Mathematical Modelling, Integrated STEM Education and Quality of Education for Linear Algebra and Vector Calculus Courses

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Abstract: Articulated pedagogical and didactic strategies allow improving academic quality and strengthen individual and institutional cognitive processes such as student retention. Mathematical modelling, articulated with complementary actions, contributes to teaching and learning processes within situations that go beyond the classroom since they are permeated by social, political, and cultural environments that globally define pedagogical work. Therefore, the objective of this article was to visualize the progress of the implementation of the mathematical modelling of integrated Science-Technology-Engineering-Mathematics (STEM) education and classroom teaching to improve the academic quality of Linear Algebra and Vector Calculus courses at Universidad de Bogotá Jorge Tadeo Lozano (Utadeo). Through a mixed methodology composed of three phases – one qualitative, one implementation, and one quantitative– the theory and practice of academic work were guided, and the strategy, classroom implementation, and work materials were designed. The results obtained, in terms of appreciation of the work in the classroom and publications, evidence the articulation of various efforts and institutional, disciplinary, personal, technological, and theoretical resources in the context of changing realities.

INTRODUCTION

The etymology of the word "quality" defines it as an abstract concept not applicable to something substantial, it is an intrinsic attribute of something that has certain characteristics. For Daros (2014), this attribute can be based on an entity, be it a person, thing, or event, which will be given a qualification related to its functional mode of being. The Aristotelian conception of quality defines it through the category of substance, stating that it cannot be spoken of without a subject, which performs or undergoes an action, whether of quality or quantity. Plato in the Protagoras, the Republic, and the Laws defines education as an essential instrument for the development of society and the State, implying that education must be of quality. Ballén (2010) highlighted that the starting points for the construction of the virtuous man are education and the academic world.

Critical, sensitive, and liberal citizenships are born from a social state of law that guarantees education, among others, as a fundamental right through public investment and State management of curricular content. At a specific moment, the actor that fosters and guides the content of educational policy through reforms and modifications is usually the leading government (Vázquez, 2015). These reforms seek to meet the educational needs of society as a priority, joining efforts that serve to expand coverage in remote regions of a nation. Likewise, its purpose is to expand intellectual capital, focusing exhaustively on the quality of education to achieve an advanced technological, industrial, environmental, and agrarian society (Cristia & Pulido, 2020; Castellar & Uribe, 2004).

The contribution of education to the development of these fields has been fundamental and it is, without a doubt, unquestionable if said growth is broken down into the educational levels of each population at the national, regional, departmental, or municipal level. In relation to this, Loaiza & Hincapié (2016) indicated that economic theory has been adjusting to the so-called economics of education, through public policies that converge in its development and promotion. In this sense, Martín (2018) asserted that business practice is based on quality standards related to market expectations. That is why the quality of education, in this area, is understood as the measurement of results. Likewise, the concept of quality comes from the neoliberal conviction that the primary task of educational institutions is not predominantly focused on education, but rather on training people to perform business functions of immediate application (Martín, 2018).

When referring to the quality of education, the concept of evaluation cannot be ignored, because there must be a measure that assigns non-generalized or homogenized standards, in specific social contexts, to the way in which said measure will be extended as a reference to the community. However, it is important to note that these standards present a bias when the actors involved seek to meet their expectations. As Cano (1998) states it,

> the concept of quality is subjective above all since each consumer or user has a different idea of what it means. However, everyone agrees that we talk about quality when we see all our expectations met, whether it is by a product or a service (p.31).

Similarly, Mosquera (2018) emphasized that the concept of quality of education has roots in the business environment, in which quality must be evaluated according to production. However, Mosquera indicates that when comparing the manufacture of products with the educational training of people, and by linking both activities as if one were the consequence of the other, the true goal and purpose of education is obscured.

The Colombian Ministry of National Education (MNE) defined the concept of quality in higher education academic programs

> as the set of articulated, interdependent, dynamic attributes, built as references by the academic community, which respond to social, cultural, and environmental demands. Such attributes allow internal and external evaluations of the institutions to promote their transformation and the permanent development of their training, academic, teaching, scientific, cultural and extension work (2019, p.4).

In Colombia, quality of education is measured with the results of standardized government tests, among others, (Melo-Becerra et al., 2021) and with student dropout rates (Ministerio de Educación Nacional, 2020). The standardized tests are called Saber tests and are applied to all students in the country under the same conditions –instrument, time, and number of questions– in accordance with their educational level (Instituto Colombiano para la Evaluación de la Educación, 2021). Particularly, at the end of basic education (primary, secondary, and mid-secondary) the Saber 11 tests, whose theoretical range of results goes from 0 to 500, are applied. The tests have a math component whose maximum score to achieve is 100 points (Instituto Colombiano para el Fomento y Evaluación de la Educación Superior, 2020). On the other hand, at the end of higher education, the Saber Pro tests, whose results range is between 0 and 300, are applied. Table 1 summarizes the scores of the two tests applied annually at the national level for the period 2018 to 2021 (Instituto Colombiano para la Evaluación de la Educación, 2022) and it shows that the overall and the mathematics results are close to the middle of the maximum possible score, which is evidence of the academic capital that students have at the beginning of higher education.

Table 1: Average results in Saber 11 for the math component, the overall score and number of students per application.

	Math Score	**Global Score**	**Number of students**
2021 2	49.66	246.02	532,979
2021 1	61.23	301.82	15.527
2020 2	51.17	249.01	504,871
2020 1	58.62	289.43	15,434
2019 2	50.52	245.34	546,211
2019 1	56.90	281.56	21,082
2018 2	50.19	250.77	549,933
2018 1	63.79	319.06	12,527

Additionally, the dropout rate by cohort is monitored to measure the long-term dropout of students who entered in the same period (Ministerio de Educación Nacional, 2009; Ministerio de Educación Nacional, 2015). This indicator for the 2016-2020 period was 9.83% and 8.38% for public and private universities, respectively (Asociación Colombiana de Universidades, 2022).

Consequently, both the initial academic conditions and the dropout rates are a challenge for any Higher Education Institution (HEI). In this sense, there are many parties interested in providing possible academic, administrative, and analytical alternatives to face the challenges (Guzmán et al, 2021; Rincón & Espitia, 2020; Radinger et al., 2018; Ministerio de Educación Nacional, 2009, 2015). The MNE (2015) recommended improving academic quality by strengthening training spaces, academic activities, and teaching resources in accordance with the specific characteristics of the institutional population (Ministerio de Educación Nacional, 2015).

