Editorial Reflection by Bronislaw Czarnocha, the Chief Editor of MTRJ

This is the 50th issue of Mathematics Teaching-Research Journal online what requires a certain amount of editorial reflection. There is a sense of accomplishment in the life history of the journal. First, it comes with the stability and continuity of our Editors' Team which meets every Saturday for the Editors' meeting discussing past reviews and distributing new authors. Everyone does it purely voluntarily! That's amazing and my gratitude goes to every member of the team as well as to the team as a whole. My special gratitude goes to Dr. Malgorzata Marciniak, the managing editor who holds the whole journal together.

Second, the journal exists for 17 years and whereas we have started with the modest beginnings of a quarterly even sometimes publishing it only twice a year, at present we produce a bimonthly with the long list of accepted authors for the next 2 issues of the journal. Of course, we are indexed in Scopus, Eric, CNKi, Cabell Journalytics.

The journal became the bridge between authors from South Asia and Africa, and the authors from the North; from Indonesia, Malayasia, Turkey, Nepal, South and West Africa and authors from Europe and Americas. The 50th issue is an example of such a bridge. We have 5 new papers from Indonesia, 3 papers from East Europe, one from Nigeria and one from Turkey.

The Editors' Team right now is also very international: the editors from Malaysia, Indonesia, from Germany, Spain, Poland, Italy, Peru, South Korea, and US of course, are on the board. The team of editors is slowly turning into the collaborative research community: we have done a small investigation collaboratively introducing and assessing an intervention in 7 countries around the world and will present the results in the summer at a major conference in Australia.

MTRJ is a teaching-research journal whose purpose is to create a bidirectional bridge between teaching and research (or research and teaching). Further in the future we envision an International Teaching-Research Conference online.

Editorial Proper

Note: The common aspect of the three presentations from Eastern Europe is their interest in the improvement of mathematics education for different types of workers entering the labor market.

One of the Ways to Organize Mathematical Training of Railway University Students in the Target Programs.

Natalya A. Arkhipova, Natalya N. Evdokimova, Tatyana V. Rudina. *Russia*

The paper of Arkhipova and colleagues from Russia describes the organization of learning in the state university of transport, and in particular the authors are interested to find out to what degree the introduction of new pedagogy of teaching mathematics will improve students' professional learning. The new pedagogy involves organization of project-based learning through professionally oriented targeted projects. An example of such a complex project is included and from its description we see the depth of possible learning by students.

Math Anxiety, Skills, and Work-related Competencies: A Study on Czech University Students

Lenka Farkačová, Mária Králová , Eva Zelendová

Farkačová and colleagues from Czechia are interested in the impact of the awareness of mathematical skills upon professional success in the labor market. They emphasize the presence of math anxiety among many students. Through the survey and structured interviews, the authors want to assess the strength of the relationships between acquisition of mathematical competences needed for the success and the awareness of their importance, as well as with the manifestation of math anxiety. Their results confirm the existence of hypothesized relationships.

Professional Development Interventions for Mathematics Teachers: A Systematic Review.

Branko Bognar, Ljerka Jukić Matić, Marija Sablić

The third paper from East Europe included in the issue is that of Branko Bodnar and colleagues from Croatia who are interested in the qualities for the most effective professional development for the teachers of mathematics. It's the metanalysis of 12 chosen literature descriptions of PD's for teachers. They conclude that to enhance mathematics learning outcomes for students, it appears that professional development interventions should provide on-site teacher support, mentoring, and feedback. Moreover, providing teachers with some form of teacher-focused resources and classroom learning materials would be of great assistance to those attempting to implement new instructional practices. They suggest the development of teaching-research school teacher teams as the way to assure a continuous professional development.

Note: The next five papers originated in Indonesia showing us the intensity of Indonesian contemporary concern with Mathematics Education.

Prediction Ability of College Students in Solving Graph Problems

Lathifaturrahmah Lathifaturahmah, Toto Nusantara, Subanji Subanji, Makbul Muksar

Lathifathurama and co-authors are interested in the assessment of mathematics' students ability to formulate well based predictions on the basis of graphs – a topic in demand in contemporary economy and politics. As the memory of Covid -19 are still very vivid with us, the presented work was based on the graph of monthly number of Covid cases in 2022. The authors use SOLO methodology in identifying levels of student responses. They discovered 4 different levels of predictions of increasing precision to predictions used by students: unistructural, multistructural, relational, and extended abstract levels. They conclude that the accuracy of students' prediction is indicative of the capacity to apply the concept in practical situations.

A Case Study on Students' Critical Thinking in Online Learning: Epistemological Obstacle in Proof, Generalization, Alternative Answer, and Problem Solving

Irena Puji Luritawaty, Tatang Herman, Sufyani Prabawanto[3]

Luritawati and colleagues, are also concerned with the effect of Covid – 19, which forced all learning to be done online. They discovered that online classes increase the difficulty of critical thinking development due to the difficulties in systematic online communication. They have characterized these difficulties as epistemological obstacles students encounter. The main encountered difficulties in usage of critical thinking were discovered while proving the relationship between two concepts, generalize relationships, seek alternative solutions, and solve problems. The authors suggest that to overcome these difficulties and to foster critical thinking skills in teaching online requires online learning to be well-prepared and structured, which can be initiated through good planning. To achieve this, lecturers can prepare comprehensive learning guidelines in e-modules or other materials that focus on understanding the relationship between concepts, the flexibility of concepts and procedures, and the habit of drawing in geometry learning.

Students' Proactive Interference in Solving Proportion Problems: How was the Met-before?
Pradina Parameswari, Purwanto, Sudirman, Susiswo

Pradina Parameswari and colleagues are focusing their attention on the well-known problem of differentiating between direct and inverse proportions - a very important kill for both radiology technicians and nursing students. In solving the problem of inverse proportion, students often use the concept of direct proportion, and the authors trace this difficulty to the habitual interference of old knowledge when it's applied to new situations. Authors characterize students on the basis of the analysis results as non-flexible and flexible types, each type displaying different relationships within their (mis) understanding of proportion.

Stages of Problem-Solving in Answering HOTS-Based Questions in Differential Calculus Courses
Eko Andy Purnomo, Y.L. Sukestiyarno, Iwan Junaedi, Arief Agoestanto

Eko Purnomo and colleagues are also interested in the development of critical thinking from the point of view of problem solving in Differential Calculus. They follow the route of stage analysis suggested by Polya method. However, they find that the Polya stages must be augmented to allow for the detailed analysis of students' work within the Higher Order Thinking Skills framework.

A Comparison of Angle Problems in Indonesian and Singaporean Elementary School Mathematics Textbooks

Yoppy Wahyu Purnomo, Antika Asri Julaikah, Galuh Candra Aprilia Hapsari,
Rina Cahyani Oktavia, Rizki Muhammad Ikhsan

An interesting paper by Yoppy Purnomo and colleagues concerns differences in pedagogical approaches to the concept of an angle as seen by the comparison of Indonesian and Singapore textbooks. The research findings show that the two textbooks introduce the angle topics in different ways. In Indonesian mathematics textbooks, the angle topics are introduced at the end of the semester, while Singaporean textbooks introduce them in the middle of the semester. Indonesian textbooks have more types of task activities than Singaporean textbooks. However, the distribution of items for each dimension in Singapore book task activities is more proportional. Other important findings: in Indonesian mathematics textbooks are still dominated in the purely mathematical category, while Singaporean mathematics textbooks are more dominated in the visual category. In other words, Indonesian mathematics textbooks place emphasis on exercise more often, whereas Singaporean textbooks are more oriented towards conceptual knowledge.

Relative Effectiveness of Formative Assessment Techniques on Students' Academic Achievement in Mathematics Classroom Teaching and Learning

Ejembi, Stephen Oche, Basil C.E. Oguguo, Cynthia O. Ezeanya, Kenneth E. Okpe, Vitalis C. Okwara, Isaac I. Adie, Bassey Bassey Ayek, Lovina Ijeoma Okpara & Blessing Ngozi Ojobo

The paper of Oguguo and colleagues from Nigeria is interested in the impact of formative assessment on student retention of the information. The authors direct their attention towards two interesting assessment techniques: quite well-known think-pair-share technique and the muddiest point assessment techniques. The latter technique asks for students to jot down at the end of the class their most confusing point in learning. The results show that the second technique is more efficient in securing students' retention than the first one.

The Effect of Computer Supported Collaborative Dynamic Learning Environment on High School Students' Success in Mathematics Classroom

Türkan Berrin Kağızmanlı Köse, Enver Tatar

We complete the issue with an important investigation of Kağızmanlı and Tatar from Turkey concerning the computer supported collaborative learning (CSCL). The authors underscore the importance of leveraging the students' agency and creativity during collaborative learning. The contribution presents the method describing the quantitative and qualitative approach to the topic of "Lines". The authors find out that CSCL approach increases students' retention rates of the subject.

Problem Corner

Ivan Retamoso

We invite the readers to try the new problem prepared by Ivan Retamoso, our Problem Corner Editor. One can look also at the interesting solutions of the previous problems.

One of the Ways to Organize Mathematical Training of Railway University Students in the Target Programs

Natalya A. Arkhipova, Natalya N. Evdokimova, Tatyana V. Rudina

Samara State Transport University, Russia

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Abstract: The article discusses the organization of the educational process of railway university students in the target programs. A successful educational process will be effective if the training provided to students of the transport university is of an acceptable quality, which will lead to a high demand for a specialist. When participating in targeted training programs, the learning process can be optimized and structured within the educational framework, thereby placing emphasis on acquiring the necessary knowledge for future professional endeavors. The objective of the article was to examine how the minor program in the Mathematics discipline for students in targeted programs enhances their professional motivation. We propose to consider the characteristics of training in targeted programs by examining the training of students in the specialty 23.05.04 "Electric power supply of railways" at the Samara State University of Railway Transport. We believe that one way to organize the mathematical training of students in targeted programs is to use professionally oriented tasks. As the objective of the study pertains to the issues of specialized university education in the field of mathematics, the paper presents a task that is geared towards professionals for students enrolled in the targeted programs of this particular field. Furthermore, this approach to the study of mathematics yields a comprehensive understanding of one's professional pursuits, implying that the integration of the educational process is essential in the preparation of a railway transport specialist. The evaluation of the proposed methodology for the instruction of students in the targeted programs has demonstrated its efficacy.

INTRODUCTION

Students entering the transport university in target programs have interests that can be considered to be clearly formed, since they have not only decided on their future profession, which they receive, but also decided on their future place of work, which they must work for at least three years. Generally, many students believe that they are not interested in general education subjects because they do not relate to the chosen type of activity. But it shouldn't be this way. When studying a course in higher mathematics, it is important to avoid the formal approach. To increase motivation for training a future specialist, we propose to include professional-oriented tasks in the learning process.

The distinctiveness of targeted training stems from the fact that this particular form of education is a prevalent phenomenon in Russia, Uzbekistan, Kazakhstan, Tajikistan.

Various aspects of the application of targeted university education were described by such researchers as V.M. Anikin, B. N. Poizner and E. A. Sosnin (2019) and others (Verhaest & Baert, 2018; Klimova et al, 2021), who consider the presented training as a purposeful system of activity.

V.M. Anikin, B. N. Poizner, and E. A. Sosnin (2019) think that the main goal of studying at the university is to make today's schoolchildren become experts in their field, as agreed upon in the contract.

According to the research conducted by S.S. Bakulina, E.A. Muzychenko, and V.E. Chernoskutov (2011), the targeted recruitment ought to be regarded as an educational program wherein the student undertakes an apprenticeship at the expense of the prospective employer, who is directly interested in a specific area of specialist training.

The history of targeted training dates back to the 1960s, when the Soviet Union established the principle of admitting applicants first to postgraduate school, and then targeted training programs were introduced into higher and secondary professional educational institutions. At the same time, the need for professional personnel in industry, agriculture, household sphere and other areas of the economy in the conditions of planned economy was taken into account (Bakulina, Muzychenko & Chernoskutov, 2011).

Thus, the order of the Ministry of Higher and Secondary Special Education of the USSR dated July 31, 1962 No. 284 "On approval of the regulations on postgraduate studies" contained a special section "Targeted Postgraduate Studies". Targeted postgraduate study was understood as a certain format of personnel training, which would later be used as employees of enterprises and higher educational institutions that do not have the opportunity to train personnel on the spot. There was a strict admissions policy for the targeted postgraduate study. As a rule, all places in full-time target postgraduate studies were occupied by secondees from the Union republics, in the programs of higher educational institutions and industrial enterprises. As per the provisions enshrined in the order at that time, the expenses associated with training were borne by educational institutions, whereas specialists who had completed postgraduate studies were to be returned to the organizations that had been designated for their services.

In 1980, on the basis of the order of the Ministry of Higher and Secondary Special Education of the USSR No. 700 dated June 19, 1980 "On approval of the regulations on postgraduate studies at higher educational institutions and research institutions", a number of postulates were changed. Now the responsibility for specialists sent for targeted postgraduate training was assigned to the ministries and departments in charge of which there were organizations giving directions for training.

By the decree of the Council of Ministers of the RSFSR dated June 10, 1987 No. 241 "On measures to radically improve the quality of training of specialists with higher education in the national economy" before the executive authorities of the RSFSR, the trajectory of the planned transition by 1993 to a new level of interaction between higher education and production was built. This decree approved the targeted training of specialists on the basis of contracts concluded between the enterprise and the higher educational institution. It was believed that the sectoral ministries would develop plans for the training of students in a number of specialties, based on the number of target contracts concluded.

A similar level of regulation of relations between students in targeted programs and organizations continued until 2002 on the basis of the Decree of the Government of the Russian Federation dated September 19, 1995 No. 942 "On targeted contract training of specialists with higher and secondary professional education".

In 2002, in p. 11 of art. 41 of the Law of the Russian Federation No. 3266-1 "On Education" dated July 10, 1992, a norm appeared, according to which state and municipal institutions of secondary and higher professional education were granted the right to carry out targeted admission of students within state tasks (control figures) in accordance with contracts concluded with state authorities, by local governments. This reception was intended to provide assistance to these institutions in the training of specialists.

Today, each enterprise is afforded the opportunity to educate and train specialists in accordance with its individual requirements. This is applicable to various universities, including but not limited to transport, medical, communications, polytechnic, and others.

With the help of targeted training, the motivation of students for learning in general is increased. The study of the motives of students' learning activities is a very important element of the teacher's activity. The students' attitude towards studying, academic performance, interest in the result, and future professionalism depend on their motives.

The activity of students at the university is of an educational and professional nature. K. Zamfir, L.B. Itelson, E.P. Ilyin, L.P. Urvantsev, O.K. Markova, O.N. Arestova, N.A. Bakshaeva, A.A. Rean and other psychologists devoted their works to the study of the motivation of educational and professional activities of students (Deleu, 1990; Urvantsev & Maleeva, 1984)

Nowadays, the Federal Law No. 273-FZ of December 29, 2012 "On Education in the Russian Federation" establishes measures to ensure targeted training of persons with higher education.

In this regard, the increase in interest and motivation in obtaining highly professional skills that should be used in the future at a particular enterprise comes to the fore.

It should be noted that the research in this area is not fully reflecting the systematic vision of the problems of implementing targeted distance learning, and further study is required.

MATERIALS AND METHODS

The objective of the article was to examine how the minor program in the discipline of Mathematics for students in targeted programs enhances their professional motivation.

This task involves drawing up a work program for mathematics students in targeted programs, the choice of a method that allows determining the importance of academic disciplines for professional training, and the development of professional motivation.

One of the methods for obtaining a high-quality mathematical education for railway transport engineers is the method of introducing tasks of a professionally oriented nature.

The object of the study was the process of organizing training in targeted programs at the Samara State University of Railway Transport in the specialty "Electric power supply of railways".

The research focuses on the organization of targeted teaching of mathematics at the Samara State University of Railway Transport in the specialty "Electric power supply of railways."

Employees of JSC "Russian Railways" have developed a target program: "Youth of JSC "Russian Railways". The proposed program promotes the professional growth of young employees of JSC "Russian Railways", and also contributes to the formation and development of professional leadership qualities and an active life position, and the production initiative of young people.

The Samara State University of Railway Transport annually allocates a certain number of places for the training of highly qualified personnel to meet the needs of the Kuibyshev, Volga, Gorky, South-Eastern, South Ural, Northern, North Caucasian railways.

All applicants who are interested in receiving a targeted referral for training have the opportunity to contact the personnel management service of the enterprise that is the customer of targeted training. If the applicant has not yet made a decision on admission to the university within the framework of the targeted admission, then he can enroll on other conditions and think about the possibility of targeted training later. Students are given the opportunity to conclude a contract for targeted training in any course.

In addition to the classes provided for by the budgetary basis in accordance with the state educational standards, students are offered additional classes, namely lectures and practical classes in disciplines that are not provided for by the federal state educational standards of higher education, but they are necessary for the graduate in the future in the performance of official duties. The universities costs for implementing additional educational services for students in targeted programs are reimbursed by the enterprises that sent them (Maleina, 2015).

It is highly valuable that during this challenging period, the customer enterprise provides the graduate with a job upon completion of their university education. With successful studies, students are awarded monthly additional scholarships. In addition, the student is not seeking an internship, but rather looking for a paid position.

After completing their university education, students are granted the status of a young specialist, which entitles them to receive relocation allowances when they are assigned to work in a different area. This allowance is intended to cover the expenses of renting a dwelling and to obtain a preferential mortgage loan to acquire a dwelling within the property.

The table shows by year the number of admission places in the specialty 23.05.04 "Electric power supply of railways" and the number of places of target destination at the full-time department of the Samara State University of Railway Transport.

	2016		2017		2018		20019		2020	
	Number of admission places	Number of places of target destination	Number of admission places	Number of places of target destination	Number of admission places	Number of places of target destination	Number of admission places	Number of places of target destination	Number of admission places	Number of places of target destination
Full-time education	122	70	100	68	100	70	100	78	90	71
Extra-mural education	60	37	75	52	60	43	27	19	27	19

Table 1. The number of admission places for applicants 2016-2020 (Compiled by the authors)

Based on the analysis of the figures presented in Table 1, it is evident that there is a consistent downward trend in the number of admission places, both for full-time and extramural education. So, since 2016, the number of admission places in the specialty "Electric power supply of railways" has decreased from 122 in 2016 to 90 in 2020 in the full-time department and from 60 in 2016 to 27 in 2020 in the extramural department. Moreover, the number of places for targeted admission in the full-time department remained at the same level.

For clarity, we present the table data for full-time education in the form of a diagram:

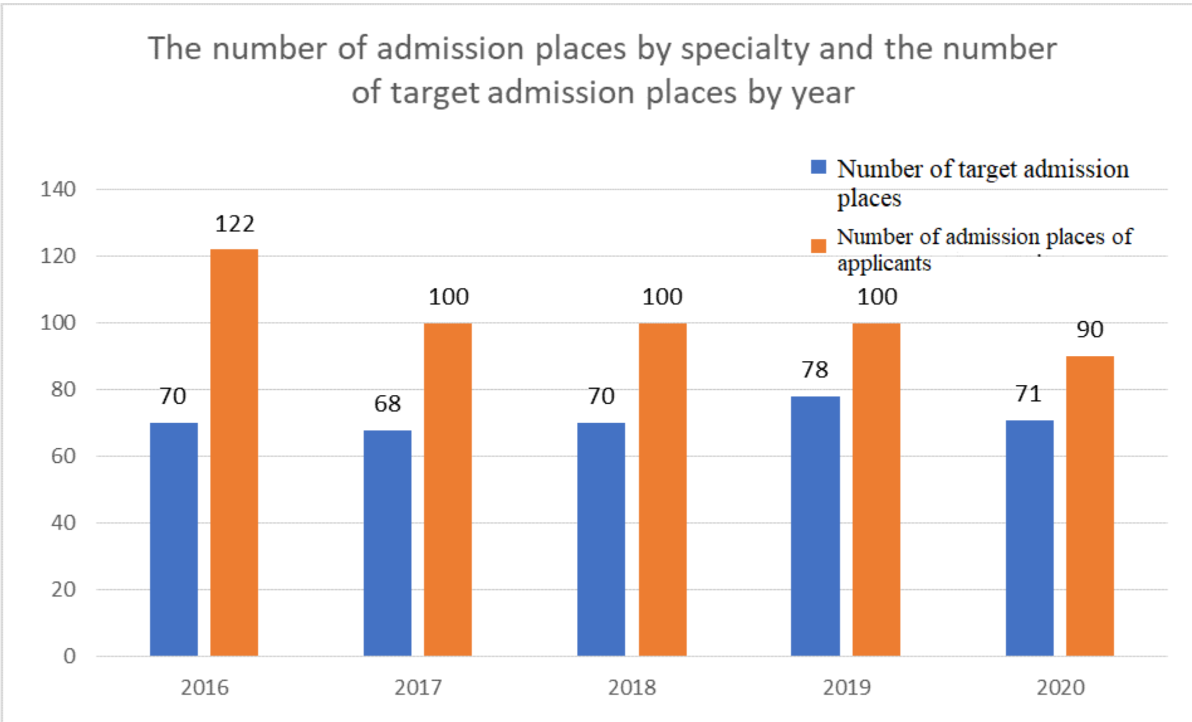

Figure 1. Diagram "Number of admission places of applicants in 2016-2020" (Compiled by the authors)

As per the current legislation, it is noteworthy that for individuals enrolled in targeted contracts, all prerequisites for free higher education have been preserved.

Candidates entering the target programs have varying levels of awareness of their future professional activities. They are both graduates of secondary schools and professional colleges. At the same time, the latter are familiar with the profession, many of them have practical experience. School graduates most often enter universities either as successors to a professional dynasty, or in company with friends, or at the insistence of their parents.

The targeted program includes both the professional cycle and the necessary disciplines for their development. Mathematics is one of those disciplines. During the creation of a mathematics course designed for targeted training programs, a variety of contemporary pedagogical technologies were used. In this study, we examine the use of professionally directed tasks in the study of mathematics. These tasks are necessary when solving problems of mathematical modeling, they are actively used in solving problems of a professional orientation using the project method (Han, Capraro, & Capraro, 2015; Hanushek et al, 2017; Verhaest & Baert, 2018).

In addition to the standard tasks that are covered in the discipline "Mathematics" for students in target programs, it is recommended to include professionally-directed tasks and use new pedagogical technologies (Klimova & Klose, 2020; Pardała, Uteeva & Ashirbayev, 2015). This approach helps to improve the assimilation of the basic special terms, and the ability to use mathematical apparatus in solving professionally-oriented problems is also developed.

The teachers' role is to help students understand mathematical concepts and understand how mathematical laws work in the professional field. We believe that the most effective way to develop students' mathematical activity is through the use of special professional tasks. Furthermore, the proposed methodology for teaching permits the exposition of the practical significance of the analyzed mathematical theory and serves as a stimulant for enhancing the cognitive abilities of students, thereby augmenting their mathematical knowledge. The use of professional tasks in mathematics helps students develop the ability to apply basic knowledge in the professional field, which in turn increases interest in studying the discipline, develops non-standard thinking, and the desire to work independently.

For example, here is an example of a professionally-oriented task for trainees in the specialty 23.05.04 "Electric power supply of railways" of the Samara State University of Railway Transport. (Arkhipova, Evdokimova & Rudina, 2019; 2020).

Find a rational option for the development of the single-track railway polygon ABCD (Fig. 2)

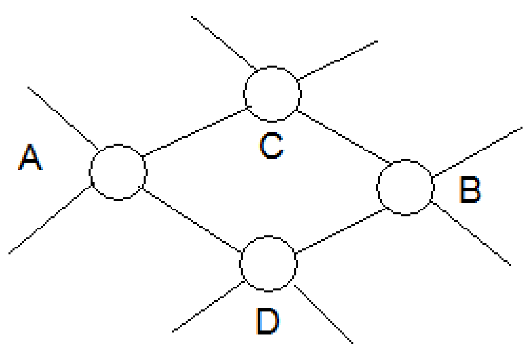

Figure 2. Scheme of single-track railways (Kaplan, Maidanov & Tsarev, 1978)

The existing capacity on all lines is 20 million tons in freight directions (from A and B to C and D). The traction on all lines is provided by diesel. Traffic-dependent costs with existing equipment everywhere are 14,000 rubles / 10 tkm.

Table 2 shows data on the polygon lines.

Line	AC	BC	AD	BD
The prospective non-distributed flow of other cargo, with the exception of the primary planned cargo (in million tons per year), back and forth	10/12	11/14	11/10	9/11
Miles of maintained track, km.	500	450	700	600

Table 2. Data on the polygon lines. *Source: Kaplan, Maidanov & Tsarev, 1978.*

Data on the stages of traffic-capacity (the possible stages are the same for all lines):

No. of the stage	1	2	3
Performing action	Lengthening of receiving and departure tracks and increase in the weight norm	Electrification	Construction of the second track
Traffic-capacity (in million tons/year) in the cargo direction	28	40	90
Capital investments (in thousands of rubles per kilometer of operational length)	20	60	120
Annual additional independent operating costs in comparison with the previous stage (in thousand rubles / km)	2	4	12
Traffic-dependent costs per 10 t km net (in thousand rubles)	1,3	1,1	1,0

Table 3. Data on the stages of increasing the traffic-capacity of lines. *Source: Kaplan, Maidanov & Tsarev, 1978.*

Normative coefficient of efficiency of capital investments $E_н$=0,12.

Data on the departure and arrival of the planned cargo (in million tons / year): departure A - 38; B - 48; arrival C - 43; D - 43.

Instructions: 1. In each variant of the polygon development, the attachment of consumers to suppliers is done according to the minimum total mileage (a criterion equivalent to the minimum total freight charges, and therefore corresponding to the self-support). It is possible to simplify the solution of this problem by applying this criterion (and not the minimum of dependent costs).

2. First, it is necessary to consider the option of strengthening the polygon, which makes it possible to implement transport links that are optimal in terms of mileage for the main cargo. For this purpose, the optimal attachment of the points of departure and arrival of this cargo is carried out without taking into account the network capacity. The resulting total cargo flows are determined, and for each line a set of reconstructive measures is established to make it possible to master these flows. For the polygon development and flow development option obtained in this way, the annual costs are calculated, including the adjusted capital investment, contingent and independent operating costs.

3. Further, the calculation is carried out on the principle of excluding individual initially established reconstructive measures with a check whether this leads to a reduction in the above costs (the principle of control reverse viewing in the ICTP method (Belov & Kaplan, 1972)). Initially, it should be noted that the activities of the final stage are excluded, specifically the construction of second tracks in distinct directions. If, for example, the original version provided for the construction of second tracks on two lines, then two new options are formed, in each of which the second tracks on one of the lines are excluded. The attachment is being made again for the main cargo - this time taking into account the capacity restrictions on the line on which the construction of the second track has been "canceled". The aforementioned expenses have been calculated for each of the options, and the option with the lowest expenses is selected, either one of the two new or original option.

Moreover, as per the ICTP methodology, in the event of selecting one of the "new" alternatives, it would be imperative to maintain the exclusion of reconstructive measures enacted by the chosen alternatives, such as evaluating the feasibility and economic efficacy of the second tracks on two lines or rescinding the electrification of any line. However, in order to reduce the amount of work in solving this problem, we will limit ourselves to only the first step - checking the effectiveness of abandoning the second tracks on one of the polygon lines.

4. Independent expenses and capital investments when choosing the i-th stage of reinforcement for this line are estimated as the sum of the expenditure standards specified in the task from the first to the i-th stage. For example, the capital investment per kilometer of operational length during the

implementation of the third stage of reinforcement is equal to 20+60+120 =200 thousand rubles. (Kaplan, Maidanov & Tsarev, 1978).

In order to assess the significance of the studied disciplines for professional training and the development of professional motivation, the authors applied the methodology of T.D. Dubovitskaya (Dubovitskaya, 2003). T.D. Dubovitskaya developed a test-questionnaire, which consists of 18 judgments and contains two scales: (1) the importance of the subject for the professional training of a future specialist and (2) the importance of the subject for the development of professional motivation. The students are presented with a total of 18 judgments, and for each one, they are required to provide an answer that aligns with the notation enclosed in brackets: true - (++); probably true - (+); probably not true - (-); incorrect - (--).

The judgments are as follows:

1. Studying this subject gives me the opportunity to learn a lot of important, valuable things for my future profession.

2. During the lessons on this topic, I am beginning to feel regret for my decision to attend this institution for study and pursue this profession.

3. Through this subject, I gained a better understanding of the complexities of my future profession.

4. What they teach me in the classroom is irrelevant to my future profession.

5. The tasks I perform develop my skills necessary for the future profession.

6. The examples provided by the instructor during class have piqued my interest in my prospective profession.

7. A significant portion of the coursework required of me in the classroom in the subject will not be applicable to my future profession.

8. During the lessons on this subject, I begin to understand the significance of my profession in the life of society and for me personally.

9. If this subject were to be excluded from the curriculum, my professional training would not suffer from its absence.

10. Subject assignments do not enhance the attractiveness of my future profession to me.

11. This subject is considered to be a crucial component of a successful preparation for a prospective profession.

12. I can hardly imagine the use of the knowledge, skills and abilities gained in the classroom in the profession.

13. During the lessons on this subject, I have thoughts that the chosen specialty is not for me and it is better for me not to work in this specialty.

14. During the lessons on this subject, I have the opportunity to present and prove myself in the role of a future specialist.

15. During the lessons on this subject, I am convinced of the correctness of the choice of my future profession.

16. The tasks performed in this subject do not give me an idea about my future specialty.

17. I do not see any connection of this subject with the future specialty.

18. Studying this subject will give me the opportunity to achieve success in the future profession.

Processing of results. The questionnaire indicators are calculated in accordance with the key, where "yes" means positive answers ("true", "probably true"), and "no" means negative ("probably not true", "wrong").

Key.

The significance of the subject for the professional training of a future specialist: Answers "yes" for the No. 1,3,5,11,14. Answers "no" for the No. 4,7,9,16,17.

The significance of the subject for the development of professional motivation: Answers "yes" for the No. 6,8,15,18. Answers "no" for the No. 2.10.12.13.

One point is awarded for each match with the key. The higher the total score, the higher the studied indicator.

RESULTS

This study was conducted at the Samara State University of Railway Transport. The tasks of professionally-oriented training were introduced into the learning process for first-year students of two groups, called experimental, in the amount of 28 and 32 people majoring in 23.05.03 "Electric power supply of railways." In the two control groups, the number of students in which was 27 and 30 people, the tasks of professionally-oriented content were not introduced into the learning process. The experimental groups consisted of students in the target programs. Mathematics is additionally allocated 18 hours for lectures, 18 hours for practical classes, and 36 hours for consultations within the framework of targeted training in the first year. The hours allocated to mathematics decreased in the second year, namely 9 hours for lectures, 18 hours for practical classes, and 36 hours for consultations. In these supplementary classes, professional tasks were taken into account. There were no additional classes in the control groups.

In all groups, there is the same work program for the basic course of mathematics. At the same time, for experimental groups consisting of students in target programs, additional hours are allocated for the discipline "Mathematics". The work program for these groups entails the investigation of mathematical subjects that are not covered in the primary curriculum. In these supplementary classes, professional tasks were taken into account.

Upon completion of both the main and additional courses in the discipline "Mathematics", a survey was conducted in the selected groups according to the methodology of T.D. Dubovitskaya. The results of this survey are presented in Table 4.

Scale	Group			
	Experimental		Control	
	No. 1 (28 students)	No. 2 (32 students)	No. 1 (27 students)	No. 2 (30 students)
The importance of the subject for the professional training of a future specialist	7,8	6,9	5,7	5,4
The importance of the subject for the development of professional motivation	5,7	5,3	4,8	4,1

Table 4. Survey results (the table shows the average score for the group).

According to Table 4, the importance of mathematics for the professional training of a future specialist in the experimental groups is higher than in the control groups. The importance of mathematics for the development of professional motivation in the experimental groups also increased compared to the control groups.

The outcomes of the experiment have demonstrated that the inclusion of tasks that are geared towards professional development in the course "Mathematics" has a positive impact on the growth of students' professional interests. The professional orientation of the fundamental sciences enables the utilization of tasks that are specifically geared towards professionals, necessitating the selection of scientific methods that have been extensively studied in any fundamental discipline, as well as methods from related disciplines, to resolve them. Such tasks should be included in the activities of establishing interdisciplinary links. Professional content tasks not only increase interest in the study of new mathematical material, but also serve to consolidate it and use it in further professional activities. The method of applying such tasks contributes to the successful formation of research competence. Therefore, it is necessary to create personal directed didactic teaching tools that allow systematically organizing the mathematics training of a future specialist. It should be noted that such tasks are effectively used not only in the study of mathematics, but also in the study of physics, chemistry and other disciplines.

For students in target programs, research according to the method of T.D. Dubovitskaya has not been previously conducted.

DISCUSSION

The researchers previously addressed the issues of implementing professionally-oriented tasks, and the results were presented in the authors articles (Arkhipova, Evdokimova & Rudina, 2019; 2020).

V.A. Shershneva (2003), N.Yu. Gorbunova (2017) and others also considered the problems of introducing professionally-oriented tasks when studying mathematics courses in various higher educational establishments, in various fields of study. The results they obtained are similar. It should be noted that such studies have not previously been conducted in the context of targeted training. Nonetheless, at present, this investigation holds significance for the education of contract students at railway universities.

The methodology employed for the introduction of tasks that are geared towards professional development in the study of mathematics for the analyzed specialty has demonstrated the efficacy of a professional orientation in education.

Furthermore, this study demonstrated that the use of professionally-oriented tasks in teaching mathematics is an effective means of implementing the professional orientation of targeted education in general.

CONCLUSION

To conclude, it is noteworthy that the incorporation of mathematical problems with a professional orientation in the training process of railway universities has a favorable impact on the standard of education. Particularly, the enhancement of mathematical reasoning is ensured, the horizons are broadened, and the proficiency in identifying the primary and discarding the secondary is honed. Students also acquire mathematical knowledge that will be useful in their future professional activities.

Hence, utilizing the instance of one of the specialties, namely 23.05.04 "Electric power supply of railways" at the Samara State University of Railway Transport, we have examined a method of instructing in targeted programs by utilizing professionally-oriented mathematics tasks. It should be noted that the described method is applicable for any other direction of training a specialist in the transport industry. We reserve the right to use the tasks of professionally-oriented content in obtaining additional specialties and training programs while conducting further research.

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Math Anxiety, Skills, and Work-related Competencies: A Study on Czech University Students

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Abstract: This article aims to identify the mutual relationship between the awareness of the importance of mathematical competencies for the acquisition of work-related skills and the perceived importance of the latter for performance in future work and successful participation in the labor market. How these issues relate to math anxiety is also explored. The above is examined on a sample of university students in the selected education institution (HEI) N =297 in the Czech Republic in 2022. Subsequently, a series of semi-structured interviews were conducted with the students in 2023. It was found that by improving work-related skills for a future career, math anxiety increases. A crucial finding is up to 47 % of the perceived importance of competencies for future job performance can be explained by understanding the importance of mathematics for their acquisition. The findings of this study have the potential to contribute to the improvement of mathematics education and the development of a more skilled and capable workforce, which can ultimately lead to greater economic growth and social progress. In addition, this article contains practical recommendations on how to use these findings in the practice of mathematics education not only in the selected higher education institution.

Keywords: Mathematics, Transversal skills, Math Anxiety, University Students, Mathematics Education

INTRODUCTION

Mathematical skills are at the core of the ability to solve problems and think logically as well as innovatively. In addition, they directly affect performance on the labor market, as suggested by many authors (Koedel & Tyhurst, 2012; the same findings were reached, e.g., by Cígler, 2018), their importance being highlighted by the International Labor Organization (cf. ILO, 2021) too. In terms of participation in the labor market, mathematical competencies influence the level of cross-cutting transversal skills that go beyond a particular field of expertise and can be exercised in various professions (e.g., Abramihin & Sarai, 2022). If the employees have acquired these skills and knowledge, then the organizational capital of the employer firm possesses sufficient flexibility supporting its competitiveness. Constant updating of knowledge and competencies in a continuous educational process become key factors for ensuring the labor market sustainability (Askar, Petryakov, Reiff-Stephan & Ungvari, 2022). Hence, the authors of the present paper consider it critical to tackle the issue of the acquisition of math skills by college students preparing for the future profession. The selection of the target group of students corresponds to the latest findings demonstrating a significant relationship between the acquired mathematical competencies and success in higher education (Delaney & Devereux, 2020), which may then positively affect the working career.

The aim of this article is to identify the mutual relationship between the awareness of the importance of mathematical competencies for the acquisition of work-related skills and the perceived importance of the latter for the performance in future work and successful participation in the labor market. How these issues relate to math anxiety is also explored. Based on the findings, specific recommendations for teachers will be formulated. The above is examined on a sample of freshman/woman undergraduate students in the selected higher education institution (HEI).

Mathematical Skills, Stress, and Anxiety

Mathematical competencies (skills) can be understood as the ability to develop mathematical thinking and use it in solving everyday problems (cf. EU Reference Framework, 2007). Such a concept is incorporated into the mathematical or competence components of most European curricula in considerable overlap with problem-solving and learning competencies. The components of communicative and work competencies are also closely related to math skills, connections with social, personal, and civic competencies being rare and rather partial (Krátká, Zelendová, Cachová & Přibyl, 2018). An adequate level of mathematics skills facilitates access to the labor market and retention of employment in high-quality and permanent positions. However, large numbers of working-age adults as well as youth (20–25 %) in elementary education lack these core skills (see 2012 PIAAC - Programme for the International Assessment of Adult Competencies and 2015 PISA test results, respectively). The enhancement of the aforementioned skills is closely related to the overall (un)popularity of mathematics. In the Czech Republic, pupils and students' attitude towards mathematics gets worse in the course of schooling. There is a significant drop at the onset of the second stage of elementary school, the relation to math remaining basically the same after the transition to high school, but the downward trend keeps going on (Chvál, 2013) A similar tendency is also observable in other countries (Gokkurt-Ozdemir, Yildiz-Durak, Karaoglan-Yilmaz & Yilmaz, 2021). The above trend was confirmed by a survey on a sample of economics students immediately after they started university (see Havlínová & Zelendová, 2019). Some student statements ("*I'm scared of the speed with which people around me can calculate*", "*Due to oral and written exam stress, I got nosebleeds, which led to ridicule and humiliation*") indicated that the dislike of mathematics could in many cases be considered math anxiety. It is not just about the manipulation of numbers, but about any problem that one considers to be mathematical. (Moore, Rudig & Ashcraft, 2015). At colleges of economics, math stress or even math phobia can pose a big problem, mathematics playing an important, although usually not a major, role in the course curriculum.

For use of the article math anxiety is understood as a psychological condition characterized by intense fear, anxiety, or apprehension when faced with mathematical tasks, resulting in reduced mathematical performance and avoidance behaviour. It can hinder an individual's ability to engage effectively with mathematical concepts and activities. The above definition is consistent with the understanding of math anxiety in recent peer-reviewed articles (Storozuk & Maloney, 2023).

The Importance of Math Skills for Successful Participation in the Labor Market

Math skills and thinking, along with the willingness to use mathematical models, have long been among the key competencies required in the labor market, as is highlighted by many studies (cf. Reynolds & Mackay, 1997) and institutions (WEF, 2015). Since mathematical skills are a prerequisite for acquiring other competencies such as analytical thinking, problem solving, etc., it is not surprising that, according to the International Labor Organization, the acquisition of math skills is crucial for the 21st century labor market (see ILO, 2021). This institutionalized framework of core skills played a key role in compiling the first part of the survey questionnaire for data collection presented below. Across the professional community, there is a consensus that mathematical skills are of considerable importance for success in the labor market. Whether the general public is aware of this, however, remains a question.

AIMS AND RESEARCH HYPOTHESES

The paper aims at identifying the relationship between the awareness of the importance of math skills for developing work-related ones and the perceived importance of these job-related skills for future career and participation in the labor market. The secondary goal is to investigate the connections between the above relations and math anxiety. Based on the findings, specific recommendations for teachers will be formulated. The recommendations will also include options for the organisation of teaching in relation to the real possibilities of classrooms for teaching mathematics at AMBIS University.

Data and Methodology

The respondents of the present survey were recruited among first-year undergraduates of AMBIS University. At the introductory lecture of the mathematics course in the 2022 winter semester, newly admitted students were administered an electronic questionnaire about their opinion on mathematics. The creation of the questionnaire, its distribution and all the evaluation were carried out by the authors of this article. The article provides analysis of original data, not yet processed in any other article. The research was undertaken at the very beginning of their studies so that they could not be influenced by the college experience. Data collection and subsequent analysis is the work of the team of authors of this article. Out of a total of 564 eligible students of the Economics and Business Management program**, 297 respondents (of all genders M/F) completed the survey questionnaire**.

After data analysis, **additional semi-structured interviews were conducted with randomly selected students**. This activity was carried out to better understand of finding from quantitative research and provide the best possible recommendations for future teaching. The qualitative investigation was more in the form of a research probe. In the case of the interviews conducted, it is not a representative sample in the static sense.

Identification of the respondent	**Gender**	**Age**	**Date of interview**	**Interviewer**
Respondent No. 1.	Female	22	07/09/2023	Farkacova
Respondent No. 2.	Male	20	08/09/2023	Farkacova
Respondent No. 3.	Male	22	11/09/2023	Farkacova

Table 1: Basic information about the respondents in the short interviews

Source: Authors' own computation

With regard to the subject of analysis, mathematical anxiety, which is partly a psychological and social phenomenon, the **combination of qualitative and quantitative investigation** seems to be optimal (Ochrana, 2015, 2022).

The definition of individual variables being based on the 2021 ILO global framework on core skills, the following three variable clusters are examined:

The first group of variables comprises those describing the degree of need that students attribute to mathematical competencies as a precondition for gaining job-related skills (hereinafter referred to as KMTS). These skills are represented by analytical and critical thinking, creative and innovative thinking, strategic thinking, learning to learn, collaboration and teamwork, and literacy in the fields of finance, culture, and science. The need for mathematical competencies to develop these skills was measured on a five-point scale from absolutely necessary (1) to absolutely unnecessary (5).

The variables in the second cluster reflect the degree of importance that students attribute to the competencies necessary for successful participation in the labor market, which also include financial, cultural, and scientific literacy (hereafter referred to as TSFC). Importance in individual monitored areas was measured on the same five-point scale as in the previous case.

Finally, the math anxiety variable (hereinafter referred to as MA) was monitored. A proven methodology was employed (cf. Hopko, Mahadevan, Bare, & Hunt, 2003) using nine standard statements illustrating the causes of math stress (see Figure 1). The degree of agreement was measured on a similar five-point scale from the most stressful attitude (1) to the most relaxed attitude (5).

The following research hypotheses were proposed:

- **H1:** The perceived need for mathematical competencies for acquiring work-related skills (KMTS) and the perceived importance of work-related skills for the performance of future work (TSFC) are associated.
- **H2:** Math anxiety (MA) and the perceived need for mathematical competencies for acquiring work-related skills (KMTS) are associated.
- **H3:** Math anxiety (MA) and the perceived importance of work-related skills for future work performance (TSFC) are associated.

For a visual overview, see Figure 1 below, methodological notes on the used CANCOR statistical model following underneath.

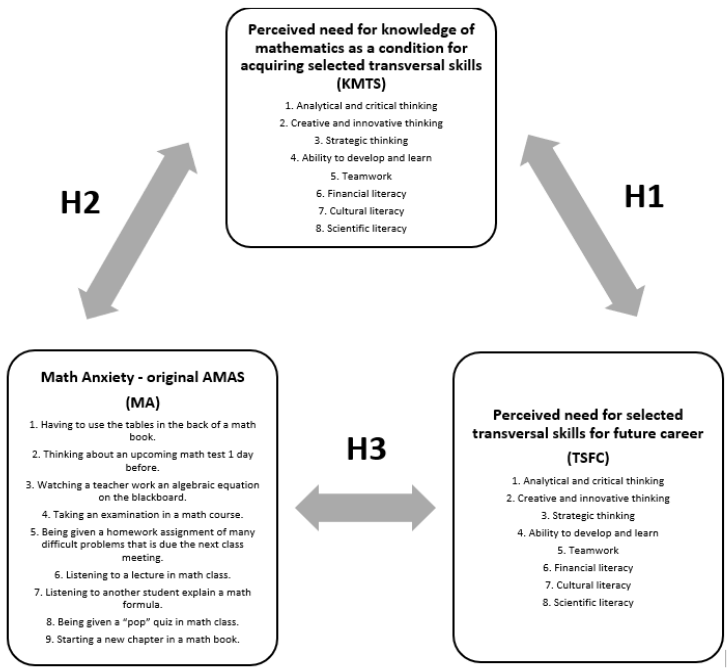

Figure 1 Graphic display of relationships between variables

Source: Authors' own elaboration

Canonical Correlation Analysis (CANCOR) is a multivariate statistics technique that analyzes correlations between two groups of variables without stating any of them as the dependent or independent one. For instance, to evaluate the relationship between the KMTS group of eight variables and the MA cluster of nine variables, the comprehensive list of all relevant "simple" correlation coefficients between two variables would present 9*8=72 correlation coefficients with no overall interpretation. Instead, CANCOR performs a limited number of canonical correlation coefficients with complex and straightforward interpretations of the correlation between the two groups of variables. To interpret the concept of canonical correlation coefficients, the concept of canonical variables is to be introduced.

The CANCOR model identifies canonical variables, i.e., linear combinations of variables in one variable cluster. Since the model focuses on correlations between two groups of variables, in fact, pairs of canonical variables are identified – one canonical variable representing the "left group", the other the "right group" of variables. If the number of variables in the left and right groups is different, then as many pairs of canonical variables are identified as there are in the smaller dataset.

The canonical variables are defined by the CANCOR model so that the two linear combinations of original variables (one in each dataset) have the largest possible correlation. Once the canonical variables are defined, the analysis can proceed with the computation of correlations between the canonical variables. Since this correlation is measured between canonical variables, it is called canonical correlation. In the given case with the KMTS cluster of eight variables and the MA group of nine variables, there are eight pairs of canonical variables and eight related canonical correlation coefficients ordered in sequence of gradually decreasing absolute values. The tests of statistical significance of individual canonical correlation coefficients can be performed consequently. (Härdle and Simar, 2015.)

RESULTS

The following section presents the results of hypotheses testing (H1–H3) and the discussion of key findings.

H1: The perceived need for mathematical competencies for acquiring work-related skills (KMTS) and the perceived importance of work-related skills for the performance of future work (TSFC) are associated.

When measuring the association between KMTS and TSFC variables, all canonical correlation coefficients proved to be statistically significant at 5% significance level, thus confirming the strong connection between KMTS and TSFC. Up to 47 % of the perceived importance of competencies for future job performance can be explained by understanding the importance of mathematics for their acquisition.

Number	Canonical Cor. Coeff	p-value
1	0.87	0.00
2	0.74	0.00
3	0.68	0.00
4	0.61	0.00
5	0.55	0.00
6	0.43	0.00
7	0.38	0.00
8	0.36	0.00

Table 2: Canonical coefficient of correlation between KMTS and TSFC variables

Source: Authors' own computation

A two-dimensional graph in Figure 2 below presents the directions of each of the original variables showing the first two dimensions. Interpreting the proximity of KMTS to TSFC variables makes it possible to understand the relation between the two data sets. Being the most important (as the first pair of canonical variables leads to the highest possible correlation), dimension 1 is plotted on the x-axis, meaning that the variables scoring high or low on this dimension will be on the right and left side of the chart, respectively.

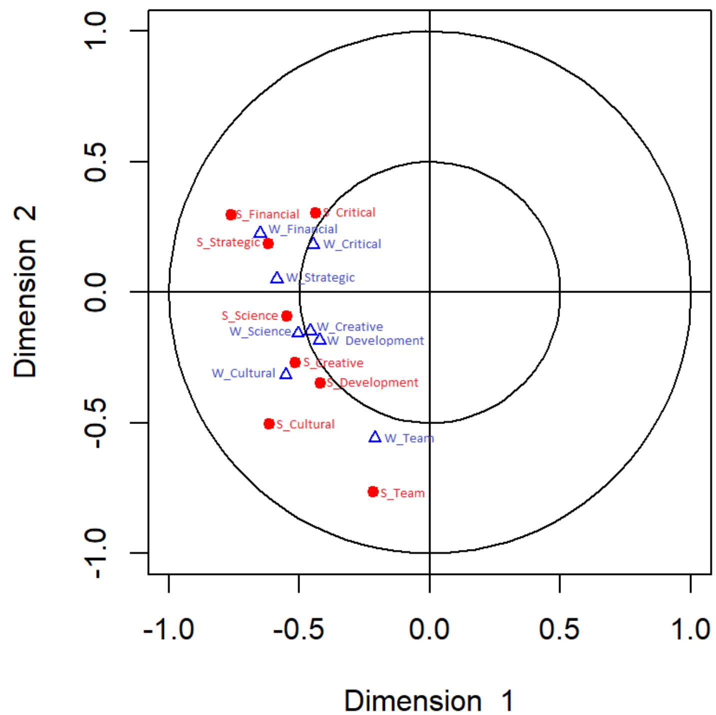

Figure 2 Two-dimensional KMTS (S_Variable) and TSFC (W_Variable) variable directions

Source: Authors' own computation

Based on the analysis of the obtained data, it was proven that the need for mathematical competencies to develop work-related skills and the importance of the latter for a future career are related. Therefore, H1 hypothesis was confirmed.

The perceived necessity of mathematical competencies for acquiring core transversal skills according to the canonical correlation coefficient is related to the perceived importance of transversal skills for future work performance. This positive relationship proves that math skills, as a prerequisite for transversal skills development, are understood by the respondents as pivotal in terms of their future career.

Indeed, as stated by the Respondent No. 2: *„I think that basic math is needed everywhere, but due to the complexity of some calculations etc., one doesn't have a chance to master a given case when technology fails. On the other hand, mathematics aids logical thinking.“* Respondent No. 3 had a similar view on the issue. Respondent No. 3 also added *"Whatever my senior position, knowledge of mathematics will be indispensable."*

Mathematical competencies are perceived as a kind of springboard for developing the skills needed for future working life and can act as a stimulus to acquire them.

H2: Math anxiety (MA) and the perceived need for mathematical competencies for acquiring work-related skills (KMTS) are associated.

Number	Canonical Cor. Coeff	p.value
1	0.36	0.03
2	0.24	0.48
3	0.23	0.62
4	0.19	0.81
5	0.15	0.90
6	0.11	0.91
7	0.08	0.87
8	0.04	0.76

Table 3: Canonical coefficient of correlation between MA and KMTS variables

Source: Authors' own computation

Only the first canonical coefficient of correlation between MA and KMTS variables is significant at a 5% level of statistical significance. However, only 4.81 % of KMTS can be explained via math anxiety.

A two-dimensional graph in Figure 3 below presents the directions of each of the original variables showing the first two dimensions. Interpreting the proximity of MA to KMTS variables makes it possible to understand the relationships between the two data sets. The direction of the first dimension is of particular importance as only the first canonical correlation coefficient proved to be statistically significant. Team, Creative, Cultural and Development variables are associated with math anxiety.

