Effects of Differentiated Instruction in Flipped Classrooms on Students' Mastery Level and Performance in Quadratic Equations

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Abstract: This study employed student choice and tiered worksheets as strategies of differentiated instruction on Quadratic Equations in addressing students' non-compliance with assignments in a flipped classroom. In each lesson, students choose among the instructional materials with guide questions to assist them in focusing on key areas during the asynchronous activities as homework. Tiered worksheets were administered in face-to-face classes based on students' readiness as reflected in pre-assessment results. Data from tiered worksheets show students' mastery levels and student engagement in online class instruction and in-class tasks. Additionally, there is a significant difference between pre-assessment and summative assessment percentage scores with a substantial effect size, implying improved student performance in solving quadratic equations.

Keywords: differentiated instruction, flipped classroom, tiered worksheets, quadratic equations

INTRODUCTION

With the formal education setup, most teachers resort to one-size-fits-all instruction in their heterogeneous classes. Some of the learning tasks and outputs tend to be too easy for exceptional students, while these can be challenging for academically struggling students. In a typical heterogeneous classroom, the teacher often instructs the intermediate learners, rather than the exceptional students (Newman, 2009) since students are treated by the teacher as if they are all the same (Valiendes & Neophytou, 2018). Consequently, students who excel academically may feel bored with easy tasks and be shortchanged of their appropriate learning competency. On the other hand, struggling students may find themselves lost if the learning tasks are difficult, leading to disadvantageous learning misconceptions. Differentiating instruction for struggling learners helps strengthen their basic skills, working memory, and fact retrieval. Similarly, differentiating instruction for high-performing students provides exemptions and cultivates their higher-order thinking skills (Laud et al., 2011). It is argued that the use of differentiated instruction as a teaching approach enables students to understand concepts efficiently by giving them appropriate learning tasks based on students' mastery level.

Differentiated instruction is an innovative and effective teaching approach (Tomlinson, 2000; Gregory & Chapman, 2013; Valiendes & Neophytou, 2018). Benjamin (2014) added that differentiated instruction refers to various classroom practices that aim to accommodate the learning differences among learners and involve balancing between content and learning competencies expected on assessments and varied activities to maximize learning.

Differentiated classrooms should be responsive to every student's learning style because students achieve better when instruction is appropriate to student's intelligence and learning preferences (Tomlinson et al., 2003). Identifying the individual student's mastery level can be applied so that the high achievers are not shortchanged by providing thought-provoking tasks. Similarly, the low-performing students should receive additional support and further remediation. Teachers are encouraged to support struggling learners through scaffolding to ensure that all students acquire the desired learning competency regardless of their cognitive skills. Especially since concepts in mathematics are built one after another; consequently, learning is more of a linear process, underscoring the importance of fundamental concepts.

Tomlinson (2000; 2005), Gregory and Chapman (2013), and Newman (2009) argue that not all students are the same and that they differ in terms of how they think and learn conceptual knowledge inside the classroom. Further, they asserted that according to existing literature, studies show heterogeneous classrooms are effective in social and academic aspects but have yet to prove the needs of all learners are being addressed. However, when teachers practice differentiation, heterogeneous classrooms are most effective (Cannon, 2017). Differentiated instruction is designed to assess students and tailor instruction to fit their varying needs, thus maximizing student's potential (Santangelo & Tomlinson, 2012). In addition, students' interest and motivation to learn are enhanced due to effective differentiation (Villamor & Lapinid, 2022), as this teaching approach enables teachers to engage all students in learning. Tomlinson (2000, p.25) states, "differentiated instruction is an approach by which teaching promotes high level and powerful curriculum for all students but varies the level of teacher support, task complexity, pacing and avenues to learning based on student readiness, interest, and learning profile."

Theoretical Basis of Differentiated Instruction

Differentiated Instruction as an approach to student diversity in the classroom is anchored on Vygotsky's Social Development Theory, specifically focused on the key concepts of Zone of Proximal Development and Scaffolding. Vygotsky's Zone of Proximal Development (ZPD) helps the students maximize their academic excellence through proper scaffolding of the "more-knowledgeable-others" (MKO). As mentioned by Eun (2017, p.3), Vygotsky (1978) defined "ZPD as the distance between the actual development level as determined by independent problem solving and the level of potential development as determined through problem-solving under guidance or in collaboration with more capable peers." If the learning task is outside the student's comfort level but can be completed with the supervision of an adult, then it enhances student learning (Newman, 2009). An increase in knowledge and understanding to the fullest extent possible can be achieved if teachers allow students to work on appropriate tasks corresponding to their level within their proximal development (Morgan, 2014).

Conceptual Framework

In the study by He et al. (2016), the open-ended student responses revealed that one of the serious implementation issues of the flipped classroom model is the non-compliance of the students with the pre-class studying, which hinders the flipped classroom model from attaining its goals and objectives in learning. Hence, to address the aforementioned weakness of this instructional technique, this study incorporated "differentiated instruction" in conjunction with the "flipped classroom model" to ensure that activities are based on students' learning preferences and that students are accountable for their own learning. Furthermore, in order to promote active learning and productive use of knowledge, the instructional materials uploaded in the pre-class study or the online asynchronous class instruction and in-class activities such as worksheets and practice exercises must be integrated and complement each other (He et al., 2016). With this premise, we posit there is meaningful acquisition of learning mathematical concepts because students must do advanced preparations prior to in-class activities.

Studies which concern teacher's way of catering to students' individual needs in a blended learning environment are still scarce (Boelens et al., 2018). Further, Boelens et al. (2018) argued that it is vital for educators to have a clear stance as regards proactively planning differentiation strategies and simultaneously responding to students' diverse needs in the context of blended learning. In this study, the researchers differentiate the content and process of learning based on students' readiness and interest as they go through the pre-class readings of instructional materials, video watching, and tiered worksheets. The study includes ways to implement the differentiation process so that mathematics teachers are given ideas to operationalize this approach in their classes. Aside from the differentiation in allowing students to choose among the varied online class tasks, differentiation in content and process is also operationalized through in-class activities using the tiered worksheets. The recall part in the in-class tiered worksheets contains examples and essential concepts which scaffold students while answering the worksheets. All those aim to cement students' knowledge further, rectify their errors and misconceptions, and maximize student engagement.

Problem Statement

The study's main purpose is to investigate the effectiveness of differentiated instruction in a flipped mathematics classroom where students vary in cognitive abilities on student performance in Quadratic Equations. Specifically, the following questions guided the conduct of the study:

1. How do students' mastery levels differ
   a. between pre-assessments and final tiered worksheets in each lesson and
   b. across tiered worksheets within each lesson?
2. Is there a significant difference in students' performance before and after the intervention?

LITERATURE REVIEW

Forms of Differentiation

Content refers to the subject matter students must learn inside the classroom (Coubergs et al., 2017) or the knowledge and skills students need to obtain to reach mastery (Boelens et al., 2018). Instruction can be differentiated in the content based on the student's interests, readiness, or prior knowledge. The first step in differentiating content is deciding which learning competencies and standards are targeted (Gregory & Chapman, 2013). Differentiating content requires different means of presenting the lesson to the students. Teachers should plan instructional materials that are exciting but challenging and timely for learners to ensure that the targeted learning competencies and standards based on the curriculum are just right on the student's mastery level (Gregory & Chapman, 2013).

On the other hand, differentiation can also be planned according to the process, or the learning activities and tasks created based on the skills of the students, level of readiness, and preferred learning style (Taylor, 2015) or how students acquire new skills learned from the content (Boelens et al., 2018). Taylor (2015) further contends that when teachers use differentiation in the classroom, they can vary the level of complexity of the learning materials (e.g., below target, on target, and above target). Coubergs et al. (2017) define a *process* as the learning track of the students, which is associated with the learning tasks and activities aligned to the content objectives. It can be in the form of varied activities such as administering worksheets to students with different difficulty levels where students practice individually, by pairs, or by groups and make sense of what they learned in the content.

Lastly, instruction can also be differentiated according to the product, wherein teachers incorporate different assessments that allow students to showcase their creativity and manifest their learning in their own ways. "Product is the learning outcomes and achievements, based on which students can prove their accomplished goals" (Tomlinson & Imbeau, 2010, as cited by Coubergs et al., 2017). It refers to how students demonstrate their knowledge (Boelens et al., 2018), tantamount to the outcome of teaching and learning processes such as summative tests, research projects, topic presentations, simulations, and portfolios.

