The Use of Calculators in Teaching Mathematics: A Survey in Vietnam

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Abstract: The purpose of this paper is to evaluate the reality of teaching and learning with the support of a calculator, examine the benefits and challenges of using a calculator in teaching mathematics in high school. The study is based on the survey data from math teachers, educational managers, and students who were selected from 24 high schools in seven provinces representing different regions of Vietnam. Initial research results show that using calculators for teachers and students in teaching and learning mathematics has been widely developed in recent times in Vietnam, especially since the implementation of the curriculum general education 2006. Calculators can support from knowledge discovery, conceptual approach to problem solving. For teachers, calculator have a positive impact on the views and attitudes of information technology applications in teaching, promoting innovation of teaching and learning methods. However, many teachers and educational managers used to worry about using calculator like affecting the students' computing skills and hindering the goal of developing mathematical thinking. In terms of organizing teaching with calculators, the main difficulties are uneven equipment, lack of teaching documents, and no mechanism on financial support teachers and students.

Keywords: Calculator, handheld calculator, mobile learning, teaching mathematics

INTRODUCTION

In high schools in Vietnam, the combination of theory teaching and practical calculation has not been promoted. Instructing students to use calculators creatively during math learning is still limited. In general, most students only use hand-held calculators at the level of performing simple calculations, but have not applied them to higher levels such as predicting results, creative reasoning to solve problems. At the same time, there is a lack of documents and research on the use of hand-held computers to support the discovery, fostering and development of mathematical competence for students at the high school level. Therefore, the use of handheld computers in

teaching math in high schools should be fully and scientifically researched in order to promote the highest efficiency in using to improve the quality of teaching and learning.

Since the invention of electronic calculators more than 40 years ago, handheld calculator technology has grown very rapidly, from simple handheld calculators to scientific calculators and graphics calculators. The main advantages of a calculator are its small size, compact size, portability, and ease of access, as well as the price of a calculator that is relatively cheaper than the cost of buying a desktop computer or laptop (Kemp, Kissane & Bradley, 1995; Kissane, 2000; Wang, 2016). A calculator is a tool to support mathematical discovery, a tool that helps teachers and students solve math problems by exploiting and using the available functions of the calculator. Many studies suggest that the introduction of calculators has profoundly affected mathematics teaching and learning (Dunham & Dick, 1994; Drijvers & Doorman, 1996; Mao et al., 2017). Regarding the benefits of using a calculator in teaching mathematics, several studies recorded the benefits that may arise from the mathematical functions installed in calculators and the rational use of calculators in the teaching process, helping teachers to innovate teaching methods, positively impacting the attitude of teachers and students and improving the effectiveness of learning mathematics (Campbell & Stewart, 1993; Carlson, 1995; Dunham, 1993; 1995; Graham & Thomas, 2000; Hembree & Dessart, 1986). The challenges of using calculators in teaching are also mentioned by some studies, such as challenges that may arise from the inappropriate method of using calculators (Barling, 1991; Barnes, 1994; Bowman, 2018). The challenge arises from a situation where students are not equipping calculators equally in the classroom or not being used regularly for long periods is also an obstacle to using a calculator in teaching and learning (Penglase & Arnold, 1996; Floris, 2017). Besides, some studies also show challenges such as lack of teaching materials, untrained teachers, training on the use of calculators, the lack of time to invest in teaching plans of teachers also affects the efficiency of using calculators (Ruthven, 1990; Clarke & Leary, 1994; Martinovic, 2018).

In Vietnam, there has not been much empirical research on the use of calculators in teaching mathematics, mainly the instructions on the use of calculators, some documents mentioning techniques of using calculators to solve math problems. Some documents related to the use of calculators in teaching, such as instructional materials for teaching mathematics at the high school, manuals for using calculators for the mathematics (Ta & Pham, 2008), a book for training pedagogical skills for teacher students including referring to teaching maths with scientific calculators (Nguyen et al., 2014). In his research, Le (2011) confirmed that the calculator is a "powerful and fast" calculation tool, replacing numbers tables, facilitating the integration of new content into high school math program. Moreover, a calculator is a pedagogical tool that helps to build teaching situations that match the characteristics of active teaching methods (Ta & Pham, 2008). Teaching mathematics with the support of calculators are favorable to apply because of the popularity of calculators in high schools. Therefore, the study on using of calculators in teaching

mathematics is necessary, as a practical basis for successful implementation of the new general education curriculum in Vietnam.

Moreover, the research results show the important role of hand-held calculators, including graphical calculators, in supporting teachers to organize discovery activities to help students learn and orientate solutions to problems. Based on these results, calculators could be encouraged to be used more in schools, including allowing students to use graphic calculators in math exams. Therefore, this research has great significance in the strong application of technology in math classrooms, thereby contributing to the development of technology skills for students.

