}{\text{number of indicators for all aspects}}$$

The assessment of material experts and media experts is as follows:

$$\bar{p} = \frac{\text{Assessment result score}}{\text{Ideal maximum score}} \times 100\%$$

Information:

$$\text{Ideal maximum score} = \sum \text{item criteria} \times \text{the highest score}$$

$$\text{Assessment result score} = \frac{\text{Total scores of material experts and media experts}}{\text{total number of indicators of experts}}$$

## Calculation of Student Opinions Regarding the Quality of RME Media Based on Virtual Exhibition

As for the results of the assessment regarding the quality of the media in the form of qualitative data, the data is then converted into quantitative data with the following conversion guidelines:

Information	Score
Very Less (SK)	1
Less (K)	2
Enough (C)	3
OK ( B )	4
Very Good (SB)	5

Table 3. Guidelines for Converting Student Opinion Qualitative Data

After obtaining data regarding the quality of medias in quantitative form, then the data is processed to determine the quality of medias per aspect and as follows:

$$\bar{p} = \frac{\text{Assessment result score}}{\text{Ideal maximum score}} \times 100\%$$

Information:

$$\text{Ideal maximum score} = \sum \text{item criteria} \times \text{the highest score}$$

$$\text{Assessment result score} = \frac{\text{The total score of the quality of the student's opinion on media of indicators for all aspects}}{\text{Ideal maximum score}}$$

The results Percentage of ideal assessment criteria can be seen in the following table:

No	Quantitative score range	Qualitative Category
1	$\bar{P} > 80\%$	Very good
2	$66,67\% < \bar{P} \leq 80\%$	Well
3	$53,33\% < \bar{P} \leq 66,67\%$	Enough
4	$40\% < \bar{P} \leq 53,33\%$	Not enough
5	$\bar{P} \leq 40\%$	Very less

Table 4. Percentage of Ideal Rating Criteria Results

#### d. Student HOTS Assessment

The student HOTS assessment is carried out through the average student score and the percentage of students who pass. The calculations are as follows:

$$\text{Average student grades} = \frac{\text{the total value of the students}}{\text{the total number of students}} \times 100\%$$

$$\text{Completed student presentation} = \frac{\text{number of students who completed}}{\text{the total number of students}} \times 100\%$$

## RESULTS

In the development of our learning media based on Realistic Mathematics Education (RME), a pivotal element was the construction of the mathematizing process, centered around the captivating concept of the area of a circle, as showcased within our virtual exhibition. Our approach sought to immerse students in a dynamic mathematical environment where they could actively grapple with this fundamental geometric concept. The selection and presentation of materials within the virtual exhibition were thoughtfully crafted to provoke profound mathematical thinking and inquiry, with the area of a circle serving as a central theme. The data on the results of each stage of the research and development procedure carried out is as follows:

### 1) Potentials and Problems

At this stage, researchers are looking for information about the media that is being developed in the world of education. One of the technologies that continue to develop and are often used in smartphones. Young people, especially school students, are now familiar with the digital world, so researchers decided to provide opportunities for students to learn interactively using virtual exhibitions.

Researchers conduct research at Widiatmika Middle School which is a target school of the Mathematics Education Study Program, Faculty of Teacher Training and Education, Universitas Mahasaraswati Denpasar where researchers teach. Researchers found a fundamental problem in grade 8<sup>th</sup> students, namely that students consider mathematics boring, difficulty understanding mathematical concepts, and the absence of mathematics learning media that is interesting, entertaining, and that teenagers currently like. The existing problems provide ideas for researchers to develop virtual exhibition-based RME learning media to improve the students' HOTS.

From Table 5 above, it is evident that media focused on circle materials had not been previously employed in mathematics learning. This pioneering research has opened up new avenues for the development of Realistic Mathematics Education (RME) media products using virtual exhibitions, particularly within the context of Widiatmika Middle School. The data presented in Table 5 underscores the unique and innovative nature of this approach, suggesting that it has the potential to introduce fresh and effective instructional strategies to enhance students' mathematical understanding and engagement. Researchers can leverage these findings to further explore and refine RME-based virtual exhibition media, thereby enriching the learning experience for students at Widiatmika Middle School and potentially in broader educational settings as well. This pioneering effort highlights the importance of continuously pushing the boundaries of educational technology to enhance mathematics education.

No	Question Topic	Conclusion Answers	Teacher	Conclusion Answers	Student
1	The learning method used	Teachers adapt to situations and conditions, most often using problem-based learning		Lectures and practice questions	
2	The difficulties faced by students	In studying circle material, namely the application of circle formulas in solving real problems and solving problems related to central angles, arc lengths, and areas of circular arcs and their relationships.		Apply the formula to the problem	
3	How to overcome student difficulties	Give lots of practice questions		Ask friends, read notebooks, or search via Google	
4.	The media used during learning	<i>Powerpoint</i>		<i>Powerpoint</i>	
5.	Utilizing smartphones for teaching	Never		Never	
6.	Applying e-learning to learning	Ever been		Ever been	
7.	Using virtual exhibition media in learning mathematics	Never		Never	
8.	Interest in and use of virtual media exhibitions	Interested		Interested	

Table 5. Results of Interviews with Teachers and Students

## 2) Data collection

Data collection is the process of finding problems and potential of an object of research so that the data obtained can be considered in the manufacture, and research, so that the data obtained can be considered in the design of a product.

Before carrying out the development of learning media, a needs analysis is carried out. A needs analysis in the form of a preliminary study was carried out on 1 August 2022 at Widiatmika Middle School. Preliminary studies were carried out when the mathematics

learning process took place, namely interviews with teachers in the field of mathematics studies.

### 3) Product Design

The design of media products includes RME learning based on virtual exhibitions. The media is adapted to the needs of students and directions from the mathematics teacher. The virtual exhibition-based RME media is divided into 4 parts: the first part contains elements related to circles, the second part contains the use of area and circumference formulas, and the third part contains the relationship between the central angle and the length of the arc, and the relationship between the angles the center with the area of the sector, and the fourth part is about solving problems related to the elements of circles, circumference, and area of circles, as well as arc length and area of circles.

Media virtual exhibition in the form of digital images of material in words which is then inserted into the digital space. The following are the general stages in making virtual exhibition-based RME media, namely:

- a. Making material containing the mathematical concept of a circle adapted to competency standards and basic competencies and syllabus based on the 2013 curriculum
- b. Preparation of dialogue between characters, images, animations, and backgrounds that will be used in making virtual exhibition-based RME media
- c. Making RME media based on virtual exhibitions made by researchers as follows:



Figure 1. Virtual Exhibition on Circle Material

### 4) Design Validation

This stage is a stage related to the assessment of the product that has been designed. Two experienced experts or experts were presented at the product validation stage to assess the

learning media that had been made by the researcher. The assessment consisted of 2 media experts, namely validation by a mathematics education lecturer, namely Mrs. Kadek Rahayu Puspawati, S.Pd., M.Pd., and validation by a grade VIII math teacher at Widiatmika Middle School, namely Mrs. Putu Yulia Prawestri, S.Pd., M.Pd. Apart from the 2 media experts, there were also 2 material experts, namely Mrs. Kadek Rahayu Puspawati, S.Pd., M.Pd. and Mrs. Putu Yulia Prawestri, S.Pd., M.Pd. The following is a presentation of the results of the validation of material and media experts.

#### a) Media Validation Results by Material Experts

We carefully curated a collection of real-world scenarios and objects that brought the concept of the area of a circle to life. Virtual exhibits included visually engaging interactive simulations, where students could experiment with changing the radius or diameter of circles, and instantly observe how these modifications affected the circle's area. We also provided historical context and real-life applications of circle area calculations, such as designing circular gardens or estimating the amount of material required for circular-shaped objects in various industries. Moreover, guided questions within the exhibition encouraged students to analyze, infer, and generalize mathematical principles from their interactions with the presented materials. This meticulous approach ensured that the mathematizing process naturally emerged as students explored the area of a circle within authentic real-world contexts, fostering a deeper understanding of mathematics grounded in practicality and relevance, while simultaneously nurturing their higher-order thinking skills, including critical analysis, problem-solving, and creative mathematical reasoning. The results of this development research are in the form of (1) a virtual exhibition-based RME learning media for students in the Circle material, and (2) student HOTS achievement using virtual exhibition-based RME learning media.

The aspects assessed by material experts are aspects of concept correctness, concept depth, concept breadth, implementability, and language.

No	Aspect	Ideal Percentage	Category
1	Concept Truth	86.66 %	Very good
2	Concept Depth	90%	Very good
3	Breadth of Concept	95%	Very good
4	Execution	93.33 %	Very good
5	language	90%	Very good
Percentage of Overall Aspects		91 %	Very good

Table 6. The Ideal Percentage of Validation Results by 2 Material Expert Validators

Based on Table 6 regarding the results of validation by material experts, it can be seen that the total average ideal percentage in material expert validation is 91%, which can conclude



that the RME media based on virtual exhibitions that have been developed are included in very good criteria for use in learning mathematics so that it does not revision is needed.

### b) Media Validation Results by Media Experts

The aspects assessed by material experts are aspects of media anatomy, image quality, and overall appearance.

No	Aspect	Ideal Percentage	Category
1	Media Anatomy	93.33 %	Very good
2	Image Quality	80%	Well
3	Full View	88%	Very good
Percentage of Overall Aspects		87.27 %	Very good

Table 7. The Ideal Percentage of Validation Results by 2 Media Expert Validators

Based on Table 7 regarding the results of validation by media experts, it can be seen that the acquisition of an ideal total average percentage in the validation of media experts, which is equal to 87.27%, can conclude that the RME media based on virtual exhibitions that have been developed are included in the very good criteria for use in learning mathematics so that no revision is needed.

### Design Revision

From the results of the validation, several comments and suggestions were obtained regarding RME media based on virtual exhibitions that had been made by researchers. From the validation results obtained, namely from material experts, among others, there were several typing errors in the media, in one of the dialogues there was a sentence that still confused students, it is necessary to be consistent in writing the multiplication symbol (x) on the media, and in the circle image, there are letters that are not clear. Comments and suggestions from media experts included changing the font color in the fourth part (part 4) of the virtual exhibition-based RME media, and improving the font size in several speech balloons. Researchers use these comments and suggestions as a reference in revising the virtual exhibition-based RME media that will be developed.

### 5) Product Trials

Products that have gone through the validation stages by material experts and media experts and have been repaired are then tested by researchers with limited field trials whose implementation aims to test the effectiveness of the product. The product trial results are as follows:

### a) Limited Trial

The limited try-out was carried out in small groups involving 10 class VIII. Widiatmika Middle School students were selected heterogeneously based on ability in class and gender using a purposive sampling technique. The limited trial was conducted in two meetings, the first meeting was used to fill out a motivational questionnaire before using the media, and carried out the pre-test, the second meeting was used to use virtual exhibition-based RME media, the post-test, and finally the virtual exhibition-based RME media assessment.

### b) Field Trials

After conducting a limited trial, the product was then tested again with a field trial involving 42 Widiatmika Middle School students from class VIII.C. This field trial was carried out to ensure the data that had been obtained by giving questionnaires and tests. The limited trial was conducted in two meetings, the first meeting was used to fill out a motivational questionnaire before using medias, and to do a pre-test, the second meeting was used to use virtual exhibition-based RME media, post-test, and finally, an RME media assessment based on virtual exhibitions.

## Classroom Activity

During limited trials and field trials, students carried out several class activities guided by the teacher. The trial steps are as follows:

### Limited Trials:

1. Introduction to RME Media: In the limited trials, the first step involved introducing the Realistic Mathematics Education (RME) media to a smaller group of students. This introduction aimed to familiarize them with the new learning tool.
2. Guided Classroom Activities: After the introduction, students engaged in various classroom activities guided by their mathematics teacher. These activities were designed to use the RME media effectively as a learning resource.
3. Teacher Support: Throughout the limited trials, the mathematics teacher played an active role in supporting and guiding students during their interactions with the RME media. This support ensured that students could effectively navigate and understand the learning content.
4. Assessment: After using the RME media, students' performance was assessed. This assessment helped researchers understand how well the media supported their learning and whether any improvements were needed.

#### Field Test:

1. **Expanding the Scope:** In the field test, the researchers expanded the scope by involving a larger group of students. This step aimed to assess the effectiveness of the RME media on a broader scale.
2. **Continued Classroom Activities:** Similar to the limited trials, students in the field test also engaged in classroom activities guided by their mathematics teacher. These activities were designed to explore the full potential of the RME media in a real classroom setting.
3. **Teacher's Integral Role:** Just like in the limited trials, the mathematics teacher remained a crucial part of the process. The teacher provided support, answered questions, and facilitated discussions related to the RME media content.
4. **Performance Evaluation:** After the field test, students' performance was evaluated once more. This evaluation helped researchers gauge the media's effectiveness in enhancing students' understanding of mathematical concepts.

In summary, the limited trials and field test involved introducing the RME media, conducting classroom activities, receiving guidance from the mathematics teacher, and assessing students' performance to determine how well the media supported their learning. These steps allowed researchers to refine the media and assess its impact on a smaller and larger scale, respectively.

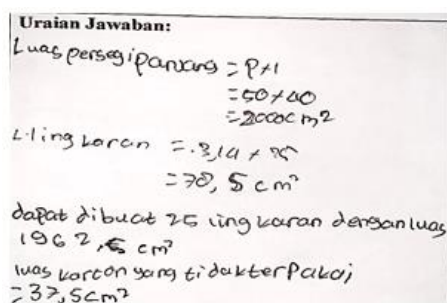


Figure 2. Students Accessing Virtual Exhibition

At the time of implementing virtual exhibition-based RME media, students carried out learning in the classroom accompanied by a mathematics teacher. Students provide laptop facilities to be able to access the internet. In the first activity, students are then asked to form groups of 3-5 people. Each group distributed worksheets containing projects that they had to complete related to the area of the circle material. In the second activity, in groups then work on the worksheets given. The material that students work on is to understand the area of a circle. Students open a virtual exhibition through the link provided. Students then go to the section on understanding the concept of the area of a circle. Students see and study the media and illustrations of the area of the circle

area concept provided in the virtual exhibition. Through these explanations and media, students then work on projects on worksheets related to the concept material for the area of a circle.

In the third activity, students were asked to find the formula for the area of a circle. This activity is also available in the student worksheet. Students then open a virtual exhibition through the link provided, and then explore material for finding the area of a circle. After that, the students together in their groups worked on the project on the worksheet. In the fourth activity, students are asked to solve several realistic problems related to the area of a circle. These problems are presented in student worksheets. Students are asked to work together with their group mates in working on these problems, helping each other, and exchanging ideas. The problems presented are problems at the higher-order thinking skills (HOTS) level. The following is an example of a problem solved by students related to the area of a circle.



Uraian Jawaban:  
 Luas persegi panjang =  $p \times l$   
 $= 50 \times 40$   
 $= 2000 \text{ cm}^2$   
 Luas lingkaran =  $3,14 \times r^2$   
 $= 78,5 \text{ cm}^2$   
 dapat dibuat 25 lingkaran dengan luas  
 $1962,5 \text{ cm}^2$   
 luas karton yang tidak terpakai  
 $= 37,5 \text{ cm}^2$

Translate:  
 Answer:  
 Area of rectangle =  $p \times l$   
 $= 50 \times 40$   
 $= 2000 \text{ cm}^2$   
 Area of circle =  $3,14 \times r^2$   
 $= 78,5 \text{ cm}^2$   
 Can be made 25 circles with area  $1962,5 \text{ cm}^2$   
 Unused carton area  $37,5 \text{ cm}^2$

Figure 3. Example of Student's Answer

The answers from students show that students are able to understand realistic problems and find appropriate ways to solve these problems, but students are not yet able to do proper reasoning. In the last activity, students jointly convey the results obtained. In this activity the teacher confirms the concepts students have learned and concludes material from the area of the circle. Students are then given a questionnaire to provide responses to the use of virtual exhibition-based RME media.

After conducting limited trials and field trials to know the effectiveness of virtual exhibition-based RME media on achieving students' conceptual understanding of circle material, it was discovered that the product being developed was effective so no further trials were carried out. Furthermore, virtual exhibition-based RME media can be utilized as a learning resource for students and teachers at Widiatmika Middle School for class VIII. A summary of student HOTS data in a limited test is presented in Table 4.6 and the field test is presented in Table 8 below.

The research results presented in the Limited Test and the Field Test unmistakably demonstrate the effectiveness of the media designed to enhance students' Higher-Order Thinking Skills (HOTS) in the context of circle materials. The analysis of the data reveals that this innovative learning tool aligns closely with HOTS indicators, as evidenced by substantial improvements in both average scores and completion percentages between the pre-test and post-test assessments.

Assessment Aspects	Limited Test		Field Test	
	Pre-Test	Post-test	Pre-Test	Post-test
Average	47,33	82.50	35,22	79,89
Completeness	1	6	3	33
The highest score	75,86	99.14	84.05	100
Lowest Value	40,52	71.98	11,21	58,62
Percentage of Completed Students	10 %	60 %	7 %	79 %

Table 8. HOTS Data in Limited Tests and Field Tests

In the Limited Test, the students initially displayed an average pre-test achievement of 47.33, with a modest completion rate of only 10%. However, after engaging with the learning media, their post-test average soared to 82.50, accompanied by a commendable completion percentage of 60%. This remarkable progress signifies that the media effectively addressed the HOTS indicators, as it prompted students to think critically, solve complex problems, and analyze mathematical concepts with depth and precision. These outcomes are consistent with established HOTS indicator references, such as Bloom's Taxonomy, which emphasizes cognitive skills such as: analyzing, evaluating, creating.

Similarly, the Field Test results mirrored the success observed in the Limited Test. Students commenced with an average pre-test achievement of 35.22 and a completion rate of just 7%. However, the post-test outcomes revealed a noteworthy transformation, with an average score of 79.59 and an impressive completion percentage of 79%. This substantial enhancement aligns with HOTS indicators, reinforcing the media's ability to stimulate higher-order thinking. It is important to note that the improvements observed encompass various HOTS domains, encompassing analysis, synthesis, and evaluation.

In conclusion, the research results underscore the media's capacity to meet and exceed HOTS indicators as defined by recognized references in educational psychology and assessment methodologies. This not only validates the educational value of the designed media but also emphasizes its potential to elevate students' cognitive abilities, ultimately fostering a deeper and more meaningful grasp of mathematical concepts, particularly in the context of circle materials.

## 7) Results and Analysis of Student Opinions Regarding the Quality of Virtual Exhibition-Based RME Media

Data on student opinions regarding the quality of virtual exhibition-based RME media for each research trial are limited and the field can be seen in the appendix. A brief description of student opinion data regarding the quality of virtual exhibition-based RME media in the limited and field tests is presented in Table 9 and the field, tests are presented in Table 9 below:

Assessment Aspects	Limited Score	Test	Field Test Score
Total Score Total	862		3489
Average	86,2		83.07
Ideal Presentation	86%		83%

Table 9. Summary of Student Opinion Data Regarding the Quality of Mathematical Media E-books in Limited Tests and Field Tests

Based on the data that has been obtained from the limited test and field test, the average score of students' opinions on the quality of RME media based on virtual exhibitions in the limited test is 86.2 with an ideal presentation of 86%, which is the ideal assessment criteria belongs to the criteria "very good" while the results of students' opinions on the quality of virtual exhibition-based RME media in the field test obtained was 83.07 with an ideal presentation of 83%, where the ideal assessment criteria belonged to the "very good" criteria-Based on the averages and ideal presentations from limited trials and field trials, it can be concluded that the assessment of students' opinions on the quality of RME media based on virtual exhibitions from limited trials and field trials is categorized as "very good" and can be used as a medium of learning on the material circle.

## DISCUSSION

In this study, we designed and developed a virtual exhibition aimed at enhancing students' high-order thinking skills (HOTS) by integrating the three principles and five characteristics of Realistic Mathematics Education (RME) (Palupi & Khabibah, 2018). We ensured that our virtual exhibition adhered to three fundamental RME principles: 'Rediscover,' 'Didactic Phenomena,' and 'Self-Developed Models.' To illustrate the 'Rediscover' principle, our virtual exhibition incorporated interactive elements, allowing students to independently explore mathematical concepts through virtual tools and simulations. Additionally, the 'Didactic Phenomena' principle was embraced by embedding real-world scenarios and phenomena throughout the exhibition, providing students with tangible contexts for mathematical exploration. Furthermore, the 'Self-Developed Models' principle was enacted through activities that prompted students to construct their mathematical models to tackle real-world problems, fostering a sense of ownership over their learning.

Our virtual exhibition also encompassed the five key characteristics of RME (Julie et al., 2014), beginning with 'Real-World Use.' We integrated practical applications of mathematics, showcasing how the mathematical concepts presented in the exhibition are employed in various real-life scenarios such as make a circular lid from cardboard, sprinkler, etc. 'Modelling' was another crucial component, as we encouraged students to create mathematical models to elucidate phenomena showcased within the exhibition. The 'Use of Production and Construction' characteristic was evident in the interactive activities provided, where students actively built their understanding of mathematics by constructing geometric shapes, creating data visualizations, and programming mathematical simulations. 'Use of Interaction' was promoted through interactive exercises and discussions that allowed students to engage directly with mathematical content and

receive timely feedback. Lastly, 'Intertwining' was a central theme, highlighting the interconnectedness of various mathematical concepts and encouraging students to explore these relationships for a deeper comprehension.