The Role of Flipped Classroom in Differentiated Instruction

With the aim to offer more flexibility in learning in terms of time and place to a group of diverse learners, blended learning is often integrated into the classroom (Boelens et al., 2018). *Blended learning* is an instructional approach that combines online and face-to-face or in-class instruction (Boelens et al., 2018). One such model of blended learning is the flipped classroom. As defined by Altemueller and Lindquist (2017), the flipped classroom reverses traditional teaching methods, aiming to deliver the instruction outside of the class, devoting the class time to other instructional tasks such as problem-solving, practice exercises, and hands-on activities.

Research says the flipped classroom model is a promising instructional technique because it frees up class time for more in-depth discussion through in-class activities such as supervised collaborative works and mastery exercises (He et al., 2016; Chen et al., 2018; Altemueller & Lindquist, 2017). This instructional model maximizes student learning through access to curated instructional materials anytime and anywhere, allowing the students more time to understand the content and more ways to support self-learning (Chen et al., 2018). The flipped classroom also enables students to have increased learning, motivation, and engagement (Rutkienė et al., 2022). The model benefits the struggling students the most because it allows them to pause, replay, and go back to the parts of the online learning videos. The same is true for reading instructional material or listening to a podcast. On the other hand, students who excel can quickly go to the challenging learning competencies and have mastery of the content knowledge.

He et al. (2016) mentioned in their study that the flipped classroom model consistently benefitted students with varied learning preferences. As a form of differentiated instruction, students may be provided online resources in different forms, such as videos, podcasts, and websites of instructional materials, and be given the liberty to choose which form of media they prefer based on their learning styles (visual, auditory, or kinesthetics).

The Role of Assessment

Effective and successful differentiation relies on the role of assessment. The data on different types of assessment employed by the teacher make informed decisions on how to help students succeed academically. Unfortunately, many teachers, students, and parents are more concerned about the scores obtained in the summative and achievement tests. Hence, students must memorize many facts and formulas such that authentic learning is often neglected. “These types of assessment teach students to memorize and encourage teachers to teach to the test, not for student understanding” (Newman, 2009, p.11). To further support this argument, Cannon (2017) said that the core of instruction must focus on meeting the needs of diverse learners in the classroom to reach for the standards and not teaching for the examination.

Tomlinson (2005) stressed that formative or ongoing assessments should not all be graded because their purpose is to help both the teacher and the student see how learning is developing and what adjustments are necessary in the instruction to ensure that learning stays on track. Assessment must be considered as a tool to underscore the strengths and weaknesses of the students so that proper remediation can be administered to rectify diagnosed misconceptions and help learners move to more advanced levels of understanding (Gregory & Chapman, 2013).

Cannon (2017) suggests determining the different mastery levels of students at the onset of differentiation to make quality instructional decisions. Particularly as a form of differentiated instruction in terms of process, a preassessment may be administered at the onset to gauge the existing knowledge and readiness of the students so that an appropriate corresponding instruction based on students' mastery level may be provided to students in a heterogeneous class. To support this, Connor et al. (2017) used assessment data to group students with common learning needs and provided individualized instruction to improve student achievement. Individualized instruction may be in tiered learning tasks administered as ongoing and formative assessments as differentiated instruction since continual assessment is an essential foundation for effective differentiation (Tomlinson, 2005; Santangelo & Tomlinson, 2012). Using tiered learning tasks is one way to support those who are low-achieving and simultaneously challenge those who are high-achieving (Laud et al., 2011).

Tiered worksheets refer to the varied learning modules with different levels of complexity to cater to the needs of the individual learners concerning their mastery levels. Using tiered worksheets conforms with Deunk et al.'s (2018) concept of mastery learning as one of the many differentiation strategies. Mastery learning uses regular assessment to check whether the students have reached a particular ability level (Deunk et al., 2018). Moreover, Huebner (2010) espoused that students must be taught at the proper instructional level responsive to their cognitive levels through different pathways to be appropriately engaged since students' attention is attracted to the idea that the learning task seems worthwhile (Cannon, 2017). Valiendes and Neophytou (2018) mentioned that tiered activities take three phases of implementation, which are (1) creating an "on-level" task based on the standards expected by the curriculum, (2) adjusting the task to create a "below-level" activity for struggling students and in the same manner, (3) adjusting the task to create an "above-level" task for advanced students.

The majority of the studies concerning differentiated instruction used flexible groupings in either homogeneous or heterogeneous ability groupings. However, according to the meta-analysis conducted by Deunk et al. (2018), simply grouping students and placing them physically together does not ensure effective differentiated teaching. Thus, individual differentiated work can be used as a strategy to differentiate instruction in a mathematics classroom as an alternative to ensure student performance improvement and progress (Nechifor & Purcaru, 2017). As an example of this practice, Hapsari et al. (2018) used tiered assignments in which the teacher created sets of questions to be answered by the students individually with varying levels of difficulty, and students liked the tiered task option because items were just appropriate to their capabilities.

METHOD

The study utilized a descriptive quasi-experimental quantitative research design. Participants comprise an intact class of 46 Grade 9 – Mathematics students from a public secondary school in the Philippines. The class was chosen using purposive sampling with an intact heterogeneous classroom as a criterion. The research participants were briefed on the nature of the research, highlighting the rationale and importance of conducting the study to obtain their full cooperation. The researchers also provided the student-respondents the extent of their participation as stipulated in the informed consent form, and they were asked to indicate their willingness, affixing their signatures, to participate in the study. All 46 students agreed to participate in the study by submitting their parent's consent form.

The intervention was implemented for almost five weeks, and the covered topics in Grade 9-Mathematics are limited to quadratic equations in the following lessons: identifying quadratic equations and solving quadratic equations by extracting the square roots, factoring, completing the square, and quadratic formula. Solving quadratic equations follows specific steps but the procedures require deep conceptual understanding. Hence, the researchers selected videos and instructional materials that ensure students' conceptual understanding of the procedures of solving quadratic equations. Some of the procedures warrant clear conceptual understanding – for example, the use of factoring as a method warrants understanding of its application of the zero-product property; the use of completing the square requires students to have a deep understanding of what makes a perfect square trinomial; and the correct numerical coefficients of a quadratic equation should be that which is written in standard form before they can use the quadratic formula because the quadratic formula was derived from the standard form. These were further processed by eliciting students' conceptual understanding in face-to-face class discussions.

We used several strategies to operate differentiated instruction as Newman (2009) suggested. We listed tasks aligned to the learning goals and competencies. Students were provided a list of activities and when each should be accomplished. Adhering to the flipped classroom model, students received pre-instruction video links and learning websites. The online materials were carefully and meticulously curated to avoid too many choices (Brandt & Columba, 2022). These materials and in-person class assessments were developed to ensure they are aligned with the intended learning competencies. To ensure students follow the given tasks, a Facebook group was set up as a communication channel where guide questions corresponding to each lesson were provided for a focused viewing and reading and for students to raise questions.

In-class activities include the administration of a pre-assessment to determine students' readiness to check whether students engaged with the given online tasks. The pre-assessment is a key tool in this study, administered at the onset of in-person class. The pre-assessment, comprising eight items, progresses in difficulty: items 1 to 3 are easy (1 point each), items 4 to 6 are of average difficulty (2 points each), and items 7 to 8 pose greater challenges (3 points each), yielding a total of 15 points. The students' mastery levels (Beginning Mastery, Approaching Mastery, High Mastery) coined by Gregory and Chapman (2013) were adopted so that appropriate tiered learning worksheets correspond to their existing learning competencies.

The tiered worksheets were designed to match varying initial mastery levels based on the students' pre-assessment scores. Focused on quadratic equations and their solution methods, the tiered worksheets tailored to the three levels of Mastery students offer scaffolding and challenge as needed. Each worksheet contains three parts of increasing complexity, contributing to a potential 15 points per student. By adhering to a passing rate of 60% of the school, students transitioned from lower to higher-tiered worksheets, fostering individual pacing while feedback and monitoring remained integral. Allotting a maximum of one hour, these worksheets reflecting the Zone of Proximal Development (ZPD) concept support students' progression toward higher cognitive abilities in quadratic equations. After addressing queries, reinforcing learning, and addressing students' common mistakes, the cycle goes on following the same process for subsequent lessons.

Following the intervention, we administered a summative assessment to measure student learning on the tiered worksheet lessons. This 40-item test mirrored the structure of the five pre-assessments, comprising 15 easy questions in Part I (1 point each), 15 average questions in Part II (2 points each), and ten difficult questions in Part III (3 points each), totaling 75 points.