METHOD

We have conducted a survey in 24 high schools of seven provinces throughout Vietnam including Thai Nguyen, Bac Kan, Bac Giang, Bac Ninh, Phu Tho, Hoa Binh, and Khanh Hoa. The high schools selected in the survey represent different types of schools from seven provinces in the country. These are provinces with different socio-economic conditions, with schools in rural, mountainous, remote areas and even schools in urban areas. Participants of the survey were 260 teachers, 40 educational managers, and 367 high school students. Teachers and students were randomly selected from high schools to participate in the survey. Questionnaires and interviews are used to collect data from these schools. The student questionnaire consists of two parts. The first part is general information about students, schools, and classes. The second part consists of open-ended questions for students to express their ideas. The teacher's questionnaire is also designed in two parts. The first part is general information about teachers. The second part contains specific information on the use of calculators in teaching, methods of using graphic calculators, assessing and evaluating the benefits, efficiency, and challenges when using a handheld calculator, including question items with open questions, answers on a five-level Likert scale, which are converted into points from 1 to 5 for each response.

Some teaching situations with graphic calculators are selected and sent to a math teacher for design. The research team conducted observing many lesson hours of math classes, conducted interviews, tested knowledge and skills using calculators for students and teachers. The research team asked the Departments of Education and Training and the principals of the schools to participate the survey to collect data. We also send the questionnaire to the corresponding respondent and conduct face-to-face interviews with students, teachers, and educational managers. Therefore, the collected data includes quantitative and qualitative data. Quantitative data is collected by using questionnaires designed for research purposes, using statistical software to obtain results on frequency, quantity, and percentage. Qualitative data comes from answering questions and interviews, categorized, and recorded in reports.

RESULTS

Using calculators in teaching and learning

We have studied and analyzed the Vietnamese general education programs from 1990 to the 2020 to clarify the role of calculators and the progress of using it in teaching and learning at schools.

For the general education program before 2000: The educational reform began in elementary school in 1980. In the primary education reform program, calculators first appeared in grade 5, to check the results of calculating when students learn about decimals. Following the elementary school program, the middle school program was implemented in 1986 and calculators continued to appear in grade 6, but with the role of support calculation. However, calculators were just introduced students to learn, not focus on skills to use a calculator in solving problem. The education reform program in lower secondary school in 1990, calculators were "forgotten" in the program.

For the adjusted program in 2000: In this program, calculators were completely forgotten. The "Mathematics Instruction Manual 10" stated: "The use of a pocket calculator to solve calculations for errors, equations, and inequalities with decimal coefficients, is very common". However, not all students can afford to buy a device, so they only rely on subjects like Physics so that students can practice. In other words, calculators are not considered a teaching content of high school level. The calculators were used to check the results of calculations.

For the program and textbooks of 2006: The high school math program was more interested in calculators to increase the use of handheld devices to mitigate unnecessary calculations. In the math textbooks 6, 7, 9, the number of explicit exercises using a calculator has increased compared to the previous books. In this program, the calculator was considered as a tool to support calculation and appeared the type of approximation task by the calculator. Currently, the use of calculators in schools is not common, even banned in exams. However, calculators are an indispensable tool for businesses, scientists, in the activities of many agencies.

For the program and textbooks of 2018: Calculators are used to handle with data, the method of using graphic calculator as a tool for calculation and solve mathematical problems. Students need to appreciate the way use these tools and methods of learning math in exploration, discovery and problem solving. It is essential that students are taught how to use the calculator. Therefore, in many categories of textbooks, they have presented how to use a calculator to calculate, solve equations and solve many other problems.

Thus, since 2006, the math program in high school has encouraged the use of graphic calculators. In the high school mathematics program, calculators have been included in the textbooks, which require practical content for students. To encourage the use of calculators in teaching and learning, the Ministry of Education and Training (MoET) has allowed the use of some kinds of calculators in tests, in exams, including national high school exams.

The approach to use of calculators

Most teachers have confirmed the use of calculators (especially graphic calculators) in teaching mathematics since the implementation of the general education program in 2006 because of the content of math textbooks that require the practice of solving math problems on calculators. Regarding the form of teacher access, equipped with the knowledge and skills to use calculators: 63% are self-studying by teachers, 15% teachers are studying from the universities, 19% teachers are taking some training courses and 3% from other forms.

