Socio-mathematical Norms Related to Problem Solving in a Gifted and Talented Mathematics Classroom

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Abstract: This study explores problem solving practices in a gifted and talented mathematics classroom in response to the calls for investigating problem solving as a sociocultural cultural activity rather than a cognitive activity of individuals. Therefore, we used a socio-mathematical norm perspective for our investigation. Data consists of forty-three mathematics lessons in a gifted and talented classroom. We used the two dimensions of a socio-mathematical norm (student and teacher) to analyze the observational data. The findings revealed a social norm regarding different solutions that reflect the classroom's micro-culture in terms of problem solving and students offered mathematically different (especially easy, simple, or effective ones) and sophisticated solutions which pointed out a socio-mathematical norm about mathematically different solutions. We observed an explicit talk on different solutions. However, the classroom community lacked a socio-mathematical norm regarding evaluations of mathematically different solutions based on criteria such as easy, simple, effective, or sophisticated. A lack of such a norm resulted in low-level problem-solving practice which was not expected from gifted and talented students. We offer practical implications for the dynamics of a classroom where gifted and talented students engage in problem solving activities and theoretical implications regarding the two dimensions of a norm.

Keywords: socio-mathematical norms, problem solving, gifted students, talented students

INTRODUCTION

Mathematical problem solving (PS) has long been an important aspect of mathematics, its learning, and teaching. Therefore, the mathematics education community has been working on understanding the nature of PS for over four decades (Carlson & Bloom, 2005; Liljedahl et al., 2016). Most mathematics educators agree that the development of students' PS abilities should be the main purpose of teaching and that a wide variety of factors and decisions must be taken into account by curriculum developers and teachers to achieve this goal (Anderson et al., 2005; Lester, 1994, 2013).

In recent years, the complex nature of PS has been accepted by the mathematics education community, and the whole picture is even more complex when social aspects of mathematics education are considered (Koichu, 2019). After his book entitled "Mathematical Problem Solving" (Schoenfeld, 1985), which was published more than 25 years ago, Schoenfeld emphasized the social aspects of PS as follows: "Individuals do not work, or learn, in a vacuum …. characterizing productive learning environments – and the norms and interactions that typify them – is an essential endeavour, if we are to improve instruction. But learning environments are highly interactive, and the ideas that individuals construct are often built and refined in collaboration with others" (Schoenfeld, 2013, p.20). Similarly, according to Cobb and Yackel (2011), the classroom micro-culture (social context) emerges from the coordinated actions of its participants (Cobb & Yackel, 2011). In every classroom, there are implicit and explicit understandings, or a set of norms, that determine the behavior of teachers and students about what other members of the classroom do and value (Makar & Fielding-Wells, 2018). As the teacher and students interpret and respond to each other's actions, normative activities of the classroom community (social perspective) emerge and are continually regenerated by its members (Cobb et al., 2011). As a collective notion, a norm is related to what is taken-as-shared by a group. It refers to expectations and obligations that are negotiated among teachers and students (Yackel, 2004). Social norms (SNs) refer to "regularities in the interaction patterns that regulate social interactions in the classroom", while socio-mathematical norms (SMNs) are specific to mathematics (Yackel & Rasmussen 2002, p.315). Normative understandings of what counts as mathematically different, efficient, or elegant, and what counts as an acceptable mathematical solution or justification are examples of SMNs (Yackel, 2000). SNs and SMNs are related to beliefs (about one's own role, others' role, the nature of a specific mathematical activity, and mathematical beliefs) and values reflexively (Cobb et al., 2011, p. 125). Therefore, the notion of a norm could be a useful construct to explore the affective aspects of a classroom.

This study aims to explore SMNs related to PS in gifted students' mathematics classrooms. Gifted students' classrooms might have a different micro-culture than others. Researchers have reported that giftedness studies had not extensively approached the issue from a sociocultural perspective (Heyd-Metzuyanim & Hess-Green, 2019). Likewise, Singer et al. (2016) suggest investigating the nature of classroom culture and the role of the teacher in fostering mathematical expertise. Furthermore, Goldin (2017) recommends the examination of "effective sociocultural norms" in these classrooms.

Although the literature extensively investigated PS as a cognitive activity of individuals, there is a gap in the literature on the nature of PS as part of classroom culture. Considering this gap in the literature, an examination of the norms related to PS in gifted students' classes has come to the fore. Therefore, this study aims to answer the following research question: What are the existing SMNs related to PS in a gifted and talented students' mathematics classroom?

LITERATURE REVIEW

Problem solving

The issue of PS, which has attracted the attention of mathematics education researchers for more than fifty years, has been generally handled in three aspects of research: "as a cognitive enterprise, as something to be taught, and as something to teach through”. However, this categorization does not fully explore the complexity of PS (Liljedahl & Cai, 2021, p.724). This approach ignores PS as a cultural activity of the classroom community and normative behaviours that support it.