RESULTS

Mastery Levels based on the Pre-Assessment and their Tiered Worksheet Progress

Figure 1 presents the changes in students' mastery levels based on the pre-assessment and the final tiered worksheet in each lesson and the students' progress in their tiered worksheets across the lessons. The pre-assessment served as the tool in differentiating students' mastery levels whether they are on the "Beginning Mastery Level" (scores from 0% to 59%), "Approaching Mastery Level" (scores from 60% to 79%), or "High Level of Mastery" (scores from 80% to 100%). Students who are at the BM level based on the pre-assessment were given T1 to work on in the class, while those at the AM level skipped working on T1 and went on to work on T2, and those at the HM level skipped working on T1 and T2 and went on ahead to work on T3. Within the given in-person class time, a student can progress from a lower-tiered worksheet to the next tiered worksheet if they get 60% of the total score in the tiered worksheet.

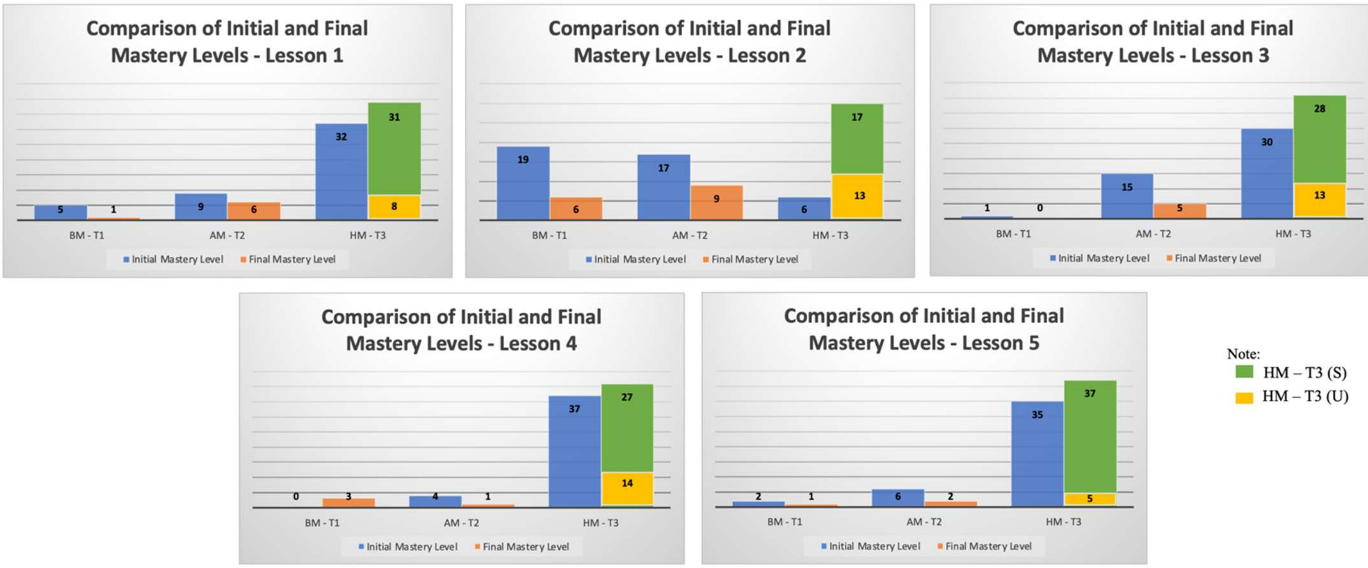

Figure 1. Graphical Representation of Students' Mastery Level at the Onset of In-Class Differentiated Instruction and their Final Tiered Worksheet Across Lessons

Legend:

BM – Beginning Mastery	T1 – Tiered Worksheet 1
AM – Approaching Mastery	T2 – Tiered Worksheet 2
HM – High Level of Mastery	T3 (U) – Tiered Worksheet 3 (Unsuccessful)
	T3 (S) – Tiered Worksheet 3 (Successful)

In Lesson 1 (Introduction to Quadratic Equations), out of 46 students, the majority of them (32 students) went straight to HM (T3) in their initial mastery level. Despite having some of the students in lower mastery levels (5 students in BM and 9 students in AM, respectively), they were able to move up to higher mastery levels in their tiered worksheet, which determined their final mastery level except for one student who remained in the same BM level. The increase of students who moved up to higher mastery levels shows that the options provided in the online class instruction and the recall part or the mini discussion contained on the front page of the tiered worksheet, which served as the scaffolding to the students really worked and were effective to increase student engagement and performance in tiered worksheets. The tiered worksheets helped students achieve optimal learning by immersing themselves in answering problems with varying difficulty levels. With this in mind, students did not get bored or frustrated because the worksheets were apt to their ability levels, and hence, their confidence in answering challenging questions but within their ZPD was enhanced as they went through the high-level worksheets. The scaffolding was paramount to increase their achievement in terms of tiered worksheet progress since they recalled concepts, which enabled them to cement their previous understanding gained from the online class instruction.

In Lesson 2 (Solving Quadratic Equations by Extracting the Square Roots), four students were unable to take the pre-assessment. Three of them came to the class the next day and were deemed regarded at the BM level and given T1 for the tiered worksheet. The majority of the students (19) were initially categorized under the BM compared with the AM (17) and HM (6) levels. Comparing it to the results in Lesson 1, the trend of the initial mastery levels of the students decreased.

When asked why they were unsuccessful in landing on an AM for T2 or HM for T3, compared to their initial mastery levels in the previous lesson, a student explained he needed help understanding the online mathematics videos in Extracting the Square Roots. Moreover, despite 45 students who saw (considered as "seen") the list of the Online Class Instruction for Lesson 2 in the created Facebook group, some students still failed to participate in the given asynchronous tasks. Another student claimed that she prefers reading to watching videos. Students with lower mastery levels further clarified with the teacher if they could still watch the off-loaded mathematics video links and other online resource materials, albeit the concluded pre-assessment. According to them, they wanted to strive harder to land on a T3 level. The purpose of this was for them to still cope with the lesson's contents and consequently progress in their tiered worksheet level since they could not go through with the online class instruction, thus getting low scores in pre-assessment and being identified as T1 level. On the other hand, students who skipped lower-tiered worksheets also professed to go through the materials seriously more than once with the questions posted on Facebook as their guide.

Hence, in the final mastery level column for Lesson 2, the students who were initially classified in the lower levels were able to move up to higher mastery levels. From the initial 19 students, it went down to 6 students classified under BM as their final mastery level. Also, from the initial 17 students under the AM level, it decreased further to 9 students, while the initial six students under HM increased to 30 students constituting the T3 - U (13) and T3 – S levels (17). Since the teacher allowed the students to watch or read the online materials, the links of which were posted in the Facebook group, students were able to move up to higher levels. Students realized that they could still participate in the online class instruction intended and given the day before the administration of the tiered worksheet.

Likewise, looking at the results from the remaining Lessons 3, 4, and 5, the same increasing trend can be observed in the number of students who moved up to higher mastery levels, specifically in the T3 level. In Lesson 3, only one student was classified under BM, 15 in AM, and 30 under HM, respectively. For the final mastery levels under Lesson 3, the one student initially regarded as BM was able to move up to higher levels. Also, the original 15 students under AM became five in the final counting, and similarly, the initial 30 HM students increased further to 41 students, T3 - Unsuccessful Level (13) and T3 - Successful Level (28) combined.

The sudden increase in the frequency of higher mastery levels confirms that students studied harder for the pre-assessment because, according to them, landing on higher mastery levels encourages them to do better, and they feel that being classified as "High Level of Mastery" for them is some accomplishment. As one student shared in class,

> *"We do not want to be in low mastery levels because it will only mean we did not prepare or study for the in-class assessment. Besides, if we get lower scores, we have many worksheets ahead of us to answer. Actually, it motivates us when the teacher announces our mastery levels that's why we always look forward to answering the Tier 3 worksheet*."

The five and three absentees in Lessons 4 and 5 were still allowed to take their tiered worksheets for BM. The final mastery level column for Lessons 4 and 5 indicates that the majority of the students were already moving to higher mastery levels and were eager to be classified under HM so that they would be able to answer the T3. The number of HM in Lesson 4 increased from 37 students to 41 in the final counting, comprising the T3 - U (14) and T3 - S levels (27).

Correspondingly, the number of students under the HM in Lesson 5 also increased from 35 to 42 in the final counting, consisting of T3 - U level (5) and T3 - S level (37). Hence, the results showed that almost all students were getting used to this type of instruction under the online class and answering the tiered worksheets based on their ability level since the number of students classified under the T3 level increased throughout the intervention. This increasing trend confirms that students are engaged when they are given tasks appropriate to their mastery levels (Huebner, 2010; Laud et al., 2011). Additionally, students' academic progress improves if the individual learning tasks are personalized to their cognitive needs (Nechifor & Purcaru, 2017).

