

## Assessing the Implemented Research Lesson Using Mathematical Quality of Instruction

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*Abstract: Fractions remain one of the most difficult topics to convey to students. Thus, employing professional development programs known for improving the teaching and learning process, such as lesson study, is deemed necessary. This study aims to assess and analyze the strengths and weaknesses of the implemented research lesson using the Mathematical Quality of Instruction (MQI) rubric. This standardized observation tool evaluates the quality of a mathematical instruction. Results indicated that the strengths of the implemented research lesson were demonstrated in the fluent utilization of mathematical language under the domain Richness of Mathematics and the minimal instances of teacher's error and imprecision. On the other hand, weaknesses were observed in student participation under the domains of 'Common Core Aligned Student Practice' and 'Working with Students' in Mathematics. Implications for educators and the various educational processes include among others the importance of lesson preparation, teaching students to articulate ideas explicitly and the continuous use lesson study as a professional learning community.*

Keywords: Lesson study, Mathematical Quality of Instruction, Operations on Fractions

### INTRODUCTION

The Philippines, as mentioned by OECD (2020), is one of the countries reported to be unprepared for online distance learning, where 30% of 15-year-old students do not have conducive homes for home study and technology inadequacy. After a year of closure, most countries returned to the normal classroom setup; however, the Philippines took two years to conduct in-person classes

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gradually indicative of educational opportunities in the country are disrupted more severely by the pandemic.

As most schools reopened in 2022, it is interesting to observe how learners engage in mathematics learning amid the transition stage. Online learning, teaching, evaluation, and engagement have all garnered positive and negative responses from students since the introduction of online classrooms during the pandemic (Stoian et al., 2022). Some research, however, has shown that online students lack a sense of community and hence struggle to learn from one another (Hollister et al., 2022), resulting in an “absence of emotional closeness” (Taufik & Effendy, 2022). Since interaction is one of the most essential variables in student experience, isolation and disconnectedness may lead to a lack of interest in learning (Bolliger et al., 2010). Teachers were also reported to be dissatisfied due to a lack of student-teacher interaction, which is essential for effective instruction (Sulaimi, 2022).

The unfavorable effects of the health crisis and school lockdowns are also beginning to reemerge. Students who had previously taken math online but had returned to traditional classroom settings performed poorly because they lacked the foundational knowledge necessary to succeed (Meniano & Tan, 2022). The study of Engzell et al. (2021) supports this claim since students make very little progress through remote learning; therefore, the learning losses are projected to be notable in countries that took a while to reopen schools. Students’ performance in the most recent grading period revealed that a percentage of the learners failed to meet at least 75% of the learning competencies set for them. The existing knowledge gap in students’ mathematical understanding has increased, and new issues concerning mathematics learning are being introduced into the education system. Public schools suffered from this negative trend especially those who have no access to internet and gadgets, and simply relied on the printed learning modules sent to their homes with parents as their home teachers during the pandemic. Teachers claimed many students struggled to learn on their own and failed to complete assignments because they could not understand the directions (Dangle & Sumaoang, 2020). In addition, most students report having difficulties with the printed learning modules adding to the existing stigma on mathematics being one of the most difficult subjects.

To lessen the disastrous effects of the pandemic, governments and stakeholders are urged to “reimagine” education and to transform the teaching and learning process as policy solutions (United Nations, 2020) especially that the resumption of traditional classroom instruction requires thoughtful consideration of the many factors involved in this shift.

### **Difficulty in fractions**

Considering this situation before the pandemic, a far worse scenario can be gleaned as learners had to learn mathematics independently. When it comes to mathematics, fractions are among the most difficult concepts for pupils to grasp and thus pose a significant barrier to their academic

success (Punzalan & Buenaflor, 2017). To this day, the ideas behind fractions remain one of the most difficult to convey to students throughout mathematical education (Africa et al., 2020). A growing body of evidence highlights the problems and hurdles children experience as they learn and comprehend fractions (Hansen et al., 2017; Lortie-Forgues et al., 2015), even though learning and teaching fractions have been regularly highlighted in a variety of mathematical standards. A nationwide study of algebra instructors in the US found that students' weakness in understanding fractions was the second most significant factor in their struggle with algebra and higher level mathematics (Hoffer et al., 2007).

Students' struggles in learning fractions and the lack of procedural fluency may be traced back to a lack of conceptual understanding, according to UNESCO's International Bureau of Education (IBE) (Africa et al., 2020). Teaching for conceptual and procedural fluency in fractions in kindergarten through eighth grade is one of the most difficult tasks for teachers. These students' difficulties in learning fractions and other mathematical content posed an emerging pedagogical challenge among teachers. Learning fractions requires students to be able to express those fractions mathematically so that they can recognize the symbols, solve the problems, and get acquainted with the procedure of solving problems (Saskiyah & Putri, 2020). The three operations on fractions—multiplication, subtraction, and division—between them posed the greatest challenge for the learners. Consequently, high school students struggle with evaluating fractions, decimals, integers, and rational expressions because they were not adequately prepared in elementary school (Punzalan & Buenaflor, 2017; Lubienski & Lubienski, 2006; Africa et al., 2020).

To help students who are having difficulty, Namkung and Fuchs (2019) recommend teachers directly instruct struggling students on how to create high-quality explanations by modeling such explanations, outlining key elements of an explanation, and giving students practice assessment and applying them in problem solving. It has been proven that the strategies and representations used in problem-solving are related to students' success in learning (Copur-Gencturk & Doleck, 2021). There is no one best way to teach, but rather a wide range of approaches, each with its own set of advantages and disadvantages, that may lead to long-term gains in students' mathematical competence. Examining education from the viewpoint of the interactions between instructors, students, and course content yields the most insightful results (Kilpatrick et al., 2001).

