Teaching the Mathematical Optimization Concept to First-year Engineering Students Using a Practical Problem

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Abstract: Optimization is a vital mathematical concept widely applied in various engineering fields. However, teaching this subject to engineering students often involves abstract materials, making it challenging for them to grasp. To address this issue, a group of five mathematics lecturers collaborated on designing a practical lesson on optimization for first-year engineering students. The aim was to present the concept through a real-world problem, making it more accessible and applicable. The lesson focused on the design of water channels used for transferring water from sources to farms, showcasing how engineers can create optimal water channels with different geometrical cross-sections for agricultural purposes. This approach allowed students to see the direct application of mathematics in the real-world. The lesson was conducted as a three-hour workshop and attended by 38 volunteer first-year engineering students at a Malaysian university. Data were collected through observations and interviews and analyzed using the thematic analysis method. Feedback from both the lecturers and the participating students indicated that this teaching method significantly enhanced their conceptual understanding of mathematical optimization. Moreover, it fostered the development of problem-solving skills in real-world scenarios, bridging the gap between engineering and mathematics.

Keywords: Optimization, Optimal water channel, Geometrical cross-section, Mathematics

1. INTRODUCTION

Mathematics holds significant importance in various engineering fields, and engineering students must possess the necessary skills to solve mathematical problems effectively. In the realm of mathematics, tasks that present challenges and are unfamiliar to students, with unknown methods of solving, are referred to as mathematical problems (Xenofontos & Andrews, 2014). Otherwise, if the tasks are straightforward and already known methods can be applied to solve them, they are simply considered as mathematical exercises (Gholami, 2021). In engineering fields, any real-world problem solved using mathematical concepts is considered a practical problem (Gholami & Sathar, 2021; Savizi, 2007). In other words, practical problems exemplify the real-life application of mathematical concepts. For instance, the given problem provided below is a practical problem (Fatmanissa et al., 2019).

> *A rectangular thin metal sheet measures 140 cm in length and 120 cm in width. Its corners are cut in the form of identical squares. The remaining sheet is then folded to form a box without a top. Determine the maximum volume of the created box.*

Therefore, once engineering students fully comprehend the practical problem, they should aim to find an optimal solution for the given scenario. By seeking optimal solutions to real-world practical problems, they prepare themselves for addressing similar challenges in the future. Engaging in discussions about small real-world projects within the classroom setting helps engineering students develop problem-solving skills that are directly applicable to real-life situations. This preparation equips them to tackle human life's problems effectively in their future careers. In engineering fields, lecturers often resort to teaching mathematical concepts using abstract materials. Unfortunately, this approach sometimes fails to establish a clear connection between real-world problems in engineering and the corresponding mathematical concepts. According to a study conducted by Harris, engineering students who encounter challenges with mathematics tend to face difficulties in understanding the connection between engineering and mathematics (Harris et al., 2015). The main aim in learning engineering mathematics is not only to practice mathematical skills but also to develop mathematical thinking for solving real-world problems (Szabo et al., 2020). Lecturers should enhance students' ability to solve practical engineering problems by emphasizing the importance of mathematical modeling, a powerful tool for addressing real-world challenges in engineering (Pepin et al., 2021). Hence, incorporating practical problems and discussing their optimal solutions in engineering classrooms is essential and unavoidable.

Optimization theory finds wide application in engineering fields, providing solutions to numerous practical problems, as engineers can employ various methods to find mathematical optimization functions for solving diverse challenges (Tsai et al., 2014). Engineering optimization forms a crucial aspect of mathematical modeling, where optimization techniques are employed to tackle specific real-world challenges in engineering disciplines (Vagaska et al., 2022).The term "optimization" shares its root with "optimal," implying the quest for the best possible outcome. Mathematical optimization, a branch of applied mathematics, finds applications in various fields like mechanical engineering, marketing, manufacturing, and economics. To enhance students' proficiency in solving real-world optimization problems, calculus serves as a fundamental course (Retamoso, 2022). However, optimization theory is extensively utilized in engineering disciplines to address a wide range of practical challenges.

Engineering optimization involves the process of optimizing (maximizing or minimizing) a modeled mathematical function, which serves as the objective function, considering a set of constraints and input parameters (Alshqaq et al., 2022). The primary objective of engineering optimization is to identify the most suitable and feasible solution for a given practical problem, taking into account the problem's assumptions, in order to save both time and money (Dastan et al., 2022). The present study aims to introduce the concept of optimization to first-year engineering students through a practical problem related to the construction of open metal or concrete channels for agricultural purposes. This problem will serve as an illustrative example to enhance their understanding of optimization principles and their applications in real-world scenarios.

1.1.Theoretical framework

Utilizing constructivist principles in mathematics teaching for engineering students presents a significant opportunity for educators to delve deeply into lesson planning, emphasizing real-world problem-solving. According to Hoover (1996), adopting a social constructivist approach to teaching mathematics enables students to actively construct mathematical knowledge rather than passively receiving it from lecturers. As Hoover suggests, educators are encouraged to involve students in appropriate mathematical problem-solving activities, enhancing their teaching methods. Social constructivism further advocates for the collaborative nature of mathematical knowledge, emphasizing its improvement through interactions and sharing between lecturers and students (Gergen, 1995). In essence, mathematical skills are fostered through group interactions involving negotiations, reflections, discourse, and explanations. Therefore, lessons prepared collaboratively by a group of lecturers are supported by a robust theoretical foundation.