Students' Progress in Tiered Worksheets in each Lesson

Figure 2 illustrates the frequency distribution of the students in terms of their tiered worksheet progress. Specifically, this shows the students' particular movement with regard to their initial to final tiered worksheet level in each lesson throughout the intervention. There is a more significant percentage of the students who have movements (1-level, 2-level, or 3-level movements combined) than the students who did not have any movements or remained in their mastery level. Correspondingly, the percentages of the students who displayed movements from their initial mastery to higher mastery levels, 1-level and 2-level movement combined, throughout the intervention are 78.26%, 68.85%, 71.77%, 68.89%, and 93.29%. However, the percentages of the students who did not have any movement are 21.69%, 31.07%, 28.3%, 31.09%, and 6.66%.

Notice that only Lesson 5 (Solving Quadratic Equations by Quadratic Formula) has the lowest percentage of students who showed no movement in their mastery level. However, Lesson 2 (Solving Quadratic Equations by Extracting the Square Roots) and Lesson 4 (Solving Quadratic Equations by Completing the Square) have the highest percentage of students remaining in worksheet mastery levels. The reasons students gave why they remained in their initial mastery levels were that they did not have any internet loads, were busy, did not have a Wi-Fi connection, and needed help understanding the lesson from the online class instruction. Thus, the success of the flipped classroom depends on these factors, among others.

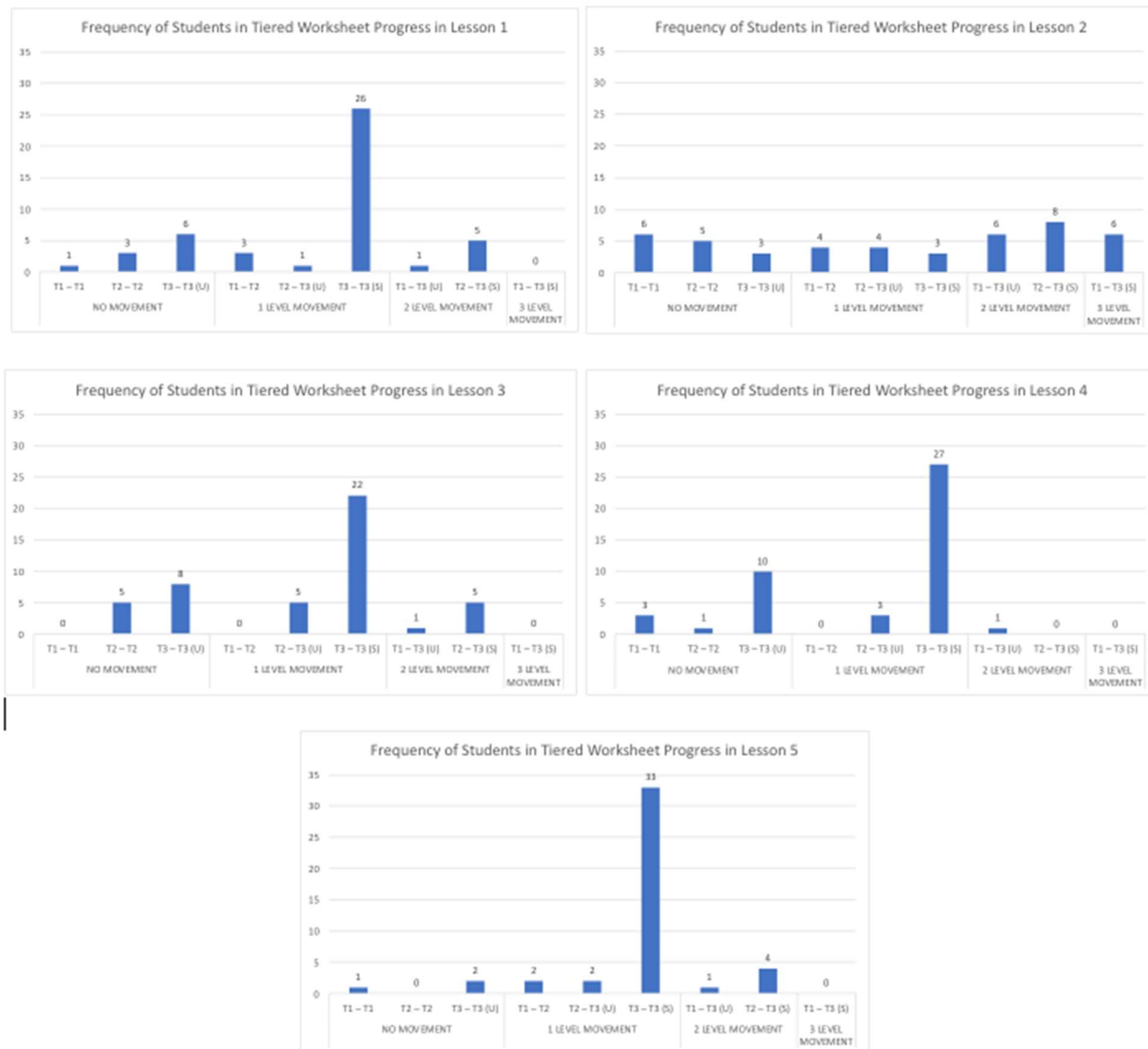

Figure 2. Frequency of Students in Tiered Worksheet Progress in each Lesson

Likewise, only Lesson 2 has the 3-level movement (13.3%). Students find it easier to extract the roots since the concepts regarding radicals were recently discussed prior to the conduct of the study based on the given curriculum. Further, it can be attributed to the nature of the topic since the process involved in Lesson 2 is more straightforward compared to the rest. Also, this can be due to time constraints since students were only given a one-hour period to accomplish the tiered worksheets. Since students under the T1 have three worksheets to answer for them to have 3-level movements, given the limited amount of time, they can only do at most 2-level movements. Hence, students initially in T1 rarely moved up to T3 – successful due to the lack of time, whereas, if the student was at T2 level at the onset, the student had a greater chance of reaching T3. Therefore, students were made to understand the importance of online class instruction to reach T3 - S. Moreover, it can be inferred that if students exert effort, they can progress to higher mastery levels and improve their performance.

In this regard, online class instruction was intended to address the limited face-to-face instruction time. Students can reach T3 (Unsuccessful or Successful), i.e., skip working on the lower-tiered worksheets if they get an AM or HM in their pre-assessment because they learn from working on the assigned asynchronous tasks during their independent learning preparation. Because of the limited time, rarely can one succeed, starting from the lower tier and achieving a higher tier level. Students can only successfully reach the higher-tiered worksheets if they study well in their online class instruction, focus on the learning materials, and follow the guide questions to answer the in-class worksheets quickly. Therefore, a flipped classroom was an excellent way to address the limited time for face-to-face instruction.

Additionally, some students struggle in Lesson 4 because they realize they must be proficient in completing the square. This process added layers of procedures, which consisted of applying the addition property of equality in coming up with a perfect square trinomial and factoring before extracting the roots to solve the quadratic equation. Most of the students were able to eventually move up from their initial tiered worksheet level because they were allowed to watch the online math videos in class, albeit reducing their time to accomplish their worksheets. With this in mind, students can comply with their assigned online tasks or risk not maximizing the in-class time because they must catch up with watching or reading the online resources.

Since learners inside the classroom were diverse, we also noticed some students relied on the scaffolding of the tiered worksheets since they preferred reading to watching videos. In contrast, others relied heavily on watching the learning videos. In a way, this validates the purpose of Differentiated Instruction in this study. Students who watched the videos because they disliked reading the printed detailed explanation of the worksheets. Others skipped the scaffolding part of the tiered worksheets since they had encountered it already from the online class instruction. At the same time, some opted to watch online mathematics videos to save time and directly answer the items in the tiered worksheets. Hence, this finding indicates that Differentiated Instruction in the flipped classroom context in this study effectively catered to students' preferred approach to learning mathematical concepts.

Students' Performance in the Pre-Assessment and Summative Assessment

Figure 3 shows the means of students' pre-assessment scores in each lesson. The highest possible score that a student can obtain from the pre-assessment is 15. Observe that there is no data for the BM – T1 for Lesson 3 because no student was categorized under that level in the final mastery level counting. On the other hand, there were no computed standard deviations (SD) for BM – T1 under Lessons 1 and 5 and AM – T2 under Lesson 4 since only one student was categorized under those levels in the final mastery level counting. Further, the means and standard deviations in Lesson 4 are 0 since the three students classified under the BM – T1 as their final mastery level were absent during the pre-assessment. Other standard deviation values range from 0 to 5.66. Also, observe that among the final mastery levels, the HM – T3 (S) always obtained higher means than the lower mastery levels except for Lesson 2, in which the HM – T3 (U) got the highest mean. This trend can be attributed to students' desire to obtain a high score in their pre-assessment so that they can skip answering T1 and T2 worksheets. One student stated during the in-class engagement:

> "*I always wanted to land on a High Level of Mastery, Sir, so that I will only answer one worksheet. Whenever I'm regarded in a lower mastery level, I feel that I wasn't able to give my best and also, I get sad and envious because my classmates are in Tier 3, while here I am, classified in Tier 1. Hence, I feel motivated to study so that I can be of equal footing with them in terms of ability level. So, I am delighted if I can land on high mastery level because it means that I learned from the videos*."

