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A Comparison of Angle Problems in Indonesian and Singaporean Elementary School Mathematics Textbooks

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***Abstract**: Textbooks are one of the main resources for teaching and learning mathematics. This study examines the presentation of angle topics in 4th-grade mathematics textbooks in Indonesia and Singapore. The analysis focused on the general characteristics of the textbook and the nature of the mathematical tasks presented. The results showed that Indonesian mathematics textbooks are more likely to provide a more ample opportunity to learn than Singaporean textbooks based on the number and description of task activities. However, the distribution of items in each task activity in Singapore mathematics textbooks is more proportionate than in Indonesian mathematics textbooks. Concerning mathematical tasks, the findings show that the form of representation in Indonesian mathematics textbooks contains a more purely mathematical form, while Singapore's mathematics textbooks are dominated by visual form. Regarding contextual features, mathematical tasks in Indonesian and Singaporean mathematics textbooks are dominated by non-application forms. Closed tasks also dominate the response type of task for both textbooks. The implications of this finding can be applied to classroom teaching activities, as highlighted in the discussion section.*

Keywords: Mathematics textbook, elementary school, angles, representation forms, contextual features, response types

INTRODUCTION

Textbooks have a significant role in supporting the learning process that's going on in schools. Textbooks strongly influence what will be taught, what students will learn, and how it will be studied (Rahmawati et al., 2020; Yang et al., 2010). A textbook is composed of specific materials according to learning objectives by referring to the curriculum that has been applied (Gracin, 2018; Usiskin, 2013). Given the relevance between textbooks and the way students learn and the learning activities they have gone through in class, textbooks are an essential means for students to obtain their learning achievements (Purnomo et al., 2019).

In various countries, textbooks are still one of the references as a source of student learning. Textbooks also show a considerable effect on teaching and learning activities and the fundamental teaching of teachers in the classroom (Yang & Sianturi, 2017). Some of the functions of textbooks in teaching and learning activities, among others helping teachers to explain the materials as exercise material for students and directing students in understanding mathematical material (Takeuchi & Shinno, 2020).

Textbooks consist of several aspects students must learn, including knowledge, skill, and attitudes to achieve predetermined competency standards and help their classroom learning process (Manopo & Rahajeng, 2020). For this reason, textbooks will affect students' ability to achieve specific competencies, one of which is the competencies tested in the PISA.

Analysis of mathematics textbooks in the last two decades has become a theme that has received increasing attention in mathematics education research (Fan, 2013; Purnomo, Shahrill, et al., 2022; Trouche & Fan, 2018), and many of them have focused on specific mathematical content or specific tasks in textbooks of two or more countries (Takeuchi & Shinno, 2020). This article reports on our research using a comparative study of Indonesian and Singaporean elementary school mathematics textbooks. Although Indonesia and Singapore are neighboring and cognate nations, Singapore is much more advanced in educational quality than Indonesia, primarily based on disparities in mathematical achievement from several international student assessments (e.g., PISA and TIMSS).

This study expanded on previous research lines that investigated geometry in mathematics textbooks (Choi & Park, 2013; Yang & Sianturi, 2017), specifically angle topics at the elementary school level in Indonesia and Singapore. Angle is one of the subsections discussed by elementary school students and has a vital position in the development of advanced mathematical concepts (e.g., trigonometric functions; proportional reasoning), the development of science (e.g., engineering, geology, architecture, physics), and solving problems of daily life (Alyami, 2020; Bütüner, 2021). Nevertheless, some empirical research highlights some of the challenges students have while learning using angles, such as typically having many misconceptions and difficulty gaining key concepts and skills in these topics (Bütüner & Filiz, 2017; Clements & Burns, 2000). Several textbook-related studies (Alyami, 2020; Haggarty & Pepin, 2002) also focus on angle topics, but the majority of the attention is on the high school level. Therefore, this research is useful to complement and fill in the gaps in textbook research on angle topics in elementary school mathematics textbooks, especially in Indonesia and Singapore.

Theoretical Framework

Features and diversity of tasks in mathematics textbooks

Lately, curriculum standards have focused and directed at domains of knowledge and skills that have also been a concern for international surveys, such as PISA, which deals with using multi-context-based problems, exploring mathematical activities, and promoting higher-level rather than low-level thinking. The implication is that textbooks are a means to implement the curriculum (Purnomo, 2023; Rahmawati et al., 2020; Valverde et al., 2002) and were asked to accommodate these domains.

Textbooks have at least two common parts, the presentation of materials and tasks. The main content of the material presentation section refers to how the content is delivered to the reader, which may include his pedagogy, the approach of the concepts used, and the related context presented. In comparison, the task section is more about how students or classes are involved in assignments, where these domains are more likely to be in this section. This section can take the form of an example, exercise, or other evaluation tools. Therefore, we focus on the task section in the textbook (examples and exercises) in promoting these domains.

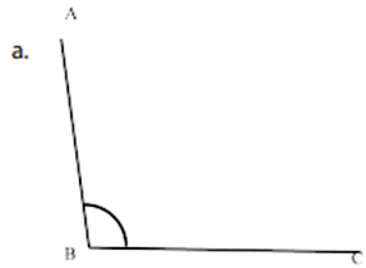
Mathematical tasks are a part of mathematics textbooks that researchers widely study because this section has an essential role in exploring knowledge and skills. Some researchers use different frameworks for mathematical analysis tasks in textbooks. We adopted a framework by Yang and colleagues (Yang et al., 2017), that includes three main focuses: representation form, contextual features, and response type. Each dimension will be elaborated into several points after this and accompanied by sample examples from our study's analysis results.

