Overview of Student’s Mathematics Reasoning Ability Based on Social Cognitive Learning and Mathematical Self-efficacy

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Abstract: Detailed mathematical reasoning abilities can help students to understand higher mathematical abilities, including proving, problem-solving, and critical thinking. However, based on surveys and research, students' mathematical reasoning abilities are low and require significant attention. Therefore, this study aims to gain a detailed picture of the mathematical reasoning abilities of students who got the social cognitive learning (SCL) model and students who got the problem-based learning (PBL) model by considering the students' mathematical self-efficacy (MSE). The study used a quasi-experimental Nonequivalent post-test-only group design with 70 students from class 11 SMA in one school in Bandung. The data collection used a Mathematical Reasoning Ability test and a mathematical Self-efficacy Questionnaire to classify MSE levels as low, moderate, or high. The data were analyzed using two-way ANOVA and a 3x2 factorial design. According to the study results, students taught using the SCL model had better mathematical reasoning abilities than those taught using the PBL model. Moreover, students with high MSE levels exceed low MSE levels in math abilities. The SCL model enhances students' mathematical reasoning abilities and expands the range of social cognitive theory's applicability of mathematics.

Keywords: Social Cognitive, Mathematical Reasoning Ability, Mathematical Self-efficacy, PBL

INTRODUCTION

The reasoning skills are needed to support higher mathematical abilities such as proof, problem-solving, and critical thinking (Krıstayulita et al., 2020; Öztürk & Sarikaya, 2021; Putrawangsa & Patahuddin, 2022). Mathematical reasoning allows students to acquire ideas, properties, and methods as logical, interconnected, and cohesive parts of mathematics, not just ordinary routines. Mathematical reasoning ability is an essential component of education, mainly needed to understand mathematics. According to NCTM (2014) and PISA (2018), mathematical reasoning ability is a way to evaluate and make arguments, evaluate interpretations and conclusions related to statements and problem solutions. Thus, mathematical reasoning skills are essential to developing at the primary, junior, and senior secondary education levels by placing students in situations where they can make, correct, and test their conjectures (Mansi, 2003; NCTM, 2020).

Students in high school are expected to be able to explain, verify, justify or validate to convince themselves, other students, and teachers about the truth of mathematical statements (Fiallo et al., 2021). If their mathematical reasoning abilities are not developed, they will see mathematics as a specialized set of rules, a collection of calculations and images produced without thinking (Payadnya, 2019). As a result, improving students' mathematical reasoning abilities in the classroom is an essential component of mathematics teaching and learning (Boaler, 2010; Mata-pereira & Ponte, 2017).

Research reports show that mathematical reasoning skills are essential in a deep understanding of mathematics (Herbert et al., 2022; Zhang & Qi, 2019). Promoting mathematical reasoning skills will also allow students to more easily understand math problems (Saleh et al., 2018; Supriadi et al., 2021) and geometric (Payadnya, 2019; Seah & Horne, 2021). Facts show that individuals with high mathematical reasoning abilities can more easily understand and solve mathematical problems (Erdem & GÜRBÜZ, 2015; Hasanah et al., 2019; Irawati & Hasanah, 2016; Rizqi & Surya, 2017). This research clearly demonstrates that mastery of mathematical reasoning abilities has far-reaching consequences for students.

Meanwhile, the facts show that the mathematical reasoning ability of Indonesian students is still low. Indonesian students are unable to compete on the international stage, as reflected in the evaluation results of TIMSS, which ranks 45 out of 50 countries (TIMSS, 2016) and the results of PISA which only ranked 63 out of 70 countries (OECD, 2018). A thorough evaluation shows that Indonesian students' mathematical reasoning abilities are low (Ayuningtyas et al., 2019; Mumu & Tanujaya, 2019; Sandy et al., 2019; Sumartini, 2015).

Developing students' mathematical reasoning abilities continues to be a current research priority. The study results explain that student activity-based learning models are the main thing in developing students' reasoning abilities (Erdogan, 2019; Masfingatin & Murtafiah, 2020; Ulya et al., 2017). However, this learning can be said to be effective when done offline. The change in the learning environment from offline-based to online-based due to Covid-19 is one factor that influences student learning success (Mukuka et al., 2021), especially students' mathematical reasoning abilities. Learning during the COVID-19 period has limited direct interaction between students and students and teachers. This change in the learning climate causes many physical, mental, and emotional responses compared to learning and teaching conditions in general (Ghazali et al., 2021). Adjusting learning models to support the learning process during the Covid-19 period is paramount. Changes in learning can be anticipated by still placing students to actively seek, listen and listen to various learning resources that can support the student's understanding process. One of the learning models that can accommodate this is the social cognitive learning model.

The social cognitive learning (SCL) model is based on social cognitive theory. In this model, students participate in learning, but they are involved in shaping themselves (Santrock, 2006). Although learning Classical and Operant Conditioning in specific ways is still a good learning pattern, most people learn about what they have learned from observation activities. (Santrock, 2006). Observational learning differs from classical and operant conditioning in that it does not involve direct personal experience with stimuli, reinforcement, or punishment. Learning through observation involves observing the behavior of others, called models, then imitating the model's behavior (Money, 2016; Nabi & Prestin, 2017). Children and adults alike learn many things from observation and imitation. For example, when children learn the language, social skills, habits, and many other behaviors, they observe their parents or older people.

Bandura (1977) states that learning through observation plays an essential role in developing a child's personality through observation. Humans acquire knowledge, rules, skills, strategies, beliefs, and attitudes by observing others. Individuals also look at models or examples to learn the usefulness and behavioral suitability of the modeled behavior; then, they act according to beliefs about their abilities and the expected results of their actions (Bandura, 1977). So that the SCL model is expected to be able to accommodate the online learning process during the Covid-19 period.