2. METHODOLOGY

In this study, five lecturers, including the first author, possessing a minimum of 9 years of experience in teaching mathematics to engineering students, participated in a collaborative effort to develop a lesson on the topic of optimization concepts. Drawing from their collective expertise and real-world applications of mathematics, they engaged in three extensive discussion sessions to plan and design the lesson. Ultimately, they chose a practical problem from the agricultural sector as the focal point for the lesson, despite initially considering various other practical problems during the first meeting. To ensure the quality and validity of the prepared lesson, it underwent content analysis and validation by two external professors not involved in the research. Upon incorporating their feedback, the refined lesson was conducted as a scientific workshop for 38 enthusiastic first-year engineering students from a public university in Malaysia.

The method of transferring the optimization concept to the students involved engaging them in group-based problem-solving activities centered on a practical problem from the agricultural sector. The lecturer facilitated the workshop using a student-centered approach, where the emphasis was on active participation and problem-solving. Students were encouraged to work collaboratively in groups, and the lecturer prompted some groups to share their methods and solutions with the rest of the class. This approach fostered a dynamic and interactive learning environment, allowing students to learn from each other's perspectives and approaches. Additionally, the lecturer made continuous observations and gathered feedback from the students during the workshop. This feedback was then used to enhance the quality of the lesson, ensuring that the content and teaching methods were better tailored to meet the needs and understanding of the students. Through this hands-on and participatory teaching approach, the optimization concept was effectively transferred, enabling students to grasp the subject matter more comprehensively.

Following the conclusion of the workshop, 38 students (referred to as $s_1, s_2, \ldots, s_{38}$ were interviewed, and their responses were digitally recorded and transcribed verbatim for analysis and reporting purposes. Thematic analysis was utilized to analyze the gathered data. Furthermore, all participants provided informed consent for their involvement in the research. The interview questions posed to the participants are as follows:

Are the contents of this lesson interesting to you?
How do you evaluate the relationship between this lesson and the real-world?
How do you evaluate the relationship between this lesson and engineering?
How do you evaluate the effect of this lesson in improving your ability to solve problems in the real world?
How does this lesson affect your understanding of solving real-world problems?
Do you have any other opinions about this educational program?
Through these interviews and thematic analysis, the study aimed to gain valuable insights into the students' perspectives on the lesson's content, its real-world applicability, its relevance to engineering, and its impact on their problem-solving abilities and understanding of real-world challenges.

2.1.Introducing the practical problem

Water wastage during the transfer of water from the source to farms through traditional channels and rivers is a significant challenge faced by the agricultural sector (Chojnacka et al., 2020). Efficient water management in agriculture, particularly in diverse climatic conditions, necessitates the optimization of water usage through scientific canalization (Vastila et al., 2021). To mitigate water losses, the adoption of metal and concrete channels is recommended by agricultural experts as a more effective alternative to traditional channels. As depicted in Figure 1 and Figure 2, the stark contrast between a traditional river and a concrete channel highlights the potential benefits of using modern canalization methods. The practical problem introduced to engineering students revolves around optimizing the design and implementation of metal or concrete channels for transferring water from sources to farms, aiming to enhance water efficiency and agricultural productivity. This practical problem is as follows:

Explain the step-by-step process that engineers can follow to design an optimal open water channel for agricultural use using a rectangular metal sheet. Additionally, discuss the significance of considering different geometrical shapes for the channel's cross-section and their impact on water volume pass.

Figure 1: Traditional River Utilized for Agricultural Purposes
Source: https://www.fwi.co.uk/business/payments-schemes/sfi-farmers-alerted-to-good-reason-clause-in-agreements

Figure 2: Concrete Channel with Trapezoidal Cross-Section for Agricultural Use
Source: [https://www.123rf.com/photo_56488343_irrigation-ditch-in-the-plain-of-the- river-esla-in-leon-province-spain.html](https://www.123rf.com/photo_56488343_irrigation-ditch-in-the-plain-of-the-river-esla-in-leon-province-spain.html)

Engineers aim to create optimal water channels that efficiently supply the required water to agricultural farms. Achieving this goal depends on various factors, including the amount of water needed for field irrigation, the distance between the water source and the farm, the channel materials, the geometric shape of the channel cross-section, and the cost of channel construction.

In this educational article, a group of lecturers focuses on teaching the concept of optimization to engineering students through a practical problem. They discuss the design of water channels with different geometrical cross-sections, aiming to maximize water volume pass. The classroom exercise involves constructing these channels to efficiently pass the maximum amount of water. Although the channels are designed using metal sheets as examples, the results can be applied to concrete channels as well. Figure 3 showcases the engineering process of curving a metal sheet to produce a water channel, while Figure 4 illustrates an open metal channel with a rectangular cross-section. Through this educational approach, future engineers gain valuable insights into channel optimization, preparing them for real-world challenges in agricultural water management and irrigation.