Early studies focused on the PS process (Polya, 1957), while more recent studies focused on the characteristics of problem solvers that promote PS success (Carlson & Bloom, 2005). The framework proposed by Schoenfeld (2013) to explain students' PS behaviors consists of the use of basic mathematical knowledge, the use of cognitive or heuristic strategies, the use of metacognition or self-regulation strategies, and students' beliefs about mathematics and PS. The framework, which consists of four basic components, has been widely used not only to reveal how successful students are in their PS attempts but also to organize and support students' development of PS experiences in the classroom (Santos-Trigo, 2014). The development of students' PS abilities is not in isolation from the learning of other mathematical concepts. Furthermore, it should be considered as a system involving “the teacher’s role” and “the classroom culture” (Lester & Cai, 2016, p.118).

Some of the beliefs and tendencies of students about PS, as revealed by Schoenfeld (1992), are as follows: there is only one correct solution to a problem, and the correct solution can only be reached by the solution shown by the teacher in the classroom, and PS is an activity that students carry out alone. Lesh and Zawojewski (2007) stated that little has changed in this regard since Schoenfeld's literature review. Many students' beliefs about PS are that problems given by their teachers should be solved expectedly (Cai, 2003; Lester & Cai, 2016). While it is considered important to reach the expected solution in a certain way in classroom environments (Mann et al., 2017), it has been reported that gifted students tend to solve problems in atypical ways (Singer et. al, 2016). In the context of PS, mathematical creativity is seen as a critical component associated with advanced mathematical thinking, which relates to one's ability to perceive original, non-algorithmic, and often insightful solutions (Leikin, 2021). It is stated that encouraging fluency, flexibility, and originality (components of creativity as reposted by Silver, 1997) can help eliminate students’ wrong beliefs about PS as mentioned above (Levenson, 2022).

PS in mathematics classrooms is a sociocultural process (Koichu, 2019) and the development of PS ability is much more socially structured and contextually situated than traditional theories assume (Lesh & Zawojewski, 2007). According to sociocultural theories, SNs and SMNs are in relation to beliefs and values, and they evolve together in the classroom micro-culture (Yackel & Rasmussen, 2002). For example, the norm “The solution must correspond to the one that the teacher has in mind" (Bingolbali, 2011; Yenmez & Erbaş, 2022) may be concerned with students' beliefs that problems should be solved as their teacher expects (Cai, 2003; Lester & Cai, 2016). On the other hand, reported teacher behaviors that reflect their beliefs show that some teachers preferred their solutions to the problems and did not value students' ideas (Rott, 2020). Some other factors outside the classroom can also affect participants' beliefs and the classroom micro-culture. For instance, preparing for national student selection exams, which require solving problems fast, could promote a normative understanding in the classroom about solving problems quickly (Authors, 2021; Yenmez & Erbaş, 2022).

The notion of a norm, similar to beliefs and values, is a useful construct to explore the affective aspects of a classroom. Investigating the norms of the classroom have the potential for revealing the teacher's expectations of the students and their wrong beliefs about PS, if they have any.

Socio-mathematical norms

The argument that knowing and doing mathematics is a social and cultural activity by its nature is increasingly accepted by the mathematics education community (Yackel & Cobb, 1996; Cobb et al., 1997). According to the interpretative framework used to understand classroom practices, the social dimensions of classroom micro-culture and the psychological dimensions of students' activities in the classroom are in mutual interaction (Cobb et al., 2011).

The concept of SMN was introduced to separate the normative aspects of mathematical discussions specific to students' mathematical activities from general social norms (Yackel & Rasmussen, 2002). Since the teacher is at the center of the "mathematical meaning, interaction and negotiation process" in a mathematics classroom, SMNs that can encourage interaction are closely linked to the teacher's beliefs about learning, teaching, and mathematics (Kang & Kim, 2016). While teachers guide the establishment of SMNs, they support students' reorganization of their beliefs and values, which can be seen as their mathematical tendencies (Cobb et al., 2011). With the emergence of the notion of an SMN, student autonomy has begun to be seen as a feature of the way of participating in the community (Cobb et al., 2011). When students participate in the negotiation of SMNs, they develop mathematical beliefs and values that enable them to become increasingly autonomous members of the classroom mathematics community (Yackel & Cobb, 1996). SMNs not only determine the quality of teaching and learning activities in a mathematics classroom but also support and guide students' participation in mathematical activities (Kang & Kim, 2016).