In previous studies, the application of social cognitive aspects in the field of Education (Ghazali et al., 2021), in the business field (Harinie et al., 2017; Healey et al., 2021; Kursan Milaković, 2021; Ng et al., 2021), in the criminal field (Proctor & Niemeyer, 2020), and the field of information and management (Lockwood & Klein-Flügge, 2020; Money, 2016; Pinho et al., 2020) have a positive impact. This study provides an overview of how social cognitive regulation can help convey positive attitudes and behaviors that researchers want to convey. However, the SCL model in mathematics learning has not been widely applied. So that researchers are interested in applying SCL in learning mathematics.

This research intends to expand on previous research (Ghazali et al., 2021; Mukuka et al., 2021), examining the SCL model's impact on students' mathematical reasoning abilities in online learning situations. Solid practical abilities in students must support this model. Self-efficacy is believed to play a vital role in student success, academic life, and career (Bandura, 1997; Kingston & Lyddy, 2013; Schunk & Dibenedetto, 2019). Self-efficacy in teaching and learning activities needs to be developed in students. The realization of this principle is to place the teacher in the leading role as a facilitator and motivator (Schunk & Dibenedetto, 2019). In detail, we expand the observational aspect in the study by (1) conducting comparisons involving the control class that applies problem-based learning (PBL); (2) including aspects of mathematical self-efficacy as aspects that influence mathematical reasoning abilities; and (3) conducting online-based learning using zoom meeting for interactive media and google classroom as a medium for organizing the results of the learning process.

This kind of research is essential in the hope that it can provide information in choosing the suitable model in responding to changes in the learning environment due to COVID-19. In addition, the results of this study are one way to introduce social cognitive concepts in mathematics learning. Therefore, this study aims to obtain a comprehensive picture of the mathematical reasoning ability of students who received the SCL model and students who obtained the PBL model by paying attention to the level of students' mathematical self-efficacy (MSE). The research questions posed are (1) Is there a difference in mathematical reasoning ability between students who get the SCL model and the PBL model? (2) Is there a difference in mathematical reasoning ability between students at different MSE levels? (3) Is there an interaction between model and MSE on students' mathematical reasoning ability (MRA)?

RESEARCH METHODOLOGY

This study uses a quantitative method with a quasi-experimental Nonequivalent post-test Control Group Design. Researchers were directly involved in treating both classes. The experimental class was given treatment by applying the SCL model, and the control class was offered treatment using the PBL model. The application of the SCL model syntax consists of (1) Attention, (2) Retention, (3) Production, and (4) Motivation (Bandura, 1977). The application of the syntax of the PBL model consists of (1) problem presentation; (2) Problem investigation; (3) Solution problems; and (4) Evaluation process (Awang & Ramly, 2008; Soden, 1994). In detail, a comparison of learning activities in the two classes is presented in Appendix 1. The fundamental difference between these two models occurs in the "retention and motivation" phase in the SCL model and the "Problem investigation and evaluation process" phase in the PBL model. The retention phase allows students to understand the problem and hear the teacher reinforce concepts. This experience was not obtained in the PBL model in the Problem investigation phase. In the PBL model, students are focused on understanding the problems given individually while the teacher only asks questions that can guide them. In the Motivation Phase in the SCL model, the teacher provides appreciation, praise, and reinforcement for what has been learned. Meanwhile, in the Evaluation process phase, the teacher asks students to reflect back on what they have learned and provides reinforcement at the end. Although the two groups of classes were given a different treatment, the material and number of meetings between the two groups of classes remained the same. The material provided is the function limit, with each session for six weeks. The learning process and the final test's implementation are conducted virtually through a zoom meeting. Student answer sheets are coordinated through Google Classroom. Before the experiment, the researcher identified the students' MSE level by distributing the MSE questionnaire through the Google form.

The research participants were 70 high school 11 students in one of the schools in Bandung, which consisted of two classes with a total of 35 each. The sampling technique used was Nonprobability Sampling with the type of Purposive Sampling. As for the considerations in choosing a class, the researcher determines the abilities possessed in the experimental and control classes in a balanced or equivalent state. The determination was strengthened by independent t-test analysis. The results of the study explained that there was no significant difference between the Experiment class and the control class (t(68) = .837 with p > .05), so the result can conclude that the two classes are in a balanced state.

They were collecting data using a mathematical reasoning ability test and MSE questionnaire. The mathematical reasoning ability (MRA) test is an essay consisting of 5 questions. Indicators of students' mathematical reasoning ability include (1) Memorized Reasoning, (2) Algorithmic Reasoning, (3) Novelty, (4) Plausible, and (5) Mathematical foundation (Jonsson et al., 2014; Lithner, 2008). The MRA test was tested for validity and reliability. To test the validity of using product-moment correlation with valid results. While the reliability test using Alpha-Cronbach with reliable results (Cronbach's Alpha reliability coefficient of 0.495). The MSE questionnaire uses 20 items, which are constructed from (1) the Magnitude dimension, (2) the Strength dimension, and (3) the Generality dimension (Bandura, 1997). Measurement of the MSE questionnaire using a Likert scale of 1-4. The questionnaire was tested for validity and reliability. The test results show that the 20 items are valid and reliable. The MSE levels in this study are grouped into three categories, namely low MSE levels, moderate MSE levels, and high MSE levels.