Figure 3: Fabrication Process of a Metal Channel
Source: https://www.shutterstock.com/image-photo/bending-sheet-metal-hydraulic-machine-factory-1771306607

Figure 4: Completed Metal Channel with Rectangular Cross-Section
Source: https://www.ebay.co.uk/itm/271175304196

The lecturers began this lesson by posing the question, "What are the common geometrical shapes of open water channels for agricultural use?" They encouraged student participation and gathered opinions on the possible geometrical shapes of such channels. Subsequently, the lecturers elucidated that the most prevalent geometric cross-sections of water channels for agricultural purposes are the triangular cross-section, rectangular cross-section, and trapezoidal cross-section, as depicted in Figure 5.

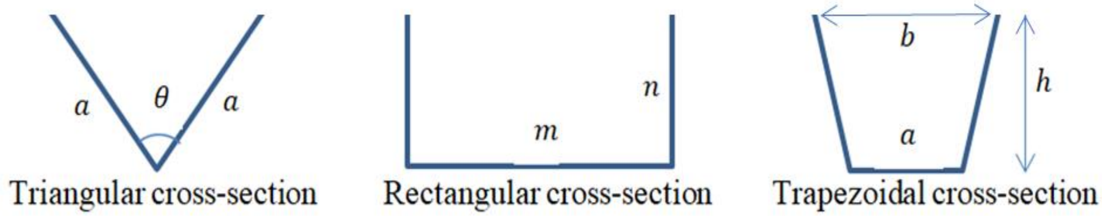

Figure 5: Common Geometrical Cross-Section Shapes for Water Channels

3. DISCUSSION ON SOLVIN THE PRACTICAL PROBLEM

In the classroom, the lecturers facilitated an interactive session where students had the opportunity to explore the design of optimal water channels with various geometrical cross-section shapes using a rectangular metal sheet with dimensions long "l" and wide "w" (Figure 6). Guided by the lecturers, students were encouraged to derive the optimal solutions for different cross-sectional shapes of water channels. Emphasizing a student-centered teaching approach, the lecturers actively discussed the optimally designed channels with the students, fostering a collaborative learning environment.

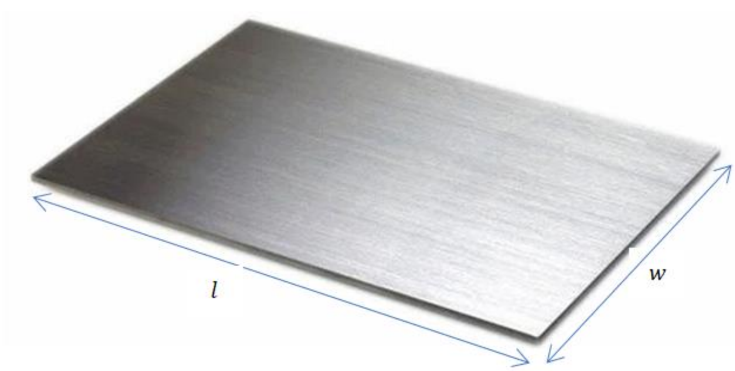

Figure 6: Rectangular Metal Sheet with Dimensions l and w

3.1.Designing an optimal water channel with triangular cross-section

Upon bending the metal sheet (Figure 6) from its wide form into a triangular channel with an angle θ, the cross-sectional area can be calculated as follows:

$$s = \frac{1}{2}\left(\frac{w}{2} \times \frac{w}{2}\right)\sin\theta \Rightarrow \mathrm{s}(\theta) = \frac{w^2}{8}\sin\theta \tag{1}$$

$$s'(\theta) = \frac{w^2}{8}\cos\theta = 0 \Rightarrow \cos\theta = 0 \Rightarrow \theta = 90. \tag{2}$$

Therefore, the maximum area value for the triangular cross-section of the constructed channel is as follows:

$$\mathrm{s} = \frac{w^2}{8}\sin\theta \Rightarrow \mathrm{s} = \frac{w^2}{8}\sin 90 \Rightarrow \mathrm{s} = \frac{w^2}{8}. \tag{3}$$

As a result, the maximum volume value of the triangular channel constructed using a rectangular metal sheet with dimensions l and w is as follows:

$$v = sl \Rightarrow v = \frac{w^2 l}{8}. \tag{4}$$

Additionally, some students arrived at the same result for this part using an alternative method, as shown below:

It is evident that for any angle θ, the inequality $-1 \leq \sin\theta \leq 1$ holds true. Hence, we can derive the inequality $\frac{-w^2}{8} \leq \frac{w^2}{8}\sin\theta \leq \frac{w^2}{8}$. Consequently, $\frac{-w^2}{8} \leq \mathrm{s}(\theta) \leq \frac{w^2}{8}$. Therefore, when $\sin\theta = 1$, the function $\mathrm{s}(\theta) = \frac{w^2}{8}\sin\theta$ attains its maximum value, which is $\frac{w^2}{8}$. This implies that the angle of the constructed channel must be 90 degrees.

