Combining flipped classroom and GeoGebra software in teaching mathematics to develop math problem-solving abilities for secondary school students in Vietnam

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Abstract: Flipped classroom is one of the teaching models that swaps students' learning space between learning in and before class. GeoGebra software is a dynamic math software that positively affects math teaching, especially geometry, and develops students' problem-solving abilities. The study was conducted to test the effect of combining flipped classrooms and GeoGebra in teaching math on students' outcomes, learning attitudes and problem-solving abilities. This study involved 74 students in 7th grade, including 37 students in the experimental group and 37 in the control group. Results from qualitative and quantitative data of pre-tests, post-tests, classroom observations and surveys show that students in the experimental group who learned with flipped classroom and GeoGebra have better problem-solving ability, results and learning attitudes. Specifically, with the significance level $\alpha = 0.05$ and degrees of freedom $df = 72$, the observed significance level (Sig. 2-tailed) is 0.010, an independent samples t-test of two groups in post-test indicates that the results of the experimental group were significantly higher than that of the control group. Besides, with the significance level $\alpha = 0.05$, the observed significance level (Sig. 2-tailed) is 0.000, and the paired t-test results reveal that the experimental group has a higher mean score in the post-test. The influence level (ES) is close to 0.64, showing that the combination of the flipped classroom and GeoGebra in the teaching of this study has a positive impact on learning outcomes and students' problem-solving ability. On the other hand, the student survey results are observed that students have a positive learning attitude toward this teaching process. In addition to the obtained results, the study also points out the remaining limitations and proposes new research directions.

Keywords: Academic achievement, Flipped classroom, GeoGebra software, Mathematical problem-solving ability

INTRODUCTION

The application of innovative teaching approaches with the support of information technology in mathematics education is an important need of the 21st century (Cevikbas & Kaiser, 2020). In particular, during the period of the global outbreak of the COVID-19 pandemic, the need for decentralized learning made the role of technology and online teaching methods fully exploited by teachers. Flipped classroom is a modern teaching method that ensures student participation through face-to-face and online learning during the pandemic (Cevikbas & Kaiser, 2022). For mathematics education, the development of digital technologies has changed the content of mathematics teaching in schools and promoted the development of students' knowledge and understanding of mathematics (Heid, 2005; Olive et al., 2009; as cited in Cevikbas & Kaiser, 2020). Nowadays, technologies have revolutionized math education by providing platforms that enable 2D and 3D graphics simulation, so teachers can teach math concepts visually and promote students' interest in learning (Chivai et al., 2022). In particular, GeoGebra is a commonly used dynamic geometry software in geometry teaching, designed to create teaching materials with various mathematical representations (Nzaramyimana et al., 2021). However, the effectiveness of using GeoGebra in teaching mathematics depends a lot on the learning content, the amount of time in class and the conditions of the facilities (computer, network connection) (Manganyana et al., 2020).

Many studies have shown that through learning geometry, students could develop deductive and reasoning thinking, logical thinking, analysis, systematization, critical thinking and creative thinking, and improve visualization and spatial thinking. At the same time, students could develop mathematical problem-solving abilities by applying mathematical knowledge and skills in solving problems (Osman et al., 2018). Many practical studies have demonstrated that developing students' ability to solve math problems is one of the important goals of education worldwide (OECD, 2019). Nonetheless, much research has not been done on using GeoGebra and flipped classrooms in math instruction to improve students' problem-solving skills. For these reasons, the study investigated the effectiveness of combining flipped classrooms and GeoGebra in teaching geometry to develop students' ability to solve math problems.

LITERATURE REVIEW

Flipped classroom

The flipped classroom is a modern teaching approach that promotes digital transformation in mathematics education (Cevikbas & Kaiser, 2022). According to Cevikbas and Kaiser (2020), the flipped classroom is a blended learning approach in which students learn theoretical content online, and face-to-face classroom time is used to cope with math problems. This approach provides more opportunities for student-centered activities such as teamwork and math problem-solving (Birgili, 2021).

Flipped classrooms allow teachers to create interactive and flexible learning environments (Cevikbas & Kaiser, 2020; Ramadhani et al., 2022). Pre-class activities like watching lecture videos and reading materials are assigned to students for independent study. In class, students engage in deep learning through discussions, problem-solving, and queries, but adopting flipped classrooms requires adjustments to traditional learning habits (Nielsen, 2020). Key requirements include well-structured out-of-class study time, teacher evaluation of pre-class activities, collaborative activities during class, and teacher support and feedback to create an organized learning environment.

With the above characteristics, the flipped classroom has gained wide popularity recently, with numerous studies highlighting its effectiveness for students and teachers. The flipped classroom enhances students' core competencies, including mathematical thinking (Cevikbas & Kaiser, 2020), critical thinking (Shaikh, 2022; Voigt et al., 2020), problem-solving competence (Cevikbas & Kaiser, 2022) and boosts student engagement, motivation, time management, teamwork, and positive learning attitude, ultimately improving student achievement (Lo & Hew, 2017; Cevikbas & Kaiser, 2022; Nugraheni et al., 2022).

Besides the undeniable benefits of math education, flipped classrooms pose many challenges for teachers and educational institutions. For teachers, the basic ability to use information technology in teaching is a fundamental requirement (Cevikbas & Kaiser, 2020; Moreno et al., 2020). Besides, the transition from traditional education to an innovative teaching method such as flipped classroom poses for teachers paradigmatic obstacles as the ability to design interactive teaching connect between out-of-class and in-class learning content and be able to encourage active social interaction in the classroom (Cevikbas & Kaiser, 2022; Swart et al., 2022). More importantly, due to the nature of the flipped classroom, teachers must regularly prepare new teaching content (e.g., lecture videos, online practice exercises, and other teaching materials), which increases the workload in the teacher's near-fixed-time budget. On the other hand, numerous studies have also mentioned technical and infrastructure challenges, such as issues with internet access, a lack of digital tools and devices, the need to create new learning materials, and the absence of guidelines for students, parents, teachers, and educational institutions (Cevikbas & Kaiser, 2022).

GeoGebra

GeoGebra is an interactive math software with features of an algebraic and dynamic geometry calculator, designed by Markus Hohenwarter, which can be accessed at https://www.geogebra.org (Ogbonnaya & Mushipe, 2020). Many studies have indicated the outstanding advantages of this software for teaching mathematics. Technically, GeoGebra (1) is free, open-source and regularly updated software, (2) can be used on many different operating systems, adapting to a variety of devices that can connect to the internet, (3) can represent a variety of mathematical representations (charts, equations, tables), (4) has a friendly, lively, easy interface, and (5) has a public community of users who can share experiences and products created when using the software. With the above characteristics, GeoGebra allows students to make a mathematical investigation and exploration easily and interactively, helping them become active actors in the knowledge-building process.

Thus, these benefits enhance interest in learning, activeness and independence in students' learning (Saputra et al., 2019). Moreover, many empirical studies have verified the effect of using GeoGebra in teaching mathematics on developing higher-order thinking skills (Ramlee et al., 2019; Wijaya et al., 2019), spatial visualization and especially mathematical problem-solving skills (Tran et al., 2014; Hernández et al., 2020). These merits make GeoGebra a highly effective teaching tool for enhancing students' math learning outcomes (Lognoli, 2017). In particular, the study of Chivai et al. (2022) emphasized the positive effect of GeoGebra on teaching concepts in geometry.