Descriptive statistical analysis includes frequency, mean and standard deviation used to describe the demographic features of students. Normality and homogeneity tests were performed as prerequisites before performing ANOVA analysis. The Shapiro-Wilk test assessed customarily distributed data (Ghasemi & Zahediasl, 2012). Observation of the normality of the data in this study, based on the standardized residual score and not normality for each data from the research variables (Everitt & Skrondal, 2010; Kozak & Piepho, 2018). Levene's test was conducted to assess the homogeneity of the research data. The analysis of the research hypotheses used a two-way ANOVA with a 3 x 2 factorial design. The post-ANOVA follow-up test used the Tukey test. The entire statistical calculation process uses SPSS 25.

RESULTS

The purpose of this study was to obtain a comprehensive picture of the mathematical reasoning ability of students who obtained the SCL model and students who obtained the PBL model by paying attention to the students' MSE level. A descriptive statistical analysis of the model's average MRA demographics based on the model is presented in Figure 1.

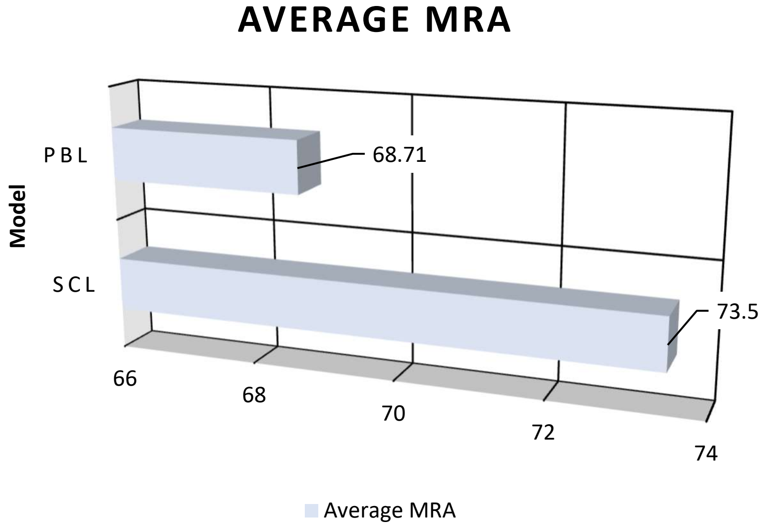


Figure 1: Mathematical Reasoning Ability Score

In Figure 1, it can be seen that the students in the SCL group ($\bar{x}_{SCL} = 73.50$) got a better average MRA than the students in the PBL group ($\bar{x}_{PBL} = 68.71$). Based on the standard deviation values in the two groups, the standard deviation of the SCL group (s = 13.03) was more significant than the standard deviation of the PBL group (s = 12.50). This indicates that students in the SCL group get different scores than students in the PBL group.

Furthermore, the demographics related to the MRA mean score based on the model and MSE are presented in Table 1, explaining that the high MSE level ($\bar{x}_{SCL} = 84.17, \bar{x}_{PBL} = 75.45$) obtained an MRA average that outperformed the MRA average at the other two MSE levels. Moderate MSE level students ($\bar{x}_{SCL} = 72.25, \bar{x}_{PBL} = 67.50$) obtained a higher MRA mean than the MRA mean of low MSE level students ($\bar{x}_{SCL} = 69.44, \bar{x}_{PBL} = 63{,}00$).

Model	MSE	Average	Std. Deviation	f
SCL	Low	69.44	11.30	9
	Moderate	72.25	13.23	20
	High	84.17	10.68	6
PBL	Low	63.00	9.77	10
	Moderate	67.50	14.77	14
	High	75.45	8.79	11
Total	Low	66.05	10.75	19
	Moderate	70.29	13.87	34
	High	78.53	10.12	17

Table 1: Descriptive statistics of MRA scores by Model and MSE

The results of the Shapiro-Wilk test show the Standardized Residual score of .968 with a significant level of .069, which is far above .05. This means that all data sets come from groups that are usually distributed. Meanwhile, in the Levene test, the Levene Statistic score was obtained at 1.282 with a significant level of .283, far above .05. Thus, the homogeneity condition is met. The hypothesis testing procedure can be carried out because the ANOVA statistical test requirements related to normality and homogeneity tests have been met. The results of the hypothesis are presented in Table 2.

Source	Type III Sum of Squares	df	Mean Square	F	sig.	Partial Eta Squared
Corrected Model	2121.539[a]	5	424.308	2.893	.020	.184
Intercept	315980.143	1	315980.143	2154.326	.000	.971
Model	671.515	1	671.515	4.578	.036	.067
Level_MSE	1678.438	2	839.219	5.722	.005	.152
Model * Level_MSE	42.035	2	21.018	.143	.867	.004
Error	9387.033	64	146.672			
Total	365800.000	70				

Table 2: Results of Analysis Tests of Between-Subjects Effects

Based on the results of data analysis in Table 2, several findings are produced. First, *F*(1,64) = 4.578 with a significance level of .036, which is far below .05. This means that there is a significant effect between the application of the SCL model and the PBL model on mathematical reasoning abilities. Second, the score *F*(2, 64) = 5.722 with a significance level of .005, far below .05. This means a significant effect between the MSE level on mathematical reasoning abilities. Based on the post hoc test in Table 3, the difference in mathematical reasoning ability occurs at the low MSE level and the high MSE level (significant level of .008, which is far below .05).