Additionally, Decree 1330 of 2019 of the MNE modified the conceptual bases for the training processes of higher-level students, moving from a system based on capabilities to one based on outcomes, under the premise that learning outcomes are unequivocal statements of what a student is expected to know and demonstrate by the time they complete their academic program (Ministerio de Educación Nacional, 2019, p. 4). To act accordingly, HEIs must work on: 1) A broad conceptual understanding of learning outcomes; 2) The appropriation and organizational incorporation of the writing of the learning outcomes; 3) The design and the adaptation of syllabus; 4) Formulation and proposal of curricular and extracurricular academic activities according to the learning levels; and 5) provision of assessments throughout the training process from admission to completion with the aim of verifying progress (Ministerio de Educación Nacional, 2020).

The institutional research in Utadeo that arose in this context has as a broad research question: how can the quality of education be improved through modelling the dynamics associated with the indicators of student dropout and the evaluation focused on learning outcomes? This article is part of the research, and its objective is to visualize the implementation progress of the mathematical modelling of integrated STEM education and classroom teaching to improve academic quality at Utadeo mathematics courses.

To fulfil the objective of this article, following this introduction, the reference framework of the integrated STEM education approach, the mathematical modelling, and its relationship with academic quality are described. Next, the implemented methodology is presented, sectioned into three non-consecutive phases: qualitative phase, implementation phase, and quantitative phase considering the courses of Linear Algebra and Vector Calculus of the Academic Area of Basic Sciences and Modelling of Utadeo. Subsequently, the results are detailed in terms of educational practice and quantitative outcomes. Finally, both the discussion and the conclusions obtained are presented.

REFERENTIAL FRAMEWORK

Conceptual Definition of Integrated STEM Education

Considering simultaneously the academic entry conditions, the dropout rates, and the learning outcomes in the context of each HEI, perspectives such as those provided by the STEM approach take a leading role (Clavijo, 2021; Moreno, 2019; García et al., 2019; Vásquez, 2014).

For Moore et al. (2014) it is essential to understand STEM education as an effort to integrate science, technology, engineering, and mathematics through a holistic and complementary approach aimed at finding connections between the topics of these disciplines and the real world. Moore et al. (2014) indicated that it is essential to understand STEM education as an effort to combine and integrate science, technology, engineering, and mathematics through a holistic and complementary approach. STEM education seeks connections between the topics of these disciplines and the real world.

The emergence of the STEM education approach, stated Moreno (2019), is underpinned by the need to work on projects related to topics in these fields of science because they potentiate industrial and economic development at different levels. It should be noted that the National Administrative Department of Statistics (DANE, in Spanish), in its technical bulletin of the training for work module in Colombia for 2021, showed a percentage reduction of 0.9 points compared to the bulletin of the year 2019, and 2.8% in comparison to 2013 in the participation of people in training areas related to engineering (Departamento Administrativo Nacional de Estadística, 2021). These data show a decrease in the interest shown by students in these training programs. The relationship that exists today between academia, industry, and the government makes it a priority to modify the curricular contents to direct them to these issues and thus promote them through the STEM approach. With this, educators would receive the necessary tools to develop and promote these skills in their students, considering the processes of modernization and industrialization, and the global boom in software development (Vásquez, 2014). The digital age which fosters STEM education justifies the implementation of computational tools, both primary and complementary, to improve the quality of education. With them, knowledge is directed to students through diverse forms and spaces that facilitate the handling of information and formulate it in a generalized and concise manner, relate it to current problems and issues and, in turn, allow it to be studied in different contexts making use of numerous platforms and tools which enable progressive digital literacy and the reception of the information contained herein (López et al., 2020).

Additionally, the STEM approach makes it possible to relate social and educational problems, such as student dropout at all levels, and understand them in various geographic, economic, and political contexts (Barragán & Guzmán, 2022). For Santamaría et al. (2020), the empowerment of women derived from the expansion of education makes STEM a driver in the development of societies and the participation of women for this purpose. An educated woman, therefore, will have greater influence in decision-making, allowing her to be part of the construction of an advanced society.

Conceptual Definition of Mathematical Modelling

Modelling is an essential practice for the study of systems and is based on the description of the properties that characterize them (Blanchard et al., 1998). It is defined as an activity that cuts across all disciplines and has different types, such as verbal, physical, mental, and mathematical models (Landriscina, 2013). The latter are worked analytically, that is, due to their complexity they require a computerized system to carry out simulations applying numerical methods (Sayama, 2015). On the other hand, modelling is an essential tool in different social, political, economic, and environmental fields, whose dynamics need to be studied (Landriscina, 2013).

The principle of mathematical modelling is to relate the macroscopic magnitudes, both extensive and intensive, typical of each system (Urquía & Martín, 2013). Such relations are mostly based on mechanistic laws or fundamental laws of nature that can become simple or complex models, depending on the different characteristics of the system such as its capacity for self-organization, its emergent properties, and the level of detail that it provides (Velten, 2009).

Oliva (2019) presents five definitions of modelling that are summarized in Figure 1. To this article, it is appropriate to understand modelling as a teaching strategy.

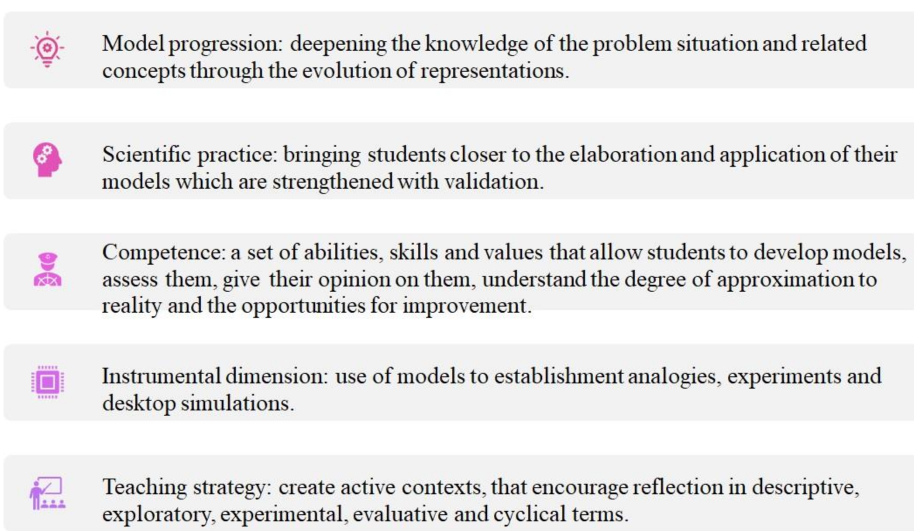

Figure 1: Modelling meanings

Source: Prepared by the authors according to Oliva (2019).