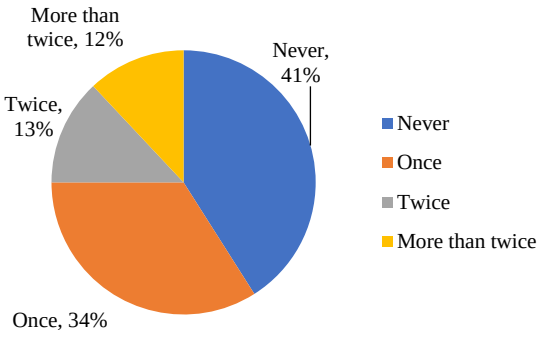

Figure 1. Percentage of teachers participating in training on calculators

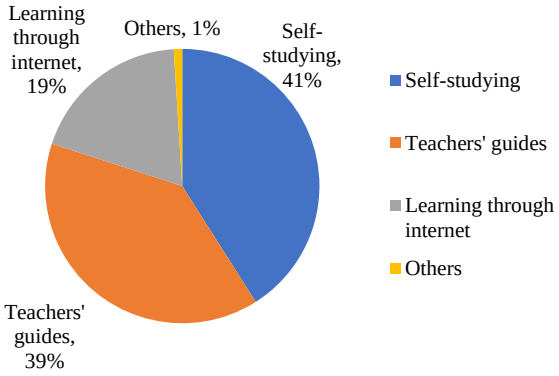

Figure 2. The form of access to a student's calculator

Regarding fostering and training for teachers using calculators in teaching, 41% of teachers have never attended refresher training courses on using calculators organized by different levels. There are very few teachers (12%) participating in training courses at least two times (see Figure 1). Regarding the form of access to calculators for learning: 41% of students are self-study, 39% of students are guided by teachers, 19% learn from the internet, and 1 % from other sources (see Figure 2).

Equipping calculators and methods of using calculators

According to the feedback of teachers, students, and assessments of educational managers of surveying schools, most math teachers are equipped with calculators to teach. Calculators are arranged in the equipment room for sharing. For students, 100% of students have a calculators to study and they buy calculators for themselves. For the type of calculators being used: 97.7% of teachers and 98.1% of students use conventional scientific calculators (this kind of calculator allowed by the MoET in exams). There are no teacher uses a calculator that has a more advanced graphing function.

Most of the teachers interviewed confirmed that they often use a calculator in teaching maths (80.5%), but 19.5% of self-assessing tablets are not regularly used; 77.8% of teachers rated that they were confident in exploiting and using calculators, 21.4% were not confident or limited their

confidence when using calculators in teaching. For students, 95% of students confirmed that they often use a calculator to study mathematics, 74% felt confident when using it (proficiently exploiting the functions of calculators) (see Figure 3 and Figure 4).

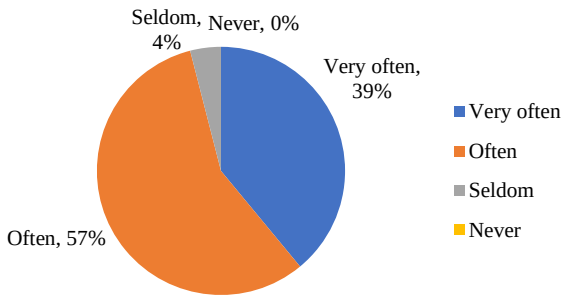

Figure 3. Frequency of students' use of a calculator

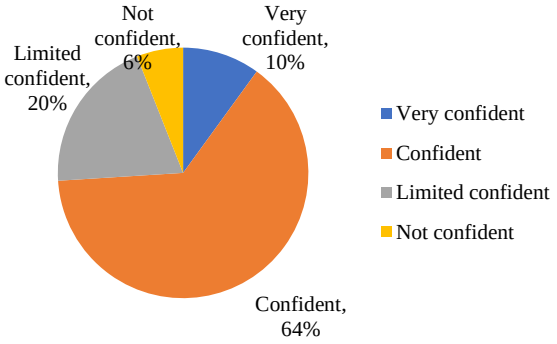

Figure 4. Students' confidence level when using a handheld calculator

Results of conducting surveys on teaching situations using calculators show that most teachers and students said that the calculators used effectively. Results in teaching situations, learning exercises, in tests and examinations. However, 100% of the teachers' opinions were asked without any confirmation that the calculator is effective for teaching mathematical concepts and theorems (see Figure 5 and Figure 6).

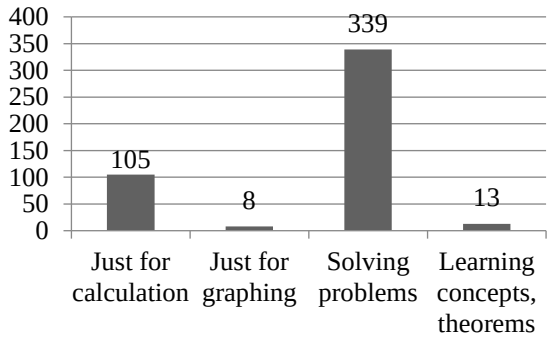

Figure 5. Using a calculator in students' learning situations

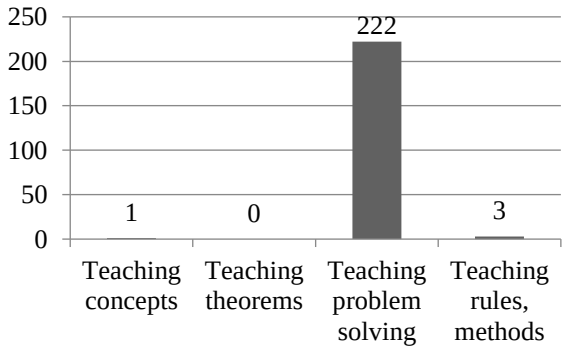

Figure 6. Using a calculator in a teacher's situation

The benefits of using a calculator in learning and teaching mathematics

This study has analyzed the benefits of using calculators in teaching and learning mathematics, including access to a calculator, the attitude of teachers and students, teaching, learning, and performance methods of the teaching process when using calculators. According to student and

teacher feedback, the following statistics (see Table 1 and Table 2) show the benefits of using a calculator in the process oflearning and teaching.