Representation Forms

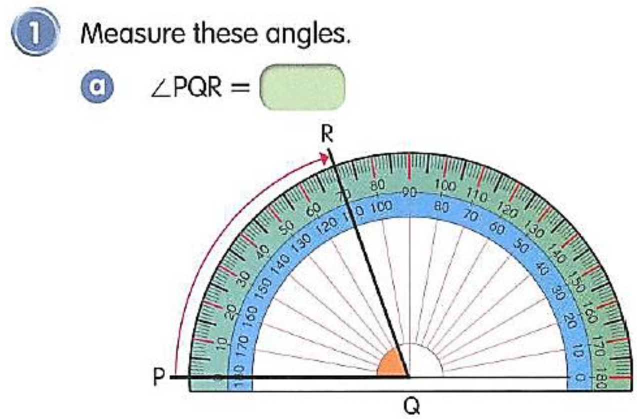
There are four forms of representation classified, namely purely mathematical (A1), verbal (A2), visual (A3), and combined (A4). Suppose the main problem presented here includes only mathematical expressions. In that case, the problem is classified into problems in purely mathematical form. If the problem presented is entirely verbal, that is, written words only, then the problem is encoded into the category of problems in verbal form. Suppose the problem presented consists only of drawings, graphs, charts, tables, diagrams, maps, and so on. In that case, such problems are classified into problems in a visual form. While problems are classified as combined if presented in two or three of the above forms. We can exemplify each of these four forms by taking the case of two textbooks in our study, namely Indonesian and Singaporean mathematics textbooks (cf., Yang et al., 2017).

Representation Form	Code	Example
Purely mathematical	A1	Indonesia: Make a shape from the following size angle. 60°, 60°, and 60° Singapore: 93° is between a 1/4 -turn and a ... -turn
Verbal	A2	Indonesia: How do you measure the angle in standard units? Singapore: Label the angles of your textbook with A, B, C, and D. How many angles are there? Name the angles
Visual	A3	Indonesia:

Find the size of the angle below!



Singapore:



Combined A4 Indonesia:

Pay close attention to the following pictures and readings!

Udin will construct the kite seen in the figure. Udin bout calculated the size of the angle at each point of the kite's angle for it to fly in balance. Angles in both kite wings at points A and C should be the same size. The magnitude of angle A is 105° as measured with a protractor. What about the sizes of angles B, C, and D?

Singapore:

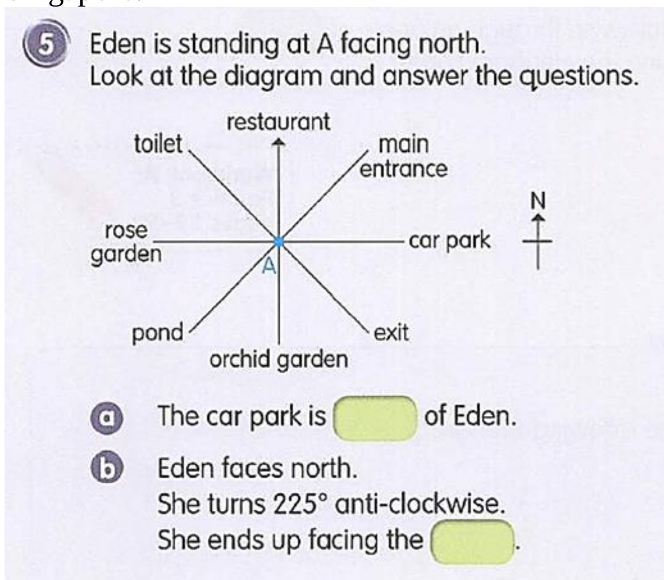
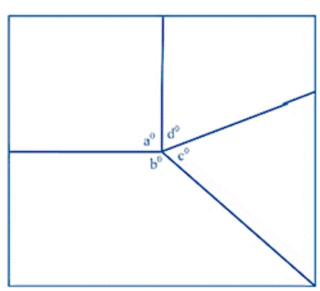


Table 1. Examples of each representation forms

Contextual Features

Two contextual features are classified: Application (C1) and Non-application (C2). Application is a problem that is presented in the context of a real-world situation. In contrast, non-application is a problem not related to the practical background in everyday life or the real world.

Contextual Features	Code	Example
Application	C1	Indonesia: Pay attention to the following figure!  Edo has a piece of cardboard that will be cut into pieces and form a flat build, as shown above. Edo wants to know

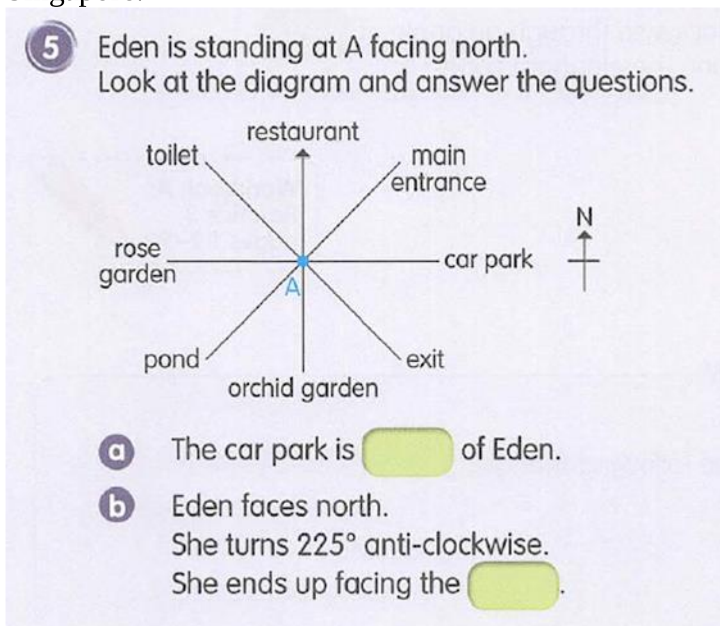
		the size of the angle at the angle points formed on the plane figure. Measure the magnitude of angles a, b, c, and d using a protractor. Singapore: 
Non-application	C2	Indonesia: At what time do the hands of the clock make an angle of 75°? Singapore: Use a ruler and a protractor to draw the following angles. a. 45° b. 80° c. 130° d. 154°

Table 2. Example of each contextual feature

Response Types

There are two types of classified responses: open-ended (E1) and close-ended (E2). Open-ended is defined as questions with many correct answers. In contrast, close-ended is defined as questions with only one correct answer.