Our findings demonstrate a significant positive impact on students' HOTS through the integration of RME principles and characteristics within the virtual exhibition. Student performance and engagement data consistently indicated that the principles of 'Rediscover,' 'Didactic Phenomena,' and 'Self-Developed Models,' along with the characteristics of 'Real-World Use,' 'Modelling,' 'Use of Production and Construction,' 'Use of Interaction,' and 'Intertwining,' collectively contributed to an enriched learning experience. These results have noteworthy implications for mathematics education, emphasizing the effectiveness of aligning instructional media with RME principles and characteristics to foster deeper understanding and higher-order thinking skills among students." Based on the findings from the previously described research, it has been established that the virtual exhibition-based Realistic Mathematics Education (RME) media effectively serves as a learning tool for enhancing Higher-Order Thinking Skills (HOTS) in the context of circle materials. The following presents the results of the HOTS post-test. This learning medium, which has successfully passed the rigorous validation processes by material and media experts, connects mathematical concepts with the circle subject matter through realistic experiences and familiar knowledge accessible to students.

The research discussed above can unequivocally be considered a resounding success, driven by the development of innovative media tailored specifically for 8th-grade students at Widiatmika Middle School. This media has demonstrated substantial improvements in students' HOTS abilities, resulting in a significant number of students meeting or surpassing the predetermined completion criteria set by Widiatmika Middle School. These achievements underscore the remarkable quality of the developed media, a fact validated by recent evaluations and assessments conducted by educational experts and the students themselves. Notably, contemporary educational research by Marzano and Kendall (2017) reinforces the importance of technology-integrated instructional media in enhancing student outcomes. Their findings emphasize the role of multimedia resources, such as virtual exhibitions, in promoting active learning, critical thinking, and engagement in mathematics education, aligning seamlessly with the outcomes observed in this study.

Furthermore, the application of the Realistic Mathematics Education (RME) approach, as endorsed by recent research by Van den Heuvel-Panhuizen and Drijvers (2014), has proven instrumental in catalyzing the positive transformation observed in students' higher-order thinking skills. The RME approach, rooted in constructivist principles and problem-solving strategies, has gained renewed attention in the context of modern mathematics education. It aligns perfectly with contemporary educational philosophies that emphasize fostering conceptual understanding, analytical thinking, and the development of 21st-century skills. In conclusion, this research stands as a testament to the potential of cutting-edge instructional media and pedagogical approaches to revolutionize

mathematics education. The incorporation of multimedia resources and the strategic application of the RME approach represent powerful tools for educators seeking to empower students with the advanced cognitive skills necessary for success in our ever-evolving educational landscape (Kilpatrick et al., 2001). The application of the RME approach has a significant positive effect on improving students' higher-order thinking skills (Anderson & Krathwohl, (2001).

The integration of Realistic Mathematics Education (RME) with technology has a profoundly positive impact on enhancing students' Higher-Order Thinking Skills (HOTS). The support from technology, such as the developed virtual exhibition media, offers students an engaging and realistic learning experience. It enables students to explore various mathematical concepts through enjoyable activities. As suggested by Papadakis et al., digital technologies can indeed play a constructive role in enhancing early mathematics skills (Sasmi et al., 2020). The utilization of mobile technologies in mathematics education further encourages meaningful student engagement by embedding the subject in real-world contexts. Ideally, mobile technologies should be seamlessly integrated into mathematics teaching and learning to create a new, more dynamic learning environment (Papadakis et al., 2021).

Moreover, RME learning complemented by virtual exhibition media presents modern, effective, and enjoyable opportunities for students to study mathematics. "Modern" signifies that students can leverage the latest virtual reality technology in their math education. "Effective" implies that virtual exhibition media can be accessed at any time and from anywhere, making learning more convenient for students. Lastly, "fun" indicates that learning activities using virtual exhibition media boost students' enthusiasm and motivation for learning mathematics. This is corroborated by the significant improvement in students' HOTS abilities and the positive feedback they provide regarding the use of RME media based on virtual exhibitions. In line with Padmasari et al., mixed-reality-based virtual exhibitions can indeed be seen as innovative solutions and positive trends in the realm of digital technology advancement (Padmasari et al., 2022).

## CONCLUSIONS

Data analysis is conducted to provide an overview of the quality of the developed learning media products. This analysis relies on quantitative data, helping to determine whether the products meet criteria for validity, effectiveness, and practicality. The validation results indicate an ideal average percentage of 87.27% from media experts and 91% from material experts, demonstrating that the RME media based on virtual exhibitions developed are of very high quality for use in mathematics education. In the Limited Test, students' Higher-Order Thinking Skills (HOTS) test completion rate was 60%, which increased to 79% in the Field Test. Furthermore, the average score from students' feedback on the quality of RME media based on virtual exhibitions is 83. These findings highlight the positive influence of combining RME learning with technology in enhancing students' HOTS.



This research makes a significant contribution by offering RME learning using virtual exhibition media, creating a modern, effective, and enjoyable approach to studying mathematics. There remains room for improvement and expansion, potentially exploring other mathematical topics to broaden the scope and innovate the design of these educational materials.

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## Characteristics of Differentiated Mathematical Creative Models in Problem-Solving Activities: Case of Middle School Students

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*Abstract: Students have varying degrees of creativity and can develop their creative abilities in specific disciplines through various stimuli. Tracing the students' mathematical creativity is very important since creativity is not a gift for specific students; rather, all students have it. The participants of this study were 170 urban and rural middle school students in Greater Malang, Indonesia. This qualitative descriptive exploratory research revealed differences in middle school students' mathematical creative models in problem-solving activities which were then used empirically to classify differences in students' mathematical creative models and provide characteristics for each of their creative models. Data were collected from problem-solving activities and semi-structured interviews. Triangulation of sources and methods was used to obtain data validity. Data analysis was performed through fixed comparison analysis. This research created meaningful and reliable differences in students' mathematical creative models, including models of imitation, modification, combination, and creation. As a conclusion, this study revealed differences in students' creative models and recommended that further research develop other problem-solving activities to promote students' creative models traceable on an ongoing basis to more varied problem themes.*

**Keywords:** creative model, mathematical creativity, mathematical creative model, problem-solving activities

## INTRODUCTION

Creativity applies previously acquired information to solve problems and create new things (Calavia *et al.*, 2021). It is a process that consists of several mental activities that people do when they create something, from identifying problems and acquiring knowledge to generating ideas and implementing them (Quiñones-Gómez, 2021). It is a sub-dimension of individual intelligence that can find unique ideas or modify existing ones (Deak *et al.*, 2004). It is a relationship between talent, method, and environment in which people or groups create new things that are understood and helpful in a social context (Schoevers, 2019; Tubb *et al.*, 2020; Hernández-Torrano & Ibraeva, 2020). One of the subcomponents of individual cognition has been recognized as general creativity that may change current ideas or generate new creative ideas (Bicer *et al.*, 2020).

The interplay of three systems: a sociocultural system with symbolic norms, a personal system that provides symbolic uniqueness, and a field expert-configured system where the creative process is produced by identifying, assessing, and validating products might be understood as the source of creativity (Aguilera & Ortiz-Revilla, 2021). Parameters commonly used to assess creativity include the Torrance Test of Creative Thinking (Schoevers, 2019), which identifies the creative process and some forms of its assessment that are still used today for general creativity assessment (Said-Metwaly *et al.*, 2018). The three most important indicators of creativity in the Torrance Test of Creative Thinking are fluency, flexibility, and originality. Fluency means the number of responses provided; flexibility means the variety of solution strategies; while originality means the uniqueness of student solutions (Torrance, 2008). According to Guilford, there are four essential sub-dimensions of creativity in the public domain: the number of ideas created, the diversity of ideas generated, the uniqueness of the ideas formed, and the number of detailed processes produced (Bicer *et al.*, 2020).

Several researchers have investigated students' creativity in open-ended problem-solving through fluency, flexibility, and originality (Kattou & Kontoyianni, 2012). Fluency is measured using a variety of approaches for tackling a particular task. The ability to change concepts to develop different finishing approaches is connected with flexibility. The introduction of fresh ideas connected to issue solutions is referred to as originality. Supporting this concept, Bezerra *et al.* (2020) stated that fluency is indicated by the number of various ideas created and the relevant solutions for issue posing; flexibility refers to the number of different categories into which the resultant solutions for each problem may be divided; and creativity refers to the number of different ideas generated and the right solutions for problem posing. Originality is defined as the non-conventionality of the ideas developed; an acceptable solution that varies from the suggested answer is deemed original. According to Aguilera and Ortiz-Revilla (2021), creativity can be evaluated through some aspects: processes, by paying attention to creative processes or something similar; procedures developed by individuals; environmental factors that act as promoters of creativity evaluated; individual creativity capacity assessed using tests or questionnaires; and the characteristics of the results are obliquely evaluated.

Creativity indicates that pupils might have varying degrees of creativity in various professions. Students can hone their creative abilities by being exposed to a variety of educational, social, and environmental stimuli. Academics, for example, have distinguished between general creativity and mathematical creativity by identifying distinguishing characteristics of persons who are creative in mathematics (Leikin *et al.*, 2013). The distinctions involving general creativity and mathematical creativity, on the other hand, may be complicated. Researchers, for example, have documented particular learning, which increases students' creative skills in mathematics, and general learning, which promotes creative abilities in any discipline (Bicer *et al.*, 2020; Sheffield, 2009), showing that overall creativity-focused instruction can boost student creativity in specialized fields, such as mathematics.

Kattou and Kontoyianni (2012) investigated the structure of the link between mathematical skills and creativity. According to the findings of the study, there were three separate types of students depending on their mathematical ability: students with high, medium, and poor mathematical talents. Meanwhile, the three groups had different levels of mathematical creativity those getting the highest scores on the mathematics test were considered the most creative. Sriraman and Hadamard (2009) investigated five mathematicians to discover the characteristics of a creative process. The results demonstrated that their creative process followed the four steps of the Wallas model: preparation-incubation-illumination-verification. In this circumstance, the creative process might occur while they studied mathematics content.

Mathematical creativity is defined as a process that produces unexpected (new) and insightful solutions to specific or comparable issues or generates new questions and opportunities that allow current problems to be seen from fresh angles and need creativity. Understanding mathematics allows students to develop creativity in mathematical tasks (Leikin & Pitta-Pantazi, 2013; Schindler & Lilienthal, 2020) based on the concept that mathematics is concerned with the construction of structures, ideas, and relationships using logic. Truth in mathematics is discovered by logical and rigorous reasoning. Mathematical activities are predominantly concerned with logical and methodical thought processes, such as looking for similarities, generalizing, establishing and testing hypotheses, drawing connections, proving theorems, building representations, and eventually, solving problems. Bicer *et al.* (2020) constructed a comprehensive concept of mathematical creative capacity by combining key elements of current definitions. The capacity to develop new mathematical concepts is referred to as a mathematical creative ability to recognize and identify relevant mathematical structures and models that are novel to some individuals.

These three sub-dimensions of creativity are most commonly researched in mathematical problem-solving and problem-posing activities, which have been recognized as mediators of mathematical creativity. In the context of problem-solving, these sub-dimensions have been interpreted as follows: solutions generated for a given problem, different approaches discovered for solving a

problem, and rare solutions produced by individuals as opposed to solutions produced by other individuals (Bicer *et al.*, 2020).

According to Collard and Looney (2014), creativity is not a gift for particular students. Each student has a different degree of creativity. The learning environment plays a role in supporting student development, productivity, identification skills, and efficiency (Davies *et al.*, 2013). Creativity involves new concepts or ideas (Leikin & Lev, 2013; Yaftian, 2015). Wallas' four stages of creative models that include preparation-incubation-illumination-verification to define creative professional mathematicians have been used in mathematics education (Schindler & Lilienthal, 2020). According to Schindler and Lilienthal (2020), school student creativity comprises phases similar to what the Wallas model has. Both still leave research gaps, thus requiring further research. In mathematics education, Pitta-Pantazi *et al.* (2018) described that a staged approach similar to Wallas' model had been recognized. However, Haavold and Birkeland (2017) assumed that distinct models are developed to accurately represent the creativity of professional mathematicians and students in order to identify creative students as a whole. The apparent differences in creative models may be investigated further based on the diverse perspectives of students analyzing the connection between general and mathematical creativity in the context of available inventions in everyday life.

To evaluate students' mathematical creativity, in general, research in mathematics education concentrated on students' creative outcomes, such as written responses (Levav-Waynberg & Leikin, 2012). The study provided uses a product-oriented approach, studying written reports with creative possibilities. All research, in this case, began with analyzing creative products, which are a general form of creativity and a tangible manifestation of the whole process. The primary goal of creative process research is to characterize the activities, and behaviors that occur during the processes of applying different ideas (Pitta-Pantazi *et al.*, 2018). Based on this background, this study aimed to describe the characteristics of the differences in students' creative models in problem-solving activities.

## METHOD

This research was an exploratory-descriptive qualitative study (Creswell, 2016). The researchers attempted to reveal the symptoms experienced by students participating in creative problem-solving activities to classify differences in students' creative models and provide characteristics for each of their creative models.

The participants of this study were 170 urban and rural middle school students in Greater Malang, Indonesia. The problems given to the students for this research were related to mathematical creativity to explore differences in students' creative models. The mathematical problems were adapted from previous research, consisting of open-start problems (approached in different ways), open-ended problems (with several possible outcomes), or a combination of them, which were considered as tasks that can promote creativity, which fulfilled the form of open-ended, connected,



visualization, extendable, and communication problems (Levenson *et al.*, 2018; Molad *et al.*, 2020; Bicer, 2021 & Bicer *et al.*, 2021; Levenson, E., 2022). These problems were then developed as mathematical problems in this study after going through content validation on problem construction and language construction (Purnomo *et al.*, 2022).

Data validation in this study was obtained using the source and triangulation method (Moleong, 2017). The source triangulation was done by comparing and examining data (information) from different students (sources). The triangulation method was carried out by examining the data from the students with different methods, namely from written tests and interviews. A fixed comparison analysis was carried out to determine the theory's reliability by comparing specific data categories with other data categories to obtain categories with the same and consistent characteristics (Hayashi *et al.*, 2019). The data analysis focused on the creativity-based problems given to students; their results or answers were then grouped based on the differences in the approaches they used. The characteristics of the creative model of at least two students were then compared to each other by looking at the similarities and differences (Belotto, 2018). Furthermore, from each group, a student who was considered to represent the group was observed. The results obtained were then used to identify characteristics based on the approach applied.

## RESULTS AND DISCUSSION

According to the responses of 170 students who completed written tasks, only 74 students employed various creative models related to these problems, while 96 students did not. Of the 74 students with different mathematical creative models, 36 were in imitation, 19 were in modification, 12 were in combination, and seven were in creation. The classification of the 74 students' responses to the creative models are shown in Table 1 below.

Characteristics	Number of students	Selected student	Student code
Imitation	36	1	I2
Modification	19	1	M1
Combination	12	1	C1
Creation	7	1	C2

Table 1: Classification of student responses based on creative problems

Differences in students' mathematical creative models could be detected in the results of their works and through in-depth interviews. The students' solutions varied depending on the type of problem (Aguilar & Telese, 2018). They used different solution pathways in response to problem-solving activities. This indicated that each problem in problem-solving activities required a solution with special characteristics in its completion. Other processes such as communicating

solving strategies or representing mathematical ideas acquired relevance in this context (Piñeiro et al., 2021). The following are the results of the interviews that show the characteristics of the students' different creative models.

### *Imitation*

Mathematical creative models in the early phenomenon were traced from the students' answers to the following problems:

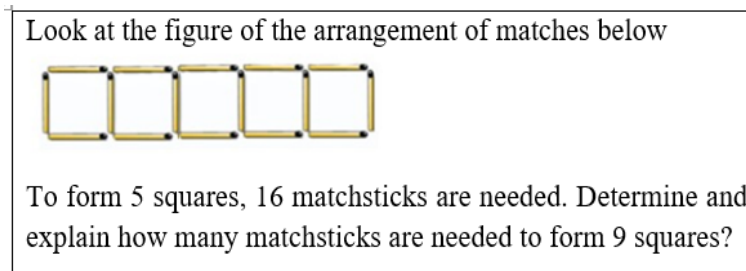


Figure 1: A creative problem adapted from Jonsson *et al.* (2016) & Norqvist (2018)

The following figure depicts a student's response to the aforementioned problem.

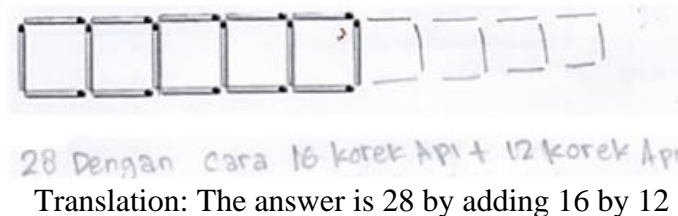


Figure 2: An imitation-based written response

The student (I2) stated that the answer was 28 by adding 16 matches to 12 matches. The student (I2) described this by adding 4 squares to the 5 ones given, resulting in 9 squares. Based on this response, an in-depth interview was then conducted by the researcher (P) with the following results:

P: What steps did you take to solve this problem?

I2: There were five squares built from 16 matchsticks; the question was how many additional matchsticks we needed to build nine squares. So, I counted these (pointing to the question figure) and added these matches (pointing to the answer figure)!

P: You got the answer by adding these squares (pointing to the student's responses); Now, please explain how you came up with such idea!

I2: I counted the matchsticks in this figure (pointing to the question figure); there were 16 matchsticks, and I continued by adding these (pointing to the answer figure) to form a total of 9 squares.

P: Pay attention to the figure you made! Explain how your steps toward the question figure so you could made your answer.

I2: There were five squares available. I followed the pattern of the sticks to get 9 squares. I counted the additional matches (pointing to the figure of the connection made) and found there were 12. additional sticks.

The student determined a new form to solve the problem by imitating the squares on the problem. The student repeated the pattern of the squares to get a total of 9 squares. The student just repeated the stick pattern and then counted the squares and the matchsticks needed to make up them. Thirty-six students did the same to solve the problem.

In this creative model, the students tried to understand the problem information and recognize the images or strategies and then imitated them. Using this model, the students imitated the image's pattern and strategy to solve the problem. In the imitation model, students can solve problems smoothly and produce correct ideas/answers. According to Rohmah *et al.* (2020), students' fluency and flexibility can be seen when they demonstrate fluency in solving a problem and offer more than one solution to one problem. In this model, students imitate the figure as a whole, and some imitate it by repeating the pattern. The imitation model is used when someone wants to make a product by duplicating an existing item. It is a logical cognitive process based on prior knowledge.

According to Purnomo *et al.* (2023), students engage in a creative process during the imitation level by attempting to make solutions by observing the problem figure accurately. After paying attention to the figure, they select the solution technique. At this stage, the imitation process is complete. Imitation occurs when students use their memories and previous experiences to solve problems following existing algorithms (Lithner, 2017). Permatasari *et al.* (2020), in their research, concluded that students, in solving geometric problems, went through a process of imitation in their creative process. Moon & Acquaah (2020) explained that imitation is vital in the creative process. The imitation strategy means that the imitator does not simply copy the attributes or practices of the original product but creatively reconfigures it with his or her distinctive characteristics. According to Lestari *et al.* (2018), imitation plays a vital role in communicating solutions. Through imitation, students can use the same methods or steps as given by the example and can apply the example in new contexts. According to Mecca and Mumford (2014), imitation occurs when an object is imitated; It, therefore, depends on how individuals provide examples. Okada and Ishibashi (2017) defined imitation as the process of copying a product and the degree of deep cognitive processes.

### *Modification*

The mathematical creative model for the following phenomenon may be traced back to the answers to the following problem:

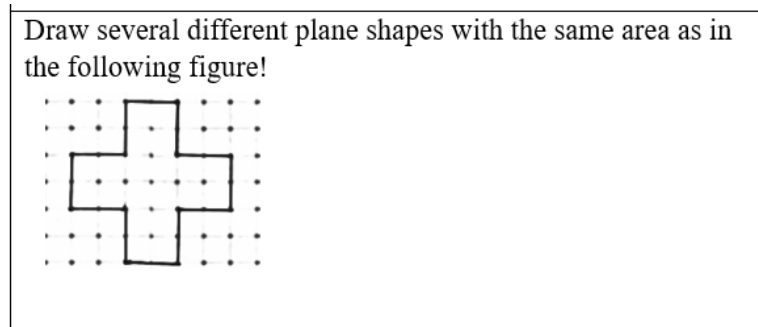


Figure 3: A creative problem adapted from Levenson et al. (2018) and Molad et al. (2020)

The following is a student's answer:

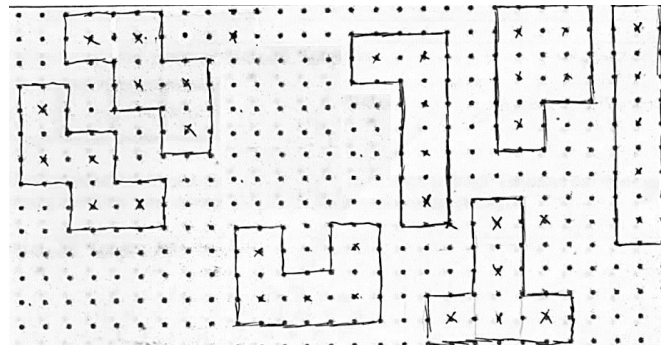


Figure 4: The modification-based written answer

Based on Fig. 4, the student solved the problem by changing the given shape into a new form. The student changed the shape by adjusting the existing area. The student calculated the area of the given shape to make the new shape. The researcher (P) then made an interview with the student (M1).