Socio-mathematical norms related to problem solving

The norms reported in the context of PS (Lopez & Allal, 2007; Roy et al., 2014; Yackel & Cobb, 1996) mostly focused on the notion of a "different solution". The SMN regarding different solutions means reaching the same result in a differently and presenting the reasons for how the methods differ from each other (Roy et al., 2014). There are no predetermined criteria for what a mathematically different solution is (Lopez & Allal, 2007; Roy et al., 2014; Yackel & Cobb, 1996). As the teacher demands different solutions from the students, along the way they develop an understanding of which solutions count as mathematically different through interaction in the classroom. A teacher's answers and actions may limit students' understanding of what a different solution is (Lopez & Allal, 2007; Roy et al., 2014; Yackel & Cobb, 1996).

Another norm reported in the literature in the context of PS is about providing easy, simple, or effective solutions to problems. According to McClain and Cobb (2001), this norm is related to the SMN, offering different solutions to problems, because the evaluation of a solution as an easy, simple, or effective solution is closely related to the understanding of mathematical differences. Researchers (Roy et al., 2014; Yackel & Cobb, 1996) stated that in addition to these norms, a normative understanding of a sophisticated solution can develop in classrooms, but it is more difficult to develop this normative understanding than a different solution. When a teacher values particular solutions proposed by students, it becomes clear to other students what the teacher values. As the teacher asks whether there is a more sophisticated (advanced) solution to the problem or if there is one that effectively solves the problem, a normative understanding may start to evolve if students also develop an awareness in the same way. In this study, we will investigate SMNs related to PS in a mathematics classroom in the case of gifted and talented students.

Theoretical stance

Our theoretical stance is based on the notion of an SMN and has two distinctive features (Authors, 2020). The first one is about what is to be taken as evidence of a norm and the second is related to the two dimensions of norms.

A classroom norm defines "regularities in the interaction patterns that regulate social interactions" (in the case of an SN) and mathematical meanings (in the case of an SMN) in the classrooms (Yackel & Rasmussen, 2002, p. 315). As these interaction patterns emerge, the classroom community develops expectations and obligations (Yackel, 2004). Researchers investigating how negotiations on expectations shape classroom culture used sociological and psychological perspectives for defining a classroom community (Cobb et al., 2011). According to the psychological perspective teachers and students interpret and respond to each other's actions in the classroom. As they interpret and respond to each other's actions, normative activities emerge (sociological perspective) as a result. Conversely, established norms within a classroom (sociological perspective) show how teachers and students interpret and respond to each other's actions (psychological perspective). Therefore, we will take both expectations and actions as evidence of a norm. Concerning expectations, an example would be a teacher's or students' expectations and awareness of these expectations about solving problems in differently. An action would be the teacher's call for different solutions to a problem. Therefore, different solutions to a problem in a classroom could be considered evidence of SMNs related to PS.

The second feature of our theoretical stance is related to the two dimensions of norms: the student dimension and the teacher dimension (Authors, 2020). Since norms are negotiated among teachers and students (Yackel, 2004), we will focus on the student dimension as well as the teacher dimension of SMNs to be able to distinguish a two-way negotiation. The teacher dimension is concerned with teachers' expectations and actions, while the student dimension refers to actions, awareness of teachers' expectations, and the students' expectations of the teacher. Concerning actions, the teacher dimension could be asking for different solutions to the problem, and the student dimension could be students' initiation of solving problems in a different way.

METHODOLOGY

Context and participants

Norms, by their nature, cannot be studied independently of their context, as they are tied to the community from which they emerge (Klosterman, 2016). As the study of SNs and SMNs requires a long-term and in-depth analysis of the real classroom environment (Yackel & Cobb, 1996) qualitatively, we conducted a descriptive case study (Yin, 2009). A single classroom was chosen, which is a fifth-grade mathematics classroom with twelve (three girls and nine boys) gifted students, to be able to reflect the characteristics of the phenomenon (Bleijenbergh, 2010).

The teacher of the class was a special education specialist and had seven years of teaching experience. The school is a private secondary school supported by a foundation for gifted and talented students. The study was conducted in a class of fifth-grade students who were diagnosed as gifted according to WISC-IV which is not specific to mathematics. Although, the school accepts students with WISC IV, they did not share students' IQ's because of ethical principles. The teacher had been teaching the same class from the beginning of the first grade. We selected this teacher because our study aims to explore sustained SMNs rather than introduced norms (Roy et al., 2014). The teacher followed the standard mathematics curriculum and the textbooks which were also used in mainstream schools. He integrated the sixth and seventh-grade mathematics curriculum subjects into his lessons considering the level of his students.

Data collection

The main data source consisted of forty-three mathematics lessons (a total of 28 h and 40 min), that were video recorded during the 2018 autumn term. Since we aimed to investigate the SNs and SMNs existing in the classroom without any intervention, the first author observed the lessons as a non-participant observer.