Types	Code	Example

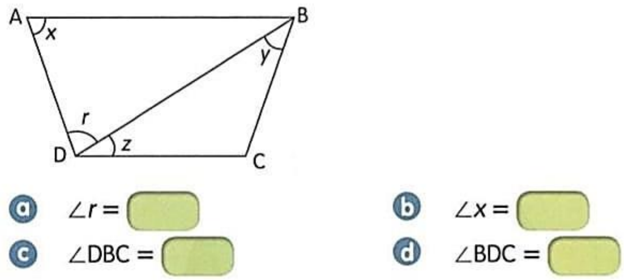
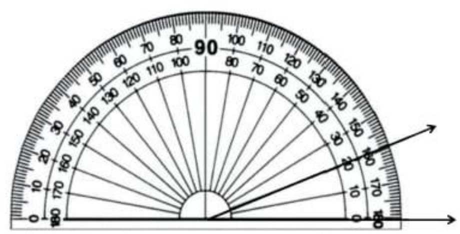
Open-ended	E1	Indonesia: At what time do the hands of the clock make an angle of 90°? Singapore: Look at the following figure. Name each marked angle in another way. 
Close-ended	E2	Indonesia: Determine the size of the angle from the measurement results below.  Singapore: 270° is equal to a ... -turn

Table 3. Examples of each response types

Comparative Study of Textbooks on the Topic of Angle

Seeing the importance of textbooks in mathematics teaching and learning, many researchers have researched and analyzed textbooks on various learning topics, including geometry and measurement topics that contain elements of angle. In their study, Choi and Park (2013) used the United States and South Korean textbooks to examine the measure of angles in plane figures. The study took math textbooks in grade 8 from both countries. Then a similar study was also conducted by Yang et al. (2017) who analyzed the topic of geometry in the middle class in textbooks in Taiwan, Singapore, Finland, and the United States. Research on mathematics textbooks on the use of the textbooks in the English, French, and German countries on the topics of angle was also carried out by Haggarty and Pepin (2002). The study reported that students from three countries have different opportunities to learn mathematics, depending on the textbook structure in combination with the teacher's use of textbooks, and the development of lessons varies according to textbooks.

Based on some of the studies mentioned above and those in the literature, at least 3 points need to be emphasized in the research. First, mathematics textbooks research has been done a lot on angle. However, there is still rare mathematics textbook research that focuses on the task section. Second, studies in mathematics textbooks have been carried out and involved various countries. However, none have been found to compare Indonesian and Singapore mathematics textbooks on angle topics. The last, according to research in mathematics textbooks, the issue of angle topic is usually addressed in middle and high school and rarely in elementary school.

The Topic of Angle in The Mathematics Curriculum of Indonesian and Singaporean Elementary Schools

In Indonesia, the angle topic was introduced in grade 4 elementary schools. Meanwhile, in Singapore, the angle topic was introduced in elementary school grades 4, 5, and 6. In grade 4, elementary school students in Indonesia learn about measuring angles in standard units with protractors and plane figure angles with protractors. While in Singapore, students learn to understand and measure angles, draw angles up to 180°, turns, and an 8-point compass in grade 4. Within the Singapore curriculum, the topic of angle is also taught in grades 5 and 6. Student grade 5 in Singapore learned about the angles properties of the 13th topic after studying rate or speed. Then, even in grade 6, students still learn about angles, namely finding unknown angles in geometric constructs. This became the last topic studied in the first semester in grade 6.

We consider that based on the level of cognition at the same grade level is more important to compare. The average age of grade 4 students in Indonesian and Singapura is also the same, 10-11 years. In addition, the sub-sub materials studied by students in Indonesia and Singapore in grade 4 are quite similar, so it is more suitable to compare. Therefore, we focus on the 4th-grade math textbook for both countries.

Research Questions

The purpose of this study was to compare the structure of units/lessons, their frequency and sequence, and the nature of mathematics tasks on angle topics between mathematics textbooks for grade 4 in Indonesia and Singapore. We focus on three analytical frameworks for the nature of mathematical tasks: the form of representation, contextual features, and types of response. The following research questions serve as a guide for this purpose:

(1) What are the differences in content structure, frequency, and sequence of angle tasks between mathematics textbooks for grade 4 in Indonesia and Singapore?
(2) Are there differences in the forms of representation (pure mathematical form, verbal form, visual form, and combined form) angle tasks between mathematics textbooks for grade 4 in Indonesia and Singapore?
(3) Are there differences in the contextual features (application and non-application) of angle tasks between mathematics textbooks for grade 4 in Indonesia and Singapore?
(4) Are there differences in the response types (open or closed) of angle tasks between mathematics textbooks for grade 4 in Indonesia and Singapore?

METHOD

Textbooks Selection

This study compared two types of mathematics textbooks used in elementary school learning in Indonesia and Singapore. The mathematics textbook from Indonesia used in this study is a textbook entitled *Senang Belajar Matematika Grade IV SD/MI* revised edition 2018, published by the Ministry of Education and Culture. Meanwhile, the mathematics textbook from Singapore used in this study is My Pals Are Here! Maths 4A published by Marshall Cavendish Education.

Senang Belajar Matematika Grade IV SD/MI was chosen because it is officially published by the Ministry of Education and Culture and used as a mandatory reference in elementary schools in Indonesia. In addition, this textbook is also following the latest revision of the 2013 curriculum. Based on the revision results, mathematics subjects at the elementary school level, and equivalent for grades 4, 5, and 6 in the 2013 Curriculum are separate from the thematic subject matter.

My Pals Are Here! Maths was chosen because it is a top-rated textbook in Singapura. The language of instruction in this textbook is English, making it easier for us to analyze it. This textbook was published by the company Marshall Cavendish Education and according to the Cambridge Curriculum for primary level schools. This textbook was chosen because 60% of Singapore schools use it (Yang et al., 2010).

Analytical Coding and Data Analysis Framework

We do data analysis with two types of analysis: horizontal and vertical (Charalambous et al., 2010). Horizontal analysis is an analysis carried out on textbooks as a whole that focuses on the characteristics of textbooks in general. We analyze Indonesian and Singaporean mathematics textbooks horizontally by focusing on the structure of units/lessons, their frequency, and sequence. Meanwhile, vertical analysis is carried out by researchers on how a single mathematical concept is treated and how to view textbooks as an "environment for the construction of knowledge" (Li, 2000). Vertical analysis is performed by adopting a framework from (Yang et al., 2017). The framework has three dimensions, namely: (1) representation forms, (2) contextual features, and (3) response types. The details of the dimensions, categorization, and coding we write in Table 4.