Nonetheless, teachers, students and educational institutions must overcome certain challenges to use GeoGebra in teaching math effectively. Students must be equipped with basic information technology skills to use GeoGebra software in learning effectively. On the other hand, exploratory learning with GeoGebra requires students to have independent learning skills, self-discipline and active learning participation, which can be difficult for some learners with low math achievement (Manganyana et al., 2020). For teachers, skills in using information technology and proficiency in lesson design with GeoGebra are two basic requirements. Also, factors such as lesson preparation time, choice of teaching methods, positive attitudes, and teachers' confidence in using GeoGebra are important challenges (Aliyu et al., 2021). Furthermore, the conditions of facilities - information technology, teaching materials, and teachers' professional knowledge and skills in using GeoGebra are also challenges for educational institutions and teacher training organizations (Ogbonnaya & Mushipe, 2020; Zengin, 2017).

Problem-solving abilities

Problem-solving is crucial in mathematics education (Purnomo et al., 2022). In general, problem-solving is a cognitive process to achieve some goal when the subject does not have a solution. Specifically, problem-solving is the application of existing knowledge and higher-order thinking skills such as visualization, relation, abstraction, understanding, application, reasoning and analysis (Nafees, 2011); to deal with new, unfamiliar situations through asking questions, analyzing situations, converting results, illustrating results, graphing and using trial and error. Problem-solving competence is the ability to apply mathematics to various mathematical situations (Osman et al., 2018). According to Osman et al. (2018), problem-solving in mathematics gives students experience using mathematical knowledge and skills to solve real-world problems.

According to Niss and Højgaard (2011), mathematical problem-solving competence is reflected in the ability to identify, formulate, delimitate, specify, and solve different types of mathematical problems such as "pure" or "applied", "open" or "closed" that do not have algorithms available, as well as analyze and evaluate how to solve their own or others' problems. In the Vietnamese educational program, MoET (2018) has clearly stated the requirements of mathematical problem-solving capacity as follows: (1) Identify and detect problems that need to be solved by mathematics, (2) Select and propose solutions to solve problems, (3) Use relevant mathematical knowledge and skills (including tools and algorithms) to solve the posed problem, (4) Evaluate the proposed solution and generalize to the same problem.

Thus, through mathematical problem-solving, learners can practice reasoning and analytical skills, form and evaluate problem-solving strategies (Osman et al., 2018) and especially contribute to developing other mathematical competencies. Besides, problem-solving helps students become more aware of problem-solving processes and strategies and improves students' skills in selecting and putting problem-solving strategies into practice (Hoon et al., 2013).

Many studies have investigated the challenges teachers and students face when teaching to develop mathematical problem-solving abilities. The main difficulties for students are understanding, analyzing, and converting the problem into a mathematical one. These difficulties often stem from students' lack of confidence, carelessness in calculations, and weak background of knowledge and experience (Angateeah, 2017; Yayuk et al., 2020). For teachers competency-based teaching, in general, requires teachers with professional knowledge and skills to design various learning activities that facilitate students to solve unfamiliar problems, real-world problems and practice exercises (Greiff et al., 2017). Therefore, a flipped classroom with a two-phase division of online and face-to-face learning can help overcome the challenge of teaching time. Additionally, it is possible to use the online learning environment to combine the use of GeoGebra to improve the effectiveness of concept teaching and students' ability to recognize, reason and solve problems. As a result, research was conducted to answer the following questions: (1) Is there a significant difference in learning outcomes between students learning with a combination of the flipped classroom and GeoGebra (experimental group) and students learning with traditional methods (control group)? (2) Is there a significant difference in students' learning outcomes in the experimental group before and after the intervention? (3) Is there any improvement in the math problem-solving ability of students learning with the combination of the flipped classroom and GeoGebra? (4) What is the attitude of students in the experimental group towards the combined learning environment between flipped classrooms and GeoGebra?

Context of the study

The research was conducted on the topic of converging lines in triangles in grade 7 in the General Education Program of Vietnam. The General Education Program in Mathematics (2018) has clearly stated the teaching content and requirements for studying this topic. Regarding the teaching content, the topic of the concurrent lines in triangles in the 7th-grade curriculum of the textbook includes the following contents: (1) the median in the triangle, the properties of the three medians of a triangle and the ratio of the distance from the vertex to the centroid to the corresponding median; (2) property of bisectors of an angle and property of three bisectors in a triangle; (3) properties of the perpendicular bisector of a line segment, property of three orthogonal lines of a triangle; and (4) properties of the altitudes in a triangle and the theorems about altitudes, medians, orthogonal, and bisectors from the vertex of an isosceles triangle (MoET, 2018). Regarding the requirements to be met, after studying this topic, students need to: (1) recognize the special lines in a triangle (median, altitude, bisector, orthogonal) and congruence. The rules of those particular lines; and (2) express geometric arguments and proofs in simple cases (e.g., argue and prove congruent line segments, equal angles from initial conditions involving triangles) (MoET, 2018).

Besides, the order of teaching organization when combining flipped classrooms with problem-solving teaching used in this study is described in the following chart (Figure 1):

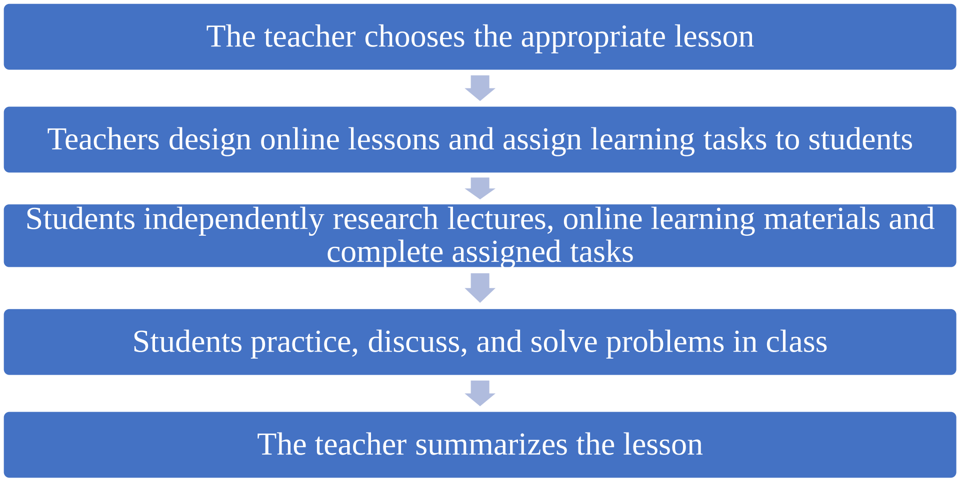


Figure 1: Teaching process

In the second step, the teacher makes requirements for solving learning problems, stating the knowledge goals to be achieved so that students have research orientation. After the learners fulfill the requirements in the third and fourth steps, the problems of the lesson are explored, discussed, resolved and commented upon by the whole class.