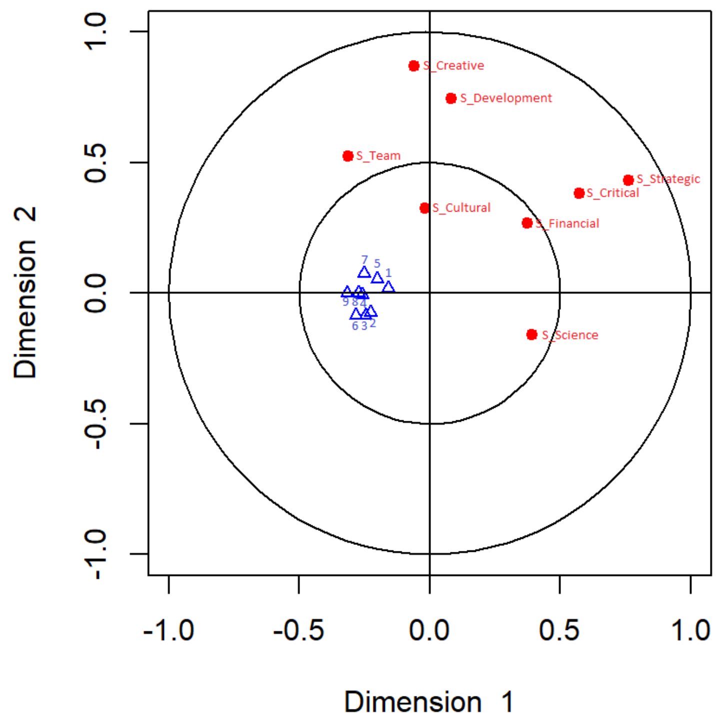

Figure 3 Two-dimensional MA and KMTS variable directions

Source: Authors' own computation

The analysis of the obtained data indicated that variables MA and KMTS are weakly related to each other only through the first canonical correlation coefficient. The hypothesis is therefore verified, but only 4.81 % of KMTS can be explained by math anxiety.

Furthermore, it was found that as the perceived importance of mathematical competencies to gain work-related skills increases, math anxiety also grows (CC1=0.36, p=0.03), which is mainly due to the relation of the four KMTS variables (Team, Creative, Cultural and Development) to MA, as can be seen from Figure 3.

The findings also confirm the viewpoint of Respondent No. 2, who sees mathematical skills as key to acquiring the skills needed for a future career. As he described, he experiences a range of emotions both positive and negative when learning mathematics, specifically stating: joy, frustration, and a sense of victory. He also added that negative emotions prevail the moment he takes longer to calculate a math problem correctly.

Respondent No. 3 adds to the situation: *"I primarily experience anxiety when solving math problems. I know I need to understand it and learn it, but at the same time it's hard and maybe I don't fully understand it, I'm pressed for time. It's such a necessary evil that I actually want to succeed in the industry. At the same time, I know that if others have mastered it, I have to master it too, so I hold out hope for a successful resolution."*

H3: Math anxiety (MA) and the perceived importance of work-related skills for future work performance (TSFC) are associated.

Only the first canonical coefficient of correlation between MA and TSFC is significant at a 5% level of statistical significance. However, only 4.26 % of TSFC can be explained by math anxiety.

Number	Canonical Cor. Coeff	p.value
1	0.33	0.03
2	0.26	0.28
3	0.22	0.52
4	0.20	0.66
5	0.17	0.77
6	0.11	0.89
7	0.10	0.80
8	0.03	0.91

Table 4: Canonical coefficient of correlation between MA and TSFC variables

Source: Authors' own computation

A two-dimensional chart in Figure 4 displays the directions of each of the original variables showing the first two dimensions. Interpreting the proximity of MA to TSFC variables makes it possible to understand the relationships between the two data sets. The direction of the first dimension is especially important as only the first canonical correlation coefficient proved to be significant at a 5% level of significance. Team and Cultural variables in particular, as well as those of Financial and Development, are associated with math anxiety.

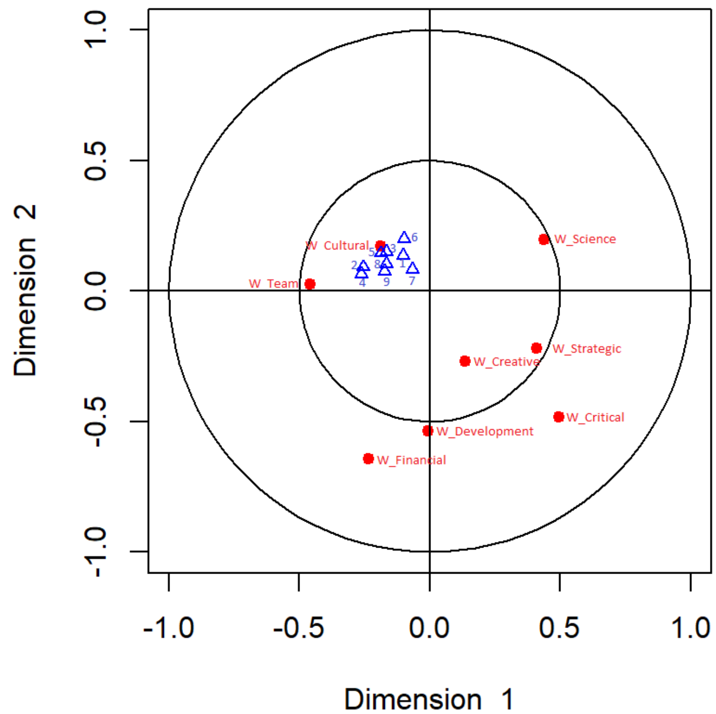

Figure 4 Two-dimensional MA and TSFC variable directions

Source: Authors' own computation

Analysis of the available data revealed a weak correlation between MA and TSFC through the first canonical correlation coefficient only. The H3 hypothesis was also confirmed, but only 4.26% of TSFC can be explained via math anxiety. In addition, it was found that with improving the work-related skills for the future career, math anxiety increases (CC1=0.36 p=0.03).

This means that those respondents who suffer more from fear of mathematics are at the same time more reflective of the importance of the given skills for their future job. This is largely caused by the relation of the four TSFC variables (Team, Cultural, Financial and Development) to MA, as can be seen from Figure 4. (The above finding is consistent with the results of a literature search in the IDEAS database conducted in January 2023.)

This relationship was also confirmed in the case of some of the respondents who took part in the semi-structured interviews. For instance, Respondent No. 2 suffered from a fear of mathematics and considered mathematics important for his future profession, Respondent No. 3 had an identical point of view. Respondent no. 1 reflected on the situation in a similar way, but she also added that "*I still hope that I will not encounter mathematics at work*".

Semi-structured interviews failed to identify the cause of this association. However, in general terms the relationship is understandable and was even suggested by Respondent No. 3. If importance of math skills and future needed skills in labour market is subjectively understood as crucial, then fear of failure grows and anxiety or fear may develop.

CONTRIBUTION OF THE FINDING - Recommendations for the Design of Classroom and Teaching Methods

The study has identified a strong association between the perceived need for mathematical competencies in acquiring work-related skills and the perceived importance of work-related skills for future job performance. This suggests that mathematics plays a pivotal role in developing the transversal skills that are important for success in the workplace. The results of this research have significant implications for educators.

Here are selected recommendations for designing the classroom and teaching methods to enhance learning, based on the research findings:

a) **Interdisciplinary connections**: Emphasize the interdisciplinary nature of mathematics by integrating it with other subjects such as economics and management. Showcase how mathematical concepts are applied in various fields, fostering connections and demonstrating the relevance of math beyond the classroom. Specifically, within the target student group, combining mathematics teaching with seminars on key subjects like economics (Ambis, 2023) would be beneficial. For instance, students could apply mathematical skills to determine price elasticity of demand and model predictions for future enterprises. It is worth noting that students already practice topics using practical examples from business economics (Zelendová, 2021), but these examples may be overly simplified, limiting the connection between the topic and real-world applications. This was also confirmed by Respondent No. 3, who stated in a semi-structured interview, *"Mathematics teaching could be less general and more connected to the student's field of study."*

b) **Contextualized learning**: Create authentic learning experiences that simulate work-related contexts where mathematics is applied. Indeed, the findings of recent studies recommend this approach in mathematics education (Wang, Lee, Zhu and Ozdemir, 2022). This can include inviting guest speakers from relevant professions, organizing field trips to workplaces, or designing simulated work scenarios. Such experiences enable students to understand the practical implications of mathematics, reinforcing the importance of mathematical competencies for success in specific careers. Given that the target student group aspires to hold positions such as Managing Director, Marketing Manager, or Chief Financial Officer (Ambis, 2023), it would be appropriate to divide students into groups based on their interests in marketing, finance, and management studies. Within these groups, assign mathematical problem-based tasks and demonstrate their practical applications with input from industry practitioners. The suggestion is supported by the viewpoint of Respondent No. 1 who stated, "*Maybe yes, involving people from practice would be good. Practice tends to be different from theory. Practice sometimes makes more sense*." In addition, Respondent No. 3 stated: *"It would change the way mathematics itself is viewed. I could see myself using it more extensively in practice."* The role of the practitioner has proven to be crucial in similar cases with university students (Farkačová, 2022). On the other hand, this solution might not suit all students. As Respondent No. 2 noted: "*I wouldn't want people from practice teaching math. I like to have the subject explained by someone who knows how to deliver and explain it*." A solution could be to pilot test this approach and then explore the benefits from the perspective of the students concerned.

Additionally, considering the physical limitations of the classrooms, it would be advisable to divide students into individual classrooms rather than accommodating multiple groups in one room. In the case of dividing the students within a single classroom, the classroom division could look like see appendix no. II. The existing classrooms have sloping floors and fixed desks (refer to appendix photos), making it impractical to rearrange them. Therefore, incorporating online classes (Tartavulea, Albu, Albu, Dieaconescu, and Petre, 2020) would be appropriate, as it allows for effective practice of newly acquired knowledge while reducing math anxiety and providing a more secure and confident learning environment. Lastly, it is worth noting that the school, as a result of the COVID-19 pandemic, is technically and skill-wise prepared for online and hybrid forms of learning.

c) **Measure the level of math anxiety regularly:** It is advisable to measure the level of math anxiety regularly to assess any improvement. It would be beneficial to test students at the beginning and at the end of the semester. This dual measurement allows for the evaluation of any changes in math anxiety. Additionally, it would be particularly useful to compare the changes in math anxiety with exam performance. An identical questionnaire and data collection instrument used in this research could be employed to measure mathematical anxiety. Considering the broad context and implications of mathematical anxiety, it is still important to compare it with achievements in subjects such as Economics, Public Finance, Banking, etc. For instance, a study conducted by Storozuk & Maloney (2023) in Canada has already demonstrated a negative association between mathematical anxiety and mathematical-financial knowledge. It would be desirable to verify these findings in the context of the Czech Republic as well.

This recommendation appears to be particularly important, as confirmed by the findings from the semi-structured interviews. All respondents were admitted to experiencing some degree of mathematical anxiety during the face-to-face interviews. Which confirms that this is a relevant topic that needs to be addressed.

LIMITATIONS OF RESEARCH AND APPLICATION OF RESULTS AND RECOMMANDATION

A certain limitation of the present study is due to the nature of the research sample, the results being indicative primarily of the situation at economic HEIs in the Czech Republic. To ensure the representativeness of the sample, other universities would have to be involved in the survey.

The main limitation of the application of the recommendations for teaching lies mainly in the additional: a) direct financial costs for practitioners, b) additional costs for measuring the level of mathematical anxiety, c) additional costs for changes in approaches to teaching (mathematics is taught simultaneously by up to four teachers who divide the teaching between them), d) additional costs for checking the compliance of the changes made and the requirements of the National Accreditation Office for higher education.

It should also be noted that the math anxiety examined here is a complex phenomenon that transcends the field of education. In the ordinary work process, mathematics is not usually applied in its pure abstract form, but rather in conjunction with other knowledge and skills. (A common example is dealing with statistics, statistical anxiety being another potential subject of further research.)

While there is a fairly extensive literature on the link between math anxiety and math avoidance (long-term processes such as career choice, college course selection, homework avoidance, etc.; cf., e.g., Hembree, 1990; Ashcraft, 2002; Skaalvik, 2018), the authors of the present article were limited by the lack of sources exploring the relationship between math anxiety and job-related competencies. Nevertheless, they identified an area of further research on the MA–TSFC relation, namely the application of expectancy-value motivation theory.

CONCLUSIONS

The purpose of the paper was to analyze the relationship between awareness of the importance of mathematical competencies for acquiring work-related skills and the perceived importance of the latter for future employment and work performance. The study also examined how the above-mentioned awareness relates to math anxiety.

The research objectives were met through a survey conducted in 2022 on a sample of economics college students. To gain a complex understanding of the relationships between the variables semi-structured interviews were then conducted in 2023.

Three clusters of variables (MA, KMTS and TSFC) were analyzed, their interrelationships having been hypothesized (see H1, H2 and H3 in the data and methodology section). The hypotheses were statistically tested using the canonical correlation coefficient, the results confirming all three predicted relationships between the variables.

Developing digital technologies are increasing the demands on the mathematical competencies of labor market participants. Therefore, the article also suggests areas of further research useful not only for modeling access to higher education, but in-house training as well.

The study provides important insights into the role of mathematical competencies in developing work-related skills and their importance for future job performance. The results of this research can be used by educators to improve the effectiveness of mathematics teaching at universities. In addition, the article contains specific recommendations for mathematics education in the areas of Interdisciplinary connections, Contextualized learning.

The findings and recommendations may be useful not only in the case of the Czech Republic, but also in other culturally close countries. To determine cultural proximity, the authors recommend using Hosftede's cultural dimensions (Kunwar Jagat Bahadur, 2021).

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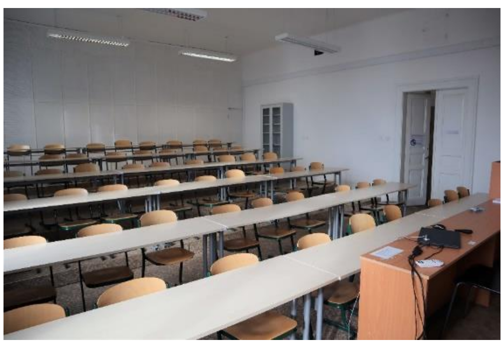
APPENDIX I.

Photos of a typical classroom where seminars are held (location: Prague, Libeň, classroom 208).

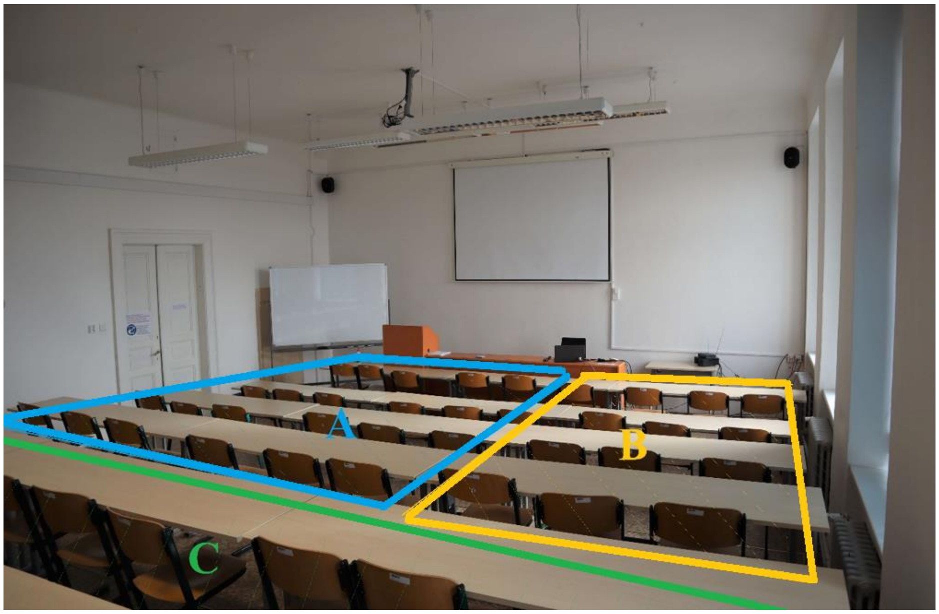


The essential specifics of the rooms for teaching mathematics are:
- the tables are fixed to the floor and cannot be manipulated,
- the floor of the rooms is not flat, it is sloping, which makes it difficult for the students and the teacher to move around the room.

Overall, the classrooms are rather adapted for frontal teaching.

APPENDIX II.

Graphical representation of the possibilities of classroom division (in case of classroom 208). In this case, students would be divided into two groups for mathematics lessons (group A and group B). Part "A" would have one career focus (for instance, Marketing = group A), and part "B" would have a second career focus (for instance, Finance = group B). Each section would be led by one teacher (a practitioner), and student collaboration would take place in groups of about 8-12 people. A mathematics teacher would assist within both sections if needed. At the same time, the teacher could use the part of the classroom marked "C" to deal with individual queries. This back part of the classroom would allow students to experience greater psychological safety and thus alleviate math anxiety.



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Professional Development Interventions for Mathematics Teachers: A Systematic Review.

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Abstract: In many countries around the world, stakeholders engaged in driving education reform policy use teacher professional development to improve the quality of teacher learning, expecting a positive effect on the quality of teaching. Given the high level of expectations for professional development, it is crucial to identify the characteristics of effective teacher professional development. Therefore, we conducted a systematic literature review of professional development interventions for mathematics teachers. We sought to identify the characteristics of interventions with positive and statistically significant effects on students' mathematics achievement. Our review includes 12 professional development interventions which included elements of structured pedagogy intervention (i.e., teacher training, on-site teacher support, and resources for teachers and students), in addition to initial professional development and follow-up workshops. Utilizing technology has proved to be beneficial for student learning, but less so for teacher learning. The results of the reviewed studies indicate that changes in instruction can be implemented incrementally, beginning with less complex interventions and progressing to those that are more complex and demanding. Furthermore, we conclude that professional development interventions that seek to improve student learning outcomes in mathematics should include on-site teacher support, mentoring and feedback, teacher-focused resources, and classroom learning materials.

Keywords: effective mathematics interventions, mathematics education, systematic review, professional development

INTRODUCTION

In many countries, education reform policymakers use teacher professional development to improve teacher learning, which is expected to improve teaching, student achievement, and teachers' long-term beliefs and attitudes. Professional development (PD) consists of unplanned and planned learning experiences and activities that enhance teachers' knowledge, attitudes, and skills, as well as their teaching practices (Day & Leitch, 2007; Avalos, 2011). Teacher professional development is defined as learning that can be either individual or collective, but should be contextualized in the teacher's workplace — the school — and contributes to the development of competencies through a variety of formal and informal experiences" (Marcelo, 2009). Professional development includes “the body of systematic activities [used] to prepare teachers for their job, including initial training, induction training, in-service training, and continuous professional development within school settings” (Hendriks et al., 2010, p. 19). Consequently, PD includes both pre-service and in-service teacher education (Bautista & Oretga-Ruiz, 2017). Guskey (2002) notes that contemporary teachers expect in-service training to provide them with concrete and specific procedures they can implement daily in the classroom.

Fullan and Hargreaves (2016) distinguish between professional development and professional learning, although some authors use the terms interchangeably. According to them, professional development consists of activities for their own sake, whereas professional learning is characterized by measurable quality, performance, and teacher impact. According to Fraser et al. (2007), professional development ensures teachers' professionalism, while professional learning results in “specific changes in the professional knowledge, skills, attitudes, beliefs, or actions of teachers.” However, Evans (2014) notes that professional development is the process by which people's professionalism can be viewed as being enhanced with a degree of permanence that exceeds transience. This paper will use the term professional development to mean planned, collective, in-service learning activities with the core aim of improving teachers’ competencies and teaching practices, and which strive to contribute to the quality of student learning.

In mathematics teacher professional development, it is vital to recognize the various types of knowledge. These include content knowledge (CK), which is mathematical knowledge, pedagogical content knowledge (PCK), which is “ways of representing and formulating the subject that make it comprehensible to others” (Shulman, 1986, p. 9), and pedagogical knowledge (PK), which is “independent knowledge of how to optimise learning situations in the classroom in general” (Krauss et al., 2008, p. 874). Ball et al. (2008) separate mathematical knowledge from pedagogical knowledge. Pedagogical knowledge encompasses content, students, instruction, and curriculum. Therefore, pedagogical content knowledge necessitates an understanding of students' mathematical reasoning, their interests, and their areas of difficulty. This latter type of knowledge enables teachers to design and implement lesson plans in accordance with the subject curriculum, taking student needs into account, using a variety of teaching methods, mathematical concept presentation methods, and suitable examples.

Given the high level of interconnected expectations currently placed on professional development, it is crucial to identify the characteristics of effective teacher professional development. Specifically, we are interested in factors that have positive and statistically significant effects on students' mathematical achievement. In order to rigorously investigate this issue, we conducted a systematic literature review focusing on randomized experimental studies conducted in the past 21 years (i.e., 1999–2020) in order to identify the characteristics of effective interventions in terms of students' mathematical achievement. Our analysis encompasses not only professional development programs but also multicomponent interventions that incorporate a variety of instructional and pedagogical techniques. Our decision to include multicomponent interventions comes from Hull et al. (2018), who contend that multicomponent interventions are particularly pertinent in real-world applications that seek to improve educational outcomes, such as student achievement. Therefore, we were interested in any randomized controlled trial (RCT) that evaluated instances in which teacher professional development led to statistically significant gains in student mathematics achievement. By conducting this systematic review, we tried to answer the following research question: What are the features of professional development interventions for mathematics teachers that have a statistically significant and positive effect size?

METHODS

The process of conducting systematic review began in the middle of 2021; therefore, we decided to include available randomized controlled trials published in English language between 1999 and 2020, carried out across all grades of elementary, middle, and high schools that include PD programs for mathematics teachers. We have chosen 1999 as our starting point because of Kennedy (1998), who conducted a systematic review of professional development programmes aimed at enhancing student learning in mathematics and science. The method used in this review is based on the emerging literature addressing this multicomponent approach (i.e., Petticrew & Roberts, 2006; Gough et al., 2017; Polanin et al., 2017; Siddaway et al., 2019), a procedure consisting of several steps. First, we formulated the research question. Second, we defined the search terms and selected appropriate databases. Third, we used inclusion and exclusion criteria to ensure the scientific quality of the relevant publications (Table 1). Finally, the data answering the research question was extracted, analysed, and interpreted.

For quantitative studies, the effect size (i.e., related to students' mathematics achievement) and its statistical significance (p value) are especially important. To assess its statistical significance, the effect size should be at least conventionally significant ($p <.05$). However, there is no clear criteria by which to determine whether the effect size is practically significant as it depends on numerous diverse factors such as sample size; research design; measure type; and differences among students, teaching subjects, schools, etc. (Hill et al., 2008; Slavin & Smith, 2009; Lipsey et al., 2012; Cheung & Slavin, 2016; Kraft, 2020), thus could hardly be unambiguously interpreted. In a bid to seek some clarity and consistency in this regard, Bakker et al. (2019) suggest 12 points for interpreting effect sizes in mathematics education journals some of which are technical (e.g. Which calculation of the effect size is used? or What is the confidence interval around the point estimate?), some methodological (e.g. To what is the effect compared? or Focus on offering or receiving?), an some empirical/ontological (e.g. What is the context? or What was the sample?). In summary, those points cover research design, alignment of the intervention and measurement, intervention duration, sample size, and context. Bearing in mind this potential for ambiguity and multiplicity, we have included various key factors in Table 2 (i.e., country where the study was conducted, study duration, sample size, and type of measure) in order to allow for different interpretations of intervention impacts. For example, if an education system is of a low standard, it may have more room for improvement (Bakker et al., 2019), meaning that interventions in such a context may result in higher effect sizes. Similarly, an intervention carried out over a longer period of time may produce a larger effect size (Bakker et al., 2019), as might a program in which the researchers develop their own outcome measure (Lipsey et al., 2012). In contrast, however, Slavin & Smith (2009) found a statistically negative correlation between effect size and sample size in their research on elementary and secondary mathematics programs.

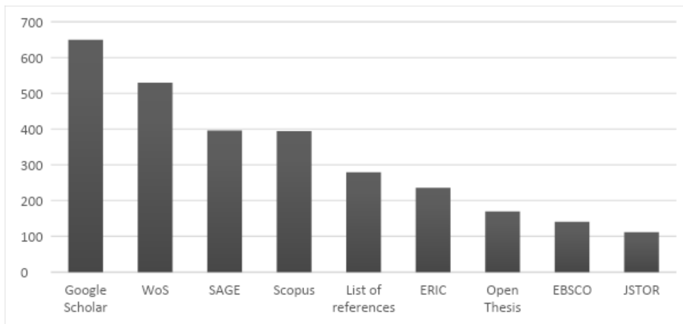

Figure 1. Number of studies included in the analysis, arranged by source.

Our main resources for finding relevant literature — empirical studies published between 1999 and 2020 — were electronic databases. We focused on peer-reviewed journal articles and non-academic research publications (commonly called grey literature). Our search for relevant publications included these databases: EBSCO, Education Resource Center (ERIC), Google Scholar, JSTOR, SAGE, SCOPUS, and Web of Science (see Figure 1). We used the following search terms in the titles, abstracts, and keywords of articles: "professional development" OR "professional learning" OR "in-service education" OR "in-service training", AND math or science AND experiment OR trial. The use of these search terms, which are in line with prior discussions in this paper, was intended to limit the data collection on the sources connected to our research questions. Due to the fact that we were conducting a systematic review of the literature regarding the professional development of science teachers at the same time (published in a separate publication), we included the keyword "science" in the search. In the subsequent analysis, the research related to science was separated from that related to the professional development of mathematics teachers. The use of above search terms was intended to limit the data collection to sources connected to our research question. In order to identify relevant grey literature for the review (e.g., reports, academic theses, working papers, etc.), we searched relevant targeted online repositories, such as Google Scholar and Open Thesis. In addition, we examined lists of references in selected articles, systematic reviews, and meta-analyses related to the professional development of mathematics teachers. As a result, we yielded 2,908 potential publications (i.e. academic papers and grey literature publications) featuring a relationship between our keywords.

Next, we carried out a preliminary review of these 2,908 publications, searching for studies that reported only on randomized design comparing treatment groups with groups using existing programs, plus that included PD focused on mathematics and students' mathematics achievements (reported either as a calculable effect size or quantitative results). We also followed Slavin's guidelines whereby selected studies should include a pre-test stage, although "randomized experiments without pre-tests are acceptable if attrition is low and equal between experimental and control groups" (2008, p. 8). Furthermore, we employed the guideline that treatment and control groups must include at least two teachers and 30 students (Cheung and Slavin, 2016), where a combination of differential and overall attrition is within an 'optimistic' boundary (What Works Clearinghouse, 2020). Lastly, we required that an intervention lasted at least 12 weeks (Pellegrini et al., 2018) and that the PD program was described in detail. Inclusion criteria can be seen in Table 1. We excluded non-randomized experiments, evaluation research, and randomized designs that did not examine students' mathematics achievement. We utilized Rayyan QCRI, a free web application designed specifically for systematic reviews and other knowledge synthesis initiatives (Ouzzani et al., 2016) for this section of the systematic review. The utilization of this application aided in the process of screening and selection of studies. The initial screening yielded 54 documents that were further examined according to the inclusion and exclusion criteria (Table 1).

Inclusion criteria	Exclusion criteria
Students in grades 1-12	Pre-kindergarten, kindergarten children as well as students in postsecondary education
In-service professional development for mathematics teachers was part of the intervention	Professional development for mathematics teacher is related to pre-service teacher education
The professional development programme is described in detail	There is insufficient information regarding the professional development programme
Randomised experiment comparing treatment groups with groups using business-as-usual or other programme already in place	Non-randomized experiment, non-experimental research, or randomized experiment without a control group
Study comprises pre-test although "randomized experiments without pre-tests are acceptable if attrition is low and equal between experimental and control groups" (Slavin, 2008, p. 8)	Randomized experiments without pre-test and with high attrition between groups along with those "in which pre-test differences are more than 50% of a standard deviation" (Slavin, 2008, p. 8)
Studies with at least two teachers and 30 students in treatment and control groups (Cheung & Slavin, 2016, p. 286)	Either treatment or control group have only one teacher
Study includes quantitative measures of students' mathematics outcomes (e.g. standardized test or a test that was developed by researcher which is fair to all treatment and control groups)	Study includes only qualitative data, or quantitative measures without students' mathematics performance, or dependant measures favour some of the treatment groups

Inclusion criteria	Exclusion criteria
Effect size for students' mathematics outcomes is calculated using appropriate analysis or it is possible to calculate it from given results	Effect size is not included, or it is calculated by inappropriate analysis and it is not possible to calculate/correct it from given results
Study had positive effect sizes.	Study had negative or statistically insignificant effect sizes.
Interventions lasting at least 12 weeks "to make it more likely that effective programmes could be replicated over extended periods" (Pellegrini et al., 2018, p. 8)	Interventions lasting less than 12 weeks
Study was published from 1999 to 2020	Study was published before 1999 or after 2020
The research could be conducted in any country, but the paper must be written in English	The paper is not written in English

Table 1: Inclusion and exclusion criteria

The inclusion and exclusion criteria (Table 1) were applied to the full-text versions of the remaining articles. This yielded 12 publications that we selected for our systematic literature review (see Table 2). For some of these selected interventions, we found additional studies that gave us better insight into PD programs. Most of the selected studies were conducted in the USA (8), and there was one study each from the following countries: Belize, China, Canada, and Pakistan. In terms of the year of publication, most studies (8) were published recently, i.e. between 2016 and 2020. Although our intention was to include studies published in the last 21 years, we did not find any published before 2007 that met the inclusion criteria. While reading the full-text versions of the selected publications, we extracted the relevant data necessary for answering the research question. For this part of the research, we used EPPI Reviewer Web (https://eppi.ioe.ac.uk/EPPIReviewer-Web), a web-based software designed for various types of literature review, including systematic reviews. This extraction of data was done using categories that we determined by analysing the selected PD programs (i.e., initial professional development, follow-up workshops, coaching, online learning, use of videos, and types of teacher knowledge).

RESULTS AND DISCUSSION

In our analysis of the included studies, we first focused on changes in teaching, because without changes in teaching it is difficult to expect any improvement in student results (Guskey, 2002; Kunter et al., 2013; Campbell et al., 2014; Kennedy, 2016). All analysed interventions have resulted in positive changes to teaching practices, student knowledge, and their mathematics achievement. Within this, we identified various teaching strategies which positioned students as active participants in the teaching-learning process: i.e., where efforts were made to increase student engagement in line with standards and curricula, providing them with optimal challenges to reach a higher level than they were previously at (Early et al., 2016). As such, teachers elicited deeper levels of student thinking, reasoning, higher-order thinking skills, and learning (Newman et al., 2012; Lewis & Perry, 2017; Chen et., al. 2020).

Title of program (Reference)	Country / Study duration / School year(s)	Aim	Baseline sample size	Type of measure, effect size†
1. eLearn (Beg et al. 2019)	Pakistan / 4 months / 2016/2017	Examining the effectiveness of short videos on student achievement in mathematics and science	60 schools, 274 eighth-grade teachers, 2,999 students	c) .26
2. Video-based teacher professional development (Chen et al., 2020)	China / 1 year / n/a	Investigating the efficacy of video-based professional development programs using a discourse visualization tool	16 schools, 54 sixth- and seventh-grade teachers, 1,507 students	c) .24
3. Every Classroom, Every Day (Early et al., 2016)	USA / 2 years / 2009/2010, 2010/2011	Instructional intervention aimed at increasing students' learning and achievement	20 high schools, n/a ninth- and tenth-grade teachers, 8,250 students	a) .15*
4. Math Pathways and Pitfalls (Heller et al., 2007)	USA / 1 year / 2003/2004	Implementing teaching materials to improve instruction	40 elementary schools, 99 second-, fourth-, and sixth-grade teachers, 1,971 students	c) .49*
5. Teacher-Led Math Inquiry (Hull et al., 2018)	Belize / 1 year / 2011/2012	Examining an effect of compound intervention on the students' mathematical skills	24 elementary and middle schools, 282 first- to eighth-grade teachers, 6,576 students	c) .27
6. Classroom Connectivity in Mathematics and Science Achievement (Pape et al., 2012, Irving et al., 2016)	USA / 1 year / 2005/2006	Using classroom connectivity technology for formative assessment	n/a schools, 82 ninth-grade teachers, 1,224 students	c) .27
7. Lesson Study (Lewis and Perry 2017)	USA / 12 weeks / 2009/2010	Investigating effectiveness of lessons supported by resource kits	39 elementary schools, 213 second- to fifth-grade teachers, 1,162 students	c) .49*

Title of program (Reference)	Country / Study duration / School year(s)	Aim	Baseline sample size	Type of measure, effect size†
8. Enhancing Missouri's Instructional Networked Teaching Strategies (Meyers et al., 2016)	USA / 3 years / 2011/2012, 2012/2013, 2013/2014	Using technology for developing student-centered instruction	60 middle schools, 100 seventh- and eighth-grade teachers, 3,072 students	a) .15*
9. Alabama Math, Science, and Technology Initiative (Newman et al., 2012)	USA / 2 years / 2006/2007, 2007/2008	Improving students' achievement by using materials, technology, and in-school support	82 elementary schools, 482 fourth- to eighth-grade teachers, 22,557 students	a) .05
10. SimCalc (Roschelle et al. 2010)	USA / 2 years / 2005/2006, 2006/2007	Using technology for learning advanced mathematics	129 middle schools, 228 seventh- and eighth-grade teachers, 2,446 students	c) .61*
11. ASSISTments (Roschelle et al., 2016)	USA / 2 years / 2012/2013, 2013/2014	Providing quality feedback and guidance for students using an online application to do homework	43 middle schools, n/a seventh-grade teachers, 2,850 students	a) .18*
12. JUMP Math (Solomon et al., 2019)	Canada / 2 years / 2013/2014, 2014/2015	Promoting a deep conceptual understanding via collaborative solving of real-world mathematical problems	41 schools, 49 fifth-grade teachers, 592 students	a) .22*
† According to Lipsey et al. (2012), educational programs with practical significance are those with effect sizes equal to or greater than: (a) .08 for broadly focused standardized tests; (b) .24 for narrowly focused standardized tests; and (c) .39 for specialized tests developed for a particular intervention. In this table we have marked statistically significant effect sizes with an asterisk (*).				

Table 2: List of papers included in the analysis.

Learning was based on prior knowledge, critical thinking, and stimulating 'creative solutions to non-routine problems and use of a variety of representations' (Heller et al., 2007, p. 2). Students had opportunities to elaborate upon their thinking, as well as discuss mathematical ideas and test their validity with peers (Heller et al., 2007; Chen et al., 2020), plus an inquiry-based approach and collaborative learning were implemented in the classroom (Meyers et al., 2016; Hull et al., 2018). Lessons utilized a fine-grained guided discovery approach, tailored according to the individual needs of students in the class (Solomon et al., 2019). Utilization of technology in the classroom also facilitated timely, supportive, and specific feedback (Pape et al., 2012; Roschelle et al., 2016), making instructional decisions easier for teachers. Teachers also used technology as an aid to clarify mathematical concepts, either by providing quality explanations using videos (Beg et al., 2019), or by supporting visualization and interaction with concrete embodiments (Roschelle et al., 2010).

Based on the papers analysed, we can conclude that changes in teaching do not always need to be comprehensive in order to lead to more effective student learning. This is confirmed in particular by the studies which reviewed the use of computer technology (Roschelle et al., 2010; Roschelle et al., 2016). These two studies found that computer-mediated mathematical content could be effective in the existing practices of most teachers, and the authors also hypothesized that some pedagogies might improve students' learning with the use of computer-mediated materials. This assumption was confirmed by Li and Ma (2010) in their meta-analysis, where they found that the use of technology has a greater effect size (i.e., 1.00 SD) when used in constructivism-based teaching.

In the selected studies, professional development hours were analyzed (Table 3). Interventions ranged from a dozen hours over a few meetings (Heller et al., 2007; Beg et al., 2019; Solomon et al., 2019; Chen et al., 2020) to 46 face-to-face sessions (a total of 240 hours) over two school years. This variation in duration suggests that effective interventions can be achieved with shorter or more intensive professional development programs and does not verify the idea that professional development must have a lengthy duration in order to be effective (Garet et al., 2001; Desimone, 2009; Darling-Hammond et al., 2017). Lynch et al. (2019) found no evidence of a positive association between professional development duration and program outcomes in their meta-analysis. If teachers and students receive additional learning incentives, the duration of professional development appears to be irrelevant (Lauer et al., 2014). The results of this analysis confirm the hypothesis posed by McEwan in an effort to answer the question: "Why are some categories [of PD intervention] apparently less effective after controlling for moderators?" (2015, p. 24). McEwan presumed that a treatment component would be more effective when combined with another complementary treatment component than either component in isolation. This assumption was proved by Snilstveit et al.'s (2016) assessment of 216 different programs in low- and middle-income countries: they found that structured pedagogy programs are the most effective of PD interventions. An ideal structured pedagogy intervention includes the following activities: "(1) teacher training, (2) ongoing teacher support, supervision and feedback, (3) provision of teacher-oriented resources or materials, [and] (4) provision of classroom learning materials" (Snilstveit et al., 2016, p. 175).

PD programme	PD duration	Structured pedagogy intervention *	Initial PD	In-year follow-up activities			Using videos	Type of teacher know.+
				Work-shops	Coach.	Online learn.		
1. eLearn	2 days	b, c	+					PCK
2. Video-based teacher PD	12 hours	a	+	+		+	+	PCK
3. ECED	5 days	a	+	+	+	+		PCK
4. MPP	8 hours	**a, b, c**	+	+			+	PCK
5. TLMI	More than 50 hours	**a, b, c**		+	+	+		CK, PCK
6. CCT	2 weeks	**a, b, c**	+	+		+	+	PCK
7. Lesson Study	7 to 42 hours	a, b		+			+	CK, PCK
8. eMINTS	240 hours	**a, b, c**	+	+	+	+	+	PCK
9. AMSTI	50 hours	**a, b, c**	+	+	+			CK, PCK
10. SimCalc	5 days	b, c	+			+	+	CK, PCK
11. ASSIST-ments	5 days	**a, b, c**	+	+	+	+		PCK
12. JUMP Math	12 hours	**a, b, c**	+	+				PCK

(See Table 2 for full versions of abbreviated titles)
* Elements of structured pedagogy intervention utilised along with teacher training: a) on-site teacher support, supervision, and feedback (multiple teacher observation and giving feedback to the teacher about his classroom action); b) resources for teachers (lesson plans, activity guides and materials, making teaching aids, etc.); and c) classroom learning materials (flash-cards, wallcharts, textbooks, workbooks, storybooks or technology etc.) (Snilstveit et al. 2016). In studies that possess all elements of structured pedagogy intervention, letters a, b, and c are marked in bold.
+ Type of teacher knowledge: Content knowledge (CK), pedagogical content knowledge (PCK), and pedagogical knowledge (PK).

Table 3: Representation of professional development (PD) elements in effective programs

Indeed, this finding was precisely the case for seven of the 12 programs listed in Table 2 which, in addition to teacher training courses and onsite support, include teacher resources, classroom materials, and technology (e.g., paper curricula, quizzes, teaching guides for lessons, videos for teachers and students, mathematics tasks for teachers and students, and visualizations and interactive technologies). Video-based teacher professional development (Chen et al., 2020) and the ECED program (Early et al., 2016) did not include materials for teachers and students, but changes in teaching were achieved through well-designed and guided PD. In Lesson Study, the teachers cooperated without guidance from coaches, mentors, or project researchers, however they did help each other to follow a detailed resource kit. The eLearn (Beg et al., 2019) and SimCalc (Roschelle et al., 2010) projects did not provide ongoing teacher support: the initial PD they received was sufficient for learning how to use the relevant technologies. While this technology-based "streamlining' may facilitate short-term achievements, more comprehensive and lasting changes require more intensive PD in order 'to sustain and expand implementations across many years" (Roschelle et al., 2010, p. 872).

Most studies (seven out of 12) reported the use of online learning and resources in teacher PD, although merely as an addition to face-to-face PD. Thus, none of the selected programs were fully based on online PD. We assume that a key reason for the dominant role of face-to-face professional learning is related to the importance of pedagogical content knowledge (PCK), on which all of the interventions were focused. PCK is rooted in adult learning principles: intrinsic motivation; self-direction; metacognition; solving practical problems idiosyncratically related to the learner; participating in communities of practitioners; deepening understandings of professional contexts; disclosing oppressive structures and practices; and transforming habits of the mind by becoming critically reflective (Lave & Wenger, 1991; Chan, 2010; Knowles, 2015). Although there are applications which can facilitate such learning in a digital space (e.g. Moodle, Zoom, and Microsoft Teams), it seems that in-person communication remains crucial in the professional learning of mathematics teachers — which (in all studies) was achieved through summer institutes or follow-up PD meetings.

Coaching was used to supplement initial teacher education in five interventions, especially those that required significant changes in teaching (e.g. ECED Math and Literacy Matters, Teacher-Led Math Inquiry, eMINTS, and AMSTI). The coach involved in this role was usually a teacher from the school where the intervention had been conducted, and who had received additional training to serve as a PD team liaison. The coaching component was accomplished both in-person and remotely. Despite the benefits that coaching provides to teachers (Campbell & Malkus, 2011; Cordingley & Buckler 2012; Darling-Hammond et al., 2017), it is a method used **in under half of the 12 programs**. This lack of utilisation may mean that the expected changes can be achieved without this component, if other aspects of the intervention allow teachers to introduce the expected changes in their teaching. However, coaching does appear to have been an important support in implementing planned changes in teaching: the results of Kraft et al. s' (2018) meta-analysis found that coaching has a significant effect on teachers' instructional practice (i.e., 0.49 SD) and on students' academic performance (i.e., 0.18 SD).

We identified video usage in six of the programmes — videos were used to communicate project, teaching, and student activities, as well as to gather qualitative data on teaching and PD in experimental studies. In the only video-based teacher PD program (Chen et al., 2020), the emphasis was on training teachers to use academically productive talk (APT) in their mathematics teaching. The Classroom Discourse Analyzer (CDA) application enabled teachers to visualize class discussions in three ways: multiple representation (e.g., teaching videos, transcripts, and visualization of the APT moves); interactive visualization (e.g., frequency of APT moves); and contextualized evidence (e.g., observation of APT moves in a certain segment of teaching). Using videos and the CDA application allows teachers to focus on their teaching which contributes to better reflection, with "CDA [being a] tool [which] can help teachers recognize how their teaching resembles or differs from one another, which empowers evidence-based discussions and collaborative learning" (Chen et al., 2020, p. 29). All three ways of watching videos of teaching — "viewing videos of unknown teacher activity", "viewing videos of peer activity", and "viewing videos of one's own professional practice" (Gaudin & Chaliès, 2015) — were only used in the Lesson Study program. Participants in this program had the opportunity to learn by watching and discussing videos of experienced Japanese teachers. In the planning phase, teachers were required to prepare and deliver a research lesson. These research lessons were recorded, reflected on in lesson study teams, and periodically mailed as video data cards to researchers. However, none of the interventions used videos of mathematics teaching practices in an online context, which may prompt us to further explore this approach.

All of the PD programmes focused on teachers' pedagogical content knowledge, plus four studies detailed efforts to improve their content knowledge (CK). The fact that in the studies analysed a greater emphasis was placed on PCK than on CK is consistent with the conclusion reached by Baumert et al. (2010), who determined that teachers' pedagogical content knowledge better predicts the mathematics outcomes of ninth-grade students than their content knowledge does. It should be noted here that none of the programs focused on general pedagogical knowledge (PK), suggesting that focussing on teachers' PCK and CK may be sufficient for increasing students' mathematics outcomes. Nevertheless, it would be wrong to conclude that general pedagogical knowledge is not important for mathematics teachers; conversely, PK has proven to be essential in students' assessment of teaching quality in vocational schools in Austria (König & Pflanzl, 2016) and a positive predictor of learning support in Germany (Baier et al., 2019). In addition, general pedagogical knowledge is an important prerequisite for the improved professionalization of the teaching vocation (Guerriero, 2017) — therefore, it is important to explore how PK may be incorporated into the professional development of mathematics teachers.

A carefully considered connection of all the elements listed in Table 2 in the intensive three-year eMINTS professional development program (Meyers et al., 2016) has led to significant improvements in students' mathematics outcomes. In this program, PD specialists provided teachers with coaching, communities of practice, and online courses. Furthermore, another important aspect of this program was school leadership supporting "eMINTS implementation and maintain[ing] a schoolwide learning environment for teachers" (Meyers et al., 2016, p. 5). Instructional changes were focused on: collaborative and inquiry-based learning; strategies that best meet learners' needs and help them learn through reflection and metacognition; using multiple data sources to present mathematics content; feedback from assessments; and technology integration. The intervention enabled more quality learning for both teachers and students, appearing to be a winning combination.

To enhance mathematics learning outcomes for students, it appears that professional development interventions should provide on-site teacher support, mentoring, and feedback. Moreover, providing teachers with some form of teacher-focused resources and classroom learning materials would be of great assistance to those attempting to implement new instructional practices. In any case, it seems unrealistic to expect an intervention to be effective if it relies solely on a one-time PD intervention conducted in the summer preceding its first term of implementation. The utilization of technology has been shown to be essential for student learning. Two programs yielded significant positive results (Roschelle et al., 2010; Roschelle et al., 2016), in which educational software played a central role in student learning, despite relatively minor changes in teaching practices. Although these interventions did not necessitate substantial changes in instruction, students' mathematics performance improved due to the success of computer applications in facilitating deeper learning. The findings of these studies may suggest that changes in instruction can be implemented incrementally, beginning with less complex interventions and progressing to those that are more complex and demanding. When teachers see the benefits of new approaches based on the results of their own practice, it is feasible to continue with a more intensive form of professional development that will prepare them for a deeper understanding and the development of innovative practices. This recommendation is consistent with Guskey's conclusion that "significant change in teachers' attitudes and beliefs occurs predominantly after they gain evidence of improvements in student learning" (2002, p. 383). Although the majority of PD was conducted face-to-face with leaders and other program participants, online platforms and videos were used as a supplement to teacher training rather than as a central component, indicating that effective, modern PD does not necessarily require the use of technology. A recommendation from our review would be to investigate whether and how technology in professional development can meaningfully assist mathematics teachers in improving student learning outcomes.

CONCLUSIONS

Professional development can be structured in various ways to enhance its effectiveness. The provision of high-quality initial PD and subsequent follow-up workshops can be effectively enhanced through the use of structured pedagogical intervention, coaching, video resources, and online learning platforms. The primary objective, as emphasized throughout this review, is to assist teachers in implementing instructional changes that improve student learning outcomes. Using the elements that have been shown to be effective in PD interventions, we propose a strategic path for teachers who are committed to enhancing their pedagogical practices despite a potential lack of high-quality opportunities for professional development: Together with school leadership, a teacher should create a community of learning that concentrates on improving his/her teaching practice. The teacher should expose students in his or her lessons to an active teaching-learning process. The teacher should use a fine-grained, individualized guided discovery approach, provide optimal challenges for each student, and could use technology for timely, supportive, and specific feedback. In addition, the teacher should videotape his/ her lessons in order to evaluate the instructional strategies employed and determine whether or not these strategies promoted student engagement and active participation. This analysis of the video should be conducted in conjunction with the school's established community of learning. Protocols from effective PDs should be utilized in this process, as they can provide precise feedback on instructional practices and student engagement. In addition, the teacher should establish a partnership with an expert teacher who could serve as a coach. If possible, from his or her own school; if not, from a nearby school. Lastly, teachers must consistently pursue the improvement of PCK, but school leadership plays a crucial role in providing support for such efforts.

STUDY LIMITATIONS AND DIRECTIONS FOR FUTURE RESEARCH

The studies in this systematic review mainly relate to the USA because eight of the 12 studies analysed were conducted there. Recognizing that teaching is culturally diverse and that the United States lacks a national curriculum (Kennedy, 2016), we have included international research. While analysing studies from various nations, we discovered a number of significant sociocultural distinctions, but they are not so substantial that research from one nation cannot inform practice and policy in others. The fact that we focused solely on randomized experiments for the purposes of our systematic review may be considered an advantage, but it is also a limitation. Quantitative results are analysed and presented in experimental studies, whereas qualitative data on intervention implementation is rarely used. However, qualitative data are frequently required to fully comprehend and replicate the effectiveness of an intervention across educational contexts. To learn more about the actual implementation and application of the programs in our analysis, we consulted other published papers, particularly qualitative research, to gather more information about the conducted PD activities (e.g., Lewis & Perry, 2014) or teaching (Bell & Pape, 2012). This additional step in our literature review demonstrates that researchers must conduct and publish qualitative and experimental research in addition to quantitative research or employ a mixed-method research design.

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Prediction Ability of College Students in Solving Graph Problems

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Abstract: The capacity to generate prediction is indispensable in daily existence, particularly amidst the swift transformations that are occurring on a global scale. Therefore, this study aimed to analyze the levels of prediction ability among mathematics students when presented with data in graphs. A qualitative approach was adopted, involving 37 mathematics students, using task-based tests and interviews as data collection techniques. The results showed that the ability of most students to make a prediction based on the Covid-19 graph was at a multi-structural level of 35.14%. This level was characterized by students making predictions based on the trends of the pandemic graph patterns, but they tended to overlook the overall patterns. The prediction generated at the *unistrutural, multistructural, relational, and extended abstract levels was considered reasonable because of the graph or data provided. These findings indicated the existence of predictive reasoning conducted by students in making predictions of problems related to the Covid-19 graph. The insights gained into the prediction ability prompted teachers to enhance graphic literacy instruction, to equip students with the skills needed to thrive in the 21st century.*

***Keywords:** Prediction, Ability, Graph, SOLO Taxonomy*

INTRODUCTION

The ongoing development of technology and information in the industrial revolution 4.0 has made data more widely available and easily accessible to many individuals (Duijzer et al., 2019; Ramadhani et al., 2022; Sima et al., 2020). This increased access has resulted in a growing demand for the processing and interpretation of data. Despite the copious amount of readily available data, its utility remains unrealized in the absence of adequate processing and analysis. Therefore, the ability to process and interpret data is becoming increasingly critical in the era of the industrial revolution 4.0 (ÇİL & Kar, 2015; Ergül, 2018a; Oslund et al., 2021).

Data are usually presented in the form of table or graph (Ergül, 2018a; Friel et al., 2001). Graphs, in particular, serve as practical forms of representation that enable the gathering and derivation of new knowledge from complex data, as well as the summarization of data sets, among other uses (Uzun et al., 2012). They possess the ability to present information quantitatively in a manner that is often more comprehensible than data presented narratively or descriptively (Kilic et al., 2012). Furthermore, graph is capable of presenting quantitative information useful for decision-making purposes (Okan et al., 2019). By aiding in the comprehension and analysis of data through visualization and representation of variations, patterns, and trends, graphs play a crucial role (Glazer, 2011; Kukliansky, 2016). Therefore, graph is an important part of daily life since it is used in different contexts (González et al., 2011).

Currently, graph has become ubiquitous in daily life, as it can be found in various mediums, including newspapers, magazines, articles, online news, and televised news (ÇİL & Kar, 2015). Moreover, it is frequently utilized to present information about election results, and public reports on topics such as education and health. Graph is also used in various subject disciplines, such as geography, economics, health, science, and mathematics (ÇİL & Kar, 2015; Ergül, 2018a; Lai & Hwang, 2016). The significance of graph in statistical education is also noteworthy (Franklin et al., 2007; NCTM, 2000), playing a fundamental role in mathematics curriculum (Ozmen et al., 2020). In mathematics, drawing, interpreting, and analyzing graphs are basic skills expected to be achieved by students (NCTM, 2000). Graph is considered as a fundamental component of mathematics, and educational programs place a strong emphasis on the need for students at all levels to develop their graphing skills (Uzun et al., 2012). These skills require abilities such as visual perception, logical reasoning, plotting po ints from data or function rules, predicting the movement of the connecting line, and determining the relationship between variables (Uzun et al., 2012). However, when invalid information is provided, a serious loss may be encountered by the individuals or organizations who make straight decisions based on the graph without conducting a proper analysis. Graph literacy skills are then needed to avoid decision making mistakes (Galesic & Garcia-Retamero, 2011).