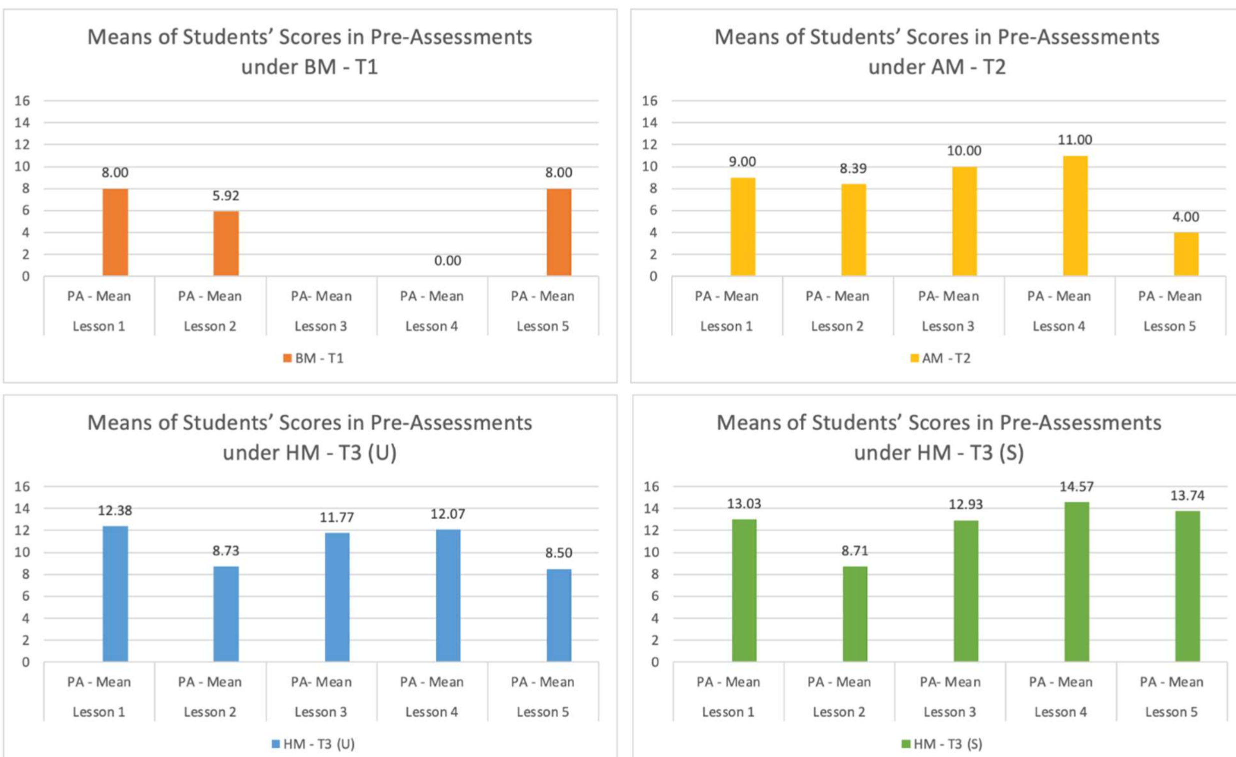

Figure 3. Means and Standard Deviations of Students' Scores in Pre-Assessments

During the intervention, students strive harder to land at higher mastery levels, reflecting how well they performed in their pre-assessment based on their understanding of the concepts gained from the online class instruction. Some students also realized that even though they were regarded at lower mastery levels in a particular lesson, they knew that they could still do better for the next lesson, where the idea of a second chance comes in. As one student said:

> "*Sir, it is okay for me if I am just in T1 level. Next time, I will surely do my best. Likewise, I can still answer the worksheets because of the mini lesson you provided there so that we can easily consult it whenever we answer the worksheets. Thus, it also helps us to familiarize ourselves in the items. Also, it's okay since I am given the chance to take three worksheets, hence, I see it as an opportunity to learn more*."

Tables 1, 2, and 3 present the Paired sample statistics: Correlation and t-test between the cumulative pre-assessment and summative assessment scores, respectively.

Percentage Scores	Mean	N	Std. Deviation	Std. Error Mean
Pre-Assessment	57.5435	46	11.09095	1.63527
Summative Assessment	62.6196	46	7.39007	1.08961

Table 1. Paired Samples Descriptive Statistics

The pre-assessment cumulative and summative assessment scores were converted to percentages. The increase in the mean percentage score indicates students' acquisition of required competency skills through the tiered worksheets and in-class remediation. As shown in Table 1, the summative percentage mean score is greater than that of the pre-assessment percentage mean score, implying an improvement in students' performance.

Percentage Scores	N	Correlation	Sig.
Pre-Assessment & Summative Assessment	46	.628	.000

Table 2. Paired Samples Correlation

In addition, as displayed in Table 2, there is a significantly strong positive correlation between the pre-assessment and the summative assessment percentage scores ($r = 0.628$) implying that students with higher pre-assessment tend to achieve more in their summative tests. This result is attributable to students' compliance with the online tasks prior to coming to class.

Based on Table 3, the negative mean difference between the pre-assessment and summative assessment scores signifies students performed better in the summative assessment than in the pre-assessment. The paired sample t-test revealed that this mean difference is significant (*p-value*=0.0002 and t=-3.982) at a 0.05 significant level. It shows that the use of the tiered worksheets was effective in increasing student achievement scores.

Paired Differences							
			95% Confidence Interval of the Difference				
Mean	Std. Deviation	Std. Error Mean	Lower	Upper	t	df	Sig. (2-tailed)
-5.07609	8.64611	1.27480	-7.64366	-2.50851	-3.982	45	.000

Table 3. Paired Samples Test

In order to check if there is a return on investment from this painstaking research process, we also computed the effect size to determine the impact of the intervention. The computed *Cohen's D* is 0.587 or approximately equal to 0.6, indicating a large effect size (Kneer, 2017). Thus, this further confirms the effectiveness of differentiated instruction, specifically the use of tiered worksheets as an intervention to boost student engagement and mathematics achievement.

DISCUSSION AND CONCLUSIONS

The results showed that there were significant changes in terms of the mastery levels of the students based on their pre-assessment and their tiered worksheet performance. Although some of the students were initially classified in lower mastery levels, most of them were able to move up to higher mastery levels in the lessons indicating that the options provided in the online class instruction and the tiered worksheets effectively addressed students' cognitive needs. Since the tiered worksheets were aligned with the online mathematics videos and reading materials, students were at ease and confident to answer the items in the tiered worksheets. Observations and interviews with students revealed that they have different learning preferences. Multiple options and opportunities for individual learning preferences were provided: online videos, online resource materials, and the mini-lessons in the worksheets. Students who missed doing their assigned online tasks were given a second chance to watch the videos or read online materials in the class to catch up. Even so, students realized the value of compliance with assigned tasks before the in-person class, that coming to class unprepared may give them limited time to move up or progress in learning due to the bounded contact time. Throughout the intervention, the number of students who were regarded in higher mastery levels increased, proving that they were intensely engaged in the online class instruction to get high scores in their pre-assessment. Likewise, the scaffolding part of the tiered worksheet (mini-lessons) also helped the students progress, especially those with lower mastery levels.

Receiving good grades can serve as an extrinsic motivation in Mathematics. However, it is also essential to balance external motivation with intrinsic motivation for a deeper understanding of concepts. Cook and Artino (2016) argued that individuals get smarter by constantly studying and practicing. In addition, even learners with low confidence and motivation in their existing abilities will choose challenging and thought-provoking tasks if they see that their "effort leads to mastery" (Cook & Artino, 2016). In this study, the researchers believe that considering students' preferences and adapting teaching methods based on their competency levels encourages and fosters an appreciation for learning mathematics. Hence, by utilizing tiered worksheets, students can experience the richness of learning mathematical concepts more deeply through problem-solving and strengthening their conceptual understanding.

The use of tiered worksheets can also be applied to lessons that require deeper and creative thinking. Nevertheless, the researchers included thought-provoking and problem-solving items throughout the intervention, specifically in Part III of each tiered worksheet. The intervention was also used to strengthen students' conceptual understanding and procedural fluency regarding quadratic equations.