### **Lesson Study for Professional Development**

Teachers must have the knowledge and teaching skills representative of the standards' depth to prepare learners to meet the necessities of educational standards successfully. Professional development and continuous professional growth help educators to improve their essential skills and learner achievement (Darling-Hammond, 1997). One of the ways teachers can participate to professional development is being part of a professional learning community such as the lesson study.

Lesson study is a collaboration-based approach originating from Japan (Murata, 2011) and is one of the growing professional development programs in the past two decades. It encourages individuals to collaborate and learn from one another's experiences and observations in one's area of specialization in teaching (Gholami, 2022; Gholami et al., 2021). A lesson study comprises teachers and educators who regularly meet to work on planning, teaching, observing, evaluating, and improving “research lessons” (Rock & Wilson, 2005). This is a cyclical process where, if desired and needed, the research lesson is revised and reimplemented for a new group of students. The principle behind the lesson study is to improve teaching, and it would best be in the context of a classroom lesson (Elipane, 2011). By observing in such a manner, multiple aspects of teaching may then be improved, such as teaching instruction, the lesson content, and the learning experiences.

With the lesson study's cyclical design, teachers can develop the research lesson continuously and indefinitely, as well as improve their teaching practices. Teachers in a lesson study also gain insights to support student learning by observing students' responses to the lesson and teaching methods. While lesson study focuses on student learning, it also has the strong involvement of teachers in the design process, which allows the design to be based on different experiences and expertise (Jansen et al., 2021). Gholami et al. (2022) observed that teachers may significantly improve their grasp of the subject matter through collaborating and sharing their experiences in the classroom. Educators develop skills in generating new challenges, debating students' misconceptions about certain subjects, enhancing their knowledge of how those topics may be used, and developing problem-based research lessons.

Teachers tend to work alone in a classroom setting, where their experiences are only obtained by themselves, and as such only improve on their own. Lesson study is an avenue for both experienced and inexperienced teachers to learn. Gholami et al. (2021) recommended using the lesson study method as a mandatory professional development strategy for teachers. The cooperative preparation, observation, and analysis of several practitioners can serve as a comparison between the point of view of a teacher who is teaching, and the points of view of teachers who are observing. With this, the teacher may become conscious of things they normally would not be conscious of due to habit (Dudley, 2011).

### **Mathematics Quality of Instruction**

The Mathematical Quality Instruction (MQI) is a framework developed by the Heather Slope and associates at the College of Michigan (Center for Education Policy Research, n.d.). The MQI was created to provide a balanced and multidimensional perspective on mathematics instruction.

As presented in Figure 1, the mathematics quality instruction is based on the different *segments* and the entire or *whole lesson*. *Segment codes* consist of the following domains: *classroom work is connected to mathematics*, *richness of the mathematics*, *working with students and mathematics*.

The *whole lesson codes* consists of *errors and imprecision*, and *common core aligned student practices*. The domain of *classroom work is connected to mathematics* measures whether the majority of time spent in the classroom was spent on activities that develop mathematical concepts. The area of the *richness of the mathematics* is divided into two components. The first is on meaning making which includes clarifications of numerical thoughts and relating various math concepts. Whereas the other is observing the different mathematical practices, such as solving using procedures and making generalizations.

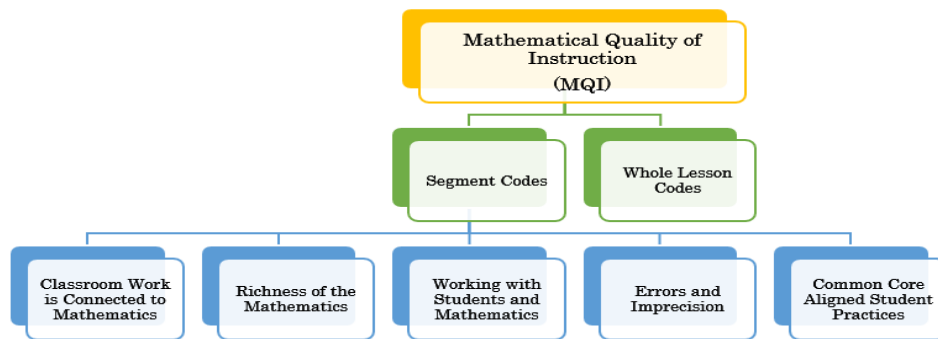


Figure 1: Domains of Mathematical Quality of Instruction

The next domain *working with students and mathematics*, focuses on how the teacher uses students' math ideas and can identify whether it is correct or not to perform immediate remediation. The subsequent domain, *errors and imprecision*, targets to check if there are math errors done by the teacher, which affects the mathematical instruction process. And lastly, *common core aligned student practices* captures how students interact with mathematical material and ideas.

### Conceptual Framework

This study makes use of the Mathematical Quality of Instruction (MQI) rubric to assess and analyze a research lesson on fractions. The MQI tool does not only analyze the content of the lesson but also the interactions and performances of the teacher and the students. This helps identify which aspects of the lesson needs improving in terms of instructional strategies effective in supporting students to understand the lessons. Adkins (2017) observed that teachers are expected to be knowledgeable of their mathematics and are skillful in providing explanations and examples of their topic. However, their ability to guide the students in making generalizations and reasoning mathematically is a relative weakness. By identifying what are the outstanding practices as well as the ones that need improvement through the lesson study approach, professional learning and development can be tailored to enhance the excellent practices and mend the weak practices. With this in mind, the purpose of this investigation was to examine and evaluate the research lesson using the Mathematical Quality of Instruction by answering the following research questions:

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1. What are the strengths of the implemented research lesson based on the MQI observation tool?
2. What are the weaknesses of the implemented research lesson based on the MQI observation tool?