Graph literacy is a very important skill in the 21st century (ÇİL & Kar, 2015; Duijzer et al., 2019; Glazer, 2011). It is part of the high-order thinking skills, required to be mastered in the current era (Duijzer et al., 2019a).Furthermore, the concept refers to a person's ability to evaluate and extract data presented in graphical form (C.M et al., 2020; Galesic & Garcia-Retamero, 2011). These include making an interpretation of graphical representations, drawing relationships between variables on the horizontal and vertical axes, such as for time and distance, making graphical representations, critically evaluating data represented in graphs, using graphs to communicate findings to others, and making comparisons (Boote, 2014). Graph literacy is also very important in mathematics lessons, such as when teaching students about linear functions (Watson & Kelly, 2008).

Assessing graph literacy involves a range of skills, encompassing not only comprehension of data but also the ability to draw inferences and identify connections between presented data, as well as extrapolate beyond the provided information (Friel et al., 2001, 2001; Zeuch et al., 2017). The aptitude for reading involves extracting relevant information from graphical representations, while the ability to read between denotes the capacity to discern correlations and interrelationships among the data displayed. Moreover, the ability to read beyond the data presented entails the capability to make informed prediction and draw well-supported conclusions (Boote, 2014; Curcio & Artzt, 1997; Friel et al., 2001; González et al., 2011; Monteiro & Ainley, 2003, Thuy & Le Phuoc, 2023).

The ability to make prediction is a crucial skill in anticipating future outcomes, particularly in the realms of education and daily life (Katarína & Marián, 2017a; G. Oslington et al., 2020). Prediction is essential for decision-making (Okan et al., 2019; Oslund et al., 2021) to potentially prevent the occurrence of future errors (Katarína & Marián, 2017b). In the context of learning mathematics, prediction-making provides an opportunity to establish connections between mathematical concepts. By predicting outcomes, students can connect prior concepts with new ones, enhancing their learning experience (Kim & Kasmer, 2007a; G. Oslington et al., 2020). Through prediction, they develop critical thinking skills by observing data and drawing conclusions from the given information (Thaiposri & Wannapiroon, 2015; Uzun et al., 2012). Students also improve their understanding of concepts and can improve students problem-solving skills by making predictions. Teachers can improve their students' understanding of mathematical concepts and enhance their problem-solving skills by promoting prediction-making on graph. Furthermore, they can enhance their teaching skills by involving students in activities related to graphing to develop learning strategies more effective and responsive to the needs of students.

The ability to predict data presented in graphical form is a valuable skill that holds significant importance for society. The process of generating patterns from graphical data remains challenging, requiring a high level of graphic literacy (Harsh et al., 2019). At both the school and college levels, students' skills in this area tend to fall into the low category. Consequently, conducting a dedicated study to assess students' prediction ability is necessary.

Various frameworks have been used to measure prediction ability, including in physics at the junior high school level (Katarína & Marián, 2017a), and at the elementary age level (G. Oslington et al., 2020; G. R. Oslington et al., 2021). To measure data prediction ability, several different frameworks are used. (Katarína & Marián, 2017a) distinguished prediction from general statements, filled in incomplete predictions, predicted answers to questions, and formulated predictions. This category can be utilized to assess prediction that has a solid foundation but remains overly generic. Another framework used (G. Oslington et al., 2020) is *AMPS* (*Awareness of Mathematical Pattern and Structure*), categorized into *prestructural*, *emergent*, *partial*, and *advanced structural*. This framework focuses on students' understanding of the statistical concepts provided. Furthermore, it applies to the framework used by (Watson & Moritz, 2001), which uses a 4-level solo taxonomy to describe students' ability to make prediction based on pictographs, namely *iconic*, *unistructural*, *multistructural*, and *relational*. The framework used by Oslington and Watson exhibits a similar attribute, intended for Elementary School children, thereby making it less applicable for use at the university level. Therefore, it is imperative to adjust the level of prediction ability under the type of task and the degree of cognitive advancement.

The framework used is SOLO taxonomy based on cognitive development used to measure students' level of understanding. Biggs and Collis (1982) created the framework for assessing the composition of observable learning outcomes (İhsan Yurtyapan & Kaleli Yilmaz, 2021). In addition, SOLO taxonomy can be used as an effective tool in determining the level of concept learning (Akbaş-Ertem, 2016; Groth & Bergner, 2006). It is a framework based on cognitive development theory by Piaget and the framework identifies a particular student's developmental stage, with the response to an assignment by a particular student (Aoyama, 2007). This taxonomy can explain the complexity of students' understanding and assess their cognitive learning outcomes (Adeniji et al., 2022). There are two main features in the taxonomy, namely the mode of thinking and the level of response (Caniglia & Meadows, 2018). Meanwhile, the level of response is the individual's ability to respond to increasingly sophisticated tasks.

Every framework possesses its own set of strengths and limitations. There is an existence of a framework explicitly designed to evaluate college-level students' graph prediction exists. However, the framework is inadequate in comprehending the intricacy of students' prediction advancement influencing graph literacy proficiencies.

This study is limited to graphs with linear curves and exclusively employs generalization-prediction task categories. Furthermore, it addresses the following research questions:

1. What levels of thinking on SOLO taxonomy do students align with when making prediction?
2. How can the prediction aptitude of mathematics students concerning a particular graph be described?

By understanding the level of prediction ability among mathematics students based on a given graph, teachers can assess their comprehension of the subject matter and determine the effectiveness of graphical representations. The rate at which students respond to graphical representations can be used to gauge their comprehension of graphs and the ability to solve problems. The accuracy of students' predictions is an indication of the concept and ability to apply it in practical situations.

METHODS

Approach and Subject

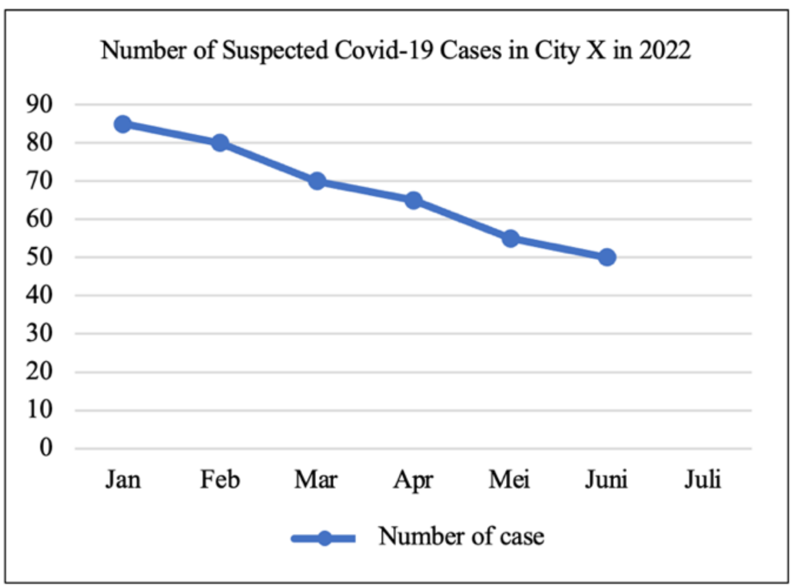
To fulfill the objective of describing students' ability to make prediction, this study employed a descriptive qualitative method. Qualitative study allowed students to furnish comprehensive reports on situations and phenomena by using various data collection tools (Creswell, 2012). The qualitative descriptive approach was preferred to present an elaborate account of students' ability to make prediction based on the data gathered.

This study comprised 37 students from the Mathematics Education Study Program at a state university in South Kalimantan. The participants were comprised of 8 male and 29 female students, with a secondary school background ranging from high school to Islamic high school. Furthermore, the average age of students was 19-20 years old. This study focused on early-semester college students who had attended calculus courses, as they possessed the necessary background knowledge required to comprehend the ideas involved in the generalization-prediction task. Students were carefully selected as subjects, given their suitability for the study objectives.

Data Collection

Data were collected using tests and interviews with students. Students worked individually to solve a generalization-prediction task. The study instrument was in the form of a problem-solving task, which was a *generalization-prediction task* based on the pattern created, as shown in Figure 1.

Data on the number of suspected Cases of Covid-19 in city X in 2022 is presented in the following picture:



Based on the graph above, make a prediction of the number of suspected cases of Covid-19 in July 2022. Explain the reason!

Figure 1. The question for predicting the Covid-19 graph.

The data collection process involves several steps by preparing study tests such as generalization-prediction tasks, with necessary tools including paper and pencils. Furthermore, this study introduces the generalization-prediction task to students and provides instructions on task evaluation. Students are then given time to work on the task individually and expected to understand the exploration of linear functions. After the assignment is completed, students are asked to answer the test questions presented in Figure 1. Subsequently, they are interviewed to evaluate their comprehension of the exploration of linear functions. Individual interviews are also performed with semi-structured questions. The results are recorded and collected before evaluating the quality of the data and identifying any problems in the collection procedure.

Data Analysis

The process of data analysis was conducted in multiple stages. Firstly, students' answer sheets were checked and then analyzed using SOLO taxonomy framework by Biggs and Collis (1982). Regarding students' prediction ability level, Watson, and Moritz (2001) used a four-level SOLO taxonomy to describe the ability to make prediction, namely iconic, unistructural, multistructural, and relational. These were used in the instrument to measure prediction using pictographs for elementary school children. This study also used the framework of Watson and Moritz (2001) based on a similar SOLO taxonomy to analyze the prediction ability of university students, with a description in line with the generalization-prediction task, as shown in Table 1.

Level	Description
Prestructural	Prediction without referring to the provided graph
Unistructural	Prediction with baseless guesses or based on a simple aspect in the graph
Multistructural	Prediction using multiple values in the graph
Relational	Prediction by integrating all information, such as using all values and trending patterns on graph
Extended Abstract	Prediction integrating all information such as values, patterns in graphs, previous knowledge, or experience, as well as generalizing structures to adopt new and more abstract features

Table 1. The Framework of Prediction Ability Classification on Graphs

The prediction results from students were coded using the framework in Table 1. These determined codes were put together based on their similarity and collected under descriptors representing the level of prediction ability. Therefore, the level of predictability is determined based on this coding. A descriptive analysis was performed to calculate the frequencies for each level of the specified outcome. The analysis gave the needed tools to calculate the frequencies for each level of predicting ability and map out how these levels were distributed among students. Consequently, deductions regarding students' overall prognostic aptitudes and any domains that may necessitate guidance or assistance have been derived from the data.

RESULT

A description of the findings obtained at the level of prediction ability is presented in the following table

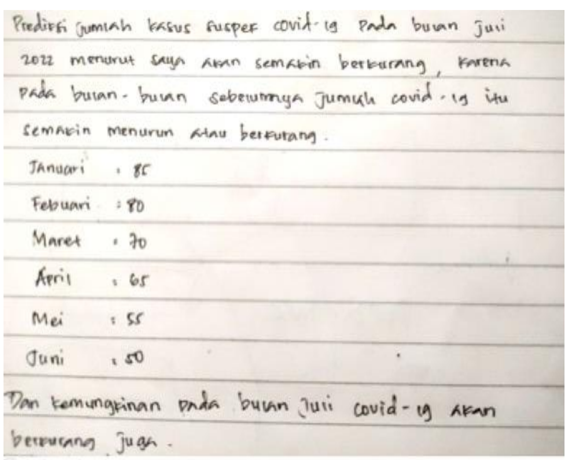
	Quantity	Percentage (%)
Unistructural	8	21.62
Multistructural	13	35.14
Relational	7	18.92
Extended Abstract	9	24.32
Total	37	100

Table 2. Students' Prediction Ability Level Based on SOLO Taxonomy

As seen in Table 2, most mathematics students belong to level 2, multistructural, accounting for 35.14%. The lowest number is found at the relational level, which is 18.92%. Since no students are encompassed within this category during the study conducted to expose the prediction levels, the ensuing depiction provides an account of the capability to make prediction for the unistructural, relational, multistructural, and extended abstract levels.

Level 2 – Unistructural

At this level, students made predictions based on a simple aspect of the graph.



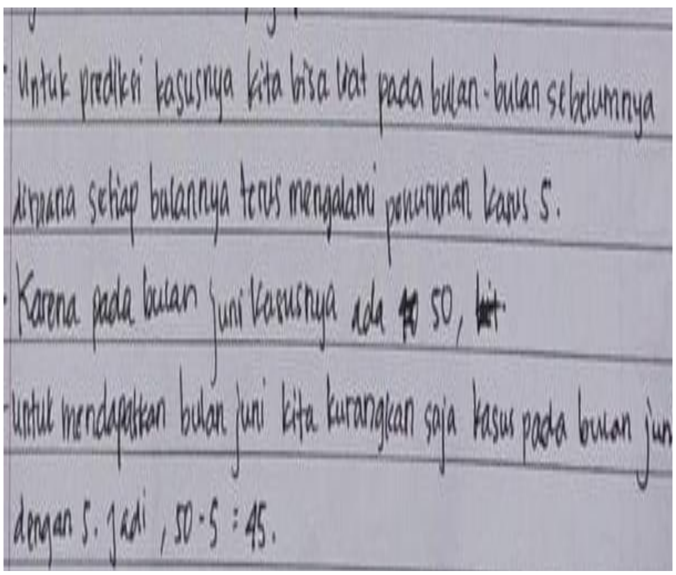
Predicting the number of Covid-19 cases in July, in my opinion, it is decreasing because in the previous months the number of Covid-19 cases is decreasing.
January: 85
February: 80
March: 70
April: 65
May : 55
June: 50
And the probability of covid-19 a in July is decreasing

Figure 2. A sample answer from a student at the unistructural level

According to Figure 2, the student predicted the number of Covid-19 cases in July by analyzing the decrease in cases for every preceding month, commencing with 85 cases in January, followed by 80 in February, 70 in March, 65 in April, 55 in May, and 50 in June. By observing the decline that transpired in all prior months, the student inferred that there would be a reduction in the number of Covid-19 cases in July.

Level 3 – Multi-structural

At this level, students make prediction using several values in the data presented in the form of graph.



Untuk prediksi kasusnya kita bisa liat pada bulan-bulan sebelumnya
dimana setiap bulannya terus mengalami penurunan kasus 5.
Karena pada bulan juni kasusnya ada 50,
untuk mendapatkan bulan juni kita kurangkan saja kasus pada bulan juni
dengan 5. Jadi, 50-5 = 45.

For the prediction of cases, we can see in the previous months, where every month it continues to experience a decrease of 5 cases. Because in each June there are 50 cases, to get june we reduce the cases in June by 5.
So 50-5=45

Figure 3. A sample of a students' answer at the multistructural level

Based on Figure 3, in making prediction the student used the number of cases in the previous months, which had been decreasing by 5 cases. For example, the number of Covid-19 cases was predicted in July by analyzing the decrease in cases from May to June, which was 55 - 50 = 5 cases. Based on this calculation, the number of monthly decline patterns was 5 cases. Therefore, when predicting for July, the number of Covid-19 cases was subtracted in the previous month with a decreasing pattern, obtaining a prediction of 50 - 5 = 45. Analyzing the answers of the student, prediction was made for July using the data from May to June, which had a decrease in the cases. Regarding the number of cases from January to February and March to April, a decrease was not reported in the pattern. Different patterns of decrease in 10 cases were reported from February to March and April to May.

Level 4 – Relational

At this level, students formulate prediction using all data values, and graph patterns exhibiting trends, or distinctive conditions within the dataset. This level involves the use of the number of cases in each month, considering the downward trend, while observing the graph trend patterns.

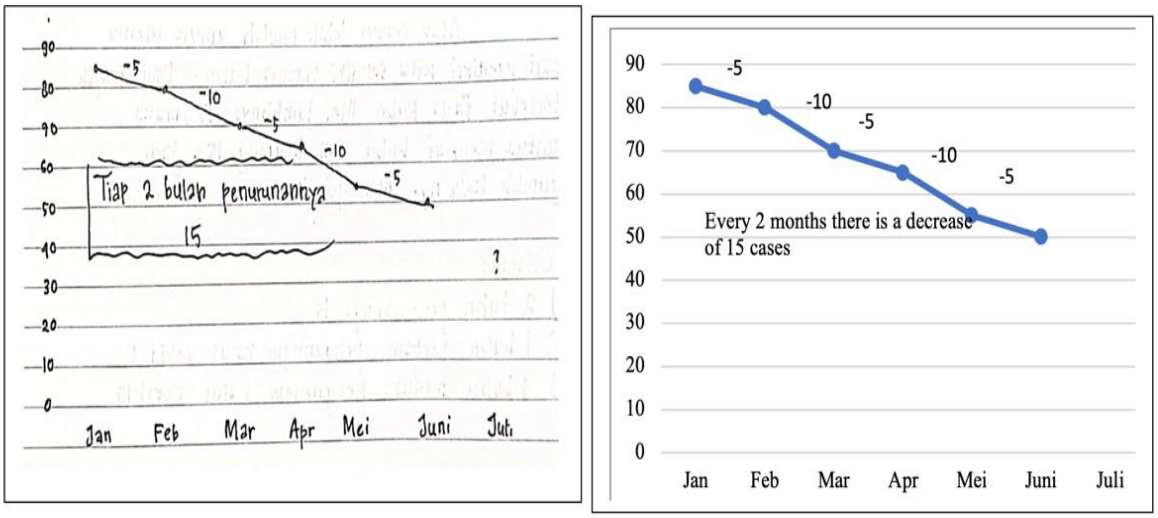


Figure 4. An example of a student's answer at the Relational level

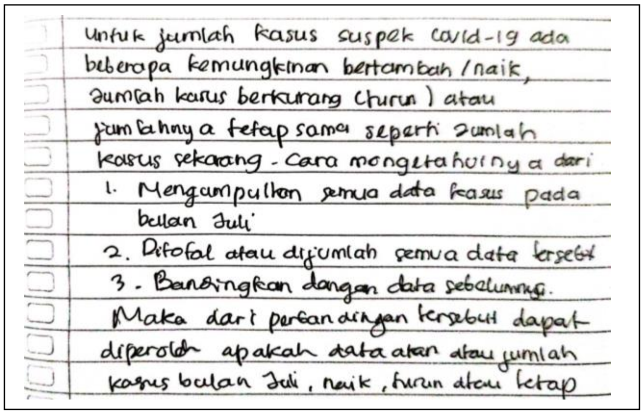
Based on Figure 4, the student made a prediction using the information in the graph, such as the number of cases in each month. The graph trend was analyzed by paying attention to the decrease every 2 months, such as from January to February, 85 - 80 = 5, which was 5 cases decrease. The decrease in the number of cases from February to March was also calculated, which were 80 and 90. The decrease from April to May, which was 5 was obtained, and the number was the same from January to February. The result showed a 10-cases decrease, which was the same from February to March. In conclusion, there was a potential trend of a decrease of 10 cases from June to July, and the predicted number was 40.

In this case, students predicted by analyzing the pattern every two months to obtain a decrease of 15 cases. Therefore, the result of the prediction in July was obtained by subtracting the 55 cases in May by 15 to obtain 40. The subject identified a pattern of decrease in the first and second months of 5 and 10 cases, hence, the pattern of decline every 2 months was 15 cases.

From these results, students predicted by using all information in each month from January to June by finding the monthly and bi-monthly patterns, and paying attention to the decreasing pattern.

Level 5 – Extended Abstract

At this level, students are expected to make prediction, integrate all information, values and patterns on graphs, and generalize structures to adopt new and more abstract features. At this stage, they can integrate the data on the graph with the existing previous knowledge.


Untuk jumlah kasus suspek covid-19 ada
beberapa kemungkinan bertambah / naik,
jumlah kasus berkurang (turun) atau
jumlahnya a tetap sama seperti jumlah
kasus sekarang. Cara mengetahuinya dari
1. Mengumpulkan semua data kasus pada
bulan Juli
2. Ditotal atau dijumlah semua data tersebut
3. Bandingkan dengan data sebelumnya.
Maka dari perbandingan tersebut dapat
diperoleh apakah data atau jumlah
kasus bulan Juli, naik, turun atau tetap

For the (predictive) number of Covid-19 cases (in July), there could be some possibilities. It is possible that the number of cases will increase, decrease, or remain the same with the number of cases now.
Ways to find out:
1. Collecting all case data in July
2. Summing all the data
3. Comparing it with previous data

Figure 5. An example of a students' answer at the Extended Abstract level

According to Figure 5, the student did not refer to the Covid-19 case graph to formulate prediction. This was corroborated during an interview, where preexisting knowledge of graphed data was used to conclude. Specifically, there were various potential outcomes for the Covid-19 case graph, such as an increase, decrease, or maintenance of the current count. Another student, upon observing a reduction in the cases in previous months, made a prediction based on observations. The students drew upon their personal experience and knowledge to generate prediction. In addition, the government's ongoing efforts to conduct vaccinations and implement health protocols across a variety of activities should reduce the number of cases.

DISCUSSION

This present study describes the level of prediction ability using the theoretical framework presented in Table 1. The proposed framework highlights the crucial role of a resource to gain insights into students' capacity and make accurate predictions based on graph analysis. The process requires a sound awareness of data patterns and trends. The results indicate that a majority of students show a multistructural level of prediction ability. This level is characterized by limited aspects or information, such as identifying the downward trend in the graph and using only one data pattern. Moreover, students at this level tend to overlook some patterns in the graph. Students possess some relevant knowledge and can accurately identify the phenomenon but fail to integrate these facts into a comprehensive whole. Therefore, this present study underscores the need for developing pedagogical strategies to enhance students' prediction ability and promote a deeper understanding of data patterns and trends (Caniglia & Meadows, 2018).

The results at level 2 *unistructural* indicate that the students predicted the downward trend of the graph. At this level, information is not related to the number in the decreasing pattern. Furthermore, students show an awareness of the downward pattern in the graph, with limited knowledge of the Covid-19 graph. A type of information, which is the downward trend of the graph, is used while failing to notice the relationship between ideas. Students do not understand problems involving mathematical concepts and environmental situations during the pandemic (Mumu et al., 2021).

At level 4, *relational*, the subjects use all information, trends, and patterns in the graph, by observing various interrelated components. Attention was given to the variables of the cases and the months in the graph to draw the connection between the two variables and obtain the number for each month. In addition, the subjects also consider the similarity of the patterns since the number of cases decreases monthly. At this level, the subjects integrate information and explain some ideas related to the topic.

The results for level 5, *extended abstract,* indicate that students refer to the data provided and also use their existing previous knowledge or information in making prediction. In addition, they also adapted the situation to their experiences as a consideration in making prediction. Individuals consider their existing knowledge and experiences acquired in their daily lives when formulating prediction (Bazinger & Kühberger, 2012; Fotou, 2014; G. Oslington et al., 2020). This finding shows that students are more aware of trends and patterns and can integrate information in the graph with previous knowledge or experience.

Based on the findings for the *unistructural, multistructural, relational,* and *extended abstract* levels, prediction can be made based on the information or data in the graph. At these levels, students perform formal thinking and the predictions generated are considered reasonable due to the use of graphs or data. This is under study conducted by (G. R. Oslington et al., 2021), where reasonable prediction is obtained from the given data or context. Furthermore, a reasonable prediction relies heavily on identifying patterns in the provided data (G. R. Oslington et al., 2021). Making reasonable predictions involving the identification of patterns reflects a process of reasoning that does not rely on feelings or experiences. Therefore, students use predictive reasoning based on the graph presented In line with (Russo et al., 2022) where prediction should lead leads to the generalization of patterns (Kim & Kasmer, 2007a).

According to Table 2, the majority of Mathematics Education students who make prediction on graphs are at the multistructural level. At this level, prediction is made based on a limited number of aspects or information, such as the downward trend in the graph and the use of only one data pattern. At this level, students can identify the phenomenon correctly, but this knowledge is not integrated as a whole (Caniglia & Meadows, 2018).By acquiring knowledge of students' prediction ability, comprehension of concepts, problem-solving skills, critical thinking ability, and graphical literacy can be augmented. Additionally, teachers can improve their pedagogical skills by engaging students in predicting graph related to mathematical concepts. This approach can assist teachers in devising and executing more efficient and pertinent strategies, catering to the diverse needs of students. Knowing the level of prediction ability of mathematics students from graphs can enable teachers to evaluate their comprehension of the subject matter and interpret graphs. The rate of response can also serve as a metric of understanding graph to tackle problems involving graphical representations. The accuracy of students' prediction is indicative of the capacity to apply the concept in practical situations.

CONCLUSION

In conclusion, the study shows that mathematics students primarily demonstrate their prediction ability at the multistructural level, accounting for 35.14%. A smaller percentage of students exhibit prediction skills at the relational level, accounting for 18.92%, where predictions are made using limited information from the graph. To enhance the teaching of mathematical concepts related to graphing, teachers can involve students in the process, allowing for the development of their skills. By leveraging knowledge of students' predictive abilities, learning activities can be better planned and executed using relevant and efficient strategies. Students who possess the capacity to make predictions at the unistructural, multistructural, relational, and extended abstract levels are capable of making reasonable predictions based on the information presented in the graph. This finding indicates the presence of predictive reasoning, which may warrant further investigation in future studies.

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A Case Study on Students' Critical Thinking in Online Learning: Epistemological Obstacle in Proof, Generalization, Alternative Answer, and Problem Solving

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Abstract: Critical thinking is a key transversal competency of the 21st century, but some students have difficulty, especially during the transition to online learning due to the COVID-19 pandemic. This study aims to identify epistemological obstacles in critical thinking related to proof, generalization, alternative answers, and problem-solving. This online learning involved 30 prospective mathematics teachers through video conferences. An exploratory case study was conducted on 9 mathematics teacher candidates with the highest exam scores. Data were collected from the results of 4 mathematical critical thinking questions. The data were analyzed and described based on a predetermined framework. The results show various epistemological obstacles in critical thinking, namely difficulty in proving the relationship between two concepts, generalizing the relationship, finding multiple alternative solutions, and solving problems. The epistemological obstacles found can be the focus of lecturers in creating more structured online learning. Online learning needs to be well-planned in terms of the use of learning resources, learning media, and integration with technology. Learning should pay more attention to understanding the relationship between concepts, the flexibility of concepts and procedures, as well as the habit of drawing in geometry learning..

Keywords: epistemological obstacle, critical thinking, proof, generalization, alternative answer, problem-solving

INTRODUCTION

The COVID-19 pandemic has led to the widespread closure of face-to-face learning activities globally (Chertoff et al., 2020; Ferrell & Ryan, 2020; Toquero, 2020). As a result, emergency instructions were issued to shift to online learning, which is a model of education that utilizes several network technologies (Moore et al., 2011), including applications such as WhatsApp, Telegram, YouTube, and video conferencing.

The adjustment to online learning implementation has led to several issues, including institutions having limited time to prepare for online learning, leading to anxiety among educators, students, and parents (Daniel, 2020). Moreover, problems arise due to the lack of experience using e-learning applications (Daniel, 2020; Zaharah & Kirilova, 2020), causing some schools to be ill-prepared to effectively implement e-learning in the learning process, particularly at the student level (Mailizar et al., 2020).

The source of problems in the learning process is related to didactic design (Suryadi, 2019), which can create epistemological obstacles when the limitations of the context used in didactic design are encountered (Sulastri et al., 2022). Epistemological obstacles can occur when students misunderstand tasks or questions given to them (Brousseau, 1997), arising from the limitation of one's knowledge in a specific context, which cannot be applied in the current situation (Suryadi, 2013).

Based on preliminary studies, several problems often occur in proof, generalization, determining alternative answers, and problem-solving (Luritawaty & Prabawanto, 2020), which are some indicators of critical thinking (Daniel & Auriac, 2011; Ennis, 2011; Sanders, 2016; Facione, 2011). This finding is in line with Safrida et al. (2018), who reported that the mathematical critical thinking abilities of pre-service mathematics teachers in one Indonesian university were still low. Moreover, students often exhibit limited critical thinking skills and slow development (Karandinou, 2012; Pascarella et al., 2011).

The critical thinking skill, which has not been optimally achieved, needs further attention. Critical thinking is a key transversal competency in the 21st century that should be owned and developed by the education system, particularly at the secondary and tertiary levels (UNESCO, 2016 ; Moore, 2013). Furthermore, it is a complex process that requires high-level reasoning to achieve desired outcomes (Da et al., 2011; Halpern, 2014). It also involves reflective and logical thinking, which focuses on deciding what to believe and do to make wise decisions through ideas or actions (Dominguez et al., 2015; Ennis, 1993; Ennis, 2015).

Critical thinking is important because it is relevant to academic fields, particularly in higher education (Wechsler et al., 2018). It also helps students to effectively engage with social events, knowledge, and practical problems in planning, managing, monitoring, and evaluating academic tasks (Peter 2012). Critical thinking skills are expected to prevent errors in decision-making (Butler, 2012). Students with these skills can engage in positive and productive activities daily. Critical thinking also significantly correlates with Grade Points (GP) (Facione, 2011).

The issues arising from online learning implementation and the importance of critical thinking skills that do not align with current achievements make this study important. Studies on epistemological obstacles need to be conducted to understand the problems that arise in the series of critical thinking processes. Some related studies on epistemological obstacles have been carried out (Obreque & Andalon, 2020; Siagian et al., 2022; Sulastri et al., 2022; Sunariah & Mulyana, 2020). However, studies on epistemological obstacles in online learning have not been widely conducted in critical thinking skills, especially in proof, generalization, determination of alternative answers, and problem-solving. Therefore, this study examined the epistemological obstacles in online learning, particularly to identify possible obstacles when students perform cognitive actions in critical thinking related to proof, generalization, determination of alternative answers, and problem-solving. This study used geometry material because previous studies had revealed that the material was relatively difficult for students (Fujita et al., 2017; Brunheira & Ponte, 2019 ; Vasilyeva et al., 2013).

METHOD

Critical thinking skills are one of the crucial goals in the curriculum. The importance of critical thinking skills is detailed in Figure 1.

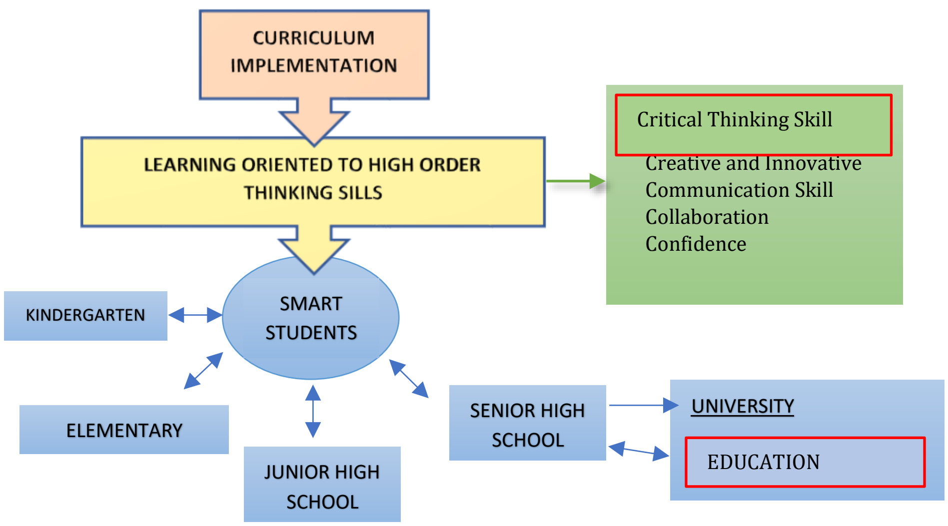


Figure 1: HOTS-oriented learning grand design (Ariyana et al., 2018)

The critical thinking skills in this study were marked by various achievement indicators, consisting of proof, generalization, determination of alternative answers, and problem-solving. The material studied was related to these indicators, such as geometry in college for the Basic Education Capita II selection course. Cube, cuboid, prism, and pyramid were used as objects. Furthermore, the concepts were taught in online learning. Learning is conducted through Zoom meetings with adapted learning steps from Arends (2012), as described in Table 1.

Table 1. Steps to Implementation of Online Learning

Steps	Activity
Introduction	- Presenting the problem through Zoom meeting - Grouping students through a Zoom meeting
Core Activities	- Delivery of materials and exploration through a Zoom meeting - Presentation and discussion through Zoom meeting
Closing	- Analysis and evaluation of the problem-solving process through a Zoom meeting

The exploration case study design was used because of its ability to manage various evidence and explore situations (Yin, 2018). Additionally, it can identify and describe epistemological obstacles in students' critical thinking processes during online learning. The preparation began with selecting the sample description, which is prospective mathematics teacher students. The selection was based on several related studies stating that their critical thinking achievement is still low.

The preparation continued with selecting the location and obtaining permission. The chosen location was one of Indonesia's educational institutions with a mathematics education program aligned with the targeted subject. Based on preliminary studies, there were still issues with developing critical thinking skills in this location. Permission was obtained from the head of the program at the institute. The study involved 30 second-year prospective mathematics teacher students aged 18-20 years old with the application of online learning. Moreover, nine students with the highest test scores were chosen as samples for further investigation. They voluntarily agreed to participate in this study. They consisted of 1 male and 8 female with a total average score of 10.5 out of a maximum score of 24.

The data in this study were collected from the mathematical critical thinking instrument test results, which were used as a reference in exploring epistemological obstacles that influenced the sample. The test was conducted at the end of the learning process after studying all the material. The essay questions form was used to test the student's ability. Unlike multiple-choice questions, essay questions allow responses to state the reasons for choosing an answer (Ennis, 1993). This also opens up opportunities for exploring the answers given. The test questions were validated by two experts, mathematics education lecturers at one of the universities in Bandung, Indonesia, before being used to measure mathematical critical thinking ability. The instrument comprised four questions representing these skills indicators (Daniel & Auriac, 2011; Ennis, 2011; Sanders, 2016; Facione, 2011).

The first question was related to the ability to prove. The subjects were given a situation about two prisms with certain volumes and conditions, and then they were asked to prove whether the three sides of the base of the first prism were always larger than the second prism or not. The second question was related to the ability to generalize. The subjects were given a situation about a cuboid and a triangular prism, and then they were asked to generalize the relationship between the two shapes. The third question was related to alternative answers. A problem was presented about fitting cube-shaped boxes into a cuboid cabinet. The subjects were asked to calculate the maximum number of boxes fitted into the cabinet using two methods. Finally, the fourth question related to the subjects' problem-solving ability. The subjects were given a picture of a pyramid-shaped building with certain conditions described. They were asked to calculate the area of material needed to construct the building according to the given conditions. The answers to all four questions are then collected as data. The data was also supplemented with interview results to identify and confirm the subjects' difficulties in critical mathematical thinking.

The data were analyzed by focusing on students' misconceptions and errors (Tan Sisman & Aksu, 2016). The first stage of the analysis was the scoring key creation. One point was given for a correct answer, and zero points were given for an incorrect answer. The second stage of data analysis was the development of an assessment framework based on indicators of mathematical critical thinking ability (Daniel & Auriac, 2011; Ennis, 2011; Sanders, 2016; Facione, 2011) with the Frisco theory, consisting of focus, reason, inference, situation, clarity, and overview (Ennis, 1991; Ennis, 1996). The Frisco theory helps demonstrate critical analysis and improve critical thinking ability (Ennis, 1996; (Dominguez et al., 2015). The third stage of data analysis was identifying student errors in their answers. This was performed by transforming the answers into Microsoft Word and categorizing them sequentially according to the framework in the second stage. The fourth data analysis stage investigated the relationship between student difficulties and the problem-solving theory framework. Next, in the fifth stage, the reasons for identifying epistemological obstacles were explained, while in the sixth stage, the results of identifying epistemological obstacles were discussed. In this final step, a descriptive analysis of student errors was conducted in relation to online learning implementation. An overview of the data analysis in this study is presented in Figure 2.

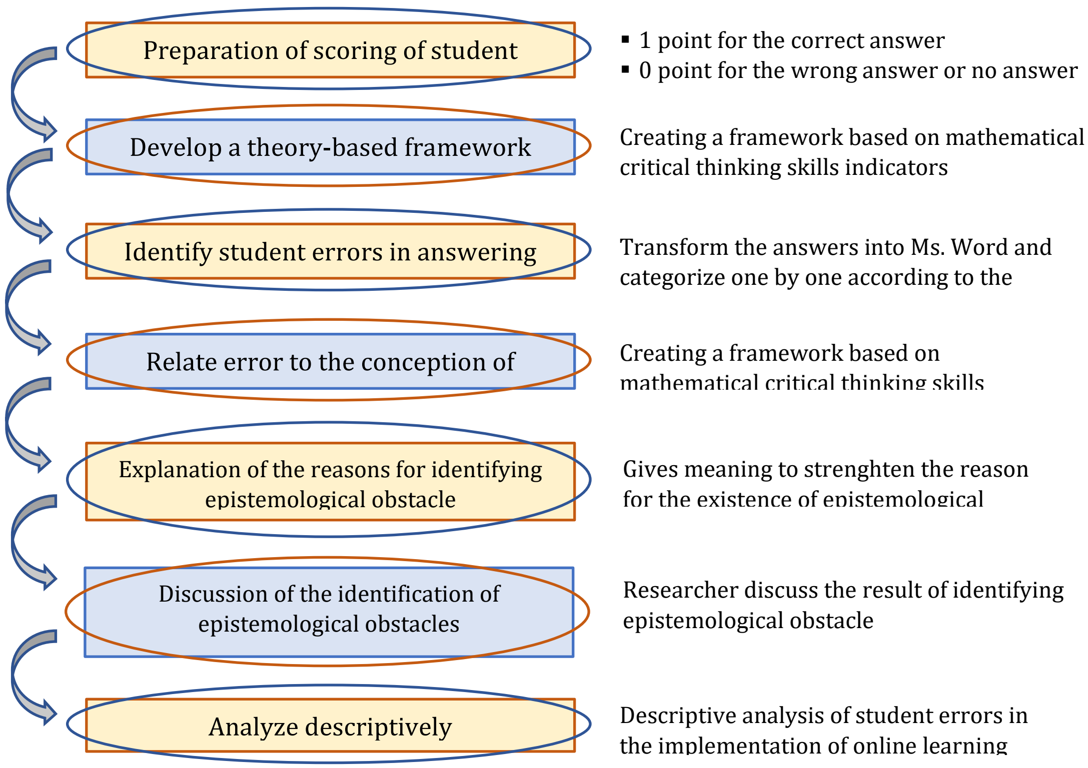


Figure 2: Data Analysis Framework

RESULTS

The analysis results of the mathematical critical thinking ability test of the nine samples in this study are empirically presented in Table 1.

Table 1. The result of critical thinking skills test

	Proof	**Generalization**	**Alternative answer**	**Problem Solving**
	Question 1	Question 2	Question 3	Question 4
N	9.00	9.00	9.00	9.00
Average	2.55	3.00	1.83	3.11

The maximum score for each question was 6. Based on Table 1, the average score for the first question was 2.55, which indicates that students have difficulty. Furthermore, the respondents found it difficult to prove the inference, situation, clarity, and overview sections. In the second question, an average value of 3.00 was obtained, indicating that they have difficulty generalizing in the situation, clarity, and overview sections.

Furthermore, the average scores obtained in the third and fourth questions were 1.83 and 3.11, respectively. This shows that students have difficulty in determining alternative answers and solving problems. They were only able to answer questions in one way and have problems in the situation, clarity, and overview sections. They also struggled with problem-solving, specifically in inference, situation, clarity, and overview.

The findings in Table 1 were strengthened by the calculation results of the percentage of students who cannot answer the questions correctly, as shown in Table 2.

Table 2. Percentage of students who cannot give the correct answer

No	**Question Indicator**	**Total Student**	**Percentage of students who cannot give the correct answer**	
			Total	(%)
1	Proving that the length of the prism base is related to the prism volume	9.00	8.00	88.89
2	Generalizing the relationship between cuboids and prisms	9.00	8.00	88.89
3	Counting the number of cubes that can be accommodated by the cuboids with at least two alternative answers	9.00	9.00	100.00
4	Calculating the surface area of a pyramid with certain conditions	9.00	7.00	77.78

Table 2 shows that the majority of students gave incorrect answers to the questions. For the first and second questions, only 1 student answered correctly, while for the third question, all answers were incorrect. Furthermore, two students answered the fourth question correctly. Epistemological obstacles in the critical thinking process occurred in the areas of inference, situation, clarity, and overview. The epistemological obstacles for each question are presented in Figure 3.

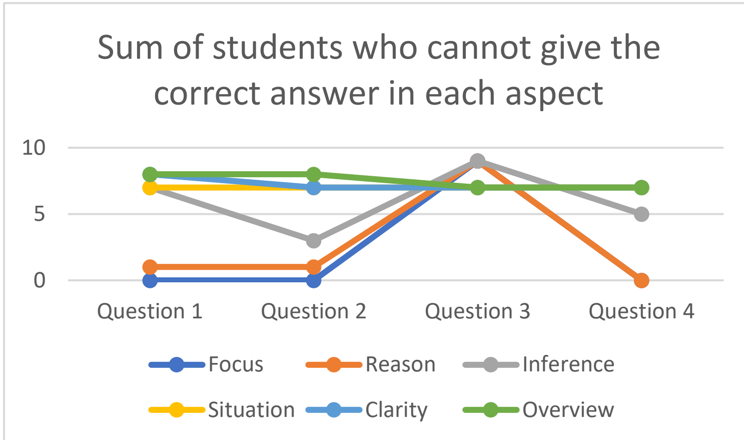


Figure 3: Epistemological obstacle in each question

Figure 3 shows five epistemological obstacles for proof in the first question. The first obstacle occurred in the reason stage, where 11.11% of students had difficulty explaining the idea or making a diagram for analysis. They directly calculated the volume of the prism without analyzing the idea. Therefore, they had difficulty continuing the problem-solving idea because the other parts were unknown. Furthermore, the second, third, and fourth types of obstacles occurred in the inference stage. In the second type, 33.33% of students failed to manipulate algebra related to the substitution of the basic elements of the prism. They only utilized the properties of an isosceles right triangle with two sides of the same length and wrote the formula for the volume ratio of the prism correctly. Students did not try to replace the area of the prism base with its details. In the third type, 11.11% had difficulty comparing volumes. They could not utilize the properties of an isosceles right triangle, where two of its sides have the same length. In the fourth type of obstacle, 22.22% had difficulty determining the formula for the prism's volume. The written formula was exchanged with the volume formula. The fifth epistemological obstacle occurred in the inference, situation, clarity, and overview stages, where 11.11%, 77.78%, 88.89%, and 88.89% of students could not answer the question. They could not understand the idea, adapt to the given situation, clarify the assumed or symbolized elements, and review the problem-solving process.

In question two, there are 5 types of epistemological obstacles to generalization. There were 11.11% of students that had difficulty in the reasoning stage. This is because they could not express the idea of analyzing the prism and the properties of the cube to determine the relationship between them. Furthermore, 22.22% of students experienced epistemological obstacles in the inference stage and made mistakes in analyzing some prism's properties. The third type of obstacle was encountered at the situation stage. Students could not determine the generalization that a cube is a four-sided prism because it does not have the properties of both a prism and a cube. The fourth obstacle was experienced by 22.22% of students at the overview stage, such as hastily drawing conclusions without reviewing the problem-solving process. The fifth type of question involved inference, situation, clarity, and overview, in which 11.11%, 77.78%, and 66.67% of students could not answer the given questions.

In question three, there were 6 types of epistemological obstacles in determining alternative answers. The first type occurred at the focus stage, where 100% of the students could not focus on identifying a problem related to at least two types of solutions. The second obstacle type was experienced at the reasoning stage, and all samples could only provide one idea related to problem-solving. Two students suggested analyzing a cube and the size of a cuboid, while seven students proposed using a formula. The third obstacle occurred at the inference stage, where 22.22% of students could only place and arrange a cube's size in a cuboid, while 77.78% performed calculations on the test material. The fourth type was experienced at the situation stage, where 77.78% of students could not connect the volume found with the problem situation. They directly divided the cube's volume by the cube's volume without considering that the latter was not water or sand, which could occupy all space. The cube was a solid object with certain dimensions, causing empty space inside the cuboid that could not be filled. The fifth obstacle occurred during the overview, where 77.78% of students drew the wrong conclusion. In the sixth type, 77.78% could not answer at the clarity stage. Additionally, they could not clarify the elements assumed or symbolized.

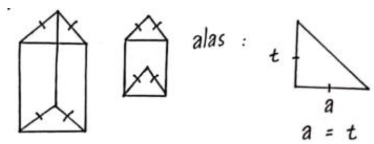
In the fourth question, there were four types of epistemological obstacles in problem-solving. The first type occurred at the inference stage, where 55.56% of students could draw the net of a pyramid and determine the part covered with an equilateral triangle glass. However, they could not calculate its area because the height was unknown. The second type of obstacle occurred at the situation stage, and 22.22% could not connect the area of the triangle with the desired condition, such as the required glass area is equal to three times the area of the equilateral triangle. The required glass area was assumed to be equal to the area of the triangle. The third type was at the overview stage, where 22.22% of students made conclusions without double-checking the problem-solving process. The fourth obstacle occurred at the situation, clarity, and image stages, where 22.22%, 77.78%, and 55.56% of students, respectively, could not answer the questions. Similar problems were also found in previous questions.

DISCUSSION

Epistemological obstacles in the proofing dimension

The findings of this study reveal that a significant percentage of students, approximately 88.89%, struggle with providing proof for a given statement. Specifically, the research explains that students face difficulties in proving the relationship between the base of a prism and its corresponding volume. To further illustrate this issue, Figure 4 provides an example of common errors made by students in their attempts to prove this relationship.

1. Two prisms are right-angled isosceles triangles, and the height of the volume of prism 1 is greater than that of prism 2. Are the lengths of the three sides of the base of Prism 1 greater than Prism 2?
   Answer:



$V_1 > V_2$

$La_1 t_1 > La_2 t_2$

$\frac{1}{2} a_1 t_1 t > \frac{1}{2} a_2 t_2 t$

$\frac{1}{2} t . t_1 . t > \frac{1}{2} t . t_2 . t$

$\frac{1}{2} t^3 > \frac{1}{2} t^3$

Wrong

Figure 4: Epistemological obstacle in proofing

In general, students demonstrate understanding of the concept of a right-angled triangle as the base of a prism, the formula for calculating volume, and the ability to write comparative formulas. However, they often make mistakes when attempting to combine these concepts with proof. For example, students may substitute algebraic expressions with equations without understanding how to integrate formulas to fit the desired situation. These errors are consistent with common mistakes made in mathematics, such as misunderstanding algebraic expressions and variable concepts (Jupri et al., 2014), substituting expressions into equations, solving and rearranging them, as well as solving equations and formulas (Rushton, 2014).

The errors made by students in combining concepts from related subjects suggest that their connection skills in mathematics may be weak. To strengthen critical thinking skills, it is necessary to establish connections to the real world, other disciplines, and other concepts (García-García & Dolores-Flores, 2021). The constraints found in the study suggest that students struggle to link one concept to another, indicating knowledge compartmentalization. This situation is a common concern where only one type of knowledge is developed separately from others (Yao et al., 2021).

Epistemological obstacles in the generalization dimension

The findings on the dimension of generalization revealed that a significant percentage of students (88.89%) could not generalize the relationship between a cuboid and a rectangular parallelepiped. While 77.78% of students understood the properties of both shapes, they had difficulty determining the relationship between them. To illustrate this point, Figure 5 shows an example of a student's incorrect answer.

2. Given: Objects with cuboid and prism shapes Question: Is every object with a cuboid shape also a prism? Answer:	
Characteristics of a cuboid: • Has a base and top with the same shape • Congruent Vertical edges • V= l x w x h • Base and top can have various shapes.	Characteristics of a prism: • Has a base and a top with the same shape • Congruent vertical edges • V = Base area x height • Base and top have only one shape
Cuboid and prism are different	

Figure 5: Epistemological obstacle in generalization

The students tend to memorize the properties of concepts without fully understanding them. Prioritizing memorization over understanding can limit their ability to think critically (Firdaus & Kailani, 2015), making it difficult for them to find connections between different concepts. This finding is consistent with the previous results regarding the weakness of students in mathematical connections, which can result in a lack of understanding in critical thinking due to the positive correlation between the two (García-García & Dolores-Flores, 2021; Cai & Ding, 2017). A student with a strong foundation in mathematical knowledge can connect ideas, concepts, procedures, representations, and meanings (García-García & Dolores-Flores, 2021).

Epistemological obstacles in the alternative answer dimension

The findings on the alternative answer dimension reveal that 100% of the students had difficulty calculating the number of cubes that a cuboid can accommodate with at least two alternative answers. Furthermore, 22.22% could find the answer but only with one method, while the remaining 77.78% who failed still used only one method. This shows that students have difficulty creating flexibility to solve a problem. Their focus is limited to one method they master without considering other alternatives. Figure 6 shows an example of a student's incorrect answer.

3. Method 1:
A cuboid-shaped cabinet, l = 50cm, w = 25 cm, h = 100cm
A cube-shaped souvenir box, V = 1 dm^3
What is the maximum number of souvenir boxes that can fit in the cabinet?
Answer:
V cuboid = l x w x h = 50 x 25 x 100 = 125000 cm^3
V cube = 1 dm^3 = 1000 cm^3
Since the units are the same, the maximum number of boxes is:
V cuboid: V cube = 125000 :1000 = 125
Method 2:

Figure 6: Epistemological obstacle in alternative answer

The concept of volume formulas for cubes and cuboids is the most common way to solve maximum capacity problems. The rigidity of conceptual knowledge in this subject is evident from the mistakes made. The volume concept is directly applied without considering the problem situation. Emphasis on theory without variation in problems is a potential source of student failure (Cai & Ding, 2017). The sample did not consider the dimensions of the binding cube, so the likelihood of not achieving the desired result increases. All procedures and rules for volume problems are directly applied regardless of the situation. Emphasis should be placed on the importance of flexibility while providing solutions. In-depth procedural knowledge, which refers to flexibility in using procedures and rules, needs further attention (Star, 2015), especially in volume. Determining the area and volume of objects is crucial for students to succeed in mathematics and science (Vasilyeva et al., 2013).

Epistemological obstacles in the problem-solving dimension

The findings on problem-solving dimensions reveal that 77.78% of students have difficulty calculating the surface area of a pyramid under certain conditions. These tasks are presented in the form of story, namely in the context of building a pyramid-shaped building where each side is covered with glass except for the front and bottom. Previous studies have revealed that difficulties often arise when solving contextual problems in narrative form (Wawan & Retnawati, 2022). This is evident from the students' challenges in understanding the problem. Figure 6 shows an example of students' errors in their answers.