Thus understood, mathematical modelling aims to provide the student with various tools that work to integrate different areas of knowledge. Modelling will promote the understanding of concepts and their relationship with their actual environment, will improve reading and writing skills, enhance their understanding and problem-solving (Purnomo et al., 2022; Villalobos, 2021), and will provide students with skills such as multidisciplinary work and research through constructivist methods applied inside classrooms (Oliva, 2019; Zaldívar et al., 2017). To formulate and apply modelling, Vergel et al. (2020) recommended considering the progressive development of mathematical thinking, based on the training and contribution of teachers to the variational thinking of students, understanding it as a mechanism of dynamic reasoning, whose purpose is to produce relationships between variables that covary through cognitive actions that distinguish between variable and constant magnitudes. Likewise, Cabezas & Mendoza (2016) identified it in the epistemological study of mathematical notions originating in infinitesimal calculus.

Quality of Education, Academic Quality, and Learning Outcomes in Linear Algebra and Vector Calculus Courses at Utadeo

In Colombia, social gaps, inequity, and inequality are the result of factors such as corruption, poor management and unequal distribution of land, lack of State presence, the concentration of wealth, centralization of resources, low institutional capacity, and deplorable levels of education in areas and regions that are far from capital cities (Cristia & Pulido, 2020). Sánchez (2017) stated that the adverse effects of the armed conflict –forced displacement, the extension of unproductive land, and drug trafficking, among others– make Colombia one of the most unequal countries in the Western Hemisphere.

Cataño (1984) highlighted that social inequality is synonymous with inequality of educational opportunities. He emphasizes the differences between classes within our society and the differences in opportunities in each of them, which has a significant impact on the country. However, in compliance with the Millennium Development Goals (MDGs), Colombia has been on a path of transformation for decades towards educational efficiency and quality, safeguarded by the Political Constitution of Colombia of 1991, which is a guarantor of this inalienable right and grants it the necessary benefits for its sustainability and promotion (Congreso de la República, 1991).

Currently, one of the factors of social inequality –understood as inequality of educational opportunities– that was added to the other factors known until now is the COVID-19 pandemic. Herrera (2020) stated that mandatory isolation intensified social disparity for vulnerable and low-income communities, since it made it impossible for them to access education, among others, due to their lack of the technological and communications infrastructure necessary to continue with the training process of girls, boys, youth, and adults at all educational levels.

In accordance with the context described above, Utadeo has provided a wide set of tools for its students to face the difficulties of Colombian society and thus deliver them a quality education that follows the guidelines of the MNE (Universidad de Bogotá Jorge Tadeo Lozano, 2022, 2011). Based on this idea and in the specific case of the Linear Algebra and Vector Calculus courses – which are part of the Basic Sciences and Modelling Area– STEM education has been implemented within the engineering and economic-administrative sciences programs. Given the recognized educational benefits of this approach, diverse technological tools were applied in the courses to facilitate communication with and among students and allow them to access the contents and material designed.

These courses were structured in accordance with the experience acquired in various contexts, including the COVID-19 pandemic. Compared to previous dynamics, this design involved the recognition and exploration of the technological tools that can be used to expand the coverage of the subject's content, diversify the ways in which knowledge is approached, strengthen the cognitive skills of the students regarding the program's content and foster the ability to address everyday problems with an emphasis on modelling and simulation.

METHODOLOGY

The methodology designed to achieve the objective of this article included three non-consecutive phases in line with the STEM educational practice, at the higher level, circumscribed to the Colombian context.

Qualitative face has two stages: Initially, a collection of literature, experiences, and good practices was made to build a frame of reference and a state-of-the-art. Subsequently, the documentation of the progress of the process was conducted along with the selection of an approach within the range of the STEM movement (Tsupros et al., 2008; Oliva, 2019; Margot & Kettler, 2019) so that it responded to the observed characteristics of the population to which it was addressed and to the institutional expectations (Arango et al., 2019; Universidad de Bogotá Jorge Tadeo Lozano, 2011, 2022).

Implementation phase: in this phase, the way in which the classroom teaching was conducted in the Linear Algebra and Vector Calculus courses of the Academic Area of Basic Sciences and Modelling of Utadeo was recorded. These classroom teaching practices were compiled both in a general way and with examples. This work was carried out on several fronts: 1) Updating the teaching skills of the teachers in charge since they require specialized knowledge in multiple aspects (e.g. pedagogy, didactics, and evaluation) (Kelley & Knowles, 2016; Margot & Kettler, 2019; Gros & Cano, 2021); 2) Designing activities, materials and resources in accordance with the cognitive and conceptual domains of each course (Mullis et al., 2009) (teaching practice); and 3) Documenting and recording the teaching and learning process in articles, book chapters, and books (teaching research).

Quantitative phase: in this phase, the pedagogical proposal with the STEM approach, the evolution of the approval rates of each course, the evaluation made by the students of the teaching practice (Universidad de Bogotá Jorge Tadeo Lozano, 2011), as well as the academic production of teachers according to the terms of the Ministry of Science and Technology, were followed and monitored. According to the ministry, the products of the Groups and Researchers are the results that they obtain in the processes of research, technological development, or innovation, and respond to work plans, lines of research, and projects (Ministerio de Ciencia Tecnología e Innovación, 2020, p. 56). Such products must be part of one of the following categories: Generation of New Knowledge, Technological Development and Innovation, Social Appropriation of Knowledge, and Public Dissemination of Science and Training of Human Resources in Science, Technology, and Innovation (Ministerio de Ciencia Tecnología e Innovación, 2020). It should be noted that university professors oversee different activities according to their missionary functions within their HEIs: direct and indirect teaching or research and social projection (Cosenz, 2022; Congreso de Colombia, 1992).