P: Explain what you know about the flat shape given in this problem, please.

M1: There were five squares (giving a cross for each square).

P: Please explain how could you make this one (pointing to the student's first answer)!

M1: I changed it to a U shape; First of all, I drew five squares, Sir, using these crosses  
It then became a U shape!

P: Explain how you got this one (pointing to another figure).

M1: I changed it to a big square (pointing to 4 squares with crosses arranged into one square), then put one small square under it.

P: For this answer (pointing to another answer), please explain how you came up with such idea!

M1: For this one, I formed the letters M and W, made two squares and then made them like stairs, and then made one more to make five squares.

The student modified the plane shape while paying attention to the area. The student changed the strategy to produce a new shape by dividing the given shape into five equal squares, breaking down the existing shape with the help of small crosses to get new shapes. Based on the results above, this creative model is called the modification model.

In this model, students tried to understand the problem information and recognize the figure or strategy, then imitated them to solve the problem. They also imitated the strategy by making the same square repeatedly. Furthermore, in this model, they changed the plane shape by paying attention to its area. They also changed the strategy by dividing the existing plane shape.

Further, they also change the strategy by breaking down the existing plane shape to get new shapes. Modification means modifying the way used to solve a problem (Singer *et al.*, 2017). According to Voica and Singer (2013), cognitive flexibility in the context of modification is an excellent predictor of mathematical creativity. Students carry out the process of modifying after recognizing that it is a more effective and efficient way to solve problems. The novelty of modification that is considered to be more effective for students is to change the completion steps and determine other strategies (Subanji *et al.*, 2021; Purnomo *et al.*, 2023). Therefore, students are challenged to develop new approaches. Yokochi & Okada (2020) explained that model modification is done by changing both the form of the strategy and the method of completion to make the product more in line with the previous idea. The main feature of the modification process is the expansion of ideas (Marhayati, 2019). The expansion of ideas causes variations in the solution form, marked by a change in the initial solution form. According to Eckert (2012), changing as little as possible is part of modification in the creative process. Creative students can modify and produce something original, meaningful, functional, and impactful. In line with the modification model, Leksmono *et al.* (2019) argued that students who can propose several solution strategies or offer a variety of solution strategies different from the commonly used ones already fulfill aspects of fluency and flexibility.

### ***Combination***

The solutions to the following question were used to trace mathematical creative models in early phenomena:

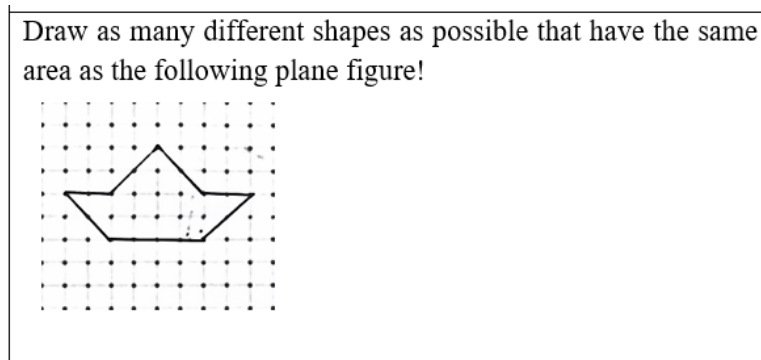


Figure 5: A creative problem adapted from Hidajat et al. (2019) and Levenson et al. (2022)

A student responded to the problem, as illustrated in the following figure:

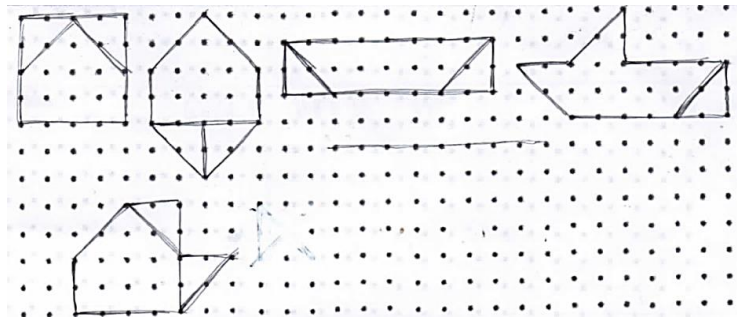


Figure 6: The combination-based written answer

The students tried to solve the problem by combining plane figures to create new figures. The students combined by distinguishing triangular and rectangular shapes. The student added a triangular plane to be placed in another position to produce a new shape. According to the answer of the student (C1), the researcher (P) performed the following in-depth interview:

P : Explain what you know about the plane figure in this problem!

C1: I was given a shape like this (showing the figure of the problem). Then, I was asked to draw another figure with the same area!

P: Explain how you got this plane figure (pointing to the first figure)!

C1: I joined the right-left side up next to the triangle above. There was a triangular plane in the boat-shaped plane.

P: What were your steps in determining these next plane figures?

C1: This one (pointing to the second figure) intersected the left and right triangles and was joined at the bottom of the rectangle. Furthermore, this one (third picture) intersected a triangle to be two equal triangles. Then put them on the right and left with another triangle.

P: How did you make this new figure (pointing to a unique figure)?

C1: I cut the right triangle on top, then added a triangle and joined it onto the right side.

The student combined some plane figures to create new ones. The student combined triangles with other triangles to produce new figures. The student also combined figures by differentiating triangles and rectangles that made up the original figure to produce a new one. The student also added triangles to the sides of other triangles to form a new figure. What the student did as described above can be classified as the creative combination model.

In this model, students understand problem information, recognize appropriate figures or strategies and use them in solving problems, and imitate the figures in solving problems. They can also imitate strategies by repeating the same squares to solve problems. Furthermore, in this model, students can change the figure by paying attention to the area. They can also change the strategy to obtain a new figure. In this model, students combine plane figures, combine strategies by differentiating plane figures, or combine strategies by adding plane figures to produce new ones.

In this model, students combine the same and different forms to produce new forms. The combination model is a creative stage through combining two or more concepts/forms into a new one. According to Edie and Krismonika (2021), combination is the process of integrating two works, both in form and function, into a work, which combines two products, or something entirely new. Creativity is generated by combining two or more concepts into a new concept and emerged various ideas (Chan & Shcunn, 2015; Rahmatina et al., 2022). Creativity results from a combination process (Yu, 2011; Kohn *et al.*, 2011). The combination also occurs by combining strategies, methods, and functions from initial representations into new representations that can occur in various cognitive domains (Hinault *et al.*, 2014). In line with the combination model, According to Rohmah *et al.* (2020), students who solve a problem smoothly produce more than one response and may supply varied and unique answers, thereby meeting fluency, flexibility, and novelty requirements.

### ***Creation***

The answers to the following problem can be used to deduce the mathematical creative model for the highest model phenomenon:

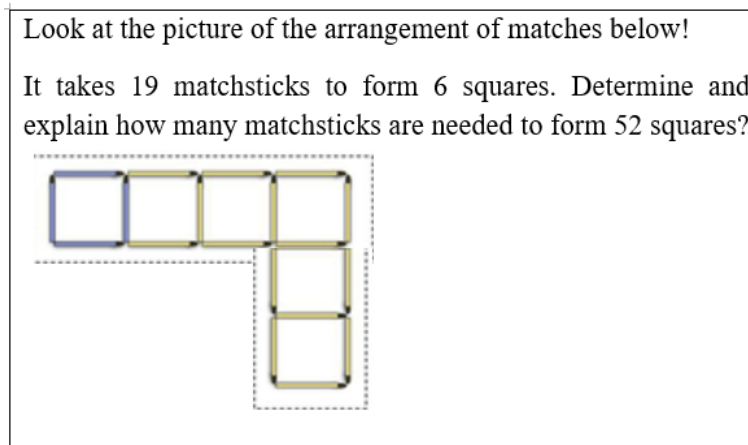


Figure 7: A creative problem adapted from Norqvist et al. (2019) & Jonsson et al. (2022) creative problem

The following the student's answer to the problem:

$$\begin{aligned}
 a=4 &= U_n = a + (n-1)b \\
 b=3 &= U_{52} = 4 + (52-2)3 \\
 n=52 &= U_{52} = 4 + (51)3 \\
 &= U_{52} = 4 + 153 \\
 &= U_{52} = 157
 \end{aligned}$$

Figure 8: The creation-based written response

The student used the arithmetic sequence formula to find the number of matchsticks that make up 52 squares. The students generated the solution based on the pattern in the figure. The student created a strategy by determining the prefix  $a$ , difference  $b$ , and problem solutions. Depending on the outcomes of the student (C2), the researcher (P) performed an in-depth interview about the mathematical model utilized.

P: Please describe the steps you took to find these 52 squares!

C2: First of all, I used the arithmetic formula and looked for the prefix, the difference, and what was asked.

P: How did you get this answer (pointing to the student's answer having a pattern)?

C2: Initially, one square had four sticks. The difference is 3; Each additional box requires three matchsticks. So, we could apply an arithmetic formula.

P: How did you find this pattern (pointing to the mathematical model/pattern made by the students)?



C2: Let's see  $Un = a + (n - 1)b$ ;  $a$  is 4,  $b$  is 3, and  $n$  is the number of squares. Then solve like this (pointing to the answer).

P: Without that formula, can you solve it?

C2: You can also use this, Sir.  $3 \dots 3 \dots 3$  and then (pointing to the figure in the question) with the first side is 1; so, 1 plus 3 times 52 makes 157.

The student developed a formal mathematical model to determine the number of matchsticks required to build 52 squares. The student created the pattern of the problem based on the first square; there are four match sticks; each additional one needs three matchsticks. The student also created new strategies by determining the pattern of the figure: the first side consists of one stick; to make a square need 3. From this pattern, the student could create a new solution to the problem.

The last creative model in solving creative problems is the creation model. In this model, students create patterns on problems to produce solutions. Students create mathematical models to solve problems. Students also create new strategies for finding solutions to problems. In this model, students create formal mathematical models with formulas, while some create mathematical models by generalizing numerical patterns. Subanji *et al.* (2023) said that students can generalize by merging several problem-solving strategies processes. Students can use formulas in solving pattern-based problems. The use of these symbolic formulas is part of functional thinking. Students use formulas by connecting in solving pattern-based problems. In Rivera and Becker's (2016) study on pattern generalization in seventh and eighth graders, students were instructed to locate two distinct continuations of figural patterns. Riviera and Becker (2016) further revealed that creation in the creative process serves to stimulate conceptual meaning in the creation of solutions and problem-solving strategies. Wilkie (2021) looked on how potential instructors discover figural patterns based on quadratic functions. According to Wilkie (2021), the act of production promotes mental meaning for linear or quadratic functions.

Purnomo *et al.* (2023) also said that the creative process in solution difference begins with students being aware of patterns and determining the structure of the eventual mathematical model. Hidayanto and Rahmatina (2020) discovered that students with strong mathematical skills had a propensity to be able to answer all sorts of difficulties and attain the highest model of mathematical thinking abilities. This problem-solving method runs smoothly and is very flexible, as shown by the use of mathematical notation and the creation of equations. Creativity is the process of creating something relatively new or unique originality or using a new style/approach that involves a process or product generation effectively and innovatively (Henriksen *et al.*, 2022). The creation process involves significant construction activities, which are categorized as a high level of creativity (Romero & Lambropoulos, 2015). The creation model is the final model of creative production that creates the final product (Jaarsveld *et al.*, 2012). Chang *et al.* (2014) explained that knowledge creation is a holistic variable and influences product novelty and suitability. The creation model involves evaluating the quality and originality of the generated ideas and selecting

the best idea from a set of alternatives accurately (Puente-Diaz & Cavazos-Arroyo, 2021). Students in the creation model are in the category of gifted students. Purnomo *et al.* (2021) & Sa'dijah *et al.* (2023) explained that gifted students in solving higher-order thinking problems are in the highest thinking model. In this term, Leksmono *et al.* (2019) and Rohmah *et al.* (2020) explained that students that fulfill the fluency, flexibility, and novelty requirements may produce various and unique answers and communicate their reasons in solving problems and drawing valid conclusions.

## CONCLUSIONS

Students have various levels of creativity in problem-solving activities. In this study, there were four different mathematical creative models: imitation, modification, combination, and creation, with their respective characteristics. In the imitation model, students imitated the shape of the figure partially/completely or imitated the strategy by repeating. In the modification model, students modified the form partially/completely or modified the strategy by dividing or parsing. In the combination model, students combined the same/different shapes or combine strategies by differentiating and adding shapes. In the highest mathematical creative model, namely the creation model, students could create patterns for the problem given and develop mathematical models. In creating new strategies to find solutions to the problem, in this model, students created symbolic formal and numerical patterns.

Furthermore, by determining the different characteristics of students' mathematical creative models, teachers can encourage students to optimize their knowledge and experience to produce creative problem-solving. In addition, by knowing the characteristics of students' mathematical creative models, teachers are encouraged to use more creativity-based tasks in learning mathematics to increase students' mathematical creativity. This research was limited to problem-solving activities by adopting four problems aimed at exploring the differences in creative models. The creative problems in this study were limited to two main problem themes: geometry and arithmetic. Further research needs to develop other problem-solving activities that promote students' creative models traceable on an ongoing basis to more varied problem themes.

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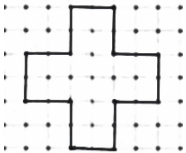
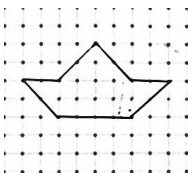
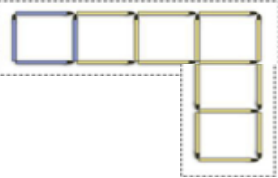
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APPENDIX

Mathematical Creative Problem	Interview questions
<p>Look at the figure of the arrangement of match</p>  <p>To form 5 squares, 16 matchsticks are needed. Determine and explain how many matchsticks are needed to form 9 squares?</p>	<ul style="list-style-type: none"> <li>● What steps did you take to solve this problem?</li> <li>● You wrote your answer by adding squares like this (pointing to the student's answers). Please, explain how you came up with this idea!</li> <li>● Look at the figure in your answer! Please explain the steps you took to produce your answer!</li> </ul>
<p>Draw several different plane shapes with the same area as in the following figure!</p> 	<ul style="list-style-type: none"> <li>● Explain what you know about the plane in this problem?</li> <li>● Explain how you produced this new figure based on the problem (point to the student's first answer)!</li> <li>● Please explain how you got this shape (pointing to another shape)!</li> <li>● You made this plane (pointing to another answer). Please explain how you came up with such idea!</li> </ul>
<p>Draw as many different shapes as possible that have the same area as the following plane figure!</p> 	<ul style="list-style-type: none"> <li>● Explain what you know about the plane in this problem!</li> <li>● Explain how you produced this new figure based on the problem (point to the student's first answer)!</li> <li>● What were your steps in making this next plane figure?</li> <li>● How did you determine the shape of this new plane (pointing the unique shape)?</li> </ul>
<p>Look at the picture of the arrangement of matches below! It takes 19 matchsticks to form 6 squares. Determine and explain how many matchsticks are needed to form 52 squares?</p> 	<ul style="list-style-type: none"> <li>● Describe the steps you took to find these 52 squares!</li> <li>● How did you come up with the idea to write like this (pointing to the patterned answer)?</li> <li>● How did you find the formula (pointing to the math model the student made)?</li> </ul>



## Effects of a Working Memory Training Program on Secondary School Students' Mathematics Achievement

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*Abstract: This paper explores the impact of a tailored working memory training program on the mathematics achievement of secondary school students in Myanmar. While previous studies have investigated the relationship between working memory and mathematics, this study focuses on a context where computerized training programs are limited. Twelve Grade 8 students participated in a six-week program conducted via Zoom due to the pandemic. A Mathematics Achievement Test, aligned with the curriculum, assessed their performance before and after the program. Semi-structured interviews were conducted to gather insights into the students' experiences. Results showed significant improvements in mathematics achievement, particularly in geometry, after the training program. Working memory strategies such as chunking, visualization, and concentration were found to enhance problem-solving abilities. This study highlights the potential of working memory techniques for enhancing mathematics education, particularly in areas with limited technology access.*

Keywords: working memory, mathematics achievement, secondary school students, teaching-learning process

### INTRODUCTION

Working memory assumes a pivotal role in the acquisition of foundational educational competencies. It encompasses the cerebral capacity to retain, manipulate, organise, and shape information before its eventual integration into long-term memory for subsequent utilisation (Johnstone, 1984). Working memory capacity has been notably correlated with children's attainment across a diverse spectrum of mathematical proficiencies (Clair-Thompson & Gathercole, 2006; Elvet & Holmes, 2005), as well as their accomplishments in the fields of mathematics (Mumtaz et al., 2018).

Previous research has examined a comprehensive exploration of the relationship between working memory and mathematics ability (Holmes & Adams, 2006). These inquiries has conducted in different educational strata, including primary school students (Caviola et al., 2020), high school students (Batool et al., 2019), and university students (Clearman et al., 2017).

Now, a number of researchers, as well as, commercial companies, highlight the profound impact of working memory, with the overarching aim of enhancing academic outcomes, particularly in mathematics and other subjects, through the deliberate cultivation of subjects' working memory. Evidently, the cultivation of working memory has emerged as the requirements for optimising the teaching-learning process. Although numerous working memory training programs are available in commercial (e.g., Lumosity, Jungle Memory, CogniFit) or clinical (e.g., Cogmed) settings, these programs are in computerised formats.

In the context of numerous developing countries, including Myanmar, the locale of this study, the utilisation of computerised programs remains limited. Therefore, in this study, a working memory program tailored to the needs of secondary school students. In addition to extend beyond program development, this study investigates the impact of working memory program on the mathematics achievement of secondary school students.

### **Working memory and mathematics achievement**

Using working memory tests, researchers have demonstrated that working memory plays an important role in mathematics calculations and achievement. Evidently, the correlation between working memory and mathematics is multifaceted and substantiated through diverse empirical observations. For instance, investigations have found the positive association between visual-spatial working memory and the mathematical performance of school-age students (Allen et al., 2019). Further insights reveal a noteworthy association between children's mental arithmetic capabilities and their capacity to retain verbal and auditory information (Jarvis & Gathercole, 2003), along with the indispensability of decision-making and information retrieval in fostering mathematical proficiency (Holmes & Adams, 2006).

This symbiotic relationship between working memory and mathematics emerges from the inherent demand of mathematical operations. Each mathematical calculation necessitates intricate working memory processes. When students grapple with mathematical problem-solving, their working memory is intricately woven into the fabric of the process. It aids in the retention of problem specifics, facilitates the retrieval of pertinent procedures, and orchestrates the transformation of these elements into tangible numerical outcomes (Dehn, 2008). Evidently, students with elevated working memory capacities tend to outperform their counterparts with limited working memory space (Alenezi, 2008), thereby underscoring the pivotal role of robust working memory in facilitating superior mathematical comprehension.

### **Need of working memory training program in the teaching-learning process**

The importance and necessity of working memory in cognitive development is very prominent. This cognitive function not only plays a crucial role in academic achievement within the teaching-learning process but is also a strong predictor thereof (Peng et al., 2018). Working memory, representing the capacity for storing and manipulating information, exhibits a steady progression from infancy through childhood to adolescence (Cowan, 2016; Gathercole, Pickering, Ambridge, et al., 2004). This progression stems from both maturation and an expansion of knowledge (Cowan, 2016; Jones et al., 2007), is further enhanced through the implementation of working memory training programs (Sala & Gobet, 2020; Von Bastian & Oberauer, 2014). The potential impact of improving students' working memory through training is considerable, with implications reaching across various academic domains and extending to cognitive and real-life activities.

The functional role of working memory is intrinsically linked to academic outcomes (Bull et al., 2008; Gathercole, Pickering, Knight, et al., 2004). It facilitates the development of intricate cognitive skills such as language and mathematical proficiency (Bull & Scerif, 2001; Colmar & Double, 2017), as well as the acquisition of new information and novel concepts (Pickering, 2006). The potential effectiveness of a training program encompassing diverse working memory strategies holds particular interest, considering the substantial body of evidence attesting to the trainability of working memory (Chein & Morrison, 2010; Jaeggi et al., 2008; Klingberg et al., 2002, 2005).

Examining the varied outcomes of working memory training, certain investigations have focused on its impact on mathematics (Gray et al., 2012; Holmes & Gathercole, 2014; Nutley & Klingberg, 2014). In 2008, Torkel Klingberg conducted a study on working memory training, employing a specific set of computerized tasks with dynamically adjusted difficulty levels based on an algorithm. This training regimen spanned five days a week for five weeks, with each daily session lasting approximately 30-40 minutes. The results revealed improvements in cognitive tasks demanding working memory and attention, translating into enhanced attention in daily life (Klingberg, 2008). In the study of Swanson et al. (2013), it became evident that working memory capacity significantly interacts with treatment outcomes, which is to improve children's experiencing difficulties with mathematics.

The training programs utilised in previous studies are not applicable within the scope of the present research due to contextual discrepancies. Thus, this study aims to develop a tailored working memory training program that aligns with the specific needs of secondary school students.

## MATERIALS AND METHODS

### Participants

A total of 12 Grade 8 students (5 males and 7 females) from Magway, Myanmar participated in this study. This study was approved by the institutional ethics board at the second author's university. Participants were recruited by using purposive sampling because of the nature of qualitative study. Delivered over five days a week, each classroom session spanned forty-five minutes, with a comprehensive duration of six weeks. Due to the prevailing pandemic circumstances, these sessions were facilitated through the Zoom platform.