METHOD

The experiment was conducted to test the effectiveness of using a flipped classroom model and GeoGebra software to teach congruent lines in triangles in Vietnamese 7th-grade math textbooks to develop students' math problem-solving abilities. The experiment was conducted from April 10, 2022, to May 14, 2022, at Cao Thang secondary school, in Vinh Long City, Vietnam, for 74 students, including 37 students of the experimental group studied with flipped classrooms and GeoGebra and 37 students from the control group studied with traditional methods. Then, the data collected from the pre-test, post-test, classroom observation and student survey were thoroughly analyzed through different data analysis methods. The study was conducted based on the consent of the Ethics Council at Can Tho University, the Board of Directors of the junior high school and the consent of parents and students at Cao Thang junior high school in Vinh Long City, Vietnam.

Design

A quasi-experimental study with a control group was conducted to test the effectiveness of the combination of flipped classrooms and GeoGebra in teaching to develop math problem-solving abilities for students. In experimental designs, a pre-test was performed on the experimental and control groups to ascertain the participants' entry scores before treatment and, at the same time, verify the equivalence between the two groups. The experimental group was taught lessons in the flipped classroom combined with GeoGebra in a problem-solving orientation, while the control group was taught with conventional instruction. Then, a post-test was performed in both groups to measure student performance when learning with the new methods (Cresswell, 2012). This experimental design was used by many previous studies on the effectiveness of flipped classrooms as well as GeoGebra in math education (Adelabu et al., 2022; Kusumah et al., 2020), and there are similarities with some studies on mathematics education in Vietnam (Tong et al., 2021). With the above design, the experimental process occurred in the following order. Before teaching experimentally, the experimental group and the control group were selected based on the results of the pre-test scores of the two groups. Based on students' learning results in the experimental and control groups, a teaching plan was prepared based on applying flipped classroom and GeoGebra and teaching traditionally.

Based on the Mathematics general education curriculum requirements (Ministry of Education and Training, 2018), a scale was developed according to each level of math problem-solving ability to evaluate students; this scale is presented in Table 1. Research by Tong et al. (2021) on teaching to develop students' mathematical competence was also achieved through the design of scales when assessing the performance of students' abilities.

Math problem-solving ability	Behavioral index	Levels	Criteria
1. Ability to recognize and understand problems	1.1 Identify the problem	1	Recognize some information about the problem but do not realize the problem.
		2	Recognize most of the information about the problem but do not understand the whole problem.
		3	Get to know the whole problem.
	1.2 Identify and interpret information	1	Identify some initial information related to the objective of the task, but the relationship between those information has not been determined.
		2	Identify the majority of information relevant to the task's objective, and understand the value of that information.
		3	Identify sufficient information relevant to the task's objectives, and understand and interpret the value and relationships between such information.
2. Set up the problem space	2.1 Select and connect information with known mathematical knowledge	1	Select and connect a small amount of task information with known mathematical knowledge.
		2	Select and correctly match most of the task's information with known math knowledge.
		3	Make accurate, complete, logical connections of task information with known mathematical knowledge

	2.2. Choose a solution to solve the problem	1	Establish a partial solution to the problem
		2	Set up most of the solution to the problem, but not exactly logically.
		3	Establish a clear, specific solution to the problem.
3. Ability to plan and present solutions	3.1. Set the execution process	1	Partially built up the implementation process.
		2	Build up the majority of the implementation.
		3	Build logical process, perfect.
	3.2. Present the solution	1	Present only some ideas of the solution, but it is incomplete or lacks logic.
		2	Present most of the logical solutions but do not solve the problem.
		3	Present complete, accurate, logical steps according to the correct solution to solve the problem.
4. Ability to evaluate and reflect on solutions	4.1. Evaluate and comment on solutions	1	Initially know how to comment on the solution, it is not accurate and right to the point.
		2	Comment and evaluate the correctness of the solution.
		3	Comment and evaluate the solution with logical and persuasive arguments.
	4.2. Reflect on the value of the solution, discover new problems	1	Know how to reflect and identify some knowledge gained from the problem-solving process.
		2	Reflect on knowledge gained from problem-solving, and suggest alternatives to similar problems.
		3	New problems can be discovered through generalization and specialization from the resolved problem.

Table 1: Scale to evaluate students' ability to solve mathematical problems

After that, the research team conducted experimental teaching with the teaching plans designed for the experimental group and taught according to the traditional method for the control group. The research team made non-public observations for the experimental and control groups during the teaching process. The content of classroom observations was analyzed according to criteria including the teacher's teaching method, the student's learning method, acquired skills, classroom atmosphere and especially the expression of competence regarding solving math problems of students in the experimental group and the control group in two time points before and after the experiment. Finally, both the experimental and control groups of students were given a post-test to assess the impact on the development of their mathematical problem-solving abilities.

Furthermore, the study surveyed students' opinions in the experimental group to assess their attitudes. A set of questions was designed as multiple-choice questions on the Likert scale with five levels of Totally disagree - Disagree - Neutral - Agree - Totally agree (Likert, 1932) to collect data on the attitude, learning motivation, interest and receptivity of experimental group students when learning with experimental designs.

Regarding the validity and reliability of instruments, the experimental teaching lesson plan was moderated by experts in mathematical education methods at Can Tho University, and the tests were verified by colleagues who were teachers. High school staff conducted experiments to ensure the lesson objectives were specified in the program. After completing the adjustments according to the experts' recommendations, the tools were confirmed to be appropriate in terms of academic content and ability to assess students' abilities and could be used when conducting experiments. Moreover, the reliability of Cronbach's Alpha was used to test the reliability of the questionnaires used in the post-test. The student attitude survey and the Pearson correlation coefficient will be calculated to determine the correlation between the scores of the experimental group.

Data collection and analysis

Data was collected from pre-test (multiple-choice test) and post-test (descriptive test), class observation results and student opinion survey results after the experiment. The collected data were analyzed quantitatively with SPSS 26 software and qualitatively. This experimental method was conducted in many studies on developing mathematical competence for students. The experimental process is depicted in Table 2 as follows:

Groups	Pre-test	Intervention	Post-test	Opinion survey
EG	x	x	x	x
CG	x	-	x	-

Table 2: Experimental design

RESULTS

Pre-test results

The study used a test consisting of 20 multiple-choice questions to test the correlation between the experimental and control group's math learning levels. The data processing results demonstrate that the test scores of the two groups are normally distributed. The results of the Shapiro-Wilk test for the observed significance level of both classes are greater than 0.05, so it can be confirmed that the pre-test scores of both groups are normally distributed. The obtained results are found in Table 3.

	Statistic	df	Sig.
EG	0.968	37	0.356
CG	0.958	37	0.173

Table 3: Shapiro-Wilk test results of pre-test scores

The independent t-test method was used to test the hypothesis that the mean pre-test scores of the experimental and control groups were not significantly different. Tables 4 and 5 indicate the results of descriptive statistics and t-tests of mean pre-test scores of the experimental group and control groups.

	N	Mean	Std Dev	Median	Minimum	Maximum
EG	37	6.135	1.553	6.000	3	9
CG	37	5.987	1.991	6.500	2	10

Table 4: Descriptive statistics of scores before the intervention

Statistical results from Table 4 are observed that the average score of 37 students in the experimental class is 6.315, and that of 37 students in the control class is 5.987. The data dispersion of the experimental class (standard deviation) is 1.553. The standard deviation of the control class is 1.991; in addition, the mean and median scores in both groups are almost the same. Also, an independent t-test was used to test the hypothesis of equality of pre-test mean scores of both classes. The test results are presented in Table 5.