Research says that when teachers respond to students' needs by readiness, like scaffolding and tiering, differentiated instruction is indeed effective (Abbati, 2012). The results of this study further validate this claim. Moreover, when students do the tasks in the class with immediate feedback from teachers, students become more focused, knowing that a more knowledgeable other is overseeing their activity and can readily provide feedback to ensure progress (National Research Council, 2004). Students are encouraged to perform well in class if the learning tasks are just right on their mastery levels. Despite learning differences especially in cognitive levels, tiered worksheets help students improve their acquisition of mathematics skills because this encourages them to have a deep understanding of mathematical concepts by going through the process from the lower level to a higher-level worksheet. Hence, alongside implementing the flipped classroom, tiered worksheets increase student participation and performance since the tasks are tailored to their mastery levels.

The pre-assessment results showing more students at a High Level of Mastery than those in the lower levels across lessons prior to tiered worksheet class activities show that the online materials have helped students learn. The significant positive correlation between students' pre-assessment and summative assessment percentage scores underscores the flipped classroom's online asynchronous tasks provision or opportunity for students to learn independently and the importance of coming to class prepared. Thus, class time is maximized. The significant difference in students' performance in pre-assessment and tiered worksheets and the computed effect size revealed the effective use of tiered worksheets. Thus, these results indicate that differentiated instruction in the flipped classroom context is effectively addresses students' diversity, specifically regarding learning interest and student readiness, in boosting student engagement and performance in the identified lessons on quadratic equations.

In this study, we tried to address the flipped classroom limitation, the students' non-compliance (He et al., 2016), by using differentiated instruction mainly by administering a pre-assessment and tiered worksheets. The paramount concern of this study is to increase student engagement and mathematics achievement by incorporating differentiated instruction strategies in the context of flipped classrooms, specifically the utilization of online class instruction. Given that students are diverse in terms of their learning preferences, learning interests, and learning readiness, the researcher addressed it by providing options to the students in the online class instruction and administering the pre-assessment for each assigned lesson to gauge student's understanding which they acquired from the online class instruction. Moreover, the pre-assessment results helped the teacher-researcher correctly identify students' mastery levels by giving their appropriate tiered worksheets commensurate to their individual readiness.

The options provided in the online class instruction effectively encouraged and engaged students to study in advance so that they could perform better in their pre-assessment. Since the initial mastery level of the students greatly depends on their score from the pre-assessment, they were encouraged and intrinsically motivated to participate in the online class instruction. The idea of having the pre-assessment excites the students because this determines whether they would land in higher mastery levels, and it was revealed that the intervention is effective in enhancing student engagement in learning. However, it was also shown that some students had inconsistent performance with regarding their initial and final mastery levels. This inconsistent performance can be attributed to the lesson's difficulty or the habitual absenteeism of some students during the administration of the pre-assessment. In general, the administration of the pre-assessment addressed the gap of the flipped classroom model since pre-assessment was used to determine whether students watched the online mathematics videos and whether they learned something from the online class instruction.

This study showed that the online class instruction maximized the time during the face-to-face instruction since the teacher only gave supplementary activities like the pre-assessment and the tiered worksheets, which strengthened the knowledge obtained by the students in their off-class engagement. The tiered worksheets were also effective in increasing student's ability in terms of critical thinking and problem-solving skills. In the study's experience, there was no idle or unused time, even for the high achievers. All students in the in-person classes were observed to spend the entire time answering their respective worksheets. The researchers noticed students who reached the highest-tiered worksheet double-checked their work and solutions. Additionally, the worksheets are challenging since the researchers ensured that the items specifically under the Tier 3 worksheets are thought-provoking and complex. For BM and AM students, since the items contained in the tiered worksheets were just right on their cognitive levels, it triggered them to be more confident in answering mathematical problems deemed challenging. Based on their pre-assessment scores, the tiered worksheets motivated students to be prepared to land at a higher mastery level. Further, the scaffolding component of the worksheets also increased their chances of getting the correct answers in the worksheet, thus helping them maximize their performance to move up to higher mastery levels.

Regarding student achievement, student absenteeism is still a perennial problem in public schools, and this immensely affected their performance in answering the tiered worksheets. Nevertheless, the intervention successfully and effectively increased students' performance for those who were at least present. Moreover, the lesson's difficulty affected whether students could have two or more movements. Hence, it was essential for the teacher to let the students have advanced readings at home since the contact hours in the face-to-face instruction were not enough to cover the topics provided the jampacked nature of the curriculum. Therefore, by looking at the data presented in this study, students must not rely exclusively on the discussion of the teacher if they want to master the lesson's contents. Considering the generation of the students, employing the responsible use of technology helped students achieve optimal learning. However, since many schools, especially public schools, cannot afford to establish their infrastructures, giving assignments and additional readings in a Facebook group and handheld devices can help students realize the flipped classroom and blended learning goals.

The researchers believe that the intervention is feasible. Nonetheless, factors such as the teacher's resources, pedagogical skills, and the specific classroom environment, including student-teacher ratio, classroom management, and, most importantly, time constraints, should be considered when planning differentiated instruction (Abbati, 2012). The intervention used in the study is challenging. However, it is doable, provided that teachers strategically group students based on the results of pre-assessments, provide well-guided instructions, and use technology effectively to assist them in differentiation. The benefits of differentiated instruction to students in this study outweighed the expended efforts. Nevertheless, we recommend varying strategies to foster student achievement and engagement, such as MyOpenMath, rotation station learning, and adaptive assessments for differentiating instruction (Benjamin, 2014; Gregory & Chapman, 2013). Nevertheless, the findings of this study have shown promise for students' mastery of learning.

Since studies concerning the effectiveness of Differentiated Instruction alongside the utilization of the flipped classroom model are scarce, this study serves as a springboard to encourage educators to adopt this teaching approach in dealing with student diversity. Considering that not all students are the same and differ in learning styles, preferences, interests, and readiness, the intervention carried out by the teacher-researcher may accommodate those needs, specifically focusing on student interest and readiness. In general, the intervention contributed significantly to the increase in student engagement both off-class and in-class, which resulted in an immense improvement in their achievement scores.

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APPENDICES

Appendix A Sample List of Online Class Instructions with Guide Questions and Pre-Assessment (Solving Quadratic Equations by Completing the Square)

LESSON: SOLVING QUADRATIC EQUATIONS BY COMPLETING THE SQUARE

A. ONLINE VIDEO INSTRUCTION:

1. https://www.youtube.com/watch?v=K_FahMr7V7k&list=PLp3WnNoPhT3KH4MUzLGWzsyR4Q33WrUj9&index=5 (URL)
   TITLE: Quadratic Equations and Inequalities - Solving Quadratic Equations by Completing the Square
   DURATION: 14 mins and 37 secs
   UPLOADER: Nick - Gurodawako – Agriam
   OTHER INFORMATION: The content of the video is based on Grade 9- Unit 1 Mathematics Module.

2. https://www.youtube.com/watch?v=QjME8FS2TtY (URL)
   TITLE: QE06 Solving Quadratic Equations - Completing the Square Part 1
   DURATION: 10 mins and 33 secs
   UPLOADER: Sipnayan
   OTHER INFORMATION: The content of the video is based on Grade 9- Unit 1 Mathematics Module and the uploader used Filipino language as his medium of instruction in the video.

3. https://www.youtube.com/watch?v=R5k96cNJXvE (URL)
   TITLE: QE07 Solving Quadratic Equations - Completing the Square Part 2
   DURATION: 6 mins and 26 secs
   UPLOADER: Sipnayan
   OTHER INFORMATION: The content of the video is based on Grade 9- Unit 1 Mathematics Module and the uploader used Filipino language as his medium of instruction in the video.

4. https://www.youtube.com/watch?v=oPE-THYrYAM (URL)
   TITLE: QE08 Solving Quadratic Equations - Completing the Square Part 3
   DURATION: 5 mins and 34 secs
   UPLOADER: Sipnayan
   OTHER INFORMATION: The content of the video is based on Grade 9- Unit 1 Mathematics Module and the uploader used Filipino language as his medium of instruction in the video.