## MEASURES

### Mathematics Achievement Test

In the present study, a Mathematics Achievement Test was developed to examine students' mathematics achievement. The test was developed based on a typical Grade 8 Mathematics curriculum textbook in Myanmar (Ministry of Education, 2019). To validate the alignment of the test with the curriculum, ten experts from the Department of Methodology (Mathematics) and the Department of Educational Psychology, Yangon University of Education, alongside three secondary school Mathematics teachers teaching Grade 8, participated in an item review process. An initial pilot study, involving 210 Grade 8 students (100 boys and 110 girls), was conducted for the 72-item mathematics achievement test. Given the relatively modest sample size, Classical Test Theory (CTT) was employed to scrutinize the pilot test outcomes. Items for the final mathematics test were selected using the same inclusion criteria as the working memory test, factoring in DI range and DP values. This meticulous curation resulted in the retention of 12 items out of the original 72. The test exhibited a commendable reliability score of 0.87, attesting to its robustness.

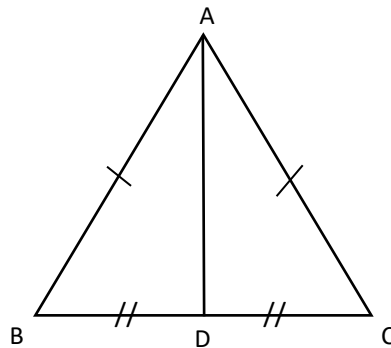
Mathematics achievement test was in line with the typical examination format in schools, Myanmar (i.e., type of questions and the number of items in each topic such as algebra and geometry). A total of 754 Grade 8 students engaged with this mathematics achievement test, each afforded 45 minutes for its completion. The test's scoring methodology allocated a maximum of 25 marks for its entirety. The test comprised four distinct sections, each carrying a designated scoring scheme. In Part 1, encompassing five items, which included both arithmetic and geometric components, the multiple-choice format with four options was employed, warranting one mark for accurate responses and zero marks for incorrect ones. Part 2 featured three arithmetic items, each attracting a score of 2 marks based on the precision of calculation steps. Part 3 incorporated a blend of two arithmetic and one geometric item, each deserving a score of 3 marks contingent upon the meticulousness of calculation steps. In Part 4, a singular geometric item was assigned 5 marks, reflecting a step-wise assessment approach. For instance, students' scores were contingent upon the correct sequence of steps, with each step meriting one mark. The following two items are the sample items that include mathematics achievement test:

Arithmetic item (1): “Calculate the problem  $\frac{x^2y^3}{x^2y} + \frac{6xy^5}{-3xy^3}$  in the simplest form by using the properties of exponents.”

Arithmetic item (2): “If  $\frac{x+y}{y} = 2.6$ , find the value of  $\frac{x}{y}$ .”

Arithmetic item (3): “Solve the problem  $\frac{2x^2}{7y^2} \div \frac{4xy}{21}$  in the simplest form by using the properties of exponents.”

Geometry item (1): “In the figure,  $\triangle ABC$  is an isosceles triangle and  $AB=AC$ ,  $AD$  is the middle line of its triangle. Prove that (i)  $\triangle ABD \cong \triangle ACD$  and (ii)  $\angle ADB \cong \angle ADC = 90^\circ$ .”



Geometry item (2): If the angles of a triangle are  $6x^\circ$ ,  $4x^\circ$  and  $80^\circ$  respectively, find the value of  $x$  by using the property of a triangle.

Geometry item (3): Write down the meaning and related properties of the following quadrilaterals. (i) Rectangle, (ii) Parallelogram (iii) Trapezoid

### Semi-structured interview

In this study, we carried out semi-structured individual interviews following the intervention program to assess the impact of the working memory program. We chose to use semi-structured interviews because they offer flexibility, allowing us to tailor the questions to match each participant's responses, as recommended by Cohen et al. (2007). These interviews consisted of open-ended questions that sought insights into the students' perspectives on the working memory training program and any changes they experienced in their working memory and academic performance, particularly in mathematics, after participating in the program. It's worth noting that we conducted these interviews in the Myanmar language, given that both the researcher and participants shared the same language background. This approach ensured smooth communication and enabled participants to express their thoughts and understandings more clearly. To maintain the accuracy and quality of the data, we employed audio recording during each interview. This

recording helped us precisely interpret the participants' opinions and perceptions, contributing to the overall reliability of our findings.

### Intervention program

The initiation of the training program mandated preliminary consent from each participant's respective parents. Given the imperative utilization of the Zoom platform due to the pandemic context, the participants were granted permission to access their own handsets or laptops during the training period. Subsequently, the researcher conducted a comprehensive briefing session, elucidating the program's intricacies, intervention protocol, and potential benefits. Active participation was encouraged, and participants were prompted to seek clarifications as needed. This initiative aligned with the predefined lesson plans, which were systematically executed. Participants were duly informed of an impending post-training program test. Carried out over six weeks, comprising 30 sessions, each lasting 45 minutes, the program's comprehensive details were meticulously outlined and presented in each session (see Table 1). Following the culmination of the intervention, the post-training mathematics achievement test was administered.

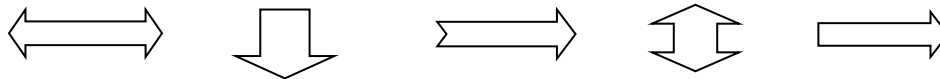
Week	Content	Topic
Week 1	Chunking Strategies	Session 1: explaining students working memory strategies, rules and regulations Session 2: practicing items by chunking method Session 3: practicing items by chunking method Session 4: practicing word sequence items Session 5: practicing items by repeating method
Week 2	Visualization Strategies	Session 6: memorizing photos and choosing the right one in pair of pictures Session 7: memorizing pictures and completing the blanks Session 8: memorizing pictures and answering the questions Session 9: memorizing pictures and choosing the right one in similar pictures Session 10: making the information change into images and memorizing
Week 3	Association Strategies	Session 11: applying the students' imagination Session 12: applying the students' creativities Session 13: memorizing the words without using association Session 14: memorizing the words using association strategies Session 15: differentiation the memorization level
Week 4	Concentration Strategies	Session 16: applying concentration

		Session 17: comparing among dependent factors such as size, shape, quantity
	Visual-spatial	Session 18: applying different colours to be concentrated
	Memory	more
	Games	Session 19: practicing “symmetry games”
Week 5	Memory	Session 20: practicing “symmetry games”
	Games	Session 21: practicing “colour challenges”
		Session 22: practicing “colour challenges”
		Session 23: practicing “right square”
		Session 24: practicing “right square”
Week 6	Memory	Session 25: practicing “risky road”
	Games	Session 26: practicing “risky road”
		Session 27: practicing “shopping list”
		Session 28: practicing “shopping list”
		Session 29: practicing “Find the new”
		Session 30: practicing “Find the new”

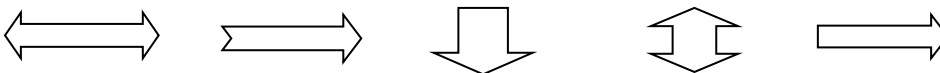
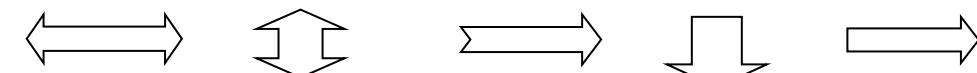
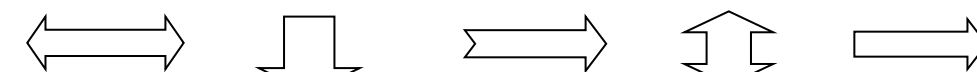
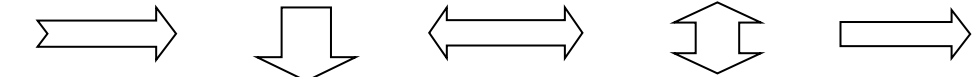
Table 1. Course content and structure of working memory strategies program

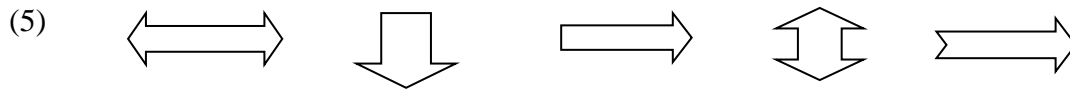
The sample items used in training for visualization and association strategies are as follows:

For visualization, the students are asked to memorize for the following picture for (20) seconds and then answer the question.



Which picture is the same as the above picture from the following ones?

- (1) 
- (2) 
- (3) 
- (4) 



For association, the students are asked to memorize the following words for (5) minutes. Break (2) minutes and then write down these words in the same order.

- |              |               |
|--------------|---------------|
| (1) Wheel    | (11) Train    |
| (2) Children | (12) Holiday  |
| (3) Orange   | (13) Golf     |
| (4) Honda    | (14) Bicycle  |
| (5) Cake     | (15) Birthday |
| (6) Banana   | (16) Nike     |
| (7) Toyota   | (17) Parents  |
| (8) Bus      | (18) Family   |
| (9) Wife     | (19) Car      |
| (10) Apple   | (20) Fruits   |

### Analysis

In order to find out the effect of training program on secondary school students' mathematics achievement before and after the intervention, two types of data analysis, quantitative and qualitative data, were conducted. The quantitative data analysis using the IBM statistical package for the social science (SPSS) was used in the Mathematics achievement data analysis. In order to examine the changes of students' mathematics achievement, paired-samples *t* test was conducted. Regarding analysing the data of semi-structured individual interviews, thematic analysis was used. Three stages of thematic synthesis as highlighted by Thomas and Harden (2008) were used. These three stages are (i) coding text: the line-by-line coding that was done using NVivo software; (ii) developing 'descriptive' themes; and (iii) generating analytical themes. The first author developed descriptive and analytical themes that were reviewed by the second authors. The results of the thematic synthesis are presented in the following section.

## RESULTS

### Improving Mathematics Achievement

To investigate the changes in students' mathematics achievement before and after the training program, we conducted a paired sample *t*-test (see Table 2). Based on the results of our statistical analysis, the paired sample *t*-test indicated a noteworthy difference in the mean scores of



mathematics achievement, with a significance level of  $p < .001$  ( $t(1,11) = -5.33$ ).

Diving into the subscales of algebra and geometry, we found a significant contrast between the pre-test and post-test scores in geometry mathematics, where  $t(1,11) = -7.92$ , and  $p < .001$ . In contrast, there was no statistically significant difference in the mean scores of algebra mathematics between the pre-test and post-test assessments.

	Intervention	Mean	Std. Deviation	Mean Difference	$t$	$df$	$p$
Algebra	Before	10.08	2.23				
	After	10.33	1.49	-0.25	-.61	11	.555
Geometry	Before	6.75	2.22		-		
	After	10.00	1.41	-3.25	7.92***	11	.000
Mathematics Achievement	Before	16.83	4.02		-		
	After	20.33	2.74	-3.5	5.33***	11	.000

Note. \*\*\* The mean difference is significant at the 0.001 level.

Table 2. Paired samples  $t$  test results of students' mathematics achievement before and after intervention

To support the analysis results of a paired sample  $t$  test, thematic analysis revealed two sub-themes arising from the overarching theme of changes in mathematics achievement: (i) algebra and (ii) geometry.

### *Algebra*

The students who actively participated in the training program uniformly reported noticeable enhancements in their mathematics achievement. One student commented her experience, attributing the development of her mathematics abilities and improved memorization techniques to the intervention program. She shared that for mathematical operations, she ingeniously employed association strategies, facilitating easy problem-solving while extending retention over time:

*“I think the level of memorization will be improved in calculating the arithmetic problems by using the working memory strategies. And also, I feel easier in solving mathematical operations. As an example, in handing the operations, there are the rules and principles to be followed. At that time, by using the association strategy, I remember the steps of operation into “PEMDAS” as an abbreviation. That is why, I never forget the steps of operation for a long time” (SS 1, L 61-69).*

Another student recounted his application of the association strategy to solve challenges related to the metric system in social arithmetic. He utilised various working memory strategies, aligning each to its corresponding context, thus enhancing his memorization capacity post-training. To illustrate, in the realm of mathematics, he memorized the metric system by adopting the association strategy, encapsulating it as "mili, centi, deci, mi, deca, heta, kilomi" for sustained recall.

*"I am able to memorize mathematical concepts and facts by using the respective working memory strategy after the training programme. As a result, I rarely forget the information that I receive shortly" (SS 3, L 247-253).*

Similarly, another participant commented her dual utilization of chunking and association strategies to streamline operations involving algebraic and rational algebraic expressions. This approach yielded quicker, more accurate solutions, as well as an increased ease of tackling mathematical challenges.

*"After attending the training program, I usually use both chunking and association strategies in solving the operations on algebraic expressions and rational algebraic expressions. Now, I know the strategies in order to memorize easily and not forget shortly in my memory. By applying these strategies, I think mathematics achievement will be improved to an appropriate level" (SS 9, L 801-809).*

This section verified the potential of working memory strategies to foster a deeper comprehension of algebraic concepts. This correlation between working memory strategies and enriched algebraic understanding hints at the prospective enhancement of overall mathematics achievement.

### *Geometry*

During interviews, a predominant sentiment emerged among participants, highlighting the efficacy of visualization and concentration strategies when confronted with geometric problems. An interviewee candidly shared her prior difficulties with geometry, largely attributed to her struggle in translating textual descriptions into coherent visual constructs. Post-training, her proficient use of working memory strategies empowered her to navigate geometric puzzles with newfound ease.

*"In the past, I did not know how to visualize the text into a figure or diagram and as a result I face difficulties in solving the geometric problems. Now, I am using both visualization and concentration strategies. After visualizing the text, the figures are coloured to see clearly. To prove the similarities of triangles, it is more prominently colouring the similar sides and similar angle" (SS 6, L 524-529).*

Another student confirmed that in proving congruence of triangles, it is necessary to find out the corresponding sides and angles. After the training, problems concerning the congruence of triangles was found easy by using both the visualization and concentration strategies.

*“I always use colour to see clearly the sides and angles. I think visualizing or colouring matches with the geometric problems” (SS 7, L 602-605).*

One of the female students stated that visualization strategy is the most favourite one for her and she applied this strategy most in figuring out and solving many geometric problems like circle problems.

*“I think visualization is very important in solving most of the geometric problems and this is also my favourite strategy. As an example, I can't find out the values if I can't figure out the circle problems” (SS 4, L 330-333).*

Hence, it becomes evident that employing working memory strategies, particularly visualization and concentration techniques, proves pivotal in resolving a majority of geometric problems. Engaging the mind in the process of visualizing textual content fosters the enhancement of students' creativity, leading to a consequent improvement in the process of memorization. This, in turn, elevates their working memory capacity to a certain degree. Additionally, the utilization of working memory strategies has already triumphed over the apprehensions related to geometry. Interestingly, it was observed that a larger number of students applied these strategies more in the domain of geometry than in algebra. As a consequence, the overall mathematics achievement of students will continue to ascend as they adeptly utilize working memory strategies in a suitable manner.

Through both quantitative and qualitative data analysis, the outcomes indicate that training secondary school students with a range of working memory strategies has resulted in improvements in their mathematics achievement compared to their state before the intervention.

## DISCUSSION

This study demonstrates the positive impact of a working memory training program on the mathematics achievement of secondary school students. The results establish the program's effectiveness in enhancing their mathematical proficiency. These findings are in line with prior research (Ayoka & Akinyemi, 2014; Dahlin, 2013) that also observed improvements in mathematics following working memory training. Moreover, these results underscore the necessity of implementing working memory strategies to elevate secondary school students' mathematics performance, aligning with the findings of Holmes and Dunning (2017). Additionally, these improvements can contribute to a reduction in students' difficulties with mathematics, as indicated by Nur et al. (2018).

Within the training program, one specific working memory strategy, known as "chunking," emerges as an essential and practical technique for enhancing academic achievement, particularly

in mathematics. Chunking minimizes the working memory space required by grouping information into easily remembered units. This approach aligns with the findings of Solopchuk et al. (2016) and Thalmann et al. (2019).

Many students derive significant benefits from visualizing the information they study. Visual aids such as photographs, charts, diagrams, and graphics in study materials capture their attention. When such visual cues are absent, especially when tackling geometric problems, students resort to creating their own. This underscores the importance of the visualization strategy, which aids in retaining information for extended periods. It serves as a foundational skill for comprehending and developing fundamental mathematical concepts and plays a pivotal role in superior problem-solving, as supported by Rabab'h and Veloo (2015).

Furthermore, this study reveals that students employ concentration strategies, including factors such as size and color, to enhance their mathematical problem-solving abilities. This finding corroborates existing literature, which posits that concentration is a trainable mental state wherein a student's senses and cognitive faculties are focused on a specific subject or information (Kumar, 2003).

However, it's important to acknowledge the limitations of this research, as is common in scientific inquiries. The sample size for the training program was constrained due to the unforeseen impact of COVID-19. Future research should prioritize larger sample size including various grades of secondary school students, to strengthen statistical power. Moreover, considering the evolving landscape of education, computer-based training programs should be considered, provided that students have access to the necessary technology and internet connectivity, to align with the changing educational needs and preferences in Myanmar.

## CONCLUSIONS

In conclusion, this study brings out the positive potential of using working memory techniques in teaching secondary school students. Importantly, this training program is easy to implement as it involves classroom strategies, and it doesn't require computers or internet access, which means it's available in all local schools in Myanmar. By taking the time to understand the benefits of enhancing working memory and using strategies that suit each student, teachers can create a more engaging and effective teaching and learning experience. This not only benefits students but also makes our educational system more dynamic and fulfilling.

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## Introducing a Teaching Technique for Reducing Students' Mistakes in Simplifying Algebraic Expressions

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*Abstract: The present study investigates the effect of the separator lines on the learning of 8th grade students in simplifying algebraic expressions with parenthesis. An experimental study was designed to achieve this goal involving 60 girl students in 8th Grade (13 and 14 years old) randomly selected and assigned to two experimental and control groups. After taking the pre-test, both groups were taught by one teacher. The control group was led as usual and the experimental group was taught by using the separator lines. As a result of the covid-19 disease, students were taught online using WhatsApp. In the end, a post-test was carried out for both groups. Data was also collected using WhatsApp. For the analysis of the data, a Covariance test was conducted. The results showed positive effects of separator lines on the student's performance, as well as reducing their mistakes when working with parenthesis.*

**Keywords:** Simplification, Algebraic Expressions, Students Mistakes, Separator Lines, WhatsApp.

### INTRODUCTION

Algebra is an essential component of the mathematics curriculum because it is the language of generalization by which we describe patterns; algebra is a language for expressing the relationship between quantities; and without algebra, we cannot understand many concepts in basic sciences

such as chemistry, physics, and geology (Kaput, 1995). Simplifying algebraic expression is one of the most important topics of algebra in middle school math textbooks (Hibi & Assadi, 2022). According to Bush (2011), simplifying algebraic expressions is a prerequisite for many mathematical subjects in higher education. Because of this, students who do not learn how to simplify algebraic expressions will have difficulty solving equations. Therefore, students' mistakes in this topic prevent them from learning other topics in math textbooks, such as equations, solving verbal problems, or calculating variable values. As a result, it is important to correct and reduce students' mistakes. Mamba (2012) and Seng (2010) found that students do poorly when simplifying algebraic expressions and make errors and misconceptions<sup>1</sup>, in using + and - symbols, parenthesis, power, addition, and subtraction of like sentences. Examples of these mistakes are given in Figure 1.

$$\begin{aligned}
 &+3(6a - 7) = -18a + 21 \\
 &4 + 5y^2 = 4 + 25y^2 \\
 &4a^2 + 3a^2 = 7a^2 = 14a \\
 &5a + 5b = 10ab \\
 &2a \times 3a = 6a \\
 &-6a + 3a = -9a \text{ or } +3a \\
 &5ab - 6 + 4ba + 7 = 9ab - 13 \text{ or } 9ab - 1 \\
 &2(3a + 2) + 3 + 4a = 6a + 4 + 6 + 8a
 \end{aligned}$$

Figure1: Examples of mistakes in simplifying algebraic expressions

Many math teachers had encountered these types of mistakes in their classroom experiences. She has observed the examples above numerous times in her classroom. There are three reasons for choosing the topic of simplifying algebraic expressions in the current study. Firstly, simplifying algebraic expressions is a topic that students face at the beginning of the middle school program. Secondly simplifying algebraic expressions is one of the most important topics at all levels of the middle school mathematics curriculum in Iran. This is dealt with in the chapter on algebra in the 7<sup>th</sup> grade mathematics books. In the 8<sup>th</sup> grade, this topic is discussed in a separate section, and the 9<sup>th</sup> grade, simplifying expressions is a prerequisite for solving equations. Therefore, if the students do not learn to simplify algebraic expressions, they will find it difficult to solve equations or systems of equations later. Thirdly, the content analysis of Iranian middle school mathematics

<sup>1</sup> . mistakes that occur due to disruption in students' conceptual and procedural understanding

textbooks (Research and Educational Planning Organization, 2020), show that simplifying algebraic expressions has the central role.

In Table 1, the relationships displayed are according to the content of the books of this educational program. As shown in the above table, the study of all these topics requires learning the topic of simplifying algebraic expressions. For example, for solving a simple inequality, the student may need to be able to solve the linear equation and this solution usually requires simplifying algebraic expressions on both sides of the equation.