Levene test	t-test			
Sig.	df	t Stat	Sig. (2-tailed)	Mean Difference
0.104	72	0.358	0.721	0.149

Table 5: Results of independent Levene and t-test of pre-test scores

Levene test results give an observed significance level of 0.104 (greater than 0.05), proving that the two groups' scores have the same variance. An independent t-test was used to test the significance of the mean difference between the experimental group and the control group. Accordingly, with a significance level of 0.05 and degrees of freedom df = 72, the critical value (Sig.) equals 0.721 (greater than 0.05). Correspondingly, there was no difference in the mean score between the experimental and control groups. In other words, the test results reveal that the qualifications of the two groups are similar.

Post-test results

The study used three essay questions to test the difference in mean scores on the experimental and control groups' post-test scores. The data processing results indicate that the test scores of the two groups are normally distributed. The results of the Shapiro-Wilk test in Table 6 show that the observed significance level of both groups is greater than 0.05, so it can be confirmed that the post-test scores of both groups are normally distributed.

	Statistic	df	Sig.
EG	0.958	37	0.178
CG	0.980	37	0.739

Table 6: Shapiro-Wilk test results of pre-test scores

The independent t-test method was used to test the hypothesis that the difference in mean post-test scores between the experimental and control groups was statistically significant. Tables 7 and 8 demonstrate the results of descriptive statistics and independent t-tests of mean post-test scores of the experimental control groups.

	N	Mean	Std Dev	Median	Minimum	Maximum
EG	37	7.588	1.630	7.500	3.75	10
CG	37	6.615	1.520	6.500	3	9.75

Table 7: Descriptive statistics of scores after intervention

Statistical results of post-test scores from Table 7 show that the mean score of students in the experimental group is 7.588 and that of students in the control group is 6.615. The data dispersion of the experimental group (standard deviation) is 1.630, and the standard deviation of the control group is 1.520; in addition, the mean and median scores in both groups are almost equal. The scores of the experimental group were 7.588 and 7.5, and in the control group, 6.615 and 6.500. This demonstrates that the mean scores of the two groups are no longer similar. An independent t-test was used to test the hypothesis of the post-test mean equality of scores of both groups. The test results are presented in Table 8 below:

Levene tets	t-test			
Sig.	df	t Stat	Sig. (2-tailed)	Mean Difference
0.569	72	2.655	0.010	0.973

Table 8: Levene test results and independent t-test of post-test scores

Levene test results give an observed significance level of 0.569 (greater than 0.05), proving that the two groups' scores have the same variance. An independent t-test was used to test the significance of the mean difference between the experimental group and the control group. Accordingly, with a significance level of 0.05 and degrees of freedom df = 72, the critical value (Sig.) equals 0.010 (less than 0.05). From that, it is concluded that the mean score difference between the experimental and control groups is statistically significant. Additionally, the experimental group's mean score in Table 7 was higher than the control group's, leading to the conclusion that the experimental group's mean score in the post-test results was higher than the control group's.

Moreover, the calculated standard mean difference (SMD) is 0.64, which is in the mean (from 0.5 to 0.79) according to the Cohen influence scale (2011), so it can be concluded that the teaching process with the combination of the flipped classroom and GeoGebra software has a moderate impact on the academic performance of students in the experimental group. On the other hand, a paired test was performed to test the improvement in the group's learning outcomes after the intervention. Figure 2 shows that the scores before and after the experimental group have a positive linear correlation; the scores are distributed in a straight line before and after the treatment, so the correlation can be quite high. Besides, a correlation test was run to confirm the reliability of the scores before and after the intervention.

	N	Correlation	Sig.
Pair of scores before and after intervention	37	0.970	0.000

Table 9: Results of the correlation test regarding the scores of the experimental group before and after the intervention

The results obtained from Table 9 give an observation value of 0.000 (less than 0.01), so the calculated Pearson correlation coefficient (0.97) is statistically significant. In other words, the correlation between scores before and after the intervention is very large. A paired-sample t-test was performed; the results are presented in Table 10. The obtained critical value is 0.000 (less than 0.05), indicating that the score difference between the experimental group before and after the impact is statistically significant. Specifically, the difference between the mean scores before and after the intervention was calculated to be 1.453. It can be revealed that the math learning results of students in the experimental group are better than before the treatment.

	Mean	df	Sig.
Pair of scores before and after intervention	37	36	0.000

Table 10: Results of t-test by pair of scores of the experimental group before and after the intervention

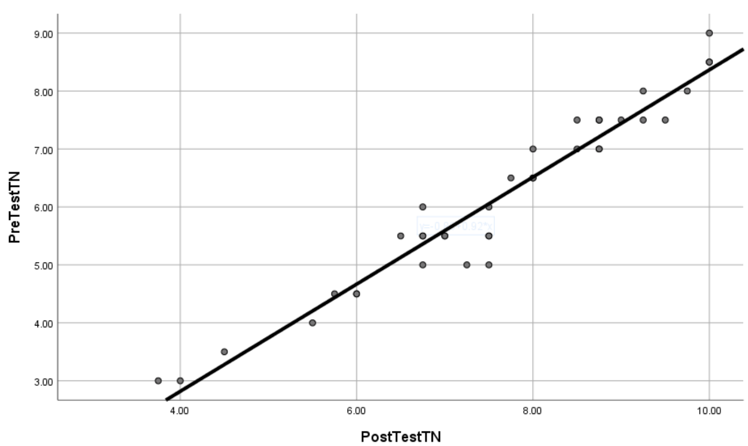


Figure 2: Scatter plot of points before and after the experiment of the experimental group

Assessment of math problem-solving abilities

Assessing the ability to solve math problems through the process of teaching in the classroom

Teachers in the experimental group carried out the teaching process in accordance with each topic, using the same teaching methodology for all topics as follows.

Step 1. Send an online lecture on the properties of the three medians of a triangle for students to self-study.

Step 2. Students go to class to discuss the content they have learned.

Step 3. The teacher concludes the problem and corrects it.

Step 4. Apply new knowledge.

Through the experimental lessons, students could identify the problem that needed to be resolved in most of the exercises; they also pointed out the information related to the goal to be directed and identified which was suitable. Nevertheless, in the proof problems, calculating the length of the line segment, some students have not been able to determine the appropriate information for the task. Most students selected and connected useful information with newly learned knowledge to solve assigned tasks when working in groups. Many students came up with most solutions to the problem, but they were not logical and precise. Some students were keen on formulating solutions and presenting them to the whole group. In the group discussions, many solutions were proposed, but there were rejections because they were not logical or did not reach the set goals.

Through the experimental process, it can be observed that students had been forming behavioral indicators in the scale of ability to cope with mathematical problems. Most students recognized the problem and identified the information, and many chose the information to resolve the problem. Many students learned and came up with the process of solving problems, knowing how to argue and refute the arguments of other members. From here, the mentioned teaching model helped students form and develop their problem-solving skills.