An official permission letter was obtained from the school directorate. The school received permission from parents and the teacher for the video recording of the lessons. We started to video-record the lesson two weeks before data collection to help students get used to the camera.

Analysis of data

Transcripts of 43 lessons were analyzed using qualitative data analysis software. The content analysis method was used. We defined the first version of the descriptors for teacher and student dimensions based on the literature on SMNs related to PS. The descriptors referred to both dimensions (teacher and student) as evidence of an SMN because norms require a two-way negotiation between the teacher and students as described in the theoretical stance section.

We analyzed the transcripts of the lesson videos first by determining episodes in videos where the class was participating in PS activities. We then analyzed the transcripts of these episodes using the first version of the descriptors of teacher and student dimensions. As we conducted the content analyses, we revised the descriptors and coded the data using the revised version as shown below:

Different solutions should be offered (SN_DS) (Yackel & Cobb, 1996)

● *Teacher dimension:* The teacher offers different solutions to problems solved in the classroom and expects the students to offer different solutions.

● *Student dimension:* Students are aware of the teacher's expectation of different solutions to problems, and offer different solutions to problems.

Mathematically different solutions should be offered (SMN_MDS) (Lopez & Allal, 2007; Roy et al., 2014; Yackel & Cobb, 1996)

● *Teacher dimension:* The teacher solves the problems in mathematically different ways and expects students to solve the problems accordingly.

● *Student dimension:* Students are aware of the expectation of their teachers to solve the problems in mathematically different ways and to explain how the solutions differ from each other mathematically. They offer solutions that are mathematically different.

Mathematically easy, simple, or effective solutions should be offered (SMN_ESES) (McClain & Cobb, 2001)

● *Teacher dimension:* The teacher offers mathematically easy, simple, or effective solutions to the problems and expects students to develop such solutions.

● *Student dimension:* Students are aware that they are expected to develop mathematically easy, simple, or effective solutions and offer such solutions.

Mathematically sophisticated solutions should be offered (SMN_MSS) (Roy & et al., 2014; Yackel & Cobb, 1996)

● *Teacher dimension:* The teacher offers mathematically sophisticated solutions to problems and expects students to offer such solutions.

● *Student dimension:* Students are aware that they are expected to offer mathematically sophisticated solutions to problems and solve problems in mathematically sophisticated ways.

An example of coding an episode is presented in the Appendix. For validity concerns, the authors discussed the first version of the codes based on the norms related to PS reported in the literature. The two authors separately coded one episode concerning each norm and negotiated on a final code to increase the reliability of the findings. We will also present the frequencies of the codes for each norm for validity concerns. We consider a high frequency as evidence of norms since norms are defined as the regularities in the interaction patterns (Yackel & Rasmussen, 2002).

FINDINGS

Analysis of the data revealed normative aspects of PS activities. We will give illustrative examples of teacher and student dimensions of each norm (one SN and three SMNs) that we observed. In line with our theoretical stance considering teacher and student dimensions, we will refer to the teacher's and students' expectations, awareness of these expectations, and also their actions.

Mathematically different solutions should be offered

In this section, we will present our findings related to different solutions to problems that point out both an SN and an SMN. The data indicated that offering different solutions to problems has become a normative activity that reflects the social perspective of the classroom community. A total of thirty instances (ten for the SN and twenty for the SMN) were coded as the student dimension and twenty-nine instances (twelve for the SN and 17 for the SMN) as the teacher dimension. We observed that the teacher and students offered not only different solutions to problems but also mathematically different ones (psychological perspective).

The SN "Different solutions should be offered", and SMN, "Mathematically different solutions should be offered" are closely related norms. The teacher's statements such as "Does anyone have a different solution to a problem?", "I listen to those who do it differently" are clear indicators of his expectation about different solutions and reflect the teacher dimension of the SN. The dialogue that illustrates different solutions offered by two of the students in the classroom is presented below:

S1: Teacher, I did it differently.
T: I was wondering how he found it.
S5: Teacher, I did it in a different way. Shall I show you what I've done?
T: Show if you did it differently, if there is a difference, please show it.

In the dialog above, it is seen that the students are aware of the teacher's expectations and stated that they want to offer different solutions (psychological perspective). These actions show the student dimension of the SN. On the other hand, the teacher's reaction to his students (in the last line) indicates the SMN “Mathematically different solutions should be offered” because the teacher questioned if there was a difference. This questioning implies a mathematical difference because different solutions offered by students might not always be necessarily mathematically different. Below, we give further examples of this SMN which we also consider as evidence for the SN related to different solutions.