3.2.Designing an optimal water channel with rectangular cross-section

In this scenario, the cross-section of the constructed water channel, utilizing a rectangular metal sheet with dimensions l and w, forms a rectangle with length m and width n as depicted in Figure 5. Hence, we can establish the relationships $2n + m = w$ and $s = mn$, where s represents the area function, and it is expressed as follows:

$$s(n) = n(w - 2n) = nw - 2n^2 \tag{5}$$

$$s'(n) = w - 4n = 0 \Rightarrow \mathrm{n} = \frac{w}{4} \Rightarrow \mathrm{m} = \mathrm{w} - 2\left(\frac{w}{4}\right) = \frac{w}{2}. \tag{6}$$

The maximum area value of the rectangular cross-section for the constructed water channel is as follows:

$$s = nm \Rightarrow \mathrm{s} = \left(\frac{w}{4}\right)\left(\frac{w}{2}\right) = \frac{w^2}{8}. \tag{7}$$

Therefore, the optimal volume value for the water channel constructed using the given metal sheet is as follows:

$$v = sl \Rightarrow v = \frac{lw^2}{8}. \quad (8)$$

3.3.Designing an optimal water channel with trapezoidal cross-section

Figure 7 illustrates the cross-section of a trapezoidal water channel, constructed using a rectangular metal sheet with dimensions l and w. The perimeter of the constructed open channel cross-section (p) can be expressed in terms of θ as follows:

$$p = a + 2(\frac{h}{\sin\theta}) \Rightarrow \mathrm{w} = a + \frac{2h}{\sin\theta}. \quad (9)$$

The area value of this trapezoidal cross-section is obtained as follows:

$$s = \frac{h}{2}(2a + 2(\frac{h}{\tan\theta})) \Rightarrow \mathrm{s} = \mathrm{h}\left(\mathrm{a} + \frac{h}{\tan\theta}\right). \quad (10)$$

Thus, we obtain the following:

$$a + \frac{h}{\tan\theta} = \frac{s}{h} \Rightarrow \mathrm{a} = \frac{s}{h} - \frac{h}{\tan\theta}. \quad (11)$$

After combining the perimeter formula and the area formula of this cross-section, we have the following:

$$w = \frac{s}{h} - \frac{h}{\tan\theta} + \frac{2h}{\sin\theta} \quad (12)$$

$$\Rightarrow \frac{dw}{dh} = \frac{-1}{h^2}s - \frac{1}{\tan\theta} + \frac{2}{\sin\theta} \quad (13)$$

$$\Rightarrow \frac{dw}{dh} = 0 \Rightarrow \frac{-1}{h^2}s - \frac{1}{\tan\theta} + \frac{2}{\sin\theta} = 0 \quad (14)$$

$$\Rightarrow \frac{dw}{dh} = \frac{-1}{h^2}\left(h(a + \frac{h}{\tan\theta})\right) - \frac{1}{\tan\theta} + \frac{2}{\sin\theta} = 0 \quad (15)$$

$$\Rightarrow \frac{-a}{h} - \frac{2}{\tan\theta} + \frac{2}{\sin\theta} = 0 \Rightarrow \frac{-2\cos\theta}{\sin\theta} + \frac{2}{\sin\theta} = \frac{a}{h} \quad (16)$$

$$\Rightarrow \frac{2(1-\cos\theta)}{\sin\theta} = \frac{a}{h} \Rightarrow h = \frac{a\sin\theta}{2(1-\cos\theta)}. \quad (17)$$

Therefore, the area formula for the trapezoidal cross-section of the water channel is as follows:

$$\mathrm{s} = h\left(\mathrm{a} + \frac{h}{\tan\theta}\right) \Rightarrow \mathrm{s} = \frac{a\sin\theta}{2(1-\cos\theta)}(a + \frac{\frac{a\sin\theta}{2(1-\cos\theta)}}{\frac{\sin\theta}{\cos\theta}}) \quad (18)$$

$$\Rightarrow s = \frac{a\sin\theta}{2(1-\cos\theta)}\left(a + \frac{a\cos\theta}{2(1-\cos\theta)}\right) \tag{19}$$

$$\Rightarrow s = \frac{a\sin\theta}{2(1-\cos\theta)}\left(\frac{2a(1-\cos\theta)+a\cos\theta}{2(1-\cos\theta)}\right) \tag{20}$$

$$\Rightarrow s = \frac{a^2\sin\theta(2-\cos\theta)}{4(1-\cos\theta)^2}. \tag{21}$$

In fact, the area formula is a function of two variables, as shown below:

$$s(a,\theta) = \frac{a^2\sin\theta(2-\cos\theta)}{4(1-\cos\theta)^2}. \tag{22}$$

The length of the metal sheet used in constructing the water channel with a trapezoidal cross-section is represented as l. Consequently, the volume value of the built channel can be calculated using the following two-variable function:

$$v(a,\theta) = \frac{a^2 l\sin\theta(2-\cos\theta)}{4(1-\cos\theta)^2}. \tag{23}$$

The lecturers emphasized an important conceptual understanding for students that the dimensions of the optimal channel with a rectangular cross-section can be determined using the formula $h = \frac{a\sin\theta}{2(1-\cos\theta)}$. To find the value of a in terms of h, it suffices to consider the angle θ equal to 90 degrees, resulting in the following relationship:

$$h = \frac{a\sin 90}{2(1-\cos 90)} = \frac{a(1)}{2(1-0)} = \frac{a}{2} \Rightarrow a = 2h. \tag{24}$$

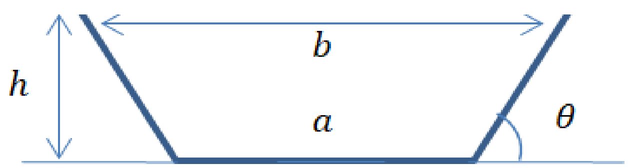

Figure 7: Cross-Section of a Trapezoidal Water Channel

3.4. Constructing an optimal water channel with trapezoidal cross-section using an optimal channel with rectangular cross-section

By altering the angle of the optimal channel walls with a rectangular cross-section, we can design an optimal channel with a trapezoidal cross-section that allows a greater amount of water to pass through. This method involves transforming the optimal water channel from a rectangular cross-section, as shown in Figure 8, into an optimal water channel with a trapezoidal cross-section.