Benefit from using a calculator in learning mathematics	Total	Agreed numbers	Ratio (%)	Mean score	Dev. Std.
1. Helping to calculate quickly and accurately	367	355	96.7%	4.43	0.23
2. Create excitement to study better	367	269	73.1%	3.78	0.19
3. Understand mathematics problems better	367	296	80.5%	3.43	0.13
4. Facilitates knowledge discovery	367	248	67.5%	3.70	0.17
5. Help to discuss and exchange groups better	367	204	55.6%	3.48	0.14
6. Helping the test / exam achieve higher results	367	326	88.9%	4.22	0.20

Table 1. The benefits of using a calculator according to student assessment

Benefit from using a calculator in teaching mathematics	Total	Agreed numbers	Ratio (%)	Mean score	Dev. Std.
1. Create opportunities for students to share ideas, allowing students to interact with challenges	260	200	76.9%	3.95	0.19
2. Bringing a new way of working for both teachers and students	260	200	76.9%	3.85	0.23
3. Not only support calculations but also support for discovering, discovering, and solving math problems	260	195	75.0%	3.87	0.21
4. Helping the conversion of mathematical representation and mathematical modeling to be performed more smoothly	260	162	62.3%	3.60	0.19
5. There is more time to focus on mathematical problems, not time on algebraic manipulations	260	195	75.0%	3.83	0.20
6. Solve some math problems that are hard to reach using algebraic techniques	260	189	72.7%	3.78	0.22
7. Students who use a handheld computer have better academic results than those not using it	260	212	81.5%	3.69	0.18

Table 2. The benefits of using a calculator according to the teacher assessment

The feedback shows that regular use of calculators helps teachers and students to be more creative in solving math problems (Walton & Wines, 1994; Ross, 2017). Teachers and students can look for alternative methods to solve a problem, thus avoiding a lot of work with paper and pens. Students can also experiment with different ways of expressing mathematical ideas while discussing with other students. Handheld calculators support the learning of math in students, such as students can calculate a large number of calculations in a given time because of the faster computing speed than calculating by paper or pen. Besides, handheld calculators are considered a common tool that students can use to save time by having to manipulate calculations, spending more time focusing on solving math problems (Hamrick, 1996; Tan, 2015; Bescherer, 2020).

Another benefit was discovered that the attitude of teachers and students also improved significantly when using a calculator in teaching and learning. Teachers' comments show that students have a positive attitude in sharing ideas, working collaboratively, make the classroom atmosphere more lively. This result is consistent with a study by Dunham (1995) confirms that using a calculator helps teachers and students feel more positive and have better attitudes in the process of teaching and learning. Regarding the effectiveness of the use of a calculator, in this study, 88.9% of students rated the use of a calculator to help the tests achieve higher results; 81.5% of teachers rated students using calculators to have better academic results than not using them. Hembree & Dessart (1993) conducted research between 1990 and 1992, the test results of 24 students using calculators were compared with those of students not using, showing that calculators had a positive impact on the improvement. High academic results for students at all levels from third through twelfth grade. This is also consistent with the results of some recent studies on the use of graphical calculators in teaching and learning mathematics (Noraini Idris, 2006; Graham & Thomas, 2000; Horton, Storm & Leonard, 2004; Berry & Graham, 2005). Empirial research by Noraini Idris (2006) investigated the efficiency of using graphical calculators in high school mathematics teaching, showing that students in the experimental group had higher results than those in a control group, confirming the use of graphing calculators in teaching and learning math which is effective in improving students' academic achievement.

The challenge of using a calculator in teaching mathematics

The requirements of mathematics education not only memorize procedures, events, formulas, and algorithms, but require creativity, thinking ability, and computational competence in learners (Kissane, 2000; Le, 2011; Nguyen & Trinh, 2015). Students need to have manipulations of thinking, reasoning, calculation, estimation, using calculation tools and measuring instruments, and proficient use of the calculator. Consistent with these requirements, this research seeks to identify the challenges faced by teachers and learners in the use of calculators in teaching and learning. The following answers to the challenges recorded by teachers and school administrators are shown in Table 3 and Table 4.