## METHODOLOGY

### Research Design

This participatory action research used an explanatory sequential mixed method design. The qualitative data consisting of observations, remarks, and transcripts offer a more in-depth explanation obtained from the MQI ratings as a means of ensuring the rigor and accuracy of the research. The combination of qualitative and quantitative research enhances the comprehension of the research problems and yields greater insight than the use of either method alone (Creswell & Plano Clark, 2017).

### Participants

The study was conducted in a select public secondary school in Mabalacat City, Pampanga, particularly in a grade 7 math class. It is a heterogenous, intact class consisting of 32 students. The study was conducted during the school year when face-to-face classes were first re-implemented, and all the students were coming from two years of distance learning. The locale was chosen since the researcher is one of the division's instructors and is in charge of the aforementioned class, as well as based on the availability of the teacher-implementer. Recording of the in-person lesson implementation was done for later observations.

### Instrument

This study utilized the Mathematical Quality of Instruction (MQI) rubric. This rubric is a standardized observation tool that evaluates the different domains of the quality of mathematical instruction. The MQI comprises segment and whole lesson codes as shown in Figure 1. Individual codes within each of the five domains contain score points that classify instruction into various quality levels. In addition, each code covers different elements for each domain to be observed. The MQI scale is specifically designed for recorded video lessons rather than in-person observations. The recorded class session was broken up into 7-minute and 30 seconds sections. The MQI rubric's descriptions and sample situations for each element were used to identify present elements of instructions in each segment.

### Rating and Analysis

The segment codes are classified into five domains, each with elements to be observed, while the whole lesson codes have ten elements. Since this study focused more on determining the

instructional strengths and weaknesses of the implemented research lesson rather than estimating its overall quality, the overall and whole lesson codes were excluded from the analysis. The recorded lesson lasted for about 80 minutes, yielding ten segments of 7 and a half minutes, and one segment came with 5 mins and 28 seconds (refer to Table 1). Segmenting the recorded class enables the observers to evaluate events as they occur by not simply recalling what happened at the end of the video. Each part was assigned randomly to two teacher-raters (See Table 1 for reference of the rater assignments), choosing among the members of the lesson study group except the teacher-implementer. Descriptions were provided for each code, which rates teacher and student actions as not present, low, middle, or high, except for the first domain, *classroom work is connected to mathematics* which only has a yes or no rating.

**Table 1**

*Segment Codes and their Assigned Raters*

Segments	Time Duration	Teacher Rater
1	0:00:00 - 0:07:30	Teachers A & B
2	0:07:30 - 0:15:00	Teachers B & C
3	0:15:00 - 0:22:30	Teachers A & C
4	0:22:30 - 0:30:00	Teachers A & B
5	0:30:00 - 0:37:30	Teachers B & C
6	0:37:30 - 0:45:00	Teachers A & C
7	0:45:00 - 0:52:30	Teachers B & C
8	0:52:30 - 1:00:00	Teachers A & B
9	1:00:00 - 1:07:30	Teachers A & C
10	1:07:30 - 1:15:00	Teachers B & C
11	1:15:00 – 1:20:28	Teachers A & C

## Procedures

There were three distinct stages to the research process: pre-implementation (planning), implementation (execution), and post-implementation (evaluation).

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## Pre-implementation

Following the lesson study process (Stepanek et al., 2006), the researchers began by familiarizing themselves with the whole lesson study cycle, procedures, and application methods. The lesson study group came up with an overarching research subject that encapsulates the learning targets for the learners. The group identified a research subject and a content area topic around which to center the lesson. Upon examining the lesson study schedule and comparing it to the K to 12 Mathematics Curriculum, one of the possible topics to be taught during the observation and debriefing is fractions (Africa et al., 2020; Namkung & Fuchs, 2019). Considering the student assessment data from the past school years, the curriculum gaps, and the existing literature on the topics that are found to be challenging to teach and learn in mathematics (Hiebert et al., 2002; Hill, Rowan, & Ball, 2005), the team decided to focus on the operations on fractions. The student assessment data obtained from the school head and mathematics department head of the participating junior high school revealed that fractions were among the least mastered skills in Grade 7 mathematics last school year 2021-2022.

Also, the period by which the lesson was implemented and the target learning competencies (Department of Education, 2019) for the quarter were considered in finalizing the research lesson topic. In the K to 12 Curriculum Guide for Mathematics, fractions were introduced as early as the 1st-grade level and this concept is being further developed across other grade levels. For instance, in grade 7 math, the focus was on mastery of the operations of fractions. Members of the group paid careful attention to the progression of ideas as they were introduced and expanded upon throughout the unit. That includes the research lesson (operation on fractions) itself as well as all lessons leading up to and after it. Since the greatest common factor (GCF) and the least common multiple (LCM) are both prerequisites of the topic of interest, the researchers decided to include them both in the review. Moreover, the researchers anticipated students having certain knowledge gaps brought by the last two years of modular distance learning. Thus, the concept of similar and dissimilar fractions was included before moving on to learning how to add and subtract fractions.

After deciding on a topic for the research lesson, the group collaborated and devised a comprehensive lesson plan noting learning goals, activities, expected student responses, and assessment questions. The lesson plan and the teaching materials were developed and crafted collaboratively by the teacher-researchers. These materials underwent evaluation and validation by four mathematics teacher experts and were revised according to the recommendations provided for the enhancement of the lesson.

The researchers secured the approval of the school head on the conduct of the study. Upon the principal's approval, students and their guardians were asked to sign an informed consent form indicating willingness to participate in the research.



## Implementation

The teacher started the lesson by checking the assignment on finding GCF and LCM. Students would raise their hands if they wanted to share their answers on specific items. Although the teacher did not ask students to write their solutions on the board, the PowerPoint presentation revealed the correct answers one by one.