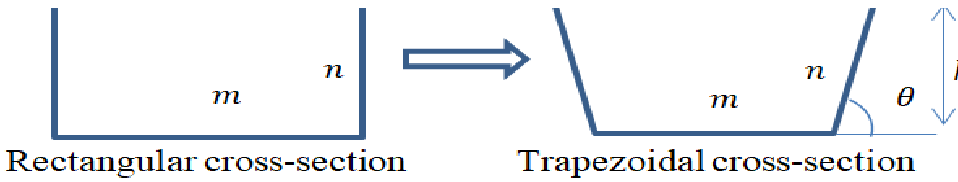

Figure 8: Transformation from Rectangular Cross-Section to Trapezoidal Cross-Section

According to Figure 8, the area value of the trapezoidal cross-section of the open channel is calculated as follows:

$$s = \frac{1}{2}(n\sin\theta)(2m + 2n\cos\theta) = \frac{1}{2}\left(\frac{w}{4}\sin\theta\right)\left(2\left(\frac{w}{2}\right) + 2\left(\frac{w}{4}\right)\cos\theta\right) \quad (25)$$

$$\Rightarrow \mathrm{s} = \frac{w^2}{16}\sin\theta\,(2+\cos\theta). \quad (26)$$

The trigonometric part of the area function is represented as $g(\theta) = \sin\theta\,(2+\cos\theta)$, and the maximum value of the function g determines the maximum value for the area function s.

$$g(\theta) = \sin\theta\,(2+\cos\theta) \Rightarrow g'(\theta) = \cos\theta\,(2+\cos\theta) - \sin\theta\,(\sin\theta) \quad (27)$$

$$\Rightarrow g'(\theta) = 2\cos\theta + cos^2\theta - sin^2\theta = 2\cos\theta + cos^2\theta - (1 - cos^2\theta) \quad (28)$$

$$\Rightarrow g'(\theta) = 2cos^2\theta + 2\cos\theta - 1 \quad (29)$$

$$\Rightarrow g'(\theta) = 0 \Rightarrow 2cos^2\theta + 2\cos\theta - 1 = 0 \quad (30)$$

$$\Rightarrow \cos\theta = \frac{\sqrt{3}-1}{2} \Rightarrow \theta = \mathrm{Arc}\cos\left(\frac{\sqrt{3}-1}{2}\right). \quad (31)$$

The volume value of the water channel constructed using the given metal sheet is calculated based on the following formula:

$$v = \frac{lw^2}{16}\sin\theta\,(2+\cos\theta). \quad (32)$$

Therefore, by utilizing the relation $\theta = \mathrm{Arc}\cos\left(\frac{\sqrt{3}-1}{2}\right)$, the volume value of the optimal channel is calculated as follows:

$$v = \frac{lw^2}{16}\sin\theta\,(2+\cos\theta) \quad (33)$$

$$\Rightarrow v = \frac{lw^2}{16}\sin\left(Arc\cos\left(\frac{\sqrt{3}-1}{2}\right)\right)\left(2 + \cos\left(Arc\cos\left(\frac{\sqrt{3}-1}{2}\right)\right)\right). \quad (34)$$

We have $\cos(Arc\cos(\frac{\sqrt{3}-1}{2})) = \frac{\sqrt{3}-1}{2}$, and by considering $\alpha = \text{Arc}\cos(\frac{\sqrt{3}-1}{2})$, we can find the value of $\sin(Arc\cos(\frac{\sqrt{3}-1}{2}))$ as below.

$$\alpha = \text{Arc}\cos(\frac{\sqrt{3}-1}{2}) \Rightarrow \cos\alpha = \frac{\sqrt{3}-1}{2}. \quad (35)$$

Therefore,

$$\sin(Arc\cos(\frac{\sqrt{3}-1}{2})) = \sin\alpha = \sqrt{1-cos^2\alpha} = \sqrt{1-(\frac{\sqrt{3}-1}{2})^2} = \frac{\sqrt{2\sqrt{3}}}{2}. \quad (36)$$

Thus, the maximum value of channel volume is as follows:

$$v = \frac{lw^2}{16}\left(\frac{\sqrt{2\sqrt{3}}}{2}\right)\left(2+\frac{\sqrt{3}-1}{2}\right) \Rightarrow v = \frac{\sqrt{2\sqrt{3}}(\sqrt{3}+3)lw^2}{64}. \quad (37)$$

3.5.Designing an optimal water channel with trapezoidal cross-section and equal dimensions