Challenges in using calculators	Total	Agreed numbers	Ratio (%)	Mean score	Dev. Std.
1. Students have not actively used calculators	260	146	56.2%	3.39	0.17
2. The ability to use a calculator in students is limited	260	190	73.1%	3.74	0.23
3. Teachers are inexperienced in using calculators	260	159	61.2%	3.56	0.20
4. The teacher has not taken time to design a lesson using the calculator	260	214	82.3%	3.25	0.16
5. Many mathematical content is difficult to use on calculator	260	183	70.4%	3.72	0.23
6. Many calculators are also prohibited from using in exams	260	135	51.9%	3.42	0.18

Table 3. Challenges in using calculators according to teachers

The challenges faced by students are the ability to exploit the functions of calculators to support mathematical discovery and problem-solving activities. The majority of students (95.5%) said that they only use calculators for normal calculations and directly use calculator functions to solve problems (such as solving equations, calculating functional values, limits, integrals, etc.), and also show that students still have difficulty using a calculator to support learning activities such as conceptual and theoretical learning (see Table 4). This was also pointed out by Ruthven (1992) who pointed out concerns about the use of calculators: students may become dependent on calculators, the use of calculators can lead to their laziness. The availability of mathematical functions of calculators can limit a student's mathematical skills. Besides, some other obstacles are the problem of equipping students with calculator skills in students: 56.2% of teachers said that students still lack calculators to practice, 73.1% of teachers' opinions that students' skill of using calculators (especially graphic calculators) are still limited.

Challenges in using calculators	Total	Agreed numbers	Ratio (%)	Mean score	Dev. Std.
1. Teachers are afraid to innovate for teaching	40	12	30.0%	2.95	0.15
2. Lack of documentation for teachers and students	40	19	47.5%	3.33	0.12
3. Teachers are rarely trained and fostered about using calculators in teaching	40	32	80.0%	3.44	0.14

4. Mathematics curriculum does not have specific requirements for using a calculator	40	18	45.0%	3.08	0.19
5. There is no policy to support and equip calculators for teachers and students	40	37	92.5%	3.69	0.15

Table 4. Challenges in using calculators according to educational managers

Challenges are evaluated by many teachers such as the problem of not spending much time on preparing lessons using calculators, the training and training are not regular, 70.4% of teachers commented that many mathematical contents were difficult to use with a calculator. This is in line with a Horton et al. (2004) study of lesson planning by high school teachers. Horton's research shows that poor teachers' preparation of lesson plans leads to ineffective use of facilities, affecting the quality of teaching. Some teachers said that the experience of using a calculator is also a challenge when using this medium in teaching, which leads to the ability to exploit the functions of computers to support activities of searching, exploring knowledge, and designing teaching situations still face many difficulties, especially in teaching concepts and mathematical theorems.

Another challenge also assessed by school administrators is the difficult and slow attitude of teachers in using information technology in teaching. In the surveyed schools, there are still about 20% of teachers who are not proficient in using calculators, so they have not exploited the benefits of calculators in teaching. Besides, the issue of calculators for students and teachers along with other policies such as the regulations of the MoET on the use of calculators in exams is also a challenge when bringing calculators to schools on a large scale. The survey results show that the use of calculators in students is entirely self-procured by students, not yet received the support of the state and calculator manufacturers. This has led to a part of students who do not have a calculator, do not have access to some advanced scientific calculators (such as graphical calculators) that are used by many countries in teaching. At the same time, some calculators are prohibited from using in exams, which limits the daily use of students. In particular, some graphic calculators are banned from national examination for high school students. In other words, the students could not use these calculators for representing graphs and mathematical modelling.

Designing some teaching situations

With the approach, calculator is the means used to analyze mathematical situations, in addition to the usual functions such as the calculator that supports calculations (algebra, trigonometry, exponential, logarithmic, etc.), solve some types of equations, systems of equations, inequalities, limits, integrals. Exploiting the functions of calculators, we offer some possibilities that calculators hands to help teachers and students discover in teaching and learning mathematics with following situations: (1) using calculators to support students in calculating and predicting rules; (2) discovering mathematical laws; (4) making and testing some mathematical hypotheses; (5)

modeling mathematical problems; and (6) exploring mathematical representations. In this study, we designed some examples to use calculators in calculations, problem solving, concept development, pattern recognition, data analysis, and graphing. Moreover, calculators are use to explore and test mathematical ideas such as predicting, finding rules, testing, proving, and disproving hypotheses.

Example 1. When solving the equation $x^2 + x = \frac{1}{x} + 2$ using the equation solver function of graphic calculator, the graph can be presented as follows:

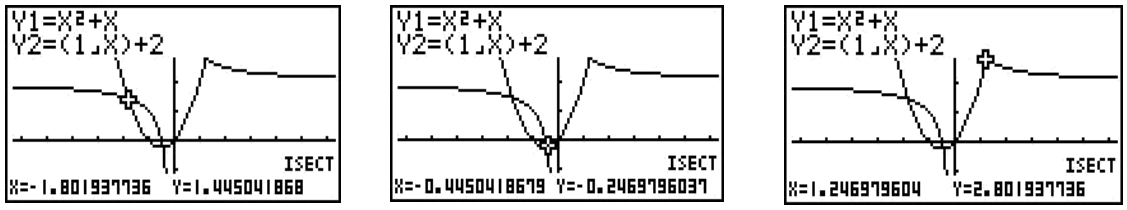

Figure 7. Approximate solution of the equation on a calculator Casio fx-9860

The following example describes a teaching task in data representation and modeling of real-world problems.

Example 2. With a graphical calculator it is possible to describe the modeling process. For illustration, choose Australia's famous Sydney Harbor Bridge. The requirement of the problem is: Consider how the shape of the bridge is a parabola with the equation?

