Students' Experience in Learning Trigonometry in High School Mathematics: A Phenomenological Study

Sandip Dhungana[1], Binod Prasad Pant[1] and Niroj Dahal[1]*

[1]Kathmandu University School of Education, Department of STEAM Education, Lalitpur, Nepal

*Corresponding Author: niroj@kusoed.edu.np

Abstract*: Trigonometry is an area of mathematics that students believe to be particularly difficult and/or abstract compared to other areas of mathematics. It is introduced as the concept in the right-angled triangle from the basic level, but the curriculum of Nepal introduced it from Grade nine onwards as a separate chapter. Its content area has a weighting of nearly one-fourth of the part in additional mathematics and around 10% in compulsory mathematics. This study aims to explore students' experiences in learning trigonometry. The data are extracted from twelve grade ten students who had chosen additional mathematics in their optional courses. Formal and informal interviews, diagnostic tests, and in-depth engagement with students in their classrooms were the major sources of data extraction. In this sense, this study adopted phenomenology as a methodological stance. The data collected from the diagnostic test were analyzed, and students' explanations of each question were discussed in this study. In doing so, this study concludes with some major findings. Students have difficulty learning trigonometry and have misconceptions about the basic concepts, producing obstacles and errors in solving trigonometric problems. The possible errors are in procedural knowledge, conceptual knowledge, or link between these two types of knowledge. It is also found that a teacher needs to incorporate the learners' everyday experiences using materials, diagrams, and equipment for meaningful learning and long-lasting knowledge. A teacher needs to be aware and responsible for students' activities inside the classroom. Healthy relationships between the teacher and the students can significantly contribute to learning trigonometry.*

Keywords: trigonometry, students' experience in trigonometry, engagement, long-lasting knowledge

INTRODUCTION

Upon reflecting on Sandip's journey in learning trigonometry, he used to belong in the category who wasn't likely found of trigonometry at the school level. Although being counted in the list of so-called good students in mathematics because of Sandip's high achieving marks/grades as compared with his classmates; trigonometry was always the most difficult area to learn. The abstract concepts, formulas, axioms, and theorems required in solving trigonometry were found straightforward in those days as we (Sandip including Sandip's friends) used to memorize most of the concepts easily and quickly. Sandip remembers his mathematics teacher who continuously repeats, "*Trigonometry is like a game of formulas unless you memorize everything in trigonometry, don't even dare to solve it*". Though as a learner in the Nepali context, where mathematics subject is considered a difficult subject (Luitel & Pant, 2019), Sandip was found to be solving problems in mathematics in those days, and normally he was much more comfortable in solving routine-based mathematics and trigonometry problems. Likewise, Sandip believes that "problem-solving ability is one of the soft skills that students should have in learning mathematics" (Nurmeidina, & Rafidiyah, 2019, p. 2). As the importance of mathematics cannot be separated from our real life, one cannot deny its importance in studying at the school level.

Next, school-level education (up to grade ten in Nepal) is considered mathematics as compulsory. The area of trigonometry as a chapter is introduced in grade nine in compulsory mathematics, though the concept of right-angled triangle and solving a right-angled triangle is also introduced in lower grades (e.g., grade seven and eight). In the case of secondary mathematics, "contents of trigonometry are taught in a compulsory mathematics course to some extent and in detail in the optional courses of mathematics" (Adhikari & Subedi, 2021, p. 90) in the Nepali school curriculum and is useful in solving mathematical problems related to height and distance, solving right-angled triangles, dealing with sine, cosine and tangent functions, measurement of angles in different measurements (degree, grade, and radian), trigonometric proof, among others.

Further, trigonometry has been considered a tough area for the majority of students; they struggle to understand the concept of trigonometry and its abstract nature (Gur, 2009; Tyata et al., 2021). The problems which Sandip had faced (as a first author) in learning trigonometry in school days were still there when Sandip started teaching at the same level. As a teacher, Sandip experienced that the problems were not only for the students but also for the teacher as they were also stuck in some points and steps in solving/proving trigonometric identities. Even we also experienced the same in our time at schooling on those days when we got trouble proving some problems in trigonometry and told students, *"I will solve it by tomorrow; please solve the next question",* and many times at the beginning of teaching career, Sandip solved the trigonometry questions first in the student's copy and later on the board. We found the problems in trigonometry which Sandip encountered as a student are not only the problems that Sandip encountered but the majority of the Nepali learners experienced in a similar pattern. For instance, many of our students also shared similar kinds of experiences in the earlier career of our teaching of mathematics it may be because of adopting a similar strategy to how trigonometry was taught, which can be considered as "teaching as a transmission of knowledge" (Pant, 2017, p. 16). At this stage, we found the delivery of knowledge in Nepali context from the teacher to his/her students is like transferring data from one electronic device to another (Dhungana, 2021). It will still be prolonged until teaching techniques won't improve.

Based on Sandip's experience, high school trigonometry is a problematic and detached area of mathematics from practical life in our context (Adhikari & Subedi, 2021, Asomah et al., 2023). Many teachers and educators might have tried different teaching methods and programs to familiarize students with trigonometry concepts, sometimes with success (Usman & Hussaini, 2017; Adhikari & Subedi, 2021, Kamber & Takaci, 2017). Arriving at this stage, this article explores secondary-level students' experiences in learning trigonometry. All of the above in responding to Sandip's experiences as a researcher, teacher, and student in mathematics education, we found most of the students in his context feel mathematics is a difficult subject. As a first author, Sandip, when involved himself in a deep investigation to find out the reason behind this difficulty, we found that many of the students feel difficulty in the chapters namely, geometry, algebra, trigonometry, and vector (to name but a few). While doing so, we also experienced most of the student's feeling difficulty in the geometry section (Adhikari & Subedi, 2021; Aminudin et al., 2019). Many of them had to repeat the same difficulty we faced as a student in our school life. They also have illusions about trigonometry's terminologies and get confused about the strategy for solving trigonometric problems (Tyata et al., 2021).