Assessment of the ability to solve math problems through the results of students' work

By teaching flipped classroom processes combined with GeoGebra software, students developed math problem-solving abilities; these competencies were analyzed based on a scale to assess math problem-solving abilities.

Math problem-solving competence	Behavioral Index	Level	Experimental group size	Control group size
1. Ability to recognize and understand problems	1.1 Identify the problem	1	0	0
		2	3	7
		3	34	30
		1	0	0

	1.2 Identify and interpret information	2 3	4 33	7 30
2. Set up the problem space	2.1 Select and connect information with known mathematical knowledge	1 2 3	3 1 33	7 0 30
	2.2. Choose a solution to solve the problem	1 2 3	3 1 33	7 0 30

Table 11: Analysis of students' ability to solve math problems in Question 1

Question 1 was given to measure students' ability to recognize and understand problems; establishing the problem space, the results of the experimental group show that most of the students of the experimental group and the control group fully understood the problem and applied the property of three medians in the triangle to fill in the blanks and identify most of the information and connect this information correctly when finding the segment length. Nonetheless, according to Table 11, three students in the control group still did not connect the information with the general knowledge; these students could not find the length of the line from the property of the three medians. This shows no significant difference in students' problem recognition in the two groups.

Math problem-solving competence	Behavioral Index	Level	Experimental group size	Control group size
1. Ability to recognize and understand problems	1.1 Identify the problem	1 2 3	0 3 34	1 7 29
	1.2 Identify and interpret information	1 2 3	3 15 19	11 18 8
2. Set up the problem space	2.1 Select and connect information with known mathematical knowledge	1 2 3	3 16 18	12 17 9
	2.2. Choose a solution to solve the problem	1 2 3	3 31 3	10 25 2
3. Ability to plan and present solutions	3.1. Set the execution process	1 2 3	7 27 3	11 24 2
	3.2. Present the solution	1 2	5 29	13 23

		3	3	1

Table 12: Analysis of students' ability to solve math problems in Question 2

Question 2 was asked to assess students' ability to recognize and understand problems, how to set up problem spaces, and ability to plan and present solutions. Most of the experimental group's students knew of the problem and could name the details important to achieving the task's goal. They also selected and connected most of the information correctly, but only three students presented the solution in the most specific and clear way. The setting up and presentation of the students' solutions were very satisfactory, but there were still some small shortcomings in the students' work. For students in the control group, only 29 students were able to identify the problem, and 26 could identify and interpret the information at levels 2 and 3. As for the identification and interpretation of the information at levels 2 and 3, only eight students in the control group could be identified; this number was quite different from the experimental group of 19 students; this shows that the experimental group had a better ability to identify the problem than the control group.

Regarding the selection of information connection and problem-solving, the experimental group only had three students at level 1; the control group had about ten students at specific level 1; 12 students selected and concluded information and ten students chose the solution to solve the problem. The control group had many students who could not select the appropriate information and connect them to handle the problem. In the ability to plan and present solutions, the experimental group had three students present their work well, while the control group only had one student present the complete solution. Table 12 indicates that the number of students who established the process and presented the solution at level 1 of the control group was still too high (over ten students).

Math problem-solving competence	Behavioral Index	Level	Experimental group size	Control group size
1. Ability to recognize and understand problems	1.1 Identify the problem	1	5	6
		2	22	23
		3	10	8
	1.2 Identify and interpret information	1	5	6
		2	22	23
		3	10	8
2. Set up the problem space	2.1 Select and connect information with known mathematical knowledge	1	7	6
		2	18	21
		3	12	10
	2.2. Choose a solution to solve the problem	1	6	6
		2	19	24
		3	12	7

3. Ability to plan and present solutions	3.1. Set the execution process	1	7	15
		2	27	21
		3	3	1
	3.2. Present the solution	1	10	25
		2	24	17
		3	3	0

Table 13: Analysis of students' ability to solve math problems in Question 3

Question 3 was given to assess students' ability to recognize and understand problems, how to set up problem spaces, and ability to plan and present solutions. Most students in both groups recognized most of the information but did not fully understand the problem. These students identified and understood the information in the problem but did not fully identify the relevant information. Since then, many students have been unable to connect this information accurately and completely. The number of students who selected the right information to use and set up the solution of both groups was also quite similar; most of them still could not connect the information logically and establish it with clear and specific solutions. In sentences c and b of Question 3, many students in the experimental group built up the process of doing the test and presented the solution, but the logic was lacking, or the process did not overcome the problem raised. For the control group, most of the children could not present the process or only presented some ideas of the solution but still had many shortcomings. According to Table 13, three students in the experimental group presented well and logically all the requirements of this problem, but in the control group, none of the students in the control group presented fully, accurately and logically according to the correct solution to the problem.

Analysis of student work

Below we present the score distribution of the control group and the group experiment and the level achieved on each pre-test question.

Classification	Poor 0.0-3.4	Weak 3.5-4.9	Average 5.0-6.4	Good 6.5-7.9	Very Good 8.0-10.0
EG	0	3	4	13	17
	0.0%	8.11%	10.81%	35.14%	45.94%
CG	2	3	9	15	8
	5.41%	8.11%	24.32%	40.54%	21.62%

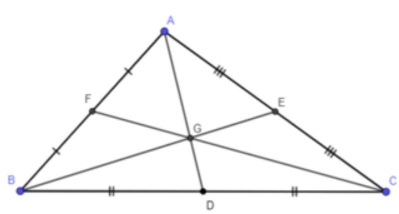
Table 14: Results of grading pre-test scores

Test scores were classified into five levels, including poor, weak, average, good and very good. In the experimental group, the excellent grade accounted for the highest percentage, including 17 out of 37 students (accounting for 45.94%). Poor and weak grades accounted for a very low percentage; specifically, there were no students with poor grades (rate of 0.0%), and average grade accounts for 8.11%, corresponding to 3 students. Thus, the total number of students below the average was three, accounting for 8.11%. Statistical results show that there were 13 students with good grades (accounting for 35.14%), an average of 4 students also accounting for 10.81%, and good grades 17 students, accounting for the highest percentage of 45.94 %. This indicates that most of the students in the experimental group achieved quite good results after this post-test. Based on statistical results from Table 14, the experimental group's post-test results were quite good, and the group's score classification was mostly good and very good.

In the control group, there were two students with poor grades, accounting for 5.41%, and three with weak grades, accounting for 8.11%. Eight students, or 21.62%, or less than half of the experimental group, were considered to be good students. Most of the students' scores were concentrated at a good average.

The two groups had a difference in the percentage of grades and the distribution of scores; specifically, the group was at a lower level than the experimental group, but there were two students in the control group, accounting for 5.41%. The number of students with weak grades in both experimental and control groups was three students, accounting for 8.11%. The experimental group's mean was 13.51% less than the control group. The good grades in the experimental and control groups were nearly equal, with 13 and 15 students accounting for 35.14% and 40.54%. In terms of good ranking, the experimental group outperformed the control group, the difference between the experimental group and the control group was nine students, accounting for 24.32% of the total students. The following are the results of the post-test for each specific question.