RESULTS

This section presents the results, initially focused on the implementation phase and then on the quantitative phase of the described methodology. It is very important to highlight that the incorporation of the STEM approach and mathematical modelling as its expression in the courses of Linear Algebra and Vector Calculus of the Academic Area of Basic Sciences and Modelling of Utadeo has been conducted for four and two years, respectively, and it is still an ongoing process.

Educational Practice: Implementation of the STEM Approach and Mathematical Modelling in Linear Algebra and Vector Calculus Courses at Utadeo

The implementation of modelling in the courses of Vector Calculus and Linear Algebra –which are part of the training axis of different engineering majors and the economic-administrative sciences of Utadeo in the foundation cycle– is conducted from different perspectives, theoretical approaches, and epistemological conceptions. Two of them will be presented below so that, without exhausting the topic, they will offer an overview of the strategies that support not only the modelling process itself but also its articulation with the general and specific objectives included in the academic syllabus as well as with the purposes of professional training that are based on the Pedagogical Model of the university (Universidad de Bogotá Jorge Tadeo Lozano, 2011). The approaches presented are generically called articulate and sedimentary to describe what happens when one of them is performed. A comparative analysis between some learning styles and performance in mathematics can be expanded in sources such as Ma (2014).

The first approach to be described is articulation. It has been named like this because it considers a general model for a specific phenomenon that articulates the concepts and processes of the discipline and the objects of study. To illustrate the how and why of this approach, the Vector Calculus course is described, and an example of the Cobb-Douglas function is used. This function is explained from different perspectives, over approximately five weeks, using the support of a sequence of seven videos available on the YouTube channel (Aya, 2021), associated with the playlist Vector Calculus - Calculus in Several Variables.

The microeconomic object is articulated for two fundamental reasons:

(1) Contribute to the economic education of citizens (basic understanding of a production model centred on capital and labour) as a priority issue in the treatment of socially relevant or sensitive concepts.

(2) Articulate the specific object of a discipline or context considered as extra-mathematical with various mathematical objects presented in the course (Dolores & García, 2017).

The first reason is understood as part of the comprehensive training of future professionals as citizens and that will allow them to understand and account for some of the social, political, economic, and cultural phenomena of their environment and of global society in general. It will also allow them to reflect on the social and cultural role of STEM education, which has been widely questioned in academic settings. Specifically, they will be able to understand that productivity depends, among other factors, on labour, and capital and that one of them is not enough. That is, they will focus on the tensions between capital (injection of money-resources) and the available labour (staff, job qualification, wages).

The second reason is based on exploring concepts typical of the economic-administrative sciences and taking advantage of the property of the Cobb-Douglas function as a function of the logarithmic-linear type in several variables (particularly ln-linear. Considering P, the total production; L, the amount of labour; and K, the amount of capital invested, thus, lnP is a linear function of lnK and lnL. In a first video, the initial development of the model and the theoretical assumptions that support it. In this process, what is proposed for the modelling phases is addressed through eight steps implemented in the format designed for the courses (which will be mentioned later). Also, technological elements associated with the use of software, such as Excel, are incorporated to perform the least squares adjustment to the proposed model. Within this phase, an analytical process is conducted on what it means to make an adjustment to a model and an analysis from basic elements of the theory of error such as absolute, relative, and percentage error that allow a reflection on the importance of considering aspects of numerical validation of the models and how it broadens the conceptual tools beyond those provided by calculus and which could well require elements of statistics or other analytical disciplines such as those offered by specialized software.
The calculation of the images serves to analyse the behaviour of the Cobb-Douglas function, when varying both capital and labour by a factor C, and shows that productivity is also modified by a factor C to break the intuitive process. If each of the variables is doubled, the function will quadruple since it is in terms of two independent variables. Contrary to what one might think, even in these courses the idea of linearity and direct proportionality is rooted.

The level curves are approached to show that, for fixed productivity, the proportionality between labour, and capital is inversely proportional and of the type

$$K \propto \frac{1}{L^{\alpha}} \tag{1}$$

taking advantage to deal with the technical conditions of the model. To support the visualization and conceptualization of the mathematical object, the use of the GeoGebra software is relevant in this approach. The partial derivatives are treated from the marginal productivities with respect to labour and capital and are interpreted considering the construction of the model and the assumptions that support it.

The conceptual development is directed towards the chain rule and an application is presented in the Cobb-Douglas model to determine the variation of productivity under the assumption that both capital and productivity are functions that depend on time. To conclude with the mathematical objects associated with the study of the function, two examples of Lagrange multipliers are discussed. The first refers to a single restriction on the function associated with capital (linkage of the system); the second deals with the situation in which productivity is constrained. In both cases, some implicit properties of the gradient and the relationship between the gradient of the objective function and that of the constraint are emphasized. It is highlighted that these treatments are not made on specific cases or values, since a generalized treatment of both the microeconomic model (Cobb-Douglas function) and the mathematical model (Lagrange multipliers method) is intended. Figure 2 presents a synthetic diagram of the developed process.