Dimension	Category and Code
Representation forms	Purely mathematical (A1) Verbal (A2) Visual (A3) Combined (A4)
Contextual features	Application (C1) Non-application (C2)
Response types	Open-ended (E1)

	Close-ended (E2)

Table 4. Vertical analysis

Guided by the categories and codes in Table 4, each researcher performed analysis and coding in both textbooks on the angle topic. After each conducted analysis and coding, it was continued with a cross-check. The agreement results were used for presentations at the Focus Group Discussion (FGD) session. The researcher chose FGD because the information obtained was more informative than data obtained through other data collection methods (Kaur et al., 2020; Purnomo, Shahrill, et al., 2022). If there is any doubt, we can get corrections and improvements right away using this method. The authors validated the results of the analysis and coding. The agreement from the coding results is used for reporting research results that are presented descriptively.

RESULTS

Textbook and Content Overview

The Indonesian mathematics textbook has 216 pages, with 20 pages (9.26%) containing angles. In contrast, Singapore's mathematics textbooks have 120 pages, with 12.5% (15 pages) containing angles. Although there are fewer pages than the mathematics textbook in Indonesia, Singaporean textbooks are more numerous in the amount of content presented. Details of the content structure of each of these textbooks can be seen in Table 5.

The Indonesian Textbooks	The Singaporean Textbooks
1. Fractions	1. Number to 100.000
2. Least Common Multiple (LCM) and Greatest Common Factor (GCF)	2. Factor and Multiples 3. Multiplication and division of whole numbers.
3. Approximation	4. Whole numbers
4. Shapes	5. Angles
5. Statistics	a. Lesson 1 Understanding and Measuring Angles
6. Angle Measurement	
a. Measurement of Angles in Standard Units with a Protractor	b. Lesson 2 Drawing Angles to 180° c. Lesson 3 Turns and 8-Point Compass
b. Measurement of the plane figure with a Protractor	6. Squares and rectangles 7. Symmetry

Table 5. Mathematics contents presented in the two mathematics textbooks

Based on Table 5, it can be seen that for the Indonesian mathematics textbook, angle topic is the last material taught in grade 4. Meanwhile, in the Singapore textbook, students learn the angles in grade 4 after learning the numbers. The angle topic is discussed in two subsections by grade 4 elementary school students in Indonesia. The first subsection measures angle in standard units with protractors, and the second is about measuring angles of shapes with protractors. While in the Singapore textbook, the material on angles is discussed in three subsections, namely (1) understanding and measuring angles, (2) drawing angles up to 180°, and (3) rotation and 8-point compass.

Mathematical Task Overview

Indonesian and Singaporean mathematics textbooks both use specific criteria to present their tasks. The mathematical tasks presented in Indonesian and Singaporean textbooks are grouped in several parts of grouping task activity based on their criteria, as shown in Table 6. In Indonesian textbooks, the tasks are grouped into seven parts. The overall types of task activities in one chapter of angle are 81 task items. While the questions in Singapore math textbooks are grouped into six parts, with all types of task activities in one chapter, there are 54 task items.

Indonesian Textbooks		Singapore Textbooks	
Task Activity	Description	Task Activity	Description
Ayo Mengamati (Let's Observe)	It contains questions for students before studying the material	Before You Learn	It contains questions for students before studying the material
Ayo Menanya (Let's Ask)	It contains activities to make questions that the students want to know related to the material	Learn	It contains questions for students and their steps
Ayo Menalar (Let's Reason)	It contains an explanation of the material accompanied by the question	Guided Practice	It contains guided question exercises in each sub-chapter of the material that has been studied
Ayo Mencoba (Fun to Try)	It contains guided question exercises that students expect to find concepts in some of the material that has been studied	Hands-On Activity	It contains steps for student activities
Tugas Proyek (Project Task)	It contains tasks that must be done in groups	Chapter Review	It contains practice questions that include material in one chapter
Latihan Soal (Practice Questions)	It contains guided question exercises that students expect to find concepts in all the material that has been studied in one chapter	Put On Your Thinking Cap!	It contains practice questions that require high reasoning

Contoh Soal disertai Jawaban (Worked Answers)	It contains examples of questions accompanied by steps of work

Table 6. Types of task activities and their descriptions

Table 6 shows that Indonesian mathematics textbooks are more likely to provide a more comprehensive opportunity to learn than Singaporean textbooks based on the number and description of task activities. For example, in Indonesian mathematics textbooks, there is a section "*Ayo Menanya*" which is an activity to make questions that students want to know related to the material being studied. This section is not described in the activities presented in Singapore mathematics textbooks.

According to the number of task items, Indonesian mathematics textbooks feature 27 more task items than Singaporean mathematics textbooks. Nonetheless, in Singaporean mathematics textbooks, the distribution of the dimension items in each task activity is proportionate to that found in Indonesian mathematics textbooks. This is more evident in the distribution table of the number of items presented in Table 7 and Table 8.

Order of Learning Activities for Indonesian Mathematics Textbooks	Number of Questions	A1	A2	A3	A4	C1	C2	E1	E2
Ayo Mengamati (Let's Observe)	4	0	0	0	4	4	0	0	4
Ayo Menanya (Let's Ask)	4	0	4	0	0	0	4	2	2
Ayo Menalar (Let's Reason)	13	1	4	1	7	7	6	1	12
Ayo Mencoba (Fun to Try)	23	16	0	6	1	1	22	8	15
Tugas Proyek (Project Task)	1	0	1	0	0	0	1	1	0
Latihan Soal (Practice Questions)	28	12	2	11	3	3	25	4	24
Contoh Soal disertai Jawaban (Worked Answers)	8	0	0	4	4	4	4	0	8
Number of Questions	81	29	11	22	19	19	62	16	65
Percentage (%)	100	35.8	13.6	27.2	23.5	23.5	76.5	19.8	80.2

Table 7 Distribution of the number of items based on analysis results for Indonesian mathematics textbook grade 4