(I) MSE Level	(J) MSE Level	Mean Difference (I-J)	Std. Error	sig.
Low	Moderate	-4.2415	3.46893	.444
	High	-12.4768*	4.04319	.008
Moderate	High	-8.2353	3.59745	.065

Table 3: Multiple Comparisons of MSE Levels

Third, *F*(2, 64) = .143 with a significant level of .867, which is far above .05. This condition explains that MSE interaction in SCL or PBL classes does not significantly affect students' MRA. As a result, students' success in MRA is only affected by their presence in SCL or PBL classes. Where the mathematical reasoning ability of high MSE level students in SCL and PBL classes outperformed low MSE level students' MRA, as illustrated in Figure 2, shows the effect of the model and MSE level on students' MRA.

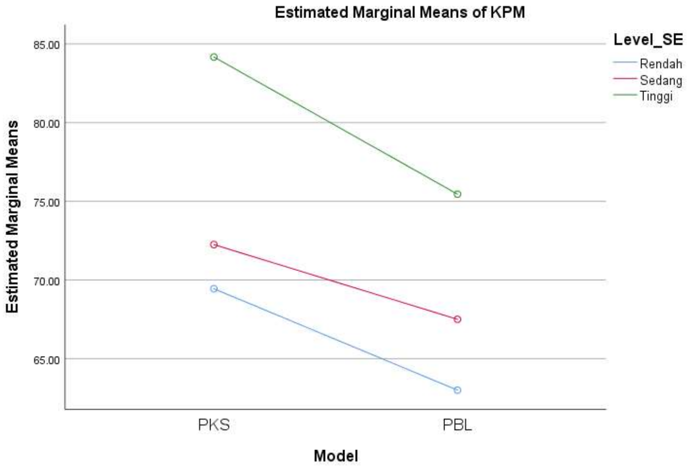


Figure 2: Graphical representation of the effect of model interaction and MSE level on MRA

Referring to the quantitative results described above, the researchers' findings were strengthened by differences in the results of the answers to students' mathematical reasoning abilities in the two classes. In the following, one of the results of students' responses in both classes is presented in solving the function limit questions in measuring students' creative reasoning, specifically on the Plausible indicator.

Indicator	Question
Indicator creative reasoning	Is known: $f(x) = \frac{bx^2+bx-21+b}{x^2+5x+6}$. If $\lim_{x\to -3} \frac{bx^2+bx-21+b}{x^2+5x+6}$ exists, then determine the limit value!

Table 4: Indicators and Problems of Mathematical Reasoning Ability

Based on the answers given in Table 5, it can be seen that there are differences in the responses provided by students in the two classes. On the plausible indicator, students studying with the SCL model can solve the questions. Students understand that the value $\lim_{x\to -3} \frac{bx^2+bx-21+b}{x^2+5x+6}$ exists, and for $x \to -3$ which will then be a Joint factor in the functions $bx^2 + bx - 21 + b$ and $x^2 + 5x + 6$. So, the student divides $(x + 3)$ by $bx^2 + bx - 21 + b$ and gets $b = 3$. Furthermore, the student determined the limit value of the requested function, which was 15. However, it was different for students who studied with the PBL model, he failed to understand the meaning of the problem, and even though he tried to solve the problem using the concept of derivative, he failed to solve the problem when the final result was is obtained in the form of "b."

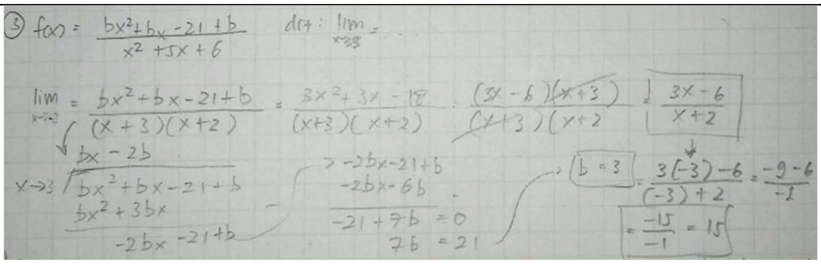
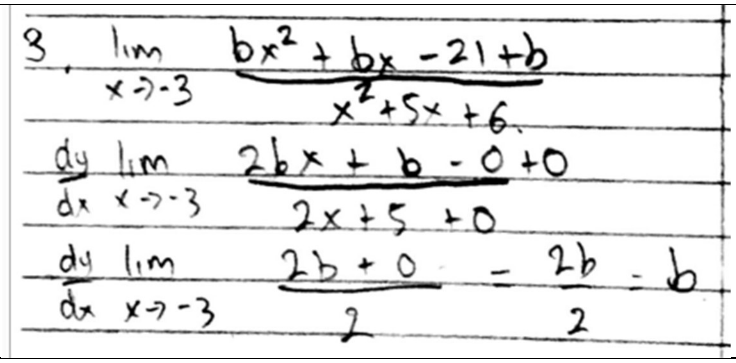
Model	Answer
SCL	 ③ $f(x) = \frac{bx^2+bx-21+b}{x^2+5x+6}$ dit: $\lim_{x\to -3} = \ldots$ $\lim_{x\to -3} = \frac{bx^2+bx-21+b}{(x+3)(x+2)} = \frac{3x^2+3x-18}{(x+3)(x+2)} = \frac{(3x-6)\cancel{(x+3)}}{\cancel{(x+3)}(x+2} = \boxed{\frac{3x-6}{x+2}}$ $x\to 3$: $bx - 2b$ $bx^2+bx-21+b$ bx^2+3bx $-2bx-21+b$ $-2bx-21+b$ $-2bx-6b$ $-21+7b=0$ $7b=21$ $\boxed{b=3}$ $= \frac{3(-3)-6}{(-3)+2} = \frac{-9-6}{-1}$ $\boxed{= \frac{-15}{-1} = 15}$
PBL	 3. $\lim_{x\to -3} \frac{bx^2+bx-21+b}{x^2+5x+6}$ $\frac{dy}{dx} \lim_{x\to -3} \frac{2bx+b-0+0}{2x+5+0}$ $\frac{dy}{dx} \lim_{x\to -3} \frac{2b+0}{2} = \frac{2b}{2} = b$