To solve this problem, a graphic calculator was used for representation and modeling to refine the original model as follows (see Figure 8):

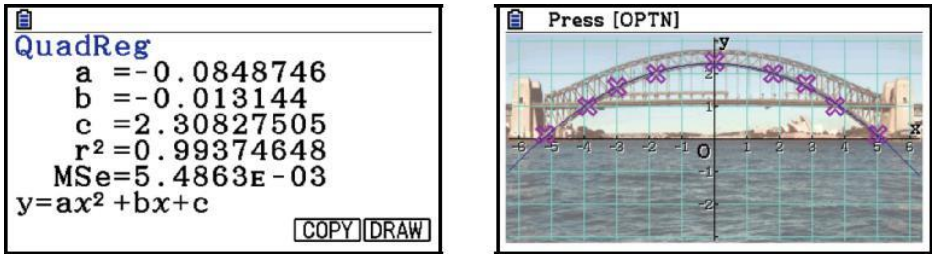

Figure 8. Modeling the situation on a calculator Casio fx-CG 20

With the error is very low, the results on the calculator indicating that the fit is very good. Therefore, a parabolic appropriate model has the equation $y = -0.08x^2 - 0.01x + 2.31$.

We also used a calculator to help students focus on developing problem-solving processes (not just computation involving problems). Robova (2002) asserted that using graphical hand-held

calculators in teaching mathematics brings new working methods, especially the ability to predict and model mathematical problems and graphical support of results obtained by algebraic operations.

Example 3. When studying the development of the city's hotel system (Y) in a number of years (X) of the twentieth century. The collected the data presented in the table:

Year (X)	1920	1928	1938	1951	1957	1964	1966	1968	1972	1982
Number of hotel (Y)	15	20	17	25	29	42	53	47	75	88

The question of the problem is the growth of the city's number of hotels can be described by (or approximated) by what function? Based on that model, predict the city's hotel development in the future?

We used a calculator Casio fx-9860 to help students solve the problems as follows:

- Step 1: Rewrite the data, with 20 being the year 1920, the order of taking the last two numbers of the years in the table, x being the variable indicating the year.
- Step 2: Re-create the data table:

Year x	Number of hotel $f(x)$
20	15
28	20
38	17
51	25
57	29
64	42
66	53
68	47
72	75
82	88

←

Year (X)	The city's number of hotels (Y)
1920	15
1928	20
1938	17
1951	25
1957	29
1964	42
1966	53
1968	47
1972	75
1982	88

- Step 3: Using a calculator to plot a scatter plot from a data table (see Figure 9).
- Step 4: Modeling the problem to explore the relationship between x and $f(x)$, here is to find functions that fit the data. Enter an initial guess for a function that might fit the data, for example $y = x$.

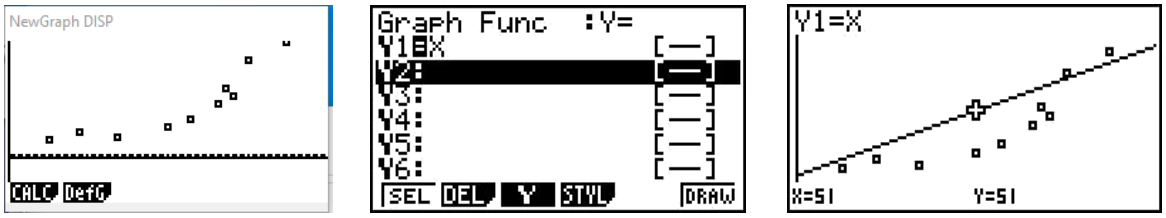

Figure 9. Approximate solution of the equation on a calculator Casio fx-9860

In the screens in Figure 9, the function $y = x$ is not good, since most of the points are below the line. A better guess might be for a line with a slightly reduced slope. The chart in Figure 10 shows one such prediction with $y = 1.2x - 20$. Using graphic calculator to calculate and plot a regression line (line of best fit through the data points). The screens below show a linear regression through the data in the form $y = ax + b$.

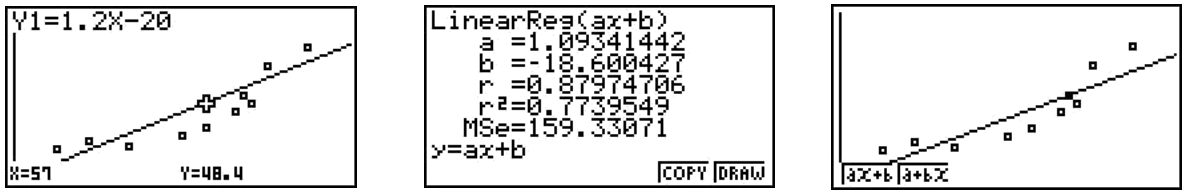

Figure 10. A linear regression through the data on a calculator Casio fx-9860

The regression coefficients a and b show that the best fitting line is given by the equation $y = 1.0934x - 18.600$, which is close to the previous prediction, but by observation still shows that the straight line is inconsistent with those this data. The student can make another choice to find a fitting data. In this case, the data suggest that a quadratic model may be the better choice:

Figure 11. The quadratic function model $y = 0.03x^2 - 1.95x + 45.5$ fits the data

- Step 5: Use the model to solve the original problem. For example, it is predicted that by 2020 (corresponding to $x = 120$) the number of hotels in city Y will be 243.