5. https://www.youtube.com/watch?v=bNQY0z76M5A (URL)
   TITLE: Solving quadratic equations by completing the square | Algebra II | Khan Academy
   DURATION: 14 mins and 5 secs
   UPLOADER: Khan Academy

6. https://www.youtube.com/watch?v=prx_Bf2hakw (URL)
   TITLE: How to Solve by Completing the Square (NancyPi)
   DURATION: 17 mins and 32 secs
   UPLOADER: NancyPi

7. https://www.youtube.com/watch?v=C206SNAXDGE (URL)
   TITLE: Completing the Square Method and Solving Quadratic Equations - Algebra 2
   DURATION: 31 mins and 54 secs
   UPLOADER: The Organic Chemistry Tutor

B. MATHEMATICS LEARNING WEBSITE:

1. https://www.khanacademy.org/math/algebra/quadratics/solving-quadratics-by-completing-the-square/a/solving-quadratic-equations-by-completing-the-square?modal=1
2. https://www.khanacademy.org/math/algebra/quadratics/solving-quadratics-by-completing-the-square/v/ex1-completing-the-square?modal=1
3. https://www.khanacademy.org/math/algebra/quadratics/solving-quadratics-by-completing-the-square/v/rewriting-quadratics-as-perfect-squares?modal=1
4. https://www.khanacademy.org/math/algebra/quadratics/solving-quadratics-by-completing-the-square/v/solving-quadratics-by-completing-the-square?modal=1
5. https://www.khanacademy.org/math/algebra/quadratics/solving-quadratics-by-completing-the-square/v/completing-the-square-to-solve-quadratic-equations?modal=1
6. https://www.khanacademy.org/math/algebra/quadratics/solving-quadratics-by-completing-the-square/v/ex2-completing-the-square?modal=1
7. https://www.khanacademy.org/math/algebra/quadratics/solving-quadratics-by-completing-the-square/a/completing-the-square-review?modal=1

C. OTHER ONLINE RESOURCE MATERIALS:

1. https://www.mathsisfun.com/algebra/completing-square.html
2. https://www.purplemath.com/modules/solvquad3.htm
3. https://www.purplemath.com/modules/sqrquad.htm
4. https://www.brainfuse.com/jsp/alc/resource.jsp?s=gre&c=35261&cc=108824

Sample Guide Questions

GUIDE QUESTIONS:
SOLVING QUADRATIC EQUATIONS BY COMPLETING THE SQUARE

NAME: DATE:

GRADE AND SECTION: TEACHER:

1. How do you describe a perfect square trinomial?
2. How are you going to express a perfect square trinomial as the square of a binomial?
3. Solve the following quadratic equations by completing the square.
   a) $x^2 + 4x = 5$
   b) $-18 + 3x + x^2 = 0$
   c) $-2x^2 = 2 - 7x$

Sample Pre-Assessment

PRE-ASSESSMENT

LESSON: Solving Quadratic Equations by Completing the Square

NAME: DATE:
SECTION: SCORE:

Directions: Answer the following items correctly. Show your solutions for those items that need to be solved. Put a box in your final answer.

Part I: For nos. 1-3, determine a number that must be added to make each of the following expression a perfect square trinomial. Express it as a square of a binomial. An example below is provided to you. (1 point each).

Example:

$x^2 + 2x +$ ____

Answer: 1
$\therefore \mathbf{x^2 + 2x + 1 = (x + 1)^2}$

1. $x^2 + 4x +$ ____
2. $r^2 + 20r +$ ____
3. $m^2 + 24m +$ ____

Part II: For nos. 4-6, solve the following quadratic equation by completing the square (2 points each).

4. $x^2 - 2x = 3$
5. $-41 - 6x + x^2 = 0$
6. $3x^2 + 5x - 2 = 0$

Part III: For nos. 7-8, answer the following questions (3 points each).

7. Peter wants to use completing the square in solving the quadratic equation, $9x^2 - 16 = 0$. Can he use it in finding the solutions of the equation? Explain why or why not.
8. If you are to choose between completing the square and factoring in finding the solution of the quadratic equation, $x^2 + 9x + 18 = 0$, which would you choose? Explain and solve the equation using your preferred method.

ADDITIONAL QUESTION:

Did you watch the off-loaded online video links?

- ☐ If your answer is YES, were you able to follow or understand the contents of the video/s?
- ☐ If your answer is NO, state your reason for not watching the online videos.

Appendix B Sample Tiered Worksheets

TIERED MATHEMATICS LEARNING WORKSHEET 1
(BEGINNING MASTERY)

LESSON: Solving Quadratic Equations by Completing the Square

NAME: **DATE:**
SECTION: **SCORE:**
RECALL:

Hi! You are answering **TIERED WORKSHEET 1!** In this section, another way of **SOLVING QUADRATIC EQUATIONS** will be introduced and that is **BY COMPLETING THE SQUARE.**

To solve quadratic equation,

$ax^2 + bx + c = 0$ **by COMPLETING THE SQUARE**, the following procedure can be followed:

1) Divide both sides of the equation by a then simplify.
2) Write the equation such that the terms with variables are on the left side of the equation and the constant term is on the right side.
3) Add the square of one-half of the coefficient of x on both sides of the resulting equation. The left side of the equation becomes a perfect square trinomial; $c = \left(\frac{b}{2}\right)^2$.
4) Express the perfect square trinomial on the left side of the equation as a square of a binomial.
5) Solve the resulting quadratic equation by extracting the square root.
6) Solve the resulting linear equation.

EXAMPLE:
Solve $x^2 + 8x - 9 = 0$ by completing the square.

Step 1: The numerical coefficient of x is 1 so, there's no need to divide 1 to both sides of the equation.

$$x^2 + 8x - 9 = 0$$

Step 2: Add 9 to both sides of the equation then simplify.

$$x^2 + 8x - 9 + 9 = 0 + 9$$
$$x^2 + 8x = 9$$

Step 3: Add the square of one-half of 8 to both sides of the equation.

$$c = \left(\frac{b}{2}\right)^2 \rightarrow \left(\frac{8}{2}\right)^2 \rightarrow (4)^2 \rightarrow 16$$
$$\therefore x^2 + 8x + 16 = 9 + 16$$
$$x^2 + 8x + 16 = 25$$

Step 4: Express $x^2 + 8x + 16$ as a square of a binomial.

$$x^2 + 8x + 16 = 25$$
$$(x+4)^2 = 25$$

Step 5: Solve $(x+4)^2 = 25$ by extracting the square root.

$$(x+4)^2 = 25$$
$$\sqrt{(x+4)^2} = \pm\sqrt{25}$$
$$x + 4 = \pm 5$$

Step 6: Solve the resulting linear equation.

$$x + 4 = 5 \text{ and } x + 4 = -5$$
$$x = -4 + 5 \text{ and } x = -4 - 5$$
$$x_1 = 1 \quad \& \quad x_2 = -9$$

SOLUTION SET: $\{-9, 1\}$

Directions: Answer the following items correctly. Show your solutions for those items that need to be solved. Put a box in your final answer.

Part I: For nos. 1-3, determine a number that must be added to make each of the following expression a perfect square trinomial. Express it as a square of a binomial (1 point each).

1. $s^2 - 16s +$ _____
2. $x^2 - 20x +$ _____
3. $y^2 + 30y +$ _____

Part II: For nos. 4-6, solve the following quadratic equation by completing the square (2 points each).

4. $x^2 + 4x = 12$
5. $x^2 - 8 = 2x$
6. $-16 - 6x + x^2 = 0$

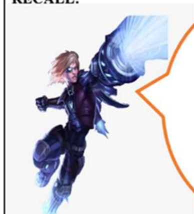
Part III: For nos. 7-8, answer the following question (3 points each).

7. Can Paul use the method of completing the square in finding the solutions of the quadratic equation, $2x^2 - 18 = 0$? Justify your answer.
8. Do you agree that any quadratic equation can be solved by completing the square? Explain your answer.

TIERED MATHEMATICS LEARNING WORKSHEET 2
(APPROACHING MASTERY)

LESSON: Solving Quadratic Equations by Completing the Square

NAME: **DATE:**
SECTION: **SCORE:**
RECALL:



Hi! You are answering **TIERED WORKSHEET 2!** In this section, another way of **SOLVING QUADRATIC EQUATIONS** will be introduced and that is **BY COMPLETING THE SQUARE.**

To solve quadratic equation,

$ax^2 + bx + c = 0$ **by COMPLETING THE SQUARE**, the following procedure can be followed:

1) Divide both sides of the equation by a then simplify.
2) Write the equation such that the terms with variables are on the left side of the equation and the constant term is on the right side.
3) Add the square of one-half of the coefficient of x on both sides of the resulting equation. The left side of the equation becomes a perfect square trinomial; $c = \left(\frac{b}{2}\right)^2$.
4) Express the perfect square trinomial on the left side of the equation as a square of a binomial.
5) Solve the resulting quadratic equation by extracting the square root.
6) Solve the resulting linear equation.

EXAMPLE:
Solve $x^2 - 5x + 2 = 0$ by completing the square.