Various water channels can be designed using a metal sheet. In this section, the lecturers discussed the process of designing an optimal water channel with a trapezoidal cross-section and equal dimensions. As depicted in Figure 8, both values m and n are equal. Consequently, the area value of the channel cross-section can be calculated as follows:

$$s = \frac{1}{2}h(m + (m + 2n\cos\theta)) = \frac{1}{2}(n\sin\theta) \quad (38)$$

$$\Rightarrow s = \frac{1}{2}\left(\frac{w}{3}\sin\theta\right)\left(2\left(\frac{w}{3}\right)+2\left(\frac{w}{3}\right)\cos\theta\right) \quad (39)$$

$$\Rightarrow s = \frac{w^2}{9}\sin\theta\,(1+\cos\theta). \quad (40)$$

So, to determine the maximum value of the trigonometric function $h(\theta) = \sin\theta\,(1+\cos\theta)$, we need to calculate it in order to obtain the maximum value for the area function of the trapezoidal cross-section of the water channel.

$$h(\theta) = \sin\theta\,(1+\cos\theta) \Rightarrow h'(\theta) = \cos\theta\,(1+\cos\theta) + \sin\theta\,(-\sin\theta) \quad (41)$$

$$\Rightarrow h'(\theta) = \cos\theta + cos^2\theta - sin^2\theta \Rightarrow h'(\theta) = \cos\theta + cos^2\theta - (1 - cos^2\theta) \quad (42)$$

$$\Rightarrow h'(\theta) = 2cos^2\theta + \cos\theta - 1 \quad (43)$$

$$h'(\theta) = 0 \Rightarrow 2cos^2\theta + \cos\theta - 1 = 0 \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = 60. \quad (44)$$

Therefore, the maximum volume value of the open channel with a trapezoidal cross-section and equal dimensions can be determined as follows:

$$v = sl = \frac{lw^2}{9}\sin\theta\,(1+\cos\theta) = \frac{lw^2}{9}\sin 60\,(1+\cos 60) = \frac{lw^2}{9}\left(\frac{\sqrt{3}}{2}\right)\left(1+\frac{1}{2}\right) \tag{45}$$

$$\Rightarrow v = \frac{3\sqrt{3}lw^2}{36}. \tag{46}$$

3.6.Designing Curved Water Channels: Optimization and Practical Considerations

The cross-sections of certain water channels used in agriculture are arcs of circles. Engineers utilize advanced machines to curve metal sheets and create these water channels. Figure 9 depicts a curved channel constructed using a metal sheet with dimensions l and w (Figure 6). According to Gholami and Sathar (2021), the circle's radius, in terms of the values h and k, can be calculated as $r = \frac{k^2+4h^2}{8h}$. The angle between two radiuses is denoted by α, and the cross-sectional area of the built open channel is computed as follows:

$$s = \frac{\alpha}{360}\pi r^2 - \frac{1}{2}r^2\sin\alpha = \frac{1}{2}r^2\left(\frac{1}{180}\pi\alpha - \sin\alpha\right) \tag{47}$$

$$\Rightarrow s = \frac{1}{2}\left(\frac{k^2+4h^2}{8h}\right)^2\left(\frac{1}{180}\pi\alpha - \sin\alpha\right). \tag{48}$$

Therefore, the volume value of the open channel constructed using a metal sheet (Figure 6) can be determined using the following formula:

$$v = sl \Rightarrow v = \frac{l}{2}\left(\frac{k^2+4h^2}{8h}\right)^2\left(\frac{1}{180}\pi\alpha - \sin\alpha\right). \tag{49}$$

In this formula, if $\alpha = 180$, then $k = 2r$ and $h = r$. Thus, the optimal volume value of the curved channel can be calculated as follows:

$$v = \frac{l}{2}\left(\frac{(2r)^2+4r^2}{8r}\right)^2\left(\frac{1}{180}\pi(180) - \sin(180)\right) \tag{50}$$

$$\Rightarrow v = \frac{lr^2\pi}{2} = \frac{(lr)(\pi r)}{2} = \frac{lrw}{2} \Rightarrow v = \frac{lrw}{2}. \tag{51}$$

In other words, the built channel has a semi-circular cross-section with a perimeter of w. Therefore,

$$\pi r = w \Rightarrow r = \frac{w}{\pi}. \tag{52}$$

The volume formula for the curved water channel is as follows:

$$v = \frac{lrw}{2} = \frac{l\left(\frac{w}{\pi}\right)w}{2} = \frac{lw^2}{2\pi} \Rightarrow v = \frac{lw^2}{2\pi}. \quad (53)$$

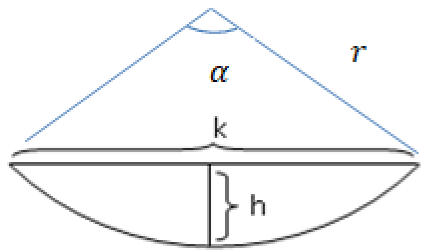

Figure 9: Cross-Section of a Curved Water Channel

4. COMPARING THE VOLUME VALUE OF OPTIMAL CHANNELS

This article delves into the diverse geometrical shapes achievable with a rectangular metal sheet of dimensions l and w. Table 1 presents the derived volume formulas for open water channels based on different sections of the article, including an illustrative example.