More so, trigonometry is considered the foundation of higher mathematics which connects peoples' day-to-day life with mathematics. The importance of trigonometry in the field of Arts and Engineering is unavoidable. Knowingly or unknowingly, the concepts of trigonometry are making people's lives easier, either in the form of carpentering or construction works or finding the height and distance without measuring the actual height. But the scenario of trigonometry learning is just the opposite of its application in our practices. Students are less motivated in learning trigonometry and are not able to show their conceptual understanding of it (Fauziyah et al., 2021, Tyata et al., 2021; Wilson et al., 2005). Orthodox pedagogies, the setting of classrooms, and the belief of teachers and students that grade points are everything could be a few examples that contribute to producing anxiety in trigonometry for students (Kamber & Takaci, 2017; Weber, 2005). In our experience, teaching and learning mathematics in Nepali schools is not as fruitful as expected. Teachers are teaching mathematics to fulfill their jobs and to complete the course context of the textbooks and the students are learning mathematics just to score marks and grades in the examinations. Believing in mathematics involves self-stabilization and seriousness, with subjective truths, and includes learning about oneself and the environment, influenced by internal and external factors (Kurniasih &Waluya, 2020). We think the teaching and learning strategies are less conceptual and less practice-oriented but more algorithm problem solving, which also raises a big problem in trigonometry learning and leads to less success in this portion of mathematics. As a result of this, achieving success in trigonometry is a nightmare for most students because of its abstractness and straightforward nature. Some problems in learning trigonometry in secondary classes might directly relate to the teachers' academic background, classroom practices, school management, and leadership. Similarly, other problems in learning trigonometry might concern the pre-knowledge of students, their learning environment, peers, and family background.

The beliefs indicating the poor performance of students in trigonometry (Gholami, 2022) underscore the importance of conducting this study. We, as a researcher, have gone through different studies like Aminudin et al. (2019), Arhin and Hokor (2021), Asomah et al. (2023), Gur (2009), Tyata et al. (2021), Mulwa (2015), Nurmeidina and Rafidiyah (2019), Usman and Hussaini (2017). In this situation, as researchers and teachers of mathematics, we found this topic relevant to study in our context. Guided by the research question--in what ways do secondary-level students learn trigonometry?, this study investigates the learning experience of grade ten students in trigonometry at the secondary level, including both the challenges and the joys they encountered. With this introduction, the paper covers methods, data analysis, interpretations, findings, and discussions.

METHODS

This section in this study gives a complete framework for this study. It also explains the design and the way through which we have designed this study. It includes a detailed description of how decisions have been made about the type of data and the data collection procedures. In this section, we have explained the method of this study with the research site and data analysis and interpretation procedure.

As a researcher in this study, we have collected the lived experiences of grade ten students in learning trigonometry. In doing this, Sandip sat together with the students in their class for two weeks, where he involved himself in extracting their real experiences of learning trigonometry. Twelve grade ten students were taken for the diagnostic test and selected from twenty-five students in the class. These twelve students are those who have taken additional mathematics as their optional courses from from grade nine. In Nepal, students have the options to choose optional subjects in grade nine, and the trigonometry section is found in additional mathematics. The selected twelve students were all the students who chose additional mathematics. Observing their copies, discussing with the students, and informal interviews were some possible strategies for data collection methods. In this sense, this study is a phenomenological study where we looked for every possibility of collecting the real experiences of the students in learning trigonometry. Phenomenology "aims to focus on people's perceptions of the world in which they live and what it means to them; a focus on people's lived experience" (Langdridge, 2007, p. 4). Likewise, Manen (1990) noted phenomenology as the appropriate method to explore the phenomena of pedagogical significance and elaborates phenomenology as a response to how one orient to lived experience and questions the way one experiences the world. In this regard, phenomenology as a research method is the best fit for this study.

This research project is qualitative. Qualitative research “aims to explore people’s perceptions and experiences of the world around them by synthesizing data from studies across a range of settings” (Ahmes et al., 2019, p. 2). Denzin and Lincoln (2000) argued that qualitative research is a situated activity that locates the observer in the world and consists of a set of interpretive, material practices that make the world visible. “Qualitative research often seeks to interpret, illuminate, illustrate, and explore meaning, context, unanticipated phenomena, processes, opinions, attitudes, actions, and to learn about people who are few or hard to reach” (Saini & Shlonsky, 2012, pp. 12-13). The argument is that a qualitative evidence synthesis provides an in-depth understanding of complex phenomena while focusing on the experiences and perceptions of research participants. In this context, we have found the qualitative manner in this study is the most appropriate research technique to find the real and expected results. This study is based on primary and secondary data sources. Students’ observations and the discussion with them are the primary data for us, and the literature from the pre-existing research is the secondary source of data. Semi-structured and unstructured questionnaires were used for the interview.

DATA ANALYSIS AND INTERPRETATIONS

This section in this study contains the analysis and interpretation of the collected data from the field which has addressed the need of the research question. Initially, after the orientation and purpose of the study, ten questions were handed out to a group of twelve students, who recorded their answers on loose sheets that were subsequently gathered. They were kept in such a way that one student in one bench and restricted from interaction during the question-solving time. Though the setting was like an examination, they were informed about the purpose and objectives of our study and were suggested to respond to the questions freely. After collecting all the worksheets, we sat individually with them to explore their experiences and assumptions on each particular problem of the question paper. In a few cases, we seat multiple times with them to conform to their ideas. In doing this, we asked about their understanding of the major concepts of that particular problem, the reason behind applying particular formula or concept, and why they were not able to solve a problem or partially solved it. The formal and informal interactions related to this study were collected in diaries, which we first coded and thematically analyzed. The diagnostic test paper was labeled from 1 to 12.

We sit together multiple times for the procedures of data analysis and interpretation. Finally, we have analyzed students' responses based on the particular problem. This section provides a depth understanding of the students on the particular problem. We have included all the student responses in each question. If they had not attempted the particular question, then it is mentioned as not an attempt. The examples presented in this study are errors committed by students, obstacles, and misconceptions that arose from secondary-level lessons in trigonometry.