Through the SMN about mathematically different solutions, students realized that they were expected not only to offer different solutions but also mathematically different ones. The following dialogue illustrates the teacher's expectations for mathematically different solutions and students' awareness of the expectation, and different solutions offered by students (action).

> T: Are the fraction 25/100 and the fraction 3/12 equivalent?
> S5: 25 is not a multiple of 3. 100 is not a multiple of 12, but when you expand 3/12 by 100, it becomes 300/1200, and when you simplify this fraction by 12, it becomes 25/100 So it's equivalent.
> S3: I did like this...
> T: It is true, there may be another way, let your friend solve it...
> S3: I simplified 25/100 as 1/4. I divided 12 by 3 and we get 1/4. So these fractions are equivalent.
> T: One of your friends did it by expansion and the other by simplification…

The excerpt above illustrates a student's offer of a different solution. In other words, the SMN is evident beyond awareness and reveals itself as an act of offering a mathematically different solution (student dimension). After the student's proposal for a mathematically different solution, the teacher's explanation to the classroom community about the mathematical difference of solutions reflects the teacher dimension of the SMN. Therefore, the existence of both student and teacher dimensions points out a two-way negotiation.

Another episode illustrates that the act of offering a mathematically different solution was initiated by a student after the teacher asked them to solve a problem in the textbook.

> Problem: The hour and minute hands of a magic clock rotate in the opposite direction. The magic clock and a normal digital clock started together. If 40 minutes later the digital clock shows 20:17, what time does the magic clock show?
> T: Yes, then Ozan is coming, we are listening (Ozan is solving the problem on the board)...Your friend subtracted 1 hour and 20 minutes from 20:17.
> (When another student says he did it in a mathematically different way, the teacher goes to the student and looks at his solution.)
> T: Mete offers an alternative way to avoid the confusion, I think you shouldn’t miss this idea... It's a great point of view... By combining the subject of hours with fractions... Bravo Mete, well done son...We applaud Mete. Mete used the following relationship: Can we express one minute as 1/60th of an hour? If the time is 20:17, then it is 20 and 17/60. It's correct. When I look at it, we can express one hour and twenty minutes as 1 and 20/60. I hadn't even thought of this solution.

In the dialogue given above, although there is no explicit demand for a mathematically different solution by the teacher, the presentation of a different solution (action) by the student points out the student dimension of the norm (psychological perspective). The solution offered by the student for this problem was considered an unexpected solution for this grade level. Although the mathematical difference was not communicated, it can be claimed that the classroom community reached an implicit agreement on this issue (sociological perspective). Expectations, awareness, and actions can be considered as evidence of the SMN in both student and teacher dimensions (psychological perspective). The absence of illustrative cases where the criteria for mathematically different solutions are explicitly discussed may be because the negotiation process of the SMN has not been observed. On the other hand, the existence of teacher and student dimensions of the SMN shows that there is a two-way negotiation.

Mathematically easy, simple, or effective solutions should be offered

This SMN is concerned with offering easy, simple, or effective solutions, and is closely related to the SMN "Mathematically different solutions should be offered". The data indicated that students were aware of the teacher's expectations and offered easy, simple, or effective solutions to the problems (psychological perspective). Ten instances were coded as the student dimension and seventeen instances as the teacher dimension of this SMN. An illustrative classroom dialog on how the teacher and students interpreted and responded to each other's actions is given below.

> T: Hande read 2/15 of a 600-page book on Monday and 7/30th on Tuesday. How many pages are left to read? Did everyone do this? Divide 600 by 15 and multiply by 2, divide 600 by 30 and multiply by 7, add the two and subtract from 600?
> S: Yes (Students responding altogether)
> T: Well that's right, how can we find a shortcut to this? She has read 2/15th and 7/30th. Guys, how can we add these fractions?
> S: The denominators must be the same (Students responding altogether)
> T: We expanded both sides, our new fraction…
> S6: Teacher, I did it in differently. Shall I show you how I did it?
> Teacher: Show if you did it differently, if there is a difference, show it.
> S6: I multiplied by 2 (expands the fraction 2/15 by 2), then I don't need to multiply with that (he says there is no need to expand the fraction 7/30).
> T: (The denominators) You made 30.
> S6: Yes, it's 30. Teacher, I multiplied by 40 and I multiplied it by 40 and found the results like that (he is trying to explain that he expanded the denominator of fractions to make 600)
> T: Yes, good.
> S1: Teacher, we can also do this with variables, right?
> T: Yes you can.