After checking assignments, examples of similar and dissimilar fractions were shown on the screen (see Figure 2). The students were instructed to analyze each set of examples and then characterize and differentiate similar and dissimilar fractions. To check students' understanding of these concepts, sets of fractions were presented on the screen where students would classify whether the fractions were similar (if so, they would stand up) or dissimilar (they would remain seated).

Similar Fractions	Dissimilar Fractions
Two or more fractions with _____ denominators	Two or more fractions with _____ denominators
Examples: $\frac{1}{3}$ and $\frac{2}{3}$  $\frac{5}{8}, \frac{2}{8}$ and $\frac{4}{8}$  $\frac{9}{35}, \frac{16}{35}, \frac{11}{35}, \frac{21}{35}, \frac{14}{35}$	Examples: $\frac{2}{3}$ and $\frac{4}{7}$  $\frac{3}{7}, \frac{2}{5}$ and $\frac{1}{7}$  $\frac{9}{17}, \frac{11}{16}, \frac{11}{25}, \frac{2}{25}, \frac{9}{10}$

Figure 2: Similar and Dissimilar Fractions

The teacher shared a real-life word problem among the students to present instances of the new lesson. Students were asked for the possible operation or solution to the problem, to which they correctly answered that they needed to add the given fractions. Consequently, the teacher presented addition and subtraction of similar fractions task, which the teacher and the students solved together using the area models (see Figure 3).

Using area models, find the sum or difference.

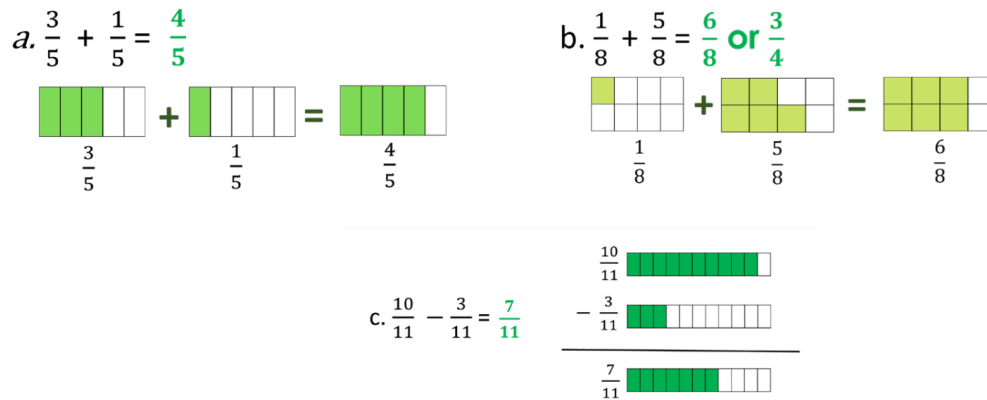


Figure 3: Adding and Subtracting Similar Fractions Using Area Models

The students were then tasked to observe these examples. Afterward, they were prompted to generalize how to add or subtract similar fractions without using area models. The area model approach in adding and subtracting dissimilar fractions (see Figure 4) was also discussed before going over the procedural process. Students worked on practice exercises to build mastery.

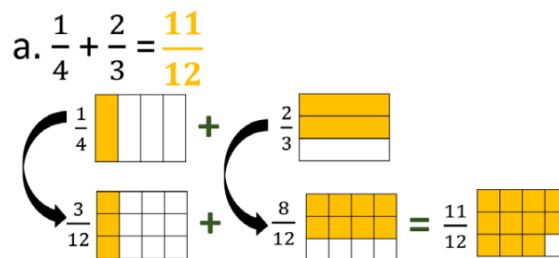


Figure 4: Adding and Subtracting Dissimilar Fractions Using Area Models

The word problem at the start of the lesson was presented again by the teacher. The students then applied the concepts and skills they learned to solve the mathematical problem. Two students presented and wrote their solutions on the board. To wrap up the lesson, students were instructed to fill in the blanks with the correct words to complete the steps in adding or subtracting fractions. For their assignment, students were tasked to create and solve their own word problem involving the addition or subtraction of fractions.

### Post-implementation

After the implementation, the lesson study group convened via an online conferencing platform. Their main agenda was to discuss the procedures on how to utilize the MQI rubric tool in analyzing

video recordings of the lesson study implementation. Following the MQI procedures, the randomly assigned lesson study members rated in each segment. These raters were given time to accomplish the MQI analysis of their designated segments individually. Each rater was unaware of the other rater's score. After the evaluation, the raters convened again to compare their ratings with their assigned partners. In cases of the assigned pairs' different ratings for a particular coded segment, all three lesson study raters discussed it further and then arrived at a final agreed rating. This manner of addressing disagreements on ratings was based on the MQI administering procedures found in other studies (Hill et al., 2012; Hill et al., 2011).

After identifying and rating the present MQI elements, the researchers recorded the frequency of each domain element's occurrences across the 11 segments. This frequency data was then organized and presented graphically to display trends not explicitly seen in the MQI analysis table. It is also important to note that the number of elements that make up each domain varies, leading to variation in each domain's highest possible frequency. For example, the domain of *richness of mathematics* has six elements, whereas there are 11 segments. Hence, 66 is its highest possible domain frequency. On the other hand, the domain of *working with mathematics* is comprised of 3 elements, leading to its highest possible frequency of 33.

## RESULTS

### Classroom Work is Connected to Mathematics

Among the domains of MQI, this area is different in terms of the manner of rating. Instead of rating as low, mid, or high, this domain simply asks whether half or more of the segment observed was related to the mathematical content. The rating shows that all the instructional time was utilized on activities to develop mathematical ideas. This further implies that the instructional time was used productively.