FINDINGS

The present study is completely based on the student's responses to the diagnostic test paper and oral interviews. There are many errors, obstacles, and misconceptions in trigonometry, and these are given in this section.

Question 1: $sin^2A + cos^2A = 1$ Why? Explain in detail.

The list of students' writing of question 1 related to the "$sin^2A + cos^2A = 1$" was coded and presented below according to the various level of understanding. Students' justification is given below.

I. *Correct answer* (12 responses out of 12)
$(p/h)^2 + (b/h)^2 = (p^2 + b^2)/h^2 = h^2/h^2 = 1$ *(If p, b, and h indicates the sides of right angled triangle where h is the longest side).*

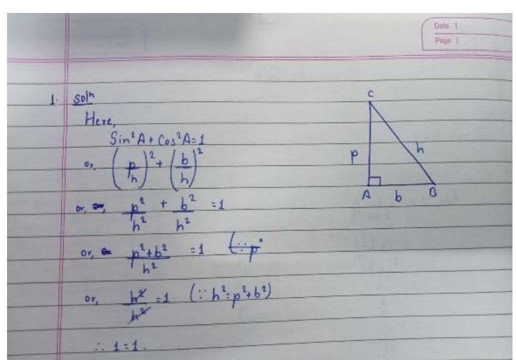


Figure 1: Sample solution for Q. No. 1

All students gave a mathematically valid explanation for why this equation was true. The alongside figure (fig.1) is a sample solution for this equation which is taken from one of the responses. Further, when we seat with them to explore their understanding of it. Sandip asked them to give examples for this equation; 9 out of 12 were able to answer it, whereas 3 were stuck with their pen on their copy. All the examples were in a pattern like $sin^2 30^o+cos^2 30^o=...=1$. Even Sandip further asked each of them to prove this equation in any alternative ways. Five of them added $sin^2A+cos^2A = sin^2A + (1-sin^2A)=1$. Three of them, out of the remaining, accepted that they had also attempted this question in alternative ways but could not remember now. The remaining students replied that this is the ultimate way for them and do not have any ideas for alternative ways of solutions. The scenario shows that all were able to answer the stepwise solutions, but only a few of them were able to explain it in detail with examples and with alternative ways.

Question 2: $tanA = \frac{1}{cotA}$ or $tanA.cotA = 1$. Explain in detail.

The list of students' writing of question 2 related to the "$tanA.cotA = 1$" was coded and presented below according to the various level of understanding. Students' justification for this question is given below.

I. *Correct answer (7 responses out of 12)*

The students' written responses show that seven students had given the correct answers. Among them, five had used the concept of the Pythagorean ratios: $tanA.cotA=(p/b)\times(b/p)=1$, one had respond $tanA.cotA=tanA\times(1/tanA)=1$ and one had respond $(tanA.cotA=(sinA/cosA)\times(cosA/sinA)=1)$. In addition, one student explains like this after the solution:"

"*cot A is in the form of divide (in this tanA=1/cotA) so if it is taken to another side it transforms into the multiplication (in this tanA.cotA=1)*"

Further, when we sat with them to know how they understood this concept, most of them replied that the product of two opposite trigonometric ratios like sine and cosecant, cosine and secant, tangent and tangent and cotangent, is always one. We found one interesting response; a boy from these seven only responds like "sirs, it is universal truth" then again, Sandip asked him why he is telling so he responded similar type of answer like others. They said that they have memorized this concept in the earlier days when they started learning trigonometry.

II. *Incorrect* (1 response out of 12)

One boy among all had responded unacceptable solution (See fig. 2 alongside) to the given idea. He performed the addition operation where he needed to use the product rule: $(p/b) \times (b/p) = (p^2 b^2/pb) = \ldots$
But when Sandip asked him why he has done like this, he was surprised with his answer and said "*sir, mistakenly happened*". Then Sandip asked him to solve it on an additional page then he was able to solve it. From this, we found that students get panic during tests/exams and such types of settings. He accepted that he was hurried to finish all the questions and didn't revisit at last as he was confident with his answer then.

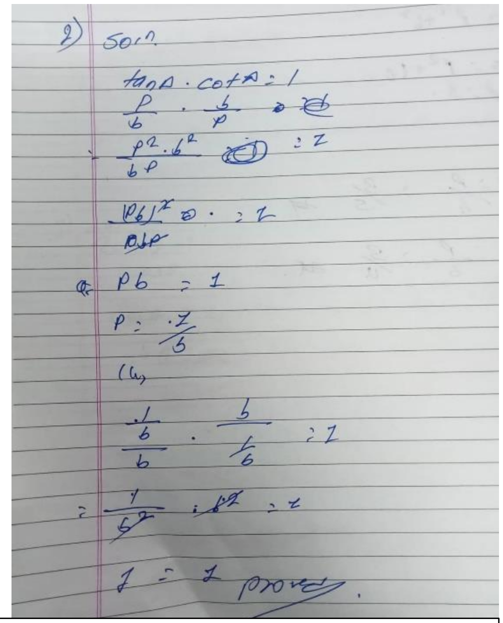


Figure 2: Sample solution for incorrect response

III. *No attempt (2 respondents out of 12)*

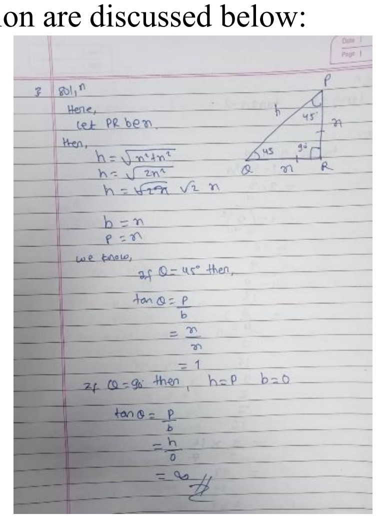
Two students out of twelve had not attempted this question. Though they had not responded to the question, Sandip tried to convince them that they couldn't solve it or there could be any other reason behind it. Both of them were able to answer correctly when Sandip provoked them by giving other examples like *sinA×cosecA=1*, and both of them accepted that they had forgotten this during the test time and also accepted that they had used these concepts many times in trigonometry lessons.