Example 4. Vietnam's population development issue: The following table records the population of Vietnam in the twentieth and early twenty-first centuries.

a) Plot the scatter plot, noting that a linear model would not be appropriate.

b) Find a natural exponential that models population growth. Plot the graph of the function you found along with the scatter plot. How well does the model fit the data?

c) Use the model you have found to predict the population of Vietnam in 2025.

We use a calculator Casio fx-9860 to help students solve this problem as follows:

Firstly, rewrite the data with 0 being 1990, 10 being 2000, x being the year variable, then for example 1995 corresponds to $x = 9.5$.

Year x	Population $f(x)$: (million people)
0	16.7
1	18.2
2	19.9
3	22.3
4	24.7
5	26.4
6	33.5
7	42.6
8	54.3
9	68.0
10	79.9
11	87.9
12	97.6

Year (t)	Vietnam's polulation (million people)
1990	16.7
1910	18.2
1920	19.9
1930	22.3
1940	24.7
1950	26.4
1960	33.5
1970	42.6
1980	54.3
1990	68.0
2000	79.9
2010	87.9
2020	97.6

Students used a calculator to plot the scatter plot from the data table (see Figure 12). The scatter plot of the data points does not lie on a straight line, so the linear model will not be suitable. The scatter plot shows that the data increases rapidly so a natural exponential model would be appropriate.

Secondly, we have a natural exponential model with the equation. The scatter plot of the points is close to the natural exponential graph found, so it fits the data (best fit the data):

Figure 12. The quadratic function model $y = 0.03x^2 - 1.95x + 45.5$ fits the data

Thirdly, the natural exponential function as a model for Vietnam's population growth is $f(x) = 14.31 \times e^{0.161x}$. In 2025, corresponding to $x = 12.5$, the population of Vietnam in 2025 is predicted to be: $f(x) = 14.31 \times e^{0.161 \times 12.5} = 107.07$ million people.

Thus, in a hand-held calculator environment, teaching and learning activities of teachers and students become more active. Hembree and Dessart (1986) have shown that students who use hand-held calculators have better learning attitudes and better ability to self-study mathematics than students who do not. Besides, Dunham and Dick (1994) also asserted that hand-held calculators with graphical functions can enable students to better solve problems, facilitate changes in student roles and teachers, leading to interactive and exploratory learning environments. In addition, Dunham and Dick (1994) and some other researchers argue that the benefits when students use graphic calculators in learning like: more success in problem-solving tests, take a more flexible approach to problem solving, willing to participate in problem solving and remember problems longer, focused on problems of mathematics and did not spend much time on algebraic transformations.

DISCUSSION

Through researching the real-life situation, acknowledging some recommendations and suggestions from teachers, educational managers, and students to improve the use of calculators in teaching. Teachers need a positive change in the use of graphic calculators. It should foster and train teachers on the use of calculators and calculators should be institutionalized in the subject curriculum. Moreover, course content, tests and assessments should be redesigned to suit the use of teaching media and digital technology. In particular, pedagogical colleges and universities need to equip students with knowledge and skills of using teaching facilities. Students need to receive instructions on how to use the calculator more from teachers and use calculators in exams and tests. Students have the freedom to use calculators whenever it feels appropriate and use more modern calculators (such as graphic calculators). Students also are encouraged to take the mathematics exam on the calculator as well as participated in forums, clubs about using calculators.

The proposed examples have ensured a close connection between theory and practice, each situation contains at the same time the training of skills in using calculators and the acquisition of students' knowledge. It creates opportunities for students to demonstrate their own abilities to an increasingly high level after a process of learning, stimulating initiative and creativity in discovering and put the acquired knowledge into practice. The proposed methods of organizing teaching by examples need to be implemented flexibly, regularly and continuously in the teaching process and can be extended to other teaching contents such as geometry, trigonometry, and statistics. Teaching means can also be replaced by graphic calculators such as using software with similar functions, using smart phones with app calculators or using mathematical software on computers. The creative application of the teachers ensures the suitability with the students and

the actual conditions of the classroom. In this study, we proposed some typical potentials of using calculators in learning and teaching high school mathematics:

Firstly, the teachers can use graphic calculators as a tool to support math teaching. Studies and practices have confirmed the benefits, roles and effects of calculators in teaching. Calculators should be considered as a tool for discovering knowledge, as a means of analyzing mathematical situations. With advantages in calculation, visual display, fast and accurate data analysis, calculators make identification and prediction easier, and at the same time, from the obtained data, suggest proving predictions, exploring and discovering new problems. In other words, the use of calculators can be viewed as an external activity (using graphs, tables, and numbers to manipulate mathematical concepts), and then transformed into an internal activity (understanding the nature of mathematical concepts).