In the dialogue above, the teacher's prompt to find a shorter solution reflects the teacher dimension of the SMN. After the teacher's question, a student offered to equate the denominators of the fractions with 600 to find the parts of the whole (a 600-page book), which is an easier solution for her. This solution, which was presented by the student as an "easier solution", can also be considered more "effective", unlike the usually preferred way (equalizing the denominators at 30). The student's offer which is an easy, simple, or effective method for adding fractions without any request from the teacher reflects the student dimension of the norm. However, what is meant by a "mathematically different solution" was not clearly stated, and what an "easier, simpler, or more effective solution" was not explicitly discussed. Although there are no clearly stated criteria in this regard, student and teacher dimensions of the SMN are evident with expectations, awareness, and actions (two-way negotiation). Therefore we can claim that the classroom community has an implicit understanding of an "easy/simple or effective solution" (sociological perspective). Another episode that illustrates this finding is presented below:

T: Serkan suggests another way for 18x9
S5: I round off 18 to 20, then multiply 9 by 20 and subtract 9 twice.
T: It's a quite complicated method…

The easy, simple, or effective solution offered above by the student is an example of using the distributive property of multiplication over subtraction. For the student dimension, we can say that the student was aware of the teacher's expectations and offered a solution that could be easy, simple, or effective (action) (psychological perspective). A similar example where a student took the initiative to offer an easier solution is presented below.

S5: Can I explain to you how I did it?
T: Tell me Serkan.
S5: Teacher, I knew that 12x12 is 144. I changed one of the factors to 13 and added 12. Then I changed the other factor 13 and added 13.
T: Good…

As can be seen above, the student proposed an easier solution which is using the distributive property of multiplication over addition as a solution method, increasing each factor by one and transforming the operation into a simpler addition operation (action). We can claim that the classroom community agreed on offering easy, simple, or effective solutions to problems (sociological perspective), and both student and teacher dimensions of the SMN are evident with expectations, awareness, and actions.

Mathematically sophisticated solutions should be offered

This norm, which is related to the SMN "Mathematically different solutions should be offered", is concerned with offering more sophisticated solutions to problems instead of a standard solution. The data analysis revealed that the teacher expected mathematically sophisticated solutions to problems, the students were aware of the expectations, and they attempted to offer mathematically sophisticated solutions (psychological perspective). An episode that illustrates this SMN is presented below:

> T: The problem is a difficult one... How many numbers are there with two natural number divisors greater than 20 and less than 50? Serkan, how did you do this question?
> S5: Since he said two natural number divisors, I understood that he meant prime numbers. I looked for prime numbers between 20 and 50.
> T: Your friend has noticed a very good point... What does it mean to have two natural number divisors, its divisors will be 1 and itself... Serkan, it is very good... We applause.

It is understood by the teacher's reaction that the problem was difficult according to the grade level. For this kind of problem, students at this grade level will usually write down numbers one by one and their divisors and then look for natural number divisors. This case reflects the student dimension of the SMN. The teacher's reaction to the student's answer shows that he considered the student's solution mathematically valuable, and sophisticated. Although the teacher did not ask for a sophisticated solution, we can claim that the student was aware of the teacher's expectations and offered a solution (action) that could be considered sophisticated.
The first dialog, which is about finding a solution to the “magic clock problem” was presented as an illustrative example of the norm "Mathematically different solutions should be offered", can also be given as an example of the norm about sophisticated solutions. The teacher's reaction indicates that he considered the solution mathematically valuable. The teacher's statement "I hadn't even thought of this solution" implies that Mete's solution was a sophisticated one. The student's solution to the problem (action) indicates the student's awareness of the teacher's expectations of offering mathematically sophisticated solutions (psychological perspective). We can infer that the teacher and students mutually agreed on offering mathematically sophisticated solutions to problems (sociological perspective), and both dimensions of the SMN are evident with expectations, awareness, and actions. This finding implies a two-way negotiation.

DISCUSSION

In this study, we investigated the SMNs related to PS in a gifted and talented mathematics classroom micro-culture as a response to calls for investigating both the social aspects of PS (Lester & Cai, 2016) and “the nature of classroom culture and the role of the teacher in fostering mathematical expertise” (Singer et. al., 2016). We observed three SMNs related to PS that were also reported in the current literature but were not examined in gifted students' classroom micro-culture.