### Richness of the Mathematics

This domain aims to determine how in-depth mathematics was offered to the students. As Figure 5 shows, most segments obtained a high level. It covers more than half of this domain's total recorded frequencies (34 out of 66). Moreover, mathematical language stands out among the elements since it gained the most frequent high-level rating. This implies that the teacher was able to use mathematical language throughout the class fluently. Meanwhile, the remaining elements still gathered a recurrence rate in the other segments rated as low, mid, and high. However, it cannot be concluded that in these elements, the quality of math richness is lacking; it is just that some of the elements can only occur in specific parts of the lesson.

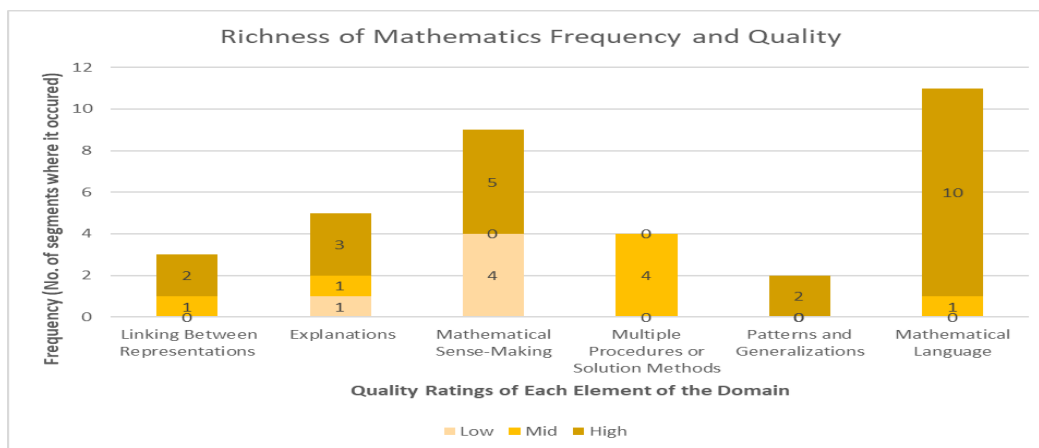


Figure 5: Richness of Mathematics Frequency and Quality

For instance, linking between representations was present since the teacher was able to use area models to represent the addition of dissimilar fractions; however, it can only be seen in segments 5 and 6 of the implemented lesson. Furthermore, this paved the way for developing the students' way of explaining contents and mathematical sense because they were able to identify and differentiate fractions. In addition, the solutions and generalizations can mostly be exhibited once the lesson is taught, which supports how it can only be present in some parts of the lesson.

### Working with Students and Mathematics

A high rating is very rare among the elements of *working with students and mathematics* as can be seen in Figure 6. It was only noticed for two segments which shows that the teacher made an effort to provide remediation in addressing students' errors and difficulties. Moreover, it was observed that more segments got a low rating than a mid-score. This means that the teacher mainly interacts with students in a usual or pro forma way and further elaboration of the content was not always evident.

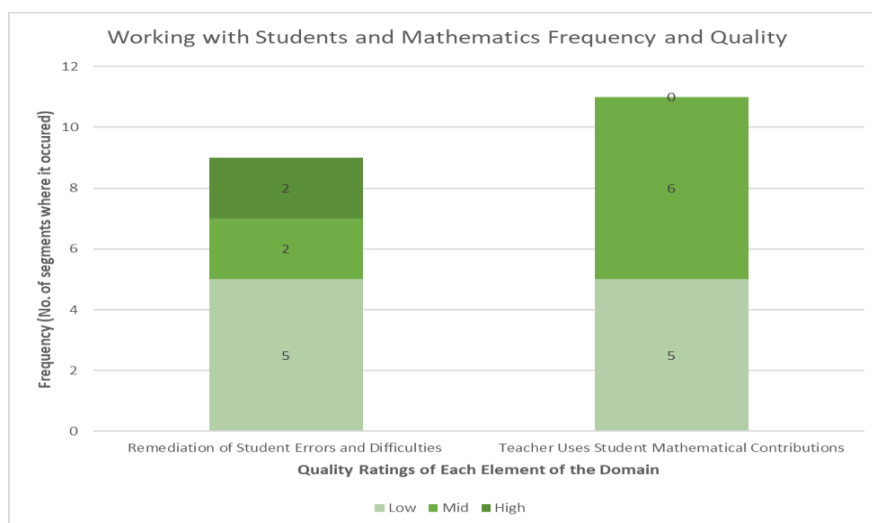


Figure 6: Working with Students and Mathematics Frequency and Quality

Here is a portion of the recorded class session that shows an example dialogue in which the students responded incompletely without providing an explanation, resulting in the teacher primarily explaining the ideas in a standard usual way.

**Excerpt 1:**

*During the discussion on adding similar fractions...*

*Teacher: But kagaya nung sinagutan natin kanina. Sa b.  $6/8$  at ginawa natin siyang  $3/4$ . Ano ang tawag natin sa fraction na iyon? (**Translation:** Just like the previous exercise, we simplify  $6/8$  to  $3/4$ . What do we call that type of fraction?)*

*Student A: Dinivide po natin (**Translation:** We divided)*

*Teacher: Dinivide natin siya by? (**Translation:** We divided by what?)*

*Student B: 2*

*Teacher: Dinivide natin siya by two which is the GCF. kapag nadivide na natin siya by two, nakakakuha tayo ng tinatawag na... (**Translation:** We divided it by two which is its GCF. If we divided it by two, the process is called?)*

*Student C: Lowest term*

*Teacher: So ano ang tawag natin sa process na yun? Lowest term or? (**Translation:** So, what do we call that process? Lowest term or?)*