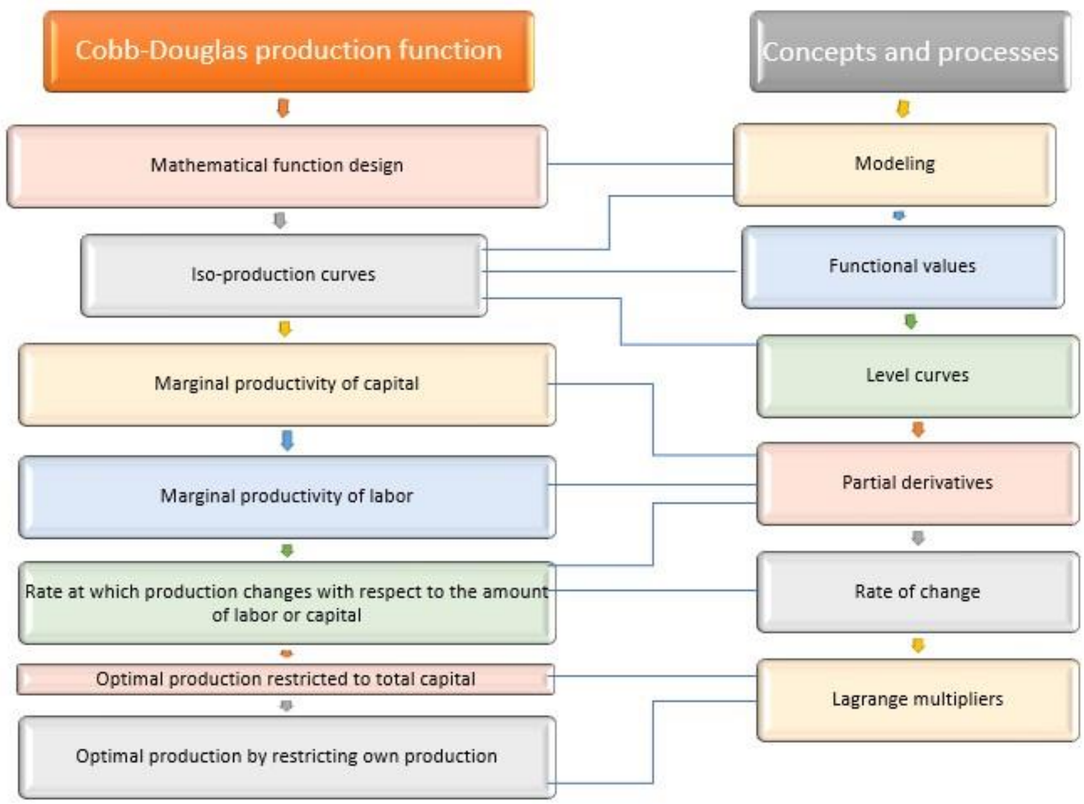

Figure 2: The microeconomic object (model to be analysed), the mathematical concepts and processes.

The second approach illustrated here is called sedimentary and is exemplified for Linear Algebra. It is understood as an action with which a series of processes and concepts are built that serve as the basis or substratum to approach a model in the long term, since its methodology, analysis and interpretation require the stabilization of certain cognitive actions –not only characteristic of human thought– such as generalizing, associating, interpreting, or visualizing. Likewise, they require a procedure that allows the analytical maturation of processes and associated concepts for the construction of tools that enable an informed approach to a situation to be modelled (specific to the area). These processes occur, according to López (2011), after a maturation that, in the case of class work, is associated with aspects such as regularity in working with processes and concepts. Of course, while working with them, an identity is formed and mediated by relationships that are established in various fields such as conceptual, procedural, or analytical. Some of these relationships may go through a state of inactive dormancy that may or may not be beneficial.

In case there is inactivity and this is of little benefit, the actions proposed in the book for Linear Algebra are activated, such as exercises solved in detail (step by step), guided exercises (with steps and key indications), proposed exercises (to be solved by the student, without guidance), use of computer and technological resources (GeoGebra®, Wolfram Alpha®, online calculators), presentations in PowerPoint format (prepared by the teachers on of the subjects for each of the class sections that are in accordance with the syllabus), support videos on two YouTube channels designed by teachers in the area for the specific purposes of the courses, and spaces such as the collaborative forum between students in the Moodle classrooms of the Tadeísta Virtual Learning Environment (Avata).

The continuous reactivation phase of the processes and concepts occurs at the end of each cycle or at the end of the academic term through two specific actions. The first is a partial exam that synthesizes the central work axes related to the mathematical objects addressed and the second focuses on the modelling process.

In the sedimentation phase, the basic elements are built with two structures: the one given by the mathematical objects and the one generated by the modelling process. For the first structure, vectors, transformations in general, linear transformations and their properties, the matrix representation of a transformation, the characteristic polynomial of a transformation, the characteristic equation, the eigenvalues, and the eigenvectors associated with a transformation are addressed. In parallel, for the case of transformations of R^n in R^n, the theorem that formulates the equivalence between: the non-zero determinant of the associated matrix, the existence of the inverse matrix, unique solution for the non-homogeneous system, trivial solution for the homogeneous system, the equivalence of the matrix by rows or columns to the identity matrix, the echelon form of the matrix has n pivots, linear independence of the rows and columns of the matrix, nullity of the transformation zero, the rank of the transformation n, and eigenvalues not null has been established.

For the second structure, it has been emphasized that the approach to both problem situations and modelling itself is intended to be systematized in classes, activities specifically aimed at modelling, as well as in videos of examples, in which the methodology of eight basic steps for the modelling process is developed. For example, in the Linear Algebra course, the work is carried out with a situation that aims to conclude a population model for a species of birds in the orientation of the Leslie Model (Grossman, 2012). For this, a discussion is made about the central hypotheses of the model (closed system, does not admit migrations, unlimited resources in the system for sustenance and survival, the predominant sex for the preservation of the species are females and particularly those that are in fertile age, the reproductive rates of young females per adult female and the survival rate from one period to the next are constant, the population of the next period depends fundamentally on the population in the current period as in the Malthusian Model). In this phase the conditions of the model are highlighted, and its limitations are evidenced to determine its scope to understand what a model is, and the context of its validity is an essential competence for the development of scientific thought, science, and technology in themselves.

Subsequently, in a second step, the unknowns, variables, and parameters are defined, as well as the technical conditions. That is, symbolize the populations of young females and adult females in each period, and in the previous period with non-negative integer values, survival rate for young and adults from one period to the next and number of new young females for each adult female (assumed constant). Next, the matrix A, associated with the transformation of the population, is constructed emphasizing the composition of the population of females, and specifically, the fertile ones, since they are the ones that essentially determine the development of the populations of the species. The composition of the population. The composition of the population constitutes a fundamental aspect of the biological sciences and of the general behaviour of dynamic population systems. With these conceptual elements, the model is established based on the assumptions associated with exponential growth, that is, in

$$P_{n+1} = A^n P_n \qquad (2)$$

The third step corresponds to a population calculation process from a given population P_0 and it is suggested to carry out some initial calculations to strengthen algorithmic processes and with this give way to projections and simulations using the software. To this end, a template was designed that, based on a population of young and adult females and the corresponding matrix A, can be modified within the technical conditions established by the model, allows the population to be measured for several successive periods and yield some results of interest to the model such as the population of total females T_n, and the ratios of the population of young females to adult females in each period

$$\frac{P_{j,n}}{P_{a,n}} \qquad (3)$$

and the ratio of the total population of females in one period to the total population of females in the previous period

$$\frac{T_{n+1}}{T_n} \qquad (4)$$

In the fourth step, two calculation processes are involved, one related to the eigenvalues associated to A and the other to the corresponding eigenvectors. This process can be done manually since, due to the order of the matrix, it leads to a quadratic equation. However, the incorporation of the use of the designed resource in Wolfram Alpha or Symbolab® is suggested. It is included here as it makes it possible to analyse and discuss the mathematical power of the software. Once these values have been evaluated, an analysis of the long-term evolution of the system and its terminal behaviour with the trend of the logistic growth curves is established. The fundamental elements were summarized in an explanatory video that is suggested at the end of the interaction work (Barragán, 2021). The analysis can be extended to a slightly more complex Leslie Model in which different factors are considered, such as the types of populations (infertile youth, fertile youth, fertile adults, non-fertile adults, with their respective survival and reproduction rates) or extended to new phenomena such as predator-prey models or competition models. Any expansion of the analysis has effects on the dimension of the associated matrix and on the use of software, among others, for both the calculation process and the analysis of results (Figure 3).