Regarding the first SMN, we observed that both the teacher and students offered mathematically different solutions. Although the evidence of the SN "Different solutions should be offered" is limited to only one classroom dialogue, we consider the existence of the SMN "Mathematically different solutions should be offered" as an indication of the fact that its related SN was a part of the classroom culture. As other researchers stated, SNs and SMNs are interdependent (Campbell & Yeo, 2021). For example, Roy, Tobias, Safi, and Dixon (2014) assert that the establishment of SNs fosters related SMNs in classrooms. In other words, the establishment of the SMN related to mathematically different solutions depends on its related SN regarding different solutions because the teacher's requests for different solutions initiate the process of comparing solutions mathematically (Yackel & Cobb, 1996). This SMN which is also associated with mathematical creativity (Leikin, 2018) allowed students to solve problems in different ways as suggested for gifted and talented students to deepen their understanding of mathematics and be experts in mathematics as much as possible (Singer et. al, 2016). This SMN has also the potential to support gifted students who might be iconoclastic problem solvers (who are apt to challenge the system in the classroom) in the context of affective aspects (Mann et al., 2017). However, for this observed norm, except for the basic ones, explicit evaluations of mathematically different solutions have not been made in the classroom. The reason for this finding might be that our study focused on established norms rather than the negotiation process of norm construction. Therefore, it was not possible to observe how the classroom members negotiated the criteria for mathematical differences. The other reason may be that the problems brought to the classroom by the teacher were not challenging enough for the students, because these kinds of problems have less potential for gifted students and their teachers to generate discussions on mathematical differences in solutions. The tasks selected by the teacher were from the textbooks that were used in mainstream schools, therefore not challenging or rich enough (Toprak & Özmantar, 2022). This might be due to his beliefs or his pedagogical content knowledge (Anderson et al., 2005; Ingram et al., 2020). Another reason may be that the teacher was not a mathematics specialist and did not tailor the tasks in the textbooks which lack the use of multiple solution strategies to meet his students' needs (Smedsrud et al., 2022).

Other observed SMNs, which are also associated with the SMN regarding mathematically different solutions are related to offering mathematically easy, simple, or effective solutions to problems. We presented the evidence of this SMN for both the teacher and student dimensions, which points out a two-way negotiation. Since the negotiations between the teacher and students were implicit, we could not explicitly distinguish solutions from each other based on the criteria such as easy, simple, or effective, but we interpreted the teacher's reactions such as his guidance and feedback as an implicit indicator (Yackel & Cobb, 1996). Since mathematically gifted children "strive for an economy of mental effort, rationality, ("elegance") in a solution" and "curtail the reasoning process, as shortening of its individual links" (Krutetskii, 1976), this SMN has the potential to support gifted students.

The third SMN observed in this study is called “mathematically sophisticated solutions should be offered”. The solutions offered by students were more sophisticated than usual solutions at this grade level. Offering mathematically sophisticated solutions to the problems was a taken-as-shared issue among the classroom participants. The teacher’s reinforcing stimulus to the student who solved the magic clock problem differently was interpreted as an implicit indicator. On the other hand, the teacher's comments on the student's solution: "It's a great point of view...combining hours with fractions...", also include an evaluation of why the solution was considered mathematically valuable. Although such evaluations of the teacher were limited, we can say that the classroom community has a taken-as-shared understanding of mathematically sophisticated solutions. Since unusual responses, thoughtful, flexible, or original solutions to a problem indicate mathematical creativity (Silver, 1997; Sriraman, 2005; Mann et al., 2017), this SMN has the potential to be supportive of gifted students.

CONCLUSION

In this study, we looked for answers to the research question “What are the existing SMNs related to PS in a gifted and talented students’ mathematics classroom?” Based on the findings, we came to some conclusions. The first conclusion is the determination of the SMN called “Mathematically different solutions should be offered”. This finding has some implications for classroom practice. More challenging problems (Leikin, 2018), “problems with multiple solutions” (Lev & Leikin, 2017), or “rich problems” that “can be solved in multiple ways and with different representations” (Ayalon et al., 2021, p.1701) could require different solution methods and strategies. Therefore, we suggest teachers select problems that may raise the need for more different solutions, and discuss clearly how the solutions differ from each other mathematically. The teacher's call for more than one different solution to such problems can support fluency in PS (Mann et al., 2017; Silver, 1997). As students offer more than one mathematically different solution to a problem, “developmentally sophisticated solutions” (Yackel & Cobb, 1996, p.466) can be seen in the classroom micro-culture.

We further suggest teachers make explicit evaluations so that students develop a clear understanding of solutions based on criteria such as easy, simple, effective, or sophisticated. Another study conducted in the same classroom revealed that problems posed by students were not evaluated explicitly by the teacher (Authors, 2020). The classroom community lacked explicit evaluations of both PS and problem posing activities, therefore an SMN regarding the evaluation of problems and their solutions. A lack of an SMN regarding evaluations of mathematically different solutions based on the aforementioned criteria resulted in low-level problem solving practice which was not expected from gifted and talented students. If a teacher endorses such an SMN to bring difficult and sophisticated problems to the classroom, it would also encourage students to offer different, effective, and sophisticated solutions. Revealing the mathematical differences between solutions is an important issue for gifted students. Therefore, we recommend their teachers include more complex and sophisticated solution processes and open-ended problems as part of more creative work (Zazkis, 2017). Furthermore, teachers should aim to develop an SMN regarding the evaluation of problems based on criteria to promote effective PS.