*Teacher: Starts with the letter s*

*Students: Simplify*

*Teacher: After you add the numerators and copy the denominator, check if you can still get the lowest term or the simplified fraction.*

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*Teacher: Those are the steps, tama po. ... we first need to add or subtract, depende on the operation. And then we need to copy the same denominator. And the last step is... (Translation: Yes, that's correct those are the steps. We first need to add or subtract depending on the given operation then we need to copy the same denominator. And the last step is...)*

*Students: Simplify*

*Teacher: Yes, when necessary. Not all the time class na you need to simplify. Because there are times you cannot simplify anymore. For example dun sa assignment ninyo. The GCF of 3 and 7 is 1. So kung 1 lang ang GCF nila, wala ka nang isisimplify. Tama po ba? (Translation: Yes when necessary. It is not all the time that we need to simplify because there are times that you cannot simplify anymore. For example, on your assignment, the GCF of 3 and 7 is 1, then if 1 is their GCF, you do not have to simplify. Is that right?)*

*Students: Opo (Translation: Yes)*

*Teacher: So, it depends. Kaya nga sabi diyan if necessary. (Translation: So, it depends, that is why it said only when necessary.)*

Thus, this shows that since students commonly use one or two-word answers, which is why most of the time the teacher is the one who completes the idea during the discussion.

### **Errors and Imprecision**

Figure 7 summarizes the frequency for each element of the *teacher's error and imprecision* domain. Aside from having the lowest recorded frequency among all MQI domains, This domain also receive a consistently low rating for all of its elements which is a good indicator of quality instruction. It means that the minor content error, a few slips in language, and a brief lack of clarity did not obscure the mathematical teaching and learning in those instances.

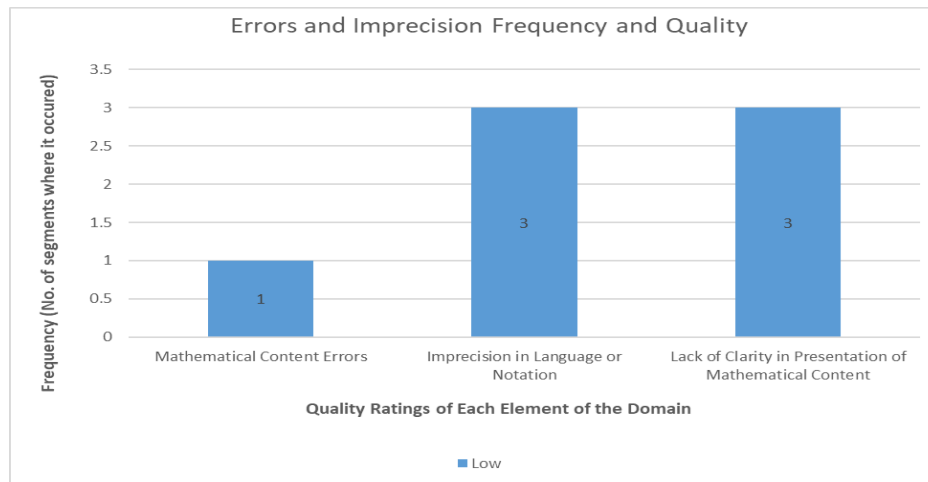


Figure 7: Errors and Imprecision Frequency and Quality

Below is an excerpt of the recorded class session, revealing an instance of teacher mathematical content error and language imprecision.

**Excerpt 2:**

*During the checking of assignment...*

*Teacher: What is the LCM of 4, 8, 12, and 16?*

*Student A: 96*

*Teacher: Do you agree?*

*Other Students: No*

*Teacher: Why not? (asks one among the students who want to answer)*

*Student B: 48 (did not answer why. Just simply stated 48)*

*Teacher: Is it 48? (asking the class)*

*Students: yes*

*Teacher: That is correct. 48 is the LCM. When we talk about LCM, it should be the least or the lowest common multiple of the given numbers. 96 is also correct but it is not the least.*

Although the teacher's elaboration and emphasis on the concept of "least common multiple" is correct, the error specifically occurred when she said, "96 is also correct, but it is not the least". What she originally meant to say is that "96 is also a multiple, but it is not the least". However, she had a spur-of-the-moment careless slip of the tongue. It resulted in a statement that might confuse learners and give them the wrong impression that 96 is acceptable while 48 is the best answer.