Câu 1 (3,5 điểm). Cho hình vẽ



a. Điền số thích hợp vào chỗ chấm

$AG = ...AD$ $\qquad$ $GE = ...BE$

$CG = ...CF$ $\qquad$ $FG = ...GC$

b. Biết $AD = 6cm, BE = 9cm$. Tính độ dài các đoạn thẳng DG và BG.

(Translate into English: Question 1 (3.5 marks). For drawing:

a. Fill in the blank with the correct number.
b. Knowing AD=6cm, BE=9cm. Calculate the lengths of the line segments DG and BG.)

Figure 3: Question 1 of the post-test

Classification	Weak 0.0 – 0.5	Average 0.6 -1.75	Good 1.75-2.75	Very Good 2.75-3.5
EG	0	0	3	34
	0.0%	0.0%	8.11%	91.89%
CG	0	0	7	30
	0.0%	0.0%	18.92%	81.08%

Table 15: Statistics on the results of students' work for Question 1

Question 1 in Figure 3 asked students to identify and understand the properties of the triangle's centroid. Table 15 demonstrates that 91.89% of the experimental group's students earned good grades; this shows that most of the students in the experimental class recognized the centroid of the triangle and understood the three properties of the median line in the triangle, and knew how to apply this property to calculations. Nonetheless, 8.11% of students could not apply the calculation but only filled in the appropriate proportion in the blank. A typical correct answer involves identifying the centroid of the triangle and using the information given in the exercise to calculate the lengths of the line segments. For the control group, the number of students with good scores was higher than in the experimental group (18.92% in the control group and 8.11% in the experimental group). Many students in the control group did not use their understanding of the properties to complete the calculation, as shown in Table 15. Through Question 1, most students in both groups could recall the property of the three medians in a triangle and use it to perform particular calculations.

The analysis of the students' work in the two groups for Question 2 of the post-test is summarized below.

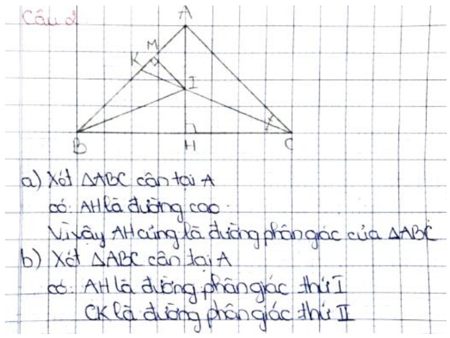
Question 2 (3.0 marks): Let ABC be an isosceles triangle at A with altitude AH and bisector CK (with K on AB). Let I be the intersection of AH and CK. IH=2cm. (a) Is AH a bisector of triangle ABC? Why? (b) Prove that BI is the bisector of angle ABH. (c) Draw IM perpendicular to AB. Calculate IM length.

Figure 4. Question 2 of the post-test

Classification	Weak 0.0 – 0.5	Average 0.6 -1.0	Good 1.1-2.0	Very Good 2.1-3.0
EG	0	3	19	15
	0%	8.11%	51.35%	40.54%
CG	1	10	18	8
	2.7%	27.03%	48.65%	21.62%

Table 16: Statistics on the results of students' work for Question 2

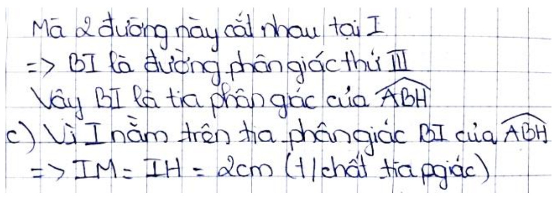
Question 2 in Figure 4 required students to apply the knowledge they have learned about concurrent lines in triangles to reason and solve the problem of geometric proofs. The majority of the experimental group's students (51.5% and 40.54%, respectively) had successful outcomes, as shown in Table 16. In the experimental group, there were no students with weak points, and the number of students with average scores accounted for a low rate (8.11%). For the control group, the student who achieved the weak point was one student, accounting for 2.7%. The number of average and good students in the control group accounted for the highest percentage (27.03% and 48.65%). In general, the results of the experimental and control groups have a clear difference through Question 2, which shows that in the control group, students did not apply the theorems and properties effectively.



(Translate into English: a) Consider triangle ABC isosceles at A; triangle ABC has AH as altitude, also the bisector of triangle ABC.

b) Consider triangle ABC isosceles at A; triangle ABC, where AH is the first interior bisector, and CK is the second interior bisector.)

Figure 5: Correct answer to Question 2 by a student in the experimental group



(Translate into English: Since these two lines intersect at I, BI is the third bisector.

c) Since the point I lies on the bisector BI of angle ABH, IM=IH=2cm.)

Figure 6: Correct answer to Question 2 by a student in the experimental group

Figure 5 and Figure 6 are students' answers in the experimental group, who captured and analyzed important information in assumptions and arguments to resolve problems. This student also used the properties of concurrent lines in calculations, but the reasoning was unclear and still needed more practice.

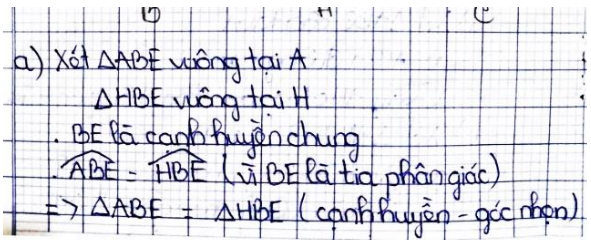
Question 3. Let ABC be a right triangle at A, where BE is the bisector of angle ABC; point E lies on AC. Draw AH perpendicular to BC, with point H lying on BC. Let K be the intersection of AB and HE. (a) Prove that triangle ABE is equal to triangle HBE. (b) Compare the length of line segment AB and the length of line segment BH. (c) Is BE the perpendicular bisector of line segment AH? Why? (d) Prove that BE is perpendicular to KC.

Figure 7: Question 3 of the post-test

Classification	Weak 0.0 - 0.5	Average 0.6 -1.0	Good 1.1-2.5	Very Good 2.6-3.5
EG	1 2.7%	3 8.1%	23 62.16%	10 27.03%
CG	1 2.7%	3 8.1%	32 86.49%	1 2.7%

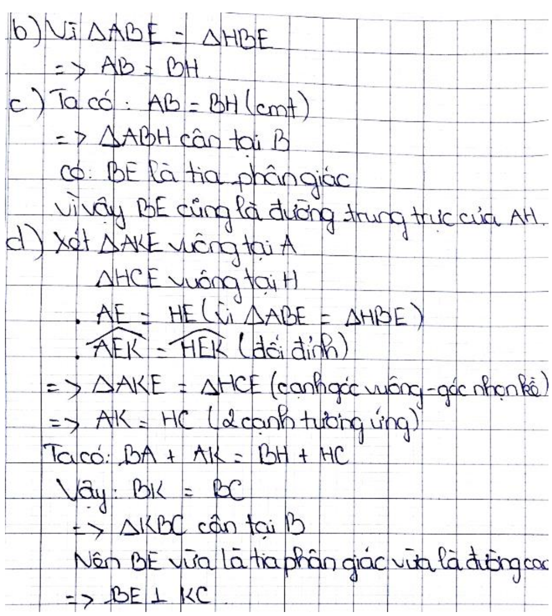
Table 17: Statistics on the results of students' work for Question 2

In Question 3 (Figure 7), the problem asked students to apply the definition and properties of the perpendicular bisector and altitude in a triangle to solve the problem. Students needed to manipulate triangles that were congruent and, from that, compared the lengths of the line segments related to two similar triangles. Students used the property of the perpendicular bisector of the line segment, the property of the three altitudes. in a triangle, or students could employ the properties of an isosceles triangle to deal with the exercise. According to Table 17, both groups' weak and averagely graded assignments for this question account for the same percentages of 2.7% and 8.11%, respectively. The control group had only one student with good scores, and the experimental group had ten students with good scores (accounting for 2.7% and 27.03%). Thus, most of the students in the experimental group could apply new knowledge in reasoning and problem-solving, and the experimental group's problem-solving ability was different from that of the control group.