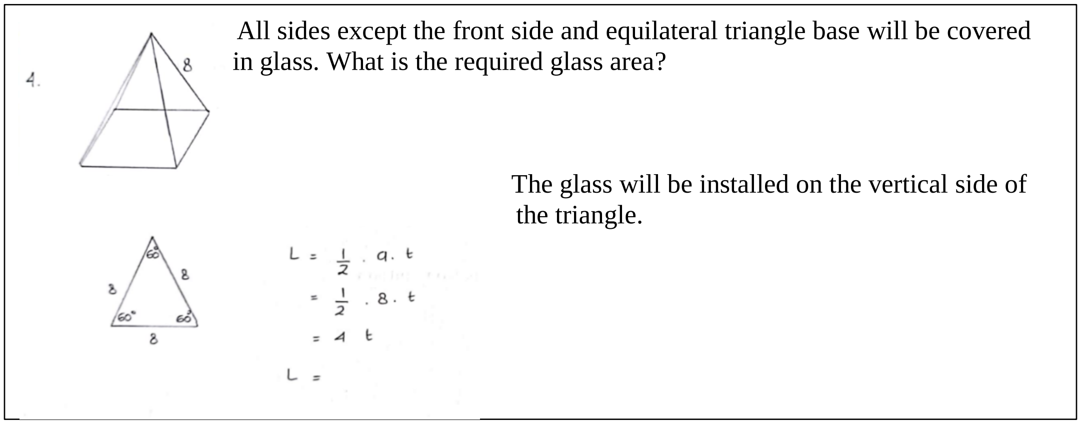


Figure 7: Epistemological obstacle in problem-solving

Figure 7 shows that the students misunderstood the required glass area as the triangle area on the pyramid side. Furthermore, 66.67% did not break down the image in the problem into pyramid nets to simplify their work. The use of images or symbols for problem-solving can help students understand mathematics (Yao et al., 2021). There is often a tendency to approach challenges when learning geometry by expressing mathematical solutions and drawing (Sumaji et al., 2019). Additionally, 55.56% of the students had difficulty calculating the area of an equilateral triangle.

The commonly used formula to calculate the area of a triangle is (1/2) x base x height. Therefore, some students make mistakes by first calculating the triangle's height using the Pythagorean theorem. Errors in determining the base of the triangle also occur when confusing it with the length of the base when calculating the height of the triangle. Only one student used another formula to find the area, for example, $\sqrt{s(s-a)(s-b)(s-c)}$, with s being $(\frac{1}{2})(a+b+c)$.

Epistemological Obstacles Causes in Online Learning

Several factors cause epistemological obstacles encountered in online learning implementation. Problems start to arise at the learning core stage, namely during the delivery of materials, exploration, presentation, and discussion. Based on interviews with prospective teachers, it is known that there are often obstacles in delivering materials in online learning due to the unpreparedness of some supporting facilities, such as unstable signals. This makes the delivery of materials often interrupted, resulting in incomplete acceptance. As a result, students only memorize concepts without a deeper understanding, which opens misconceptions possibility. Special attention is needed to overcome difficulties and misconceptions in learning materials (Sebsibe, et al., 2019). Supporting tools for online learning need to be prepared carefully as an anticipation form for possible obstacles, such as complete electronic guidebooks that can be accessed anytime and anywhere.

Another issue that arises in the exploration stage is suboptimal learning conditions. The lack of interesting and interactive online learning media introductions and presentations can decrease students' motivation to explore. Therefore, a comprehensive study is needed to identify various media or applications that can be used in online learning. The lack of experience among teachers in using online applications can also create difficulties in online learning (Zaharah & Kirilova, 2020).

To make mathematics learning more interesting, teachers can utilize various technologies in online learning. Technological advancements can create interesting stimuli to promote active and creative learning activities (Papadakis et al., 2016). The use of technology can also help overcome obstacles in using images or symbols for problem-solving by making mathematics more concrete. Technology-based mathematics applications can help students create, analyze, and clarify images. However, teachers need to choose the appropriate tool and ensure its accuracy in representing mathematical concepts in the classroom (Sulastri et al., 2022).

In addition to using technology, teachers can also provide stimuli for students to explore deeply using various online learning sources. This can help students overcome epistemological obstacles by creating flexibility in problem-solving. Students will not only rely on one method they have mastered but will also consider other alternatives through exploration and knowledge-building.

The next problem occurs during the discussion phase, which can be quite challenging. Minimal interaction becomes a challenge in online learning. Basically, online learning is very flexible and provides opportunities for more interaction (Chertoff et al., 2020). Therefore, it is essential to educate prospective teachers about the benefits of online learning and stimulate their participation in discussions through engaging questions or interesting games. This approach aims to hasten the adaptation process to online learning.

CONCLUSION

In conclusion, the epistemological obstacles related to critical thinking were discovered in online learning, with students struggling to prove the relationship between two concepts, generalize relationships, seek alternative solutions, and solve problems. These difficulties arose partly because the steps of online learning had not been fully optimized. At the time, online learning emphasized virtual learning through live video conferencing, which became a hindrance for students when they encountered signal issues. As a result, they missed some parts of their learning, which caused confusion when they could rejoin the class but with new material. Additionally, the lack of information regarding the use of interactive, technology-based online learning media made the exploration activity, which is at the core of learning, less effective. Students tended to be passive, not exploring much information by browsing online, but only using available learning resources. However, in-depth exploration activities could stimulate the development of critical thinking skills. Students also appeared to be perplexed when interacting with other students. The difficulty in adapting to online learning was still apparent.

To foster the development of critical thinking skills, various enhancements are necessary in online learning. For this purpose, online learning should be well-prepared and structured, which can be initiated through good planning. To achieve this, lecturers can prepare comprehensive learning guidelines in e-modules or other materials that focus on understanding the relationship between concepts, the flexibility of concepts and procedures, and the habit of drawing in geometry learning. Furthermore, lecturers can provide ample information about the use of learning sources, learning media, and appropriate technology to support online learning.

Well-prepared online learning, combined with appropriate technology and correctly delivered to students, can make learning mathematics meaningful. Students' exploration of their abilities can be maximized by the opportunities provided by online learning, both from unlimited learning resources and technology that can be integrated. Students can also learn without spatial and temporal limitations. These things can certainly stimulate students to improve their critical thinking skills.

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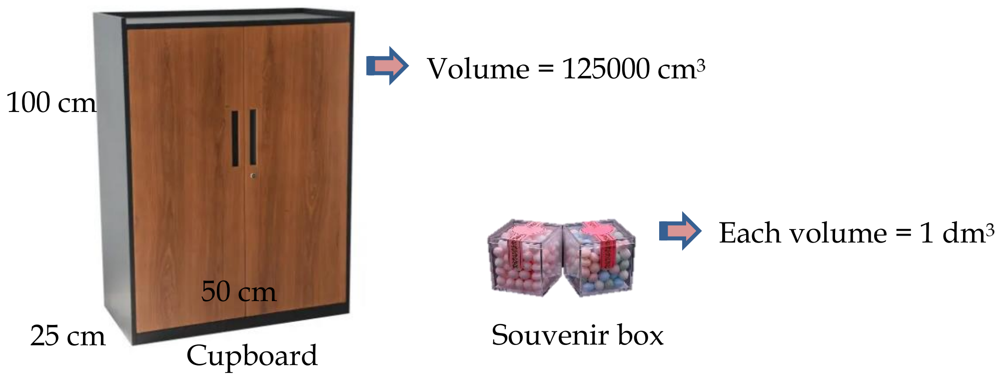
APPENDIX

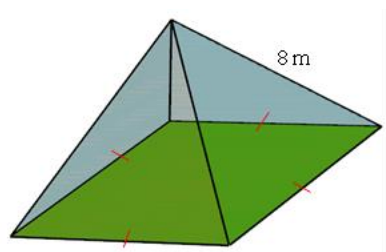
MATHEMATICAL CRITICAL THINKING TEST QUESTIONS

1. Two upright prisms of the same height have a base in the form of an isosceles triangle. If the volume of the first prism is greater than the second prism, is it certain that the three sides of the base of the first prism are larger than the second prism? Prove it!
2. There are objects in the form of cuboid and prisms with a volume of 60 cm^3 each. Must every cuboid-shaped object be a prism? Explain your reasons in detail!
3. A storage cupboard and a souvenir box are shown as follows.



If the cupboard will be used to store several souvenir boxes, count in at least two different ways how many maximum souvenir boxes the cupboard can accomodate!

4. Look at the following picture.



Garden Building Design

A company wants to build an indoor park according to the design of a garden building. All the vertical sides will be made in the form of equilateral triangles. Furthermore, the building will be completely covered with glass, except for the front which is left open. Flowers and grass will be planted at the base. Calculate the area of glass needed to make the building according to the desired design!

Students' Proactive Interference in Solving Proportion Problems: How was the Met-before?

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Abstract: Students' difficulties in differentiating the direct proportion and inverse proportion problems cause interference. Proactive interference is the error that occurs when old information (concept of direct proportion) interferes with new information (concept of inverse proportion). In solving the problem of inverse proportion, students often use the concept of direct proportion. The student's mental structure regarding the concept of proportion as a result of previous learning is referred to as met-before. Therefore, this study aims to describe the met-before of students who experience proactive interference. This research is a case study involving 32 8th-grade students in Malang, Indonesia. These subjects were students who experienced proactive interference with specific fluency of communication and willingness. Data was collected through proportion problems and interviews. Students' work was analyzed based on the description of the met-before. The results showed that students who experienced proactive interference with the non-flexible type had suppressed problematic, while students with the flexible type have focus supportive met-before in solving direct proportion problems. Both students with non-flexible type and flexible type have focus problematic met-before when solving inverse proportion problems. This is because met-before about cross multiplication strategy interferes with students' problem-solving.

Keywords: proactive interference, direct proportion, inverse proportion, met-before

INTRODUCTION

Thinking carries an important role in the process of understanding and acquiring new knowledge (Sanjaya et al., 2018; Tohir et al., 2020), as well as facing and solving a problem (Hobri et al., 2021; Mairing, 2016; Tekin et al., 2021), and also reasoning (Faizah et al., 2022). Thinking is also related to mathematics and problem-solving. The tasks and exercises provided in the process of learning mathematics can be in the form of problem-solving. Solving mathematics problems is an important part of mathematics education research (Akyüz, 2020) and learning (Izzatin et al., 2021; Szabo et al., 2020) globally (Rahayuningsih et al., 2020). Problem-solving serves as the foundation (NCTM, 2000; Reys et al., 2009) and the heart of mathematics (Barham, 2020). Baraké et al. (2015) asserted that problem solving has been and still remains the basis for learning mathematics.

In addressing a particular problem, one needs to recall knowledge in their long-term memory. This act of recalling information in long-term memory is known as retrieval (Ormrod, 2020; Slavin, 2017). McBride and Cutting (2018) described retrieval as the process of calling/removing information from memory. However, one can experience failure when doing the retrieval process. In information processing theory, this retrieval failure is called interference (Slavin, 2017; Sternberg & Sternberg, 2012).

Interference is a disturbance or error that occurs because the process of calling one information interferes with other information (Slavin, 2017; Solso et al., 2014; Sternberg & Sternberg, 2012). Anderson (2020) and Slavin (2017) described that this interference refers to forgetting events caused by disruption in the information retrieval process. Besides, interference can also occur when existing information is mixed with other information (Ormrod, 2020; Sternberg & Sternberg, 2012). Therefore, interference is often defined as interference that occurs because one information interferes with other information and the mixing of information due to the similarity of the received information.

In the field of mathematics, interference is related to the failure of students to recall concepts that have been learned and are being studied. Sternberg and Sternberg (2012) states that interference occurs when students have an understanding of two or more different concepts where these concepts are interrelated. The same thing was expressed by Sukoriyanto et al. (2016) this interference is in the form of errors that occur due to conceptions that interfere with each other, so that one concept interferes or interferes with other concepts.

Interference in thinking is divided into retroactive interference and proactive interference (Georgiou et al., 2021; McBride & Cutting, 2018; Mercer, 2014; Slavin, 2017; Solso et al., 2014; Sternberg & Sternberg, 2012). Interference is said to be retroactive when new information interferes with the ability to recall old information. Furthermore, Anderson (2020) states that retroactive interference is defined as forgetting that arises as a result of new learning. In other words, someone who experiences this retroactive interference usually forgets old information, highlighting the process where learning a new task leads to forgetting previously learned information.

Conversely, proactive interference occurs when old information interferes with the ability to remember new information (McBride & Cutting, 2018; Slavin, 2017; Solso et al., 2014; Sternberg & Sternberg, 2012). In line with this, Anderson (2020) defines proactive thinking interference as forgetting that arises as a result of previous learning. Therefore, when a person experiences this proactive interference, memories that have been stored for a long time in long-term memory interfere with new information being entered into memory. Both forms of interference occur when the information received occurs in close temporal proximity.

For example, the mathematical materials that are often presented in close time proximity is the material for direct and inverse proportion (Sukoriyanto et al., 2016). Those two materials have similar problem structures, leading students to frequently experience interference (Irfan et al., 2019a). Following the curriculum guidelines, teachers typically introduce the topic of direct proportion as the initial material (Ben-Chaim et al., 2012; Billstein et al., 2016; Petit et al., 2020;

Walle et al., 2020). Subsequently, inverse proportion material is taught after students learn direct proportion. As the first information received by students is direct proportion material, this material becomes old information for students. Meanwhile, information regarding inverse proportion material is seen as new information. Consequently, when the student's firmly embedded memory is the concept of inverse proportion, they may solve the problem of direct proportion by using the concept of inverse proportion, thereby, they experience retroactive interference. On the other hand, when a student's strong memory is the concept of direct proportion, they solve the inverse proportion problem using the concept of direct proportion. Therefore, the student experiences proactive interference.

Mathematical materials that possibly cause interference with students include greatest common factors and least common multiple, direct and inverse proportion, arithmetic sequences and series, geometric sequences and series, and permutations and combinations (Sukoriyanto et al., 2016). In this study, we focus on direct and inverse proportion material in tracing the occurrence of interference. The concept of proportion is important in an education setting (Andini & Jupri, 2017; Artut & Pelen, 2015; Buforn et al., 2022; Diba & Prabawanto, 2019; Dougherty et al., 2016; Perumal & Zamri, 2022). Proportional material serves as the foundation for studying more advanced mathematical material (Dougherty et al., 2016; Misnasanti et al., 2017; Vanluydt et al., 2021; Weiland et al., 2021) such as algebra, geometry, statistics, and so on (Beckmann & Izsák, 2015; Misnasanti et al., 2017; Vanluydt et al., 2021). Apart from being important in learning mathematics, this proportion concept is also useful in everyday life (Phuong & Loc, 2020).

Research on direct and inverse proportion mostly focuses on proportional reasoning (Artut & Pelen, 2015; Castillo & Fernandez, 2022; Öztürk et al., 2021; Pelen & Artut, 2016; Tjoe & de la Torre, 2014). Irfan et al. (2019a) examined the interference that occurs when students solve proportion problems in terms of APOS theory. Then, Irfan et al. (2019b) examined semantic and procedural interference. Meanwhile, our observation conducted at Junior High School 3 Malang revealed that students were confused and interfered with when solving two problems (direct and inverse proportion). Most of the students solved the problem of inverse proportion with direct proportion concepts. This phenomenon is known as proactive interference. Proactive interference occurs when someone's old knowledge interferes with new knowledge.

In addition, interference is related to the process of recalling information in students' memory (Slavin, 2017; Solso et al., 2014; Sternberg & Sternberg, 2012). The thinking process of these students can be traced through their met-before. Met-before refers to a mental structure that a person currently possesses as a result of previously encountered experiences (McGowen & Tall, 2010; Mowahed & Mayar, 2023; Tall, 2013). Chin and Jiew (2019) elaborate that met-before refers to the results of previous student experiences that influence their current thinking and shaping mathematical conceptions. Through met-before, students' learning problems can be identified. This is in accordance with the statement of Tall et al. (2014) that previous learning experiences and prior knowledges (Martin & Towers, 2016; Wakhata et al., 2023) can affect a person's cognition. Previous learning experience used in current learning is also known as met-before (Mowahed & Mayar, 2023; Tall et al., 2014).

Research related to met-before was conducted by McGowen and Tall (2010), focusing on met-before, which caused students difficulties in studying algebra in college and problems related to the minus sign (-). Specifically, met-before can be supportive and problematic (McGowen & Tall, 2010; Mowahed & Mayar, 2023). Met-before becomes supportive when old ideas can be used in new contexts in a plausible way (McGowen & Tall, 2010; Mowahed & Mayar, 2023). Conversely, met-before becomes problematic when students cannot use the ideas or knowledge they have previously learned (McGowen & Tall, 2010; Mowahed & Mayar, 2023). This cases also often causes cognitive conflict for student which becomes problematic because of the difference between new information and existing mental structures (met-before) (HR et al., 2023). Similarly, Chin et al. (2019) described that supportive conceptions refer to old conceptions that have been studied before and are applicable to new contexts. In contrast, problematic conceptions refer to previously learned conceptions that are non-applicable in new contexts. The conception described by Chin et al. (2019) is a form of met-before (Chin & Pierce, 2019). In other words, a supportive met-before will aid and help students use their existing knowledge in learning or understanding new knowledge. Meanwhile, the problematic met-before will become an obstacle for students in learning further knowledge. This supportive and problematic is then examined by Chin et al. (2019), where the construction of this supportive and problematic conception can assist researchers in understanding the process of assimilation and accommodation occurs in the human mind.

Supportive met-before does not always offer a supportive nature, and it sometimes becomes an obstacle. According to Chin and Jiew (2019) Jiew and Chin (2020), supportive met-before may contain problematic aspects which are then referred to as suppressed problematic. Conversely, problematic met-before may contain supportive aspects which are then referred to as *suppress supportive*. Those forms met-before is illustrated in Figure 1.

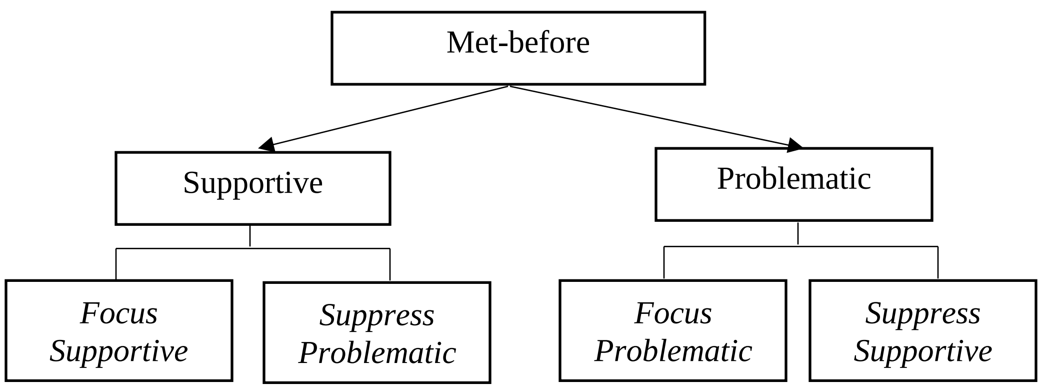


Figure 1: Supportive and Problematic Met-before
Source: (Chin & Jiew, 2019)

As presented in Figure 1, supportive met-before can be *focus supportive* and *suppress problematic*. According to Chin and Jiew (2019), *focus supportive* is a conception or met-before held by students that is supportive and applicable in new contexts. As an illustration, the met-before student about "concepts multiplication is repeated addition." That met-before will *focus supportive* in natural numbers, for instance, $3 \times 1 = 1 + 1 + 1$; $2 \times 5 = 5 + 5$, etc. However, supportive met-before about "concepts multiplication is repeated addition" may contain problematic aspects or are called *suppress problematic*. In this case, when the multiplier of multiplication is negative numbers, such as to solve "-4×2," attempting to represent it as repeated addition (e.g., "$2 + 2 + 2 + 2$") proves challenging and impractical. However, there is a case study where students can remove the problematic aspect by using their knowledge about the commutative property of multiplication ($p \times q = q \times p; p, q \in R$) (Jiew & Chin, 2020). Therefore, "$-4 \times 2 = 2 \times -4$" can be written as "$2 \times -4 = (-4) + (-4) = -8$". From these examples, *suppress problematic* is supportive met-before, which may contain problematic aspects. However, there may be a possible approach to remove problematic aspects using other knowledge that assists in effectively addressing the problem.

Besides being supportive, the met-before also contains problematic met-before, namely *focus problematic* and *suppress supportive*. Chin and Jiew (2019) define *focus problematic* as a problematic conception or met-before, which hinders or is non-applicable in new contexts. For example, with the same case for met-before about "concepts multiplication is repeated addition". This met-before becomes problematic when students apply it to the calculation of fractions such as "$\frac{1}{2} \times \frac{1}{4}$". Students can't write down."$\frac{1}{2} \times \frac{1}{4}$" as repeated addition. However, this problematic met-before also contains supportive aspects, referred to as *suppress supportive*. As an illustration, when students decide to use "concepts of multiplication as repeated addition" in the multiplication of fractions, they can solve "$\frac{1}{2} \times \frac{1}{4}$" with $\frac{1}{2}$ of $\frac{1}{4}$ or "$\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$".

Following up on the results proposed by Chin et al. (2019); Chin and Jiew (2019); and Jiew and Chin (2020) research, this study explores those forms met-before illustrated in Figure 1, specifically on students who experience interference. This study focuses on investigating the occurrence of met-before in students who experience proactive interference when solving problems of direct and inverse proportion. Students who are suspected of experiencing proactive interference present the ability to solve the problem of direct proportion using the concept of direct proportion. However, students solve the problem of inverse proportion with the concept of direct proportion.

Interference has been studied by several other researchers (Babai & Lahav, 2020; Hidayanto & Budiono, 2019; Irfan et al., 2019; Jayanti et al., 2018; Maulyda et al., 2020; Stavy & Babai, 2010; Sukoriyanto et al., 2016; Visscher et al., 2015). However, those studies mainly focused on students with dyscalculia (Babai & Lahav, 2020; Stavy & Babai, 2010; Visscher et al., 2015) and problem-solving (Hidayanto & Budiono, 2019; Irfan et al., 2019; Jayanti et al., 2018; Maulyda et al., 2020; Sukoriyanto et al., 2016). Existing research has not investigated the causes of interference through met-before. Through the students' met-before, their stored knowledge can be analyzed more effectively (Chin, et al., 2019; Chin & Jiew, 2019). Therefore, it is essential to examine the interference of students when solving proportion problems through their met-before. Therefore, the problem in this study is "how was students' the met-before who experience proactive interference in solving problems of direct and inverse proportion?".

METHOD

Research design

This research was designed using the case study research type. The approach was selected based on the findings of researchers regarding met-before students who experience proactive interference in solving proportion problems. Therefore, the researchers used a mathematical test on direct and inverse proportion problems and an interview guide. The test was used to identify the met-before and proactive interference that occurs in students. Meanwhile, the interview was used to confirm and deepen the understanding of the thinking processes of students who experience proactive interference. The results of the student's work were analyzed based on the student's work process, which was adjusted to the alternative answers prepared by the researcher. We analyzed the process of students' work indicated experiencing interference. This study adopts a case study following the assertion from Creswell and Creswell (2018) that case studies are applicable for describing and exploring a unique case in a particular phenomenon. The case study was performed specifically to deepen the understanding of a phenomenon for the general public (Bloomberg & Volpe, 2019). In this study, we describe the students' met-before who experience proactive interference.

To achieve this goal, we used the guidelines shown in Table 1.

Stage 1	**Stage 2**	**Stage 3**	**Stage 4**	**Stage 5**
A preliminary study conducted observations of students in accelerated classes but there were indications of proactive interference when solving direct and inverse proportion problems.	We gave two mathematical problems (direct and inverse proportion) to 32 8^{th}-grade students in Malang.	Researchers analyzed students' work and selected students who experienced proactive interference. Researchers consider the fluency of student communication and the student's willingness to be used as research subjects.	The researchers conducted interviews with two research subjects who had been selected based on the results of the researcher's analysis from Table 2. The researchers conducted interviews with the two research subjects outside of mathematics class hours.	The researchers triangulated data from the results of the research subject's work and the results of interviews to provide conclusions regarding the students met-before who experienced proactive interference.

Table 1: Research Stages

As described in Table 1, the data were collected from various sources, containing of student work and recorded interviews to obtain accurate results. After collecting the data, the findings were analyzed from the students' work through the indicators of the met-before presented in Table 2. After the analysis, we drew conclusive insights on the met-before of students experiencing proactive interference.

This research was conducted on students attending class category, thereby, they are regarded as having high abilities. This choice was made to clearly identify that interference does not exclusively occur in students with low or medium mathematical abilities, underscoring the need to investigate and address this phenomenon across a diverse spectrum of mathematical proficiency.

Research Subject

This research involved 32 8th-grade students in Malang. The selection of 8th-grade students was based on a preliminary study reporting indications that students experienced proactive interference in solving problems of direct and inverse proportion. The proactive interference being investigated in this study pertains to students who solve the problem of inverse proportion using the concept of direct proportion. To ascertain that proactive interference is occurring, we prepare a direct proportion problem. The problem of direct proportion serves as an instrument for identifying whether the interfering concept observed is related to direct proportion. Therefore, the selected research participants are those who correctly complete direct proportion problems but use the concept of direct proportion in solving inverse proportion problems. We also consider the fluency of student communication and student willingness in the selection of research subjects.

From these considerations, we determined two research subjects, with the first subject coded as S1 and the second subject as S2. S1 was a student with a non-flexible type, and S2 was a flexible type. This classification was made based on students' answers in solving direct proportion problems. They are classified as non-flexible when they make mistakes in algebraic algorithms, while flexible students can do the algebraic calculation process properly.

Data Collection and Data Analysis

We gave two proportion problems (direct and inverse proportion) to 32 8th-grade students. The problems are presented in Figure 2.

1. If the salary of 12 workers for 5 days is IDR 9,000,000.00. What is the salary received by 15 workers for 3 days assuming the performance of each worker is the same?
2. The project can be completed by 8 workers in 6 hours per day for 10 days. How long will it take 4 workers in 8 hours per day to complete the project? The performance of each worker is considered the same.

Figure 2: The Problem of Direct and Inverse Proportion

The results of student work were analyzed using the rubric of alternative answers. The results of this analysis will suggest the students who experience proactive interference.

In exploring the met-before of students who experienced proactive interference, we used a description of the met-before classification shown in Figure 1 (Chin and Jiew, 2019) and described in Table 2.

Met-before	Description
Focus Supportive	Supportive conceptions refer to old conceptions (direct proportion) that have been studied before and are applicable in new contexts (inverse proportion). Met-before is supportive of all concepts used in solving problems.
Suppress Problematic	Supportive conceptions contain problematic aspects and possible ways to remove problematic aspects by using other knowledge that is useful to solve the problem.
Focus Problematic	Problematic conceptions refer to previously learned conceptions (direct proportion) that are not applicable in new contexts (inverse proportion). Met-before is problematic in all the concepts used in solving problems.
Suppress Supportive	Problematic conceptions contain supportive aspects and use other knowledge that can be used to solve the problem and make sense.

Table 2: Description of Met-before

The collected data from students' works were analyzed following the description provided in Table 2. Subsequently, the interview was conducted. This interview was a semi-structured interview, allowing for adjustments based on the specific findings from the initial analysis. This interview aims to explore the met-before students who experience proactive interference. Researchers also triangulated data from the results of student work and interviews.

RESULT

From 32 students who solved the problem 2 presented in Figure 2, there were 3 students with correct answers, while 29 students answered incorrectly, as presented in Figure 3.

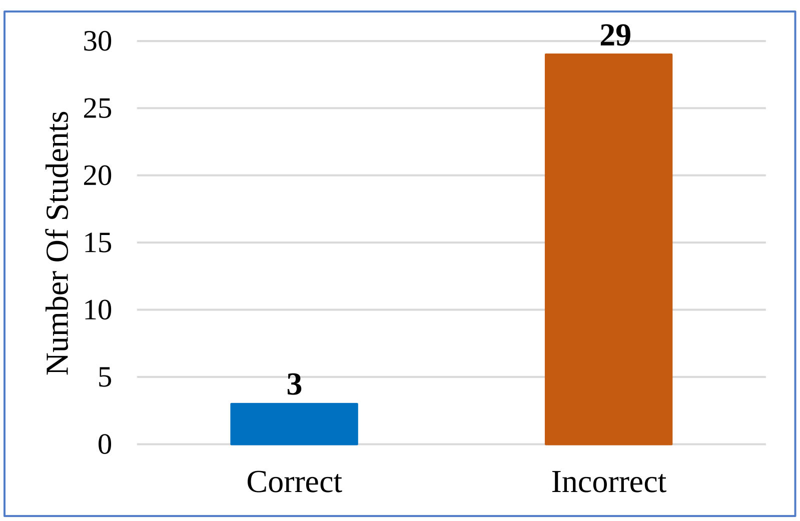


Figure 3: Students' Answer

Based on Figure 3, there are 29 students who are still wrong in answering the problem 2. Of the 29 students, 15 students did not answer the problem using the concept of proportion, 3 students were indicated as having retroactive interference, and 11 students experienced proactive interference. The results of these data are illustrated in Figure 4.

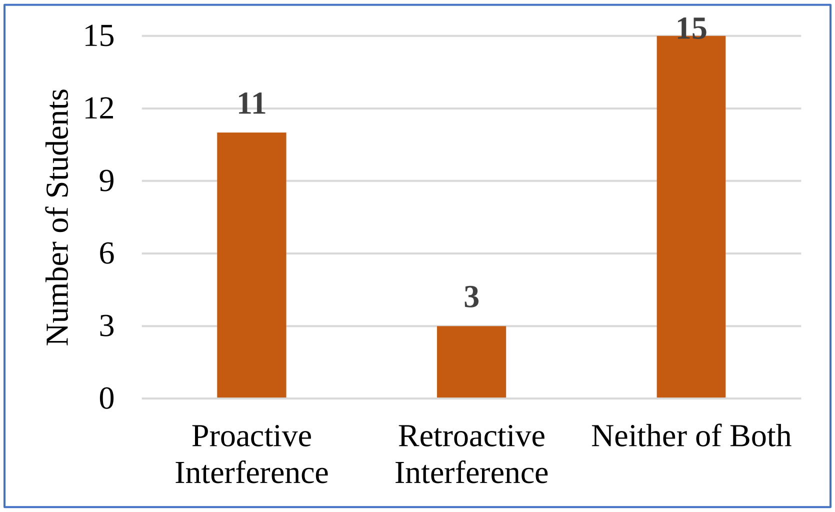


Figure 4: Types of Students' Errors

As shown in Figure 4, this study centered on students who experienced proactive interference. The work of the three students experiencing retroactive interference was unable to be explored as they had limited ability to articulate why they applied the concept of inverse proportion to solve problem number 1. Besides, the students' work also didn't show clear results. Therefore, to explore the cause of the problems faced by students encountering interference, we selected those who experience proactive interference.

The results from this study are manifest in a description of the met-before from students who experience proactive interference in solving direct and inverse proportion problems. Specifically, the proactive interference in focus pertains to students solving the problem of inverse proportion using the concept of direct proportion. In essence, this signifies that the memory and understanding of direct proportion interfere with their ability to correctly utilize the concept of inverse proportion.

The research data was collected from the results of student work during tests and interviews. In the presentation of the research data, we present the subject's correct answers in working out the direct proportion problem and wrong answers (interference) in working out the inverse proportion problem. This presentation of correct answers shows that the proactive interference experienced by the subjects is due to the concept of direct proportion being stronger in memory subjects. In other words, the concept of direct proportion interferes with the concept of inverse proportion. The following describes the proactive interference experienced by research subjects in solving problems of direct and inverse proportion.

S1 Work Results (Non-Flexible)

In problem 1, S1 performs calculations by multiplying the number of workers by the worker's time, thereby, $12 \times 5 = 60$ and $15 \times 3 = 45$, as presented in Figure 5.

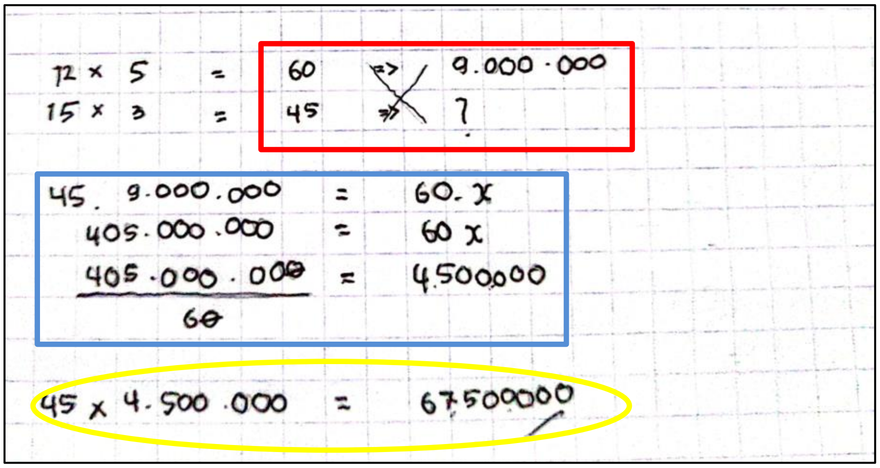


Figure 5: S1's Correct Answer in Problem 1

As shown in Figure 5, S1 carries out the calculation process with cross multiplication, as illustrated in the red box. To identify the process of working on the red box, we conducted interviews with S1. The following is an excerpt of the transcript of the researcher's interview with S1.

Q : *From your work on problem number 1, what is the meaning of writing "60 →9,000,000 and 45 →?" (while showing S1 work)*

S1 : *Oh, yes, ma'am. After I multiply 12 by 5, we get 60.*
So 60 gets 9,000,000. So if it's 45, how much will the worker get the money?
Then all cross I multiplied, as usual, ma'am.

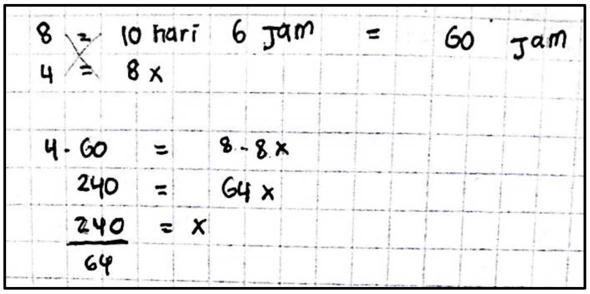
Through the interview, it was revealed that the length of work is 60, and the salary received is 9,000,000. However, when inquired about the salary for a working length of 45, S1 applied the cross multiplication method, as illustrated in the blue box. Regrettably, S1 writes in the last line $\frac{405.000.000}{60} = 4.500.000$. Ideally, the result from dividing 405,000,000 by 60 should be 6,750,000. However, when asked directly, S1 stated that the result of dividing 405,000,000 and 60 is 4,500,000. Then, S1 multiplies 45 by 4,500,000, resulting in 67,500,000. During the interview, S1 did not realize that the result of multiplying 45 by 4,500,000 was not 67,500,000.

From the data showing S1 work, S1 used the concept of direct proportion in solving the problem of direct proportion. However, S1 still made mistakes in calculating the results. Upon careful examination, S1 does not experience interference in solving the problem of direct proportion.

However, there were challenges in S1's met-before concerning the algebraic calculation process for determining the value of x. Even though the met-before of S1 supports the concept of direct proportion, it is still problematic for the concept of algebra. In other words, students' met-before is solving problem 1 is classified as met-before *suppress problematic*.

Then, we suspect the presence of interference when S1 solves the second problem. In this second problem, the inverse proportion problem is observed. However, when working on problem number 2, S1 still uses the concept of direct proportion, similar to when working on question number 1.

The results of S1's work on problem 2 is shown in Figure 6 below.



Translate:
8 = 10 days 6 hours = 60 hours
4 = 8x
4 . 60 = 8 . 8x
240 = 64x
$\frac{240}{64} = x$

Figure 6: S1's wrong answer in Problem 2 (Proactive Interference)

Referring to the information in Figure 6, S1 states "8 = 10 days 6 hours = 60 hours." S1 describes that with eight workers, the task can be completed within 60 hours.

In order to comprehend S1's approach to problem 2, researchers engaged in interviews with S1, and the interview transcript is shown in the following.

Q : *From problem number 2, how do you obtain the value of 60 hours? How do you solve this problem?*
S1: *Hmm.. 60 hours, I multiply 10 days by 6 hours.*
Q : *Why is that?*
S1: *Yes, ma'am, in that question, it said there were 8 workers. Then from those 8 workers, they work for 6 hours per day, and there are 10 days. It means total the time that the worker completed was 60 hours, ma'am.*
Q: *Then what does it mean 8 = 10 days 6 hours = 60 hours? (while designate S1 work)*
S1: *This means that if there are 8 workers, they complete their work in 60 hours, ma'am. So that's the same as this 4 = 8x, that's what was asked. If 4 workers, how long will it take? So it's the same as question number 1 it is equally cross-multiplied.*

From the interview excerpt, S1 stated that the value of 60 was obtained by multiplying the number of days by the time worked in a day. Then S1 writes " $4 = 8x$". S1 asserts that if there are 4 workers, the time to complete the work is 8 x, with x representing the length of time it takes for 4 workers to complete the work in 8 hours per day. Similar to the procedure adopted in solving problem 1, S1 solves problem 2 with cross multiplication of $\frac{8}{4} = \frac{60}{8x}$, thereby, the result is $x = \frac{240}{64}$. When asked by the researcher, S1 clarified that the results of the division of 20 and 64 were not whole, so S1 only wrote them in fractional form.

The results of S1's work on problem 2, make it apparent that S1 still experiences interference on the concept of direct proportion from his work on problem 1. From further analysis, at the initial stage, S1 begins to read and understand the problems. S1 assimilates the provided information, representing it in textual form as "$8 = 10\ days\ \ 6\ hours = 60\ hours$" and "$4 = 8x$" (Figure 7). In this case, S1 sorts out the information for solving the problem. S1 experiences interference when S1 multiplied the time "6 hours per day" with the information "10 days". Because S1 assumes that these two things represent time, thus, the result is 60 hours. This pattern repeats in the subsequent step, where S1 formulates "$8x$". This highlights that the student's met-before is still problematic, especially when understanding the meaning of the problem and connecting it to a comparison problem. The thinking structure of S1 in solving problem 2 is shown in Figure 7.

The following is the thinking structure of S1 when solving problem 2 as shown in Figure 7.

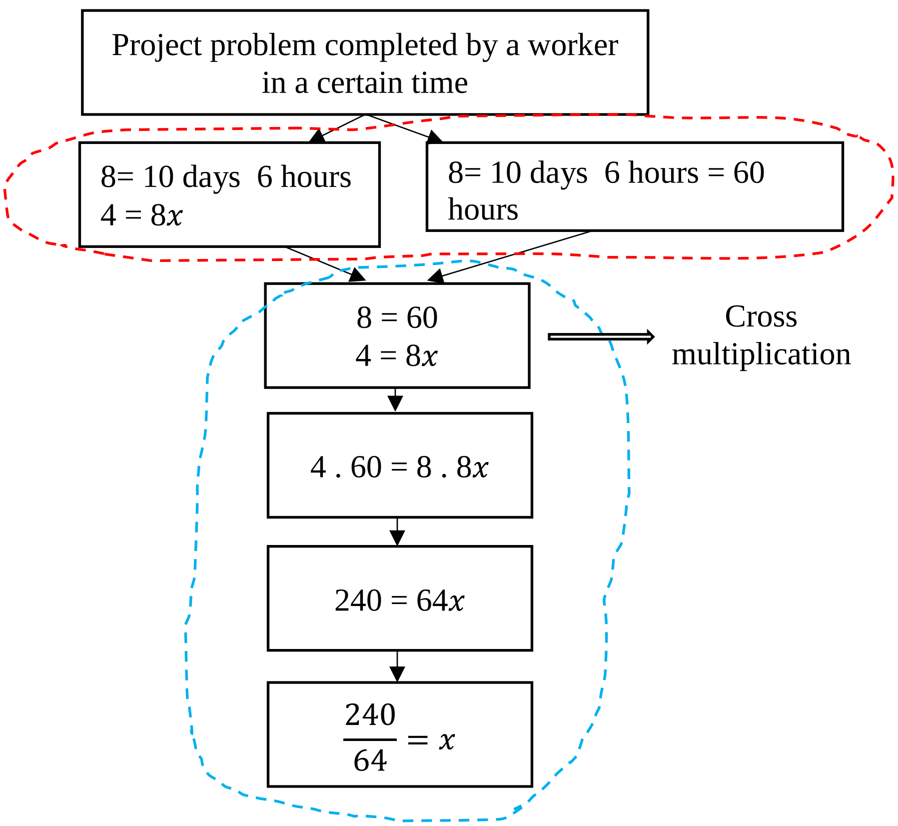


Figure 7: Thinking Structure of S1 when solving Problem 2

Note:

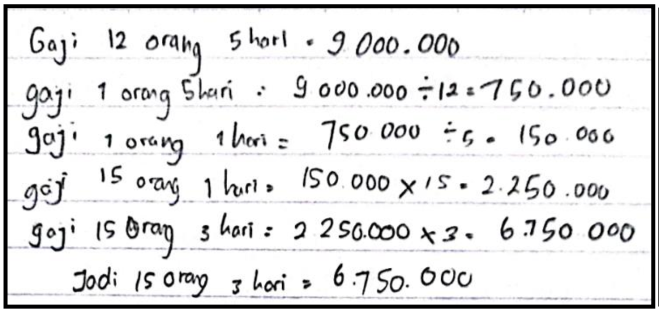
= Interference

= met-before

The met-before of students concerning the concept of comparison $\frac{a}{b} = \frac{c}{d}$ in such a way that $bc = ad$, proves to be problematic (Figure 7). This issue manifests particularly in solving problem 2 following the procedure for solving the concept of direct proportion. S1 assumes that in solving each proportion problem, the problem should be made into a solution model $\frac{a}{b} = \frac{c}{d}$. Therefore, S1 experiences interference in solving problem 2.

S2 Work Result (Flexible)

In problem number 1, S2 solves the given problem using the concept of direct proportion. The results of S2's work on problem number 1 is presented in Figure 8.



Translate:
Salary 12 peoples 5 days = 9000000
Salary 1 people 5 days = 9000000 : 12 = 750000
Salary 1 people 1 day = 750000 : 5 = 150000
Salary 15 peoples 1 day = 150000 × 15 = 2250000
Salary 15 peoples 3 days = 2250000 × 3 = 6750000
So, 15 peoples 3 days = 6.750.000

Figure 8: S2's Correct Answer for Problem 1

In Figure 8, S2 systematically writes down the steps for calculating workers' salary. This is evident when S2 writes information on the problem regarding the salary of 12 workers for 5 days is Rp. 9,000,000.00, by "salary of 12 people five days = 9,000,000". Then, S2 determines the salary of 1 person for 5 days to be " 9,000,000 : 12 = 750,000". Consequently, the daily salary for 1 person is determined as " 750,000: 5 = 150,000". Then, on the salary of 15 workers for 3 days, S2 writes down "salary of 15 people for 1 day = 150,000 ×15 = 2,250,000". Thus, the salary of 15 people for 3 days is "2,250,000 ×3 = 6,750,000".

The met-before S2's on problem 1 is *focus supportive*, whereas S2's understanding related to direct proportion is supported by well-structured problem-solving procedures. Therefore, S2 can solve problem 1, correctly. However, when presented with a problem similar in structure but involving a different mathematical concept in problem 2, S2 experiences interference with the direct proportion formula. The results of S2's work on problem 2 are shown in Figure 9.

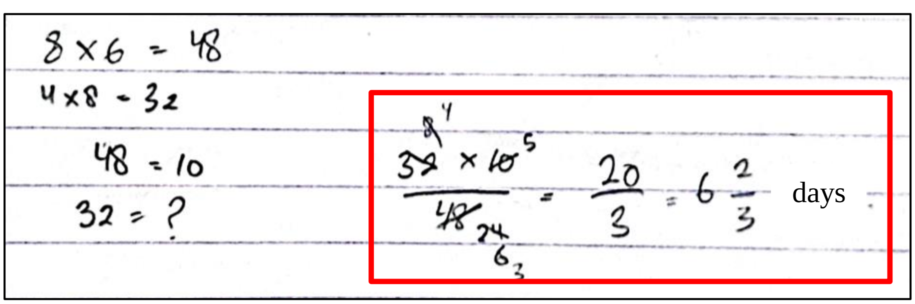


Figure 9: Wrong Answer of S2 in Problem 2 (Proactive Interference)

In Figure 9, S2 directly multiplies the number of workers by the time of workers ($8 \times 6 = 48$ and $4 \times 8 = 32$). To find out the S2's process of thinking in understanding problem 2, we conducted interviews with S2. The excerpt of the interview with S2 is presented in the following.

Q : *From problem number 2, can you tell me about the initial process of working on the problem?*

S2 : *First, I multiply 8 and 6, then 4 times 8, Ma'am. If I get the result, I write 48 = 10, then what about the 32 =?*

Q : *Hmm, what do you mean about 8×6 and 4×8?*

S2 : *From that problem, there are 8 workers who work for 6 hours so $8 \times 6 = 48$. Meaning that 48 will equal 10 days. Thus, similar for $4 \times 8 = 32$ equals how many days? After that, I just count, as usual, ma'am.*

Based on the excerpt from the interview with S2, S2 multiplies the information from the problem. Then, S2 writes 48 = 10, implying that with 48 hours of work, the work will be completed within 10 days. Therefore, S2 assumes that if there are 32 hours of work, then the problem is to find the duration of completing the work. At this stage, S2 performs calculations with cross multiplication (red box). The results obtained from these calculations are $6\frac{2}{3}$ days.

Figure 10 illustrates the S2's thinking structure when solving problem 2.

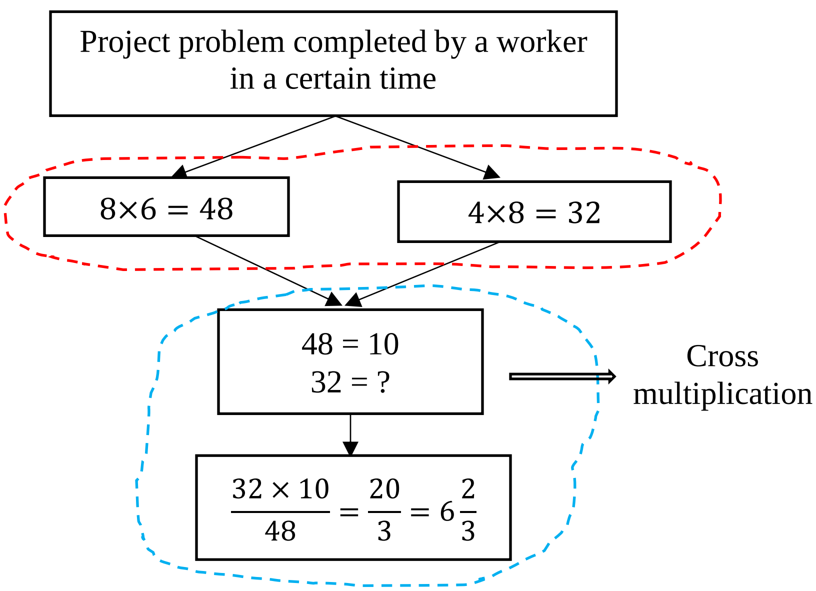


Figure 10: Thinking Structure of S2 when Solving Problem 2

Note:

= interference

= met-before

The results of S2's work on problem 2 suggested that S2 experiences proactive interference. From further analysis, initially, S2 interprets the problem as a matter of direct proportion. S2 states that problems number 1 and 2 are the same, both revolving around the completion of work. Because S2 initially perceived problem 2 as a direct proportion, S2 begins to assume that the total hours of the number of workers are obtained by multiplying the number of workers by the hours worked per day ($8 \times 6 = 48$ and $4 \times 8 = 32$) (see Figure 10).

In S2's thinking framework (met-before), this problem can be brought into a cross-multiplication formula as he has learned before. Thus, S2 performs the cross-multiplication calculation operation with the form "$\frac{32\times10}{48}$" with the obtained results of $6\frac{2}{3}$ days. Essentially, S2's met-before on the concept of comparison $\frac{a}{b}=\frac{c}{d}$ and cross-multiplication strategy ($bc = ad$) proves to be problematic. In addition to S2's problematic concept of proportion, problem 2, which revolves around workers, also poses difficulty for S2. S2 perceives problems related to work as direct proportion problems akin to problem 1. The summary of research results is shown in Table 2.

Proactive Interference	Question Number	Met-before
Non Flexible	1	Suppress problematic
	2	Focus problematic
Flexible	1	Focus supportive
	2	Focus problematic

Table 2: Met-before Student who Experienced Proactive Interference

DISCUSSION

The analysis results suggested that the proactive interference experienced by S1 and S2 has differences and similarities. In the met-before of S1 and S2 on problem 1 we observed differences. In problem 1 (direct proportion problem), met-before of S1 includes *supress problematic*, while the met-before of S1 is supportive of the concept of proportion. However, this supportive does not always aid S1 to solve the problem properly. During the process, S1 experiences problematic algebraic calculation procedures. This is in accordance with the statement of Chin and Jiew (2019) that a supportive conception may contain problematic aspects which is referred to as suppress problematic. In addition, McGowen and Tall (2010); Mowahed & Mayar (2023); Tall (2013); and Tall et al. (2014) also described that met-before can be supportive in certain concepts and problematic in other concepts.

In contrast to S1, S2 presents a *focus supportive* on problem 1. This is evident from met-before supportive of S2 in the process of solving problems. Consequently, S2 presents a correct answer. The supportive met-before in S2 facilitates the appropriate problem-solving procedure. In accordance with Chin et al. (2019), that supportive met-before can support the process of generalization and problem-solving.

However, when S1 and S2 experience proactive interference in problem 2, the met-before of both S1 and S2 is *focus problematic*. This problematic met-before leads to errors in solving problems 2. Chin et al. (2019) and Tall et al. (2014) asserted that problematic met-before can result in difficulty and confusion when facing math problems. In problem 2, proactive interference is not only limited to retrieval; it extends to when students understand the problem or receive incoming information. This is corroborated by the research from Irfan et al. (2019a) that the interference can be caused by students misunderstanding the meaning of the questions. This error causes students to incorrectly call the knowledge possessed by students. Irfan et al. (2019b) further categorized this misunderstanding as semantic interference.

Problem number 2 also has similarities with problem number 1, as the materials for direct proportion and inverse proportion have similar problem structures (Irfan et al., 2019a). Redick et al. (2020) described that problems with a similar structure, both in terms of content and processing procedures, are called near transfers.

Sometimes, specific information on the problem can mislead students to wrong perceptions. For instance, when S1's process in solving problem number 2, assumes that 6 hours per day with ten days are equivalent, so S1 multiplies the two numbers. In addition, the embedded met-before in the minds of students suggests proportion problems can be solved using the comparisons $\frac{a}{b} = \frac{c}{d}$ with $bc = ad$. It further confuses students, leading to interference every time they find a proportion problem. Students often assume that the problem can be changed in the form of this proportion. This problematic met-before caused students to experience interference. Thus, through met-before, the causes of the problems can be traced. Met-before can be used as a measuring tool or an analytical tool to analyze students' thinking processes and sense-making in solving problems (Chin et al., 2019; Chin and Pierce, 2019).

In addition to being related to mathematic concepts, the problem-solving process also involves problem-solving procedures and experience working on similar problems. In this study, the problem-solving approach employed by both S1 and S2 is notably centered around the cross-multiplication strategy. This aligns with the results of previous studies reporting that students often use cross-multiplication strategies (Avcu & Doğan, 2014; Ayan & Isiksal-Bostan, 2019; Öztürk et al., 2021; Parameswari et al., 2023; Tunç, 2020). For the cross-multiplication strategy, students cross the denominator and multiplier of the multiplication form $\frac{a}{b} = \frac{c}{d}$ such that $bc = ad$ (Çalışıcı, 2018; Im & Jitendra, 2020; Parameswari et al., 2023).