## Common Core Aligned Student Practices

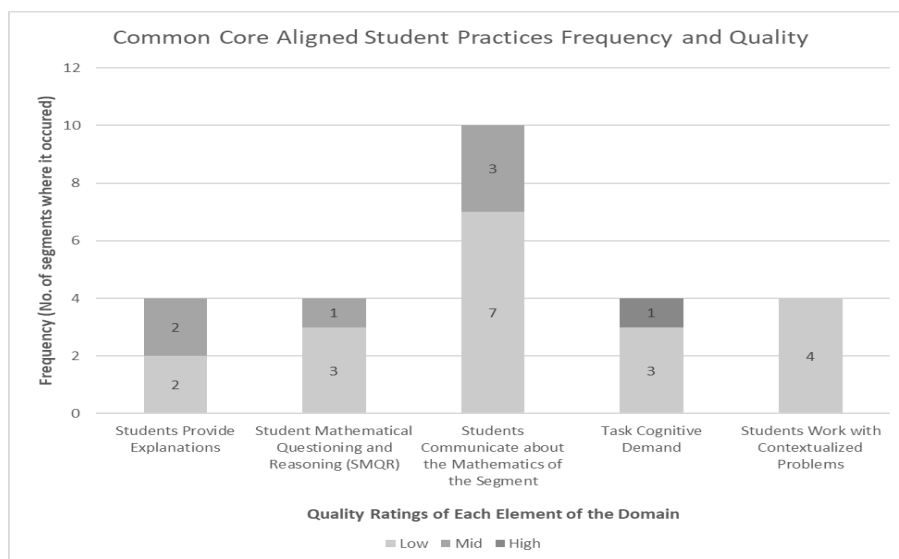


Figure 8: Common Core Aligned Student Practices Frequency and Quality

*Common core aligned practices* is the MQI domain that aims to capture evidence and describe the extent of students' meaningful engagement in “doing” mathematics during the instruction. Figure 8 shows that elements of students' involvement and participation were indeed present, but the majority of these instances leaned towards a low rating. Low ratings were given in students' explanation, questioning and reasoning, and communication of math due to the brevity of students' responses and talks in one-or-two-word answers. There were few instances of complete sentence responses, yet the delivery of ideas still needs improvement.

The teacher tried to elicit students' explanations through various strategies, such as asking why and how questions about an idea, procedure, or solution. However, either the students did not respond to these questions (refer to Excerpt 2) or they only provided single word or brief phrase and sentence-length explanations. There were some instances of more sustained student explanations (rated as mid-quality) but these were only obtained through the teacher's follow-up questions.

Mathematical questioning refers to instances where students ask questions that lead to the development of mathematical ideas for the lesson. Although the teacher often told the class just to ask her in case there were questions about the lesson, they did not ask anything. Hence, the teacher was the one who raised questions to elicit students' ideas and reasoning in moving forward with the lesson. An example of this case is when the teacher asks a series of probing questions to lead students in making a generalization about adding or subtracting similar fractions without the use of area models. Please refer to the excerpt below.

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### Excerpt 3:

a.  $\frac{3}{5} + \frac{1}{5} = \frac{4}{5}$

b.  $\frac{1}{8} + \frac{5}{8} = \frac{6}{8}$  or  $\frac{3}{4}$

c.  $\frac{10}{11} - \frac{3}{11} = \frac{7}{11}$

d.  $3\frac{6}{7} - 1\frac{2}{7} = 2\frac{4}{7}$

*Teacher: Without using the area models, how would you get the sum or difference of similar fractions?*

*Teacher: Do you see any patterns? What generalization can we make out of this pattern that you noticed?*

*Student C: Ipag-a-add po natin ang numerators and then ika-copy po natin ang denominator. (Translation: We will add the numerators of the given fractions and then copy their common denominator.)*

When a contextualized problem was used to introduce the lesson about adding and subtracting fractions, the teacher asked the students, “what mathematical operation are we going to use to solve the problem?”. They answered correctly, but they did not solve this problem yet. They only went back to solve this problem after the lesson discussion. This one contextualized problem covered 4 out of the 11 lesson segments. However, all of these instances were rated low. This is due to their rote or routine nature of learning and heavy scaffolding by the teacher. It is a far-fetched opposite of students performing the cognitive load of solving problems with greater autonomy and less to no help from the teacher - the characterizing feature for the high-quality manifestation of students’ work on contextualized problems.

Most of the tasks involved were undemanding activities such as applying procedures discussed previously or within the current segment. Some activities were not routine but hints were mostly provided by the teacher. The only highly cognitively demanding task was when the students were asked to observe the examples and make conjectures based on patterns they noticed.

## DISCUSSION AND CONCLUSION

The research has uncovered several significant findings and highlighted some aspects deserving of further investigation.

### Strengths of the Implemented Research Lesson

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Among all the domains of MQI, all segments under *error and imprecisions* obtained a consistently low rating. Errors and ambiguity made by educators in solving problems, defining concepts, introducing tasks, and using the correct notation are all within the scope of this domain. The low remarks indicate rare instances of teacher errors and imprecision, which is ideal and highly desirable in the teaching and learning process.

Moreover, it is also notable within this domain that *mathematical content errors* are the least common among the recorded data. Teacher content-related errors in the classroom is a sign that they are unsure about the material they are teaching, revealing a lack of mathematical competence (Center for Education Policy Research, n.d.). Thus, the low scores achieved under this segment imply that the teacher committed few content errors in instruction and highlights the level of conceptual understanding and proficiency of the teacher-implementer of the subject matter.

Hill et al. (2008) emphasize the favorable correlation between a teacher's level of mathematical knowledge for teaching and the quality of the mathematics education students receive. However, the results obtained in this undertaking are not sufficient to support this claim. On the other hand, Ronda & Adler (2019) explained that teachers' subject-matter knowledge was connected to teachers' present instructional practice but not to its development; rather, an increase in instructional quality was associated with teachers' vision of excellent teaching. This illustrates how well the educator-implementer of the topic knows the material and how well she can explain it to students.

Notable content errors occurred once when the instructor failed to correctly state that only one definition could be met by a given value. Thus, it was recorded by the raters as an error and imprecision. Being specific helps communicating correct mathematical ideas with pupils since it eliminates room for misunderstanding (Bieda & Voogt, 2019). Mathematical mistakes instructors make during education, especially if persistent, may expose knowledge gaps. Overall, even though there were some momentary and minor mistakes, the teacher-implementer did not make it hard to understand the concept of adding or subtracting fractions in the segments.

Another remarkable result obtained based on the coded data is that the teacher-implementer's *mathematical language* received a consistently high rating for almost all the segments. Thus, this implies that the teacher's skills in appropriately using mathematical terms to explain and discuss the content with the students are commendable. The ability to think critically, a solid understanding of the English language, a well-built number sense, and a strong background in mathematical content and pedagogy are all necessary for mathematical communication (Riccomini et al., 2015). Also, as Smith (2017) mentioned, proficiency in the instructional language is essential for comprehending the contents being taught. Therefore, being able to identify that the teacher-implementer's mathematical language with a high rating indicates quality of instruction.