Question 3: $tan\ 45 = 1\ and\ tan\ 90$ is undefined. Explain in detail.

Common errors, obstacles, and misconceptions that students make with the above question are highlighted. Students' justifications and responses to this question are discussed below:

I. *Correct answer* (6 responses out of 12)

In responding to this question, six students answered correctly (See the sample solution in fig. 3 alongside), where all of them had used the concept of a right-angled isosceles triangle. All of them had drawn pictures with a reference angle of 45°.
tan45=p/b=p/p=1 where p and b represent the adjacent equal sides of a right-angled isosceles triangle.
In responding to the second part, four students used the concept of shortening the base to make the hypotenuse equal to perpendicular in a right-angled triangle. Then,
tan90=p/0=Undefined. Among the two of them had written the conclusion as well, which is like:



"The value of tan90 is p/0. If something is divided by zero then it is undefined."

Figure 3: Sample solution for question 3

The other two correct responses were:

Tan90=sin90/cos90=1/0=undefined (Any number divided by zero is undefined)

The interaction with these students also gives a clear picture of what they have presented in their solution.

II. *Partially correct* (1 response out 12)

One student among them responded: "*Here, the actual value of tan90 is 1/0 and if you divide anything y zero then it is undefined. So, the value of tan90 is undefined.*"

He just answered this for the above question and was not able to answer anything for *tan45=1*.

III. *No attempt* (5 respondents out of 12)

Five students among them had not responded to this question. When Sandip asked the reason behind this two of them replied that they had forgotten, whereas the remaining told him that they might not have heard the reasons behind those expressions. However, all of them accepted that they had applied those concepts in solving trigonometric expressions. Sandip interaction with those questions also shows that students have something for particular concepts, but they are not able to express it in systematic form.

Question 4: Simplify: $(\boldsymbol{sin^2 A})^2 - (\boldsymbol{cos^2 A})^2$ =?

The information collected for question four from the students' responses is presented below.

I. *Correct answer* (9 responses out of 12)

In responding to this question, nine students respond correctly where $(sin^2A)^2 - (cos^2A)^2$ = *(sin^2A+cos^2A) (sin^2A-cos^2A) = 1. (sin^2A-cos^2A) = (sin^2A-cos^2A)* was the common method. Five among these nine ends with this answer, whereas the remaining four reached up to *(sin^2A-cos^2A)=(sinA+cosB)(sinA-cosB).* See fig. 4 for a sample solution from one of the students.

II. *Incorrect* (3 responses among 12)

Three responses from the participants ends with this solution: *(sin^2A+cos^2A) (sin^2A-cos^2A) = (sinA+cosB)(sinA-cosB).(sin^2A-cos^2A)*

The above scenario shows how students have performed in their paper tests, but when we interacted with them, the scene

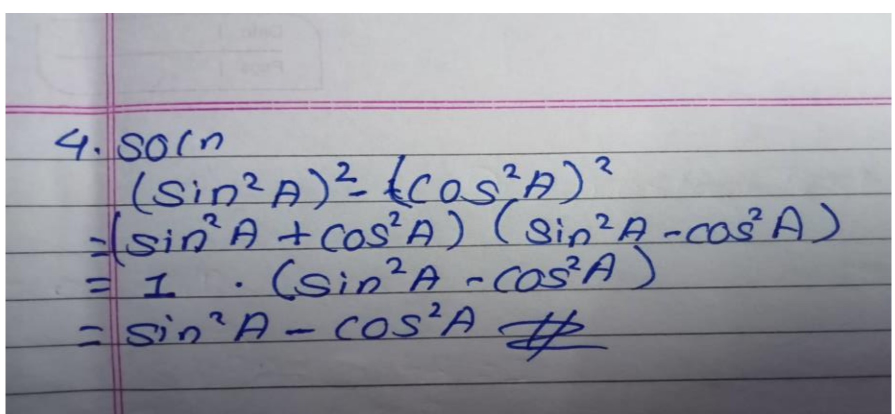


Figure 4: Sample solution for question 4

was different. One girl from these three who attempt incorrect answers was found clear with the concept and solved the question again in front of us without any hesitation and answered all the questions Sandip asked but the remaining two were found confused there as well. From this, we found that some students can not attempt the correct solutions in the examinations though they know it. Two among the nine who attempted the correct answer were not able to explain their solution and the remaining were found clear with their solution to this question.

Question 5: $\boldsymbol{tanA = \frac{3}{4}}$ **then** $\boldsymbol{tan2A = ?}$

Common errors, obstacles, and misconceptions that students make with the proof "$tanA = \frac{3}{4}$ *then* $tan2A$" equations are highlighted. Students' responses to this question are discussed below:

I. *Correct answers* (9 responses out of 12)

Nine responses were found with correct answers and the correct process for solving the above question (Question 5). They used the formula $tan2A=2tanA/1-tan^2A$ and then inserted the value of *tanA*, which gives the output of 24/7. See fig. 5 for a sample solution.

II. *Incorrect answer (2 responses out of 12)*

Two students were found to have the incorrect answer to the solution to the above question (Question 5). However, they have inserted the correct formula and correct values for tanA. Both of them attempt mistakes in subtracting during the calculations.

III. *No attempt* (1 response out of 12)

One student was found not attempting this question test paper. In the interview round, when Sandip asked the reason behind her difficulty, she replied: "Sir, I forgot the formula".