Order of Learning Activities for Singaporean Mathematics Textbooks	Number of Questions	A1	A2	A3	A4	C1	C2	E1	E2
Before You Learn	4	1	2	1	0	2	2	1	3
Learn	10	0	2	1	7	1	9	0	10
Guided Practice	21	9	0	8	4	4	17	8	13
Hands-On Activity	3	1	0	0	2	1	2	1	2

Chapter Review	13	2	0	9	2	2	11	5	8
Put On Your Thinking Cap!	3	0	1	1	1	1	2	0	3
Number of Questions	54	13	5	20	16	11	43	15	39
Percentage (%)	100	24.1	9.3	37	29.6	20.4	79.6	27.8	72.2

Table 8 Distribution of the number of items by analysis results for Singaporean mathematics textbook grade 4

Representations Forms

There are four forms of representation classified, namely Purely Mathematical (A1), Verbal (A2), Visual (A3), and Combined (A4). Tables 7 and 8 show that the number of task items in Indonesian mathematics textbooks on angle content is 81. In comparison, the number of task items in Singapore mathematics textbooks is 54.

Table 7 shows that the task items included in representation forms in Indonesian mathematics textbooks are more dominated by the purely mathematical (A1) category than the verbal (A2), visual (A3), and combined (A4) categories. There are 29 items (35.8%) in the purely mathematical category. This category in Indonesian mathematics textbooks is widely spread in *Ayo Mencoba* activity, namely as many as 16 task items, and the rest is only spread in *Ayo Menalar* activity (1 item) and *Latihan Soal* or Practice Questions (12 items). The purely mathematical category in Indonesian mathematics textbooks can be exemplified on page 191 in *Ayo Mencoba* (Fun to Try).

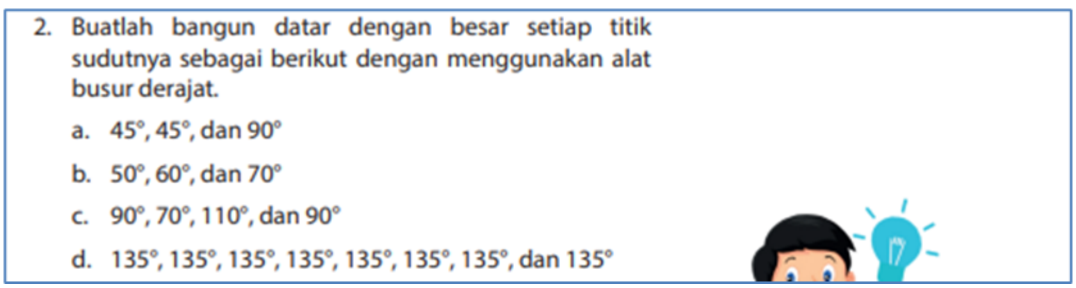

2. Buatlah bangun datar dengan besar setiap titik sudutnya sebagai berikut dengan menggunakan alat busur derajat.
a. 45°, 45°, dan 90°
b. 50°, 60°, dan 70°
c. 90°, 70°, 110°, dan 90°
d. 135°, 135°, 135°, 135°, 135°, 135°, 135°, dan 135°

Figure 1. Sample item of a purely mathematical task in the Indonesian math textbook

Translate:
Make a shape with the size of each angle as follows using the protractor tool.
a. 45°, 45°, and 90°
b. 50°, 60°, and 70°
c. 90°, 70°, 110°, and 90°
d. 135°, 135°, 135°, 135°, 135°, and 135°

After the purely mathematical category, the task items in the form of the representations are spread into other categories in this Indonesian mathematics textbook, namely 22 items (27.2%) for the visual category and 19 items (23.5%) for the combined category. The least number of items found in the verbal category is 11 items (13.6%).

The findings are different for Singaporean mathematics textbooks shown in Table 8. The task items included in the representation forms in this textbook are more dominated by the visual category (A3). There are as many as 20 (37%) visual task items among the 54 task items in Singaporean mathematics textbooks. The visual category in Singaporean mathematics textbooks can be exemplified on page 97 of the Chapter Review.

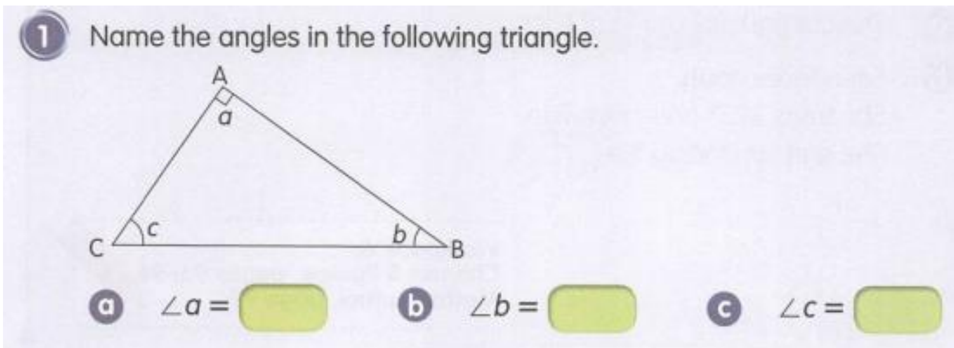


Figure 2. Sample item of a visual task in the Singaporean math textbook

As with Indonesian mathematics textbooks, the verbal category (A2) is the lowest than other categories (5 items or 9.3%). The combined category (A4) consists of 16 items (29.6%), and the purely mathematical category (A1) of 13 items (24.1%).

Contextual Features

Two contextual features are classified: Application (C1) and non-application (C2). Application is a problem presented in the context of a real-world situation. In contrast, non-application is a problem not related to a practical background in everyday life or the real world. Table 7 shows the distribution of contextual features in each type of activity in Indonesian mathematics textbooks. In contrast, Singaporean mathematics textbooks can be seen in Table 8.

Table 7 shows that of the 81 task items in Indonesian mathematics textbooks, it provides contextual feature task items, most of which are classified as non-application. The task items included in this non-application comprised 62 items (76.5%). The distribution of the 62 items is mostly found in the *Latihan Soal* activity (25 items), then continued in the *Ayo Mencoba* activity (22 items), the rest are scattered in other learning activities, except *Ayo Mengamati*. The non-application category in Indonesian mathematics textbooks can be exemplified on page 193 of the *Latihan Soal*.