Aside from mathematical language, other elements of richness, such as linking representations, explanations, sensemaking, and generalizing patterns, notably received high ratings for quality. These specific elements were coded in segments 2 through 6, where the concept-building part of the lesson occurred. Thus, reflecting the level of mathematical depth explored during the development of the lesson.

In the research lesson, area models were explicitly used to represent the addition and subtraction of similar and dissimilar fraction problems. Several studies revealed that using multiple representations improves students' learning of mathematical content such as fractions (Flores et al., 2019; Fyfe & Nathan, 2019). However, teachers must also help students understand how these representations are connected and elaborate on when relationships exist between these representations (Blanton, 2008). Furthermore, exposing learners to pattern generalization tasks does not only help them make sense of the operational procedures, it also develops their mathematical thinking in general and, more specifically, their algebraic thinking later on (Demonty et al., 2018).

### **Weaknesses of the Implemented Research Lesson**

On the other hand, the recorded data resulted low on the *common core aligned student practices*. This means the students did not contribute much to meaning-making or reasoning during the lesson. From obtaining a mostly low rating for this domain, it reflects that the students were not participating at a moderate or high cognitive level. Some students provided brief answers, and their responses were not substantial. It is not to say that the students were not engaging or participating, but rather their responses were mostly at an undemanding cognitive level, including routine and procedural responses. Although there were tasks that were not completely routine, it was heavily scaffolded by the teacher. The common core was made to improve the standards for student achievement (Marchitello & Wilhelm, 2014). Lynch (2015) also said that the *common core state standards* were designed to give students a more profound conceptual understanding of mathematics, not just procedural literacy. In this case, students are expected to develop and express conceptual ideas and understanding instead of just reporting procedural steps in solving equations or problems.

There were also limited occurrences of high ratings in the domain of *working with students in mathematics*. This domain pertains to how the teacher understands and responds to student's mathematical contributions or mathematical errors. There exists comprehensive research evidence to support that interactive dialogue between students and teachers can be a great source of drive for the development of student performance (Mercer & Howe, 2012). Several of the ratings were low, as the teacher mostly provided brief conceptual and procedural remediations, re-demonstrating procedures, or asking the students again so they might be able to correct themselves. The use of student contributions was also minimal, mostly because they were answering calculations or giving short definitions. Solomon and Black (2008) claimed that student

participation is significant for learners in building their understanding through verbal interactions with the teacher as well as their fellow students during the mathematical discourse. There were occasions of extensive remediation or great use of student contributions, highlighting said contributions to develop the mathematics during the discussion. However, although the teacher and student interactions go beyond pro forma, there was a mix of high and low elements, which resulted in ratings not qualifying as high.

The level of teachers' use of students' contributions might be attributed to the quality of students' contributions. If students are only providing one-to-two-word answers or just simply enumerating the steps, this does not give sufficient ideas for teachers to use in moving the lesson forward. Classroom discourse is the major setting in most mathematics classrooms, and some studies show that communication inside the classroom can help improve student achievements (Thompson, 2007). However, as the student's contributions lacked additional ideas to advance the discussion, the teacher had to put the ideas in to proceed with the lesson. The teacher provided ample opportunities for the students to give their thoughts, explanations, and justifications, with only several students able to speak their minds with responses suitable to further the discussion.

### **Limitations**

The research lesson's MQI ratings and analysis were administered by the lesson study group members except the teacher-implementer. However, they were also the ones who collaborated in conceptualizing the lesson's instructional plan. Hence, this study's findings are limited to the group's self-evaluation. Nonetheless, the MQI observational tool helped the group to thoroughly scrutinize several aspects of mathematics instruction, a vital part of improving teaching practices.

Since the lesson study aims to improve the education process, other areas, such as student academic performance, can be explored to measure the effectiveness of a lesson study. Lesson study that is well-executed and useful for teachers boosts their expertise in both subject and pedagogy, which eventually benefits students' educational experiences.

Lastly, the results may not be generalizable to different contexts, given that this research examined a single video lesson involving a specific mathematics subject, instructor, and learners. Nonetheless, suggestions for improvement in the research lesson particularly in addressing its weaknesses could be considered in the next cycle of the research lesson revision and implementation.

### **Implications**

Despite some limitations, the findings of this research have implications for educators and the educational system.

First, committing minimal mathematical errors and imprecision illustrate the teacher's level of content mastery and how well the teacher can facilitate learning among the students.

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Communicating with students more clearly and precisely helps students learn because it leaves little opportunity for ambiguity (Bieda & Voogt, 2019). Teachers' skills in appropriately using mathematical terms to explain and discuss the content with the students are important in promoting high-quality instruction.

Second, in terms of the research lesson on operations on fractions, teachers must explain to students the connections that may exist between the various fraction representations and aid them in recognizing those links when they are present (Blanton, 2008). Students benefit in more ways than one by being exposed to pattern generalization problems beyond just better understanding of the functional methods. It prepares pupils for subsequent success in algebra by strengthening their ability to reason mathematically (Demonty et al., 2018).

Third, the quality of student work may influence how teachers draw ideas. Getting learners' one- or two-word responses is not helpful for teachers in advancing the subject. It is suggested that teachers explicitly teach students how to provide high-quality explanations. Students must create and articulate concepts and knowledge instead of just describing the processes they used to solve equations or problems. Explicit teaching entails modeling high-quality explanations, emphasizing key aspects of the explanation, and giving challenging practice exercises.

Finally, since the MQI domains highlight the importance of lesson preparation, we hope that teachers seek opportunities to work together to improve mathematics education in their classrooms and beyond.

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