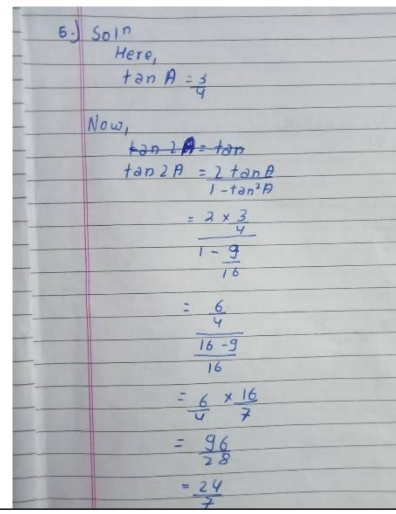


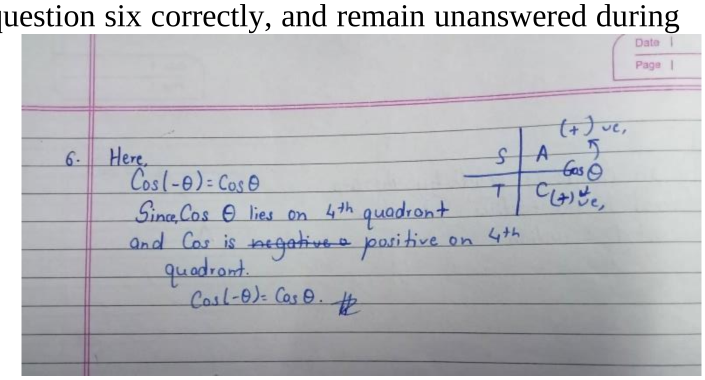
Figure 5: Sample solution for question

Question 6: $\boldsymbol{\cos(-\theta) = cos\theta}$**. Why?**

I. *Correct answers* (0 responses out of 12)

None of the students could explain question six correctly, and remain unanswered during the interview also.

"Sir, I know this concept, and I have applied it many times, but I don't know its explanation", one of the students said on the interview. See fig. 6 for a sample solution.



II. *Incorrect answer* (4 responses out of 12)

Four students responded to this question but were not able to justify the question. Among them, three had responded like "*cos is positive in the fourth quadrant, so* $cos(-\theta) = cos\theta$" which is a partial understanding of this question. During the interaction also, they were not able to reply more than this.

III. *No response* (8 responses out of 12)

Eight students were found unanswered with question six. During the discussion and interview time, they replied: "Sir, no idea!" In addition, they added that they had applied this idea to solving/proving trigonometric concepts, this idea is not new to them, but they don't know how $\cos(-\theta) = cos\theta$?

Figure 6: Sample solution for question 6

Question 7: Can you identify the figure below? What is the y-intercept of this function?

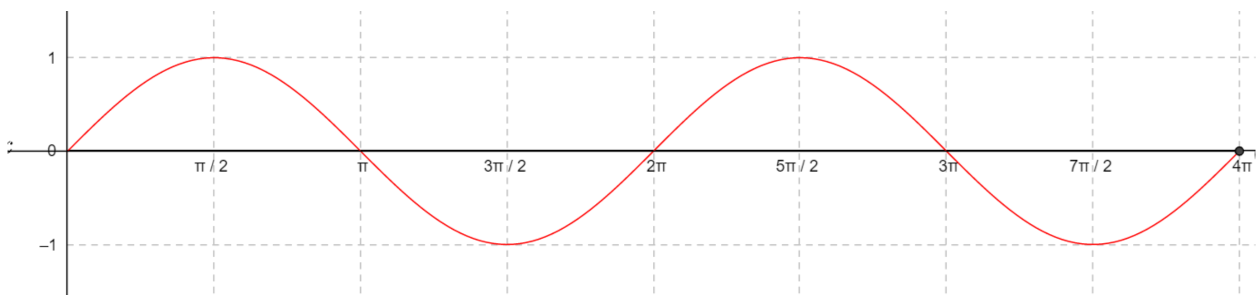


Figure 7: Sample picture of trigonometric curve

I. *Correct answer* (7 responses out of 12)

"*The curve represents the sine curve, and its y-intercept is 0*" (Student 10 replied in the interview). This is a student's response coded as 10 in his copy. Each of these seven students responded to those questions in their own way and language and was able to explain in the interaction as well. This shows their clear concept of trigonometric curves. See fig. 7.

II. *Incorrect answer* (3 responses out of 12)

One student among these three responded with an incorrect answer, whereas the remaining two responses were found with partial understanding. Among the two, one mentioned the correct name of the curve and a mistake in the y-intercept, whereas the next was vice-versa.

III. *No response* (2 responses out of 12)

Two students were found unanswered with this question. During the interview, they were also found confused with the sine and cosine graph. However, one of these two was able to tell the y-intercept but has not mentioned it in his copy. He replied that he was anxious while responding to these questions and forgot to mention this idea.

Question 8: In which quadrant lies the angle 480°?

The responses of the students the question 8 are discussed below:

I. *Correct responses* (8 responses out of 12)

It lies in the second quadrant because *480 = (6×90-60)*, and counting from the first quadrant as 1, it only reaches the second quadrant at six and minus 60o lie there. One of the students explained her solution this way when Sandip requested that she explain question eight. Among these eight students, three were found without explanation in their copies but could explain during the interview.

II. *No responses* (4 responses out of 12)

Four students were found without a solution to this question. During the explanation, one could answer after counting the graph on the side of his copy and telling the second quadrant, but he was unsure about his answer. The remaining three remained silent and just said, *"I don't know sir", "I forgot sir"*.

Question 9: Solve $2cos\theta + 1 = 0$ over the interval [0^0-360^0).

The list of students' responses related to "*Solve* $2cos\theta + 1 = 0$ *over the interval [0; 360)"* was coded and discussed below:

I. *Correct answer* (2 responses out of 12)

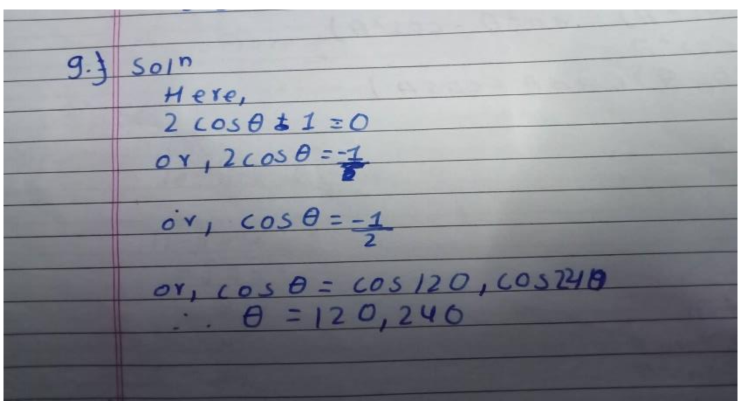


Figure 8: Sample solution for question 9

$2cos\theta+1=0$, $cos\theta=-1/2$,

Cosine is negative in the second and the third quadrant so we need to look for the possible values in the second and third quadrants. Thus, the correct answer is 120^o and 240^o. These two students ended up with a correct answer and could explain it during the interview. See fig. 8 for a sample solution.