There are several reasons for the frequent usage of cross-multiplication strategy. One significant reason is that students are often taught cross-multiplication strategies in solving comparison problems (Öztürk et al., 2021). Linearly, Andini and Jupri (2017) described that students only remember the methods or procedures given by the teacher. In addition, proportion problems, such as direct and inverse proportion, are often associated with multiplication (Vanluydt et al., 2021). Therefore, students automatically solve the proportion problems with cross-multiplication strategies (Parameswari et al., 2023).

IMPLICATION FOR LEARNING ACTIVITY

This research focuses on the interference of students when solving problems of direct and inverse proportion. There are several alternatives that can be used to prevent this interference. First, the teacher provides a peer-assessment form to give students a chance to analyze each other work and find potential fruitful errors. Some leading questions can be very helpful in spotting mistakes. Second, teachers must provide meaningful learning to students. For example, learning that usually occurs in class is when the teacher gives a problem: “If a vehicle travels a distance of 50 km, then the vehicle has 2 liters of fuel. How far can the vehicle travel if it consumes 5 liters of fuel?”. The completion process is usually given as follows:

$2\ liters\ \rightarrow 50\ km$

$5\ liters\ \rightarrow ?\ km$

Then, the above problem is completed with $\frac{5}{2} \times 50 = 125$ km.

The previously mentioned solution primarily relies on procedural learning through symbols without conveying meaningful understanding. Teachers should help students understand each problem sentence used and not rely on the use of algebraic symbols (Edo & Tasik, 2022). Therefore, teachers should intervene in learning by providing the following directions: “If 2 liters of fuel can be used to cover a distance of 50 km, then 1 liter of fuel can be used to cover a distance of 25 km. So, if there are 5 liters of fuel, it can be used to cover a distance of $5 \times 25 = 125$ km”.

While the outcomes in both instances are identical, the process for finding the results is different. The intervention provided by the teacher makes learning more meaningful rather than providing formulas that confuse students, resulting in interference.

Third, the teacher can give some alternative problem-solving strategies, especially for students showing interference. The example questions include: have you tried to study another problem with some easier numbers, what would you expect to happen if one of the numbers approaches zero or is it consistent with your current numerical result or what you expected.

CONCLUSIONS

Based on the analysis results, the met-before of students experiencing proactive interference can be classified into non-flexible and flexible types. For the first problem (direct proportion), students with non-flexible type have *suppress problematic* met-before because students are able to solve direct proportion problems using the appropriate concept, but students experience problems in the completion procedure. It is evident that student’s understanding of the concept of direct proportion is supportive but it contains problematic aspects in the problem-solving process. On the other hand, students with a flexible type were categorized under the *focus supportive* category. This classification is attributed to their comprehensive understanding of the concepts and adeptness in the procedural aspects, enabling effective problem-solving.

For the second problem (inverse proportion), both students with non-flexible type and flexible type have *focus problematic* met-before because they assume that the first and second problems are the same. The problematic concept arises when students automatically resort to the cross-multiplication strategy ($\frac{a}{b} = \frac{c}{d}$ such that $bc = ad$). Students also cannot reason or link relationships between existing information. Thus, students experience problems determining the direction of changes in quantity in proportion problems.

The met-before that happened to students who experienced proactive interference turned out to be problematic. Accordingly, further research can examine the causes of met-before problematic further. This can be an input for educators to prepare learning that can prevent problematic met-before on students. In addition, this study is centered on proactive interference, while interference is very likely to occur retroactive interference or mixed (proactive and retroactive interference) so it is suggested for further research to examine the met-before students who experience retroactive and mixed interference (proactive and retroactive interference). Material that students have the potential to experience interference is not only material for direct and inverse proportion. Therefore, future researchers can analyze interference in other materials which allows for more variants of met-before.

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Stages of Problem-Solving in Answering HOTS-Based Questions in Differential Calculus Courses

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Abstract: Students are required to have the ability to implement mathematics in solving everyday life problems. A good solving process will produce an excellent solving ability. The existing problem-solving stages cannot be used in solving problems with the Higher Order Thinking Skills (HOTS) category. The aims of this research are 1). to know students' problem-solving process in solving HOTS category questions; 2). To design the stages of problem-solving that can be used to solve HOTS category questions. Twenty-four students who took the differential calculus course in mathematics education at one of Indonesia's private universities made up the study group. The solving process is divided into high, medium, and low. The descriptive and qualitative research method describes the Polya stages' problem-solving approach. Researchers used four sets of problem-solving by making nine indicators. Based on the analysis results, it can be concluded that several indicators of the problem-solving stages have not been appropriately implemented, including I4, I8, and I9. In addition, the steps at the Polya problem-solving stage of devising a plan and looking back must be improved. Based on the findings above, research recommendations design the stages of problem-solving consisting of 6 steps. Future research will develop steps of problem-solving with the characteristics of the HOTS category questions.

Keywords: HOTS, Problem-Solving, Polya

INTRODUCTION

Problem-solving skill is the highest skill in mathematics. It is one of the mathematical skills that students must possess (Akay & Boz, 2010; Dagan et al., 2018; Nurkaeti, 2018; Osman et al., 2018; Eichmann et al., 2019; Rott et al., 2021; Sulistyaningsih et al., 2021; Purnomo et al., 2022). Problem-solving requires applying several mathematical principles and knowledge to non-routine, open-ended, real-life problems, which helps solve everyday problems. Problem-solving is how to solve the non-routine problems (Temur, 2012; Güner & Erbay, 2021; Pardiansyah et al., 2021) and complex problems (Greiff, S & Fischer, 2013) to find the most appropriate solution (Greiff, S & Fischer, 2013; Yayuk & Husamah, 2020). The importance of this ability means that every student-teacher candidate must have good solving skills (Barnett & Francis, 2012; Purnomo & Mawarsari, 2014; Santoso, 2016). The problem-solving process requires the ability to understand the problem and the willingness to face the problem and solve the problem (Bingolbali, 2011; Dostál, 2015; Purnomo et al., 2020; Thienngam et al., 2020). Problem-solving skills are used in applying mathematics in everyday life. Students who have good problem-solving skills will quickly and precisely solve problems.

Many research results that the problem-solving abilities are still low (Hidayat & Irawan, 2017; Kusuma et al., 2017; Kilic & Sancar-Tokmak, 2017; Yeni et al., 2020; Aziz et al., 2021; Güner & Erbay, 2021). Factors causing low problem-solving abilities include students who are not used to solving problems (Abdullah et al., 2015), especially those involving real-life situations (Aziz et al., 2021; Puteh et al., 2017). Solving problems has many obstacles, including the problem-solving process containing errors that do not follow the legal problem-solving stages (Rott et al., 2021). Problem-solving has many problems for students, but not much research has been done on developing problem-solving models.

Improving problem-solving stages will make it easier for students to solve problems. Previous research with the problem-solving theme only described the stages of Polya's problem-solving (Pardiansyah et al., 2021), described the steps in terms of conceptualization and problem-solving abilities (Delahunty et al., 2020), knowing the impact of students' abilities in solving non-routine math problems (Saadati & Felmer, 2021), analyzed the problem-solving process in terms of the language of mathematics (Strohmaier et al., 2019), investigated critical thinking and problem-solving abilities used by students (Shanta & Wells, 2020), students' cognitive barriers in the process of solving mathematics problems (Antonijević, 2016), describing mathematics teaching through issue solving (Zhang & Cai, 2021), fourth and eighth-grade students' misconceptions in solving issues (Delahunty et al., 2020). There are no recommendations to improve the problem-solving stages based on previous research.

One of the subjects that frequently applies problem-solving is the Differential Calculus course. The Differential Calculus course is a material that is rarely used to solve problems in implementing derivative materials in everyday life. Applying mathematics in this course can be in economics, engineering, chemistry, and others. Based on observations in Differential Calculus courses, many students do not have good problem-solving skills. The difficulty in solving student problems means that the questions apply derivatives in the High Order Thinking Skills category. In solving problems in the application of results, it is necessary to have stages of solving according to the characteristics of the HOTS category questions.

Problem-solving is a complex activity that includes higher-order thinking skills (Simamora et al., 2018; Gursan & Yazgan, 2020). Problem-solving is an activity that encourages students to use HOTS. It is referred to as the level of thinking needed to shape the 21st-century generation that has the potential to compete globally with intelligence, creativity, and innovation (Hamzah et al., 2022). The achievement of the HOTS thinking process includes high knowledge, which provides for analytical, evaluative, and synthetic thinking levels. According to Bloom, education should focus on competence (subject mastery) and higher-order thinking outcomes. HOTS focuses on developing students' ability to analyze and evaluate by inferring existing information and creating (synthesizing) something new (Anderson et al., 2001; Wilson, 2016). A student can't differentiate and HOTS if he succeeds in solving the top four Bloom taxonomic indicators (Aziz et al., 2021).

Based on research, many students have low HOTS abilities (Misrom et al., 2020; Kim How et al., 2022; Andin & Aziz, 2019; Abdullah et al., 2015). Based on the research results, High Order Thinking Skill needs to be developed in problem-solving (Aziz et al., 2021; Osman et al., 2018; Mohd Rusdin et al., 2019; Ismail et al., 2022). Low problem-solving ability causes students to have still difficulty solving questions in the HOTS category (Amir, 2015; Karimah et al., 2018; Santoso, 2016; Misrom et al., 2020). Improving problem-solving abilities can be done by being trained (Abosalem, 2015), getting used to interacting with problem-solving problems (Barnett & Francis, 2012; Andin & Aziz, 2019), and getting used to solving complex problems (Purnomo et al., 2014; Yeni et al., 2020; Purnomo et al., 2022), and integrating digital literacy (Kim How et al., 2022). Future research explores possible patterns of teachers dealing with problem-solving and identifies the most effective discourse patterns when teaching mathematics through problem-solving (Zhang & Cai, 2021), focusing on the integration of digital literacy in improving HOTS (Kim How et al., 2022)

In this study, the stages of solving Polya's problem is elaborated. There are four stages of Polya problem-solving recently, however it less relevant to solve HOTS-based questions. For that reason, researchers modify and add several stages to make it more relevant. Many researchers can understand the profile of issue-solving abilities through these indicators. Based on the analysis of problem-solving steps, the students' errors in solving problems will be seen. Through the mistakes made, the root of the problem is students' inability to solve problems. The situation suggests a new completion stage to improve problem-solving, especially HOTS category questions.

LITERATURE REVIEW

Polya Problem Solving

Problem-solving arises from psychological and pedagogical confusion in mathematical problems (Schoenfeld, 1987). Problem-solving is based on various cognitive processes, such as metacognitive processes (Schoenfeld, 1992), attention, memory, and language (Jitendra et al., 2015). Numerous theories propose the phases of problem-solving in the problem-solving model.

The first experts to introduce problem-solving were Dewey, Wilson, et al., Bransford & Stein, Schoenfeld, Burton & Stacey, and Polya. Each theory of solving stages has its characteristics. Based on the analysis, each approach has its advantages and disadvantages. This study will look at students' problem-solving abilities profiles based on the theory of Polya's problem-solving stages..

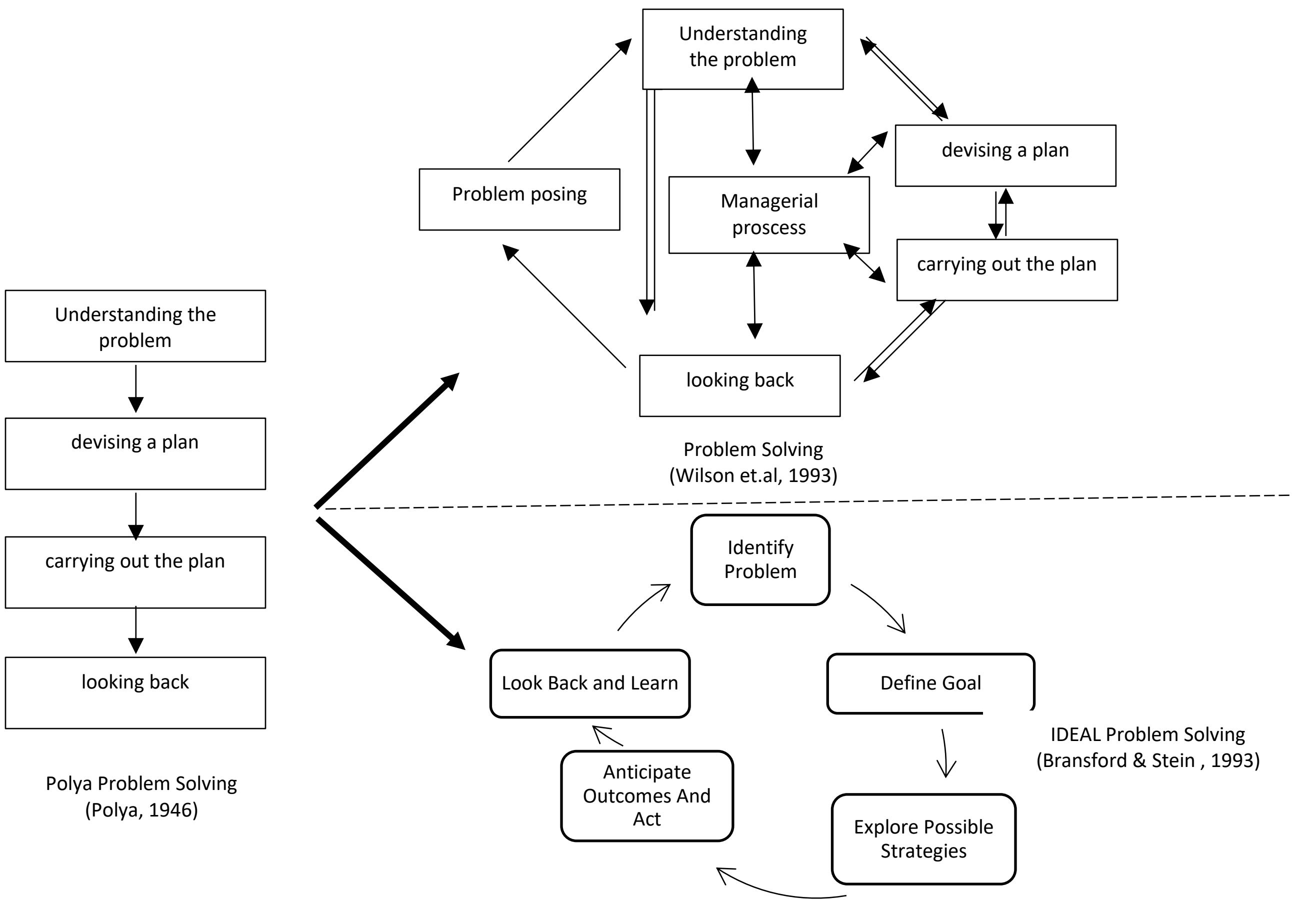


Figure 1. Modify Polya problem-solving stages

There are four stages of problem-solving skills: understanding the problem, devising a plan, carrying out the project, and looking back (Robson & Polya, 1946). Polya problem solving has been modified by several researchers, including (Wilson et.al, 1993) and (Bransford & Stein, 1993). An overview of Polya's troubleshooting modification can be seen in Figure 1 above. Figure 1 shows that the modification of problem-solving carried out by Wilson et al. (1993) is still in the same stages as Polya. The looking backstage in Polya is divided into two phases: anticipate outcomes & act, and look back & learn. Wilson et al. added four steps plus the managerial part of the process and problem posing. Bransford & Stein (1993) added five stages. Using the findings from examining the Polya solving phases, it can still be modified and adapted to problem-solving characteristics.

Higher Order Thinking Skills (HOTS)

Students must think higher in receiving new information, organizing and storing information in long-term memory, connecting details and existing knowledge, and processing data to solve a problem. This ability is known as HOTS. It is a method of creative thinking, problem-solving process, and critical situations in making good decisions (Behar-horenstein, 2011) in complex cases (Andin & Aziz, 2019). HOTS in Bloom's revised Taxonomy, namely application skills, analyzing, evaluating, and creating (Anderson et al., 2001; Wilson, 2016).

Mathematical Modelling

Mathematical modeling is a tool for deciphering a system's dynamics and forecasting future outcomes (Varaki & Earl, 2006). Mathematical modeling has been positioned at the forefront of many levels of education globally as modeling strengthens purposeful problem-solving skills, connects mathematics to the real world, and makes mathematics more meaningful and relevant. Mathematical modeling is identifying a situation in the real world, confirming assumptions and choices, and then using a mathematical model to derive a solution that can be translated back into the real world. Using mathematical modeling in learning mathematics allows students to understand mathematics more meaningfully, learn mathematics by relating it to real life, and eliminate the inadequacy of available problems (Yasa & Karatas, 2018). The Figure below illustrates that mathematical modeling involves a multistep and iterative process.

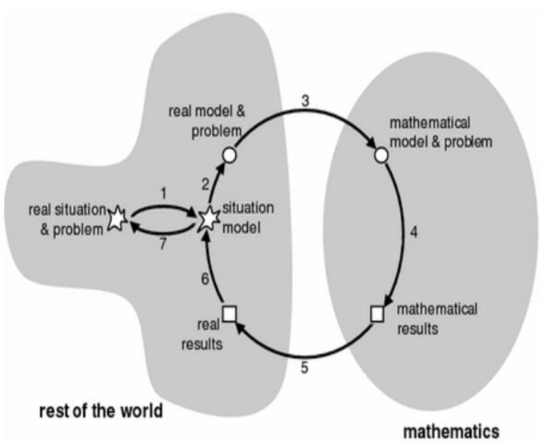


Figure 2. Mathematical modeling process

RESEARCH METHODOLOGY

This research employed a qualitative descriptive study that described students' problem-solving abilities. Students from the mathematics education study program in Central Java, Indonesia, made up the research group. The sample of this study was 24 students who had done educational internships. The research sample was selected using the cluster sampling technique to obtain three categories, namely high ability (A1), moderate ability (A2), and low ability (A3). There were three categories of questions such as easy, medium, and complex types. Problem-solving problems in this study were chosen as non-routine problems that could not be solved directly (Saadati & Felmer, 2021). Problem-solving questions consist of the HOTS category with the ability to analyze, evaluate, and create. The instrument employed in this study was a problem-solving test consisting of three items tested by expert validation with an assessment of content, context, and language aspects. The results of the validation showed that the average overall score was 4.5 (out of a total score of 5), which include very valid category. The following are indicators of problem-solving abilities in this study.

HOTS Ability Category	**Question**	**The Steps in Answering the Questions**
Analyzing (C4) Easy	An open box is made of a zinc sheet in a square measuring 12 cm inside. By cutting at each end of the congruent squares, determine the size of the maximum volume of the box!	Problem Solving Stages: 1. Describing the requested conditions 2. Determining the mathematical model 3. Solving with derivatives
Evaluating (C5) Medium	The operating cost of a truck is estimated to be (30+v/2) cents per mile when driven at v miles/hour. Drivers are paid 14 dollars per hour. With a speed limit of $40 \leq v \leq 60$. At what rate would it be cheapest to ship to a city k miles away?	Problem Solving Stages: 1. Describing the requested conditions 2. Determining the mathematical model 3. Connecting variables with each other 4. Solving with derivatives
. Creating (C6) Difficult	Finding the size of a rectangular, cylindrical cylinder with as large as possible can be placed inside a rectangular cone!	Problem Solving Stages: 1. Describing the requested conditions 2. Setting the variables 3. Finding a mathematical model, then determine the maximum quantity 4. Connecting variables with each other 5. Solving using derivatives

Table 1. Problem Solving Questions using HOTS Category

The researchers collected the data by triangulation, namely evaluation tests, observations, and in-depth interviews (Creswell, 2014; Sukestiyarno, 2020). The evaluation test has three levels: easy, medium, and high. Determination of problem-solving indicators employs Polya indicators (Robson & Polya, 1946; Argarini, 2018; Puspa et al., 2019; Nurkaeti, 2018). Based on the indicators of these studies, the indicators are shown in Table 2.

Problem Solving Steps	Indicator	Code
Understanding the problem (2 indicators)	1. Analyzing question 2. Focusing question	I1 I2
Devising a plan (2 indicators)	3. Associating the relationship between known data to find things that are not known. 4. Determining the method used to solve the problem	I3 I4
Carrying out the plan (2 indicators)	5. Executing the solution plan and checking every step. 6. Determining the solution to the problem and writing down the solution or answer to the problem 1. Determining the solution to the problem and writing down the solution or answer to the problem	I5 I6
Looking back (3 indicator)	2. Determining conclusion 3. Is there an alternative to get a different result? 4. Checking the accuracy of answers to questions	I7 I8 I9

Table 2. Polya Problem Solving Indicator

Data from tests, interviews, and observations were analyzed by content analysis consisting of three activities: data reduction, data presentation, and conclusion (Creswell, 2012; Miles et al., 2014). By coding the interview files, data was reduced. Coding is used to make monitoring crucial information about how exposed existing data is more accessible. The next step is to show the data after it has been reduced. The following stage is to use in-depth interviews to confirm the data. Whereas, concluding the field data is the final stage. To create a comprehensive picture, data analysis employs an inductive method where conclusions are made after a thorough investigation of small cases (Sukestiyarno, 2020).

FINDINGS

In this study, to see the profile of problem-solving abilities by giving three questions with easy, medium, and high difficulty levels. The results of student work were investigated and compared according to their level of knowledge. The work results were analyzed using the problem-solving ability stages whose indicators have been determined. The analysis results had been a reference in looking at the profile of students' problem-solving abilities.

The first analysis used the easy category problem with the question "*An open box is made of a square sheet of zinc measuring 12 cm sides. By folding at each the end of the congruent squares, please, determine the maximum volume of the squares*!" The results of students' answers in easy category questions can be seen below.

Stage 1: *understanding the problem*

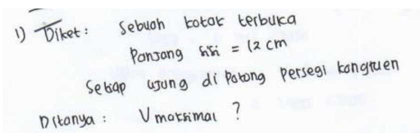
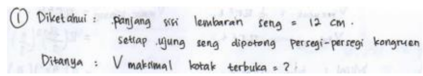
High Skill (A1)	Medium Skill (A2)	Low Skill (A3)
		Do not write
Translation : Given : • An open box with a side length of 12 cm • Each end of the square is cut with a congruent square shape Asked: Maximum Volume?	**Translation :** Given : • Zinc sheet side length = 12 cm • Each zinc end is cut into a congruent square Asked: Max volume of the open box?	

Figure 3 : The results of student work on the analyzing (C4) questions in the problem solving stage 1

In stage 1, indicator 1 (I1), students A1 and A2 completely write everything known in the questions. For A3, students do not write what is known in the questions. Based on the interview results, A3 did not write down what was known in the questions because A3 felt that this was not necessary to answer the questions. What is known is that there is a box measuring 12 cm in length, while the question is the maximum volume. In stage 1, indicator 2 (I2), students do not write down that are not known. Students should be able to write down something they don't know because later, it will become a basis for planning in answering questions.

An in-depth interview was conducted to find out why A3 students ignored to write what information was known. The results of interviews showed that students actually were able to read the questions well. However, their absence of writing the information because they consider it as unnecessary and unimportant. The students considered that solving problems could be directly solved as long as they understand the concepts and do the calculations well.

Stage 2 : *devising a plan*

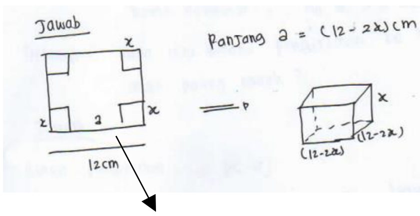
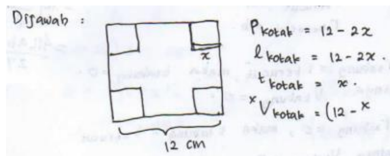
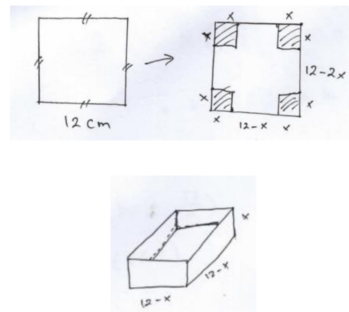
High Skill (A1)	Medium Skill (A2)	Low Skill (A3)
 a		
Translation : Length a = (12-2x) cm	**Translation :** • box length = (12-2x) cm • box width = (12-2x) cm • box height = x cm • box volume = 12-x cm	

Figure 4: The results of student work on the analyzing (C4) category questions in the problem-solving stage2

In stage 2, indicator 1 (I3), students A1, A2, and A3 have found a relationship between the data and the unknown, namely the shape's size and its pictures. A1, A3 are written in full from the net and its size and the shape of the space formed. Even A3 students wrote down from the initial process that it was known that there was a box with a height of 12 cm, then each corner would be cut along the x length and the space that would be formed. Student A2 wrote down only the picture and its measurements. The interview results with student A3 showed that students were able to correlate the unknown information from the questions, so they could find new information. However, they could not visualize it in pictures. Indicator 2 stage 2 (I4) all students do not write explicitly in the answers, but only in the students' thoughts. The solving steps for students A1, A2, and A3 are the same: finding the mathematical model of the volume of space, finding the stationary point, and substituting the critical point in the volume equation.

From the results of in-depth interviews with students A1, A2, and A3, it was found that all students successfully identified the stages of solving the problem well; however, solution plans they had in mind were not written down. They ignored the plan and tended to write the solution directly to save the time.

Stage 3: Carrying out the plan

High Skill (A1)	Medium Skill (A2)	Low Skill (A3)
$V_{\text{kotak}} = p \times \ell \times t$ $= (12-2x)(12-2x)(x)$ $= (144-24x-24x+4x^2)x$ $= (4x^2-48x+144)x$ $= 4x^3-48x^2+144x$ $V_{\text{maksimal}} \longrightarrow V' = 0$ $12x^2-96x+144=0 \quad :12$ $x^2-8x+12=0$ $(x-6)(x-2)=0$ $x-6=0 \vee x-2=0$ $x=6 \qquad x=2$ untuk $x=2$ $V(x) = 4x^3-48x^2+144x$ $V(2) = 4(8)-48(4)+144(2)$ $= 32-192+288$ $= 32+96$ $= 128 \text{ cm}^3$ Untuk $x=6$ $\cancel{V(6) = 4)}$ $V(x) = 4x^3-48x^2+144x$ $V(6) = 4(6)^3-48(6)^2+144(6)$ $= 4(216)-48(36)+864$ $= 864-1728+864$ $= 0$	$V_{\text{kotak}} = p \times \ell \times t.$ $= (12-2x)(12-2x)x.$ $= (144-24x-24x+4x^2)x$ $= 144x+48x^2+4x^3$ $V = 144x-48x^2+4x^3$ Syarat $V'=0.$ $144-96x+12x^2=0. \quad :12$ $12-8x+x^2=0.$ $(x-6)(x-2)=0.$ $x=6 \vee x=2.$ $x=2 \longrightarrow V$ $V = 144x-48x^2+4x^3$ $= 144(2)-48(2)^2+4(2)^3$ $= 288-192+32$ $= 128 \text{ cm}^3.$ $x=6 \longrightarrow V$ $V = 144x-48x^2+4x^3.$ $= 144(6)-48(6)^2+4(6)^3$ $= 864-1728+864$ $= 1728-1728.$ $= 0 \text{ cm}^3$ Jadi, volume maksimal kotaknya adalah 128 cm^3.	$V = p\ell t$ $= (12-2x)(12-2x)x$ $= (144-48x+4x^2)x$ $= 4x^3-48x^2+144x$ $V' = 12x^2-96x+144$ $0 = x^2-8x+12$

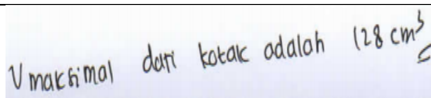
		
Translation : The maximum volume of the box is 128 cm^3	**Translation :** The maximum volume of the box is 128 cm^3	

Figure 5: The results of student work on the analyzing (C4) category questions in the problem-solving stage 3

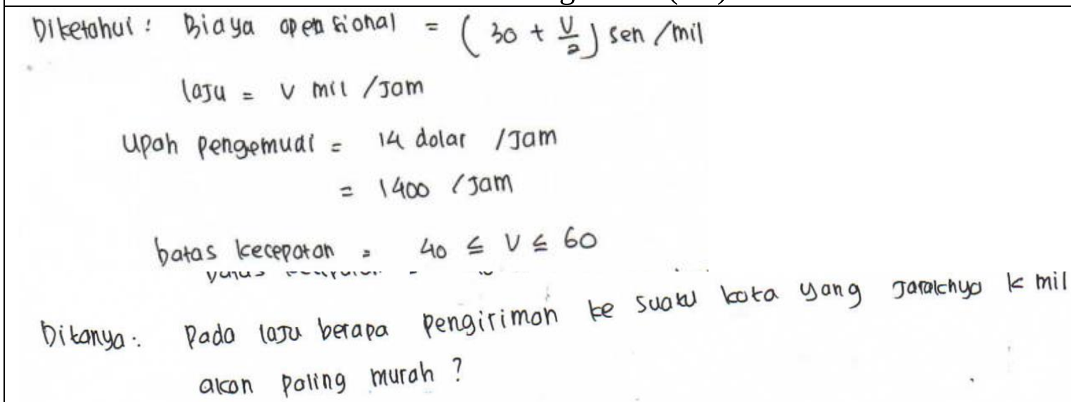
In stage 3, indicator 1 (I5), All students have found the correct volume of space shapes, and all are correct. The next step is to see the conditions for getting to the critical point with the condition $V' = 0$. In this step, all students have implemented it, but A3 does not explicitly write it down. A1 and A2 get the critical point, namely $x = 6$ or $x = 2$; A3 doesn't get the crucial point. To get the maximum volume, A1 and A2 plug the two critical points into the volume equation. The two students answered the questions correctly, but A3 was only up to determining the requirements for getting the maximum volume. The interviews result with A3 students showed that students needed more time to complete them. In stage 3 indicator 2 (I6), students A1 and A2 have written the correct answer, such as the maximum volume of 128 cm^3. The results of interviews with A3 students showed that students had not accomplished the question because they did not have enough time.

Stage 4: looking back

In stage 4, there are three indicators I7, I8, and I9. In indicator I7, students have done it, but it is not written explicitly in the answer. At I8, only A1 students tried to find other alternative solutions. The result of interview with student A1 showed that A1 was able to solve the problem well, however she/he was little hesitated when writing the solution. At that time, A1 substituted the x value in the length, width, and height equations to figure out the volume. Whereas, the interview result with student A2 stated that students were pretty sure to write the answer and o not even think about other alternative ones. Therefore, in stage I9 both A1 and A2 have successfully carried out this step and answer the question accurately. On the other hand, student A3 only answered the questions up to the maximum volume requirement. According to the result of interviews with students A1 and A2, the answers they provided were in accordance with the questions such as determining the volume and the value of the volume. The medium category problem is given the following questions: "The operating costs of a truck are estimated to be $(30 + \frac{v}{2})$ cents per mile while driving by mile/hour. Drivers are paid 14 dollars per hour. With speed limitation at $40 \leq v \leq 60$. At what rate would shipping costs to a city mile away be cheapest? " The results of student answers can be seen below.

Stage 1: ***understanding the problem***

High Skill (A1)



Diketahui : Biaya operasional = $\left(30 + \frac{v}{2}\right)$ sen/mil
laju = v mil/jam
Upah pengemudi = 14 dolar /jam
= 1400 /jam
batas kecepatan = $40 \le v \le 60$
Ditanya : Pada laju berapa pengiriman ke suatu kota yang jaraknya k mil akan paling murah ?

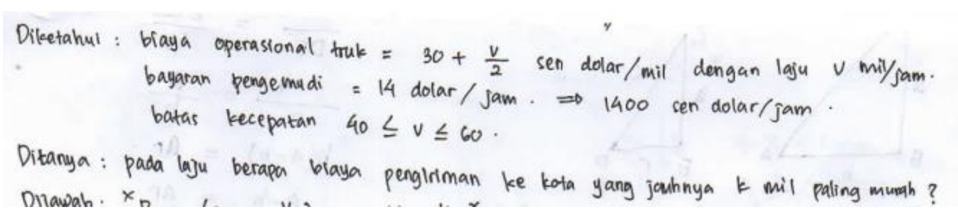
Translation :

Given :

- operating costs = (30+v/2)cents/mile
- Speed = v miles/hour
- Driver fare = 14 dollars/hour = 1400/hour
- Speed limit = 40≤v≤60

Asked : At what rate will shipping to a city k miles away be cheapest?

Medium Skill (A2)



Diketahui : biaya operasional truk = $30 + \frac{v}{2}$ sen dolar/mil dengan laju v mil/jam.
bayaran pengemudi = 14 dolar/jam. ⇒ 1400 sen dolar/jam.
batas kecepatan $40 \le v \le 60$.
Ditanya : pada laju berapa biaya pengiriman ke kota yang jauhnya k mil paling murah ?

Translation :

Given :

- Truck operating costs = (30+v/2) cents/mile at the rate of v miles/hour
- Driver fare = 14 dollars/hour = 1400/hour
- Speed limit = 40≤v≤60

Asked : At what rate is the cheapest delivery to the city k miles away?

Low Skill (A3)

D1. Biaya operasional sebuah ~~truk~~ truk diperkirakan sebesar $(30+\frac{v}{2})$ sen dolar setiap mil
Pengemudi dibayar 14 dolar setiap jam
Pembatasan ∀ kecepatan $40 \le v \le 60$

D2 ~~Biaya~~ Pada laju berapa biaya pengiriman ke suatu kota yang jauhnya k mil akan paling murah?

D3. Biaya total dalam sen = biaya pengemudi + biaya operasi

Translation :
D_1 (given)
- The operating cost of a truck is estimated at (30+v/2) cents/mile
- Drivers are paid 14 dollars every hour
- Speed limiting $40 \le v \le 60$

D_2 (Asked): at what rate will it be cheapest to a city k miles away?
D_3 (answer): Total costs in cents = driver fees + operational costs

Figure 6: The results of student work on the evaluating (C5) category questions in the problem-solving stage 1

In stage 1 indicator 1 (I1), students A1, A2, and A3 have written everything that is known, namely operational costs $(30 + \frac{v}{2})$ cents, driver's wages 14 dollars per hour, and speed limitation at $40 \le v \le 60$. In stage 1, indicator 2 (I2) of students A1, A2, and A3 has found a relationship between operational costs and wages and results. Through this relationship, a mathematical model will be found. The result of the interview showed that all students had successfully analyzed the information and focused on the questions, so they could properly write down the known information and questions.

Step 2: *devising a plan*

High Skill(A1)	Medium Skill (A2)	Low Skill (A3)
Jawab Biaya Pengiriman	$B = (30+\frac{v}{2})k + 1400\frac{k}{v}$. $= 30k + \frac{1}{2}vk + 1400v^{-1}k$.	

$= P(v)$ $=$ upah pengemudi + biaya operasional $= \frac{k}{v}(1400) + k(30 + \frac{v}{2})$ $= 1400k\,v^{-1} + 30k + \frac{kv}{2}$		$= \frac{k}{v}(1400) + k(30 + \frac{v}{2})$ $= 1400\,k\,v^{-1} + (\frac{k}{2})v + 30k$
Translation : Total costs = $P(v)$ = driver fare + operational costs $= \frac{k}{v}(1400) + \mathrm{k}\left(30 + \frac{v}{2}\right)$ $= 1400kv^{-1} + 30k + \frac{kv}{2}$	**Translation :** $\mathrm{B} = \mathrm{k}\left(30 + \frac{v}{2}\right) + \frac{k}{v}(1400)$ $= 30k + \frac{1}{2}vk + 1400\,v^{-1}k$	**Translation :** $= \frac{k}{v}(1400) + \mathrm{k}\left(30 + \frac{v}{2}\right)$ $= = 1400kv^{-1} + \frac{kv}{2} + 30k$

Figure 7: The results of student work on the evaluating (C5) category questions in the problem-solving stage 2

In stage 2, indicator 1 (I3), All students have found a relationship between known and unknown data by looking for shipping costs. Shipping costs $p(v) = 1400\,v^{-1} + 30k + \frac{kv}{2}$.. In stage 2, indicator 2 (I4) is not written explicitly in the answer. Students already know the flow of solving existing problems. The steps are to find the critical point in the obtained mathematical model. The results of the interviews showed that students had already known the flow of solving the existing problems. This stage focused on looking for critical points in the mathematical model. However, they failed to write it down because they thought that this activity was unnecessary to be accomplished. Whereas, the most prominent was going straight to solve the problem.

Step 3: Carrying out the plan

High Skill (A1)	Medium Skill (A2)	Low Skill (A3)

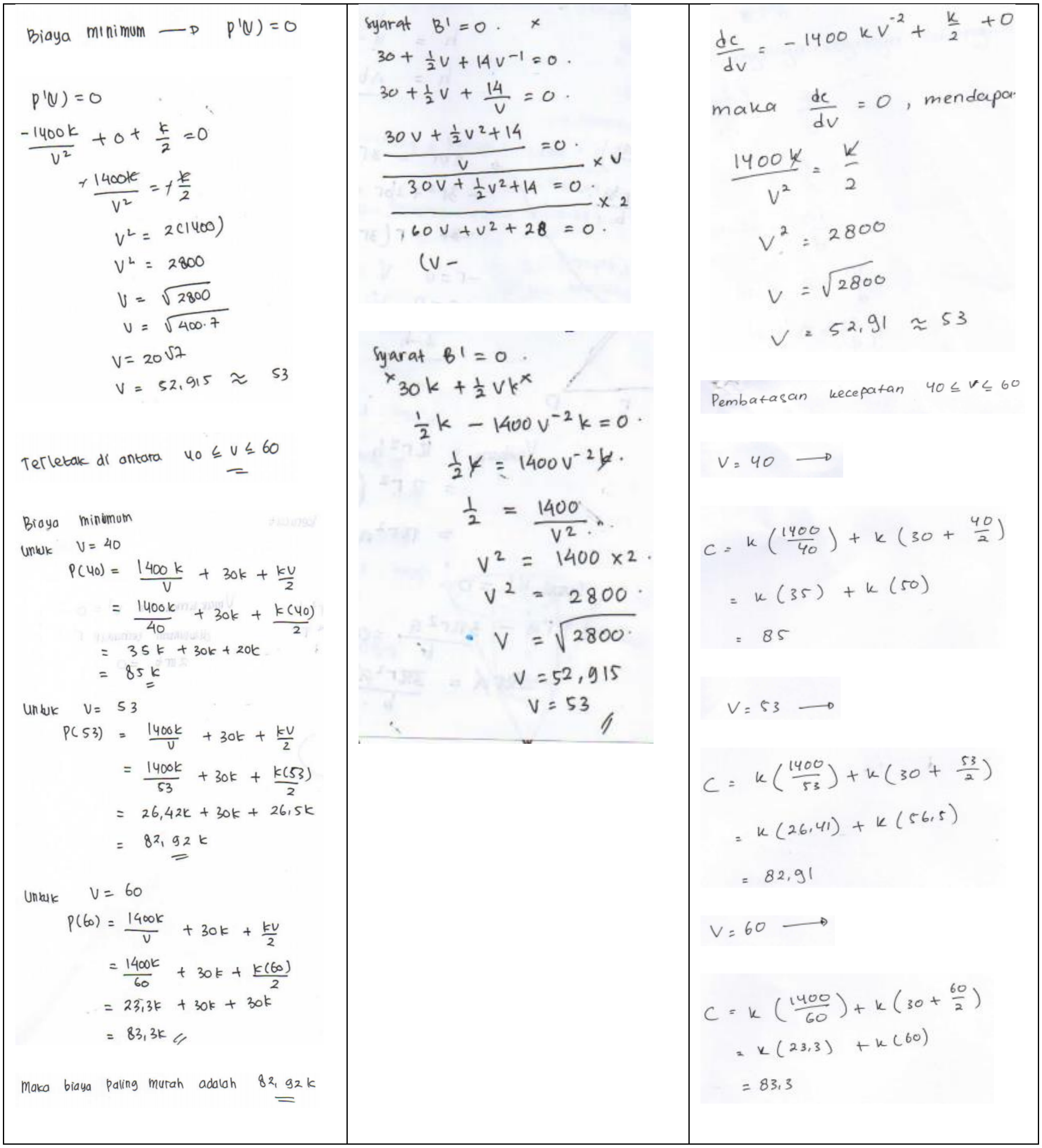
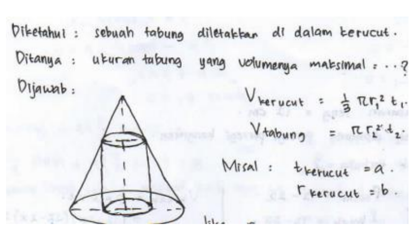
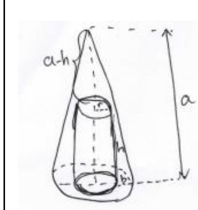


Figure 8: The results of student work on the evaluating (C5) category questions in the problem-solving stage 3

In stage 3, indicator 1 (I5), students A1, A2, and A3 had received the minimum fee. Minimum cost provided that $p'(v) = 0$, and all correct answers were $52{,}91 \approx 53$. A1 and A3 were able to determine the speed limit, $40 \le v \le 60$, so three critical points could be easily found such as 40, 53, and 60. In addition, A2 did not specify a speed limit. Whereas, A1 and A3 had incorporated a tipping point into the shipping cost equation, resulting in shipping costs of $85\text{k}, 82{,}92\text{k},$ and $83{,}3\text{k}$. Finally, A1 and A3 concluded a minimum fee of $82{,}92\text{k}$. Then in stage 3, indicator 2 (I6), all students failed to write down explicitly in their answers. Moreover, the results of the interviews showed that students had already known the flow of solving the existing problems. This stage focused on looking for critical points in the mathematical model. However, students failed to write it down since they thought that it was unnecessary. In fact, the most prominent was going straight to solve the problem.

Stage 4: looking back
In stage 4, indicator 1 (I7) was carried out by students but explicitly unwritten in their answer. In stage 4 indicator 2 (I8), only A1 attempted to find other alternative solutions to the questions. In stage 4 indicator 3 (I9), A2 students did not undergo this stage, while A1 and A3 only attempted to find the minimum cost. A1 and A3 failed to list the rate at which shipping costs to a miles away city will be the cheapest. In fact, they should had written the cost per mile in the general equation, namely $\frac{C}{k} = 1400\, v^{-1} + \frac{v}{2} + 30$. In the other hand, students should had written down the cost per mile in the general equation, namely C/k=1400 v^(-1)+v/2+30. Finally, the result of the interview supports the finding such as students A1 and A3 only wrote down the cheapest cost values but had not answered the question about finding the rate position to get the cheapest cost values. In addition, students also did not try to determine or ensure that the answers written are correct. The next problem is a problematic category: "Find the size of a vertical circular tube with the largest possible volume that can be placed in a vertical circular cone!" The outcomes of the student's efforts are listed below.

Stage 1: ***understanding the problem***

High skill (A1)	Medium skill (A2)	Low skill (A3)
		

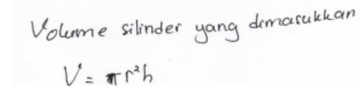
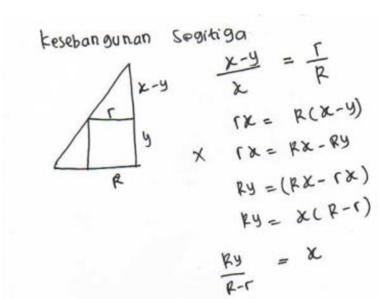
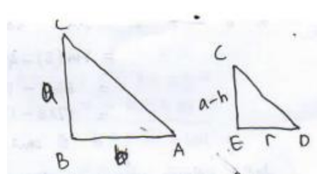
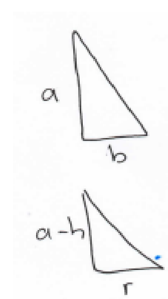
		
Translation : Given : There is a tube inserted into the cone Asked: Maximum tube volume Answer : Cone radius = R Tube radius = r Cone height = x Tube height = y	**Translation :** Given : A tube is placed inside the cone Asked: What is the maximum volume of the tube? Answer :	**Translation :** Introduced cylinder volume

Figure 9: The results of student work on the problematic questions in the problem-solving stage 1

In stage 1, indicators 1 (I1) of students A1 and A2 had written correctly what was known in the problem. A3 student could not find out the information from the questions and what to do to solve the problem. Through the consideration, finally, the lecturer gave direction to A3 regarding the purpose of the questions and the stages of solving the questions. In stage 2, indicator 2 (I2) is not written explicitly in the answer. *The results of interviews with A1, A2, and A3 showed that students A1 and A2 were able to analyze and focus questions by writing them well. In A3, students can analyze the questions and focus on questions from the questions given, so they can visualize cones and tubes and exemplify their sizes but are not given information on the size problems that have been written.*

Stage 2: ***devising a plan***

High Skill (A1)	Medium Skill (A2)	Low Skill (A3)
		

$\frac{x-y}{x} = \frac{r}{R}$ $x-y = \frac{xr}{R}$ $x = y + \frac{xr}{R}$ $y = x - \frac{xr}{R}$	$\frac{EC}{DE} = \frac{BC}{AB}$ $\frac{a-h}{r} = \frac{a}{b}$. $b(a-h) = ar$. $a-h = \frac{ar}{b}$. $-h = \frac{ar}{b} - a$. $h = a - \frac{ar}{b}$. $h = \frac{ab-ar}{b}$.	$\frac{(a-h)}{r} = \frac{a}{b}$ $a-h = \frac{a}{b} r$ $a-h = \frac{ar}{b}$ $h = a - \frac{ar}{b}$ $h = a - \left(\frac{a}{b}\right) r$

Figure 10: The results of student work on the creating (C6) category questions in the problem-solving stage 2

Stage 2 indicator 1 (I3) students A1, A2, and A3 had found a relationship between the known and the unknown. Students A1 and A3 were able to write the visualization of the shapes in question and their measurements (variables). While, student A2 wrote down the visualization of the image only, but the size was quite clear. A2 jot down several alternative sizes of the radius of the tube and cone with three alternatives, such as a). radius of cone = radius of tube consequently t tube = 0 consequently v tube is 0. b). The radius of the tube = 0, then t tube = t cone consequently v tube = 0. c). $0 < r\ tube\ < r\ cone$, the tube would experience a certain volume on the third point, which is an alternative answer. Stage 2 indicator 2 (I4) was not written explicitly in the answer. The results of interviews A1, A2, and A3 showed that all of them were able to find and relate what was known to the problem and find problems to find solutions using the concept of triangular congruence. They thought about how to solve the problem, unfortunately it was not written down because it was unnecessary. They thought that focusing more on the solution of the problems is more important.

Stage 3: ***carrying out the plan***

High Skill (A1)	Medium Skill (A2)	Low Skill (A3)
	$V_{tabung} = \pi r^2 h$. $= \pi r^2 \left(\frac{ab-ar}{b}\right)$ $= \pi r^2 a - \frac{\pi r^3 a}{b}$.	

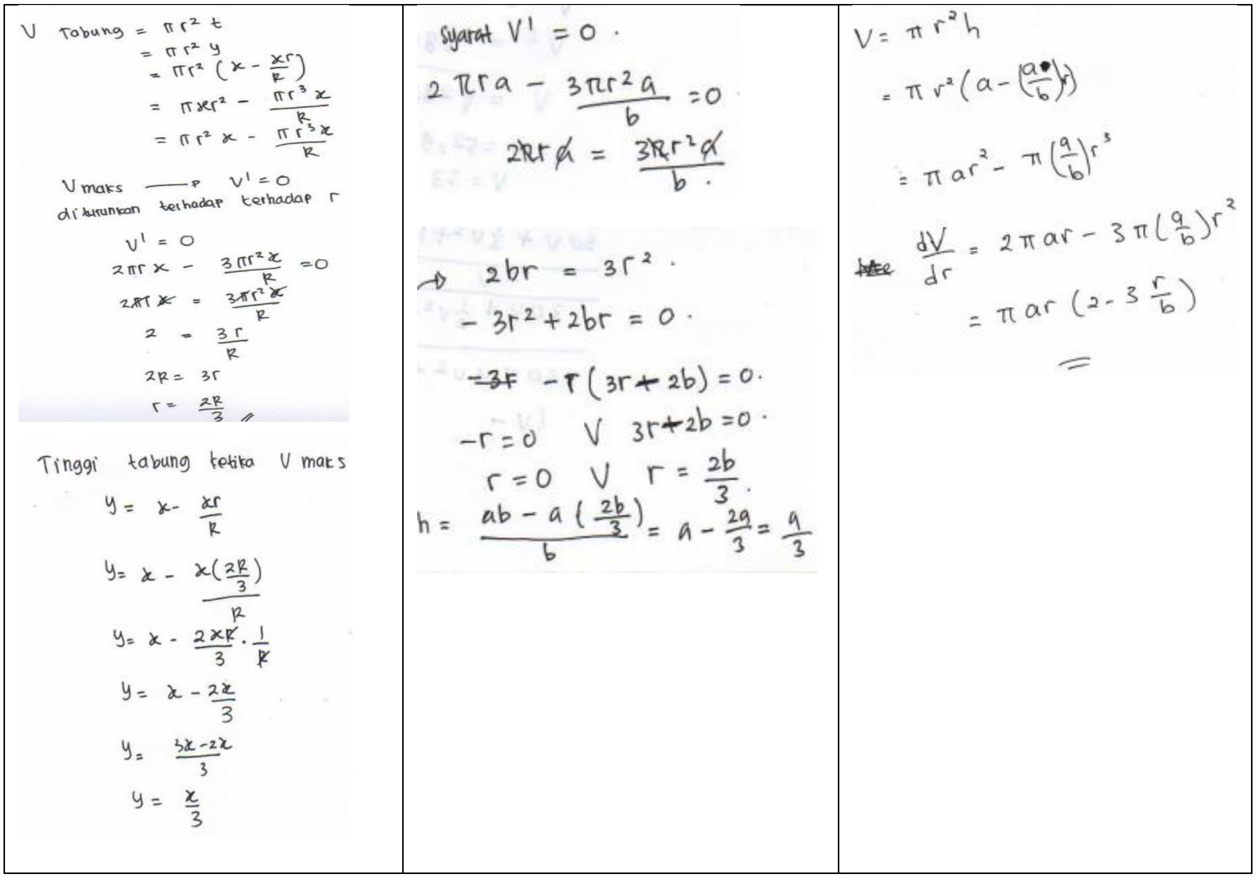


Figure 11: The results of student work on the creating (C6) category questions in the problem-solving stage 3

In stage 3, indicator 1 (I5), students A1, A2, and A3 used a similarity triangle to find the radius ratio to the tube's height. A1, A2, and A3 figured out that the ratio of height to the radius, and all of them were correct, i.e., $h = a - \frac{a}{b}r$. In this stage, A1, A2, and A3 attempted to find the volume of the cylinder $v = a\pi r^2 - \pi\frac{a}{b}r^3$. Then, A1, A2, and A3 tried to find the maximum volume with $v'(r) = 0$ that was $v'(r) = \pi ar\left(2 - \frac{3}{b}r\right)$. However, only A1 and A2 produced a stationary point $r = 0$ or $r = \frac{2b}{3}$ and A3 only up to $v'(r)$. On the other hand, A1 and A2 did not determine the critical point that should have 3, namely $r = 0$ or $r = \frac{2b}{3}$ and $r = b$. Finally, A1 and A2 tried to find the tube size $r = \frac{2R}{3}$ and the tube height $y = \frac{x}{3}$. In stage 3, indicator 2 (I6) was not written explicitly in the answer. These results indicated that A3 failed to solve the problem properly. Then, an interviews with A3 students was carried out and it showed that students were unable to construct their thoughts to relate to the concepts they had in solving problems. In stage 3 indicator 2 (I6), students A1 and A2 could determine solutions to problems and write down solutions to the problems. Unfortunately, student A3 did not explicitly wrote the answers. Based on the interviews result, it showed that students were unable to construct their thoughts to relate the concepts they had in solving problems, so that the answer was not written.