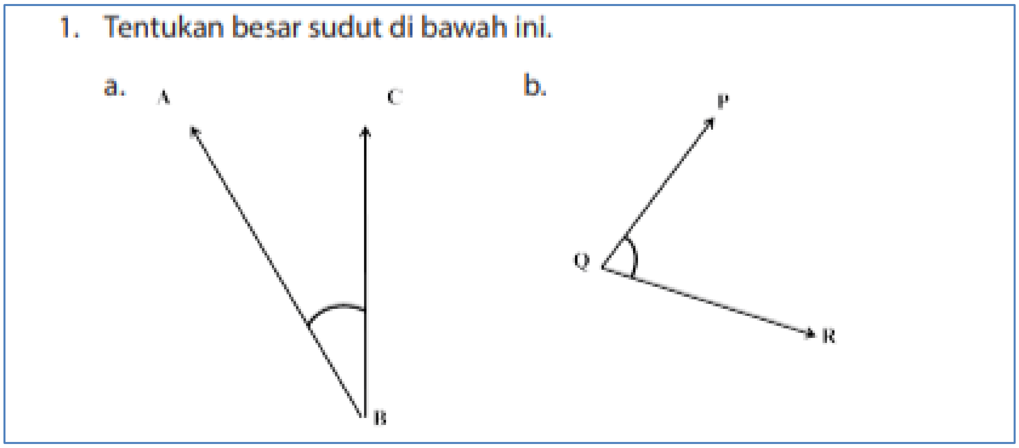


Figure 3. Sample item of non-application task in the Indonesian math textbook

Translate:
1. Determine the size of the angle below.

The application category in the Indonesian mathematics textbook consists of 19 items (23.5%), with the most shares spread out in the *Ayo Menalar* (7 items). The rest are scattered in other learning activities except the *Ayo Menanya* and *Tugas Proyek* sections.

As with Indonesian mathematics textbooks, most of the problem items in Singaporean mathematics textbooks are also classified as non-application (see Table 10). Of the 54 task items in Singaporean mathematics textbooks, there are 43 (79.6%) non-application task items. The most distribution is in Guided Practice activities (17 items), followed by Chapter Review (11 items). The rest are nine items for Learn, two items for Before You Learn, Hands-On Activity, and Put On Your Thinking Cap!

Similar to Indonesian mathematics textbooks, the application category in the Singaporean mathematics textbooks has fewer items than category C2 or non-application. The application category in the Singaporean mathematics textbook can be exemplified on page 98 of the Chapter Review.

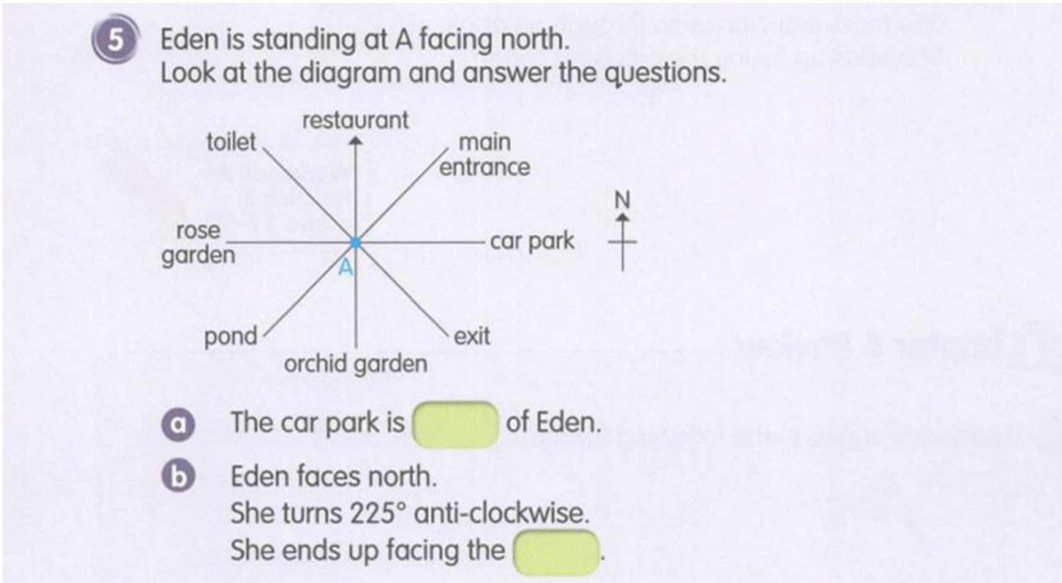


Figure 4. Sample item of application task in the Singaporean math textbook

Response Types

There are two response types: open-ended (E1) and close-ended (E2). Based on Table 7, it can be seen that the task items included in the response types are dominated by the close-ended category in Indonesian mathematics textbooks. Sixty-five items (80.2%) belong to the close-ended category out of 81 task items in this textbook. The most items section is found in the *Latihan Soal* activity, which is 24 items, and the rest is spread out in other activities except *Tugas Proyek*. The close-ended category in Indonesian mathematics textbooks can be exemplified on page 194 in the *Latihan Soal*.

Figure 5. Sample item of a close-ended task in the Indonesian math textbook

Translate:
4. Determine the size of the angle from the measurement results below.

The open-ended category in Indonesian mathematics textbooks consists of 16 items (19.8%). The activity section in the textbook is mostly contained in the *Ayo Mencoba* activity (8 items). The rest is spread on other learning activities except *Ayo Mengamati* and Worked Answers.

Table 8 shows that close-ended categories dominate the task items included in the response types in the Singaporean textbook. This category consists of 39 items (72.2%). The most common items encountered were in Guided Practice activities, with 13 items and the rest scattered in other activities.

The open-ended category in Singaporean mathematics textbooks consists of only 15 items (27.8%), with the most items encountered in Guided Practice activities (8 items) and the rest spread on other activities except Learn and Put On Your Thinking Cap! This category can be exemplified on page 87 in Guided Practice.