II. *Partial Understanding* (8 responses out of 12)

This level of understanding contains the majority of the students' responses to this question, where they had forgotten to look at the given interval where they need to calculate the value of θ. They have resulted in 120°, which is the correct answer but failed to see in the third quadrant (as cosine is negative in the second and third quadrants). During the discussion majority of them accepted that they had forgotten to see the interval; otherwise, the concept was familiar to them.

III. *Incorrect responses* (2 responses out of 12)

Out of the twelve responses, two are found incorrect as the value θ was not calculated in the right quadrant. The interaction with these students also shows their misconceptions regarding such ratios. They are confused with the negative and positive signs of the values, which they need to compare. Though they reached up to *-1/2* in the calculation but ended with 60^o and $(360-60)=300^o$ as the answer, which is not true.

Question 10: If 5 cosθ = 4, find the trigonometric ratios sin θ and tan θ.

The list of students' writing of the question "*If 5cosθ = 4, find the trigonometric ratios sin θ and tanθ"* was coded and discussed below under the different levels of understanding.

I. *Correct answer* (12 responses out of 12)

If 5cosθ = 4 then cosθ=4/5=b/h (b=base and h=hypotenuse)⟹b=4k and h=5k then perpendicular (p) = 3k (by using Pythagoras Theorem). Hence, sinθ=p/h=3k/5k=3/5 and tanθ=p/b=3/4. See fig. 9 for a sample solution.

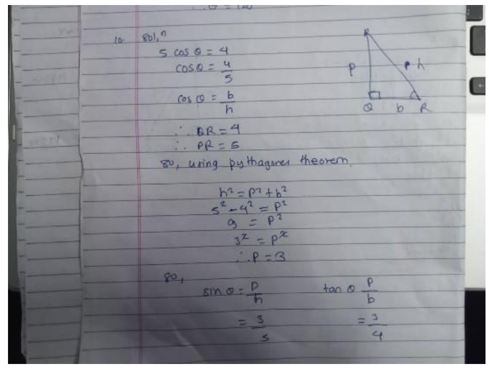


Figure 9: Sample solution for question

This was the common method in those correct responses. Ten students out of twelve had got the correct answer in this way whereas remain two had done like:

$sin\theta = \sqrt{1 - cos^2\theta}$ =3/5 and tanθ=$sin\theta/cos\theta$

During the interaction, all the students were able to explain their solution to this question and found a clear concept with such ratios in trigonometry.

DISCUSSION ON THE STUDENTS' EXPERIENCES

In serving the purpose and the need of the research question of this study, we have presented the findings of this study in the above section in descriptive form. The result of this study shows that students are having difficulty learning trigonometry. From the diagnostics test, we found that students have misconceptions about basic concepts and are even unable to explain what they had written in their copies. The formal and informal interactions with the students show that they are less confident with what they learned. They know the concepts like $sin^2A+cos^2A=1$, $cos(-\theta)=cos\theta$, but they are not able to answer how these concepts are derived. In analyzing the result section, we found that the students had problems finding the connections between the idea they learned and the context of the problem. They even share that they are not able to apply the right formula in the correct place though they know it. They know the chunk of formulas, but the connection among them is unclear for any of them. Do they know that $cosec^2A-cot^2A=1$ but are not able to answer $cosec^2A-1$=? We found that there are considerable research of such kind where students' difficulties and errors are explored, and even though a teacher of each scenario might be aware of the difficulty and possible errors made by the students in their classroom but "students have not been encouraged to take advantage of errors as learning opportunities in mathematics instruction" (Gray & Tall, 1994, p. 166). It is also found that students' errors are the symptoms of misunderstanding (Lai, 2006). The causes of systematic errors may relate to students' procedural knowledge, conceptual knowledge, or links between these two types of knowledge (Manandhar et al., 2022). Errors can be mistakes, blunders, miscalculations, or misjudgments; such a category falls under systematic errors (Muzangwa & Chifamba, 2012). Even our discussion with the students and their solutions reveals different types of errors (reading error, comprehension error, transformation error, process skills error, and encoding error), as argued by Newman (1977). A teacher can significantly minimize such errors and familiarize students with trigonometric concepts.

In addition, some students during the interactions replied like; "*sir, we have learned it that I can remember but I cannot answer it*", "*sir has taught this to us but I forgot*", and "*I had attempted this question in last class/exam but now I forget it*". Such replies from the students showed us that the students cannot understand the concepts clearly; they just memorize them for the exam or classroom purpose and forget after that. Their learning might be limited at the cognitive level and produces lower-order thinking skills. At this moment, we remember a research Luitel (2019) where he concluded that "how people explain the nature of mathematics explains how they understand it" (p. 7). To minimize such scenarios, the teacher can engage students in activities and projects so that they can learn from their own experiences and produces higher-order thinking skills. I believe that engaging students in so-called progressive ideas in teaching and learning like collaborative learning, group work, and pair work can also give lifelong learning and understanding to the learners. Three students from this group were found less motivated toward trigonometry learning. They told us in an informal discussion, "*Sir, I never understand these trigonometric concepts and felt difficulty in solving even easy problems which make me uncomforted in front of my teacher and friends*" they even added that they can't ask about their difficulty with the teacher thinking that "*if the teacher asks any other simple idea, then we don't know even that*". Further, they added that the teacher offered the general rules, ideas, and concepts in any topic which the high-achievers can easily grab, and they continued solving the exercises. "*Though our teacher makes groups in our class where we low-achievers are kept with high-achievers but instead they guide us, they give their copies, and we copy from there*". As for the disadvantages of pair work and group work, we found such a scenario in this group of students. If a teacher is less active and does not give attention to the students' jobs and activities they are doing within themselves, it also creates learning problems. We found such students when we were with them in their classes; a student has a good mathematical understanding but needs to poke by the teacher each time for the learning. After listening to the students' experiences, We found that a teacher needs to be aware of the student's activities and figure out their individual learning styles. We found that engagement in classroom activities also might not be enough all the time, but effective engagement is required (Lamichhane & Dahal, 2021). The teacher needs to be aware of such scenarios and need to make students motivated and responsible for their learning.