Algebraic topics	Prerequisite
Number patterns, the Nth sentence, the idea of a variable	Arithmetic in school elementary
Algebraic expressions, monomials, and like sentences	Number patterns
Simplification of algebraic expressions	Like sentences, monomials
Finding the numerical value of algebraic expressions	Simplification of algebraic expressions, variable expressions
Linear Equations	Simplification of algebraic expressions
Identities	Simplification of algebraic expressions
Factorization	Identities, Simplification of algebraic expressions
Inequalities	Linear Equations, Simplification of algebraic expressions

Table 1: Algebraic topics in middle school and their prerequisites

According to Fleisch (2008), some algebraic errors made by students are the result of improper teaching methods. A teacher's main goal is to help students understand and use mathematical concepts and procedures correctly. Research in other countries has recently focused on the professional knowledge of teachers and found that one of the most important predictors of student success is the knowledge and teaching method of teachers (Hill, Rowan & Ball, 2005). To improve learning, teachers need to use new teaching methods. In most classrooms, teachers still use traditional methods and are unmotivated to teach differently. Norton and Irvin (2007) demonstrated that traditional methods are not useful, thus other teaching methods may be able to improve students' learning. Perhaps teachers' inappropriate teaching methods contribute to students' poor performance in simplifying algebraic expressions. The above content created the question in the minds of the authors how we can improve the performance of students on this topic and reduce their mistakes?

Having found that students' mistakes in simplifying algebraic expressions are an obstacle to learning mathematics in higher grades, and on the other hand, the research conducted in this field has focused on students' mistakes and their origin, without proposing a way to reduce them. Therefore, the authors of the current research suggest separating algebraic expressions by separator

lines to reduce students' mistakes. In an algebraic expression, separating the sentences involves putting a slant line (/) before each positive and negative. Positive and negative are not considered inside the parenthesis (See formula 1).

$$3x - 4x(2 + 8x) - 7x^2 = 3x / -4x(2 + 8x) / -7x^2 \quad (1)$$

When we use slant lines, an algebraic expression is divided into several parts, and we know which parts to operate on. In the formula 1, the separated parts are  $3x$  and there is no need to operate on this part since it has no parenthesis. In the next part, we have  $-4x(2 + 8x)$  that there is a parenthesis in this part, so we have to remove it. To do this, we have to multiply the sentence before the parenthesis in it. As a result, the expression  $-4x$  is multiplied by  $2$  and  $+8x$  upon the associate property. Finally, for  $-7x^2$ , there is no need to operate on this part since it has no parenthesis. Therefore, separator lines divide algebraic expressions into several parts to recognize which parts should be operated on. Essentially, the separator lines indicate what should be multiplied in parenthesis and in which sentences it should be performed. So, below research question lead the current study.

- Do separator lines affect the learning of students to simplify expressions with parenthesis and reduce their mistakes?

The reason for choosing algebraic expressions with parenthesis is that most expressions in the algebra chapter of the math textbooks are in many countries. Due to this, parenthesis plays an important role in simplifying algebraic expressions, because students cannot add or subtract similar sentences until parenthesis are removed. In other words, removing parenthesis is the first step toward simplifying algebraic expressions with parenthesis. According to Seng (2010) when deleting parenthesis, students make mistakes, such as, they do not recognize what and how should multiply in parenthesis (2), in which sentences they should multiply (1), or to which sentence they should continue multiplying (3). Examples of these mistakes are given in Figure 2.

$3x - 4x(2 + 8x) - 7x^2 = 3x - 8x + 8x - 7x^2 = 3x - 7x^2$
$3x - 4x(2 + 8x) - 7x^2 = 6x - 32x^2 - 7x^2 = 6x - 39x^2$
$3x - 4x(2 + 8x) - 7x^2 = 3x - 8x - 32x^2 - 28x^3 = -5x - 32x^2 - 28x^3$

Figure 2: examples of mistakes in simplifying algebraic expressions with parenthesis

## LITERATURE REVIEW

So far, no research has been done on teaching algebraic expressions differently from the textbook, and most research has focused on students' misconceptions of algebra. In this part, we discuss some challenges in learning algebra, particularly simplifying algebraic expressions, as well as the role of teaching methods in education. Mamba (2012) pointed out that understanding and working with algebraic expressions by accepted rules, procedures, and algorithms create challenges for students because of using letters. Students are faced with a lot of mistakes in the multiplication of algebraic expressions and equations, because of the lack of understanding of symbols and letters (Hall, 2002b). Some students are not even able to read algebraic expressions. Many students do not understand the idea of letters as a number, they tend to interpret the letters as a particular number and believe that different letters must necessarily represent different numbers (Seng, 2010). Using letters has caused students to find algebra challenging, according to Booker (1987). Students are unsuccessful in algebraic topics such as recognizing sentences, decomposing, simplifying, and calculating the numerical value of algebraic expressions, and for this reason, they have difficulty in solving verbal problems, forming, and solving equations.

A mistake is a simple lapse of care or concentration which almost everyone makes at least occasionally (Marpa, 2019). Misconceptions are one of the most important challenges in learning algebra. Russell, Odwyer, and Miranda (2009) believe the deep misconceptions that are created in algebraic concepts are not accidental and they lead to mistakes made by students. Sisman and Aksu (2016) while examining the mistakes and misconceptions of students presented them as evidence of a lack of basic concepts and lack of learning. Welder (2012) pointed out that student mistakes in algebra may be because the knowledge of students is not complete or is not understood well. Misconceptions of students have roots in the mental structures of individuals, for this reason, knowing how the effects of mental schemas in creating misconceptions can improve learning. Booth, Barbieri, Eyer, and Pare-Blagoev (2014) identified six mistakes categories when students solved algebraic Questions that include: Misconception of the variable concept; Use of the negative sign; False solving equation and inequality; Incorrect application of features such as displacement law; Fractions; and Do not observe the order of operation. Jupri, Drijvers, and Van den (2014) divided the mistakes in the initial algebra into seven categories which are applying an arithmetic operation, understanding the variable concept, understanding the concept of algebraic expressions, mathematization which goes back to the mistakes in describing the real situation mathematically, understanding the meaning of the sign of equality involves mistakes about the meaning of this sign in arithmetic and algebra, understanding the concept of algebraic expressions, and finally understanding the meaning of the sign of equality that in arithmetic this sign requires a numerical answer, but in algebra, it may require an algebraic expression.

Simplifying an algebraic expression means calculating a simpler and shorter algebraic equation than the original expression (Owusu, 2015). Students must understand that the question and answer of the algebraic expression can be variables and that the letter in the answer of an algebraic

expression is a generalized number, which means that the letter can be any number (Hall, 2002b). Kieran (1992) observed that many people see a letter as a generalized number hardly, and this may be because the letter in the solution of the equation is only one specific number. In Malaysia, which calculated the frequency of student mistakes when simplifying algebraic expressions, was observed that the mistake in the order of the operation and the negative sign have the most frequent among the mistakes (Seng, 2010). Reyes (2012) finds that the common mistake of most students in simplifying is that they equalize algebraic expressions with zero. In other words, students know algebraic expressions are incomplete. For example, students cannot accept the expression of  $6 + 8x$  as a solution to a question, (See formula 2).

$$6 + 5x + 3x = 6 + 8x \Rightarrow 6 + 8x = 0 \Rightarrow x = \frac{-6}{8} \quad (2)$$

This mistake may be due to the lack of understanding of the difference between the algebraic expression and the equation. Other mistakes are also made by students in simplifying algebraic expressions which we quote them below.

- Students tend to add and subtract similar sentences while simplifying algebraic expressions, for example;  $5x+3=8x$ . The reason for this mistake may be that students collect non- similarity sentences based on the concept of the + sign, to write a reply.
- Sometimes a student writes  $3x-2x = 1$ , whose reason may be that the student tends to deal with numbers and letters separately; hence  $3-2=1$  and  $x-x=0$  (Matz, 1980, cited in Gunawardenna, 2011).
- In simplifying the sentence,  $a \times a$  to the form  $2a$  and  $a + a$  to the form  $a^2$ , the student may be wondering about the rules related to collecting and multiplying and remind them (Owusu, 2015), But the other reason for this mistake is the application of correct rules in inappropriate situations (Matz, 1980, cited in Gunawardenna, 2011).

The reason to make mistakes in simplifying algebraic expressions is that mathematics teachers do not care about the origin of these mistakes (Guler & Celik, 2016). Mathematics teachers not only need to have arithmetical skills, but also must have the ability to think, select aimful training strategies, and curriculum algebraic ideas to help students move from arithmetic to algebra (Anne Hayata, 2012). In most classrooms, traditional methods dominate, and these traditional approaches have failed to teach algebra. (Norton & Irvin, 2007). Studies repeatedly highlight the role of teaching as an important variable, which affects learners' performance in mathematics as well as an effective factor in causing student mistakes (Shulman, 1987). It is reasonable to say that traditional methods do not provide meaningful educational options for addressing learners' mistakes in mathematics, and particular algebra (Owusu, 2015). Creating a new educational method that improves students learning (Doerr, 2004).

## METHOD

The quantitative method was found to be appropriate for this study. To evaluate the effectiveness of the designed method in simplifying algebraic expressions compared to traditional methods, an experimental method was employed with two groups of control and experiment, as well as pre-test and post-test. The innovation of this work is in providing an educational method for simplifying algebraic expressions that reduces students' mistakes. Since other research has focused on students' mistakes and their causes. Furthermore, WhatsApp was used to collect data and teach students during this research. Due to its features such as the possibility of two-way communication, the ability to form groups, and the ability to share files, the WhatsApp environment is convenient for teaching (Barhoumi, 2015). Users can send text, audio, and video messages with this messenger. According to the study by Rafiepour, Abdolapour, and Farsani (2021), the WhatsApp environment can be used to teach mathematical concepts.

As the statistical population, we selected eighth-grade students in the academic year 2021-2022 who were 13 and 14 years old. The sample consists of students from a girls' school in Kerman. This school had 150 eighth-graders, 60 of whom were randomly selected and placed in control and experimental groups. The Covid-19 pandemic caused education in villages and city schools to be virtual, making it difficult to reach students from other schools.

To determine the effects of separator lines on learning to simplify algebraic expressions, an experimental study was designed and conducted in which 8th grade students were divided into two experimental and one control group .60 students were randomly divided into two groups of 30 each. The teacher who teaches control and experimental groups of students has 14 years of experience teaching mathematics at middle school. In 6 sessions of 45minutes, she taught both groups how to simplify algebraic expressions. According to the experimental design, the experimental group was taught using the new method, while the control group was taught using the traditional method. Teacher in both groups recalled concepts that students had previously read and needed to simplify algebraic expressions, such as how to multiply and empower parenthesis. The teaching method was flipped classroom. In this way, the teacher sent clips of the content of each group to the students in every session of class. Students were given time to watch the clips. Then, the questions of the students were answered through a conversation. An example of a flip classroom assignment and communication about that come in appendix (figure 3). In the pre-test and post-test Students sent photos of their answers on WhatsApp.

The data were collected through tests designed by the researchers. First, In the WhatsApp environment, all students took a pre-test related to simplifying algebraic expressions taught in the 7<sup>th</sup> grade. In the pre-test, 5 problems about simplifying algebraic expressions from the 7<sup>th</sup> math book were raised. In the classroom, students worked individually. If a student asked a question, everyone could answer it. After teaching simplifying expressions, In the WhatsApp environment, the students again completed a post-test. Based on the training content, five questions were raised

about simplifying algebraic expressions in the post-test. Five experienced math teachers evaluated and approved the validity of the tests. Cronbach's alpha was used to determine the reliability of the test, and the result was  $\alpha=0.79$ . To analyze the quantitative data for investigating the effect of the method on students we carried out an analysis of covariance (ANCOVA) using SPSS 20.

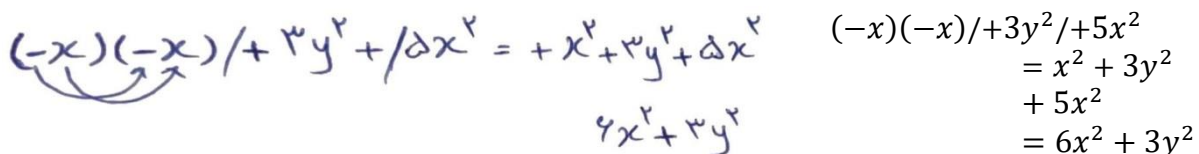
## RESULTS

The mean and standard deviation of students' scores in the control and experimental groups for the pre-test and post-test are displayed in Table 2. As shown in the above table, the difference in the mean post-test and pre-test scores is much greater for the experimental group than for the control group.

Group	Test stage	Number	Mean	Std. Deviation
Experimental	pre-test	30	16.46	3.60
	post-test	30	18.06	2.90
Control	pre-test	30	16.93	3.74
	post-test	30	17.31	2.83

Table 2: Descriptive Statistics

For checking the research hypothesis, we used to analyze covariance (ANCOVA) with the post-test score as the independent variable, pre-test score as covariate, and group as a fixed factor. We first need to check the main assumptions needed. Figures 3, 4, and 5 show examples of the answers given by the experimental group's students.



$$(-x)(-x)/+3y^2/+5x^2 = +x^2+3y^2+5x^2$$

$$(-x)(-x)/+3y^2/+5x^2 = x^2 + 3y^2 + 5x^2 = 6x^2 + 3y^2$$

Figure 4: The answer of student number 10 in the experimental group at the post-test

As shown in Figure 4, the student has drawn a slant line before the positive sign to separate the sentences of the algebraic expression. His operation has been performed correctly on the first part, which has parenthesis, of the algebraic expression that is divided into three parts.



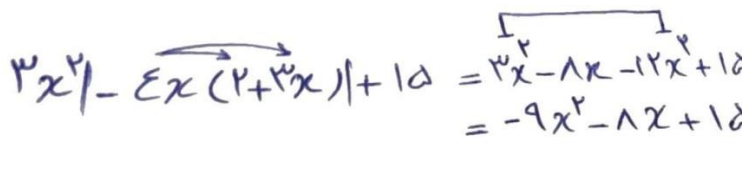
$$\begin{array}{l}
 3x^2 / -4x(2 + 3x) / +15 \\
 = 3x^2 - 8x \\
 - 12x^2 + 15 \\
 = -9x^2 - 8x \\
 + 15
 \end{array}$$


Figure 5: The answer of student number 14 in the experimental group at the post-test

This algebraic expression is divided into three parts because of the slant line used by the student in Figure 5. The student correctly recognized that the second part requires an operation and correctly multiplied  $-4x$  in parenthesis.

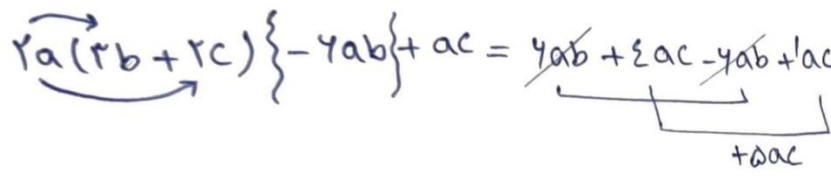
$$\begin{array}{l}
 2a(3b + 2c) / -6ab / +ac \\
 = 6ab \\
 + 4ac \\
 - 6ab + ac \\
 = 5ac
 \end{array}$$


Figure 6: The answer of student number 26 in the experimental group at the post-test

6 shows the student divided the algebraic expression into three parts by drawing a line and correctly recognized that the first part required multiplying  $2a$  by two sentences inside the parenthesis. The research hypothesis was then tested.

Research Hypothesis: Compared to the traditional method, the explained method has a greater impact on students' learning of simplifying algebraic expressions with parenthesis in 8th grade. In this paper Kolmogorov-Smirnov test was used for checking the normality of data and obtained  $F = 1.39$  and  $p = 0.083 > 0.05$ ; also the Levene test was used for checking Homogeneity (equality) of variances and obtained  $F(1, 58) = 0.63$  and  $p = 0.281 > 0.05$ . Furthermore, the sampling method in this study guarantees Random sampling. The random sampling, normality, and homogeneity of variances allowed the necessary assumptions for an analysis of covariance to be made, thus this test was conducted. Table 3 shows the results of the analysis of covariance (ANCOVA).

According to Table 3,  $F = 4.34$  and  $p = 0.02 < 0.05$  for groups, i.e. after removing the effect of the covariate, there is a significant difference between the mean post-test scores of the two groups (experiment and control). So, the hypothesis of the research i.e. "effect on learning in middle school students when our method is used for teaching simplification of algebraic expressions is greater than when using traditional methods" is confirmed.

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Pre-test	292.341	1	292.341	89.72	.000
group	14.820	1	14.820	4.34	.02
Error	185.711	57	3.258		
Total	491.828	60			

Table 3: Results of analysis of covariance (ANCOVA)

## DISCUSSION AND CONCLUSIONS

According to the findings of the research in Table 3, separating sentences in algebraic expressions by separator lines facilitates the learning of how to simplify algebraic expressions and reduces girl students' mistakes in performing algebraic expression calculations. The innovation of separator lines explains this positive effect. By separating algebraic expressions with slant lines, students were able to perform algebraic calculations correctly. See, for example, figures 3 and 2. You can see the role of separator lines in these figures. It seems that these lines caused the students to pause, or reflect, to focus on the calculation of the expression that has priority over other parts, leading to more accurate calculations and fewer errors. In general, Separator lines for separating algebraic expressions have a positive effect on solving algebraic expressions and also on reducing students' errors. The findings of this study are consistent with the results of Seng (2010) and Sisman and Aksu (2016). In their research, these people stated that it could be effective in improving algebra learning by students. Although these researchers did not examine the effects of a new method in their research, they claimed that new teaching methods can improve algebra learning in students. New methods of teaching can enhance students' problem-solving skills and improve their academic performance (Seng, 2010). Sisman and Aksu (2016) point out that despite recent advances in learning theories and the emergence of constructivist approaches, most mathematical teaching methods are traditional and new ones are seldom utilized.

The findings of the study are related to the subject of algebra. About mathematical content, algebra occupies a very special place, both because it is a powerful tool for solving problems or modeling situations (Watson, 2016), and also because it is an essential building block for learning other types of mathematics and the various sciences (Chea & Baba, 2021). Capraro and Joffrion (2006) observed in their research that one of the obstacles to learning mathematics among middle school students was that they could not make a connection between their knowledge of arithmetic and the knowledge needed to understand algebraic concepts. When algebraic topics are discussed in

middle school, students encounter many challenges and mistakes. Mistakes seem a common phenomenon among students with algebra. Mistakes have been a concern for researchers and mathematics teachers (Kshetree et al., 2021). Therefore, this study can provide a solution to these challenges and reduce mistakes.

The next topic addressed in this study was simplifying algebraic expressions with parenthesis. When simplifying algebraic expressions with parenthesis, students make more mistakes. The reason is that they don't know what to multiply in parenthesis or from which sentence to start and which to continue this multiplication. Therefore, the present study examined the effect of separator lines on the learning of 8th grade students in simplifying algebraic expressions with parenthesis. This way, clips were sent on What Sapp to two control and experimental groups, and the data was collected by pre-and post-tests that were taken on WhatsApp. The findings of the teaching of this topic by separator lines have shown its positive effect on improving 8th grade students' learning. Using this method to simplify algebraic expressions helped students make fewer mistakes while removing parenthesis. They learned how to divide algebraic expressions into parts and which to operate on. Students learned what should be multiplied in parenthesis or in which sentences the sentence before parenthesis should be multiplied. As a result of this success in algebra, students will be able to recognize the number of sentences in algebraic expressions, solve verbal problems, and solve equations and systems of equations. Hence teachers should help students to learn topics of algebra. Therefore, teachers should determine the most appropriate way to facilitate students' learning based on purposes, content, students' needs, and available resources (NCTM, 2000).

Mathematical books in all countries cover simplifying algebraic expressions (NCTM, 2000). We hope that this research has helped the international mathematics education society and the mathematics performance of students in all countries. It is suggested that those interested in mathematics education research develop new teaching methods to reduce students' misconceptions in other algebra topics such as addition and subtraction of like sentences.

### **Disclosure statement**

No potential conflict of interest was reported by the authors.

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## Appendix

In one of the sessions of the experimental group, after sending the educational clip and watching it for 15 minutes by the students, the algebraic expression  $2x - 3(x - 2y) + 2y$  was sent on WhatsApp and discussed with the students about it; a part of this conversation is below. Of course, in the educational clip, simplifying 5 algebraic expressions was taught using separator lines and the method of using these lines was fully explained to the students.

Teacher: Maryam, how many separator lines did you use to simplify this algebraic expression?

Maryam: 2

Teacher: Anahita, where did you draw the separator lines?

Anahita: before  $-3$  and before  $+2y$

Teacher: Nasim, dividing lines divide the algebraic expression into how many parts?

Nasim: in 3 parts

Teacher: Saba, says the three parts

Saba:  $2x, -3(x - 2y), +2y$

Teacher: Are all three parts single?

monomial expression?

Hasti: No, we have to remove the parenthesis, that is, multiply  $-3$  by  $x$  and by  $-2y$  to convert them into a monomial expression.

Teacher: Nazanin, what will be the result of this multiplication?

Nazanin:  $-3x - 2y$

Teacher: Fatima, is Nazanin's answer correct?

Fatima: No,  $-3x + 6y$

Teacher: Yes, Fatima's answer is right. Now identify similar terms.

Yasna: Now we have  $2x - 3x + 6y + 2y$ , so  $2x$  and  $-3x$  are similar terms and  $6y$  and  $2y$  are similar terms to each other.

Teacher: What is the answer of  $-3x + 2x$ ?

Maryam:  $-1x$

Teacher: That's right. What is the result of  $2y + 6y$ ?