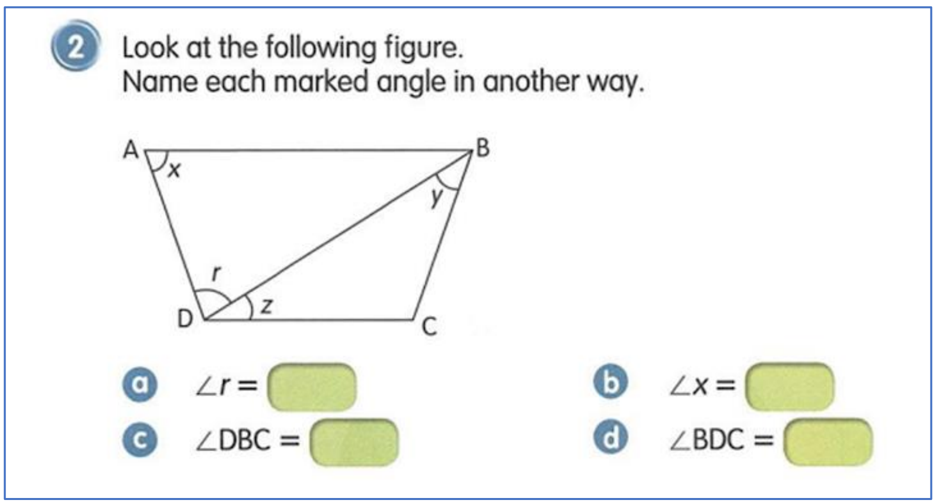


Figure 6. Sample item of an open-ended task in the Singaporean math textbook

According to the findings in each textbook, both are dominated by the closed response type. Nonetheless, Singapore mathematics textbooks provide more proportionate opportunities for students to handle open-ended questions. This interpretation is supported by the percentage comparison achieved by each textbook. The Singapore textbook provides 27.8% of the questions with an open response type, which is more proportional than the Indonesian textbook, which only provides 19.8%.

DISCUSSION

One of the most significant effects on student math comes from textbooks. Both students learning mathematics and teachers planning and delivering mathematics classes use textbooks as essential resources. In many instances, the mathematical problems and exercises provided in textbooks serve as the foundation for and the primary means of delivery for mathematics classroom instruction. Thus, one of the most direct influences on how education is practiced is the use of textbooks (Lepik et al., 2015).

Analysis of tasks in textbooks has the potential to reflect a picture of student engagement with mathematical problems (Yang & Sianturi, 2022). The study of Purnomo, Pasri, et al. (2022) on students' work in dealing with the multiplication of fractions. Students are limited to only realizing that the multiplication of fractions is repeated addition. This limitation causes them to face the multiplication problem of fractions as part of a quantity and other types. This finding also indicates that classroom teacher practices are limited to those contained in textbooks. Other studies also noted findings that corroborated that mathematics textbooks affect teacher instruction in mathematics classes (Hemmi et al., 2014).

Our findings indicate that Indonesian mathematics textbooks are more likely to provide more opportunities to exercise than Singaporean textbooks based on the number and description of task activities. However, the distribution of items in each task activity in Singapore mathematics textbooks is more proportionate to each dimension than that found in Indonesian mathematics textbooks. It is important to note that the effectiveness of a textbook cannot be solely determined by the number of task activities and their distribution. Teaching methodology and teacher activity should consider how evaluating the quality of mathematical tasks in a relevant context.

In the content structure, the angle is the last material taught to grade 4 students in Indonesia, which includes using protractors and applying protractors to measure angles in plane figures. Meanwhile, in the Singapore textbook, angles are studied it after the topics of the numbers and before studying plane figures. Moreover, in contrast to the curriculum in Indonesia, which is limited to only grade 4, the angles and types were previously introduced in grade 3. The mathematics curriculum in Singapore places angles at various grade levels so that the distribution is more evenly distributed for each level, namely grade 4 (understanding, measuring, and drawing angles), grade 5 (angles properties), and grade 6 (measuring angles in geometric figures). This approach allows students in Singapore to have a deeper understanding of angles and their properties, which is essential for higher-level mathematics. By introducing angles at different grade levels, the curriculum ensures that students have a solid foundation before moving on to more complex concepts.

The results of the vertical analysis of mathematical tasks in the two textbooks show that Indonesian and Singaporean mathematics textbooks have similarities in the dominance of categories, especially in 2 dimensions: contextual features and response types. Contextual features are dominated by non-application, and close-ended forms dominate the response types. It becomes clear that these two characteristics are interconnected, with closed forms frequently dominating non-application math tasks and vice versa, making it challenging to solve problems involving application features in closed tasks. Therefore, an additional empirical study is required to examine how the two are related

The contextual features of Indonesian and Singaporean mathematics textbook task items present more task items in the non-application category, that is, problems that are unrelated to practical backgrounds in everyday life or the real world. This indicates a potential gap between mathematics education and its application in real-life situations, which may affect students' ability to transfer their mathematical knowledge and skills to solve practical problems. Similar findings can be identified in studies Purnomo et al. (2019) on middle-grade geometry materials in Indonesia and algebra in elementary schools in Indonesia and Singapore (Yang & Sianturi, 2022) which is more dominant in tasks with an intra-mathematical context. The opinion Gracin (2018) corroborates the statement that intra-mathematical tasks (non-application tasks) dominate textbook task items more. Further, according to Gracin (2018), almost all textbook tasks that include non-application contexts require low cognitive demands. This suggests that teachers should be trained to incorporate both types of features (i.e. non-application and application) into their lessons to ensure a well-rounded education for their students.

Teaching angles will be more meaningful if teachers can help students grasp what they are studying, identify their learning objectives, and recognize that the problems they experience are close to their environment. Some expert opinions state that mathematics comes from social and environmental needs (Ernest, 1991, 1998; Purnomo et al., 2016). Therefore, it is appropriate to present mathematics in the context of social and environmental needs in teaching design and when students interact with the textbook. This approach to teaching mathematics can help students understand the relevance and practical applications of mathematical concepts in their daily lives. It can also promote a deeper appreciation for the subject and increase student engagement and motivation. Textbooks mediate the design of teaching and how students are involved in it (Rahmawati et al., 2020).