Secondly, the teachers can use graphic calculators in teaching mathematical content knowledge. Specifically, calculators are used to analyze mathematical situations, not just to perform calculations or just use the built-in functions of the calculator to perform mathematical calculations. However, it is necessary to avoid the abuse of calculators to practice calculation skills and to avoid over-expectancy and dependence on calculators in students.

For high school students, the basic requirements of calculation (arithmetic, algebra, geometry) and skills in calculation such as mental calculation, expression transformation, equation solving, simple inequalities, etc. has been equipped from the lower classes, so the use of graphic calculators to support normal calculations in math learning activities will help students get accurate results, save time for mathematical exploration, discovery and problem-solving activities.

Thirdly, high schools should organize training courses for teachers and students on the use of calculators in teaching and learning. Teachers are practitioners of instruction who need to determine their ability and confidence in using calculators and accept the positive advantages of calculators in order to promote the effective use of technology as a tool for teaching mathematics. For these reasons, teachers need to strive to embrace technology and make it a regular part of their classroom practice. Currently, in pedagogical universities calculators have been included in the curricula for pedagogical students, but the time and practice requirements are not much, in practice, many teachers still lack the necessary knowledge for effective use of calculators in teaching a lesson. Therefore, teachers need to be trained on how to use calculators effectively. Hence, it is important to train math teachers with skills in exploiting the functions of calculators, designing teaching situations in the environment of using calculators.

Fourthly, MoET needs to develop mechanisms and policies to develop a teaching environment using graphic calculators. It is necessary to develop the course curriculum to better exploit the potential of calculators. The mechanisms for curricula to adapt to the availability of new technologies depend on the structure of the education system in each country. In countries that

have used calculators in teaching, the curriculum is designed to adapt to the technology by making changes to some mathematical content, such as redesigning the content to ensure that the requirements are met theory and practice, redesigning test content, and methods of assessing students. Teachers play an important role in classroom organization using graphic calculators. Therefore, teachers are encouraged to make good use of calculators before they are confident to use these devices in their classroom. Moreover, in order to bring graphic calculators into teaching in Vietnam, it is necessary to have a strategy of cooperation and companionship between educational management agencies, scientists and manufacturers to produce new products with mathematical functions suitable for subject education programs, with prices suitable to socio-economic conditions. Besides, educational administrators need to monitor the effectiveness of calculators use in the teaching of mathematics and develop timely interventions to address the challenges that may encounter with implementation.

In summary, using a graphic calcultor discovery case benefits students and teachers alike. However, the abuse of calculators can bring about negative effects such as reducing basic calculation skills in students, or being too dependent on calculators in the learning process. Therefore, rational use of graphic calculators, combined with paper and pens in the classroom is necessary, and at the same time, the use of graphic calculators in teaching mathematics needs to be further researched, especially research to build teaching situations to promote the benefits of graphic calculators, improve the quality of teaching mathematics in high schools.

CONCLUSION

The requirement for using media to support math teaching in high schools is an urgent issue in the context of current fundamental and comprehensive educational renovation. Teaching mathematics with graphic calculators has not been paid enough attention by teachers in high schools for many different subjective and objective reasons such as: only considering graphic calculators as the tool only has a calculation function, has not yet exploited the graphic calculators to become a tool to access knowledge, a means to analyze mathematical situations, teachers have not yet designed teaching situations in the environment for using graphic calculators.

Although the use of graphic calculators in the classroom is still being debated by many educators, the results of this study confirm that the use of calculators has many benefits for students and teachers. However, the misuse of calculators can have negative effects such as reducing basic computing skills in students or being too dependent on calculators in the learning process. Therefore, the rational use of calculators, combined with paper and pens in the classroom is a necessity, and the use of calculators in teaching mathematics should be further studied to promote the benefits of calculators, improve the quality of teaching mathematics. Further study is to examine how to use a calculator in teaching situations such as designing teaching situations for solving problems, teaching mathematical concepts, theorems, rules, and laws. It is necessary to continue proposing measures to put a calculator in a new general education curriculum such as the

design of textbooks and school teaching materials. In particular, MoET should promulgate policies to connect scientists, educational managers, and businesses to produce calculators suitable to the financial capacity of teachers, students; support mechanism for students who are in difficult circumstances and who cannot yet equip graphic calculators themselves.

It can be said that the appearance of graphic calculators has had an impact on teaching and learning mathematics. It has been more than 40 years since many countries with advanced education systems such as the United States, Australia, Canada, United Kingdom, etc. have allowed it. Using graphic calculators in high schools and increasingly widely used in schools, along with the continuous change and improvement of technology, the calculators are increasingly meeting the needs of students in learning. However, in the process of studying the use of graphic calculators in teaching mathematics, we found that basically the studies only focused on affirming the benefits or challenges that calculators can bring during the teaching process. Therefore, the paper still opens the judgments about the effects of graphic calculators in teaching mathematics without much in-depth research on how to use calculators. It is more necessary to examine how to be highly effective in teaching, how are teaching situations designed to use calculators as a means of helping to explore, discover and solve mathematical problems.

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