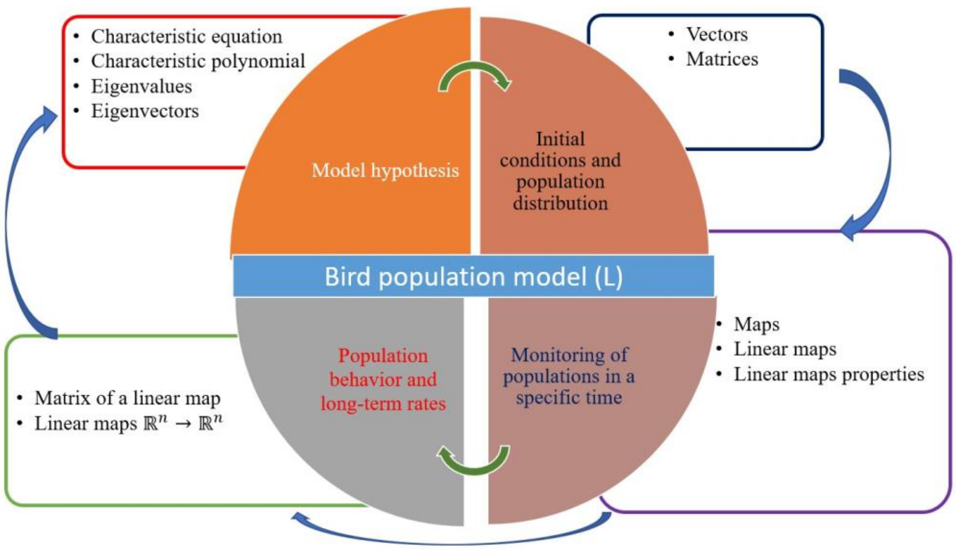

Figure 3: Synthesis of the treatment of the bird population model.

It is essential to mention that the evaluation, within its holistic process, refers to:

(1) The level of academic achievement of students, measured with assessments applied by teachers. A total of sixteen evaluations are applied per academic term with different exam formats, such as quizzes on platforms like Socrative and Kahoot and modelling workshops. Mock tests are used to familiarize students with the settings and types of questions. The questions used in the midterms are standardized so that the level of demand for all groups is the same.

(2) The developed academic processes, measured with instruments designed by administrative instances, filled out by the students about the teachers. Throughout the academic period, Utadeo applies two evaluations, one called Early Evaluation to timely identify aspects to improve in teaching, methodology and evaluation. The second is applied at the end of the course and allows students to evaluate teachers through a structured survey. The results are systematized, and the corresponding administrative instances deliver the results to the teachers, who can access them at any time.

(3) The academic facilities and the support provided by the professors as well as the monitors linked to the teaching practice, through structured formats. The teacher-coordinator jointly evaluates the performance of the teachers in their practice and that of the outstanding students who support them through academic monitoring.

(4) The syllabus and resources used in each of the courses. During and at the end of each academic period, teachers jointly review the achievements made, contrasting them with what is proposed in the syllabus and the relevance of the resources used, fundamentally seeking not only to improve them but also to adjust them to the changing needs and the dynamic conditions of teaching and learning processes.

As mentioned at the beginning of this section, the diversity of strategies or approaches transcends what is summarized here. Such diversity is reflected in the curricular designs, the training and information objectives, the learning approaches, the materials, and the available resources, as well as in the capacity for innovation and reflection on each constitutive element of the pedagogical proposal, which is enriched with the incorporation of articulated elements of the STEM approach to teaching mathematics at university.

Classroom Teaching: Virtual Educational Spaces and Evaluation

As previously indicated, various tools were applied during the development of Vector Calculus and Linear Algebra courses, such as 1) Videos that are specifically designed for the courses, 2) training and peer evaluations conducted on Socrative and Kahoot platforms, 3) modelling situations designed with specific objectives, 4) evaluations implemented in the Moodle platform in the virtual learning environment. The tools were articulated in each specific class session in accordance with the syllabus or following the consensus reached among professors.

Next, the way in which a sequence of classes was developed in the virtual spaces in which the proposal was implemented is described. The case of a Linear Algebra class is presented as an example. Initially, several virtual educational spaces were set in the Avata (a virtual learning environment). The first was the Class material space (Figure 4), in which the general presentations were published four days in advance to guide the students in the topics that would be addressed each week (students had four weekly hours of class accompanied by the professor). The idea was to encourage the students to follow the principle of autonomy.

Figure 4: General scheme of the structure of the course in the Avata and a prototype of a presentation prepared for each of the class sessions.
Note: The course was delivered in Spanish.

A second space, named Support materials (Figure 5), included a base of 51 problems from various application contexts (both interdisciplinary and mathematics). Some were solved as class examples, others in workshops, and others individually by students.

21. Determine el polinomio $p(x) = ax^2 + bx + c$ que satisfaga las condiciones $p(1) = f(1), p'(1) = f'(1)$ y $p''(1) = f''(1)$, donde $f(x) = e^{-5x}Cos(\pi x)$.

12. Un dietista desea planear cierta dieta con base en tres tipos de alimentos. Una onza de cada tipo de alimento contiene unidades de proteínas, carbohidratos e hierro como aparecen en la siguiente tabla.