Bahar:  $8y$

Teacher: what is the final answer?

Fatima:  $-1x + 8y$

Figure 3: An example of a flip classroom assignment and communication about that



## The Effect of Attitude Towards School on the Students' Happiness: The Moderating Role of Math Anxiety

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*Abstract: This study aims to examine the moderating role of math anxiety in the effect of the attitude towards school on the students' happiness. In the semester of 2021-2022, 415 students at the 8<sup>th</sup> grade of secondary schools, participated in the study. In this study a correlational survey model was employed. "Scale for Attitude Towards School", "Math Anxiety Scale", and "Adolescent Happiness Scale" were used to collect data. In the analysis of data, SPSS Process Macro extension was utilized. This study reveals that attitude towards school positively affects students' happiness while math anxiety negatively affects students' happiness. Furthermore, the study demonstrates that math anxiety has an effect that could decrease the effects of attitude towards school on students' happiness. Therefore, it was determined that there is a moderating role of math anxiety in the effect of the attitude towards school on the students' happiness.*

Keywords: Happiness, attitude towards school, math anxiety.

### INTRODUCTION

Happiness affects the behaviors, dreams and targets as well as the decisions of people because it is a state of willingness to reach, and when it is reached, the state of happiness is not wanted to be lost. Since ancient times, people have been trying to understand what the happiness is, and how it is reached, and they have suggested ideas to explain the concept of happiness. The dynamics of life have led to a change in the meaning of happiness concepts in time. Therefore, happiness and its effects on people have been discussed in many disciplines such as philosophy, psychology, biology, medicine and economy nowadays (Kesik & Aslan, 2020; Yalvaç Arıcı, 2019).

Happiness is a concept that is related to satisfaction with life, pleasure in life, life quality and being good (Bora & Altınok, 2021; İme & Öztosun, 2020). In literature, the focus of the claims and recognized approaches to explain the happiness is on the self-realization of individuals and satisfaction of individuals. Happiness, which is explained by the self-realization of an individual

and the emergence of his/her whole potential, is conceptualized as psychological well-being. Drawing from this perspective, happiness means living in accordance with the nature of individuals to achieve targets in order to actualize his/herself. Therefore, happiness is based on the concept of living with wisdom, ethics, meanings and virtues (Deci & Ryan, 2008). Based on the approach of “psychological well-being”, happiness is defined as a permanent situation where individuals feel in tune with their targets, needs and their surroundings (Fidan, 2020).

According to the approach which explains the happiness as depending on the satisfaction of individuals, the pleasures that an individual takes as a result of his/her behaviours are an indicator of happiness. Happiness depends on pain and pleasure (Epicurus, 1949). Based on this approach conceptualized as subjective well-being, the majority of positive thoughts and emotions in their judgments in the field of work, school, marriage, and lack of negative emotions and thoughts indicate happiness (Diener, 2000). According to this approach, happy individuals experience positive emotions such as confidence, joy, fun, and hope much more than negative emotions such as anger, hate, anxiety, fear, hopelessness and sadness (Eryılmaz, 2011). In this regard, happiness can be defined as living the moment and having leisure with life, feeling good, being happy and healthy, and having a lovely family life that is admired by surroundings (Gökdemir-Dumludağ, 2011).

United Nations Education, Scientific, and Cultural Organization (UNESCO) accepted the search for happiness as a fundamental aim. The individual has a tendency to accept actions that have positive emotions, and has also a tendency to escape from actions that have negative emotions. Moreover, an individual’s evaluation of his/her life leads to positive/negative emotions. These emotions affect individuals’ both cognitive structure and behaviours (Salovey et al., 1995). Therefore, it can be said that happiness affects individuals in many ways. In literature, there are studies on happiness indicating that it is related to mental, psychological, social and general health. Happiness is associated with mental factors such as academic success (Bücker et al., 2018; Kirkcaldy, Fumham & Siefen, 2004; Winarso & Haqq 2019; Yıldırım & Turaç, 2020), exam or test anxiety (Feta, 2019; Steinmayr et al., 2016); creative thinking and creativity (Conner, DeYoung & Silvia, 2018; Flor et al., 2013; Soleimani & Tebyanian, 2011; Tan & Majid, 2011); motivation for participation in the class (Eryılmaz & Aypay, 2011). Furthermore, it has been determined that happiness is related to psychological factors such as, hope (Çankaya & Meydan, 2018; Namdar, 2018); stress (King et al., 2014; Monfort, Stroup & Waugh, 2015; Tan et al., 2019); anxiety (Dilmaç & Baş, 2019; Feta, 2019; Crego et al., 2021; Steinmayr et al., 2016; Milić et al., 2019; Takebayashi et al., 2018; Wasil et al., 2021); satisfaction of basic and psychological needs (Eryılmaz & Atak, 2011; İlhan & Özbay, 2010; Telef & Ergün, 2013; Türkdöğän & Duru, 2012). Happiness is also related to social factors: loneliness (Baltacı, 2019; Traş, Öztemel & Koçak, 2020; Yavuz, 2019); interpersonal relationship (Baytemir, 2019; Köse, 2015; Xu & Huang, 2021); social anxiety (Baltacı, 2019; Dilmaç & Baş, 2019; Maričić & Štambuk, 2015; Son & Kim, 2020), social appearance anxiety (Seki & Dilmaç 2015); social talents (Canbay, 2010; Telef & Ergün, 2013)

and family (Demir, 2020a; Mertoğlu, 2020). Besides, there is a relationship between happiness and healthy life (Lin, Pan & Yi, 2019; Mahon, Yarcheski & Yarcheski, 2005; Mehrabi, Ghazavi & Shahgholian, 2017). Therefore, when the aim is to develop individuals' cognitive, social, and affective skills in schools (Waters, 2011), it shouldn't be indifferent to students' happiness. In fact, governments transfer an important part of their budgets to education for the happiness of society. Families shape their life-style according to their children's education and spend most of their savings on their education. However, people shouldn't neglect their happiness at the moment while they about the future happiness of their children. Considering students' spending most of their life at school, their happiness shouldn't be ignored. Some thinkers consider that people concentrate on academic success, and happiness is neglected so it affects students' development negatively (Guilherme & de Freitas, 2017; Noddings, 2003; White, 2011).

The schools are the places where planned programs of education are achieved and where students are required to have the knowledge, emotions, and behaviours though planned actions and experiences (Demirel, 2021). Students' volunteer joining the social and educational activities is very important to achieve the targets which are needed to be acquired. Therefore, it can be said that attitude towards school is very important in the success of educational services. Attitude towards school can be defined as a cognitive, affective and behavioural attitude, and reaction and tendency that are formed in the minds of individuals based on previous experiences, knowledge and motivations towards school (İnceoğlu, 2010). Attitudes affect the definition of behaviours (Hortaçsu, 2012). Hence, attitude towards school is the predictor of the students' behaviours towards the school. Students, who have a positive attitude towards a school, attend the social and educational activities at school voluntarily. This provides a contribution to the school to achieve its mission. A negative attitude towards the school causes a loss of motivation for school and a perception that a school is boring. This results in seeing school as a place for a source of unhappiness. Seen as a place for the source of unhappiness, school affects the students academically, socially, and psychologically in a negative direction. In studies, there is a relationship between happiness and loyalty to school (Özdemir, 2017), academic procrastination (Demir, 2020b), truancy behaviour (Gülcemal, 2019), and school burnout (Koç, 2019). The students' school experience in the growth and developmental period has the potential to affect not only the academic success of students, but also all of their life as students. Thus, the effect of the attitude towards the school (as the determinant of the behaviours towards the school) on the happiness of the students should be searched.

Negative emotions (fear, unease, sadness), which are contrary to positive emotions (joy, fun, happiness) observed in the happiness, are seen in the state of anxiety. Anxiety which has an important role in the formation of characters and behaviours of individuals means thoughts, sorrow, and sadness that causes anxiety (Kartopu, 2012; Manav, 2011; Turkish Language Society (TDK), 2022). Anxiety is an emotional state that depends on an unreasonable cause of affairs (Gall, 2012). In fact, low level of anxiety accelerates reflexes and it plays a role in protecting individuals

in case of danger by providing concentration. On the other hand, excessive anxiety causes numerous problems in individuals. In the case of anxiety, physical behaviours such as dried mouth, and swearing, and psychological behaviours: fear, unease, anger and thrilling as well as stuttering are seen (Geçtan, 2005). Hence, it can be said that there is an inverse relationship between happiness and anxiety. In literature, research studies demonstrate that there is a relationship between happiness and anxiety (Crego et al., 2021; Dilmaç & Baş, 2019; Feta, 2019; Steinmayr et al., 2016; Milić et al., 2019; Takebayashi et al., 2018; Wasil et al., 2021), social anxiety (Baltacı, 2019; Dilmaç and Baş, 2019; Maričić & Štambuk, 2015; Seki & Dilmaç 2015; Son & Kim, 2020) and exam anxiety (Feta, 2019; Steinmayr et al., 2016). When the students need to do the duties about math, the feelings such as sadness, hopelessness, stress, and fear as well as displeasure (Ma&Hu, 2004) and the lack of math performance (Bayırlı, Geçici & Erdem, 2021; Kesici & Aşlıoğlu, 2017; Ma, 1999) come from math anxiety that affects the students' happiness.

The physical structure of the school, and the communication and relationships among teacher, students, administrator and parents affect the students' attitudes towards school (Adıgüzel, 2012). Attitudes towards school develop depending on the experiences (İnceoğlu, 2010). Therefore, lessons at school can also affect the attitudes towards school. Math lesson is a very important part of the school programme. The subjects of mathematics are abstract objects. For this reason, math is perceived as a difficult lesson due to its nature. Considering the attention paid to math, math could be effective in the formation of the attitudes towards school. Math anxiety, which results from students' negative experiences about math, causes negative feelings such as inadequacy, anxiety, decline in success, guilt and shame. Furthermore, math anxiety can affect the whole life, leading to choices where mathematics is absent or less involved in the selection of a school and profession (Kesici & Aşlıoğlu, 2017; Ma, 1999; Namkung, Peng & Lin, 2019). The feeling of anxiety and tension which prevents the manipulation of numbers and solving of math problems (Richardson & Suinn, 1972) and the math anxiety which Newstead (1998) defines as a feeling of fear, tension and restlessness that affects the mathematics performance may be related to the attitude towards school. Accordingly, the interaction of mathematics anxiety and attitude towards school, which are related to each other and thought to have an effect on happiness separately, may also affect student happiness.

The search of happiness is a humanistic and existential state. Education aims to enable individuals to actualize themselves and maintain social order and peace by improving their cognitive, social, and psychological aspects. Considering the nature of humans, education needs to achieve educational targets. Therefore, educators pay attention to students' happiness. This study which aims to investigate the attitude towards school and math anxiety on students' happiness answers these research questions below:

1. Is there an effect of the attitude towards the school on the students' happiness?
2. Does math anxiety affect student happiness?

3. Is there a moderating role of math anxiety in the effect of the attitude towards the school on the students' happiness?

As it is thought that taking into consideration the students' happiness could help in using the labour and capital effectively and efficiently, this research study may contribute to determining the educational policy and preparing the educational plans, programmes and implications about education.

## METHOD

### Research Design

The aim of this study is to investigate the moderating role of math anxiety in the effect of the attitude towards school on the students' happiness. Correctional survey model is used to determine the existence and degree of change among variables (Karasar, 2014).

### Population and Sample

Participants of this study were 8<sup>th</sup> grade students who were aged at 14 were in secondary schools in Siirt Province, Turkey in the semester of 2021-2022, 415 students at the 8th grade from 6 secondary schools were selected by the cluster sampling method. 195 students are males and 220 are females.

### Data collection instruments

*Scale for Attitude Towards School:* To determine the students' attitude towards the school, "The Scale for Attitude Towards School" which was developed by Alici (2013) was used. The scale has 20 items with 5-point Likert-type. The scale explains 51.67% of variance. Cronbach Alpha coefficient is calculated as .907. There are three sub-dimensions of scale: "school as a barrier in personal development", "supports for personal development" and "school as a missing place". In this study, Cronbach Alpha coefficient of "The Scale for Attitude Towards School" is calculated as .925.

*Math Anxiety Scale:* Students' math anxiety was determined by using the "Math Anxiety Scale" which was developed by Bindak (2005). The scale has 10 items with 5-point Likert-type. The scale has only one dimension and Cronbach Alpha coefficient is calculated as .84. The scale explains 51.7% of variance. In this study, Cronbach Alpha coefficient is calculated as .905 for Math anxiety scale.

*Adolescent Happiness Scale:* To determine the level of the students' happiness, the "Adolescent Happiness Scale" developed by Işık and Atalay (2019) was used. Based on the students' self-evaluation, the scale, which explains their cognitive and affective judgments, and life satisfaction, has only one dimension and 15 items with a 5-point Likert-type. The scale explains 50.41% of variance and Cronbach Alpha value is calculated as .91. For this study, Cronbach Alpha coefficient of Adolescent happiness scale is calculated as .93.

## Collecting Data

The data were collected in the lesson by students' teachers. The teachers working at those schools were informed about the study. Teachers were also asked to inform their students about the study and involve the volunteer students in the study. Additionally, the ethical approval was taken from Siirt University Ethics Committee for this study (Date: 29.03.2022, Number: 2442).

## Analysis of Data

The collected data were analysed by using SPSS programme. First of all, mean scores of attitudes towards school, math anxiety, happiness were converted into z scores and 4 outlier values which were not between the levels of (-3, +3) were identified. These the data set were removed from the study. After that, skewness and kurtosis coefficients for the remaining 411 data were identified between the levels of (-1,+1). Thus, it was determined that data provides the normal distribution. In this study, descriptive statistics (mean, standard deviation, kurtosis and skewness) were used in the analysis of the data. Furthermore, it was determined that there is a linear relationship among variables by using the correlation analysis and the level of the relationship among variables do not cause a multicollinearity problem ( $r < .80$ ) (Can, 2014; Büyüköztürk, 2011). The moderating role of math anxiety in the effect of the attitude towards school on the students' happiness was analysed by using SPSS Process Macro extension. Moderation analysis is used to determine in which situations the relationship between two variables increase, decrease or change directions (Bayram, 2016; Gürbüz, 2021).

## RESULTS

In this study, mean scores of the attitude towards school, math anxiety, happiness level of participants, standard deviation, skewness and kurtosis coefficients were calculated. To determine the level of the relationship among variables, Pearson correlation coefficients were calculated through the correlation analysis. Results are shown in Table 1.

	N	Mean	Sd.	Skewness	Kurtosis	(1)	(2)	(3)
Attitude Towards School (1)	411	3.90	.78	-.46	-.72	1		
Math Anxiety (2)	411	2.62	1.04	.19	-.86	-.38**	1	
Happiness (3)	411	3.47	.95	-.26	-.72	.45**	-.47**	1

\*\*  $p < .01$

Table 1. Descriptive statistics and correlation coefficients

As seen in Table 1, the mean scores of participants' attitude towards school, math anxiety, happiness were calculated as 3.90 (sd.= .78), 2.62 (sd. = 1.04), 3.47 (sd. = .95) over 5.00 respectively. Kurtosis and skewness coefficients were between -1 and +1 values. There is a positive, and statistically significant relationship between attitude towards school and happiness ( $r=.45$ ;  $p<0.01$ ). There is a negative, significant relationship between attitude towards school and math anxiety ( $r=-.38$ ;  $p<0.01$ ). Also, there is a negative, and significant relationship between happiness and math anxiety ( $r=-.47$ ;  $p<0.01$ ).

To determine the moderating role of math anxiety in the effect of attitude towards school on students' happiness, regression analysis was computed. Before regression analysis, values of estimated variable (attitude towards school) and moderator variable (math anxiety) are standardized. Based on the regression analysis, model statistically and significantly explains 30% of happiness of students ( $R^2=.298$ ;  $F=57.71$ ;  $p<.001$ ). Regression analysis results are shown in Table 2.

	$\beta$	S.E.	$t$	$p$	Confidence Interval	
					Lower Bound	Upper Bound
Constant	3.43	.04	81.35	<.001	3.35	3.51
Attitude Towards School	.34	.04	7.85	<.001	.25	.42
Math Anxiety	-.29	.04	-6.71	<.001	-.37	-.20
Interaction (moderating effect)	-.12	.04	-3.25	<.002	-.20	-.05

$\beta$ : Unstandardized regression coefficients; S.E.: Standard error.

Table 2. Results of regression analysis

As seen Table 2, attitude towards school has a positive and significant effect on students' happiness ( $\beta= .34$ ;  $p<.001$ ). Math anxiety affects students' happiness in a negative direction and significantly ( $\beta=-.29$ ;  $p<.001$ ). Attitude towards school and math anxiety's common interaction (moderating effect) affect students' happiness in a negative direction and significantly ( $\beta= -.12$ ;  $p<.001$ ). Therefore, study shows the moderating role of math anxiety in the effect of attitude towards school on students' happiness. In order to analyse the moderating effect in depth, slopes analysis was calculated. In case of low, medium, and high level of math anxiety, the effect of the attitude towards school on students' happiness is presented as follows in Table 3.

Predictor Variable	Dependent Variable	Level of Math Anxiety	$\beta$	S.E.	$t$	$p$	Confidence Interval	
							Lower Bound	Upper Bound
Attitude Towards School	→ Happiness	Low Level	.48	.06	7.43	<.001	.35	.61
		Medium Level	.34	.04	7.90	<.001	.25	.43
		High Level	.20	.06	3.50	<.002	.10	.31

$\beta$ : Unstandardized regression coefficients; S.E.: Standard error

Table 3. Regression analysis which was done to show moderating role of math anxiety in the effect of attitude towards school on students' happiness

As seen table 3, there is a positive and significant effect of attitude towards school on students' happiness in case there is low, medium, and high level of math anxiety. In case of a low level of math anxiety, the effect of attitude towards school on students' happiness is  $\beta=.48$  ( $p<.001$ ). In case of a medium level of math anxiety, the effect of attitude towards school on students' happiness is  $\beta=.34$  ( $p<.001$ ). And, the effect of attitude towards school on students' happiness is  $\beta=.20$  ( $p<.002$ ) in case there is a high level of math anxiety. Moderating role of math anxiety in the effect of attitude towards school on students' happiness is shown in the graphic in the Figure 1.

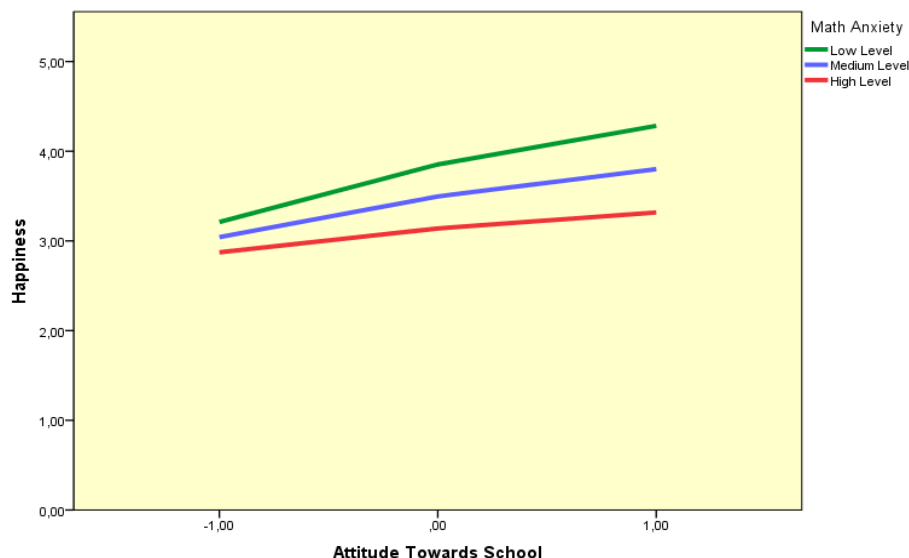


Figure 1. Graphic illustration of moderating role of math anxiety



As seen in Figure 1, when math anxiety is at low level, the effects of attitude towards school on students' happiness are higher than medium and high level of math anxiety. When math anxiety is at medium level, the effects of attitude towards school on students' happiness are more effective than the high level of math anxiety. Math anxiety changes the attitudes towards school on the students' happiness. It was determined that math anxiety has lessened the positive effects of attitudes towards school on students' happiness. This shows the moderating role of math anxiety.

## CONCLUSION, DISCUSSION AND SUGGESTIONS

This study was conducted to examine the direct and common interaction effects of attitude towards school and math anxiety on students' happiness. The model which was created to explain the effects of attitude towards school and math anxiety on students' happiness significantly explains approximately 30% of changes in student happiness. The study shows that students' attitudes towards school statistically affect their happiness in a positive and significant way. As a consequence of this, when students' attitude towards school increases, their happiness level will increase.