(Translate into English: a) Consider triangle ABE right-angled at A; triangle HBE right-angled at H; BE is the common hypotenuse. So angle ABE is equal to angle HBE. Therefore, triangle ABE is equal to triangle HBE.)

Figure 8: Correct answer to Question 3 by a student in the experimental group



(Translate into English:

a) Because triangle ABE is equal to triangle HBE, AB=BH

c) We have AB=BH, so triangle AABH is isosceles at B; BE is the bisector. So, BE is also the perpendicular bisector of AH.

d) Triangle AKE is right angled at A; Triangle HCE is right angled at H; AE=HE; angle AEK is equal to angle EHK; so AK=HC; we have: BA+AK=BH+HC, so BK=BC. Hence, triangle KBC is isosceles at B, so BE is both the bisector and the altitude; infer that BE is perpendicular to KC).

Figure 9: Correct answer to Question 3 by a student in the experimental group

Figure 8 and Figure 9 are examples of the correct work of students in the experimental group, who proved that the triangle is congruent, deducing the pair of equal sides in question (b). In questions

(c) and sentences (d), students applied their knowledge of perpendicular and bisectors in an isosceles triangle to prove the orthogonal line.

Classroom observation results

After teaching the lessons on the converging line in the triangle, the results of the experimental group and the control group's observations were analyzed and compared based on the factors of the teaching methods, learning methods, acquired skills, learning content and students' interest in learning. The observed results are shown in Table 18.

Factors	Experimental group	Control group
Teaching methods	- Ask questions related to the lesson in videos sent to students. Class activities are discussed and explained, and do exercises based on the teacher's suggested questions. - Student-centered. - Teachers use GeoGebra software to form concepts and properties with visual images and perform measurements. - The teacher presents the combined slide show using GeoGebra. Use the visualization software GeoGebra to comment on the theorem. - The teaching process follows the process of the reverse classroom combined with GeoGebra software.	- Explain and present. - Teacher-centered. - The teacher presents concepts and properties; proves properties. - The teacher presents the board. - The teaching process starts from explaining concepts and properties to solving examples and exercises in textbooks.
Learning methods	- Discuss together, work in groups, collaborate, observe, predict, present, and critique. - Work individually and in groups. - Learn new knowledge through problem-solving. Explore knowledge under the guidance of teachers and self-study about new knowledge before going to class.	- Work individually, listen to the teacher's questions and express the opinions - Expressing opinions and working individually. - Acquiring new knowledge imparted by the teacher.
Acquired skills	- Achieve teamwork, communication, questioning, problem-solving, presentation, analysis, prediction, and criticism skills.	Ability to comment, adjust math solutions, and prove formulas.
Learning content	- Know and draw a triangle's medians, medians, and bisectors.	- Know and draw a triangle's medians, medians, and bisectors.

	- Understand and apply properties of concurrent lines to calculate angle measures and find side lengths. - Understand the properties of concurrent lines in triangles and apply them to complex problems. - Know several practical problems related to concurrent lines to apply to solving real-life problems.	- Understand and apply properties of concurrent lines to find the measure of an angle, the length of the side, and the ratio of the sides.
Student attitude	- Students actively participate in group activities and exchange ideas. - Students enthusiastically expressed, commented and absorbed ideas.	Students are still passive; a few students actively voice their opinions.

Table 18: Classroom observation results between the experimental group and control group

Classroom observation results show that this teaching and learning method positively impacted students' skills, learning content and learning attitude in the experimental group. When learning new knowledge with the support of GeoGebra, students in the experimental group had access to real things and visual images to new knowledge, helping students to be more excited about new content. Because the students in the experimental group were studied before coming to class, they were more confident when expressing their opinions, asking their questions as well as commenting on the opinions and questions of others. Besides, the length of lessons devoted to problem-solving was relatively large, enabling students in the experimental class to practice the steps of problem-solving in learning mathematics. Thereby forming the quality of confidence and self-control for students in problem-solving was observed.

Results of the survey of students' opinions after the experiment

After completing the lessons in the experimental group, the research team surveyed the opinions of students in the experimental group through a set of multiple-choice questions, according to the Likert scale. The survey was conducted to investigate the students' attitudes towards the combined learning method of the flipped classroom and GeoGebra and the students' assessment of the learning effectiveness and capacity building in math problem-solving after the experiment. Statistical results of the responses are as follows.

Items	Totally disagree	Disagree	Neutral	Agree	Totally agree
1. I like the lessons about concurrent lines in triangles.	0 0%	0 0%	1 2.7%	13 35.1%	23 62.2%
2. I find that the learning process following these lessons helps me study more effectively.	0 0%	0 0%	3 8.1%	14 37.8%	20 54.1%

3. I find that the learning process following these lessons helps me access new knowledge more effectively and easily.	0 0%	0 0%	2 5.4%	13 35.1%	22 59.5%
4. I find that the learning process following these lessons helps me to solve problems similar to those approached before.	0 0%	1 2.7%	4 10.8%	15 40.5%	17 46.0%
5. I want to take similar lessons in another lesson.	0 0%	0 0%	1 2.7%	11 29.7%	25 67.6%

Table 19: Students' feedback on questions of the survey

In Item 1, 100% of students answered fully; specifically, 23 students (accounting for 62.2%) agreed that they would like to learn lessons about convolutions using a combination of the flipped classroom and GeoGebra software, 13 students agreed (accounting for 35.1%), and one student (accounting for 2.7%) chose the normal answer, no student disagreed or totally disagreed with the above question. Based on the above data, Table 19 shows that students were interested and interested in experimental lessons in class when learning the topic of congruent lines in triangles by combining flipped classroom model and GeoGebra software.

In Item 2, most of the students totally agreed and agreed that organizing the activities of the experimental class helped them learn more effectively (accounting for 54.1% and 37.8%), and besides, three students chose the normal answer to the question (accounting for 8.1%). No student chose to disagree or completely disagree. Based on the data mentioned above, it can be concluded that students are more adaptable to a new teaching model that incorporates GeoGebra software and flipped classrooms than they are to the traditional classroom-based teaching method. It also contributed to motivating teachers when applying other teaching methods.

Item 3 was given to survey students' opinions about finding that accessing new knowledge by combining the flipped classroom model and GeoGebra software was more effective and easier to achieve the expected results. Specifically, 22 students (accounting for 59.5%) completely agreed with the given statement, 13 students (accounting for 35.1%) agreed, and two students (accounting for 5.4%) gave the opinion that it was normal. In this question, no student disagreed or completely disagreed with the statement. Thus, through collecting opinions from students, research shows that this teaching method is one of the methods that can help students acquire knowledge effectively and easily.