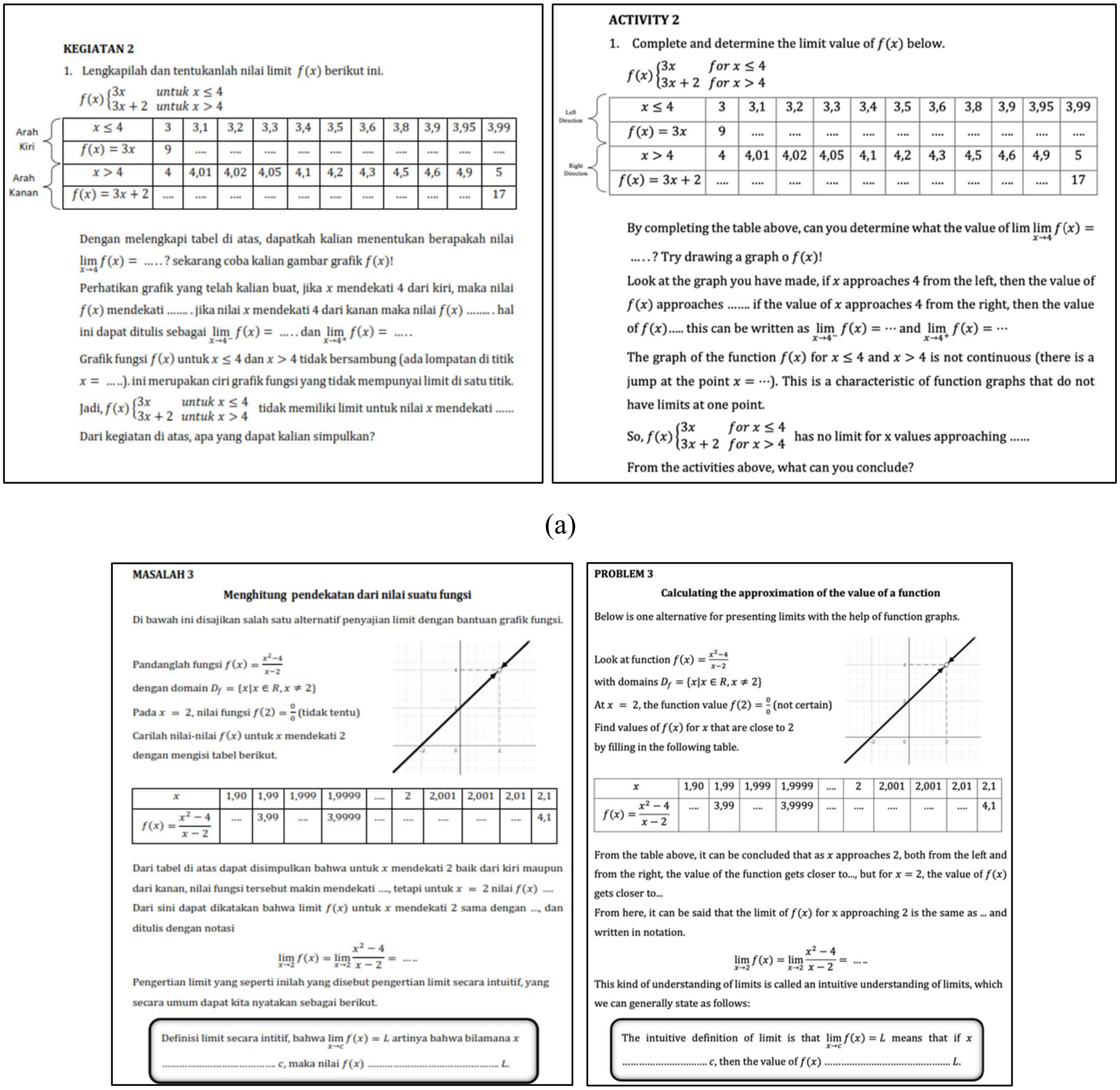
Table 5: Students' answers in the two classes in solving creative reasoning questions on plausible indicators

This indicator requires students to solve problems by paying attention to mathematical characteristics or giving reasons acceptable (plausible), namely utilizing information from the problem which states that ***the limit value of the given function exists***. Based on the answers given, students who study with the SCL model are able to understand the meaning of the problem, and understand the ***concept of algebraic limits***, so that for x to approach a (a certain value) it is a common factor in the given algebraic function. Understanding this concept makes it easier for students to solve the problems given. The demonstrated concept of creative thinking refers to the development of adaptable ideas and task solutions based on sound arguments and inherent mathematical features. This creative reasoning does not refer to high or extraordinary intelligence, but rather to easy and new answers to mathematical problems (Lithner, 2006). so that students in the SCL class have creative reasoning abilities compared to students in the PBL class.