There are some studies in the literature that show that different factors about school are related to happiness and which the findings of this study. It was determined that there is a positive relationship between happiness and school satisfaction (Chen & Lu, 2009; Long et al., 2012) and school alienation affects students' happiness negatively (Natvig, Albrektsen & Qvarnström, 2003), and there is a positive relationship between loyalty to school and happiness (Bora & Altınok, 2021; Vuorinen, Hietajärvi, & Uusitalo, 2021). Furthermore, it was found that there is an invert relationship between school happiness and school burnout (Akpınar, 2016; Aypay & Eryılmaz, 2011; Özhan & Yüksel, 2021) and good school atmosphere has a positive effect on students' happiness (Asıcı & İkiz, Aldridge et al., 2016; 2019; Suldo et al., 2013). As a result of the effects of the attitude towards the school on students' happiness in this study, these findings are essential in the student-centred education today. Today, education accepts the approach of the constructivist paradigm as it is more relevant to the nature of learning. Therefore, it is seen that such applications as multiple intelligences, problem-solving, and cooperation approaches and techniques which put the student in the centre are used more in education (Kesici, 2019). In addition to student-centred cognitive approaches which have harmony with human nature, considering the affective aspects for students' happiness are also effective and efficient in education. In educational planning and implications, the aim of happiness should be addressed as the necessity of human nature. To make students happy, the social and educational activities should be more for students so that they could develop their autonomy, establish positive social relationships with their environment and be aware of their talents and limits (İme & Öztosun, 2020). Therefore, schools should be turned into places where students can develop their hobbies. In doing so, students' self-actualization can be supported. In order to help students be happy, their attitudes towards school should be improved. Positive attitude towards school has a positive effect on academic success. Activities that students love should be decided and suitable ones should be employed. Establishing healthy social

relationships with surroundings affects students' happiness. To develop social skills, workshops should be done and their social development should be followed.

The study reveals that math anxiety affects students' happiness negatively. When math anxiety is high, students' happiness will be affected negatively. This finding can be explained as the negative emotions that math anxiety creates in students affect happiness negatively, because these negative emotions are the opposite of the emotions that are felt at the time of happiness. This finding is consistent with the findings that there are inverse relationships between happiness and anxiety (Crego et al., 2021; Dilmaç & Baş, 2019; Feta, 2019; Milić et al., 2019; Steinmayr et al., 2016; Takebayashi et al., 2018; Wasil et al., 2021), social anxiety (Baltacı, 2019; Dilmaç & Baş, 2019; Seki & Dilmaç 2015; Son & Kim, 2020; Maričić & Štambuk, 2015), and exam anxiety (Feta, 2019; Steinmayr et al., 2016). (Wasil et al., 2021; Crego et al. 2021; Steinmayr, Crede, McElvany & Wirthwein, 2016; Feta, 2019; Milić et al., 2019; Takebayashi, Tanaka, Sugiura & Sugiura, 2018; Dilmaç & Baş, 2019), social anxiety (Baltacı, 2019; Son & Kim, 2020; Maričić & Štambuk, 2015; Dilmaç & Baş, 2019; Seki & Dilmaç 2015), test anxiety (Steinmayr, Crede, McElvany & Wirthwein, 2016; Feta, 2019) in the literature.

This study shows the moderating role of math anxiety in the effects of attitude towards school on students' happiness. In the case of low math anxiety, the positive effects of positive attitude towards school on students' happiness are higher than medium and high levels of math anxiety. In the case of medium math anxiety, the positive effects of positive attitude towards school on students' happiness are higher than in the case of the high level of math anxiety. Math anxiety decreases the positive effects of positive attitudes towards school on students' happiness and this shows the moderating role of math anxiety.

In the study, it was defined that there is an inverse relationship between math anxiety and attitude towards school, and math anxiety is a predictor that affects attitude towards school negatively. Researchers who have studied math education pay attention to math anxiety. In meta-analysis studies, there is an inverse and meaningful relationship between math anxiety, math performance (Zhang, Zhao & Kong, 2019) and math success (Barroso et al., 2021; Bayırlı, Geçici & Erdem, 2021). Therefore, math anxiety is an important problem for math teaching. However, the study reveals that besides the math lesson, math anxiety affects various things such as attitude towards school and students' happiness. Therefore, the solution to the problem is important not only for an individual's math success but also for his/her whole education. According to Namkung, Peng and Lin (2019), math anxiety can cause low math success. And, low math success can cause math anxiety. However, both math anxiety and math success can cause the formation of a circle that affects each other. If this circle is not broken, there can be negative results. For this reason, it should be put into an effort to break the circle between performance and anxiety. In this regard, it should be taken measures to develop math performance of students who have low math performance. To lessen the negative effects of anxiety, guidance should be offered to students who have the high math anxiety.

This study was conducted with 8<sup>th</sup> grade students at secondary schools. In Turkey, the high-school entrance exam is organized by central mechanisms. Families, school managements, and teachers take this exam seriously to the extent that it can pressurize students. As a result of this, the condition which renders the sample of the research specific might be considered as the limitation of this study. In further research, research studies, which aim to examine the happiness of students at different levels and classes, could be conducted. This study investigates the effects of the attitude towards school and math anxiety on students' happiness. There can be various factors that affect students' happiness with regard to school. There could be numerous school-related factors that have an effect on students' happiness. Thus, further studies, which aim to investigate the school-related factors that affect students' happiness, can contribute to the field of study.

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## Appendix: Items of the scales

### *Scale for Attitude Towards School*

- |   |   |
|---|---|
| 1. Okula gitmek benim için işkence gibi.                        | 11. Okulda öğretilenler bana çok şey katıyor.                         |
| 2. Elimden geldiğince okuldan kaçırım.                          | 12. Okul, psikolojik ve sosyal gelişimi destekler.                    |
| 3. Okul denince hep soğuk duvarları olan bir bina düşünürüm.    | 13. Okulsuz bir eğitim düşünemiyorum.                                 |
| 4. Okulda olmaya katlanamıyorum.                                | 14. Okulsuz bir toplum düşünemiyorum.                                 |
| 5. Okul olmasa daha eğlenceli bir çocukluk geçiririm/ geçirdim. | 15. Okulda benim için yararlı olduğunu düşündüğüm şeyler öğreniyorum. |
| 6. Okulda kendimi hapsedememiş gibi hissedirim.                 | 16. Okul, bilgi ve becerilerimi arttırdığı yerdir.                    |
| 7. Diplomaya ihtiyacım olmasa okula gitmek istemezdim.          | 17. Her gün aynı heyecanla okula giderim.                             |
| 8. Okulların kapatılması gerektiğini düşünüyorum.               | 18. Keşke okullar hiç kapanmasa.                                      |
| 9. Okulun kişisel gelişimim için önemli olduğunu düşünüyorum    | 19. Tatilde okulun açılmasını dört gözle beklerim.                    |
| 10. Okul, zihinsel ve bedensel gelişimi destekler.              | 20. Okula gittiğimde saatlerin nasıl geçtiğini anlamam.               |

### *Math Anxiety Scale*

- |  |   |
|--|---|
| 1. Matematik denince aklıma karmaşık, anlaşılmaz şeyler gelir.               | 6. Matematik yüzünden sınıfımı geçemeyeceğinden korkuyorum.           |
| 2. Matematik derslerinde tahtaya kalkmak bana zor geliyor.                   | 7. Matematik dersine girdiğimde kendimi korkudan büzülmüş hissedirim. |
| 3. Matematik derslerinde bana daima soru sorulacağından endişelenirim.       | 8. Matematik sınavlarına nasıl çalışacağımı bilemiyorum.              |
| 4. Şimdi matematik anlıyorum fakat giderek zor olacağından endişe duyuyorum. | 9. Benim için matematik çok eğlencelidir.                             |
| 5. Matematik sınavlarından korktuğum kadar diğer hiçbir şeyden korkmam.      | 10. Matematik dersinde soru sormaktan korkuyorum.                     |

### *Adolescent Happiness Scale*

- |   |  |
|---|--|
| 1. Hayatın tadını çıkaran biriyim.                  | 9. Her ne olursa olsun hayatımın eğlenceli olduğunu düşünürüm. |
| 2. Yapmaktan keyif aldığım etkinlikler var.         | 10. Kendimi çok dinç hissediyorum.                             |
| 3. Huzurlu bir hayatım var.                         | 11. Kendimi mutlu bir insan olarak tanımlanırım.               |
| 4. Hayatımda güvenebileceğim insanlar var.          | 12. Hayatın güzel yanları üzerine odaklanırım.                 |
| 5. Kendimi güvende hissediyorum.                    | 13. Hayatım tam da istediğim gibi gidiyor.                     |
| 6. Gelecekte iyi bir yaşamım olacağını düşünüyorum. | 14. Hayatımda her şey yolunda gidiyor.                         |
| 7. Geçirdiğim her anın keyfini çıkarmaya çalışırım. | 15. Geleceğe umutla bakarım.                                   |
| 8. Mutlu olmak için pek çok sebebim var.            |  |

## Content Analysis of Students' Argumentation Based on Mathematical Literacy and Creation Ability

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*Abstract: This study investigated the argumentation of 27 junior high school students in building their creative reasoning and the relationship between students' argumentation and their mathematical literacy skills. The data collected using mathematical literacy test and assessment rubric and were analyzed using qualitative content analysis. The study found that students tended to use simple statements than complex statements in explaining their mathematical arguments. Furthermore, the study also showed that students, in constructing their arguments, used syntax to support their problem-solving reasoning structure though it does not necessarily strengthen their correct final answer. Students with developed mathematical literacy skills tended to be creative in building their arguments, where they used not only statements but also pictures to strengthen their constructed problem-solving arguments.*

### INTRODUCTION

The ability of argumentation in learning mathematics is important in building students' mathematical abilities. Arguments are at the core of scientific thinking (Cross, 2009; Hidayat et al., 2018b) and knowledge in argumentation are also important for logical understanding and effective communication (Lin, 2018). Argumentation in mathematics is an important part of the discipline of mathematics and a key indicator of mathematical competence (Graham & Lesseig, 2018). In the process of building arguments and criticizing the reasoning of others, students develop their understanding of the underlying mathematical ideas and engage in critical thinking activities (Graham & Lesseig, 2018; Yackel & Hanna, 2003).

Arguments are commonly perceived as a statement expressing a viewpoint, backed by logical reasoning (Hidayat et al., 2018b). Viewpoints (Soekisno, 2015) describe arguments as a person's justification for addressing issues, problems, and debates. Argumentation aims to provide a solution to a problem, consisting of claims supported by different principles (assurances), evidence, and counterarguments (objections) (Hidayat et al., 2018b). Arguments are considered to be a result of the reasoning process. Therefore, it can be inferred that arguments arise from the process of reasoning (Dawson & Venville, 2010; Mercier & Sperber, 2013; Soekisno, 2015).

The ability of students to present mathematical arguments is supported by their creative motivation to provide logical and mathematical explanations to solve a given problem (Walter & Barros, 2011). Mathematical creativity is crucial for the growth of mathematics as a whole. Laycock (1970) defined mathematical creativity as the ability to approach a problem from different perspectives, recognize patterns, differences, and similarities, generate multiple ideas, and choose appropriate methods to tackle unfamiliar mathematical situations.

According to Poincaré (as cited in Nadjafikhah et al., 2012), mathematical creativity involves making useful combinations of ideas while avoiding useless ones. The focus on creativity is to encourage students to not only solve problems given to them but also generate creative ideas that can be used in their solutions and provide creative reasoning (Kozlowski et al., 2019).

In this context, creative reasoning refers to the thought process used to make statements and reach conclusions while solving problems. The reasoning is not always based on formal logic and may even be incorrect as long as there are plausible reasons to support it. The term "reasoning" is used broadly in this framework to include both high-quality and low-quality arguments, and the quality of the argument is evaluated separately. The data used in this investigation are behavioral data, and any underlying thought processes can only be speculated (Vinner, 1997).

Students aim to develop their reasoning skills through their mathematical literacy knowledge. Mathematical literacy is a necessary competency for students, encompassing abilities such as communication, representation, reasoning, and problem-solving strategies, among others (OECD, 2010; Nasrullah & Baharman, 2018). Despite this, students need to improve their use of mathematical literacy to solve problems effectively. The Organization for Economic Co-operation and Development (OECD) framework identifies seven mathematical abilities that comprise mathematical literacy, including critical and creative thinking, communication, and assessment. This framework highlights the importance of mathematical literacy in contemporary society, as it (OECD, 2016) is seen as being as significant as literacy.

Despite the fact that students struggle to develop these abilities, it's important to understand how they respond to using mathematical literacy skills, which are crucial not just for completing competencies but also for problem-solving. However, utilizing mathematical literacy is a challenging task as it requires a level of knowledge and awareness to link that knowledge to real-life phenomena, even though such phenomena can be used to motivate students to learn mathematics (Sembiring & Hadi, 2008). Therefore, given problems can serve as stimuli for students to enhance their mathematical literacy skills (Eerde & Van Galen, 2019).

Everyday life phenomena and contexts are fascinating and can be used to aid students in building their knowledge. However, teachers face difficulties in teaching mathematical literacy, as many still apply mechanistic mathematics learning in their classes (Bustang, 2022). Additionally, people may not comprehend the significance of studying mathematics in everyday life because the subject matter is often dominated by mathematical formulas and modeling.

Creating mathematical models actually originates from real-life situations and contexts. This is an important aspect of mathematics education and the development of mathematical literacy skills. To foster these skills, it is essential to connect students' knowledge with various contexts from their personal lives, communities, workplaces, and sciences.

Observations of school learning activities show that students lack opportunities to develop their mathematical literacy skills. These opportunities require allocated learning time, and are related to the process of how individuals acquire mathematical knowledge (Carroll, 1963; Cogan & Schmidt, 2015). Learning opportunities include various factors that affect teacher practices and student learning, such as content coverage and emphasis (Barnard-Brak et al., 2018). Building mathematical arguments requires both creative encouragement and emphasis on content (Stevens & Grymes, 1993). However, the learning outcomes achieved do not always result in improved learning outcomes. Therefore, it is important for teachers to provide structured reinforcement through learning activities that contain exercises to develop mathematical literacy skills. The study reported in this article sought to answer the following questions:

1. What is the content of students' arguments and what does an analysis of this content as well as the syntax used inform on their reasoning, specifically their concept development as they struggle to create, and communicate meaning for their responses?
2. Is there a relationship between the content of the argument and the mathematical literacy ability of students?

## METHOD

This research is based on a descriptive qualitative method and involved 27 junior high school students from Toli-Toli City, Central Sulawesi, Indonesia. The students were provided with a mathematical literacy test that required them to provide explanations for their answers. The test was assessed using a rubric that included representation, interpretation, and argumentation as assessment indicators. For the content of the argumentation, qualitative content analysis was used to analyze the text data. Research using qualitative content analysis focuses on the characteristics of language as communication by paying attention to the content or contextual meaning of the text (Budd et al., 1967; Lindkvist, 1981; McTavish & Pirro, 1990; Tesch, 1990). Text data may be in verbal, printed, or electronic form and may be obtained from narrative responses, open-ended survey questions, interviews, focus groups, observations, or printed media such as articles, books, or manuals (Kondracki et al., 2002). Qualitative content analysis goes beyond simply counting words to intensively examine language with the aim of classifying large amounts of text into an efficient number of categories that represent the same meaning (Weber, 1990). This category can represent explicit communication or inferred communication. The aim of content analysis is “to provide knowledge and understanding of the phenomenon under study” (Downe-Wamboldt, 1992). In this article, qualitative content analysis is defined as a research method for the subjective

interpretation of text data content through a systematic classification process, coding and identifying themes or patterns.

This study conducts a content analysis by examining the argumentation keywords present in the responses provided by the students. The analysis identifies three types of keywords: simple statement keywords, complex statement keywords, and statement keywords with syntax. Simple statement keywords are the fundamental ideas that form the basis of the reasoning presented in the responses. If a keyword contains more than one important idea, it is referred to as a complex statement keyword. Additionally, some students use syntax, such as the "known-asked-answered" pattern, when constructing their arguments. If these statements contain fundamental ideas within the reasoning, they are classified as statement keywords with syntax.

## RESULTS AND DISCUSSION

This section is divided into multiple parts which include 1) issues, 2) illustrations of student responses, and 3) a compilation of key argument statements. The explanations are presented as follows.

### Problem

The image displays the Balre Masigi traditional house of Tolitoli, which is typically utilized for meetings of regional officials. Alongside the house, a garage for official vehicles is planned to be constructed, which will feature one door and one window.



Figure 1: House of Tolitoli (Balre Masigi)      Figure 2: Illustration of a garage

Using the images provided in figures 1 and 2, identify the matching picture of the garage building as seen from the back. Justify your selection.



Figure 3: Back view of a garage

**Example of students' answers**

***Student's answers with incorrect simple statements***

Based on the questions given, one of the student's answers which are included in the argument is a simple statement which is shown as follows.


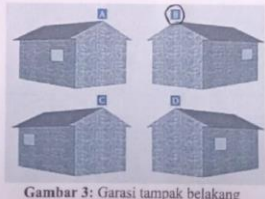
Student's Answers	Description
<p>Jika bangunan garasi tersebut nampak seperti pada gambar berikut.</p>  <p>Gambar 2: Ilustrasi Garasi</p> <p>Jika bangunan garasi tersebut digambarkan tampak dari belakang. Lingkari gambar yang sesuai dengan gambar di atas! Berikan alasanmu!</p>  <p>Gambar 3: Garasi tampak belakang</p> <p>Jawaban: karena garasi B bayangannya berlawanan arah sehingga pada di belakang</p>	<p>The student's reasoning is based on the observation that the shadow of the garage is cast in the opposite direction, indicating that the orientation of the garage is different from the house. Specifically, when viewed from behind, the garage should be on the right side of the house. Therefore, the student selected option B as the correct match.</p>

Table 1: Example of student's answers with incorrect simple statements

***Student's answers with correct simple statements***

Based on the questions given, one of the student's answers which are included as correct with arguments in the form of simple statements is shown as follows.


Student's Answers	Description
 <p>Garasi tampak dari Samping =</p> <p>Menurut Saya jawabannya c</p> <p>Karena jika Garasi tampak dari belakang Jendela yang berada di Samping Pada gambar c tentu letak akan berubah tempat</p>	<p>The student's answer argues that the correct option is C because the window on the side in picture C remains in the same position when viewed from the back. The argument is based on the location of the window, and the student first describes the position of the garage from the side to strengthen their argument.</p>

Table 2: Examples of student's answers with correct simple statements

***Student's answers with correct complex statements***

Based on the questions given, one of the student's answers which is included is correct with an argument in the form of a complex statement is shown as follows.

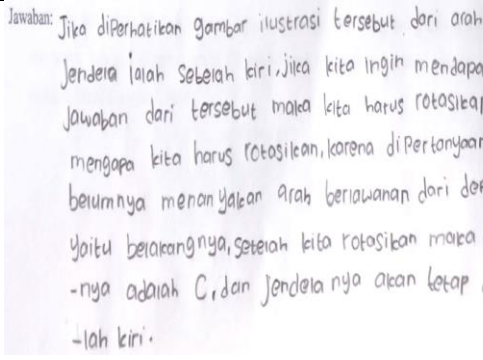
Student' Answers	Description
	<p>The student's argument in this answer involves not only the window's location but also the concept of rotation. The student notes that the previous question asked for the opposite direction from the front, which is the back. To obtain an answer, the student suggests rotating the illustration. After the rotation, the window remains on the left, leading to answer C. The concept of rotation is relevant to the problem because it addresses the requirement to consider the opposite direction.</p>

Table 3: Examples of student's answers with correct complex statements

***Student's answers with incorrect syntax statements***

Based on the questions given, one of the student's answers which was included as an argument was in the form of a statement equipped with syntax as in Table 4.

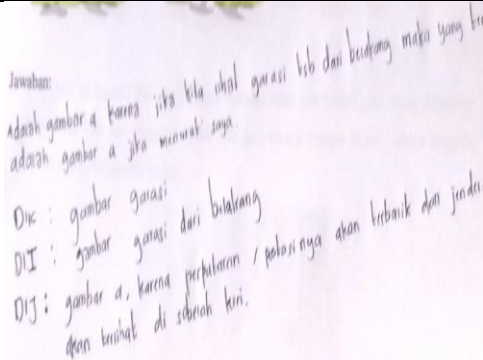
Student' Answers	Description
	<p>The student's argument in their answer is that the garage in image A is the one that would be seen when viewed from behind.</p> <p>Known: garage drawing</p> <p>Question: Picture of the garage from behind</p> <p>Solution: Picture a, because the rotation will be reversed, and the window will be visible on the left.</p> <p>The syntax used in building the argument is based on the Known, Asked, and Answered stages, which are seen as steps in preparing arguments. The argument used in this case involves a complex concept, which not only considers the window's location but also the idea of rotating or turning the object. However, the interpretation of the object's rotation does not necessarily support the correctness of the chosen answer.</p>

Table 4: Examples of student's answers with incorrect syntax statements

*Student's answers with correct syntax statements*

One of the answers provided by a student is deemed correct based on the given questions, and the argument provided is presented in the form of a statement accompanied by syntax.

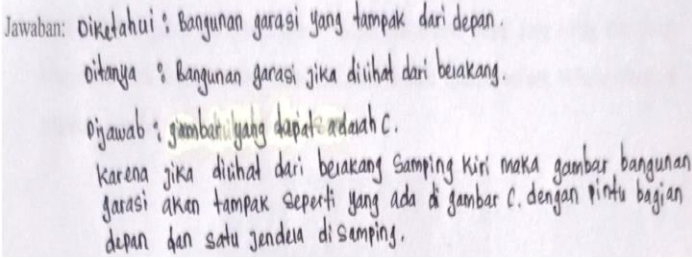
Student' Answers	Description
 <p>Jawaban: Diketahui : Bangunan garasi yang tampak dari depan. Ditanya : Bangunan garasi jika dilihat dari belakang. Dijawab : gambar yang tepat adalah C. Karena jika dilihat dari belakang samping kiri maka gambar bangunan garasi akan tampak seperti yang ada di gambar C. dengan pintu bagian depan dan satu jendela di samping.</p>	<p>Known: Garage building as seen from the front</p> <p>Question: Garage building when viewed from behind</p> <p>Solution: the right picture is C. Because if you look at it from the back on the left, the garage building will look like the one in picture c. With a front door and a side window.</p> <p>The syntax used in constructing arguments includes the stages of Known, Asked, and Answered. The argument is based on a complex concept that involves not only the window location but also the rotation of the object. However, the interpretation of the object's rotation does not support the correct answer.</p>

Table 5: Examples of student answers with correct syntax statements

**A collection of core arguments**

The study identified four groups of presentations based on the collected data: 1) answers with no argumentative statement, 2) answers with simple statement keywords, 3) answers with complex statement keywords, and 4) answers with keyword statements arranged through syntax. It is noteworthy that two students did not provide any reasoning in their answers, despite one of them getting the right answer while the other got the wrong answer. Although it is not clear how one of the students arrived at the correct answer without providing any reasoning, it is possible that their surroundings may have played a supportive role.