Stage 4: looking back

In stage 4, indicator 1 (I7) was carried out by students although it was not explicitly written in the answer. In stage 4 indicator 2 (I8) there was only A1 who find other alternative solutions, while A2 and A3 did not try to find alternative explanations. Stage 4 indicators 3 (I9) both A1 and A2 underwent this step and made the exact answer without considering to write down the answer.

DISCUSSION

The first question showed that students A1 and A2 answered the questions accurately. In answering the questions, both students A1 and A2 had carrried out many problem-solving stages but they failed to write the answers explicitly as in indicator I4. Students did not write down the strategic plan to solve the problem. This finding was in line with a study conducted by (Özdemir, Furkan, and Çelik, 2021). According to the result, it would affect the accuracy of answering questions. Students who plan strategies appropriately will carry out the problem-solving stages well (Ersoy, 2016; Eichmann et al., 2019). In fact, A1 student was the only one who attempted to seek alternative answers to the first question (indicator I7). While, student A2 and A3 did not carry out indicator activities I7 because they did not have enough time to accomplish and could not think about indicator I7. For A3 students, they were unable to complete the answers due to the constrained processing time that runs out. All stages had been undergone according to the analysis results on the easy category questions, but some indicators missed. It happened because students were not get used to answer the questions using problem-solving steps. Thus, it is necessary for the students to get used to work on non-routine questions (Riastini & Mustika, 2017; Özdemir, Furkan, and Çelik, 2021) and problem-solving with students (Purnomo et al., 2014; Ersoy, 2016; Gogo et al., 2021; Purnomo et al., 2022).

The medium category questions illustrated that there were no stages of problem-solving written explicitly in the answers, such as indicator I4. Students A1 and A2 searched for alternative solutions (indicator I8), while A3 did not even think about alternative answers. At this category, student A3 still faced difficulty answering questions and did not have enough time accomplish the questions. All students did not correctly carry out the Looking Backstage, especially indicator I9. They were quite unsure about what was being asked in the question. Finally, all answers found a tipping point where the cost wasminimal. In the question, it was clear that what wasbeing asked was a general mathematical model so that the prices were minimum. This happened due to weak problem-posing skills (Science & Soyba, 2018; Tabak, 2019) and inappropriate problem-solving processes (Rohmah & Sutiarso, 2018). The conclusion was that re-checking what was asked and the answer given was prominent thing. In addition, students must accomplish and understand questions thoroughly.

Faced on the problematic questions, it was found that student A3 failed to understand the questions. It could be obviously seen from how they deal with and accomplish the problem. The information in the question coud not be described adequately. Student A3 did not understand the questions (Yeni et al., 2020) and were not get used to solve questions with high difficulty category questions. So the new A3 students could get information from the questions and the flow of solving the questions. In the second stage, all students did not write their answers explicitly (indicator I4). In indicator I8, A1 students was the one to carry out this stage, while A2 and A3 did not perform it well. It happened because A2 and A3 had difficulty in dealing with the questions. Besides, time limit was also become their consideration. From the explanation, it could be concluded that there was a need for habituation in working on questions in the high difficulty category (Abdullah et al., 2015). Therefore, it is needed to assist students in analyzing problems by modeling the situation into a mathematical model (Varaki & Earl, 2006; Yasa & Karatas, 2018; Schukajlow et al., 2018). For that reason, using mathematical modeling can also help students in planning problem-solving strategies.

Again, in this finding it was found that students still experienced difficulties at all stages of Polya (Abdullah et al., 2015; Yayuk & Husamah, 2020; Pardiansyah et al., 2021). There were several indicators of the stages of problem-solving. Indicators that were not carried out including 1). determine the method used to solve the problem, 2). the existence of alternatives to get different results, 3). check the accuracy of answers to questions, 4). modeling the problem into a mathematical model situation. There were still errors when planning for the completion and implementation of the project (Pardiansyah et al., 2021). Students should expressed how the planning process in their minds when solving problems, for example, by telling the steps in their minds. For instance, performing prior planning such as outlining the information provided in mathematical form and implementing strategies during the processes and calculations (Yayuk & Husamah, 2020).

Polya's problem-solving design stages included the Problem Posing Learning Model (PPLM) (Örnek & Soylu, 2021), IDEAL problem solving (Bransford & Stein, 1984; Bransford & Stein, 1993) and Wilson's problem solving (Wilson et.al, 1993) and problem-solving evaluation design (Gebel & Kuzle, 2020). Based on these findings, Polya's problem-solving stages could be modified to be appropriate to solve problems with the HOTS type. The characteristics of HOTS-type questions are problems in complex situations (Andin & Aziz, 2019) that cannot be translated directly. Solving HOTS questions require a more thorough problem-solving stage and the proper steps. The locations of solving the Polya problem require modifications to be used to solve HOTS questions. Errors in student solutions are in the last stage of trial solving. In general, students need to defend the chosen plan at this stage. Alternative solutions would minimize errors by improving the stages of problem-solving (Wee, 2007) and modeling the problem into a mathematical situation. In the findings of this study, the Polya stages need to be developed must be improved by inserting mathematical modelling stages and returning answers to existing problems. Problem-solving designs need to be made so that problem solving, especially HOTS type questions, can be adequately solved. Four factors served as the foundation for the new problem-solving phases' design, including 1). to find the root of the problem in the field, 2). Literature review related to difficulties in solving problems, 3). Stages of problem-solving that have developed, and 4). Mathematization process and mathematical modeling. The study results showed that it was necessary to design a problem-solving step with the characteristics of solving HOTS questions. In solving the HOTS problem, a need for additional stages were strongly necessary. For instance, when expressing the situation mathematically and comprehending how to apply math to real-world situations or issues. The steps of problem-solving were developed by various experts, including Dewey (1910), then Polya (1945), Mason, Burton & Stacey (1982), Schoenfeld (1985), Wilson et al. (1993), and Yimer & Ellerton (2010). Based on the analysis, it can be concluded that the stages had several advantages and disadvantages. In addition, the results of the research can be used to add new problem-solving stages. The mathematization process consists of four steps, including 1). Horizontal mathematization: organizing, reflecting, compiling problems, and identifying aspects of the problem mathematically to find the rules or relations; 2). Vertical mathematization: formalizing and abstracting mathematical concepts to give birth to mathematical concepts; 3). Applying in different situations: apply in various problems and cases, and 4). Back to reality: back to the real problem. The illustration below shows how the phases of problem-solving are laid out.

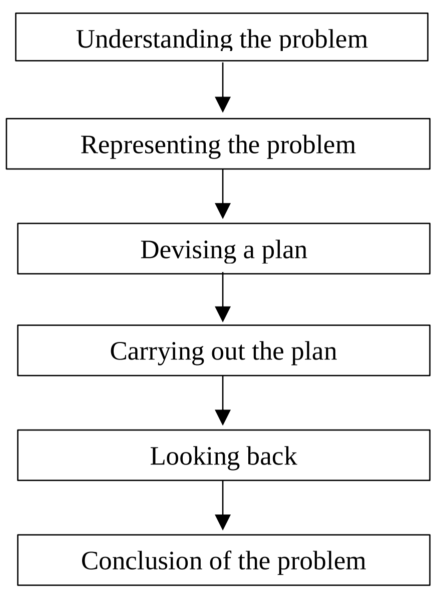


Figure 12: Design new problem-solving stages

Based on the analysis, the stages of the mathematization process can be included in the new problem-solving steps—the addition of solving stages in the 2nd stage, such as; representing the problem. In the last stage, there is also an additional stage, namely conclusion of the problem. The design results of the solving stage consist of 6 steps. Each stage of problem-solving consists of indicators. The design of the problem-solving stage consists of 6 stages and 11 indicators. More specifically, each indicator in the problem-solving stage can be described in Table 3.

Step	Problem-Solving Step	Indicator Description
Step 1	Understanding the problem	• Analyzing the problem by identifying what is known and asked • Focusing on the problem
Step 2	Representing the problem	• Presenting statements in the form of pictures or example variables • Finding the relationship between known data to find the unknown
Step 3	Devising Plan problem solving	• Determining the method used to solve the problem • Looking for various alternative solutions to problems
Step 4	Carrying out the plan	• Executing the solution plan and checking every Step • Determining solutions to problems and writing solutions or answers to problems
Step 5	Looking back	• Looking for alternatives get different results • Checking the accuracy of answers to questions
Step 6	Conclusion of the problem	• Returning the results of the answers to the context of questions or everyday life

Table 3 lists the phases of problem-solving and provides an indicator description.

A design for a problem-solving stage is created based on the analysis. Problem-solving stages using six groups of techniques, such as 1). understand the problem, 2). representing the problem, 3). devising plan problem solving, 4). carrying out the plan, 5). looking back, and 6). conclusion of the problem. The limitation of this study is that the research sample is still in the scope of mathematics education study program students. Subsequent further research will be more sophisticated when it is carried out in other study programs and at high school level. Future research can also be conducted to test the effectiveness of the problem-solving model stage design. By conducting the effectiveness test, a better problem-solving stage design will be obtained.

CONCLUSION

Based on the elaboration in the previous part, it can be clearly concluded that several indicators in the stages of problem-solving are not performed well. In indicator I4 for instance, all students did not write a work plan for solving the problem. Indicator I4 was also not carried out perfectly that caused less-than-optimal completion result. However, in indicator I8 only high-ability students encouragingly attempted to find other alternative answers. In fact, indicator I8 is important to be carried out by students to stimulate other alternative answers and avoid the wrong answers. Whereas, all students did not perform well in indicator I9 (medium category questions). It is as important as the previous one, this indicator needs to be performed by students to match between questions and answers. This study come to new paradigm that the design of problem-solving stages now can be categorized into 6 steps, including 1). understand the problem, 2). representing the problem, 3). devising plan problem solving, 4). carry out the plan, 5). looking back, and 6). conclusion of the problem. In addition, this study focuses that there is a need to develop problem-solving stages. By adding the stages of the problem-solving model, students can cover the shortcomings of the previous problem-solving steps. This new problem-solving stage, in addition, will be much more necessary to solve problems with the HOTS category.

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A Comparison of Angle Problems in Indonesian and Singaporean Elementary School Mathematics Textbooks

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***Abstract**: Textbooks are one of the main resources for teaching and learning mathematics. This study examines the presentation of angle topics in 4th-grade mathematics textbooks in Indonesia and Singapore. The analysis focused on the general characteristics of the textbook and the nature of the mathematical tasks presented. The results showed that Indonesian mathematics textbooks are more likely to provide a more ample opportunity to learn than Singaporean textbooks based on the number and description of task activities. However, the distribution of items in each task activity in Singapore mathematics textbooks is more proportionate than in Indonesian mathematics textbooks. Concerning mathematical tasks, the findings show that the form of representation in Indonesian mathematics textbooks contains a more purely mathematical form, while Singapore's mathematics textbooks are dominated by visual form. Regarding contextual features, mathematical tasks in Indonesian and Singaporean mathematics textbooks are dominated by non-application forms. Closed tasks also dominate the response type of task for both textbooks. The implications of this finding can be applied to classroom teaching activities, as highlighted in the discussion section.*

Keywords: Mathematics textbook, elementary school, angles, representation forms, contextual features, response types

INTRODUCTION

Textbooks have a significant role in supporting the learning process that's going on in schools. Textbooks strongly influence what will be taught, what students will learn, and how it will be studied (Rahmawati et al., 2020; Yang et al., 2010). A textbook is composed of specific materials according to learning objectives by referring to the curriculum that has been applied (Gracin, 2018; Usiskin, 2013). Given the relevance between textbooks and the way students learn and the learning activities they have gone through in class, textbooks are an essential means for students to obtain their learning achievements (Purnomo et al., 2019).

In various countries, textbooks are still one of the references as a source of student learning. Textbooks also show a considerable effect on teaching and learning activities and the fundamental teaching of teachers in the classroom (Yang & Sianturi, 2017). Some of the functions of textbooks in teaching and learning activities, among others helping teachers to explain the materials as exercise material for students and directing students in understanding mathematical material (Takeuchi & Shinno, 2020).

Textbooks consist of several aspects students must learn, including knowledge, skill, and attitudes to achieve predetermined competency standards and help their classroom learning process (Manopo & Rahajeng, 2020). For this reason, textbooks will affect students' ability to achieve specific competencies, one of which is the competencies tested in the PISA.

Analysis of mathematics textbooks in the last two decades has become a theme that has received increasing attention in mathematics education research (Fan, 2013; Purnomo, Shahrill, et al., 2022; Trouche & Fan, 2018), and many of them have focused on specific mathematical content or specific tasks in textbooks of two or more countries (Takeuchi & Shinno, 2020). This article reports on our research using a comparative study of Indonesian and Singaporean elementary school mathematics textbooks. Although Indonesia and Singapore are neighboring and cognate nations, Singapore is much more advanced in educational quality than Indonesia, primarily based on disparities in mathematical achievement from several international student assessments (e.g., PISA and TIMSS).

This study expanded on previous research lines that investigated geometry in mathematics textbooks (Choi & Park, 2013; Yang & Sianturi, 2017), specifically angle topics at the elementary school level in Indonesia and Singapore. Angle is one of the subsections discussed by elementary school students and has a vital position in the development of advanced mathematical concepts (e.g., trigonometric functions; proportional reasoning), the development of science (e.g., engineering, geology, architecture, physics), and solving problems of daily life (Alyami, 2020; Bütüner, 2021). Nevertheless, some empirical research highlights some of the challenges students have while learning using angles, such as typically having many misconceptions and difficulty gaining key concepts and skills in these topics (Bütüner & Filiz, 2017; Clements & Burns, 2000). Several textbook-related studies (Alyami, 2020; Haggarty & Pepin, 2002) also focus on angle topics, but the majority of the attention is on the high school level. Therefore, this research is useful to complement and fill in the gaps in textbook research on angle topics in elementary school mathematics textbooks, especially in Indonesia and Singapore.

Theoretical Framework

Features and diversity of tasks in mathematics textbooks

Lately, curriculum standards have focused and directed at domains of knowledge and skills that have also been a concern for international surveys, such as PISA, which deals with using multi-context-based problems, exploring mathematical activities, and promoting higher-level rather than low-level thinking. The implication is that textbooks are a means to implement the curriculum (Purnomo, 2023; Rahmawati et al., 2020; Valverde et al., 2002) and were asked to accommodate these domains.

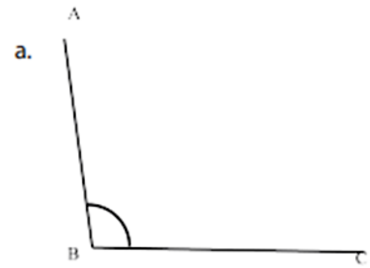
Textbooks have at least two common parts, the presentation of materials and tasks. The main content of the material presentation section refers to how the content is delivered to the reader, which may include his pedagogy, the approach of the concepts used, and the related context presented. In comparison, the task section is more about how students or classes are involved in assignments, where these domains are more likely to be in this section. This section can take the form of an example, exercise, or other evaluation tools. Therefore, we focus on the task section in the textbook (examples and exercises) in promoting these domains.

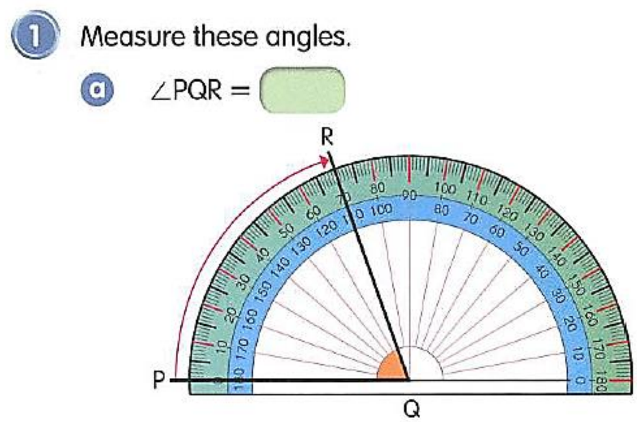
Mathematical tasks are a part of mathematics textbooks that researchers widely study because this section has an essential role in exploring knowledge and skills. Some researchers use different frameworks for mathematical analysis tasks in textbooks. We adopted a framework by Yang and colleagues (Yang et al., 2017), that includes three main focuses: representation form, contextual features, and response type. Each dimension will be elaborated into several points after this and accompanied by sample examples from our study's analysis results.

Representation Forms

There are four forms of representation classified, namely purely mathematical (A1), verbal (A2), visual (A3), and combined (A4). Suppose the main problem presented here includes only mathematical expressions. In that case, the problem is classified into problems in purely mathematical form. If the problem presented is entirely verbal, that is, written words only, then the problem is encoded into the category of problems in verbal form. Suppose the problem presented consists only of drawings, graphs, charts, tables, diagrams, maps, and so on. In that case, such problems are classified into problems in a visual form. While problems are classified as combined if presented in two or three of the above forms. We can exemplify each of these four forms by taking the case of two textbooks in our study, namely Indonesian and Singaporean mathematics textbooks (cf., Yang et al., 2017).

Representation Form	Code	Example
Purely mathematical	A1	Indonesia: Make a shape from the following size angle. 60°, 60°, and 60° Singapore: 93° is between a 1/4 -turn and a ... -turn
Verbal	A2	Indonesia: How do you measure the angle in standard units? Singapore: Label the angles of your textbook with A, B, C, and D. How many angles are there? Name the angles
Visual	A3	Indonesia:

<table>
<tr>
<td></td>
<td></td>
<td>Find the size of the angle below!


Singapore:

</td>
</tr>
<tr>
<td>Combined</td>
<td>A4</td>
<td>Indonesia:
Pay close attention to the following pictures and readings!

Udin will construct the kite seen in the figure. Udin bout calculated the size of the angle at each point of the kite's angle for it to fly in balance. Angles in both kite wings at points A and C should be the same size. The magnitude of angle A is 105° as measured with a protractor. What about the sizes of angles B, C, and D?</td>
</tr>
</table>

Singapore:

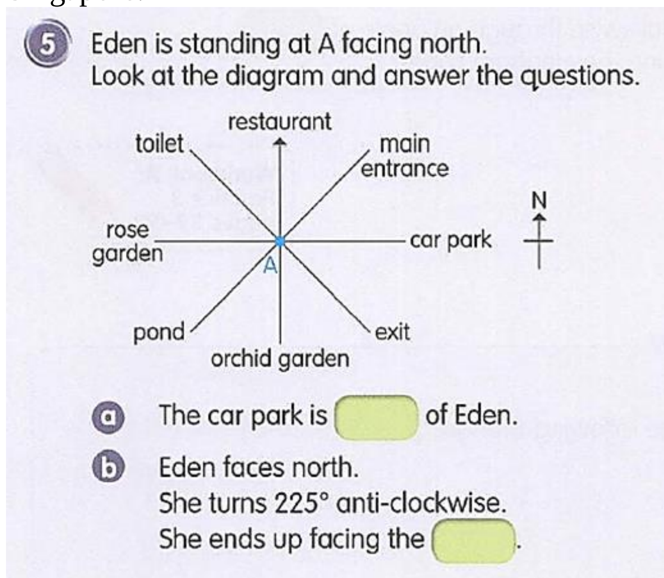
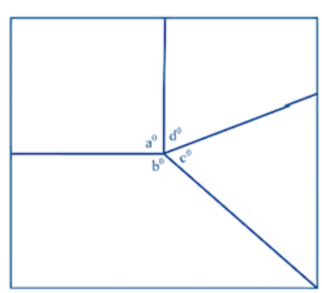


Table 1. Examples of each representation forms

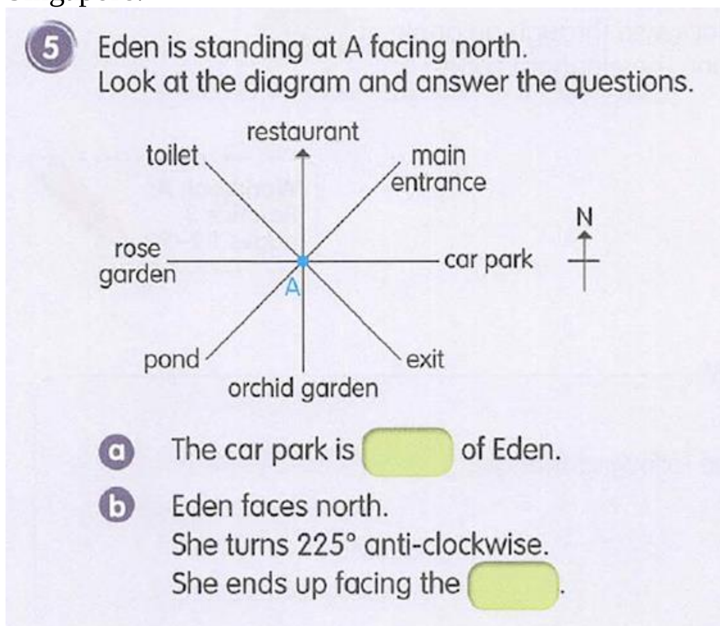
Contextual Features

Two contextual features are classified: Application (C1) and Non-application (C2). Application is a problem that is presented in the context of a real-world situation. In contrast, non-application is a problem not related to the practical background in everyday life or the real world.

Contextual Features	Code	Example
Application	C1	Indonesia: Pay attention to the following figure!  Edo has a piece of cardboard that will be cut into pieces and form a flat build, as shown above. Edo wants to know

<table>
<tr><td></td><td></td><td>the size of the angle at the angle points formed on the plane figure. Measure the magnitude of angles a, b, c, and d using a protractor.

Singapore:



</td></tr>
<tr><td>Non-application</td><td>C2</td><td>Indonesia:
At what time do the hands of the clock make an angle of 75°?

Singapore:
Use a ruler and a protractor to draw the following angles.
a. 45° b. 80°
c. 130° d. 154°</td></tr>
</table>

Table 2. Example of each contextual feature

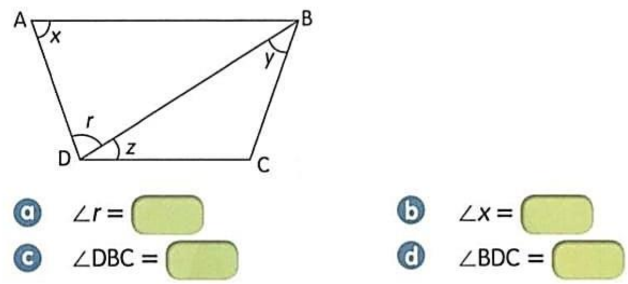
Response Types

There are two types of classified responses: open-ended (E1) and close-ended (E2). Open-ended is defined as questions with many correct answers. In contrast, close-ended is defined as questions with only one correct answer.

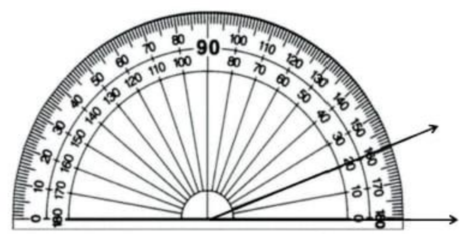
Types	Code	Example

<table>
<tr>
<td>Open-ended</td>
<td>E1</td>
<td>Indonesia:
At what time do the hands of the clock make an angle of 90°?

Singapore:
Look at the following figure.
Name each marked angle in another way.



</td>
</tr>
<tr>
<td>Close-ended</td>
<td>E2</td>
<td>Indonesia:
Determine the size of the angle from the measurement results below.



Singapore:
270° is equal to a ... -turn</td>
</tr>
</table>

Table 3. Examples of each response types

Comparative Study of Textbooks on the Topic of Angle

Seeing the importance of textbooks in mathematics teaching and learning, many researchers have researched and analyzed textbooks on various learning topics, including geometry and measurement topics that contain elements of angle. In their study, Choi and Park (2013) used the United States and South Korean textbooks to examine the measure of angles in plane figures. The study took math textbooks in grade 8 from both countries. Then a similar study was also conducted by Yang et al. (2017) who analyzed the topic of geometry in the middle class in textbooks in Taiwan, Singapore, Finland, and the United States. Research on mathematics textbooks on the use of the textbooks in the English, French, and German countries on the topics of angle was also carried out by Haggarty and Pepin (2002). The study reported that students from three countries have different opportunities to learn mathematics, depending on the textbook structure in combination with the teacher's use of textbooks, and the development of lessons varies according to textbooks.

Based on some of the studies mentioned above and those in the literature, at least 3 points need to be emphasized in the research. First, mathematics textbooks research has been done a lot on angle. However, there is still rare mathematics textbook research that focuses on the task section. Second, studies in mathematics textbooks have been carried out and involved various countries. However, none have been found to compare Indonesian and Singapore mathematics textbooks on angle topics. The last, according to research in mathematics textbooks, the issue of angle topic is usually addressed in middle and high school and rarely in elementary school.

The Topic of Angle in The Mathematics Curriculum of Indonesian and Singaporean Elementary Schools

In Indonesia, the angle topic was introduced in grade 4 elementary schools. Meanwhile, in Singapore, the angle topic was introduced in elementary school grades 4, 5, and 6. In grade 4, elementary school students in Indonesia learn about measuring angles in standard units with protractors and plane figure angles with protractors. While in Singapore, students learn to understand and measure angles, draw angles up to 180°, turns, and an 8-point compass in grade 4. Within the Singapore curriculum, the topic of angle is also taught in grades 5 and 6. Student grade 5 in Singapore learned about the angles properties of the 13th topic after studying rate or speed. Then, even in grade 6, students still learn about angles, namely finding unknown angles in geometric constructs. This became the last topic studied in the first semester in grade 6.

We consider that based on the level of cognition at the same grade level is more important to compare. The average age of grade 4 students in Indonesian and Singapura is also the same, 10-11 years. In addition, the sub-sub materials studied by students in Indonesia and Singapore in grade 4 are quite similar, so it is more suitable to compare. Therefore, we focus on the 4th-grade math textbook for both countries.

Research Questions

The purpose of this study was to compare the structure of units/lessons, their frequency and sequence, and the nature of mathematics tasks on angle topics between mathematics textbooks for grade 4 in Indonesia and Singapore. We focus on three analytical frameworks for the nature of mathematical tasks: the form of representation, contextual features, and types of response. The following research questions serve as a guide for this purpose:

(1) What are the differences in content structure, frequency, and sequence of angle tasks between mathematics textbooks for grade 4 in Indonesia and Singapore?
(2) Are there differences in the forms of representation (pure mathematical form, verbal form, visual form, and combined form) angle tasks between mathematics textbooks for grade 4 in Indonesia and Singapore?
(3) Are there differences in the contextual features (application and non-application) of angle tasks between mathematics textbooks for grade 4 in Indonesia and Singapore?
(4) Are there differences in the response types (open or closed) of angle tasks between mathematics textbooks for grade 4 in Indonesia and Singapore?

METHOD

Textbooks Selection

This study compared two types of mathematics textbooks used in elementary school learning in Indonesia and Singapore. The mathematics textbook from Indonesia used in this study is a textbook entitled *Senang Belajar Matematika Grade IV SD/MI* revised edition 2018, published by the Ministry of Education and Culture. Meanwhile, the mathematics textbook from Singapore used in this study is My Pals Are Here! Maths 4A published by Marshall Cavendish Education.

Senang Belajar Matematika Grade IV SD/MI was chosen because it is officially published by the Ministry of Education and Culture and used as a mandatory reference in elementary schools in Indonesia. In addition, this textbook is also following the latest revision of the 2013 curriculum. Based on the revision results, mathematics subjects at the elementary school level, and equivalent for grades 4, 5, and 6 in the 2013 Curriculum are separate from the thematic subject matter.

My Pals Are Here! Maths was chosen because it is a top-rated textbook in Singapura. The language of instruction in this textbook is English, making it easier for us to analyze it. This textbook was published by the company Marshall Cavendish Education and according to the Cambridge Curriculum for primary level schools. This textbook was chosen because 60% of Singapore schools use it (Yang et al., 2010).

Analytical Coding and Data Analysis Framework

We do data analysis with two types of analysis: horizontal and vertical (Charalambous et al., 2010). Horizontal analysis is an analysis carried out on textbooks as a whole that focuses on the characteristics of textbooks in general. We analyze Indonesian and Singaporean mathematics textbooks horizontally by focusing on the structure of units/lessons, their frequency, and sequence. Meanwhile, vertical analysis is carried out by researchers on how a single mathematical concept is treated and how to view textbooks as an "environment for the construction of knowledge" (Li, 2000). Vertical analysis is performed by adopting a framework from (Yang et al., 2017). The framework has three dimensions, namely: (1) representation forms, (2) contextual features, and (3) response types. The details of the dimensions, categorization, and coding we write in Table 4.

Dimension	Category and Code
Representation forms	Purely mathematical (A1) Verbal (A2) Visual (A3) Combined (A4)
Contextual features	Application (C1) Non-application (C2)
Response types	Open-ended (E1)

	Close-ended (E2)

Table 4. Vertical analysis

Guided by the categories and codes in Table 4, each researcher performed analysis and coding in both textbooks on the angle topic. After each conducted analysis and coding, it was continued with a cross-check. The agreement results were used for presentations at the Focus Group Discussion (FGD) session. The researcher chose FGD because the information obtained was more informative than data obtained through other data collection methods (Kaur et al., 2020; Purnomo, Shahrill, et al., 2022). If there is any doubt, we can get corrections and improvements right away using this method. The authors validated the results of the analysis and coding. The agreement from the coding results is used for reporting research results that are presented descriptively.

RESULTS

Textbook and Content Overview

The Indonesian mathematics textbook has 216 pages, with 20 pages (9.26%) containing angles. In contrast, Singapore's mathematics textbooks have 120 pages, with 12.5% (15 pages) containing angles. Although there are fewer pages than the mathematics textbook in Indonesia, Singaporean textbooks are more numerous in the amount of content presented. Details of the content structure of each of these textbooks can be seen in Table 5.

The Indonesian Textbooks	The Singaporean Textbooks
1. Fractions	1. Number to 100.000
2. Least Common Multiple (LCM) and Greatest Common Factor (GCF)	2. Factor and Multiples 3. Multiplication and division of whole numbers.
3. Approximation	4. Whole numbers
4. Shapes	5. Angles
5. Statistics	a. Lesson 1 Understanding and Measuring Angles
6. Angle Measurement a. Measurement of Angles in Standard Units with a Protractor	b. Lesson 2 Drawing Angles to 180° c. Lesson 3 Turns and 8-Point Compass
b. Measurement of the plane figure with a Protractor	6. Squares and rectangles 7. Symmetry

Table 5. Mathematics contents presented in the two mathematics textbooks

Based on Table 5, it can be seen that for the Indonesian mathematics textbook, angle topic is the last material taught in grade 4. Meanwhile, in the Singapore textbook, students learn the angles in grade 4 after learning the numbers. The angle topic is discussed in two subsections by grade 4 elementary school students in Indonesia. The first subsection measures angle in standard units with protractors, and the second is about measuring angles of shapes with protractors. While in the Singapore textbook, the material on angles is discussed in three subsections, namely (1) understanding and measuring angles, (2) drawing angles up to 180°, and (3) rotation and 8-point compass.

Mathematical Task Overview

Indonesian and Singaporean mathematics textbooks both use specific criteria to present their tasks. The mathematical tasks presented in Indonesian and Singaporean textbooks are grouped in several parts of grouping task activity based on their criteria, as shown in Table 6. In Indonesian textbooks, the tasks are grouped into seven parts. The overall types of task activities in one chapter of angle are 81 task items. While the questions in Singapore math textbooks are grouped into six parts, with all types of task activities in one chapter, there are 54 task items.

Indonesian Textbooks		Singapore Textbooks	
Task Activity	Description	Task Activity	Description
Ayo Mengamati (Let's Observe)	It contains questions for students before studying the material	Before You Learn	It contains questions for students before studying the material
Ayo Menanya (Let's Ask)	It contains activities to make questions that the students want to know related to the material	Learn	It contains questions for students and their steps
Ayo Menalar (Let's Reason)	It contains an explanation of the material accompanied by the question	Guided Practice	It contains guided question exercises in each sub-chapter of the material that has been studied
Ayo Mencoba (Fun to Try)	It contains guided question exercises that students expect to find concepts in some of the material that has been studied	Hands-On Activity	It contains steps for student activities
Tugas Proyek (Project Task)	It contains tasks that must be done in groups	Chapter Review	It contains practice questions that include material in one chapter
Latihan Soal (Practice Questions)	It contains guided question exercises that students expect to find concepts in all the material that has been studied in one chapter	Put On Your Thinking Cap!	It contains practice questions that require high reasoning

Contoh Soal disertai Jawaban (Worked Answers)	It contains examples of questions accompanied by steps of work

Table 6. Types of task activities and their descriptions

Table 6 shows that Indonesian mathematics textbooks are more likely to provide a more comprehensive opportunity to learn than Singaporean textbooks based on the number and description of task activities. For example, in Indonesian mathematics textbooks, there is a section "*Ayo Menanya*" which is an activity to make questions that students want to know related to the material being studied. This section is not described in the activities presented in Singapore mathematics textbooks.

According to the number of task items, Indonesian mathematics textbooks feature 27 more task items than Singaporean mathematics textbooks. Nonetheless, in Singaporean mathematics textbooks, the distribution of the dimension items in each task activity is proportionate to that found in Indonesian mathematics textbooks. This is more evident in the distribution table of the number of items presented in Table 7 and Table 8.

Order of Learning Activities for Indonesian Mathematics Textbooks	Number of Questions	A1	A2	A3	A4	C1	C2	E1	E2
Ayo Mengamati (Let's Observe)	4	0	0	0	4	4	0	0	4
Ayo Menanya (Let's Ask)	4	0	4	0	0	0	4	2	2
Ayo Menalar (Let's Reason)	13	1	4	1	7	7	6	1	12
Ayo Mencoba (Fun to Try)	23	16	0	6	1	1	22	8	15
Tugas Proyek (Project Task)	1	0	1	0	0	0	1	1	0
Latihan Soal (Practice Questions)	28	12	2	11	3	3	25	4	24
Contoh Soal disertai Jawaban (Worked Answers)	8	0	0	4	4	4	4	0	8
Number of Questions	81	29	11	22	19	19	62	16	65
Percentage (%)	100	35.8	13.6	27.2	23.5	23.5	76.5	19.8	80.2

Table 7 Distribution of the number of items based on analysis results for Indonesian mathematics textbook grade 4

Order of Learning Activities for Singaporean Mathematics Textbooks	Number of Questions	A1	A2	A3	A4	C1	C2	E1	E2
Before You Learn	4	1	2	1	0	2	2	1	3
Learn	10	0	2	1	7	1	9	0	10
Guided Practice	21	9	0	8	4	4	17	8	13
Hands-On Activity	3	1	0	0	2	1	2	1	2

Chapter Review	13	2	0	9	2	2	11	5	8
Put On Your Thinking Cap!	3	0	1	1	1	1	2	0	3
Number of Questions	54	13	5	20	16	11	43	15	39
Percentage (%)	100	24.1	9.3	37	29.6	20.4	79.6	27.8	72.2

Table 8 Distribution of the number of items by analysis results for Singaporean mathematics textbook grade 4

Representations Forms

There are four forms of representation classified, namely Purely Mathematical (A1), Verbal (A2), Visual (A3), and Combined (A4). Tables 7 and 8 show that the number of task items in Indonesian mathematics textbooks on angle content is 81. In comparison, the number of task items in Singapore mathematics textbooks is 54.

Table 7 shows that the task items included in representation forms in Indonesian mathematics textbooks are more dominated by the purely mathematical (A1) category than the verbal (A2), visual (A3), and combined (A4) categories. There are 29 items (35.8%) in the purely mathematical category. This category in Indonesian mathematics textbooks is widely spread in *Ayo Mencoba* activity, namely as many as 16 task items, and the rest is only spread in *Ayo Menalar* activity (1 item) and *Latihan Soal* or Practice Questions (12 items). The purely mathematical category in Indonesian mathematics textbooks can be exemplified on page 191 in *Ayo Mencoba* (Fun to Try).

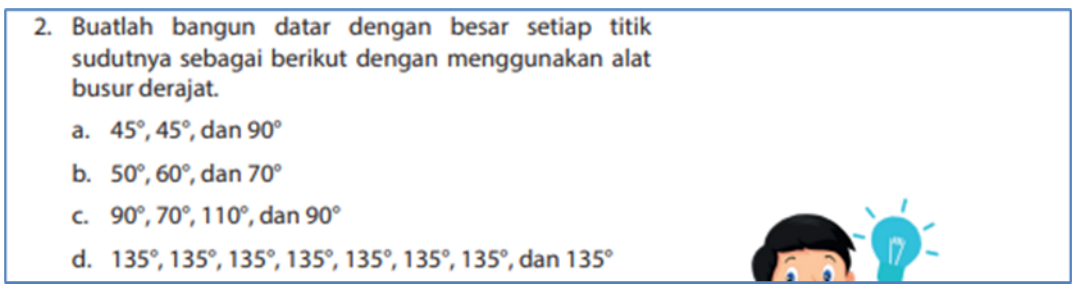

2. Buatlah bangun datar dengan besar setiap titik sudutnya sebagai berikut dengan menggunakan alat busur derajat.
   a. 45°, 45°, dan 90°
   b. 50°, 60°, dan 70°
   c. 90°, 70°, 110°, dan 90°
   d. 135°, 135°, 135°, 135°, 135°, 135°, 135°, dan 135°

Figure 1. Sample item of a purely mathematical task in the Indonesian math textbook

Translate:
Make a shape with the size of each angle as follows using the protractor tool.
a. 45°, 45°, and 90°
b. 50°, 60°, and 70°
c. 90°, 70°, 110°, and 90°
d. 135°, 135°, 135°, 135°, 135°, and 135°

After the purely mathematical category, the task items in the form of the representations are spread into other categories in this Indonesian mathematics textbook, namely 22 items (27.2%) for the visual category and 19 items (23.5%) for the combined category. The least number of items found in the verbal category is 11 items (13.6%).

The findings are different for Singaporean mathematics textbooks shown in Table 8. The task items included in the representation forms in this textbook are more dominated by the visual category (A3). There are as many as 20 (37%) visual task items among the 54 task items in Singaporean mathematics textbooks. The visual category in Singaporean mathematics textbooks can be exemplified on page 97 of the Chapter Review.

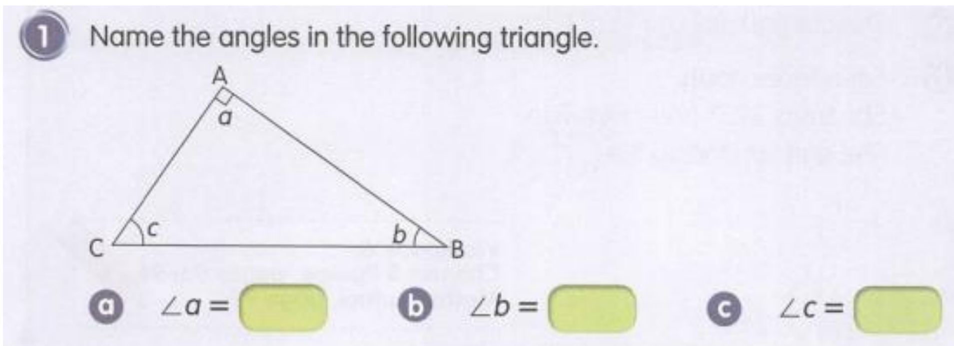


Figure 2. Sample item of a visual task in the Singaporean math textbook

As with Indonesian mathematics textbooks, the verbal category (A2) is the lowest than other categories (5 items or 9.3%). The combined category (A4) consists of 16 items (29.6%), and the purely mathematical category (A1) of 13 items (24.1%).

Contextual Features

Two contextual features are classified: Application (C1) and non-application (C2). Application is a problem presented in the context of a real-world situation. In contrast, non-application is a problem not related to a practical background in everyday life or the real world. Table 7 shows the distribution of contextual features in each type of activity in Indonesian mathematics textbooks. In contrast, Singaporean mathematics textbooks can be seen in Table 8.

Table 7 shows that of the 81 task items in Indonesian mathematics textbooks, it provides contextual feature task items, most of which are classified as non-application. The task items included in this non-application comprised 62 items (76.5%). The distribution of the 62 items is mostly found in the *Latihan Soal* activity (25 items), then continued in the *Ayo Mencoba* activity (22 items), the rest are scattered in other learning activities, except *Ayo Mengamati*. The non-application category in Indonesian mathematics textbooks can be exemplified on page 193 of the *Latihan Soal*.

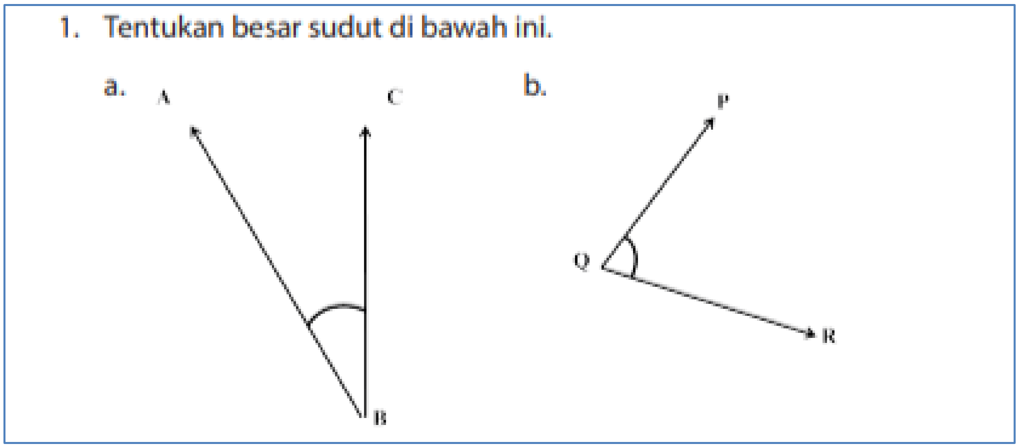


Figure 3. Sample item of non-application task in the Indonesian math textbook

Translate:
1. Determine the size of the angle below.

The application category in the Indonesian mathematics textbook consists of 19 items (23.5%), with the most shares spread out in the *Ayo Menalar* (7 items). The rest are scattered in other learning activities except the *Ayo Menanya* and *Tugas Proyek* sections.

As with Indonesian mathematics textbooks, most of the problem items in Singaporean mathematics textbooks are also classified as non-application (see Table 10). Of the 54 task items in Singaporean mathematics textbooks, there are 43 (79.6%) non-application task items. The most distribution is in Guided Practice activities (17 items), followed by Chapter Review (11 items). The rest are nine items for Learn, two items for Before You Learn, Hands-On Activity, and Put On Your Thinking Cap!

Similar to Indonesian mathematics textbooks, the application category in the Singaporean mathematics textbooks has fewer items than category C2 or non-application. The application category in the Singaporean mathematics textbook can be exemplified on page 98 of the Chapter Review.

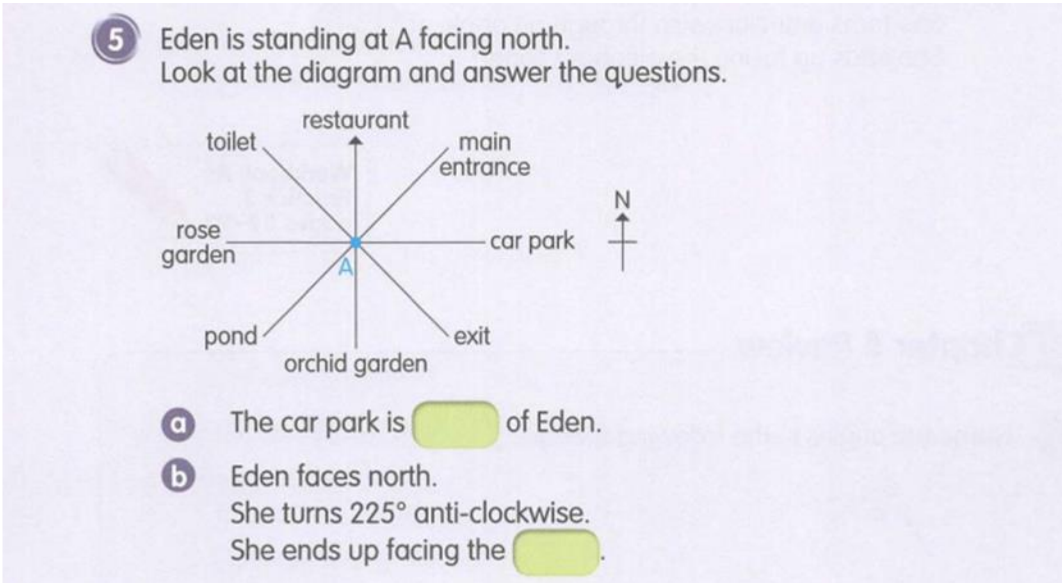


Figure 4. Sample item of application task in the Singaporean math textbook

Response Types

There are two response types: open-ended (E1) and close-ended (E2). Based on Table 7, it can be seen that the task items included in the response types are dominated by the close-ended category in Indonesian mathematics textbooks. Sixty-five items (80.2%) belong to the close-ended category out of 81 task items in this textbook. The most items section is found in the *Latihan Soal* activity, which is 24 items, and the rest is spread out in other activities except *Tugas Proyek*. The close-ended category in Indonesian mathematics textbooks can be exemplified on page 194 in the *Latihan Soal*.

Figure 5. Sample item of a close-ended task in the Indonesian math textbook

Translate:
4. Determine the size of the angle from the measurement results below.

The open-ended category in Indonesian mathematics textbooks consists of 16 items (19.8%). The activity section in the textbook is mostly contained in the *Ayo Mencoba* activity (8 items). The rest is spread on other learning activities except *Ayo Mengamati* and Worked Answers.

Table 8 shows that close-ended categories dominate the task items included in the response types in the Singaporean textbook. This category consists of 39 items (72.2%). The most common items encountered were in Guided Practice activities, with 13 items and the rest scattered in other activities.

The open-ended category in Singaporean mathematics textbooks consists of only 15 items (27.8%), with the most items encountered in Guided Practice activities (8 items) and the rest spread on other activities except Learn and Put On Your Thinking Cap! This category can be exemplified on page 87 in Guided Practice.

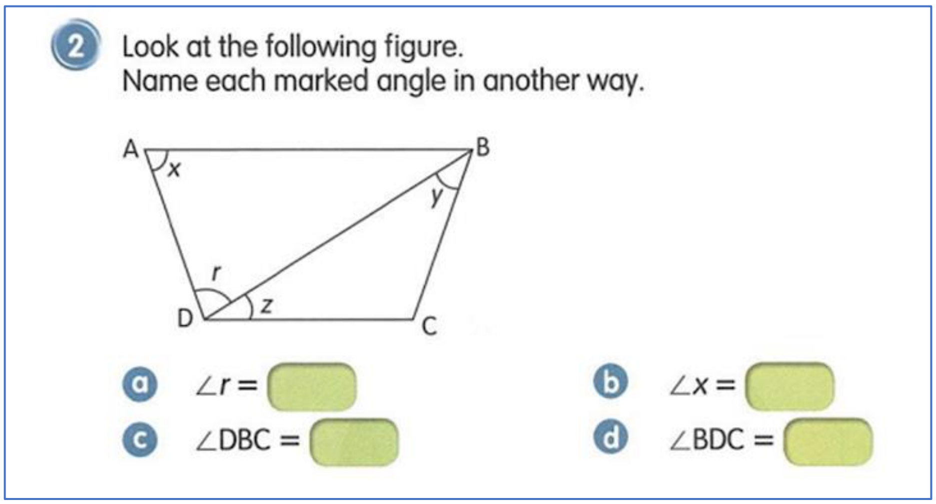


Figure 6. Sample item of an open-ended task in the Singaporean math textbook

According to the findings in each textbook, both are dominated by the closed response type. Nonetheless, Singapore mathematics textbooks provide more proportionate opportunities for students to handle open-ended questions. This interpretation is supported by the percentage comparison achieved by each textbook. The Singapore textbook provides 27.8% of the questions with an open response type, which is more proportional than the Indonesian textbook, which only provides 19.8%.

DISCUSSION

One of the most significant effects on student math comes from textbooks. Both students learning mathematics and teachers planning and delivering mathematics classes use textbooks as essential resources. In many instances, the mathematical problems and exercises provided in textbooks serve as the foundation for and the primary means of delivery for mathematics classroom instruction. Thus, one of the most direct influences on how education is practiced is the use of textbooks (Lepik et al., 2015).

Analysis of tasks in textbooks has the potential to reflect a picture of student engagement with mathematical problems (Yang & Sianturi, 2022). The study of Purnomo, Pasri, et al. (2022) on students' work in dealing with the multiplication of fractions. Students are limited to only realizing that the multiplication of fractions is repeated addition. This limitation causes them to face the multiplication problem of fractions as part of a quantity and other types. This finding also indicates that classroom teacher practices are limited to those contained in textbooks. Other studies also noted findings that corroborated that mathematics textbooks affect teacher instruction in mathematics classes (Hemmi et al., 2014).

Our findings indicate that Indonesian mathematics textbooks are more likely to provide more opportunities to exercise than Singaporean textbooks based on the number and description of task activities. However, the distribution of items in each task activity in Singapore mathematics textbooks is more proportionate to each dimension than that found in Indonesian mathematics textbooks. It is important to note that the effectiveness of a textbook cannot be solely determined by the number of task activities and their distribution. Teaching methodology and teacher activity should consider how evaluating the quality of mathematical tasks in a relevant context.

In the content structure, the angle is the last material taught to grade 4 students in Indonesia, which includes using protractors and applying protractors to measure angles in plane figures. Meanwhile, in the Singapore textbook, angles are studied it after the topics of the numbers and before studying plane figures. Moreover, in contrast to the curriculum in Indonesia, which is limited to only grade 4, the angles and types were previously introduced in grade 3. The mathematics curriculum in Singapore places angles at various grade levels so that the distribution is more evenly distributed for each level, namely grade 4 (understanding, measuring, and drawing angles), grade 5 (angles properties), and grade 6 (measuring angles in geometric figures). This approach allows students in Singapore to have a deeper understanding of angles and their properties, which is essential for higher-level mathematics. By introducing angles at different grade levels, the curriculum ensures that students have a solid foundation before moving on to more complex concepts.