Unidades de	Alimento I	Alimento II	Alimento III
Proteínas	2	3	2
Carbohidratos	3	2	1.5
Hierro	0.5	2	1

Si la dieta debe proporcionar exactamente 25 unidades de proteínas, 24 unidades de grasa y 21 unidades de carbohidratos, ¿cuántas onzas de cada tipo de alimento deben utilizarse?

Figure 5: Interdisciplinary or mathematical context problems prototypes.
Note: The course was delivered in Spanish.

The space named Computer resources (Figure 6) is the inventory of videos that are used for three purposes: 1) Exemplifying specific aspects of the mathematical objects addressed, 2) supporting students and help them deepen in some respects, and 3) supporting the modelling process. The activity that uses the inventory is conducted following the scheme referred to above. The activity is used to collect the written production of the students and help them develop skills and competencies for modelling in different contexts and problem situations.


Cuarta situación problema: Ejercicio adaptado del libro de Barragán, Melo y Aya (Barragán, Melo, & Aya, 2021, pág. 96).
A partir de una mezcla de 4 gramos de uvas pasas rubias, 6.5 gramos de maní sin sal y 14 gramos de almendras y de otra mezcla de 6 gramos de uvas pasas rubias, 5 gramos de maní sin sal y 5.5 gramos de almendras se quiere obtener un nuevo producto con las siguientes condiciones: 1 gramos de uvas pasas rubias, 6 gramos de maní sin sal y 12 gramos de almendras. ¿Es posible hacer esta mezcla de mezclas con las especificaciones dadas? Sugerencia: consultar Mezcla de mezclas: https://youtu.be/KOmGN19tkzM (5:12)

Figure 6: Elements of the proposal: Modelling process and resources available on YouTube.
Note: The course was delivered in Spanish.

Figure 7 shows an open question exercise for a linear transformation that produces changes in the polygon appearance and the answer elaborated by a student. The answer to the question required calculations, graphs, interpretation, and the student's understanding of the context.

Primera situación problema:
Se tiene una transformación

$$T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$$
$$\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

y se tiene un polígono cuyos vértices son $A(3,2), B(4,0), C(-2,3), D(-4,0), E(-1,-1)$.

Determine e interprete los eigenvalores y los eigenvectores asociados a la transformación. Realice la transformación aplicada al polígono e interprete los resultados obtenidos.

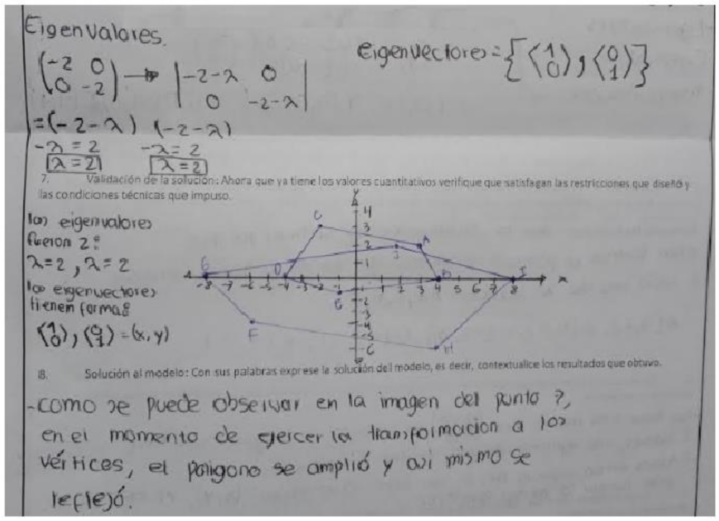

Figure 7: Example of a modelling phase with a fragment of a student's production.
Note: The course was delivered in Spanish.

Classroom Teaching Results

The implemented methodology received high levels of approval. In 2022, 242 students from first and second semesters enrolled in the Linear Algebra course (176) and in the Vector Calculus course (66). 218 approved (90%) and 24 failed (10%). The average grades for these two categories were 3.8 and 1.6 correspondingly, on a 0 to 5 scale in which the minimum passing grade for each course was 3.0.

To understand the levels of approval of the courses, it is essential to discriminate the characteristics of the academic results. According to Figure 8, in the Linear Algebra course, the students achieved an average grade of 3.8 in the two semesters of 2022. Also, in both semesters, 25 % of the students obtained a grade below 3.5. In the case of the Vector Calculus course, a significantly different behaviour was identified: in the first semester of 2022, an average grade of 3.7 was observed, while in the second semester, the average grade was 4. The students who failed some of the courses were classified as outliers and were located below the respective interquartile ranges in each semester.

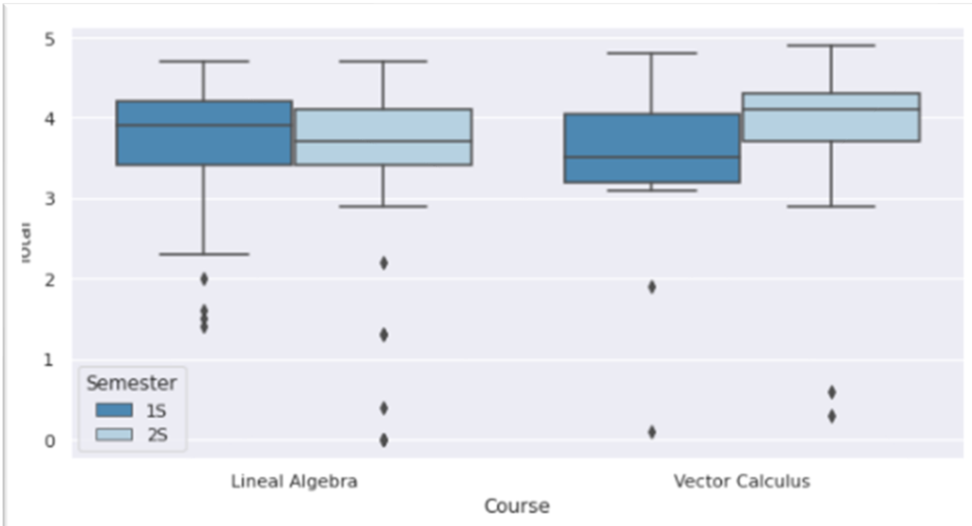

Figure 8: Grades distribution in 2022.

Using the STEM approach oriented to mathematical modelling has made it possible to condense the set of results obtained in the approval category. Figure 9 shows the significant relationship between the grades of the modelling test and the results obtained in each course. A correlation was observed in most cases that was higher than 50% in the first semester of 2022. An analogue behaviour was observed in the second semester. Such appropriation of the concepts makes it possible to relate reaching the general objective of the mentioned courses as the acquisition of knowledge applied to a real-life problem.