### Simple statement

For simple statement keywords, a collection of statements put forward by students is shown in the table as follows.

No	Statement	Respond
1	<ul style="list-style-type: none"> <li>• Place windows</li> <li>• Viewed from behind</li> <li>• Looking back</li> <li>• Position windows by direction</li> <li>• Location of windows, from the front of the window in front</li> <li>• Window position from the front and from the back</li> <li>• Window shape and location</li> <li>• A, B, D do not match if the photo is from behind</li> <li>• The location of the window determines the image from the back</li> <li>• Decisive window</li> <li>• Image C is correct when photographed from behind and photos A, B, D are not correct when photographed from behind</li> <li>• Looking at the garage from behind, windows visible from behind</li> </ul>	Correct Answer
2	<ul style="list-style-type: none"> <li>• The shadow is in the opposite direction</li> </ul>	Incorrect Answer

Table 6: Simple statement keywords

Table 6 shows that students presented 13 variations of statements in constructing their answers. One of the statements directed the students by stating "The image is in the opposite direction". However, the chosen answer was not correct or as expected, indicating that the arguments built were not in line with the context of the picture. This reflects the students' weak mathematical literacy skills, particularly in reading and communicating information to draw accurate conclusions.

According to the data in the table, there were 12 variations of statements used by students in constructing their arguments to support their chosen answer. Two forms of ideas were identified from these statements: 1) referencing the location or position of the window, and 2) testing the answer choices based on the location or position of the window. These strategies were used by students to arrive at the correct answer.

### Complex Statement

For complex statement keywords, a collection of statements put forward by students is shown in the table as follows.

No	Statement	Respond
1	<ul style="list-style-type: none"> <li>• Front view and back view</li> <li>• Window front direction, rotate, opposite direction</li> <li>• View from the rear, rotates <math>180^0</math>, the window moves to the front on the left</li> <li>• Using pictures for representation, the garage seen from behind the side window</li> </ul>	Correct Answer

Table 7: Complex statement keywords

Table 7 displays the results of student work, which revealed that all answers were correct and met the expectations. The complexity of student responses can be attributed to the concepts of orientation, rotation, and visualization used, such as considering the perspective from multiple sides, understanding direction and rotation, and considering image representation, view, and position. The ability of students to provide logical reasoning and make appropriate decisions based on the information provided demonstrates that their mathematical literacy skills vary, with some students being more proficient than others.

### Statements with Syntax

While it is not uncommon for students to use syntax when solving mathematical problems, it appears that the syntax pattern of Known, Asked, and Answered is commonly used in the mathematics learning they follow. This pattern of steps is used to solve the given problem, but only a few students use it in their answers to this particular problem. When constructing their answers, they formulate the statement in the following form.

Table 8 presents the results of analyzing student answers using the syntax approach, and it was found that one student provided an incorrect answer. Despite using the concepts of orientation and rotation, the student's answer was not presented in a clear and hierarchical manner. On the other hand, other students who provided the correct answer used a more structured and organized syntax, mainly based on the concept of orientation from two different sides. Their argumentation and reasoning were also strong and supported their answer choice.

Based on the explanation stated above, various arguments built by students involving various mathematical concepts show the progress of their way of thinking. By placing some mathematical concepts in the construction of solving these problems, it is their way of being creative to achieve the targeted goals (Poincaré, as cited in Nadjafikhah et al., 2012), such as making pictures or making an overview of how to view the garage building from various directions or positions. This combination is a creation that is used to solve a given problem, but also offers creative ideas that are used in creative arguments (Hidayat et al., 2018a).

It seems that spatial reasoning (holistic, analytic and pattern-based) (Hsi et al., 1997) was used in the construction of the solution, although analytic and pattern-based spatial reasoning was not well

developed. In general, students' ability to utilize holistic spatial reasoning can be seen from the way they interpret a given mathematical literacy problem. In other words, students can interpret the problem well if it is supported by mathematical literacy skills which also develop well following the given problem, although it was found that 2 students did not show the argumentation ability well.

No	Statement	Respond
1	<ul style="list-style-type: none"> <li>• Front view and back view (Arranged with known, asked, and answered patterns)</li> <li>• Seen from the rear left side, the picture of the garage building will look like picture C, the front door and one side window (Arranged with known, asked, and answered patterns)</li> <li>• The garage building that is visible from the back because of the location of the window from the front on the left after behind the garage it appears that the window has moved to the right because it is viewed from behind (Arranged with known, asked, and answered patterns)</li> </ul>	Correct Answer
2	<ul style="list-style-type: none"> <li>• Viewed from behind, the rotation will be reversed, and the window is visible on the left (Arranged with known, asked, and answered patterns)</li> </ul>	Incorrect Answer

Table 8: Keyword statements with syntax

However, if it is seen from what has been shown by students in their work, the ability of students to argue using complex statements is able to demonstrate the mathematical concepts used to obtain the correct answer. Meanwhile, the use of syntax in constructing answers does not ensure that students can get the correct answer as expected. In this problem, the use of simple statements is more used than complex statements and statements using syntax. The variety in constructing the answer cannot be separated from factors or dimensions or moments that do not change, such as cultural or habitual, social, and even individual factors (García et al., 2006).

Based on this review, several skill-oriented transformations toward a problem-based reform approach (Sembiring & Hadi, 2008), in the form of mathematical literacy problems are intended to highlight mathematical skills and understanding that are useful in future life so that they are used as preparation for using mathematics in learning high-level technical profession (Stacey, 2011).

## CONCLUSIONS

Based on the results of the research presented above, some conclusions are obtained as follows, 1) Students use simple statements more than complex statements in building mathematical arguments, 2) In building arguments, students use syntax to support problem solving reasoning

structures even though does not necessarily strengthen the correct final answer, and 3) Students who develop mathematical literacy skills can be creative by building their arguments not only with statements, but also through pictures to strengthen the constructed problem-solving arguments.

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## The Problem Corner

Ivan Retamoso, PhD, *The Problem Corner* Editor

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The Purpose of **The Problem Corner** is to give Students and Instructors working independently or together a chance to step out of their “comfort zone” and solve challenging problems. Rather than in the solutions alone, we are interested in methods, strategies, and original ideas following the path toward figuring out the final solutions. We also encourage our Readers to propose new problems. To submit a solution, type it in Microsoft Word, using math type or equation editor, however PDF files are also acceptable. Email your solution as an attachment to The Problem Corner Editor [iretamoso@bmcc.cuny.edu](mailto:iretamoso@bmcc.cuny.edu) stating your name, institutional affiliation, city, state, and country. Solutions to posted problem must contain detailed explanation of how the problem was solved. The best solution will be published in a future issue of MTRJ, and correct solutions will be given recognition. To propose a problem, type it in Microsoft Word, using math type or equation editor, email your proposed problem and its solution as an attachment to The Problem Corner Editor [iretamoso@bmcc.cuny.edu](mailto:iretamoso@bmcc.cuny.edu) stating your name, institutional affiliation, city, state, and country.

Greetings, fellow problem solvers!

I'm happy to share that I've obtained answers for both Problem 18 and Problem 19. I'm pleased to report that not only were all the solutions accurate, but they also demonstrated the application of effective strategies. My primary objective is to present what I consider to be the best solutions to contribute to the enhancement and elevation of mathematical knowledge within our global community.

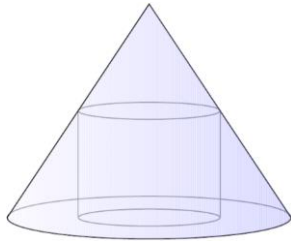
## SOLUTIONS TO PROBLEMS FROM THE PREVIOUS ISSUE

### Interesting “Cylinder inside Cone” problem

#### Problem 18

Proposed by Ivan Retamoso, BMCC, USA.

What are the dimensions of the cylinder that can be placed inside a right circular cone measuring 5.5 feet in height and having a base radius of 2 feet to maximize its volume?



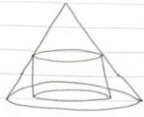
Note: Round your answers to three decimal places.

#### First solution to problem 18

By Manvinder Singh, Borough of Manhattan Community College, India.

*Our solver skillfully applies the essential relationship between the radius and height of the cylinder, along with the corresponding dimensions of the cone. This proportional connection is vital for placing the cylinder correctly inside the cone. Subsequently, our solver uses the derivative from Calculus to optimize the cylinder's volume.*





Height of the cone  $h_{\text{cone}} = 5.5$  feet

Base radius of the cone  $r_{\text{cone}} = 2$  feet

Objective  $\Rightarrow$  To find the dimensions of a cylinder (radius  $r_{\text{cyl}}$  and height  $h_{\text{cyl}}$ ) that can be placed inside the cone to maximize its volume.

Relationship b/h cone & cylinder  $\Rightarrow$

For the cylinder to fit inside the cone its dimensions must be scale with the cone's dimensions. The cylinder's height decreases as its radius increases, maintaining the slope of

the cone's side.  
Therefore,

$$h_{\text{cyl}} = h_{\text{cone}} - \frac{h_{\text{cone}}}{r_{\text{cone}}} \cdot r_{\text{cyl}}$$

Volume of the cylinder  $\Rightarrow$

The volume of the cylinder is given by the formula  $V = \pi r_{\text{cyl}}^2 h_{\text{cyl}}$ .

Substituting the expression for  $h_{\text{cyl}}$  gives:

$$V = \pi r_{\text{cyl}}^2 \left( h_{\text{cone}} - \frac{h_{\text{cone}}}{r_{\text{cone}}} r_{\text{cyl}} \right)$$

$$V = \pi r_{\text{cyl}}^2 \left( 5.5 - \frac{5.5}{2} r_{\text{cyl}} \right)$$

$$V = \pi r_{\text{cyl}}^2 (5.5 - 2.75 r_{\text{cyl}})$$

Maximizing the Volume  $\Rightarrow$

To find the maximum volume, we differentiate  $V$  with respect to  $r_{\text{cyl}}$  and set the derivative equal to zero.

$$\frac{dV}{dr_{\text{cyl}}} = \frac{d}{dr_{\text{cyl}}} \left[ \pi r_{\text{cyl}}^2 (5.5 - 2.75 r_{\text{cyl}}) \right]$$

$$\frac{dV}{dr_{\text{cyl}}} = \pi (2r_{\text{cyl}}(5.5 - 2.75 r_{\text{cyl}}) - 2.75 r_{\text{cyl}}^2)$$

$$\frac{dV}{dr_{\text{cyl}}} = \pi (11r_{\text{cyl}} - 8.25 r_{\text{cyl}}^2)$$

$$\text{Set } \frac{dV}{dr_{\text{cyl}}} = 0:$$

$$\pi (11r_{\text{cyl}} - 8.25 r_{\text{cyl}}^2) = 0$$

$$11r_{\text{cyl}} - 8.25 r_{\text{cyl}}^2 = 0$$

$$r_{\text{cyl}} (11 - 8.25 r_{\text{cyl}}) = 0$$

This gives us two solutions:

$$r_{\text{cyl}} = 0 \quad | \quad r_{\text{cyl}} = \frac{11}{8.25}$$

Solving for  $r_{\text{cyl}}$   $\Rightarrow$

$$r_{\text{cyl}} = \frac{11}{8.25} = 1.333 \text{ feet}$$

using this in the height equation:

$$h_{\text{cyl}} = 5.5 - 2.75 \times 1.333$$

$$h_{\text{cyl}} = 1.83425$$

$$h_{\text{cyl}} \approx 1.833 \text{ feet}$$

Conclusion  $\Rightarrow$

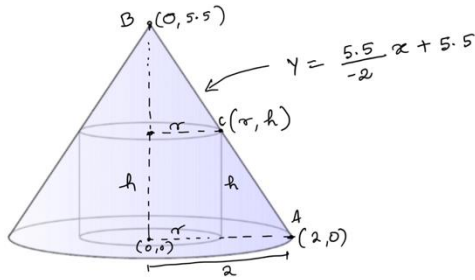
The cylinder that maximizes volume inside the given cone has a radius of approximately 1.333 feet and a height of approximately 1.833 feet.

**Second solution to problem 18**

**By Aradhana Kumari, Borough of Manhattan Community College, USA.**

*Our alternate solution is characterized by a meticulous attention to detail, a strong organizational structure, and a comprehensive justification for every step taken towards the ultimate solution. The sign of the second derivative is utilized to demonstrate that the volume of the cylinder reaches its maximum at the critical point.*

Solution: Consider the picture below.



The equation of the line passing through the points

A (2,0) and B (0,5.5) is given as  $y = \frac{5.5}{-2} x + 5.5$

Since the point C (r, h) lies in the above line we have:

$$h = \frac{5.5}{-2} r + 5.5$$

$$h = -2.75 r + 5.5 \dots\dots\dots(\text{eq 1})$$

The Volume  $V$  of a cylinder with radius  $r$  and height  $h$  is given as

$$V = \pi r^2 h$$

Substituting the value of  $h$  in the formula for volume of cylinder we get

$$V = \pi r^2 (-2.75 r + 5.5)$$

$$V = -2.75 \pi r^3 + 5.5 \pi r^2 \dots\dots\dots(\text{eq 2})$$

We differentiate equation given by (eq 2) with respect to  $r$  we get

$$\frac{dv}{dr} = -2.75 \pi (3r^2) + 5.5 \pi (2r) \dots(\text{eq 3})$$

For Maxima or minima, we have  $\frac{dv}{dr} = 0$

$$\text{i.e } -2.75 \pi (3r^2) + 5.5 \pi (2r) = 0$$

$$\pi r [-2.75 3r + 11] = 0$$

Therefore, we have  $\pi r = 0$  or  $[-2.75 3r + 11] = 0$

$$\text{Since } r \neq 0 \text{ we have } r = \frac{11}{8.25} \approx 1.33$$

We differentiate equation given by (eq 3) we get

$$\frac{d^2v}{dr^2} = -2.75 \pi (6r) + 5.5 \pi (2) \dots\dots(\text{eq 4})$$

Substituting the value of  $r = \frac{11}{8.25}$  in the equation given by (eq 4) we get

$$\begin{aligned} \frac{d^2v}{dr^2} &= -2.75 \pi (6 \times \frac{11}{8.25}) + 5.5 \pi (2) \\ &= -22\pi + 11 \pi \\ &= -11\pi < 0 \end{aligned}$$

Hence  $r = \frac{11}{8.25}$  is a point of maxima.

Substituting the value of  $r = \frac{11}{8.25}$  in the equation given by  $h = -2.75 r + 5.5$

$$\text{We get } h = -2.75 \left(\frac{11}{8.25}\right) + 5.5 \approx 1.83$$

Therefore, radius of the required cylinder is  $r = \frac{11}{8.25} \approx 1.33 \text{ ft}$

Height of the required cylinder is  $h = -2.75 \left(\frac{11}{8.25}\right) + 5.5 \approx 1.83 \text{ ft}$

**“Largest cord in a circle” problem.**

**Problem 19**

Proposed by Dr. Michael W. Ecker, (retired) Pennsylvania State University, USA.

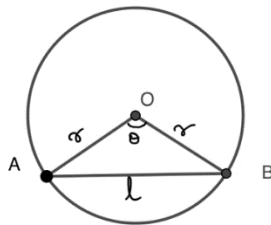
Prove that the diameter of a circle is the largest possible size of a chord of said circle.

### First solution to problem 19

By Aradhana Kumari, Borough of Manhattan Community College, USA.

Without loss of generality, our solver cleverly positions a cord of the circle with arbitrary length “horizontally” and finds its length in terms of a central angle and the radius of the circle. Finally, using the derivative from Calculus our solver maximizes the length of the cord, showing that it is indeed equal to the diameter of the circle.

Solution: Consider below Circle C with center O and radius r. Let AB be a chord of length l.



Let  $\angle AOB$  be  $\theta$

Using Cosine rule we have:

$$l^2 = r^2 + r^2 - 2r^2 \cos \theta, \quad 0 < \theta < 360^\circ$$

$$l = \sqrt{r^2 + r^2 - 2r^2 \cos \theta} = \sqrt{2r^2 - 2r^2 \cos \theta} = \sqrt{2r^2(1 - \cos \theta)} = \sqrt{2r^2} \sqrt{(1 - \cos \theta)}$$

Differentiate both side with respect to  $\theta$  we get

$$\frac{dl}{d\theta} = \sqrt{2r^2} \times \frac{1}{2} \times (1 - \cos \theta)^{-1/2} \sin \theta$$

For maxima and minima, we equate  $\frac{dl}{d\theta} = 0$

$$\sqrt{2r^2} \times \frac{1}{2} \times (1 - \cos \theta)^{-\frac{1}{2}} \sin \theta = 0$$

$$\sqrt{2r^2} \times \frac{1}{2} \frac{\sin \theta}{\sqrt{(1 - \cos \theta)}} = 0$$

Hence  $\sin \theta = 0$ ,

$$\theta = 180^\circ$$

$$\frac{d^2l}{d\theta^2} = \frac{d}{d\theta} \left( \sqrt{2r^2} \times \frac{1}{2} \times (1 - \cos \theta)^{-\frac{1}{2}} \sin \theta \right)$$

$$= \frac{\sqrt{2r^2}}{2} \left( (1 - \cos \theta)^{-\frac{1}{2}} \cos \theta + \left( \sin \theta \left( \frac{-1}{2} (1 - \cos \theta)^{-3/2} \sin \theta \right) \right) \right)$$

When  $\theta = 180^\circ$  we get

$$\begin{aligned} \frac{d^2l}{d\theta^2} &= \frac{\sqrt{2r^2}}{2} \left( (1 - \cos 180^\circ)^{-\frac{1}{2}} \cos 180^\circ \right. \\ &\quad \left. + \left( \sin 180^\circ \left( \frac{-1}{2} (1 - \cos 180^\circ)^{-3/2} \sin 180^\circ \right) \right) \right) \\ &= \frac{\sqrt{2r^2}}{2} \left( (1 - \cos 180^\circ)^{-\frac{1}{2}} \cos 180^\circ \right) \\ &= \frac{\sqrt{2r^2}}{2} 2^{-1/2} (-1) < 0 \quad (r > 0) \\ \frac{d^2l}{d\theta^2} &< 0 \end{aligned}$$

Hence  $\theta = 180^\circ$  is a point of maxima.

Substituting the value  $\theta = 180^\circ$  in below equation we get

$$\begin{aligned} l &= \sqrt{r^2 + r^2 - 2r^2 \cos 180^\circ} \\ l &= \sqrt{r^2 + r^2 + 2r^2} \\ &= \sqrt{4r^2} \\ &= 2r \\ &= \text{diameter of the Circle C.} \end{aligned}$$

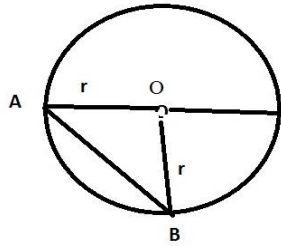
Hence diameter of the circle is the largest possible chord of said circle.

### Second solution to problem 19

**By Dr. Michael W. Ecker (The proposer)**  
**(retired) Pennsylvania State University, USA.**

*The proposer's solution takes a distinct approach, omitting the use of Calculus. Instead, it capitalizes on an essential condition regarding the lengths of a triangle's sides, specifically, that the length of one side cannot be greater than the sum of the lengths of the other two sides.*

Typical chord  $AB$  is shown in circle  $O$ . If  $AB$  is not a diameter, then drawing radii  $AO$  and  $BO$  results in a triangle,  $AOB$ . The length of  $AB$  then is smaller than the sum of the lengths of the two other sides of triangle  $AOB$ . Those two sides have a total length of twice the radius, or  $2r = d$ .



Hence,  $AB < d$ , as claimed. (Note: It does not matter how you draw AB. It's the argument, the proof, that matters here.)

## NEW PROBLEMS

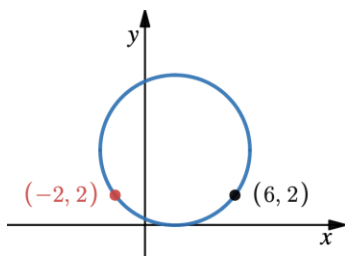
*Dear fellow problem solvers,*

*I am confident that the resolution of problems 18 and 19 not only provided you with enjoyment but also granted valuable insights. Now, let's progress to the next two problems to continue this journey of exploration and learning.*

### Problem 20

Proposed by Ivan Retamoso, BMCC, USA.

Find the radius and the equation of the circle shown below.



### Problem 21

Proposed by Ivan Retamoso, BMCC, USA.

Solve the equation below to find all real numbers  $x$  that satisfy:

$$\frac{8^x + 27^x}{12^x + 18^x} = \frac{7}{6}$$