The results of the vertical analysis of mathematical tasks in the two textbooks show that Indonesian and Singaporean mathematics textbooks have similarities in the dominance of categories, especially in 2 dimensions: contextual features and response types. Contextual features are dominated by non-application, and close-ended forms dominate the response types. It becomes clear that these two characteristics are interconnected, with closed forms frequently dominating non-application math tasks and vice versa, making it challenging to solve problems involving application features in closed tasks. Therefore, an additional empirical study is required to examine how the two are related

The contextual features of Indonesian and Singaporean mathematics textbook task items present more task items in the non-application category, that is, problems that are unrelated to practical backgrounds in everyday life or the real world. This indicates a potential gap between mathematics education and its application in real-life situations, which may affect students' ability to transfer their mathematical knowledge and skills to solve practical problems. Similar findings can be identified in studies Purnomo et al. (2019) on middle-grade geometry materials in Indonesia and algebra in elementary schools in Indonesia and Singapore (Yang & Sianturi, 2022) which is more dominant in tasks with an intra-mathematical context. The opinion Gracin (2018) corroborates the statement that intra-mathematical tasks (non-application tasks) dominate textbook task items more. Further, according to Gracin (2018), almost all textbook tasks that include non-application contexts require low cognitive demands. This suggests that teachers should be trained to incorporate both types of features (i.e. non-application and application) into their lessons to ensure a well-rounded education for their students.

Teaching angles will be more meaningful if teachers can help students grasp what they are studying, identify their learning objectives, and recognize that the problems they experience are close to their environment. Some expert opinions state that mathematics comes from social and environmental needs (Ernest, 1991, 1998; Purnomo et al., 2016). Therefore, it is appropriate to present mathematics in the context of social and environmental needs in teaching design and when students interact with the textbook. This approach to teaching mathematics can help students understand the relevance and practical applications of mathematical concepts in their daily lives. It can also promote a deeper appreciation for the subject and increase student engagement and motivation. Textbooks mediate the design of teaching and how students are involved in it (Rahmawati et al., 2020).

It is important to initiate the problem-solving process with real-world problems, as it acquaints students with activities related to mathematical representation and modelling. The process of mathematical modelling begins with a real-world problem, which is subsequently formulated in mathematical terms. After the mathematical problem is solved, the solution must be interpreted to provide an answer to the real-world problem, and it should be verified for its adequacy (Rafiepour & Farsani, 2021).

Another finding, task items in both textbooks are equally dominated by close-ended categories. 80.2% of task items in Indonesian mathematics textbooks and 72.2% of task items in Singapore textbooks belong to the close-ended category. These results correspond to some of the findings of previous studies (Fan & Yan, 2000; Yang & Sianturi, 2017) that close-ended is most commonly found in problems found in mathematics textbooks. This makes students have more experience in solving problems with close-ended response types and relatively less experience they gain on problems with open-ended response types. This type of close-ended response tends to emphasize a low level of thinking, so that students' argumentation and reasoning skills cannot develop optimally (Gracin, 2018). The same is expressed by Yang and Lin (2015) that students who have too much experience with close-ended problems and have limited experience with open-ended problems can cause them difficulty in solving open-ended problems. Moreover, the limited focus on closed-ended problems directs classroom teaching to mechanistic teaching and is teacher center in which the focus of proposing problems is more on result orientation than process. This often leads to a deadlock strategy and, in turn, does not enjoy the beauty of mathematics.

Elementary school students need to be given more opportunities to handle open-ended type task items and to discuss non-unique mathematical problems. This is intended to provide more opportunities for students to try to solve problems with higher-level mathematical thinking (Cai, 1995) and help foster students' divergent thinking skills, including fluency, flexibility, and originality of their response types (Kwon et al., 2006). Thus, the philosophy of humanizing humans can be implemented. Basically, humans have their explorations to achieve goals, so teaching supported by open-based problems allows students to be more creative, not easily discouraged, think critically, and finally become problem solvers.

Our findings, which state that the two textbooks have similarities in the dominance of contextual feature categories toward non-application and the type of response towards closed-ended responses, contradict the results of international surveys in both countries. However, it is interesting that although textbooks are very closely related to how teacher instruction is guided and how students learn, many factors influence the performance of the two countries in the survey results, including Indonesia's and Singapore's culture and education systems. Therefore, future comparative studies can target how cultural reviews are in textbooks and curricula. Apart from that, expanding the scope of the unit of analysis of the two textbooks can strengthen the generalization of the findings so that future researchers can complement the findings of this study by using a broader topic of mathematics.

A striking difference from our findings is the opportunity to learn angles in representational variation. The distribution of most task items in the Indonesian mathematics textbook is more toward the purely mathematical category. In contrast, in the Singaporean mathematics textbook, the task items in the form of representations are dominated by the visual category. The teaching implication is that Singaporean students are given more opportunities to learn with visual objects that they can imagine to help them understand concepts (Yang & Sianturi, 2022), whereas the Indonesian textbook emphasizes the formal aspects of mathematics and is dominated by procedural knowledge. This is in line with the volume and variety of issues, which emphasize more frequent formal mathematics exercises that commonly dominate Indonesian mathematics instruction (Purnomo, 2015, 2016) and are depicted in the mathematics textbooks (Purnomo, Shahrill, et al., 2022). Some researchers agree that procedural knowledge that is not based on strong conceptual knowledge can cause stagnation in problem-solving, and it is difficult to evaluate the location of procedural errors carried out (Byrnes & Wasik, 1991; Purnomo et al., 2014) and also, in turn, causes frustration.

As the thinking stages of elementary school students are still concrete and the characteristics of corner topics that require visual illustrations, visual representations help students and teachers develop the concept of this topic more. Sweller et al. (1990) stated that mathematics curricula (e.g., textbooks) should allow students to solve any [representation] problem and facilitate students' conceptual understanding. National Council of Teacher Mathematics (2000) emphasize that students must be exposed to all kinds of representations of problems in learning mathematics. Therefore, textbooks must provide students with sufficient opportunities to practice solving all kinds of problem representations consisting of task items in the form of purely mathematical, verbal, visual, and combined categories to ensure that students can understand the fundamental structure of different types of questions representation.

Although our findings do not specifically focus on the mathematical activities presented in textbooks, at least three focuses of our analysis relate to these competencies. In several contexts, the findings in the sample analysis found several cases that were significantly different. For example, in case A2 in Table 1, the Singapore version involves students in the action of identifying and measuring the angles, whereas the Indonesian textbook asks a general question quite removed from the process. Furthermore, example C1 in Table 2 also shows that the Singapore problem involves the student directly in the process of solving it, whereas the Indonesian example describes the situation and gives the task. Again, the distance between thinking and doing is larger in the Indonesian example than in the Singapore example, so students can get engaged much easier. The Indonesian questions have a more theoretical flavor, while the Singaporeans are more practical and engaging through action. To improve education, we must look for what engages students the most. Future research could focus on how mathematical activities are presented in textbooks to directly identify the expected directions of teaching practice.

CONCLUSIONS

This study analyzes and compares Singapore and Indonesian mathematics textbooks based on the general characteristics of the textbooks and the nature of the mathematical task on the angle topics. The research findings show that the two textbooks introduce the angle topics in different ways. In Indonesian mathematics textbooks, the angle topics are introduced at the end of the semester, while Singaporean textbooks introduce them in the middle of the semester. Indonesian textbooks have more types of task activities than Singaporean textbooks. However, the distribution of items for each dimension in Singapore book task activities is more proportional.

As for other findings in this study, namely that the task items in Indonesian mathematics textbooks are still dominated in the purely mathematical category, while Singaporean mathematics textbooks are more dominated in the visual category. In other words, Indonesian mathematics textbooks place more emphasis on exercise more often, whereas Singaporean textbooks are more oriented towards conceptual knowledge. To facilitate students and teachers recognizing problem patterns with different representational variations, textbooks must be proportionately constructed with consideration to various mathematical representations.

Lastly, both textbooks are still lacking in the application category and present far fewer forms of answers in the open-ended category. These findings have implications for students' experiences in solving problems that are presented contextually, and more complex problems are less honed, problem-solving tends to be carried out without being based on realistic conceptual knowledge, and the emphasis is more on tasks on tasks with low order thinking skills level.

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Relative Effectiveness of Formative Assessment Techniques on Students' Academic Achievement in Mathematics Classroom Teaching and Learning

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Abstract: This mathematics classroom based experimental study determined the relative effectiveness of formative assessment techniques on students' academic achievement in Mathematics. Factorial research design was adopted for the study involving 100 students from 2 co-educational secondary schools drawn through multistage sampling procedure. Mathematics Achievement Test was the instrument used for data collection. The reliability of the instrument was established using Kudder Richardson (KR_{20}) formula and a reliability coefficient of 0.92 was obtained. Descriptive statistics of mean and standard deviation were used to answer the research questions and Analysis of Covariance (ANCOVA) was employed to test the hypotheses at 0.05 level of significance. The findings of the study revealed that the muddiest point formative assessment technique proved to be more effective in improving students' academic achievement in Mathematics than the think-pair-share technique. The findings also showed that the mean achievement scores of male and female students in Mathematics are not statistically significant. The muddiest point technique should be employed regularly by teachers for classroom assessment of the students since it is more effective in improving students' academic achievement in Mathematics.

Keywords: Classroom teaching & learning, formative assessment, think-pair-share, muddiest point, mathematics education, relative effectiveness

INTRODUCTION

Mathematics plays an important role in the growth, development and sustenance of a nation. Jayanthi (2019) observed Mathematics as the bedrock for nation building; it determines the level of science and technological components and development of any nation, which is a prerequisite for its development. Mathematics education is an essential component of secondary school since it provides students with the basic knowledge required for success in higher education and a variety of jobs (Mario de la & Helen, 2023). Consequently, Mathematics is one of the compulsory subjects offered by all students in secondary schools (Olasen & Lawal, 2020). Mohd (2016) pointed out that Mathematics has always been given extraordinary consideration in school because the nature of the subject is deep rooted in many other fields and disciplines. It as the foundation of all subjects thereby making the teaching and learning of Mathematics inevitable

(Faluyi, 2016). Despite the relevance of Mathematics due to its importance in the development of a country both on national, educational, social and individual level, the records of students' academic achievement in Mathematics over the years is still unsatisfactory.

Academic achievement refers to the results of an individual's performance that show how far he or she has progressed toward a given objective. The breakeven point of academic growth is referred to as academic achievement (Samuel & Ramon, 2020). Kpolovie, et al, (2014) viewed academic achievement to comprise students' ability and performance that linked to human growth, cognitive, emotional, and socio-physical development. Academic achievement, according to Onukwufor and Ugwu (2017), is defined as a score that shows a learner's level of success following teaching and learning. It's a way of demonstrating whether or not learning has occurred, as well as how much learning has occurred. Therefore, the performance of students in terms of grade or score in a Mathematics test or examination can be characterized as students' academic achievement in Mathematics. However, students' academic achievement in Mathematics over the years has been unsatisfactory. In Nigeria, especially at the secondary school educational level, one of the crucial problems faced is the students' unsatisfactory achievement in Mathematics (Sulieman & Hammed, 2019). Moyosore (2015) associates the unsatisfactory students' academic achievement in Mathematics to factors such as attitudes, interest, instructional strategy, classroom environment, and nature of assessment employed in the classroom. The author stressed that among this factor, the nature of assessment practices in the classroom appeared to be the most crucial as less attention have been paid to formative assessment techniques. These records of students' unsatisfactory achievement in Mathematics can be seen from public examination like West African Senior School Certificate Examination (WASSCE) conducted by West African Examination Council (WAEC). Owan (2020) analyzed students' achievement at credit level in Mathematics in May/June WASSCE from 2009 to 2018 and discovered that students' achievement at credit level was below average (50%). This implies that the percentage of students who obtain at least credit pass in Mathematics is below 50%.

The issue of these inconsistent and unsatisfactory students' achievement in Mathematics is also evident in the West African Examination Council (WAEC) Chief Examiner's report of 2016, 2017, 2018, and 2019. From the report it can be deduced that students have some misconceptions in some topics and found most topics difficult. The report also showed that students exhibit consistent weaknesses in most topics that contribute major questions in West African Senior School Certificate Examination (WASSCE) in Mathematics. Topics such as approximation and rounding up of numbers, graph drawing and reading from it, and statistics (measures of dispersion) that have been stated more than twice from 2016 to 2019 by the Chief Examiner, are topics that contribute considerable number of questions in WASSCE in Mathematics.

LITERATURE REVIEW

Formative assessment is an ongoing assessment. According to Adejor and Obinne (2013), it takes place while teaching and learning are still running. The purpose is to find out whether after a learning experience, students are able to do what they were unable to do previously. Formative assessment is defined as a type of assessment that is used to provide immediate feedback to the teachers in order to plan remedial action before the completion of the course or program (Nworgu, 2019). Therefore, formative assessment is a progress checking tool during instruction. According to Ugodulunwa and Uzoamaka (2015), the implementation of quality assessment techniques and the subsequent use of the information obtained from these assessments to enhance teaching and learning instructions are the two most important conditions for successful formative assessment.

Formative assessment provides immediate feedback to teacher and students during assessment. Spector et al. (2017) stressed that actual point or grade, short conferences to measure student progress and provide assistance or peer discussion and peer review are all ways in which feedback can take place. Written feedback from the teacher on assignments, identifying successes and areas of concern, is another typical kind of feedback. According to Clark (2012), feedback should instill confidence in students, particularly when it comes to problem-solving ability. Ajogbeje (2012) stated that formative assessment serves three purposes: planning corrective action for overcoming learning inadequacies, motivating learners and increasing learning retention and transfer. Ajogbeje further posited that the feedback from students' responses to a formative assessment may be analyzed to determine groups and individual faults that need to be corrected.

Formative assessment is useful to both students and teachers. Ojugo (2013) posited that formative assessment is beneficial to both students (for diagnosing learning difficulties and prescribing alternative remedial measures) and teachers (for locating specific difficulties that students are experiencing within subject matter content and forecasting the appropriate techniques to help the students understand the difficulties in the contents in order to facilitate improvement). Formative assessments in the classroom provide timely feedback on the learning process to both the teacher and the learners. Formative feedback shows the gap between what a student already understands and what the teacher expects from that knowledge and understanding (Afemikhe, 2018). Even more impressively, formative assessment closes the achievement gap by assisting the lower achievers (Black, 2012). However, it is argued that formative assessment techniques are missing or lacking from many classrooms for students' assessment (Black & Wiliams, 1998; Keeley, 2018). As a result, much still needs to be done by using formative assessment techniques in assessing students in classrooms in order to prepare them for the task ahead; educationally or career wise or both. The think-pair-share technique blends communication and thinking. This was introduced when science educators sought for a way to shift assessment from teacher centered to student centered (Eze & Obiekwe, 2018). The researchers further stressed that think-pair-share as a formative technique engross self-reflection, collaboration and multiple interactions (teacher to students, students to students, students to teacher) between students and teachers. According to Ifamuyiwa and Onakayo (2013), the think-pair-share is in three stages (think, pair and share) that begin when students are given an open-ended item to consider and are given time to think and, if necessary, scribble down their responses. After which, the students are then assigned (pair) to partner(s) with whom they will discuss and clarify their thoughts. Students are encouraged to present (share) their thoughts to the class or larger group. By going through this process, students are able to solidify and refine their thinking before having to share their answers with the whole class (Alison, 2011). This allows the teacher to spot any flaws in their thinking and rectify them on the spot. The think-pair-share technique is an excellent way to assess students' understanding levels (Bamiro, 2015). This is because the teacher moves round the class as students are sharing their thoughts and ideas to assess the overall depth of understanding.

The think-pair-share technique has been argued by researchers to have contributed to students' improvement and their academic achievement over the years. Eze and Obiekwe (2018) found the technique to be of immense contribution to students' achievement in Chemistry. This is in line with Nwaubani, et al. (2016) who found that think-pair-share significantly improve students' achievement in Economics. Furthermore, a study by Bamiro (2015) found that the think-pair-share's contribution to students' achievement is unmatched compared to the other techniques. For the sole reason that students tend to forget what they learn easily, as well as having troubles recalling information from memory in Mathematics over a period of time, this called for an assessment technique that might enhance their retention capacity and as well might reduce the cognitive load on the working memory. The think-pair-share technique might be suited for that purpose. This is because the think-pair-share technique might be suitable for schema (the way the brain stores information in the long-term memory) development for easy remembering. This is because it involves three vital stages (think, pair and share) that might be crucial in schema development for long term information storage in the long-term memory.

As a result, it was used in this study because it might reduce the cognitive load on the working memory of students, and probably ensure the development of a schema which might in turn enhance students' retention capacity in Mathematics. More so, there was a need to make sure that students develop this schema during assessment in Mathematics lesson by pinpointing and planning remedial instructional objectives for the areas they still find difficult in the lesson, this necessitated the need for the muddiest point formative assessment technique.

Muddiest point formative assessment technique is a technique in which students are asked during or at the end of a lesson to jot down the section of the lesson that they are most confused about. This allows learners to reflect on their own learning and what they find difficult or simple to comprehend. The core benefit of this technique is that it provides students who are reluctant to speak out with an opening to let their difficulties known in an easy way (Alison, 2011). Index-card size pieces of paper are provided for use for this assessment procedure. The students are asked to describe the part of the lesson that they find the most difficult to understand. Saleem, et al. (2021) stressed that asking students to identify which part of the lesson they least understood or most difficult is a fascinating tale and potentially powerful integrative exercise. This is because it first calls for students to reflect on their understanding across several parts of the lesson to identify gaps and secondly, to contemplate, if only briefly, why or which one particular part of the lesson should be selected as the least understood. Their written replies on the piece of papers are collected for review by the teacher (King, 2011). The responses gathered enable the teacher to identify areas where the students might be struggling, which can be addressed immediately or in the next lesson.

The muddiest point formative assessment technique has been argued by researchers to play a significant part in students' improvement. In a study conducted by King (2011), the result showed that students' achievement in Chemistry improved immensely when assessed using muddiest point technique. This is in line with the findings of Akhtar and Saeed (2020) who found that the mean achievement scores of students assessed with the muddiest point technique was greater compared to other techniques. Also, a research study by Carberry, et al. (2013) on unmuddying course content using muddiest point reflection, proved to have positive impact on students' interest and achievement. More so, the technique might be of immense importance because it tends to elicit responses from all ability groups, more especially the low ability group without them having the fear of being laughed at.

The muddiest point technique was used in this study to assess students in Mathematics because it appeared that it doesn't discriminate among the ability levels (high, medium and low) as it tends to provide an opportunity for 100% engagement for all the groups. More so, the technique might tend to pin-point areas of misconception and difficulty in a lesson as well as identifying gap in students' understanding. By asking students to identify areas they are yet to understand might help reduce the cognitive load on their working memory. Since Mathematics has been perceived by students as a difficult subject over the years, it is crucial to use a technique that might pin-point areas they still find difficult in a lesson, so as to provide remedial instructional objectives immediately. Possibly, the most suitable technique for that is the muddiest point formative assessment technique, thus it was employed in this study.

Gender is a social construct that defines male and female identities. According to Ezeh (2013), gender is described as relative power, influence, roles and expectation (feminity and masculinity) that society ascribes to the two sexes on a differential basis. Therefore, gender was seen as any distinguishing feature, or characteristics that separate males from females in the society. The influence of gender on students' achievement and retention in science in general and Mathematics in particular, has been a global concern for science educators and researchers over the years. Consequently, there is no agreement among researchers on the outcomes of studies on gender influence. Where some researchers found a significant difference in students' academic achievement and retention in favour of males (Augustinah & Bolajoko, 2014; Allahnana, et al., 2018), others found in favour of females (Owodunni & Ogundola, 2013; Kwame, et al., 2015; Amalu, 2017). Also, studies have shown no significant difference in Mathematics achievement scores of male and female students (Oluwatayo & James, 2011; Ajai & Imoko, 2015). Moyosore (2015) concluded that when students are constantly introduced to formative assessment, there is no gender influences on their academic achievement.

Evidence abounds in literature of the unsatisfactory senior secondary school students' academic achievement and gender disparity in WASSCE in Mathematics. Among the many factors identified to be responsible for this ugly state of affairs; the most crucial has been the nature of assessment practices by teachers. As it appears that the nature of assessment practices by most teachers in nearly all the public secondary schools in Nigeria appears to be summative assessment. Summative assessment is a teacher centered assessment and as a result, might not give the students the opportunity to participate in the assessment process, interactive learning, self-reflection, identifying gaps in understanding (areas of difficulties and misconceptions), assessing one another nor inculcating the spirit of cooperative learning in tackling problems. The purpose of this study was to determine the relative effectiveness of formative assessment techniques on students' academic achievement in senior secondary Mathematics. The following questions were addressed;

1. What are the mean achievement scores of students taught and assessed in Mathematics using think-pair-share and muddiest point formative assessment techniques?
2. What are the mean achievement scores of students taught and assessed in Mathematics based on gender?

Hypotheses

The null hypotheses formulated were tested at P<0.05, level of significance:

$\mathbf{H0_1}$: there is no significant difference in the mean achievement scores of students taught and assessed using think-pair-share and muddiest point formative assessment techniques.

$\mathbf{H0_2}$**:** there is no significant difference in the mean achievement scores of male and female students taught and assessed in Mathematics.

RESEARCH METHODOLOGY

Design and Participants

The researchers adopted factorial research design. Particularly, the 2x2 factorial research design. According to Cheng (2016), a factorial research design is a type of design whereby the investigator manipulates two or more independent variables at the same time in order to investigate the independent effects of each variable on the dependent variables, as well as the effect caused by interactions among the variables. The design also enables the researchers to investigate the interactions of independent variables with one or more other variables, at times known as moderator variable(s). This study used 2x2 factorial research design because it had two treatment groups (think-pair-share and muddiest point) and a moderator variable (gender) with two levels (male and female). Furthermore, the students were not randomly assigned to classes by the researchers, implying that there might be some relevant confounding variables that the researchers were unable to control. This study used intact classes that were randomly assigned to the experimental groups (two treatment groups). The study participants comprised of 100 senior secondary two (SS2) students from two intact classes using multistage sampling procedure. The treatment group for the think-pair-share technique comprised of 26 males and 22 females whereas the muddiest point comprised of 20 male and 32 female students.

Research Instrument

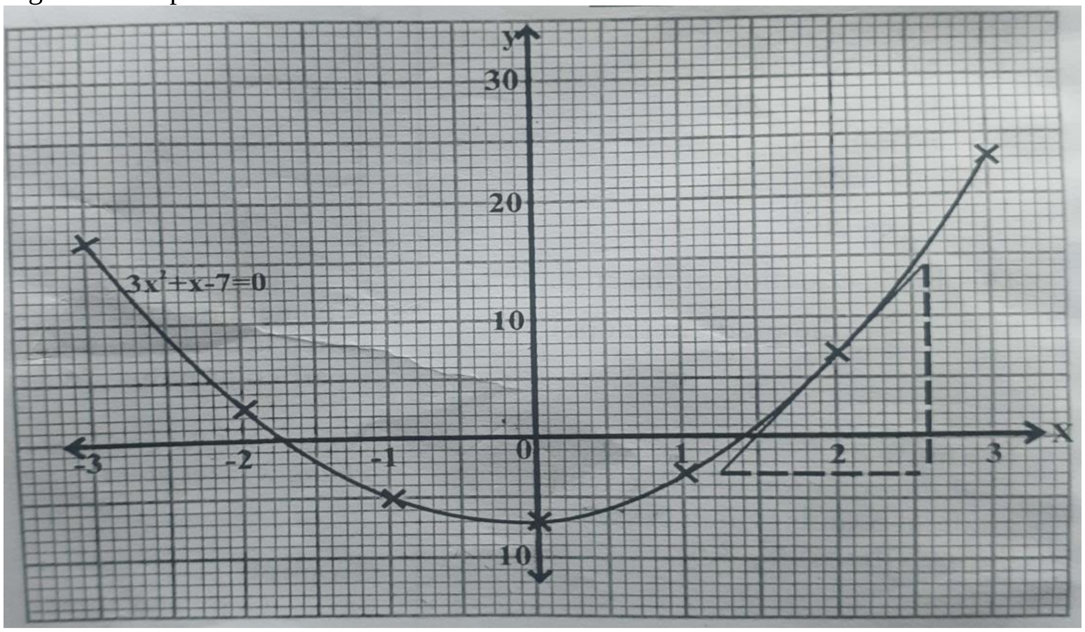
Mathematics Achievement Test (MAT) developed by the researchers was used for data collection. The instrument was 50 multiple-choice items with options A-D. The MAT is divided into two sections: A and B. Section A features demographic data of the respondents, whereas Section B contained information on the mathematics achievement test items. In the instrument, correct and incorrect responses attract one (1) mark and zero (0) mark respectively. Also, the highest possible score is 50 and the lowest possible score is 0. The scoring guide for the instrument was equally developed. The MAT's items were subjected to both face and content validation. The content validation was done using table of specification to determine the instrument's validity. Items were formed from three SS2 Mathematics topics based on the respective emphases placed on each of the topics in SS2 Mathematics curriculum as shown in the table of specification on Table 1. The MAT's items measure only objectives in the cognitive domain employing the revised Bloom taxonomy of educational measures. Approximations with 16 items; Gradient of a curve with 18 items and Measures of dispersion with 16 items were the topics which accounted for the 50 items in MAT. Three specialists face validated the instrument. The specialists performed a face validation of the instrument with respect to clarity of the items, suitability and effectiveness of the items for the study. Their corrections and modification on phrasing of the items, language level and the objectives led to the final production of MAT. The MAT sample is shown in figure 1.

Table 1. Table of specification for mathematics achievement test

COGNITIVE LEVELS / CONTENT AREAS	REM. 14%	UND. 30%	APP. 26%	ANA. 18%	EVA. 6%	CRE. 6%	Total 100%
Approximations 32%	2(1&2)	5(3,4,5,6&7)	4(8,9,10&11)	3(12,13&4)	1(15)	1(16)	16
Gradient of a curve 36%	3(17,18& 19)	5(20,21,22,23&24)	5(25,26,27,28&29)	3(30,31&2)	1(33)	1(34)	18
Measures of dispersion 32%	2(35&36)	5(37,38,39,40&41)	4(42,43,44&45)	3(46,47&8)	1(49)	1(50)	16
Total (100%)	7	15	13	9	3	3	50

From the graph, determine the roots for the equation y=$3x^2$ + x -7.

Figure 1: Sample of the Mathematics Achievement Test



Reliability of the Instrument

Thirty-five copies of the instrument were administered to SS2 students that are not part of the area under investigation but has similar features to the area under study in order to ensure the instrument's reliability. The results of the administered instrument were recorded and subjected to Kuder-Richardson formula 20 (KR_{20}) in order to obtain the reliability index. The reliability index was found to be 0.92. The Kuder-Richardson formula 20(K_R_{20}) was used because it is a measure of internal consistency reliability for measures with dichotomous choices

Classroom Teaching Procedure

On the basis of assignment, two intact classes from two schools were randomly assigned into treatment groups (think-pair-share and muddiest point). One of the schools was used for think-pair-share treatment group and the other was used for muddiest point treatment group. For the sake of clarity, the schools were labeled. School for the think-pair-share treatment was named T while the school for the muddiest point treatment was named M. This was done to allow the researchers to have more control over the interactive effect. The experiment lasted for five weeks. The first week was dedicated to research assistant training and pretest. The research assistants administered the instrument as a pretest at the end of the training. For three weeks, the three-week lesson planned were covered during which students were taught and assessed using the assessment for learning technique allocated to each school. Students were assessed in first topic in the lesson plan (approximations) by research assistants in each school (T, M) using the assessment technique assigned to each school during the second week. The second topic (gradient of a curve) also followed that sequence in the third week, and the third topic (measures of dispersion) equally followed that pattern in the fourth week of the experiment. During each task in the classroom, the research assistant in the think-pair-share treatment school (T) guided students on how they could assess and share knowledge with peers, provide feedbacks to peers and coming to a consensus before sharing their ideas with the entire class. Also, the research assistant in the muddiest point treatment school (M) guided the students on how to reflect and assess their own understanding in order to identify gaps in learning, guide the students on how to write muddiest points by using precise and concise phrases. More so, providing feedback, students assessed the feedback received and finally enhanced their own work. The teacher guides the students on weekly classes conclude lesson by going through the core points of the lesson, such as approximating numbers to ten, hundred, thousand, million, billion, and trillion.

Figure 2: Teacher sharing the lessons learned during the week

Think Pair Share:

Teacher's Activities

The teacher guides the students through the solution using the logarithm table and calculator and makes a comparison of the results obtained from both approaches. The teacher observes, provides feedback, and reinforces their answers.

Students' Activities

Students think individually and pair with peers, forming 3 groups. Students make use of the logarithm table to solve log2468. Students are expected to carry out the task with their peers as follows: Using the table, the characteristics are 3. To get the mantissa, check "24" under "6" difference "8" which is to yield 3923 = 3.3923. The students are to equally compare the calculator value with the table value to see if they yield the same results. A representative from each group shares their answers with the entire class. The students still in the group solve more problems and equally employ the calculator to make a comparison of the results obtained.

Figure 3. Think Pair Share

Muddiest Point:

Teacher's activities
The teacher guides the students through the solution of log2468 using the logarithm table and calculator and makes a comparison of the results obtained from both approaches.

Students' Activities
Students are expected to solve log2468 using a table, calculator and compare the results as follows: Using table, the characteristics is 3, to get the mantissa, check "24" under "6" difference "8" which yielded 3923 = 3.3923, Also, students are expected to punch log2468 and compare the value obtained with the table. The students are to solve more problems and equally employ the calculator to make a comparison of the results obtained. The students use 2 minutes to think individually in order to identify areas of difficulty or misconceptions, and after that, they write it down on the index card part of the lesson they still find difficult and hand it in to the teacher. The students use the feedback from the teacher and enhance their understanding, thereby clearing up the areas of misconception and difficulties.

Figure 4. Muddiest Point

The instrument that was administered to the students as a pretest was reshuffled and administered as a post-test to the two schools (T, M) two days after completing the teaching and learning (lessons), which was the fifth week of the experiment. The research assistants scored the scripts using the marking guide.

Method of Data Analysis

Descriptive statistics of mean and standard deviation were used to answer research questions and analysis of covariance (ANCOVA) was used to test the hypotheses at 0.05 level of significance. The ANCOVA was employed because it adjusts post-test scores using the pretest score to resolve variances in post-test scores that might have arisen from non-equivalence. If found that the probability value (p-value) is more than 0.05, the null hypothesis was not rejected. Otherwise, the null hypothesis was rejected. Glimpses of the experimental classes:

Figure 5. Experimental classes

Ethical Approval and Consent to Participate

The ethics committee at the school where the research was conducted granted ethical approval. Before the commencement of the study, the students were presented with informed consent forms to fill and sign. The informed consent forms were properly filled and signed. This research was carried out in accordance with the principles outlined in the ethical policy. The data collection from the entire sample was in accordance with the ethical standards of the researchers' institutional research committee and with the 1964 Helsinki declaration.

RESULTS

Result on Table 2 shows the pretest and post-test mean achievement scores of students taught and assessed in Mathematics using different forms of formative assessment technique. The result shows that the students who were taught and assessed using think-pair-share assessment technique had a pretest achievement mean score of 17.46; with a standard deviation score of 4.66 and a post-test achievement mean score of 30.50; with standard deviation score of 1.74. The mean gain score was 13.04. The result also shows that the group taught and assessed in Mathematics using muddiest point assessment technique had a pretest mean achievement score of 17.00, with standard deviation score of 4.30 and a post-test achievement mean score of 37.37, with standard deviation score of 2.71. The mean gain score was 20.37. The results of the study show that the students taught and assessed in Mathematics using muddiest point assessment technique achieved better than those assessed in Mathematics using think-pair-share technique. This result therefore shows that muddiest point assessment technique proved to be more effective.

Table 2. Scores of Students Taught and Assessed using Formative Assessment Techniques.

Assessment Techniques	Pre-test	Post-test

	N	$\bar{x}$	SD	$\bar{x}$	SD	Mean Gain
Think-pair-share Assessment	48	17.46	4.66	30.50	1.74	13.04
Muddiest Point Assessment	52	17.00	4.30	37.37	2.71	20.37

The result in Table 3 shows the ANCOVA result of the significant difference in the mean achievement scores of students taught and assessed in Mathematics using formative assessment techniques. Result shows that an F-ratio of $F(1, 95) = 216.095$, $p<0.05$, $\eta^2_p = 0.695$ was obtained. There is significant difference in the mean achievement scores of students taught and assessed in Mathematics using formative assessment techniques with those taught and assessed using muddiest point assessment technique having a higher mean gain. The result of the study further shows that the effect size, as indicated by the corresponding partial eta squared value of 0.695 shows how much of the variance in the dependent variable that is explained by the independent variable. The partial eta square of 0.695 which translate to 69.5% implies that 69.5% of the variance in students' achievement in Mathematics is accounted for by the forms of formative assessment.

Table 3. Effectiveness of Formative Assessment Techniques on Students' Achievement in Mathematics.

Source	Type III Sum of Squares	Df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	1190.340[a]	4	297.585	56.297	.000	.703
Intercept	6531.379	1	6531.379	1235.599	.000	.929
PRETEST	13.371	1	13.371	2.530	.115	.026
GROUP	1142.280	1	1142.280	216.095	.000	.695
GENDER	.676	1	.676	.128	.721	.001
GROUP * GENDER	.010	1	.010	.002	.965	.000
Error	502.170	95	5.286			
Total	117769.000	100				
Corrected Total	1692.510	99				

Result on Table 4 shows the mean achievement scores of male and female students taught and assessed in Mathematics. The result shows that the male student had a pretest achievement mean score of 17.46, with a standard deviation of 5.08 and a post-test achievement mean score of 33.41, with a standard deviation of 4.25. The mean gain score was 15.95. The result also shows that the female students had a pretest achievement mean score of 17.02, with a standard deviation of 3.90 and a post-test achievement mean score of 34.63, with a standard deviation of 3.99. The mean gain score was 17.61. Mean gain score of 15.95 and 17.61 for male and female students respectively imply that the female students achieved slightly better than the male students.

Table 4. Achievement Scores of Male and Female Students in Mathematics

Variable Gender	N	Pretest $\bar{x}$	SD	Post-test $\bar{x}$	SD	Mean Gain
Male	46	17.46	5.08	33.41	4.25	15.95
Female	52	17.02	3.90	34.63	3.99	17.61

The result in Table 3 shows the ANCOVA result of the influence of gender on students' achievement in Mathematics. The result shows that an F-ratio of $F(1, 95) = 0.128$, $p>0.05$, $\eta^2_p = 0.01$ was obtained. The associated probability value of 0.72 is greater than 0.05 set as level of significance. The mean achievement scores of male and female students in Mathematics is not statistically significant. This implies that gender is not a significant factor in determining students' achievement in Mathematics. The result of the study further shows that the effect size, as indicated by the corresponding partial eta squared value of 0.01 shows how much of the variance in the students' achievement in Mathematics that is influenced by gender. The partial eta square value of 0.01 which translate to 1.0% implies that 1.0% of the variance in students' achievement in Mathematics is influenced by gender. This result shows that gender does not significantly influence students' achievement in Mathematics when taught and assessed using different forms of formative assessment.

DISCUSSION

The result showed that muddiest point assessment technique improved students' achievement better than think-pair-share assessment technique. This result therefore shows that muddiest point assessment proved to be more effective. However, the standard deviation score of the think-pair-share assessment technique shows more homogeneous responses from the students than the muddiest point assessment technique. The finding further revealed significant difference in the mean achievement scores of students taught mathematics and assessed using formative assessment techniques with those taught and assessed using muddiest point assessment having a higher mean gain. There is no doubt therefore that the muddiest point assessment technique is more effective. The finding of the study agrees with King (2011), who conducted a study by using clickers to identify students' muddiest point in Chemistry and the result showed that students' achievement in Chemistry improved immensely. The study is also consistent with Akhtar and Saeed (2020a) who carried out a study on the effect of frayer model, choral response and muddiest point on students' academic achievement and found that muddiest point assessment technique enhanced students' achievement more than the other two assessment techniques. The result of the study also reinforces the earlier claims made by Carberry, et, al. (2013) on unmuddying course content using muddiest point reflection, which proved to have a positive impact on students' interest and achievement and the fact that many students want the muddiest point assessment technique to be continually used during instruction. It is therefore evident that muddiest point assessment technique proved to be more effective in enhancing students' achievement in Mathematics. The findings on the think-pair-share assessment technique is consistent with that of Hamdan (2017) who carried out a research work on the effect of think-pair-share on the achievement of third grade students in sciences in the education district of Irbid in Jordan and found a significant difference in the grades of students in favour of the think-pair-share technique. Furthermore, the findings on the think-pair-share assessment technique agrees with Akhtar and Saeed (2020b) who conducted a study on assessing the effect of agree/disagree, exist ticket and think-pair-share on students' academic achievement at undergraduate level and found among other things that there was a significant difference in the mean achievement scores of students assessed with think-pair-share assessment technique and the other techniques in favour of think-pair-share assessment. The result of the think-pair-share assessment technique also strengthens earlier claims made by Sampsel (2013) who maintained that think-pair-share technique greatly improve students' confidence and participation. This is because students think about their response to a question, discuss their response in pair and share their response or ideas with larger group or class. However, the results of the study show that muddiest point assessment technique proved to be more effective than think-pair-share assessment technique.

The study showed that female students achieved slightly better than the male students when taught and assessed using forms of formative assessment technique. The study further revealed that gender does not significantly influence students' achievement in Mathematics when taught and assessed using forms of formative assessment techniques. This implies that students' gender is not a significant determinant of academic achievement in Mathematics. The finding of the study is consistent with Oluwatayo and James (2011) who conducted a study on gender difference in Mathematics of which the outcome showed no significant difference in the achievement of male and female students. This is in line with the research work by Ajai and Imoko (2015) on gender differences in Mathematics achievement and retention using Problem-Based Learning (PBL) which revealed no gender difference in students' Mathematics achievement and retention scores. The findings of this study reinforce the claim by Moyosore (2015) who concluded that when students are constantly introduced to formative assessment, there is no gender influences on academic their achievement. The finding of this study is in variance with a research work by Allahnana et, al. (2018) who found that male students excel in Mathematics more than their female counterparts. Also, a research work by Amalu (2017) found higher academic achievement in English language and Mathematics in favour of the female students. However, the finding of this study shows that there is no significant gender difference in students' academic achievement in Mathematics. This implies that students' achievement in Mathematics does not depend on whether the student is a male or female.

CONCLUSIONS

Students taught and assessed using muddiest point assessment technique achieved better than those taught and assessed using think-pair-share formative assessment technique. This result therefore shows that muddiest point formative assessment technique is more effective than the think-pair-share formative assessment technique which implies that the forms of formative assessment technique are effective in assessing students. There was significant difference in the mean achievement scores of students taught and assessed using think-pair-share and muddiest point formative assessment technique with those taught and assessed using muddiest point formative assessment having a higher mean gain. The difference in the mean achievement scores of male and female students in Mathematics was not statistically significant. Gender is not a significant factor in determining students' achievement in Mathematics. The findings of the study showed that the students taught and assessed using muddiest point formative assessment technique achieved better than those taught and assessed using think-pair-share formative assessment technique. This implies that the muddiest point formative assessment is more effective in improving students' achievement in Mathematics. The result of the study showed that the difference in the mean achievement scores of male and female students in Mathematics is not statistically significant. This implies that gender is not a significant factor in determining students' achievement in Mathematics.

LIMITATIONS OF THE STUDY

1. Even though the research assistants were trained on how to deliver the lesson that was incorporated with the assessment techniques, other intervening variables like mastery of content, intellectual abilities and teaching experience of the teachers were not totally controlled and these might have influenced the result of the study.
2. It is also possible that due to time frame, the research assistants might not have understood the instructional packages to the last details and as a result, the process of administration of the instructional packages could have not been followed effectively and that might have affected the outcome of the study.

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Appendix

MATHEMATICS ACHIEVEMENT TEST (M AT)

Section A: Demographic data of respondents

Please tick (✔) appropriately: Male ☐ Female ☐

Section B: Item statements
Instruction: Choose the correct answer from letter A-D by ticking (✔) the correct option.
Do not carry out rough work on this paper.
Time allowed: 1 hour, 35 minutes

1. A number which is 5 and above after a decimal point is rounded as……….
   (a) 0.5
   (b) 1
   (c) 0
   (d) 1.5
2. A number which is 4 or below after a decimal point is rounded as………..
   (a) 1.5
   (b) 0
   (c) 1
   (d) 0.5
3. Express 4764.457 to 1 and 2 decimal places
   (a) 4764.5 and 4760.50
   (b) 4764.5 and 4764.45
   (c) 4765.0 and 4764.46
   (d) 4764.5 and 4764.46
4. Round off 400,453 to the nearest thousand
   (A) 400,050
   (B) 400,000
   (C) 400,100
   (D) 401,000
5. Approximate 678,502, 453 to the nearest million
   (A) 679,000,000
   (B) 679,100,000
   (C) 678,000,000
   (D) 678,500,000
6. Round off 174,667,232,999 to the nearest billion
   (A) 174,000,000,000
   (B) 175,700,000,000
   (C) 175,000,000,000
   (D) 174, 700,000,000
7. Estimate 283,644,234,999,701 to the nearest trillion
   (A) 283,000,000,000,000
   (B) 283,000,000,010,000
   (C) 284,000,000,000,000
   (D) 284,100,000,000,000

8. A rope of length 11.8m was measured by a girl to be 10.9m. calculate the percentage error
   (A) 7.6%
   (B) 7.0%
   (C) 8.5%
   (D) 9.0%
9. A 3% error was committed by a carpenter in measurement of a ladder of actual length 45cm. Find the absolute error
   (A) 1.5
   (B) 1.0
   (C) 1.6
   (D) 1.8
10. The absolute error when Peter measured his father's staff is 2.4m and the percentage error is 15%. Calculate the actual length of the staff.
   (A) 15
   (B) 14
   (C) 12
   (D) 16
11. A chef underestimated his expenses by 3.5% but actually spent N400, what was his estimate?
   (A) N385
   (B) N380
   (C) N386
   (D) N384
12. How many hours to the nearest hour is in 4 hours 25 minutes
   (A) 5 hours
   (B) 6 hours
   (C) 4 hours
   (D) 7 hours
13. How many hours to the nearest hour is in 8 hours 45 minutes
   (A) 9 hours
   (B) 7 hours
   (C) 8 hours
   (D) 6 hours
14. A 3% error was made in the measurement of a wood of actual length 98cm. calculate the absolute error to the nearest whole number
   (A) 3cm
   (B) 4cm
   (C) 5cm
   (D) 3.1cm
15. Calculate the percentage error correctly to 1 decimal place if a bag of beans which weighed 15kg is recorded to have weighed 15.9kg
   (A) 6.0%
   (B) 6%
   (C) 6.1%
   (D) 6.10%
16. Arrange 23.02, 23.1 and 23.330 in ascending order
   (A) 23.02, 23.1 and 23.330
   (B) 23.1, 23.02 and 23.330
   (C) 23.330, 23.1 and 23.02
   (D) 23.1, 23.330 and 23.02
17. In a straight line graph, $\frac{y-axis}{x-axis}$ is referred to as ……..

(A) Intercept
(B) Graph line
(C) Slope
(D) Curve

18. The gradient of ………. Changes from point to point
    (A) Line
    (B) Straight line
    (C) Curve
    (D) Slope
19. The gradient of a straight line at any too points are always……..
    (A) Straight
    (B) Equal
    (C) Different
    (D) Curve
20. The gradient of the line described by the given points; X(8,4) and Y(10,6) is …….
    (A) 0
    (B) 2
    (C) 3
    (D) 1
21. Given the points; T(-4, 8) and J(8, -4), the gradient of the line is…….
    (A) 1
    (B) 0
    (C) -2
    (D) -1
22. A point where a line cut the y-axis is called……..
    (A) Gradient
    (B) Intercept
    (C) Slope
    (D) Curve
23. Given 4x+y = 14, the gradient of the equation is…….
    (A) -4
    (B) 4
    (C) -5
    (D) 5
24. If y_2 = 4, y_1=2, x_2=5 and x_1=1 calculate the slope
    (A) $\frac{1}{3}$
    (B) $\frac{1}{4}$
    (C) $\frac{1}{2}$
    (D) $-\frac{1}{3}$
25. What is the gradient and y-intercept of equation 8x-2y=10
    (A) 4,5
    (B) 4,-5
    (C) -4,-5
    (D) -4,5
26. Given that m =2, y_2=7, y_1 = -8, x_1 = -1.5, calculate the value for x_2
    (A) -6.5
    (B) 6.5
    (C) -6
    (D) 6

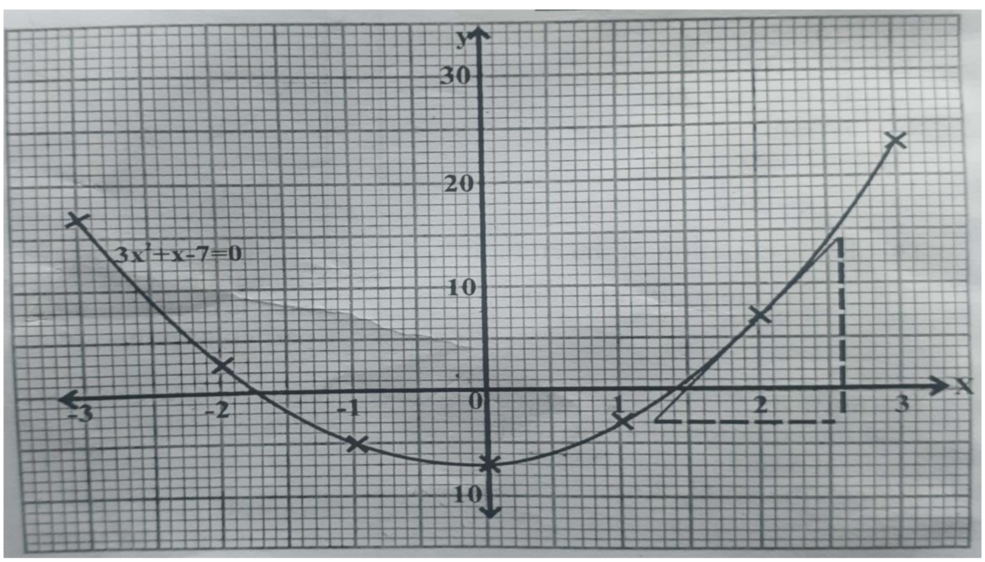
27. The equation of the line A(3,7) and B(4,9) is …….
   (A) y=2x-1
   (B) y=2x+1
   (C) y= 3x-2
   (D) y= 2x-2
28. The equation of a line with gradient 10 units which passes through (4,3) is …..
   (A) y=9x+36
   (B) y=10x-37
   (C) y=10x+37
   (D) y=-10x-37
29. A tangent of a curve is a straight line which is drawn to touch the … at a particular point.
   (A) Line
   (B) Curve
   (C) Graph
   (D) Intercept

Use the information in Table 1 and the Graph to answer question 30 to 34 given that $y = 3x^2 + x - 7$

X	-3	-2	-2	0	1	2	3
Y	P	3	Q	Z	W	7	23

Table 1.

30. Calculate the value of PQZW respectively
   (A) -17,5,-7,3
   (B) 17,-5,7,-3
   (C) 17,-5,-7,-3
   (D) -17,-5,7,-3



31. From the graph, determine the roots for the equation $y=3x^2 + x - 7$.
   (A) -1.4cm or -1.7cm
   (B) 1.4cm or 1.7cm
   (C) 1.4cm or -1.7cm
   (D) -1.4cm or 1.7cm
32. What is the scale on the graph for y-axis

(A) 2cm:10units
(B) 10cm:10units
(C) 10cm:2units
(D) 2cm:2units

33. The least value for the equation y=3x^2 + x -7 is ………
    (A) -7.0
    (B) -7
    (C) 7.01
    (D) 7.0
34. The gradient of the graph at the point when x = 2 and y = 7 is……..
    (A) $12\frac{1}{6}$
    (B) $11\frac{1}{7}$
    (C) $-12\frac{1}{7}$
    (D) $12\frac{1}{7}$
35. The difference between the highest value and lowest value in a given set of numbers is…
    (A) Range
    (B) Mean deviation
    (C) Variance
    (D) Standard deviation
36. The extent to which variation in a given distribution is shown is referred to as…….
    (A) Range
    (B) Mean deviation
    (C) Variance
    (D) Standard deviation
37. The individual spread of numbers in a set of numbers from a mean is known as…….
    (A) Central tendency
    (B) Deviation
    (C) Range
    (D) Assume mean

Given the data 2,4,6,8,10. Use the data to answer question 38 to 41

38. What is the range of the distribution
    (A) 7
    (B) 8
    (C) 6
    (D) 5
39. What is the sum of the deviation from the mean
    (A) 1
    (B) 0
    (C) -1
    (D) 2
40. The variance of the distribution is……
    (A) 6.7
    (B) 8
    (C) 9
    (D) 7
41. The standard deviation to 1 decimal place using the value of variance in question 40 is…
    (A) 2.8
    (B) 2.6
    (C) 2.5
    (D) 2.3

Elizabeth measured some cups of rice as follows; 2 cups, 4 cups, 8 cups and 10 cups. Use the information to answer question 42 to 45.

42. The range of the measurement is …..
    (A) 8 cups
    (B) 8
    (C) 8 cup
    (D) 7 cups
43. The variance in measurement to the nearest whole number is….
    (A) 10
    (B) 12 cup
    (C) 10 cups
    (D) 11 cups
44. The standard deviation in the measurement to 1 decimal place is….
    (A) 3.1
    (B) 3.0
    (C) 3.2
    (D) 4
45. The difference between range and standard deviation to the nearest whole number is...
    (A) 5
    (B) 4
    (C) -4
    (D) -5

Given the data 3,1,4,2,5,6,7,9,10,8,12, 11 Use the information to answer question 46 to 50.

46. Calculate the value for the first quartile
    (A) 4.5
    (B) 6.5
    (C) 9.5
    (D) 3.5
47. Calculate the value for the third quartile
    (A) 5.5
    (B) 7.5
    (C) 4.5
    (D) 9.5
48. The sum of the first quartile and the third quartile is …..
    (A) 13
    (B) 12
    (C) 14
    (D) 11
49. What is the value for the inter-quartile range
    (A) 6
    (B) 7
    (C) 5
    (D) 4
50. The semi-inter quartile range to the nearest whole number is ….
    (A) 4
    (B) 5
    (C) 3
    (D) 2

The Effect of Computer Supported Collaborative Dynamic Learning Environment on High School Students' Success in Mathematics Classroom

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Abstract: This research discuss and considers possible components in computer supported collaborative dynamic learning environment in mathematics classroom. The aim of the research is to observe the applicability of this environment to the courses and to determine its effect on student success. The research group consisted of 68 high school students and their mathematics teacher. Accordingly, in the learning environment in the teaching of the first objective, it was observed that the collaborative teams were working tangentially. Upon examining the results observed in the teaching of the other three objectives, it was observed the behaviors of the collaborative teams were carried out in accordance with the environment and that the teams and the course teacher were more effective. Teaching through computer supported collaborative dynamic learning had a more positive effect on the success of high school students.

Keywords: computer supported collaborative learning. dynamic learning. GeoGebra. instruction of lines. worksheets

INTRODUCTION

This research how to design the technological environment for collaboration and how teachers and researchers learn in the context of collaborative activities. Teachers are considered to be key figures in ensuring the use of computer-based cognitive tools in mathematics teaching (Umay, 2004). Accordingly, the question of when and how teachers and students use this tool for effective and long-lasting learning becomes crucial. It is possible to ensure students' effective and permanent learning through social interactions in computer-supported collaborative environments. Johnson and Johnson (1994) state in their study that collaborative learning is used not only as a learning process but also as a form of classroom management and that the best way to manage technology-supported instruction is through collaborative learning. Computer Supported Collaborative Learning (CSCL) is a combination of the concepts of computer, support, cooperation, and learning (Jones, Dirckinck-Holmfeld, & Lindström, 2006). It is possible for teachers to integrate computers and computer technologies into their lessons and for students to do their learning activities collaboratively in the classroom environment with learning environments prepared in accordance with the purpose of the lesson.