It is important to initiate the problem-solving process with real-world problems, as it acquaints students with activities related to mathematical representation and modelling. The process of mathematical modelling begins with a real-world problem, which is subsequently formulated in mathematical terms. After the mathematical problem is solved, the solution must be interpreted to provide an answer to the real-world problem, and it should be verified for its adequacy (Rafiepour & Farsani, 2021).

Another finding, task items in both textbooks are equally dominated by close-ended categories. 80.2% of task items in Indonesian mathematics textbooks and 72.2% of task items in Singapore textbooks belong to the close-ended category. These results correspond to some of the findings of previous studies (Fan & Yan, 2000; Yang & Sianturi, 2017) that close-ended is most commonly found in problems found in mathematics textbooks. This makes students have more experience in solving problems with close-ended response types and relatively less experience they gain on problems with open-ended response types. This type of close-ended response tends to emphasize a low level of thinking, so that students' argumentation and reasoning skills cannot develop optimally (Gracin, 2018). The same is expressed by Yang and Lin (2015) that students who have too much experience with close-ended problems and have limited experience with open-ended problems can cause them difficulty in solving open-ended problems. Moreover, the limited focus on closed-ended problems directs classroom teaching to mechanistic teaching and is teacher center in which the focus of proposing problems is more on result orientation than process. This often leads to a deadlock strategy and, in turn, does not enjoy the beauty of mathematics.

Elementary school students need to be given more opportunities to handle open-ended type task items and to discuss non-unique mathematical problems. This is intended to provide more opportunities for students to try to solve problems with higher-level mathematical thinking (Cai, 1995) and help foster students' divergent thinking skills, including fluency, flexibility, and originality of their response types (Kwon et al., 2006). Thus, the philosophy of humanizing humans can be implemented. Basically, humans have their explorations to achieve goals, so teaching supported by open-based problems allows students to be more creative, not easily discouraged, think critically, and finally become problem solvers.

Our findings, which state that the two textbooks have similarities in the dominance of contextual feature categories toward non-application and the type of response towards closed-ended responses, contradict the results of international surveys in both countries. However, it is interesting that although textbooks are very closely related to how teacher instruction is guided and how students learn, many factors influence the performance of the two countries in the survey results, including Indonesia's and Singapore's culture and education systems. Therefore, future comparative studies can target how cultural reviews are in textbooks and curricula. Apart from that, expanding the scope of the unit of analysis of the two textbooks can strengthen the generalization of the findings so that future researchers can complement the findings of this study by using a broader topic of mathematics.

A striking difference from our findings is the opportunity to learn angles in representational variation. The distribution of most task items in the Indonesian mathematics textbook is more toward the purely mathematical category. In contrast, in the Singaporean mathematics textbook, the task items in the form of representations are dominated by the visual category. The teaching implication is that Singaporean students are given more opportunities to learn with visual objects that they can imagine to help them understand concepts (Yang & Sianturi, 2022), whereas the Indonesian textbook emphasizes the formal aspects of mathematics and is dominated by procedural knowledge. This is in line with the volume and variety of issues, which emphasize more frequent formal mathematics exercises that commonly dominate Indonesian mathematics instruction (Purnomo, 2015, 2016) and are depicted in the mathematics textbooks (Purnomo, Shahrill, et al., 2022). Some researchers agree that procedural knowledge that is not based on strong conceptual knowledge can cause stagnation in problem-solving, and it is difficult to evaluate the location of procedural errors carried out (Byrnes & Wasik, 1991; Purnomo et al., 2014) and also, in turn, causes frustration.

As the thinking stages of elementary school students are still concrete and the characteristics of corner topics that require visual illustrations, visual representations help students and teachers develop the concept of this topic more. Sweller et al. (1990) stated that mathematics curricula (e.g., textbooks) should allow students to solve any [representation] problem and facilitate students' conceptual understanding. National Council of Teacher Mathematics (2000) emphasize that students must be exposed to all kinds of representations of problems in learning mathematics. Therefore, textbooks must provide students with sufficient opportunities to practice solving all kinds of problem representations consisting of task items in the form of purely mathematical, verbal, visual, and combined categories to ensure that students can understand the fundamental structure of different types of questions representation.

Although our findings do not specifically focus on the mathematical activities presented in textbooks, at least three focuses of our analysis relate to these competencies. In several contexts, the findings in the sample analysis found several cases that were significantly different. For example, in case A2 in Table 1, the Singapore version involves students in the action of identifying and measuring the angles, whereas the Indonesian textbook asks a general question quite removed from the process. Furthermore, example C1 in Table 2 also shows that the Singapore problem involves the student directly in the process of solving it, whereas the Indonesian example describes the situation and gives the task. Again, the distance between thinking and doing is larger in the Indonesian example than in the Singapore example, so students can get engaged much easier. The Indonesian questions have a more theoretical flavor, while the Singaporeans are more practical and engaging through action. To improve education, we must look for what engages students the most. Future research could focus on how mathematical activities are presented in textbooks to directly identify the expected directions of teaching practice.

CONCLUSIONS

This study analyzes and compares Singapore and Indonesian mathematics textbooks based on the general characteristics of the textbooks and the nature of the mathematical task on the angle topics. The research findings show that the two textbooks introduce the angle topics in different ways. In Indonesian mathematics textbooks, the angle topics are introduced at the end of the semester, while Singaporean textbooks introduce them in the middle of the semester. Indonesian textbooks have more types of task activities than Singaporean textbooks. However, the distribution of items for each dimension in Singapore book task activities is more proportional.

As for other findings in this study, namely that the task items in Indonesian mathematics textbooks are still dominated in the purely mathematical category, while Singaporean mathematics textbooks are more dominated in the visual category. In other words, Indonesian mathematics textbooks place more emphasis on exercise more often, whereas Singaporean textbooks are more oriented towards conceptual knowledge. To facilitate students and teachers recognizing problem patterns with different representational variations, textbooks must be proportionately constructed with consideration to various mathematical representations.

Lastly, both textbooks are still lacking in the application category and present far fewer forms of answers in the open-ended category. These findings have implications for students' experiences in solving problems that are presented contextually, and more complex problems are less honed, problem-solving tends to be carried out without being based on realistic conceptual knowledge, and the emphasis is more on tasks on tasks with low order thinking skills level.

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