Step 1: The numerical coefficient of x is 1 so, there's no need to divide 1 to both sides of the equation.

$$x^2 - 5x + 2 = 0$$

Step 2: Add -2 to both sides of the equation then simplify.

$$x^2 - 5x + 2 - 2 = 0 - 2$$
$$x^2 - 5x = -2$$

Step 3: Add the square of one-half of -5 to both sides of the equation.

$$c = \left(\frac{b}{2}\right)^2 \rightarrow \left(\frac{-5}{2}\right)^2 \rightarrow \frac{25}{4}$$
$$\therefore x^2 - 5x + \frac{25}{4} = -2 + \frac{25}{4}$$
$$x^2 - 5x + \frac{25}{4} = \frac{-8+25}{4}$$
$$x^2 - 5x + \frac{25}{4} = \frac{17}{4}$$

Step 4: Express $x^2 - 5x + \frac{25}{4}$ as a square of a binomial.

$$x^2 - 5x + \frac{25}{4} = \frac{17}{4}$$
$$\left(x - \frac{5}{2}\right)^2 = \frac{17}{4}$$

Step 5: Solve $\left(x - \frac{5}{2}\right)^2 = \frac{17}{4}$ by extracting the square root.

$$\left(x - \frac{5}{2}\right)^2 = \frac{17}{4}$$
$$\sqrt{\left(x - \frac{5}{2}\right)^2} = \pm\sqrt{\frac{17}{4}}$$
$$x - \frac{5}{2} = \pm\frac{\sqrt{17}}{2}$$

Step 6: Solve the resulting linear equation.

$$x - \frac{5}{2} = \frac{\sqrt{17}}{2} \text{ and } x - \frac{5}{2} = -\frac{\sqrt{17}}{2}$$
$$x = \frac{5}{2} + \frac{\sqrt{17}}{2} \text{ and } x = \frac{5}{2} - \frac{\sqrt{17}}{2}$$
$$x_1 = \frac{5+\sqrt{17}}{2} \quad \& \quad x_2 = \frac{5-\sqrt{17}}{2}$$

SOLUTION SET: $\{\frac{5-\sqrt{17}}{2}, \frac{5+\sqrt{17}}{2}\}$

Directions: Answer the following items correctly. Show your solutions for those items that need to be solved. Put a box in your final answer.

Part I: For nos. 1-3, determine a number that must be added to make each of the following expression a perfect square trinomial. Express it as a square of a binomial (1 point each).

1. $s^2 - 7s +$ ____
2. $x^2 + 11x +$ ____
3. $y^2 - 15y +$ ____

Part II: For nos. 4-6, solve the following quadratic equations by completing the square (2 points each).

4. $x^2 - 3x - 3 = 0$
5. $x^2 - 5x = 6$
6. $x^2 - 26 = 11x$

Part III: For nos. 7-8, answer the following question (3 points each).

7. If you are to choose between completing the square and factoring in finding the solution of the quadratic equation, $x^2 + 11x + 30 = 0$, which would you choose? Explain and solve the equation using your preferred method.
8. Luke tried to determine a number that must be added to make $x^2 - \frac{4}{5}x$ a perfect square trinomial. The parts of his solutions are shown below.

$$x^2 - \frac{4}{5}x \longrightarrow x^2 - \frac{4}{5}x + \_\_\_\_\_$$
$$c = \left(\frac{b}{2a}\right)^2 \longrightarrow \left(\frac{\frac{-4}{5}}{2(1)}\right)^2$$
$$\left(-\frac{4}{5(2)}\right)^2 \longrightarrow \left(-\frac{4}{10}\right)^2 \longrightarrow \left(-\frac{2}{5}\right)^2 \longrightarrow \frac{4}{25}$$
$$x^2 - \frac{4}{5}x + \frac{4}{25} \longrightarrow \left(x + \frac{2}{5}\right)^2$$

Do you think Luke arrived at the correct answer? Explain your answer.

TIERED MATHEMATICS LEARNING WORKSHEET 3
(HIGH LEVEL OF MASTERY)

LESSON: Solving Quadratic Equations by Completing the Square

NAME: **DATE:**
SECTION: **SCORE:**

RECALL:

Hi! You are answering **TIERED WORKSHEET 3**! In this section, another way of **SOLVING QUADRATIC EQUATIONS** will be introduced and that is **BY COMPLETING THE SQUARE.**

To **solve quadratic equation,**

$ax^2 + bx + c = 0$ **by COMPLETING THE SQUARE**, the following procedure can be followed:

1) Divide both sides of the equation by *a* then simplify.
2) Write the equation such that the terms with variables are on the left side of the equation and the constant term is on the right side.
3) Add the square of one-half of the coefficient of x on both sides of the resulting equation. The left side of the equation becomes a perfect square trinomial; $c = \left(\frac{b}{2}\right)^2$.
4) Express the perfect square trinomial on the left side of the equation as a square of a binomial.
5) Solve the resulting quadratic equation by extracting the square root.
6) Solve the resulting linear equation.

EXAMPLE:
Solve $2x^2 + 11x + 15 = 0$ by completing the square.

Step 1: Divide both sides of the equation by 2 then simplify.

$$\frac{2x^2+11x+15}{2} = \frac{0}{2}$$
$$x^2 + \frac{11}{2}x + \frac{15}{2} = 0$$

Step 2: Add $-\frac{15}{2}$ to both sides of the equation then simplify.

$$x^2 + \frac{11}{2}x + \frac{15}{2} - \frac{15}{2} = 0 - \frac{15}{2}$$
$$x^2 + \frac{11}{2}x = -\frac{15}{2}$$

Step 3: Add the square of one-half of $\frac{11}{2}$ to both sides of the equation.

$$c = \left(\frac{b}{2}\right)^2 \to \left(\frac{\frac{11}{2}}{2}\right)^2 \to \left(\left(\frac{11}{2}\right)\left(\frac{1}{2}\right)\right)^2 \to \left(\frac{11}{4}\right)^2 \to \frac{121}{16}$$
$$\therefore x^2 + \frac{11}{2}x + \frac{121}{16} = -\frac{15}{2} + \frac{121}{16}$$
$$x^2 + \frac{11}{2}x + \frac{121}{16} = \frac{-120+121}{16}$$
$$x^2 + \frac{11}{2}x + \frac{121}{16} = \frac{1}{16}$$

Step 4: Express $x^2 + \frac{11}{2}x + \frac{121}{16}$ as a square of a binomial.

$$x^2 + \frac{11}{2}x + \frac{121}{16} = \frac{1}{16}$$
$$\left(x + \frac{11}{4}\right)^2 = \frac{1}{16}$$

Step 5: Solve $\left(x + \frac{11}{4}\right)^2 = \frac{1}{16}$ by extracting the square root.

$$\left(x + \frac{11}{4}\right)^2 = \frac{1}{16}$$
$$\sqrt{\left(x + \frac{11}{4}\right)^2} = \pm\sqrt{\frac{1}{16}}$$
$$x + \frac{11}{4} = \pm\frac{1}{4}$$

Step 6: Solve the resulting linear equation.

$$x + \frac{11}{4} = \frac{1}{4} \text{ and } x + \frac{11}{4} = -\frac{1}{4}$$
$$x = -\frac{11}{4} + \frac{1}{4} \text{ and } x = -\frac{11}{4} - \frac{1}{4}$$
$$x = -\frac{11+1}{4} \text{ and } x = -\frac{11-1}{4}$$
$$x_1 = \frac{-10}{4} \ \& \ x_2 = \frac{-12}{4}$$
$$x_1 = -\frac{5}{2} \ \& \ x_2 = -3$$

SOLUTION SET: $\{-3, -\frac{5}{2}\}$

Directions: Answer the following items correctly. Show your solutions for those items that need to be solved. Put a box in your final answer.

Part I: For nos. 1-3, determine a number that must be added to make each of the following expression a perfect square trinomial. Express it as a square of a binomial (1 point each).

1. $w^2 - \frac{2}{3}w +$ ____
2. $r^2 + \frac{3}{4}r +$ ____
3. $x^2 - \frac{5}{2}x +$ ____

Part II: For nos. 4-6, solve the following quadratic equations by completing the square. (2 points each).

4. $x^2 + 7x = \frac{51}{4}$
5. $4x^2 - 20x = 11$
6. $\frac{2x+3}{3x+2} = \frac{2x-7}{x+2}$

Part III: For nos. 7-8, formulate a quadratic equation for each word problem and solve for its solution using completing the square (3 points).

7. If a number is added to its square, the result is 42. Find the number.
8. The sum of the squares of two consecutive integers is 85. What are the numbers?