DISCUSSION

The results of the research related to students' mathematical reasoning abilities based on the learning model found that there were differences in MRA in the two groups. Based on the mean MRA scores of students presented in Figure 1, it can be seen that the MRA of students who received the SCL model (73.5) was higher than the MRA of students who received the PBL model (68.71). In the learning process, the SCL model provides opportunities or space for students to develop MRA compared to students who learn to use the PBL model, as shown in Figure 1. It can be seen that each student in each class is given worksheets that guide in conveying the one-sided limit concept. In the SCL model (figure a), students are given activities to determine the value of the function $f(x)$ with different domains. Next, students are led to try to describe the function $f(x)$ based on the value of the function that has been determined. At the end of the activity, students are asked to be able to draw conclusions based on the value $f(x)$ that has been obtained and an image of the function $f(x)$, whether the given function $f(x)$ has a limit value or not. Different from the PBL model (figure b), students are asked to be able to define the limit of a function based on the problem given. students are asked to observe the image of the function, the behavior of the function, and the value of the function at the given points. This situation requires students to understand the function in trying to make conclusions about the definition of the limit of the function.

In the SCL model, learning situations are carried out through observation. Students can develop their mathematical reasoning abilities by observing the teacher solve problems in this observation process. Through this observation, students freely determine whether the strategy can solve the following problem (Mata-pereira & Ponte, 2017). Even in this condition, students who want to confirm the truth of their reasons observe the collection of opinions from their friends. So that reasoning can be developed by asking students to explain the proof or justification for a mathematical concept. This is different from the PBL environment, where students solve problems independently without any previous experience. So that students' mathematical reasoning abilities are not well developed. In connection with these results, in the findings of previous studies, no research has examined the effect of these two learning models on mathematical reasoning abilities. However, specific findings regarding the application of the PBL model (Aslan, 2021; Bosica et al., 2021; Evendi et al., 2022; Ping et al., 2020) explain that this model can develop mathematical abilities, which lead to aspects of students’ mathematical reasoning abilities. The problems' characteristics at the beginning of learning become a stimulus for students to link the problem with the mathematical concepts being studied. So that students' success depends on the ability of students to connect information that has been previously owned. The researchers' findings are slightly different, where the effect of the PBL model is not better than the SCL model in developing students' mathematical reasoning abilities.



KEGIATAN 2

1. Lengkapilah dan tentukanlah nilai limit $f(x)$ berikut ini.

$$f(x)\begin{cases}3x & untuk\ x \leq 4\\ 3x+2 & untuk\ x > 4\end{cases}$$

	$x \leq 4$	3	3,1	3,2	3,3	3,4	3,5	3,6	3,8	3,9	3,95	3,99
Arah Kiri	$f(x) = 3x$	9	....	....	....	....	....	....	....	....	....	....
Arah Kanan	$x > 4$	4	4,01	4,02	4,05	4,1	4,2	4,3	4,5	4,6	4,9	5
	$f(x) = 3x + 2$	....	....	....	....	....	....	....	....	....	....	17

Dengan melengkapi tabel di atas, dapatkah kalian menentukan berapakah nilai $\lim_{x\to 4} f(x) =$ ......? sekarang coba kalian gambar grafik $f(x)$!

Perhatikan grafik yang telah kalian buat, jika x mendekati 4 dari kiri, maka nilai $f(x)$ mendekati .... jika nilai x mendekati 4 dari kanan maka nilai $f(x)$ .... hal ini dapat ditulis sebagai $\lim_{x\to 4^-} f(x) =$ ..... dan $\lim_{x\to 4^+} f(x) =$ .....

Grafik fungsi $f(x)$ untuk $x \leq 4$ dan $x > 4$ tidak bersambung (ada lompatan di titik $x =$ .....). ini merupakan ciri grafik fungsi yang tidak mempunyai limit di satu titik.

Jadi, $f(x)\begin{cases}3x & untuk\ x \leq 4\\ 3x+2 & untuk\ x > 4\end{cases}$ tidak memiliki limit untuk nilai x mendekati ......

Dari kegiatan di atas, apa yang dapat kalian simpulkan?

ACTIVITY 2

1. Complete and determine the limit value of $f(x)$ below.

$$f(x)\begin{cases}3x & for\ x \leq 4\\ 3x+2 & for\ x > 4\end{cases}$$

	$x \leq 4$	3	3,1	3,2	3,3	3,4	3,5	3,6	3,8	3,9	3,95	3,99
Left Direction	$f(x) = 3x$	9	....	....	....	....	....	....	....	....	....	....
Right Direction	$x > 4$	4	4,01	4,02	4,05	4,1	4,2	4,3	4,5	4,6	4,9	5
	$f(x) = 3x + 2$	....	....	....	....	....	....	....	....	....	....	17

By completing the table above, can you determine what the value of lim $\lim_{x\to 4} f(x) =$ ......? Try drawing a graph o $f(x)$!

Look at the graph you have made, if x approaches 4 from the left, then the value of $f(x)$ approaches .... if the value of x approaches 4 from the right, then the value of $f(x)$...... this can be written as $\lim_{x\to 4^-} f(x) = \cdots$ and $\lim_{x\to 4^+} f(x) = \cdots$

The graph of the function $f(x)$ for $x \leq 4$ and $x > 4$ is not continuous (there is a jump at the point $x = \cdots$). This is a characteristic of function graphs that do not have limits at one point.

So, $f(x)\begin{cases}3x & for\ x \leq 4\\ 3x+2 & for\ x > 4\end{cases}$ has no limit for x values approaching ......

From the activities above, what can you conclude?

(a)

MASALAH 3

Menghitung pendekatan dari nilai suatu fungsi

Di bawah ini disajikan salah satu alternatif penyajian limit dengan bantuan grafik fungsi.