Another area of weakness that students revealed during the interaction with them when we asked the reason behind their errors and misconception about trigonometry is the teacher-student relationship (Dahal, 2013; Dahal et al., 2019). The teacher's strict nature and appearance in the classroom prevent them from asking their queries. "*Though our math sir seems friendly and encouraging teacher in the classroom, his appearance and his voice vibrate my heart, I cannot even speak with him in general topics; mathematics is already tough for me*". Such statements from the students make us stubborn for some time. We remembered our school days when the scenario was similar. These students' experiences are not only those of the students from one particular institution but of most of the schools in Nepal. There is huge room for changes in teaching-learning activities in mathematics and other subjects, but in our opinion, teacher-student relations should be the first one. The research like Dahal (2020) concluded that "teacher-student bonding has to be considered a vibrant feature in the school hall of both remote and urban institutions in Nepal where Nepalese mathematics teachers have not been able to link bonding of teachers and students not as such" (p. 72). But at the same time, it should be considered that "the sociocultural viewpoint of both the teacher and students respectively influence the social and cultural interactions in a mathematics classroom" (Dahal et al., 2019, p. 119). Lack of healthy relations and good rapport between the teacher and students create communication gaps, and students cannot share their difficulties with the teacher/facilitator. The interaction with each student also results in the same issue, and all the students want their teacher to be frank in case of teaching-learning activities. Classroom communication in mathematics teaching and learning is thus an essential factor.

CONCLUSIONS AND IMPLICATIONS

This qualitative research paper focuses on Nepali students' challenges, hindrances, and misunderstandings while solving and/or studying trigonometry concepts. The impetus for this paper stems from the challenges teachers face in elucidating trigonometry contents and the difficulties students encounter in comprehending the abstract nature of trigonometric concepts. The above discussion concludes that students are facing difficulty in learning trigonometry. Errors, obstacles, and misconceptions are the significant domains for many students in trigonometry. They even have misconceptions about the basic concepts, leading to errors in learning/solving trigonometric problems. The possible errors might be in procedural knowledge, conceptual knowledge, or a link between these two types of knowledge. It is also found that students may feel comfortable learning trigonometry with materials, diagrams, and equipment. The connection of abstract context with the learners' context might be another possibility that students shared in the interview. This study highlights the significance of phenomenology in mathematics education. We as authors have developed guidelines based on their own teaching experiments, which can be utilized by other instructors when teaching trigonometry independently. The author, like Usman and Hussaini (2017), revealed that Africa's context seems almost similar to the Nepali context, i.e., students, regardless of their varying cognitive abilities, are prone to making errors when solving trigonometry problems. Unlike, Usman and Hussaini (2017) quantitative study, this qualitative study explored the errors that occurred both in conceptual and procedural understanding. A study like Rosjanuardi and Jupri (2022) also highlights the procedural errors in trigonometry and trigonometric functions, whereas this study concludes that both conceptual and procedural aspects are equally responsible.

This study also uncovers the limitation of collaborative learning in mathematics classrooms. Students are copying and engaging in unnecessary activities during the group/paired work. The study demonstrates that simply encouraging group or paired work among students may not always be sufficient and that regular supervision and periodic monitoring are also necessary. We also realized that rapport among the teachers and students is essential while teaching mathematics in general and trigonometry in particular. Teacher-student relationships can be another factor that contributes to learning difficulty in trigonometry. It is found that a healthy relationship between teacher/facilitator and students can create a harmonious and favorable environment for learning. Such an environment motivates students in the learning process and promotes meaningful learning.

LIMITATIONS AND FUTURE RESEARCH

This study was limited to twelve grade ten students as participants based on ten trigonometry questions from a single private school in Kathmandu, Nepal. Further research can be conducted in schools with many participants from private and public schools.

REFERENCES

[1] Adhikari, T. N., & Subedi, A. (2021). Difficulties of grade X students in learning Trigonometry. *Siddhajyoti Interdisciplinary Journal*, *2*(1), 90-99.

[2] Ames, H., Glenton, C., & Lewin, S. (2019). Purposive sampling in a qualitative evidence synthesis: A worked example from a synthesis on parental perceptions of vaccination communication. *BMC Medical Research Methodology*, *19*(1), 1-9.

[3] Aminudin, M., Nusantara, T., Parta, I. N., Rahardjo, S., & As'Ari, A. R. (2019). Engaging problems on trigonometry: Why were student hard to think critically? In *Journal of Physics: Conference Series* (Vol. 1188, No. 1, p. 012038). IOP Publishing.

[4] Arhin, J., & Hokor, E. (2021). Analysis of high school students' errors in solving trigonometry problems. *Journal of Mathematics and Science Teacher*, *1*(1).

[5] Asomah, R. K., Agyei, D. D., & Ntow, F. D. (2023). Developing in-service mathematics teachers' pedagogical content knowledge and skills to teach trigonometry: Using cooperative teaching and learning approach. *Contemporary Mathematics and Science Education*, *4*(1), ep23001.

[6] Dahal, N. (2013). *Teacher-students relationship and its potential impact on mathematics learning: An autoethnographic inquiry* [Unpublished Master Dissertation]. Kathmandu University School of Education, Dhulikhel, Nepal.