The second conclusion is related to our theoretical stance which deals with what is to be taken as evidence of a norm (expectations, awareness, and actions). It provided us with an operational data analysis framework with descriptors of teacher and student dimensions. The teacher's expectations and students' initiations (as actions) were considered as evidence of these SMNs. Unlike other SMNs in which we observed the teacher and student dimensions, for the SMN called "Sophisticated solutions should be offered", we did not observe the teacher's expectations regarding such solutions. Although the teacher did not explicitly ask for sophisticated solutions, the students offered these kinds of solutions. The teacher not asking for such solutions might be concerned with the lack of explicit evaluations of mathematically different solutions in the classroom because the SMN about mathematically different solutions "provide a basis for the subsequent emergence of the norms of what counted as a sophisticated solution" (McClain & Cobb, 2001, p.263). Therefore, we take students' solutions to the problems as the only evidence of taken-as-shared understanding of sophisticated solutions in the classroom micro-culture without the teacher's explicit talk about these solutions. Considering this finding, we suggest future research elaborate descriptors of teacher and student dimensions of SMNs or SNs.

The findings of this study should be considered with their limitations. The first limitation concerns the case selection. Since we aimed to examine existing (sustained) SMNs, we selected a classroom whose teacher had been teaching since the first grade. On the one hand, our choice has increased the possibility of observing existing SMNs, which were already established in the classroom. On the other hand, due to this preference, we could not observe the norm construction process. This was beyond our aim but one should consider this limitation when interpreting our findings. The second limitation, which is also connected with the first one, is about evaluations of mathematically different solutions. Explicit talks on the criteria such as easy, simple, effective, or sophisticated solutions to problems could be observed better during the negotiation process of SMNs. However, since this process is beyond our aim and our data are limited to the teacher's fifth year of teaching in the classroom, we could not observe the process of how the classroom members negotiated on the aforementioned criteria. The third limitation is that the teacher's approach was teacher-centered. Therefore, interaction among the classroom community was limited to teacher-student interaction and our observational data lacked student-student interaction because collaboration was not promoted.

We conclude with further suggestions for future research. First, since our findings are limited to one mathematics classroom, further studies can investigate SMNs regarding PS in different classrooms of gifted and talented students. Secondly, for an investigation of norm construction processes, researchers can observe the micro-culture of gifted classrooms from the first lesson which could guide researchers in SMNs' negotiation process with explicit talks of mathematical differences among solutions. A further investigation of SMNs regarding PS in other contexts could increase our understanding of the social aspects of, and beliefs about PS

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Appendix: Example of coding of an episode

Classroom episode and dialog	Discourse	SMN	Dimensions of the SMN
Problem: The hour and minute hands of a magic clock rotate in the opposite direction. The magic clock and a normal digital clock started together. If 40 minutes later the digital clock shows 20:17, what time does the magic clock show? T: Yes, then Ozan is coming, we are listening (Ozan is solving the problem on the board)...Your friend subtracted 1 hour and 20 minutes from 20:17. (When **another student says he did it in a mathematically different** way, the teacher goes to the student and looks at his solution.) T: **Mete offers an alternative way** to avoid the confusion, I think you shouldn't miss this idea... **It's a great point of view... By combining the subject of hours with fractions...** Bravo Mete, well done son...We applaud Mete. Mete used the following relationship: Can we express one minute as 1/60th of an hour? If the time is 20:17, then it is 20 and 17/60. It's correct. When I look at it, we can express one hour and twenty minutes as 1 and 20/60. **I hadn't even thought of this solution.**	(When **another student says he did it in a mathematically different** way, the teacher goes to the student and looks at his solution) **Mete offers an alternative way** to avoid confusion. **"It's a great point of view... By combining the subject of hours with fractions…"** **"I hadn't even thought of this solution."**	***Mathematically different solutions should be offered*** ***Mathematically sophisticated solutions should be offered***	The student told the teacher that he had an alternative way for solution (Student dimension of the SMN regarding different solution coded as SMN_MDS_S) The teacher brought the student's different solution suggested by the student to the classroom agenda (Teacher dimension of the SMN regarding different solution coded as SMN_MDS_T) The student used fractions for expressing the time in the problem (He expressed one minute as 1/60th of an hour. Then the time 20:17, turned to 20 and 17/60.) (Student dimension of SMNs regarding both different solutions and sophisticated solutions coded two times as SMN_MDS_S and SMN_MSS_S) The teacher gave feedback about the solution which showed he found it valuable (Teacher dimension of the SMN regarding sophisticated solutions coded as SMN_MSS_T)