Item 4 was interested in whether the students themselves saw an improvement in computation, resolved problems similar to those approached before, and obtained the following results: there were 17 students (accounting for 46.0%) out of 37 students who completely agreed with this, 15 students (accounting for 40.5%) agreed, four students (10.8%) had no opinion, and one student (accounting for 2.7) %) disagreed with the statement given. Thus, most students saw progress when learning with the combined learning method of flipped classroom model and GeoGebra software, and besides, there were still some students who did not notice any progress in learning practice with this method.

In this last question, students were asked if they would like to take similar lessons in other lessons instead of only learning in lessons on the topic of concurrent triangles and congruences. The results were as follows: There were 25 students (accounting for 67.6%) who totally agreed, 11 students (accounting for 29.7%) who agreed, and one student (accounting for 2.7%) who was normal (Table 19). Thus, most students wanted to learn other lessons by this method and could extend to other subjects.

DISCUSSIONS

The experimental results answered the research questions posed. Regarding the learning results, the two groups before the experiment had similar qualifications, and following the experiment, there was a difference between the mean scores of the experimental control groups. Specifically, with the significance level of 0.05 and degrees of freedom df = 72, the critical value (Sig.) equal to 0.010 shows that the mean score of the post-test results of the experimental group is higher than that of the control group. Also, the influence level of 0.64 according to the Cohen influence scale (2011) shows that the pedagogical experiment has an average impact on students' learning outcomes in the experimental group. Besides, the experimental group's test of two sets of scores before and after the treatment reveals that, at a significance level of 0.05, the critical value is obtained at 0.000, and the difference between the mean scores before and after the intervention's impact is 1.453, indicating that the math learning outcomes of the experimental group's students improved since before the experiment. Thus, the experiment has confirmed the effectiveness of applying the combined teaching process of the flipped classroom and GeoGebra software in teaching the converging lines in the triangle for students' learning outcomes. This result is similar to the conclusions of (Cevikbas & Kaiser, 2022; Birgin & Acar, 2020). Regarding the development of students' ability to solve math problems through the built-in scale, the research team analyzed students' work based on how well students achieved the criteria of the scale and based on student performance in class. Through the analysis of the post-test work of the students in the experimental group, it can be seen that the majority of students in the experimental group performed well in the requirements of problem recognition and understanding (over 90%), choosing and connecting information with the learned knowledge and selecting problem solutions (over 70%), and planning and presenting solutions to solve problems (over 60%).

Moreover, the classroom observation results show that with the combined teaching method of the flipped classroom and GeoGebra, students had access to real things and visual images for new knowledge, helping students to be able to learn more excited about new content. At the same time, because the students in the experimental group were studied before coming to class, students were more confident when expressing their opinions and offering solutions to solve problems because the duration of the sessions was devoted to their problem-solving relatively much, creating conditions for students in the experimental group to practice skills in the process of problem-solving. Thus, it can be concluded that integrating GeoGebra into the flipped classroom in teaching math contributes to training students' problem-solving abilities. This result is consistent with studies on the effectiveness of GeoGebra (Thapa et al., 2022; Tran & Nguyen, 2020) and flipped classrooms (Cevikbas & Kaiser, 2020; Lo & Hew, 2017) for the development of math problem-solving. Besides, the survey results of students in the experimental group show that students had a positive attitude towards the combined teaching method of the flipped classroom and GeoGebra and realized that they made progress in learning. with this method (Lo & Hew, 2017; Cevikbas & Kaiser, 2022).

During the experiment, the research also recorded positive results when applying the teaching process that combines flipped classrooms and GeoGebra software into teaching. Experimental results show that students grasped the knowledge of the lesson content and applied knowledge to problems with a high degree of application. Indeed, they learned new knowledge with the combined teaching process of the flipped classroom and GeoGebra software to help students retain knowledge more deeply because the knowledge was thoroughly researched at home with the support of teachers and was reinforced in classroom learning, evident in classroom sessions.

Nonetheless, applying flipped classrooms and GeoGebra in teaching mathematics had certain difficulties. Firstly, students did not have the same level of knowledge and skills in using information technology; some did not learn new knowledge at home, making it difficult to keep up with other students. Secondly, teaching according to a process different from the traditional method made it difficult for some students who could not adapt in time and could not yet study independently. Third, teachers had to prepare new learning content thoroughly in advance, which took time and increased teachers' workload (Cevikbas & Kaiser, 2022). Consequently, in order to overcome the challenges above, the study recommends that educational researchers and teachers thoroughly equip themselves with the knowledge and abilities to use information technology for students, guide students about the learning process with flipped classrooms, and organize training for teachers on subject matter expertise as well as methods of organizing teaching models (Cevikbas & Kaiser, 2022). Furthermore, teachers can track students' learning progress during the teaching process by letting students use GeoGebra math software to answer questions in the middle of lessons and collect statistics on lesson outcomes. Teachers can also create lesson-related questions using other online tools so that students can complete practice exercises and assess the information they learned at home.

Besides the obtained results, the study still has many limitations. With a small experimental sample and short experimental time, the experimental results may be partial and not comprehensive due to public health issues in preventing COVID-19. Hence, the study's conclusions may be more representative, and the experimental influence may be investigated more deeply if the sample size is larger and the experimental time is longer. At the same time, requiring students to be exposed to a new learning environment that is different from what is done in a regular classroom makes some students unable to adapt to the classroom activities. The post-test results in the experimental class were quite high, but the study has shown a moderate impact effect; some students still did not achieve good results. In addition, the tools to assess the ability to solve math problems do not reflect students' achievement levels. On the other hand, the study has not yet explored the difficulties teachers and students face in the flipped classroom's combined learning environment and GeoGebra, focusing on developing math problem-solving abilities. However, this information also contributes to deepening the conclusions of the study.

CONCLUSIONS

Results from this experimental study show that combining flipped classrooms and GeoGebra improves students' math problem-solving abilities, learning outcomes and attitudes. Analysis of the post-test results found that the experimental group had significantly higher scores than the control group (Sig. 2-tailed=0.010 with $\alpha = 0.05$ and degrees of freedom $df = 72$). Additionally, the mean score after the intervention in the experimental group was higher than before (paired t-test showed a significance level of 0.000). The combined approach had an effect size (ES) of 0.64, positively affecting learning outcomes and mathematical problem-solving abilities.

For new studies in the future, the research team proposes several related research directions such as (1) applying a combination of the flipped classroom and GeoGebra in teaching various math topics and in order to develop other math competencies for students; (2) researching and applying a combination of the flipped classroom and GeoGebra with other active teaching methods such as problem-based learning and project-based learning; (3) research on the influence of some factors on the development of student's ability to handle math problems such as academic levels of students; and (4) study the long-term effects of using a combination of the flipped classroom and GeoGebra. On the other hand, in terms of research design, the research team proposes new studies organized with large sample sizes and long observation time to assess the effectiveness and challenges of combined flipped classrooms comprehensively and GeoGebra in teaching math.

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