Geometrical cross-section	Volume formula	Volume value of a channel with $l = 3$ and $w = 1.8$ meters
Triangular	$v = \frac{w^2 l}{8}$	$1.215m^3$
Rectangular	$v = \frac{w^2 l}{8}$	$1.215m^3$
Trapezoidal ($m = 2n$)	$v = \frac{\sqrt{2\sqrt{3}}(\sqrt{3}+3)lw^2}{64}$	$1.336m^3$
Trapezoidal ($m = n$)	$v = \frac{3\sqrt{3}lw^2}{36}$	$1.402m^3$
Semi-circular	$v = \frac{lw^2}{2\pi}$	$1.547m^3$

Table 1: Comparison of Volume Values for Different Geometrical Water Channels

In Table 1, the optimal channel with a semi-circular cross-section allows the maximum water flow, but certain limitations hinder its widespread use. Interestingly, some students mistakenly assumed that changing a rectangular cross-section channel to a trapezoidal one automatically increases its volume value. To illustrate this, the lecturers considered an example: "In Figure 8, assuming $m = 2n$ and constructing a rectangular channel with a metal sheet having dimensions $l = 3m$ and $w = 1.8m$, converting it to a trapezoidal channel with $\theta = 45$ degrees. We can compare the volume values of the rectangular and trapezoidal cross-section channels." This example serves as a valuable exercise for students, promoting critical analysis and the identification of optimal solutions for real-world challenges. In this case, the volume value of the built channel with a rectangular cross-section is calculated as follows:

$$v = \frac{w^2 l}{8} = \frac{(1.8)^2 \times 3}{8} = 1.215m^3.$$

Whereas, the volume value of the built channel with a trapezoidal cross-section is determined as follows:

$$v = \frac{w^2 l}{16} \sin\theta\,(2 + \cos\theta) = \frac{(1.8)^2 \times 3}{16} \sin 45\,(2 + \cos 45)$$

$$\Rightarrow v = \frac{(1.8)^2 \times 3}{16}\left(\frac{\sqrt{2}}{2}\right)\left(2 + \frac{\sqrt{2}}{2}\right) = 1.162m^3.$$

The lecturers found that employing this practical problem focused on optimizing agricultural water channels was highly effective in helping students develop a profound understanding of the concept of optimization. By using a student-centered teaching method centered around problem-solving, the students were drawn to the subject due to the clear connection between engineering and mathematics. This approach demonstrated the significance of mathematics in solving real-world challenges through engineering methods. Furthermore, it fostered a deeper appreciation for how mathematical principles can be applied to practical engineering situations, showcasing the tangible benefits of mastering such skills. To further enhance the teaching approach, incorporating more real-world examples and hands-on experiences could help students grasp the practical applications of the concepts and further engage them in the learning process.

Collaborative work and knowledge-sharing among the lecturers significantly enhanced their teaching capabilities, particularly in designing mathematical lessons that bridge the gap between mathematics and real-world challenges. Through collaborative efforts, the lecturers were able to pool their diverse experiences and expertise, fostering a more comprehensive and engaging learning environment for students. This approach enabled them to incorporate practical examples and applications of mathematical concepts, making the lessons more relevant and applicable to real-life situations. As a result, students were better equipped to understand the importance of mathematics in addressing real-world problems and could perceive its direct relevance to their future careers in engineering and other fields. To further bolster this approach, continued professional development and ongoing sharing of best practices could lead to even more effective teaching strategies and improved student learning outcomes. One of the lecturers provided the following explanation:

In the past, I used to teach the concept of optimization to students using abstract materials from textbooks. However, being part of this collaborative program has greatly enhanced my teaching approach in conveying the optimization concept. By incorporating real-world problems that are well-suited to the students' abilities, I found that it is an effective teaching method that creates a strong connection between mathematics and engineering. Moreover, exploring various mathematical models for the practical problem discussed in this program has not only improved my lecturing skills but also enhanced the students' proficiency in solving real-world challenges. This experience has reinforced the importance of integrating practical applications into mathematical lessons, making the learning process more engaging and relevant for the students. As I continue to develop as an educator, I look forward to exploring more hands-on approaches and expanding my repertoire of real-world problem-solving activities to better equip my students for their future endeavors in various fields.

5. STUDENTS' PERSPECTIVES

According to the feedback from the students who participated in this study, they found the mathematical materials and teaching methods to be enjoyable. All students agreed that this workshop significantly enhanced their proficiency in solving real-world problems. Table 2 provides an analysis of the students' responses to the first five interview questions.

Question	Very little	Little	Not sure	Much	Very much
1	0	0	1	4	33
2	0	0	0	0	38
3	0	0	0	3	35
4	0	0	0	7	31
5	0	0	0	5	33

Table 2: Summary of Students' Responses to Interview Questions One to Five

Based on the data presented in Table 2, it is evident that the workshop had a highly positive impact on the participants. Approximately 97% of the students found the prepared lesson to be enjoyable, with 33 of them finding it very interesting and an additional 4 finding it interesting. All participants unanimously agreed (100%) that the lesson effectively connected with real-world applications, emphasizing optimization concepts through practical problems rather than abstract mathematical concepts. The primary objective of teaching engineering mathematics—to familiarize students with real-world problem-solving—was clearly met. An impressive 92% of the students recognized the deep connection between the lesson's contents and engineering.