Pandanglah fungsi $f(x) = \frac{x^2-4}{x-2}$

dengan domain $D_f = \{x|x \in R, x \neq 2\}$

Pada $x = 2$, nilai fungsi $f(2) = \frac{0}{0}$ (tidak tentu)

Carilah nilai-nilai $f(x)$ untuk x mendekati 2

dengan mengisi tabel berikut.

x	1,90	1,99	1,999	1,9999	....	2	2,001	2,001	2,01	2,1
$f(x) = \frac{x^2-4}{x-2}$	....	3,99	....	3,9999	....	....	....	....	....	4,1

Dari tabel di atas dapat disimpulkan bahwa untuk x mendekati 2 baik dari kiri maupun dari kanan, nilai fungsi tersebut makin mendekati ...., tetapi untuk $x = 2$ nilai $f(x)$ .... Dari sini dapat dikatakan bahwa limit $f(x)$ untuk x mendekati 2 sama dengan ..., dan ditulis dengan notasi

$$\lim_{x\to 2} f(x) = \lim_{x\to 2}\frac{x^2-4}{x-2} = \ldots..$$

Pengertian limit yang seperti inilah yang disebut pengertian limit secara intuitif, yang secara umum dapat kita nyatakan sebagai berikut.

Definisi limit secara intitif, bahwa $\lim_{x\to c} f(x) = L$ artinya bahwa bilamana x .... c, maka nilai $f(x)$ .... L.

PROBLEM 3

Calculating the approximation of the value of a function

Below is one alternative for presenting limits with the help of function graphs.

Look at function $f(x) = \frac{x^2-4}{x-2}$

with domains $D_f = \{x|x \in R, x \neq 2\}$

At $x = 2$, the function value $f(2) = \frac{0}{0}$ (not certain)

Find values of $f(x)$ for x that are close to 2

by filling in the following table.

x	1,90	1,99	1,999	1,9999	....	2	2,001	2,001	2,01	2,1
$f(x) = \frac{x^2-4}{x-2}$	....	3,99	....	3,9999	....	....	....	....	....	4,1

From the table above, it can be concluded that as x approaches 2, both from the left and from the right, the value of the function gets closer to..., but for $x = 2$, the value of $f(x)$ gets closer to...

From here, it can be said that the limit of $f(x)$ for x approaching 2 is the same as ... and written in notation.

$$\lim_{x\to 2} f(x) = \lim_{x\to 2}\frac{x^2-4}{x-2} = \ldots..$$

This kind of understanding of limits is called an intuitive understanding of limits, which we can generally state as follows:

The intuitive definition of limit is that $\lim_{x\to c} f(x) = L$ means that if x .... c, then the value of $f(x)$ .... L.

(b)

Figure 3: (a) Student worksheet on the SCL model (b) Student worksheet on the PBL model

Although the application of the SCL model in mathematics learning has not been widely investigated, the application of the SCL model in the online learning environment in the COVID-19 situation in this study explains how the learning process occurs. The cognitive aspect filters students' thinking processes in observing cognitive behavior presented by students or teachers in online learning situations (Bandura, 1977). Cognitive engagement refers to the cognitive processes that enable students to absorb information. Model figures in this study, namely teachers, students, and exciting teaching materials, can arouse students' curiosity, can focus students' attention so that the learning process can flow, and students can concentrate (Kemp et al., 2019). In this research process, online presence is defined as the behavior of teachers with students in distance learning that intentionally involves, attends, or at least listens during online classes by utilizing the zoom meeting platform. Teacher behavior in explaining material concepts can contribute to student behavior to take action and be used as a problem-solving process (Tajudeen, Madarsha, Suryani, & Badariah, 2011). The research findings strengthen the research results conducted by Ghazali et al. (2021). His research findings explain the relationship of a social cognitive theory that consists of behavior, cognitive and situational factors described through social, cognitive, and teacher presence, as a case study framework on online and distance learning. In which it can support decision-making in education during the COVID-19 pandemic. Based on these results, the study concludes that applying the SCL model contributes to students' mathematical reasoning abilities and broadens the scope of the application of social cognitive theory in mathematics learning.

The following finding is related to students' mathematical reasoning abilities based on the MSE level. These results conclude that there are differences in students' mathematical reasoning abilities based on the MSE level. A significant difference occurred between the MRA of students who had a low MSE level and a high MSE level. Based on Table 1, it is known that students' MRA scores at the high MSE level (78.53) are higher than the MRA scores at the low MSE level (66.05). This result aligns with the research findings (Schöber et al., 2018; Yelorda et al., 2021), which states that higher academic achievement can be expected from students with high self-efficacy than students with low self-efficacy levels. Self-efficacy is based on social cognitive theory (Bandura, 1997), Individuals are viewed as proactive agents in controlling their cognitions, motives, behaviors, and emotions, according to this theory (Mayer, 2002). So self-efficacy becomes a factor in controlling perceived behavior (Patricia, 2020).

Meanwhile, the comparison between MRA for low MSE level students and moderate MSE level students has no difference. Likewise, with the MRA for students at the moderate MSE level and students at the high MSE level. This finding is slightly different from the research results (Ma, 2021; Schöber et al., 2018), where students with a high MSE level outperform those with a low MSE level. These findings can be used as a basis for further research studies; namely, online-based learning can accommodate students with low MSE levels to compete with students with high MSE levels.

Finally, the findings explain no interaction between model variables and MSE on MRA. From the fact that there is no such interaction, it can be concluded that the differences between the MSE levels (low, moderate, and high) for each learning model are the same. These characteristics are, of course, the same as the total mean characteristics at the MSE level. As seen from the total mean in Table 1, the MRA of students at the high MSE level is higher than the MRA of students at the low MSE level. Because there is no interaction, the same applies to the students who were given the SCL model and the PBL model. This means that in the SCL model, the MRA of students at the high MSE level is also higher than students at the low MSE level.