[7] Dahal, N., Luitel, B. C., & Pant, B. P. (2019). Teacher-students relationship and its potential impact on mathematics learning. *Mathematics Education Forum Chitwan*, *4*(4), 35–53. https://doi.org/10.3126/mefc.v4i4.26357

[8] Dahal, N. (2020). Teacher-students bonding in mathematics learning: An exploration of past, present and future. *AU eJournal of Interdisciplinary Research*, *5*(1), 63-76.

[9] Dahal, N., Luitel, B. C., & Pant, B. P. (2019). Understanding the use of questioning by mathematics teachers: A revelation. *International Journal of Innovative, Creativity and Change*, *5*(1), 118-146.

[10] Denzin, N. K., & Lincoln, Y. S. (2008). Introduction: The discipline and practice of qualitative research. In N. K. Denzin & Y. S. Lincoln (Eds.), *Strategies of qualitative inquiry* (pp. 1–43). Sage Publications, Inc.

[11] Dhungana, S. (2021). *Problems in learning geometric theorems in secondary schools: A mixed method study* [Unpublished master's dissertation]. Kathmandu University, Dhulikhel, Nepal

[12] Fauziyah, N., Asari, S., Ma'Rifah, U., Uchtiawati, S., & Husniati, A. (2021, February). Improving Students' Creativity through Sharing and Jumping Task in Mathematics Lesson Study Activity. *In Journal of Physics: Conference Series* (Vol. 1764, No. 1, p. 012100). IOP Publishing.

[13] Gray, E. M., & Tall, D. O. (1994). Duality, ambiguity, and flexibility: A perceptualal" view of simple arithmetic. *Journal for Research in Mathematics Education*, *25*(2), 116-140.

[14] Gholami, H. (2022). Improving the performance of mathematics teachers through preparing a research lesson on the topic of maximum and minimum values of a trigonometric function. *Mathematics Teaching Research Journal*, *14*(2), 26-43.

[15] Gur, H. (2009). Trigonometry Learning. *New Horizons in Education*, *57*(1), 67-80.

[16] Kamber, D., & Takaci, D. (2017). On problematic aspects in learning trigonometry. *International Journal of Mathematical Education in Science and Technology, 5211*(August), 1– 15. https://doi.org/10.1080/0020739X.2017.1357846

[17] Kurniasih, M. D., & Waluya, S. B. (2020, April). The analysis of learning obstacle of mathematics belief. In *Journal of Physics: Conference Series* (Vol. 1511, No. 1, p. 012106). IOP Publishing.

[18] Lai, C.-F. (2006). Error analysis in mathematics. *Behavioral Research and Teaching*.

[19] Lamichhane, B. R., & Dahal, N. (2021). Engaged mathematics learning as/for a transformative praxis. *Mathematics Education Forum Chitwan*, *6*(6), 1–7. https://doi.org/10.3126/mefc.v6i6.42395

[20] Langdridge, D. (2007). *Phenomenological psychology: Theory, research and method*. Pearson.

[21] Luitel, L. (2019). Nature of mathematics and pedagogical practices. A conference paper. Proceedings of the *Tenth International Mathematics Education and Society Conference Hyderabad*, India, 2019

[22] Luitel, L., & Pant, B. P. (2019). Images of the mathematics curriculum held by school mathematics teachers: Mapping the road for transformative pedagogies. In *Empowering science and mathematics for global competitiveness* (pp. 510-516). CRC Press.

[23] Manandhar, N. K., Pant, B. P., & Dawadi, S. D. (2022). Conceptual and procedural knowledge of students of Nepal in algebra: A mixed method study. *Contemporary Mathematics and Science Education*, *3*(1), ep22005. https://doi.org/10.30935/conmaths/11723

[24] Mulwa, E. C. (2015). Difficulties encountered by students in the learning and usage of mathematical terminology: A critical literature review. *Journal of Education and Practice*, *6*(13), 27-37.

[25] Muzangwa, J., & Chifamba, P. (2012). Analysis of errors and misconceptions in the learning of calculus by undergraduate students. *Acta Didactica Napocensia*, *5*(2), 1-10.

[26] Newman, A. M. (1977). Analysis of sixth-grade pupils' errors on written mathematical tasks. *Victorian: Institute for Educational Research Bulletin*, 31-43.

[27] Nurmeidina, R., & Rafidiyah, D. (2019, August). Analysis of Students' Difficulties in Solving Trigonometry Problems. In *PROFUNEDU 2019: Proceedings of the 4th Progressive and Fun Education International Conference, Profaned 2019, 6-8 August 2019, Makassar, Indonesia* (p. 9). European Alliance for Innovation.

[28] Pant, B. P. (2017). Doing, teaching, learning and thinking about mathematics–On becoming a transformative teacher. *Journal of Education and Research*, *7*(1), 11-24.

[29] Rosjanuardi, R., & Jupri, A. (2022). Epistemological obstacle in learning trigonometry. *Mathematics Teaching Research Journal*, *14*(2), 5-25.

[30] Saini, M., & Shlonsky, A. (2012). *Systematic Synthesis of Qualitative Research*. OUP USA.

[31] Tyata, R. K., Dahal, N., Pant, B. P., & Luitel, B. C. (2021). Exploring project-based teaching for engaging students' mathematical learning. *Mathematics Education Forum Chitwan*, *6*(6), 30–49. https://doi.org/10.3126/mefc.v6i6.42398

[32] Usman, M. H., & Hussaini, M. M. (2017). Analysis of students' error in learning of trigonometry among senior secondary school students in Zaria Metropolis, Nigeria. *IOSR Journal of Mathematics*, *13*(2), 1-4.

[33] Van Manen, M. (1990). Beyond assumptions: Shifting the limits of action research. *Theory into Practice, 29*(3), 152-157.

[34] Weber, K. (2005). Students' understanding of trigonometric functions. *Mathematics Education Research Journal*, *17*(3), 91-112.

[35] Wilson, P. S., Cooney, T. J., & Stinson, D. W. (2005). What constitutes good mathematics teaching and how it develops: Nine high school teachers' perspectives. *Journal of Mathematics Teacher Education, 8*, 83-111.