Furthermore, 82% of the students acknowledged that the course not only enhanced their understanding of real-world problems but also improved their problem-solving skills under diverse assumptions. Remarkably, 87% of the participants believed that the teaching method greatly influenced their comprehension of real-world issues, helping them select appropriate strategies and consider necessary assumptions when tackling problems.

However, the overwhelming majority (90%) of students strongly agreed that this mathematical lesson was not only interesting and enjoyable but also highly effective in enhancing their abilities to solve real-world problems. The positive feedback from the participants underscores the workshop's success in achieving its goals and fostering an engaging and practical learning environment.

During the interview process, the sixth question stood out for its flexibility, allowing engineering students to freely express their opinions about the presented lesson without any specific constraints. The researchers used the thematic analysis method to carefully analyze the data collected from these interviews. They diligently identified recurring ideas and patterns in the interview transcripts, employing a six-step process of cognition, coding, producing themes, checking themes, naming themes, and writing up the report to discover general themes in the data analysis (Kiger & Varpio, 2020).

The obtained general themes from the interviews' transcripts highlight the role of this lesson in enhancing students' ability to solve real-world problems. Firstly, understanding the real-world problems emerges as a foundational aspect of effective problem-solving, enabling students to devise appropriate strategies using mathematical models. Secondly, the translation of real-world problems into mathematical scenarios facilitates systematic analysis and solution development. Thirdly, the importance of making reasonable assumptions in constructing mathematical models for real-world challenges is emphasized. Flexibility in employing assumptions, the fourth theme, is crucial to adapt to dynamic real-world conditions. Furthermore, the fifth theme emphasizes exploring diverse mathematical models to find different solutions for real-world problems, fostering creativity and broadening students' problem-solving capabilities. The sixth theme emphasizes the significance of critically evaluating and comparing the mathematical models developed by students to select the most appropriate one based on accuracy and applicability.

Regarding students' learning experience, the seventh theme highlights the effectiveness of practical, real-world problems in teaching optimization concepts, as they provide context and relevance. The eighth theme emphasizes the fundamental role of mathematics in addressing real-world challenges, demonstrating its practical applications to students. The ninth theme recognizes the value of errors in the learning process, promoting resilience and critical thinking in students when tackling real-world problems. Lastly, the tenth theme underscores the positive impact of the lesson on students' self-confidence. As students gain competence in solving real-world problems using mathematical tools, their self-assurance grows, motivating them to tackle more complex challenges confidently. Therefore, this study identifies a comprehensive set of factors contributing to students' improvement in solving real-world problems. Understanding these themes enables educators to design more effective lessons that empower students with essential skills and confidence to address future challenges successfully.

For example, student s_7 explained that:

> *The approach to teaching the concept of optimization is fascinating to me due to my practical experience in optimizing various models to address real-world challenges. By comparing these optimized models, I have gained a profound understanding of how to effectively tackle real-world problems.*

Student s_{16} stated that:

> *The design and implementation of agricultural water channel projects can be affected by errors arising from the geometric shape of their cross-section. Notably, the objective function for different cases of this problem is a one-variable function. However, when additional factors are taken into account, such as the distance of the water source to the farm, the type of water channel, and the construction cost, checking the error rate and optimizing the objective function becomes more complex. The materials presented in this lesson, along with the transferring method, proved to be highly engaging and interesting for me.*

6. CONCLUSION

Optimization is a crucial concept in engineering fields, empowering engineers to solve real-world problems based on specific conditions. Engineering students must grasp this concept to effectively connect mathematics with engineering. This article's results have been instrumental in aiding engineering students to design a variety of optimal open channels with different cross-sections, utilizing rectangular metal sheets for agricultural purposes. While the article focused on conceptually teaching the design of optimal agricultural water channels, students can readily extend these findings to build concrete channels. Moreover, the results can be incorporated into engineering classes for designing and constructing industrial concrete channels, especially for flood control purposes.

The prepared lesson tackles a practical problem, encouraging students to enhance their mathematical reasoning, writing, and project skills through collaborative interaction (Van Lierde, 2022). This approach to teaching mathematics in engineering classes not only fosters conceptual understanding but also enables students to establish connections between mathematics and engineering, thus equipping them to tackle real-world challenges with enthusiasm. Incorporating practical problems into engineering classrooms, rather than relying solely on abstract materials, proves to be invaluable as students gain essential skills in handling large engineering projects. As such, instructors are encouraged to embrace real-world problems in their teaching approach (Larina, 2016).

7. LIMITATIONS AND RECOMMENDATIONS

One limitation of this study is the relatively small sample size of 38 first-year engineering students from a single public university in Malaysia. While the participants' perspectives and feedback provided valuable insights into the effectiveness of the optimization lesson, the findings may not be fully representative of the broader population of engineering students. To enhance the generalizability of the results, future research could include a more diverse and larger sample of students from various universities or educational institutions. Additionally, the study's focus on a specific practical problem from the agricultural sector might limit the transferability of the optimization concept to other engineering domains. Exploring different practical problems and their applications in various engineering fields could offer a more comprehensive understanding of the lesson's adaptability and impact. Addressing these limitations would strengthen the study's significance and broaden its implications for mathematics teaching in engineering education.

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