Similarly, it was concluded that in the PBL model, students at the high MSE level were also higher than students at the low MSE level. Furthermore, another implication of the absence of interaction is that the characteristics of the different learning models will be the same at each MSE level and will also be the same as the characteristics of the total mean. This means that, in general, the SCL model is better than the PBL model; if it is reviewed by students at low MSE level only, then the conclusion will apply that low MSE level students who receive the SCL model have a higher MRA than low MSE level students who receive the PBL model. Likewise, if viewed from students at the moderate MSE level and at the high MSE level. The findings of this study are in line with the results of research conducted by Jannah et al. (2019), which concluded that there is no interaction between the model and the classification of self-efficacy on understanding mathematical concepts, and there are differences in understanding of mathematical concepts based on the classification of self-efficacy. Likewise, Fajri et al. (2016) research show no interaction between the model and self-efficacy towards increasing spatial ability. A similar study conducted by Chotima et al. (2019) explained no interaction between the model and self-efficacy in solving mathematical problems. Differences in self-efficacy influence mathematical problem-solving. Based on the research conducted by several researchers above and associated with the findings of this study, it can be concluded that the research findings are in line with the findings of this study.

CONCLUSIONS

The results showed that students who received learning using the SCL model had a higher MRA than students who received learning using the PBL model. Furthermore, students with a high MSE level have higher mathematical reasoning abilities than those with a low MSE level. Meanwhile, the mathematical reasoning ability between students with high MSE level and moderate MSE level and students with moderate MSE level with low MSE level are not significantly different. The results of this study have limitations; namely, the research subjects are only 11 high school students, and these findings still need to be reviewed so that they can be generalized to lower or higher school levels. This study only focuses on mathematical reasoning abilities, so studies on other mathematical abilities of students need to be explored. As a suggestion for further research, the application of social cognitive learning models and observations of mathematical self-efficacy aspects in online learning situations can be used as alternative learning models in responding to changes in the learning environment today.

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Appendix. Comparison of learning activities in the two models.

Table 1. Learning Stages, Activity Descriptions, and Aspects of Capabilities expected in the SCL Model

Stage	Activity Description	Emerging aspect
Initial activity		
Phase 1. Attention	1. The teacher conveys the learning model that will be used. 2. The teacher conveys the learning objectives. 3. The teacher gives appreciation in the form of limit terms that are commonly found in everyday life and asks students to give their opinions. Do you often hear/see the sentences "He almost fell", "His credit card has a limit" and "30 km/hour speed limit".	
Core activities		
Phase 2. Retention	1. The teacher associates these everyday problems with the mathematical concepts that will be studied, namely the limits of algebraic functions. 2. The teacher displays an illustration of the value of a function at a point. 3. The teacher asks students to look at the ACTIVITIES contained in the Worksheet. 4. The teacher guides students by giving questions that can arouse students' reasoning in solving problems on student worksheets. 5. The teacher repeats the meaning of limits for each problem that has been solved by students.	*Memorized reasoning* *Algoritmic Reasoning*
Phase 3. Production	1. The teacher asks students individually or in groups to complete the exercises contained in the Worksheet. 2. The teacher asks one of the students to explain the answers they have found, 3. The teacher asks other students to pay attention and provide feedback.	Creative reasoning
Phase 4. Motivation	1. The teacher gives praise for each student's response 2. The teacher provides reinforcement of students' answers that are still lacking.	*Memorized reasoning* *Algoritmic Reasoning*

Closing Activities		
	1. The teacher directs students to draw conclusions about the activities that have taken place (attention and retention). 2. The teacher gives homework assignments a few questions regarding the material that has been studied (production). 3. The teacher ends the learning activity by delivering material to be discussed at the next meeting and giving a message to repeat the concepts that have been learned and always learn.	

Table 2. Learning Stages, Activity Descriptions, and Aspects of Capabilities expected in the PBL Model.

Stage	Activity Description	Emerging aspect
Initial activity		
Phase 1. *Problem Presentation*	1. The teacher greets and opens the lesson. 2. The teacher conveys the learning model that will be used. 3. The teacher conveys the learning objectives. 4. The teacher gives appreciation in the form of limit terms that are commonly found in everyday life. 5. Do you often hear/see the sentences "He almost fell", "His credit card has a limit" and "30 km/hour speed limit". 6. Students are invited to examine the problem given and asked to express their opinion.	
Core activities		
Phase 2. *Problem investigation*	1. The teacher associates these everyday problems with the mathematical concepts that will be studied, namely the limits of algebraic functions. 2. The teacher asks students to look at the PROBLEMS in the Worksheet. 3. The teacher guides students by giving questions that can arouse students' reasoning in solving problems on Worksheet.	*Memorized reasoning* *Algoritmic Reasoning*
Phase 3. *Solution problem*	1. The teacher asks students individually or in groups to complete the EXERCISES contained in the Worksheet.	Creative reasoning

	2. The teacher asks students to explain the answers they have found. 3. The teacher asks other students to pay attention and provide feedback.	
Phase 4. *Evaluation process*	1. The teacher helps students reflect on the process and results of their investigation 2. The teacher provides reinforcement of students' answers that are still lacking.	*Memorized reasoning* *Algoritmic Reasoning*
	Closing Activities	
	1. The teacher directs students to draw conclusions about the activities that have taken place. 2. The teacher gives homework assignments a few questions regarding the material that has been studied. 3. The teacher ends the learning activity by delivering material to be discussed at the next meeting and giving a message to repeat the concepts that have been learned and always learn.	