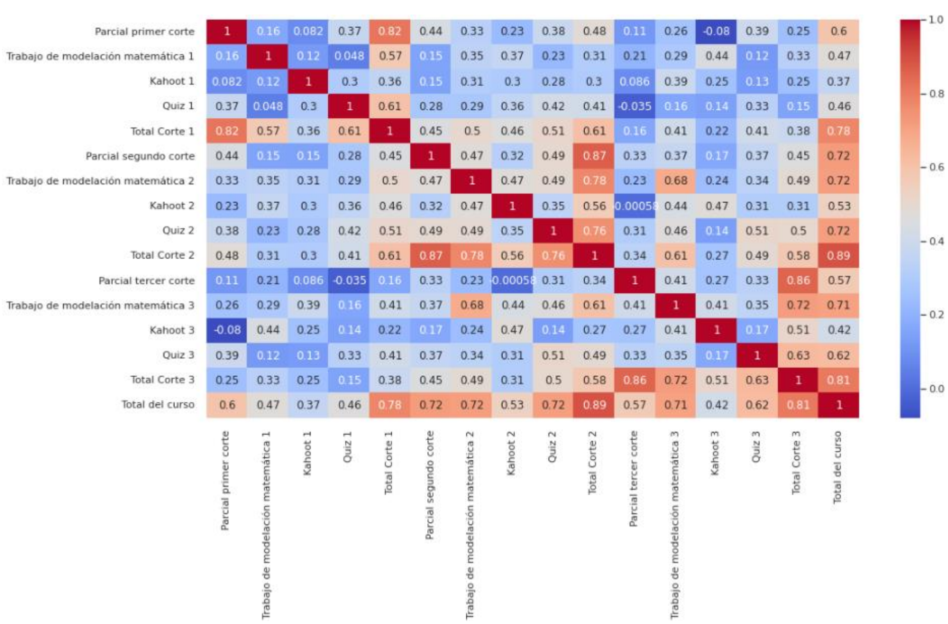

Figure 9: Correlation matrix for evaluation activities in the first semester of 2022.
Note: The course was delivered in Spanish.

Results in Terms of Didactic Material Production and in Pedagogical Research

Table 2 summarizes the results of the preparation of the material that condenses the pedagogical proposal and methodology from the STEM approach and from the adopted expression that is mathematical modelling. It is important to emphasize that this material has several direct interlocutors such as the students, the professors who oversee the courses, the monitors, the teaching interns, and the subject coordinators, who support the development of the academic work and therefore have different orientations and scopes.

Table 2: Results of the preparation of the material on the STEM approach.

Material produced	Linear Algebra	Vector Calculus
Slide presentations	13	13
Instructional brochure for assessment	9	9
Assessment item on institutional Learning Management System	352	273
Assessment test	9	9
Textbook	1	0
Protocol for professors	2	2
Test through platforms (Kahoot or Socrative)	11	8
Files with simulations, projections, or animations	3	65
Explanatory videos	93	102
Virtual classrooms	1	1

On the other hand, the academic production of the professors of the courses that underpins and supports all the academic work from the point of view of pedagogical research is gathered in Table 3.

Table 3: Results of the pedagogical research.

Academic production	Total
Published research articles	2
Submitted research articles	2
Textbooks	1
Published research book chapters	2
Research projects	3
International conferences	2

Regarding the evolution of the approval percentages for the first academic period of 2022, consisting of 16 weeks of classes, it was found that 83.09% of the Linear Algebra students passed the course with an average of 3.9 on a scale of 0.0 to 5.0. While, in the Vector Calculus course, the passing rate was 84.61% with an average passing rate of 3.7. To these results it can be added that, in the first academic period of 2022, 226 hours of extracurricular academic support were offered for each course, which corresponds to 14 hours per week on average.

The evaluation of the academic and general processes developed by the professors was answered on average by 61.6% of the students. The average rating was 4.27. The following comments from students were highlighted:

1. Real life problems are studied, in which we applied what we learned in class.

2. Studying algebra with "real" life problems is a different way of understanding algebraic problems.

3. We did many exercises and we tried to make the most difficult, this allows us to understand a little better the cases in which the problems are very complex and to develop the simplest ones more easily.

CONCLUSIONS

At Utadeo, an eight-step sequence applicable to exercises related to modelling, and simulation was developed, guided by the application and development of conceptual, analytical, and computational procedures. This sequence provides students with a mechanism that allows them to systematically break down the proposed application problems, aimed at achieving the general and specific objectives established in the Linear Algebra and Vector Calculus courses.

Implementing an integrated STEM approach has led to the applying graphical and algorithmic tools that have fostered or developed new skills and competencies in students oriented toward the learning outcomes of these subjects. Academic work in this approach is not a finished matter, it requires constant updating of teaching skills and the design of appropriate pedagogical and didactic material and educational research products that support the process. Therefore, the teaching and learning processes are carried out in the long term and in a sustained manner.

Additionally, many of the stages of the teaching and learning process require specific and timely evaluation methods that offer an opportunity for improvement at the right time. Thus, the holistic evaluation within the academic process has made it easier for all interested parties to recognize the degree of progress and identify the achievements to be attained. The way in which the knowledge imparted in the Linear Algebra and Vector Calculus courses is evaluated has highlighted the relevance of using virtual tools and environments for these processes, since they diversify the structure of the items, their scope, and the possible answers, which, therefore, has led to a satisfactory evolution of the evaluation of academic results.

What has been described so far demonstrates the fulfilment of the objective of visualizing the progress of the implementation of the mathematical modelling of integrated STEM education to improve academic quality in Utadeo mathematics courses. At the same time, it was evidenced that the implementation of mathematical modelling is not a completed stage but rather requires academic work that must continue to be strengthened because it requires the maturation of teaching skills and students' competencies and learning outcomes. The integrated STEM approach has not only resulted in a decrease in dropout rates and repetition but also, by integrating various methodological and strategic elements, has allowed students to integrate the interdisciplinary knowledge articulated by math skills. Simultaneously, this approach has increased teaching sensitivity towards their practice and towards written academic production, with which their role as university professors and researchers has been strengthened.

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