In the field of CSCL various approaches are applied to explain when and for whom collaborative learning can be benefcial as well as how the design of CSCL can be improved (Schnaubert & Vogel, 2022). CSCL is found in a wide range of fields, from education to sociology, architecture to economics. CSCL, the effects of which have been investigated in different cultures, leads to the emergence of new concepts with each new research (Jones et al., 2006). Although much progress has been made, questions remain about the sequencing of education and curriculum to support learning and collaboration in the classroom, and in particular about the approaches used in these classrooms (Lee, Chan, & Aalst, 2006).

Today, CSCL has emerged as a combination of collaborative learning and the use of computers and computer-related technologies such as the Internet (Stahl, Koschmann, & Suthers, 2006) or software created for many different infrastructures and purposes in the learning environment. Lipponen, Rahikainen, Lallimo, and Hakkarainen (2003) state that collaborative learning is supported by computers and the question of how students who form collaborative learning groups can learn better by working interactively through computers has formed CSCL. According to Adams (2004), collaborative learning has been examined by researchers in terms of different variables affecting learning such as gender, the number of students required to be in the group, talent, collaborative learning skills, and other ability factors. However, with the increasing use of computers in classroom environments, these variables, which have been investigated in collaborative learning, moved to this new field with the combination of computers and collaborative learning. Researchers have begun to answer many of these questions in the field of computer-supported collaborative teaching (Adams, 2004).

Ching Sing et al. (2011) state that the teacher's educational and pedagogical skills play an important role in creating the learning environment, conducting activities, and engaging the students. Accordingly, teachers should support students in deciding with whom and with which educational tools they want to work. This will enable students to interact more dynamically and give them responsibility so that they learn to build knowledge across communities (Ching Sing et al., 2011). Stahl et al. (2006) stated that CSCL supports individual learning, but that learning cannot be reduced to this and that one of the effects of group work is to enable individual learning. From the student's point of view, team building brings with it some problems. These problems can be listed as difficulties in interacting, difficulties in motivation, and the provision of the necessary educational tools (Ching Sing et al., 2011). Thus, it is necessary to provide the technological infrastructure that will enable teachers and groups to work together interactively and enable groups to easily access learning products. In such learning environments, there are debates about how one should comprehend the educational content and the approaches taken to define the analytic structure applied (Arnseth & Ludvigsen, 2006). In the literature, it is possible to come across more studies that examine CSCL's synchronized collaborative teams working together in the internet environment, while CSCL's collaborative studies created in the classroom environment are less common. However, how one handles the computer-aided structure applied during collaborative work in mathematics education is important for students and teachers to adopt this learning style. If the technological structure applied in Computer Supported Collaborative Learning (CSCL) environments is dynamic, this method can be expressed as Computer Supported Collaborative

Dynamic Learning (CSCDL). Considering the dynamic structure of mathematics, CSCL is applied as CSCDL in mathematics education. Considering that mathematical expressions contain variables and that students need to learn different situations and properties within each concept, the importance of integrating dynamic learning in CSCL environments becomes apparent. Accordingly, one can define the CSCDL presented in this study as a learning method in which students form collaborative teams and reach generalizations of the definitions and concepts expected from them through the dynamic mathematics software they use on computers, while their teachers guide them with hints. Accordingly, there are a wide variety of studies in the literature on the use of technologies created by GeoGebra as a computer and dynamic software in classroom environments (Adelabu, Marange & Alex, 2022; Joshi & Singh, 2020; Khalil, Khalil & Haq, 2019; Thapa, Dahal, & Pant, 2022; Uwurukundo, Maniraho & Rwibasira, 2022). As a matter of fact, one cannot ignore the suggestions of mathematics educators about the introduction of technology into the classroom environment. However, with the advancement of technology, it is seen that people become lonely and are only interested in technology. In this case, in the coming years when face-to-face interaction will always remain valid, there may be a generation that is not really socialized and that spends time only on the internet through social media. It is possible to reverse this situation by using technology in a useful way in terms of education. In other words, it is possible to raise individuals who both use advanced technologies and truly cooperate and interact with each other.

A core challenge is understanding how the open-ended, ever-evolving process of collaborative inquiry can be organized, regulated, and supported in a manner that leverages students' agency and creative imagination (Tao & Zhang, 2021). According to Mapile & Lapinid (2023), students interaction with each other, more ideas are created, student assistance, encouragement among learners, development of skills, timeefficiency in finishing a task, and better output produced. Before explaining the role of technology in collaborative learning, it is possible to summarize the features of collaborative learning (Ching Sing, et al., 2011). Student Teams-Achievement Divisions (STAD) which is the technique of collaborative learning was developed by Slavin (1990). This technique was used in this study. Determining that STAD is the most researched technique among the results of collaborative learning, mathematical operations and applications are suitable for carrying out concepts in science, language use and mechanics, geography and map coverage (Slavin, 1994). This technique has five elements: teams, presentation, quizzes, individual progress points, and team reward.

In this technique, students are first divided into heterogeneous groups of three, four or five in terms of achievement level, gender, and race. Students then work as a team on learning materials such as worksheets and support each other's learning. Dynamic materials and worksheets were used in this research. A new CSCDL environment was created by combining the dynamic learning environment with collaborative learning. The goal here is for the whole team to work together to ensure that other team members learn the subject thoroughly. Students work for the team and teams work for their members. Finally, students take the exam individually. The individual progress score of each student in the group is determined. Team scores are obtained by averaging the progress scores of the students on each team. Thus, the student competes with himself (Açıkgöz, 1992; Slavin, 1994). It aims to develop learning tools and to be aware of the relationships between concepts in order to effectively support the learning of group members consisting of students (Buder, 2011).

Analytic geometry, which provides spatial thinking and schematic visualization, deals with the nature of two- and three-dimensional shapes, spatial concepts, and planar, deductive, and coordinate geometries (Chinnappan & Lawson, 2005). According to Young (1909), analytical geometry, which is the basic idea of graphically representing the relationship between two or more variables, is embedded in all professional fields. Many people find themselves seriously inadequate due to their lack of knowledge of analytic geometry, and the industrial world states that the solution to difficult problems cannot be overcome without the help of analytic geometry (Young, 1909). Pritt (2010) emphasized that thanks to analytic geometry, mathematicians can work in more dimensions than one can imagine. He also stated that scientists can solve the problems encountered in writing secret coded messages in cryptography, physicists can base their vector and analysis studies on analytical geometry, astronomers can check the point location of telescopes with the help of trigonometry, and biologists can solve equations obtained from theories by using spectroscopy (Pritt, 2010). What can be done to help students learn analytic geometry topics better? Considering that knowledge of analytic geometry is necessary for all professions where mathematics is required (Young, 1909), students need to learn analytic geometry in learning environments where they can create products thanks to technology and collaborative learning. Accordingly, the basic structure of this research is to create a useful and applicable dynamic learning environment in which collaborative teams and computer-assisted instruction are integrated into teaching analytic geometry to teachers and students.

The aim of this research is to observe the applicability of the CSCDL environment to the courses and to determine its effect on student success. In this context, this study seeks answers to the following questions:

1. What are the applicability situations of the CSCDL environment in teaching the "Lines" unit?
2. Is there a significant difference between the effect of teaching in the CSCDL environment and teaching using only dynamic materials on the interactive whiteboard on students' academic success and retention of the "Lines" unit?

METHOD

Research Design

The study used a mixed research method. This approach, which supports the research problems both quantitatively and qualitatively by not limiting the collection of data to a single type of method, provides a comprehensive picture by explaining the process of the research, its initial situation, and external influences during the process (McMillan & Schumacher, 2010). Accordingly, the study used embedded design, one of the mixed research approaches. In the embedded design, the researcher collects quantitative and qualitative data simultaneously or sequentially and explains the data they collect in a way that one supports the other. Supporting data can be quantitative or qualitative (Creswell, 2011).

For this study, the researcher preferred the embedded design since it aimed to examine the use of the CSCDL environment, which combines the computer-assisted mathematics teaching method and collaborative learning approach, on the subject of "Lines" through qualitative and quantitative data. This study including the Experiment 1 and Experiment 2 groups, collected quantitative data to determine the effect of CSCDL on students' success. The researcher collected qualitative data throughout the process to illustrate how CSCDL was implemented in the lessons. Therefore, this research revealed the effect of CSCDL on both the observation process and student achievement.

Study Group

The research group consisted of 68 high school students studying in Turkey and their mathematics teacher. The Experiment 1 group consisted of 35 students and the Experiment 2 group consisted of 33 students. The participants were chosen using the convenience sampling method. This sampling method is preferred because of students' convenient accessibility and proximity to the researchers (Yıldırım & Şimşek, 2011). The students in the study had not learned the GeoGebra software from any source before. Previously, the math teacher participated in the CSCDL workshop conducted by the researcher.

Data Collection Tools

Using the data obtained with different methods increases the reliability and validity of the results obtained (Yıldırım & Şimşek, 2011). Therefore, both qualitative and quantitative data collection tools were used in the research study. The study used the Lines Knowledge Test (LKT) and Observation Form as data collection tools. These are presented below.

Lines Knowledge Test

To determine the academic success of the students in the "Lines" unit taught in the research, the researcher prepared the LKT to be used in both experimental groups. The researcher developed this test to be used as a pre-test, post-test, and retention test. The researcher used the high school geometry textbooks (TMoNE (2013); Aksoy & Görçe, 2013) and the geometry course 9-10th grade curriculum (TMoNE (2010) in the development of the test. Care was given to ensure that the items in the knowledge test included the learning objective in the "Lines" unit, that is, to ensure content validity.

The researcher developed the test as 15 open-ended items and gave it to four researchers who are experts in mathematics field education, two doctoral students, one in mathematics education and the other in mathematics, and one mathematics teacher for review. The test, which was revised according to expert opinions, consisted of 14 open-ended items, and the time required for answering the test was determined as 50 minutes.

The pilot study of the prepared knowledge test was applied to 75 11th grade students. The pilot study was conducted in two high schools different from the high school where the research took place According to the data obtained here, Cronbach's α-reliability coefficient, which is the internal consistency coefficient of LKT, was 0.814. This indicates that the scale is highly reliable (Kalaycı, 2010). The study used the final version of the LKT.

Observation Form

In the study, all lessons were observed from the beginning to the end to reveal the applicability conditions of the CSCDL environment. The researcher created the observation form before the implementation process started. First of all, the researcher tried to determine which important situations would shape the characteristics of the CSCDL environment that was going to be observed in general terms by considering the relevant literature. Then they examined the existing literature and the mathematics curriculum (TMoNE (2013).

After the draft form was created, it was presented to two researchers who specialized in field education and one researcher who specialized in education. After reorganizing the form in line with the feedback received, an expert researcher in field education checked it. The form was then used in the pilot study. In the pilot study, the researcher observed 3 objectives in the "Coordinate Systems" unit taught through CSCDL for 4 lesson hours. During the pilot lessons, the researcher observed the lesson by being present among the students. The researcher tried to determine how the students constructed computer-assisted and collaborative learning among themselves and the situations they paid attention to. This way the researcher tried to analyze both the existing situations in the observation form and new situations that may occur. The researcher recorded the data observed for each objective on the form. During the lessons conducted in the pilot study, the observation form was updated and finalized. This study used this final version of the observation form. The observation form consisted of 34 codes under 9 categories, each of which indicated a behavior. The categories included in the form are "teacher's role", "team reward/joint reward", "positive dependency", "individual assessability", "face-to-face (supportive) interaction", "social skills", "assessment", "equal opportunity for success", and "learning environment". While creating the categories, it was taken into consideration that group work should have these characteristics (Açıkgöz, 1992) for it to be collaborative learning. In the creation of the codes, the researcher tried to combine both the features that should be under the relevant category and the dynamic learning features and thus create codes/behaviors that would serve as examples of collaborative dynamic learning.

Teaching the "Lines" Unit with CSCDL

To determine whether the applicability of teaching through CSCDL affects success and retention, the researcher selected the "Lines" unit from among the analytic geometry units. In the study, 4 objectives in the 10th grade geometry course "Lines" unit were taught with CSCDL and only using dynamic materials on the interactive whiteboard in 12 lesson hours. The 6-week implementation of the study took place with the course teacher and 35 Experiment 1 group students and 33 Experiment 2 group students.

In the two weeks before the implementation, the researcher met with the teacher of the course. They analyzed the methodology of the research and the equality between the Experiment 1 and Experiment 2 groups. For this purpose, the researcher analyzed the data obtained through the pre-tests, which were the Geometry Knowledge Test (GKT) and the LKT, which were conducted by the Provincial Directorate of National Education to determine the success of 10th grade students in the geometry course and which contained 15 multiple-choice items. The results of the Kolmogorov-Smirnov test were: ($p_{gkt\text{-}Experimental\text{-}I} < .05$; $p_{gkt\text{-}Experimental\text{-}II} > .05$; $p_{pretest\text{-}Experimental\text{-}I} < .05$; $p_{pretest\text{-}Experimental\text{-}II} < .05$). Measurement results of both groups have to exhibit a normal distribution to choose parametric tests (Büyüköztürk, 2010). Q-Q, histogram, box plot, detrended normality plot, kurtosis, and skewness values had the same trend as the Kolmogorov-Smirnov test. Therefore, the Mann–Whitney test was used for analyzing the quantitative data. According to Mann Whitney U-Test results ($U_{gkt}=473.500$, $p_{gkt} > .05$; $U_{lkt}=531.000$, $p_{lkt} > .05$), there was no significant difference. Based on these data, the class of 35 students was randomly selected as the Experiment 1 group and the class of 33 students was randomly selected as the Experiment 2 group. According to the study plan, in the Experiment 1 group, the lessons were based on CSCDL, while in the Experiment 2 group, the lessons were carried out using only dynamic materials on the interactive board.

Now it was time to form the collaborative teams of the Experiment 1 group. The course teacher determined the starting scores of the students in the Experiment 1 group by taking the average of the students' previous geometry exam grades and their scores on the GKT out of 100 points. Accordingly, they formed heterogeneous collaborative teams according to each student's initial score. They formed teams of 3 people. Accordingly, there was a total of 11 collaborative teams. When forming the teams, the teacher first ranked the students according to how high their initial scores were. The teams were numbered 1, 2,...,11, and the first 11 students with the highest success scores were placed in the 1st, 2nd,..., 11th teams respectively. Then, the second 11 students ranked in the initial score list were placed in the 11th, 10th,...,1st teams respectively, while the third 11 students were placed in the 1st, 2nd,...,11th teams respectively. The last two students remaining in the initial score list were placed in two teams with the appropriate average score. The LKT was administered to both experiment groups as a pre-test the week before the implementation. Both groups had 50 minutes for the knowledge test. Afterward, the students received a briefing about the teams. Students were given the CSCDL Team Study Guide. Students were asked to decide on the team name, team color, and team emblem. Before the start of the course, a poster describing the CSCDL environment prepared by the researcher was placed in the laboratory where the implementation would take place, and the GeoGebra software was installed on the computers and interactive board. Additionally, the researcher uploaded the materials related to the learning objective to the computers before the implementation each week. In the Experiment 2 group, the course teacher taught the objectives using only dynamic materials on the interactive whiteboard. Students did not use computers during the implementation, they learned the unit by listening to their teachers in their own classrooms and occasionally practicing the materials on the interactive board. In both test groups, the same course teacher taught the unit objectives in the same week and for the same duration. Every week the researcher and the course teacher reviewed the dynamic materials and worksheets to be used in teaching the learning objectives during the implementation period. In the Experiment 1 group, each team worked on its own computer and worksheet. Before teaching each learning objective, the teacher went over reminders for the students by using the materials on the interactive board from time to time, taking into account the readiness required for the objective, and initiated the collaborative work in the teams. After each lesson, the teacher collected the worksheets of the teams to check them and provide feedback. These worksheets were photocopied and the original copies were given to the researcher to use in the study, and the worksheets were returned to the teams the following day after examining the photocopies. This way the teams had the opportunity to re-evaluate the feedback they received. During the applications, follow-up tests were conducted after the teaching-learning of both learning objectives. These follow-up tests can be seen as small quizzes. Students participated in the follow-up tests individually. The researcher prepared the follow-up tests by examining the relevant books (TMoNE, 2010; Aksoy & Görçe, 2013). The teacher of the course examined the appropriateness of the follow-up test. Each test was completed in 15 minutes. The purpose of conducting follow-up tests is to determine the scores of the collaborative dynamic groups. Accordingly, two important scores stand out. The starting score and individual progress score. The starting score (SS) was determined as the students' previous geometry written grades. The individual progress score was determined by evaluating the results of the follow-up tests according to the starting scores. The individual progress score was determined as follows: If the result of the follow-up test was 10 points lower than the achievement score, 0 points were given, if 1-10 points were lower, 10 points were given, if 1-10 points were higher, 20 points were given, and if 10 or more points higher, 30 points were given. Accordingly, after the completion of the teaching of the learning objectives with CSCD, each team's score was calculated by averaging the individual progress scores of the students in the team and the successful team was determined. The teams were congratulated in the classroom for their success and the most successful team received a gift and a certificate. The researcher conducted 12 lesson hours of observations using the observation form in the Experiment 1 group. Also, they observed the lessons in the Experiment 2 group without a form. The teacher and students carried out the implementations and the researcher was present in the learning environment only to make observations. The practices were carried out with the following learning objectives:

1. **Parametric and closed equations of a line (Line equation with one known point and straight line vector, line equation with two known points, and line equation with one known point and normal vector).** These were taught through CSCDL in 4 lesson hours. Since one observation form was used to observe each two-hour lesson, 2 observation forms were used for these subjects. Thus, the "observation status" determined for the evaluated codes was calculated by taking the average of the two forms.

2. **The state of two lines according to one another.** Teaching this subject through CSCDL took 2 class hours.

3. **The slope of a line (The slope of a line and the angle between two lines).** This was taught through CSCDL in 4 lesson hours. Since one observation form was used to observe each two-hour lesson, 2 observation forms were used for these subjects. Thus, the "observation status" determined for the evaluated codes was calculated by taking the average of the two weeks.

4. **The distance of a point to a line (The distance of a point to a line, the distance between two parallel lines, and the geometric location of points equidistant from two parallel lines)**. These objectives were taught through CSCDL in 2 class hours.

The findings present the observation form data according to these objective numbers. The LKT was administered to both experimental groups as a post-test the week after completing the implementations. In addition, 8 weeks after the completion of the study, the LKT was applied to both groups as a retention test. The CSCDL process can be summarized as follows: (see Fig 1)

The course teacher develops dynamic materials and worksheets on the subject.
Determination of students' starting scores.
Formation of heterogeneous collaborative teams based on starting scores.
Informing students about the teams. Presenting the CSCDL Team Work Guide.
Teams agree on the team name, team color, and team emblem.
The course teacher ensures that the dynamic materials are loaded on the teams' computers and provides teams with the worksheets.
The course instructor creates an environment of readiness for the lesson using dynamic materials and initiates the collaborative work of the teams.
Each team works on its own computer and worksheet.
The teacher conducts individual follow-up tests at regular intervals.
Determination of individual progress score based on follow-up tests.
Determination of team score by averaging individual progress scores.
Announcing the successful team and congratulating them with certificates and small gifts.

Figure 1: The CSCDL process

Dynamic Materials and Worksheets

CSCDL was conducted with the 4 objectives mentioned above. Dynamic materials and worksheets that enable students to work in teams were developed for these objectives. Eleven dynamic materials and worksheets was created. An example from the worksheets which were used with the dynamic materials are given the Appendix.

While creating the materials and worksheets, the literature was examined and related books (TMoNE, 2013; Aksoy & Görçe, 2013; Çelik, 2013; Larson, Boswell, Kanold, & Stiff, 2004; Sullivan, 2008; Beecher, Penna, & Bittinger, 2007; Timmons, Johnson, & McCook, 2010; Lial, Hornsby, & McGinnis, 2012; Hungerford & Shaw, 2009; Boyd, Cummins, Malloy, Carter, & Flores, 2008; Burger, et al, 2008), the GeoGebra official website (www.geogebra.org), openly available dynamic materials.

The prepared material and worksheets were presented to a researcher specialized in field education on a computer and corrections were made with the feedback they gave. Additionally, before teaching the units, the researcher and the teacher worked on the materials and worksheets together and made arrangements where necessary.

Data Analysis

The researcher obtained observation data during the teaching of each objective with CSCDL according to the observation form created before the implementation of the study. Accordingly, data analysis occurred under the categories of learning environment, teacher behaviors, and student team behaviors. The study analyzed the observation results obtained from CSCDL environment under the subcategories of "learning environment"; "teacher's role" and "individual assessability"; "team reward/joint reward", "positive dependency", "face-to-face (supportive) interaction", "social skills", "assessment", and "equal opportunity for success". The researcher observed the "Observation Status Coding" for each code under these defined categories during the lesson. Accordingly, a value of "0" was given when the defined behavior was not performed in the classroom environment, "1" when the defined behavior was performed slightly, and "2" when the defined behavior was performed in accordance with the CSCDL environment. The researcher observed the behaviors defined in each code separately for the teams and determined the observation status of each team. The teams were numbered and the same numbered team was observed in each lesson. The observation states obtained were added together for each behavior and divided by the number of teams. This way, for each code, an observation situation was identified in which each team was observed in the classroom and each team was evaluated. Accordingly, the researcher interpreted the data according to the situation defined by the "0", "1", and "2" values in the "Observation Status Coding". An observation form was used to observe each two-hour lesson. For this reason, when more than one observation form was used for a learning objective, the "observation status" for the evaluated codes was determined by taking the weighted average of the observations.

The evaluation of the 14 open-ended items in the LKT, from which the researcher obtained the experimental data of the study, occurred based on the levels of correct, partially correct, and incorrect. A score of "2" was given for a correct answer, "1" for a partially correct answer, and "0" for an incorrect or blank answer. Accordingly, the highest score that one can obtain from the test is 28 and the lowest score is 0. The SPSS 16.0 package program was used for the analysis of the data obtained from the knowledge test. To determine the test to be used in analyzing the data, the researcher first performed a normality analysis of the data. Since the sample numbers were 35 and 33, a Kolmogorov-Smirnov test was used for the normality analysis (Kalaycı, 2010). In addition, Q-Q, histogram, box plot, detrended normality plot, kurtosis, and skewness values were examined to see whether the data distribution was normal (Field, 2009; Kalaycı, 2010). The results of the Kolmogorov-Smirnov test were as follows ($p_{posttest\text{-}Experimental\text{-}I} > .05$; $p_{posttest\text{-}Experimental\text{-}II} < .05$; $p_{retention\text{-}Experimental\text{-}I} > .05$; $p_{retention\text{-}Experimental\text{-}II} < .05$). In addition, the Q-Q, box and whisker plots, detrended normality plot, kurtosis and skewness values were analyzed to determine whether the measurement results exhibited a normal distribution or not (Field, 2009). Measurement results of both groups have to exhibit a normal distribution to choose parametric tests (Büyüköztürk, 2010). Therefore, the Mann–Whitney test was used for analyzing the quantitative data. The study accepted $\alpha=0.05$ as the significance level, which is the most commonly used level in educational studies. The provincial Directorate of National Education gave the results of the GKT used in the study to the high school directorate where the study was conducted. Since the items in the test are multiple-choice, the number of correct and incorrect answers for each student is evident. Accordingly, the researcher determined the geometry scores of each student based on the results obtained to be used in the study. While determining the net score, the basis was that 4 wrong answers eliminated one right answer.

RESULTS

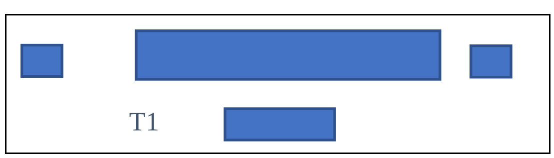
This heading presents the findings obtained from teaching the "Lines" unit in the CSCDL environment.

Table 1 and Figure 2, present the results of the observations regarding the physical environment and layout of the classroom where the "Lines" unit was taught with CSCDL.

The Physical Environment of the Classroom
• The study took place in the information technologies classroom.
• There are 11 teams in the class.
• Two teams consist of 4 people, and the other teams consist of 3 people.
• There is 1 interactive board and 11 computers in the classroom.
• Each team works with 1 computer on a work desk and worksheets.
• The temperature and lighting in the classroom are sufficient.
• The size of the classroom is not sufficient for the number of teams.

Table 1: Observation results regarding the physical environment of the classroom

The figure presents the layout of the teams.



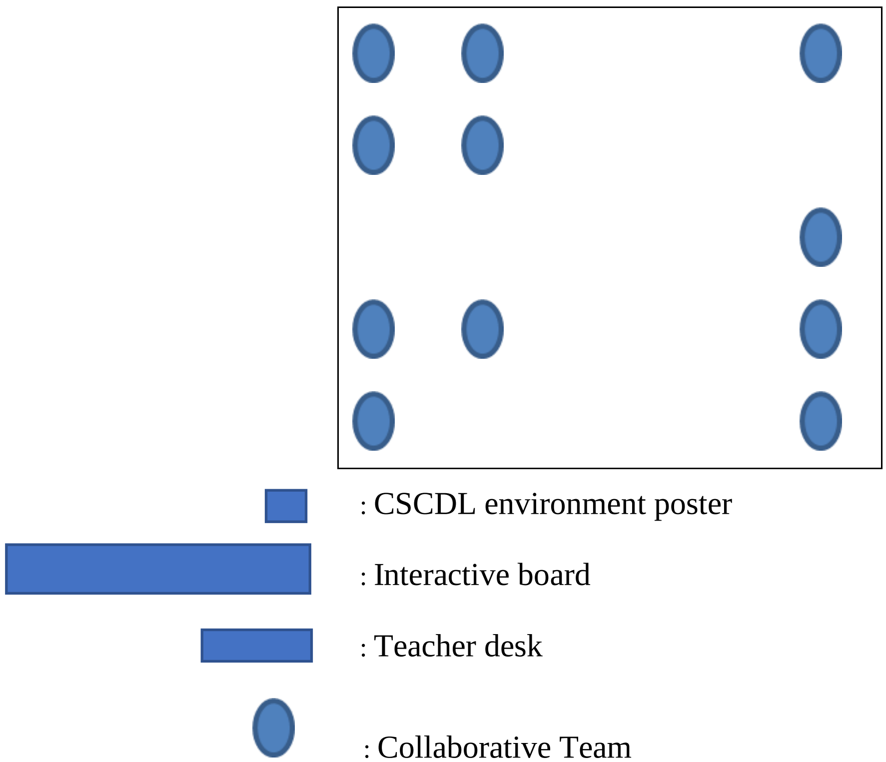


Figure 2: The Layout of the teams

		Behavior	1	2	3	4
Learning Environment	**Learning Environment**	Computers were used effectively in the creation of the CSCDL environment.	1.5	2	2	2
		The interactive whiteboard was used effectively to create the CSCDL environment.	1.5	2	2	2
		Collaborative teams worked effectively in building the CSCDL environment.	1	2	2	2
Teacher Behavior	**Role of the Teacher**	They create a learning environment that enables active participation in accordance with students' levels and interests.	1	2	2	2
		They give students hints instead of giving them information directly.	1.5	2	2	2
		They provide feedback to support learning.	2	2	2	2
		They enable students to share with others.	1.5	2	2	2
		They enable students to make active use of interactive whiteboards and dynamic materials.	1.5	2	2	2

		The teacher facilitates and accelerates the exercises.	0.5	2	2	2
		They question the appropriate level of readiness that involves the objective.	1	2	2	2
		They remind students of the basic concepts and terms related to the subject.	1	2	2	2
		They use dynamic materials effectively on the interactive whiteboard.	1	2	2	2
	Individual Assessability	In the dynamic environment, every student participates in the lesson.	1	2	2	2
		Each student's level of success in generalizations is monitored.	1.5	2	2	2
Student Team Behaviors	Team Award\Joint Award	Team members work to increase team success (a collaborative work structure).	0.95	1.4	1.4	1.5
		Team members can use dynamic materials to reach the generalizations on the worksheets (collaborative reward structure)	1.2	1.5	1.3	1.5
		Team members can to identify algebraic expressions of generalizations using the materials.	0.9	1.3	1.3	1.5
	Positive Dependency	Team members fulfill their responsibilities.	1.35	1.6	1.35	1.5
		Team members act as if they are responsible for each other's learning.	1.35	1.5	1.3	1.5
		Team members use dynamic materials and worksheets effectively.	1.1	1.6	1.35	1.5
		Team members act in cooperation and unity.	1	1.5	1.35	1.5
	Face-to-face (Supportive) Interaction	Team members interact face-to-face (helping).	1.2	1.5	1.35	1.5
		Team members encourage each other (trust).	1.15	1.5	1.4	1.5
		They identify parts of the topic that are not clear (giving feedback).	1.05	1.2	1.15	1.5
		Team members explain the use of dynamic materials to each other.	1.3	1.4	1.25	1.7
		Students correct each other's mistakes.	1.15	1.3	1.15	1.5
		Team members discuss and examine the dynamic materials together and solve the problems they encounter in the generalizations.	0.95	1	1.1	1.5
	Social Skills	Team members stay together for a while before the activities start.	1.35	1.4	1.35	1.4
		Team members can build good relationships with each other.	1.45	1.5	1.35	1.5
		Team members listen to each other.	1.4	1.5	1.35	1.5

Evaluation	Team members evaluate their generalizations at the end of the activity.	1.1	1.4	1.2	1.3
	Team members examine the generalization obtained at the end of the activity together with other teams.	1.2	1.5	1.2	1.5
Equal Opportunity for	Team members improve themselves and thus contribute to generalizations.	1.05	1.5	1.15	1.3
	The contribution of each student is taken into account when making generalizations.	1.1	1.4	1.25	1.5

Observation Status Coding: 0: The described behavior was not performed in the classroom environment, 1: The described behavior was performed partially, 2: The defined behavior was performed in accordance with the CSCDL environment.

Table 2: Observation of applicability conditions of the CSCDL environment

Upon examining Table 2 and the observation data obtained from the teaching of the "Lines" unit in the CSCDL environment, the learning environment and the behaviors of the course teacher were suitable for the CSCDL environment after the first 4 lessons, and the behaviors of the students forming the teams developed in accordance with the CSCDL environment.

This section presents the findings obtained for the statement "Is there a significant difference between the effect of teaching in the CSCDL environment and teaching using only dynamic materials on the interactive whiteboard on students' academic success and retention in the "Lines" unit?"

Table 3 presents the results of the Mann-Whitney U-Test used to determine whether there was a difference between the post-test scores of the Experiment 1 and Experiment 2 groups.

Group	n	Rank Mean	Rank Sum	U	p
Experiment 1	35	45.76	1601.50	183.500	.000
Experiment 2	33	22.56	744.50		

Note: The maximum score for this test is 28.

Table 3: The Mann Whitney U-Test results of post-test scores of the groups

Upon examining Table 3, there was a significant difference between the post-test scores of the Experiment 1 and Experiment 2 group students in favor of the Experiment 1 group ($U=183.500$, $p< .05$).

Table 4 presents the results of the Mann-Whitney U-Test used to determine whether there is a difference between the retention test scores of Experiment 1 and Experiment 2 groups.

Group	n	Rank Mean	Rank Sum	U	p

Experiment 1	35	47.40	1659.00	126.00	.000
Experiment 2	33	20.82	687		

Note: The maximum score for this test is 28.

Table 4: The Mann Whitney U-Test results of retention test scores of the groups

Upon examining Table 4, there was a significant difference between the retention test scores of the Experiment 1 and Experiment 2 group students in favor of the Experiment 1 group ($U=126.00$, $p< .05$).

As a result, it became evident that teaching the subject of "Lines" through CSCDL positively affected students' achievement and ensured the retention of their learning.

DISCUSSION AND CONCLUSION

The study analyzed the observation results obtained from teaching the "Lines" unit in the CSCDL environment under the subcategories of learning environment, teacher behaviors, and student team behaviors.

Accordingly, in the learning environment, in the teaching of the first learning objective of the unit through CSCDL for 4 lesson hours, it was observed that the collaborative teams were working tangentially, and the computer and interactive board were used a little more effectively in the CSCDL environment. Upon examining the results observed in the learning environment in the teaching of the other three objectives, the researcher observed the behaviors of the collaborative teams were carried out in accordance with the CSCDL environment and that the teams and the course teacher were more effective. Accordingly, one can say that the teams and the course teacher behaved in accordance with the CSCDL environment after the first 4 lessons.

According to the analysis of the teacher behaviors, some of the teacher behaviors observed in the 4 lesson hours of the first learning objective of the unit were tangential and some of them were close to being suitable for CSCDL. Additionally, it was observed that in the first 4 lessons, the teacher gave feedback to support learning in accordance with the CSCDL environment, but did not play a role in accelerating and facilitating the work. Upon examining the results observed in teacher behaviors in the teaching of the other three objectives, it was determined that all behaviors were performed in accordance with the CSCDL environment. Accordingly, one can say that the course teacher acted in accordance with the CSCDL environment after the first 4 lessons in teaching this unit.

Upon examining the student team behaviors separately for each objective, the researcher observed that the teams performed most of the behaviors tangentially in teaching parametric and implicit equations of a line. While it can be said that the teams' ability to reach the generalizations in the worksheets was tangential, their ability to define the algebraic expressions of the generalizations using the materials and to solve the problems they encountered in the generalizations was close to tangential. Accordingly, one can say that the teams were able to reach generalizations in teaching parametric and implicit equations of a line, but they had difficulty in defining algebraic expressions of generalizations.

The teams performed some behaviors tangentially during the teaching of the states of two lines with respect to each other, and at the same time, the teams were close to performing most of the behaviors in accordance with CSCDL. Accordingly, one can say that the teams behaved in a manner that was close to being in accordance with CSCDL when learning the states of two lines with respect to each other. Additionally, the level of teams performing all behaviors in accordance with CSCDL increased compared to the previous objective.

It was concluded that the teams performed all of the behaviors partially in teaching the slope of a line. Accordingly, one can state that the teams acted less in accordance with CSCDL in the teaching of this objective compared to the previous objective. However, the teams had the same level of reaching the generalizations in the worksheets and defining the algebraic expressions of the generalizations using the materials. While the level of the teams' ability to reach the generalizations in the worksheets by using dynamic materials decreased compared to the previous learning objective, the level of defining the algebraic expressions of the generalizations did not change.

In the teaching of the distance of a point to a line, the teams performed only three behaviors partially, and most of the behaviors were close to being performed in accordance with CSCDL. Accordingly, one can state that in teaching the distance of a point to a line, the teams were close to acting in accordance with CSCDL. Additionally, the level of teams performing all behaviors in accordance with CSCDL increased compared to the previous objective.

Accordingly, based on the general evaluation of the observation results obtained from teaching the "Lines" unit in the CSCDL environment, the learning environment and the behaviors of the course teacher were suitable for the CSCDL environment after the first 4 lessons and the behaviors of the students forming the teams developed in accordance with the CSCDL environment.

There was a significant difference in favor of the Experiment 1 group between the post-test scores of the Experiment 1 and Experiment 2 group students who were taught using CSCDL and dynamic material only on the interactive board in the "Lines" unit ($U=183.500$, $p< .05$). Accordingly, one can derive that teaching through CSCDL in the "Lines" unit had a more positive effect on the success of high school students. This result is similar to the result of Takači, Stankov, and Milanovic, (2015) who found that the success of the students who applied computer-supported collaborative learning using GeoGebra on the subject of analyzing and graphing functions was higher than the success of the students who only applied collaborative learning. This result is in line with the results of Chiu, Kessel, Moschkovich, and Munoz-Nunez, (2001) and Moschkovich (1999) who found that the use of software is more effective in the subject of lines.

The study determined that there was a significant difference between the retention test scores of the Experiment 1 and Experiment 2 group students in favor of the Experiment 1 group in the teaching of the "Lines" unit ($U=126.00$, $p<.05$). Accordingly, one can derive that the CSCD teaching in the "Lines" unit enabled the retention of high school students' learning. Birgin and

Topuz, (2021) found that this environment increased seventh grade students' geometry achievement.

This result is in line with the result of Takači, Stankov, and Milanovic's (2015) study in which they determined that the retention of the learning of the students who used computer-supported collaborative learning using GeoGebra was higher than the learning of the students who only used collaborative learning. Eshuis, Vrugte Anjewierden, Bollen, Sikken, & Jong, (2019) showed that students from the instruction with tool condition out performed the other students as far as their collaborative behavior and their domain knowledge gains. In addition, this result is similar to the result of Ubuz, Üstün, and Erbaş (2009). They found that the retention of seventh-grade students' learning of the concepts of lines, angles, and polygons when they used Geometer's Sketchpad, a dynamic geometry software in the computer laboratory, was higher than traditional learning. This research shows the methods for evaluating CSCDL in the classrooms and the observation form contains CSCDL components in the classroom. Working with scripted activities, students' self- and shared regulation are often limited to understanding the requirements, dividing up the given tasks, and meeting the requirements (Rogat & Linnenbrink-Garcia, 2011); rarely do they have the chance to make transformative changes in inquiry directions and group structures based on emergent interests (Tao & Zhang, 2021). Considering that mathematical expressions contain variables and that students need to learn different situations and features within each concept, the importance of integrating dynamic learning in CSCL environments emerges. According to this, if it is desired to explain the CSCDL revealed in this research; it can be defined as a learning method in which students form collaborative teams and reach generalizations of the definitions and concepts expected from them thanks to the dynamic mathematics software they use on computers, and their teachers guide them with clues. It is hoped that CSCDL will be increased learning success in the classrooms.

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Ethical Statement

Ethical guidelines in this study were observed by the researchers. From the initial and implementation phase up to the final writing phase, ethical considerations were strictly followed. Securing necessary permits to observe the clasrooms and implement the lines knowledge test from respondents underwent proper procedure through the assistance of the Turkish Ministry of National Education. The researcher keeps her documents and permissions and can submit the permission documents at any time. During the conduct of the study in this CSCDL environment, observations made orientation on the nature, purpose, and objectives of the study as well as their roles. The students and teacher were not forced to implement the environment.

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Appendix

Worksheet

Objective: The slope of a line. Tools Used: Computer and GeoGebra software

Make sure each of your teammates has learned the subject. You can get help from your teacher.

Group name: ....

Group members:

1)…....................................... 2)... 3)...

Open the slope of a line.ggb file saved on the desktop.

1. Drag the A and B points on the d line on the screen to get the desired values below. (4 different examples are presented consecutively on this tab. But here is one given.)

Example-1

- for the d line passing through the **A = (0, 0) and B= (4, 2)** points
  - ✓ Find the coordinates of the other F, G and D points on the line.
  - ✓ Write **the number** of **horizontal units** (increasing or decreasing according to right or left progression) and **vertical units** (ascending or descending according to progress up or down) from A to B?
  - ✓ Write the number of horizontal and vertical units for the other desired points in the table?

	A=(0, 0) and B=(4, 2)	**B=(4, 2) and F=(..., ...)**	**F=(..., ...) and G=(..., ...)**	**D=(..., ...) and A=(..., ...)**
Horizontal progression				
Vertical progression				
$\frac{\text{Vertical progression}}{\text{Horizontal progression}}$				

What can you say about the ratio of vertical advances to horizontal advances that you found for each pair of points on the line?

....
....

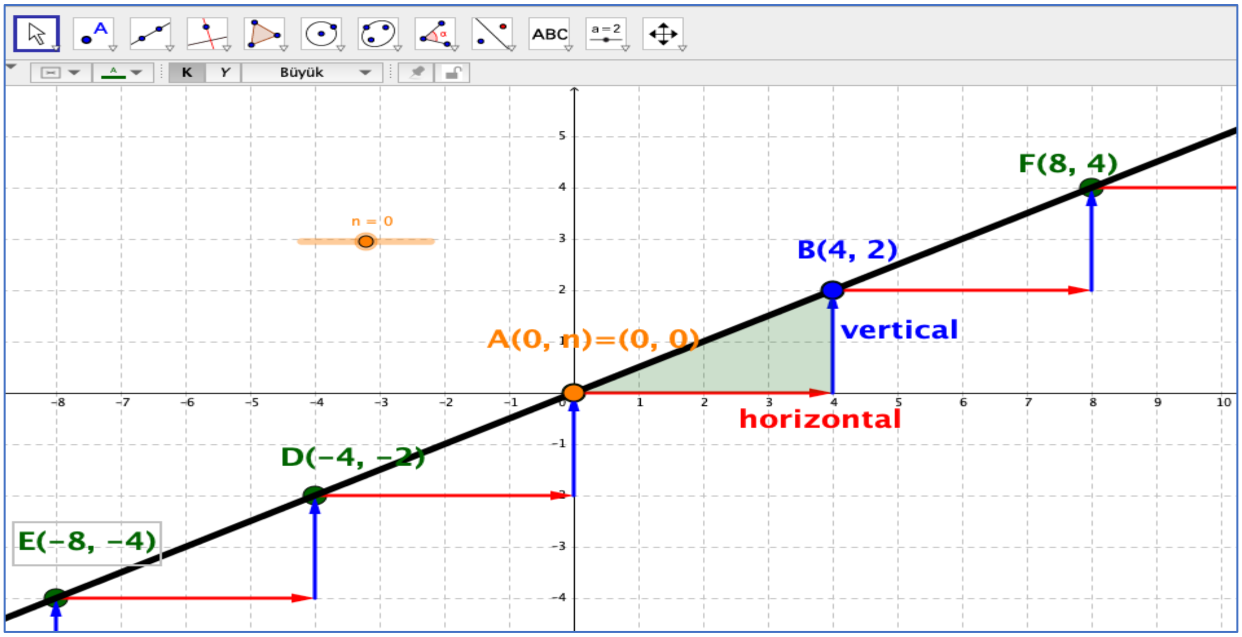
2. According to this, the ratio of vertical advances on the line to horizontal advances is .... to each other. This ratio is called the slope of the line. It is denoted by m.

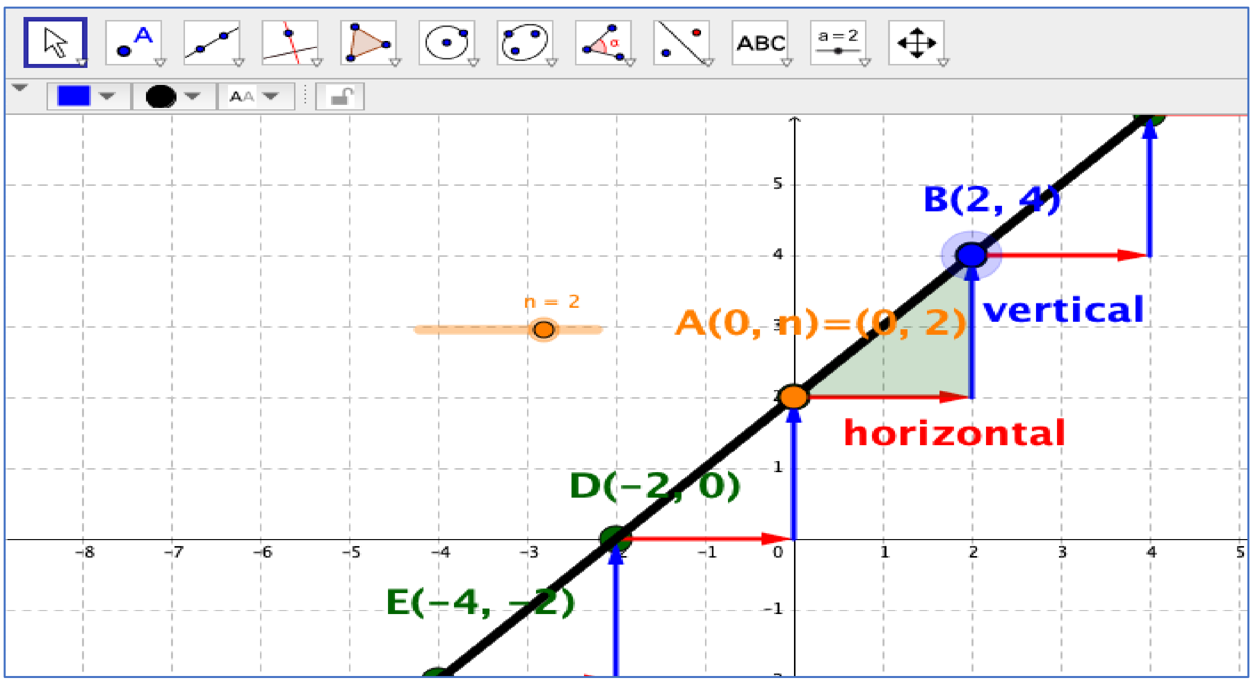
3. Accordingly, let's take the points A= (x_1, y_1) and B= (x_2, y_2) as any two points on a line. Explain how the slope of the line is calculated with the help of these points?

....

Dynamic Material

Screenshots of the dynamic material that are used in worksheet are given below.


Büyük
n = 0
F(8, 4)
B(4, 2)
vertical
A(0, n)=(0, 0)
horizontal
D(−4, −2)
E(−8, −4)


ABC
a = 2
n = 2
B(2, 4)
vertical
A(0, n)=(0, 2)
horizontal
D(−2, 0)
E(−4, −2)

The Problem Corner

Ivan Retamoso, PhD, *The Problem Corner* Editor

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The Purpose of **The Problem Corner** is to give Students and Instructors working independently or together a chance to step out of their "comfort zone" and solve challenging problems. Rather than in the solutions alone, we are interested in methods, strategies, and original ideas following the path toward figuring out the final solutions. We also encourage our Readers to propose new problems. To submit a solution, type it in Microsoft Word, using math type or equation editor, however PDF files are also acceptable. Email your solution as an attachment to The Problem Corner Editor iretamoso@bmcc.cuny.edu stating your name, institutional affiliation, city, state, and country. Solutions to posted problem must contain detailed explanation of how the problem was solved. The best solution will be published in a future issue of MTRJ, and correct solutions will be given recognition. To propose a problem, type it in Microsoft Word, using math type or equation editor, email your proposed problem and its solution as an attachment to The Problem Corner Editor iretamoso@bmcc.cuny.edu stating your name, institutional affiliation, city, state, and country.

Greetings, fellow problem solvers!

As the editor of **The Problem Corner**, I'm delighted to announce that I've successfully obtained answers for both Problem 20 and Problem 21. I'm pleased to report that all solutions were not only accurate but also showcased the effective application of strategies. My main goal is to present what I consider the best solutions to contribute to the enhancement and elevation of mathematical knowledge within our global community.

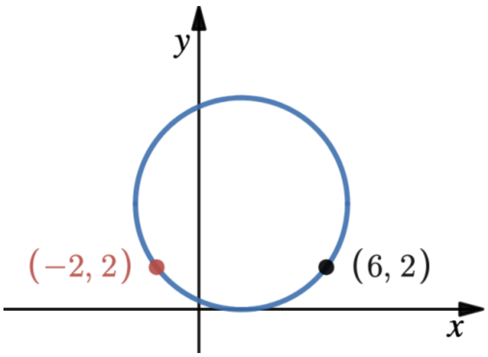
Solutions to **Problems** from the Previous Issue.

Engaging "Circle" puzzle.

Problem 20

Proposed by Ivan Retamoso, BMCC, USA.

Find the radius and the equation of the circle shown below.

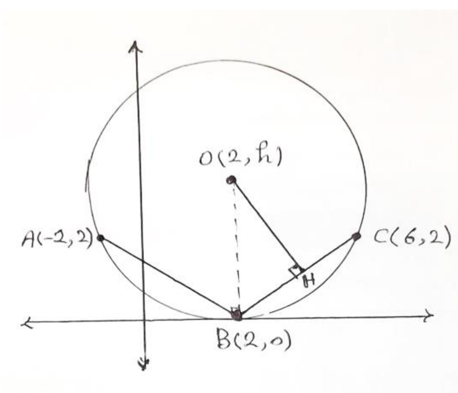


First solution to problem 20

By Dr. Hosseinali Gholami, University Putra Malaysia, Serdang, Malaysia.

This solution stands out for its remarkable simplicity and elegance, enjoy it.

We consider the following shape based on the given information and some geometric theorems.



$H(4, 1)$ is the middle point of BC.

$$a_{BC} = \frac{y_C - y_B}{x_C - x_B} = \frac{2-0}{6-2} = \frac{1}{2} \rightarrow a_{OH} = -2$$

Therefore, the equation of OH is as below.

$$y = ax + b \rightarrow y = -2x + b$$

The coordinates of $H(4, 1)$ satisfy in the equation OH, hence we have:

$$1 = -2(4) + b \rightarrow b = 9 \rightarrow y = -2x + 9$$

The coordinates of the center $O(2, h)$ satisfy in the equation OH.

$$h = -2(2) + 9 \rightarrow h = 5$$

It means $O(2, 5)$ is the center of the circle.

The value of radius is calculated as follows:

$$r = OC = \sqrt{(x_C - x_O)^2 + (y_C - y_O)^2} = \sqrt{(6-2)^2 + (2-5)^2} = 5$$

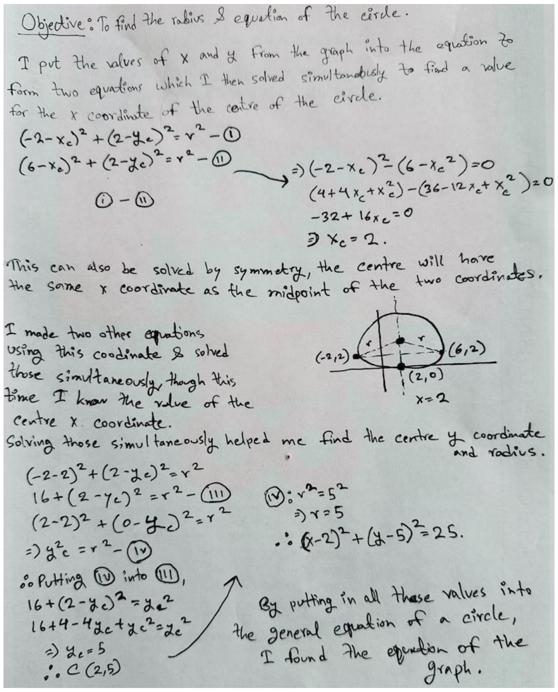
Therefore, according to the standard form of circle equation, the equation of the given circle is as follows:

$(x - 2)^2 + (y - 5)^2 = 25.$

Second solution to problem 20

By Abir Mahmood, Borough of Manhattan Community College, Queens, USA.

Our solver employs symmetry to its fullest extent, employing various but equivalent methods to derive the equation of the circle. Notably, it offers both algebraic and geometric solutions in parallel, thereby enhancing the overall depth and richness of the solution.



Objective: To find the radius & equation of the circle.

I put the values of x and y from the graph into the equation to form two equations which I then solved simultaneously to find a value for the x coordinate of the centre of the circle.

$(-2-x_c)^2+(2-y_c)^2=r^2$ — (i)

$(6-x_c)^2+(2-y_c)^2=r^2$ — (ii)

(i) − (ii)

$\Rightarrow (-2-x_c)^2-(6-x_c{}^2)=0$

$(4+4x_c+x_c^2)-(36-12x_c+x_c^2)=0$

$-32+16x_c=0$

$\Rightarrow x_c=2.$

This can also be solved by symmetry, the centre will have the same x coordinate as the midpoint of the two coordinates.

(−2,2) (6,2) (2,0) x=2

I made two other equations using this coodinate & solved those simultaneously, though this time I know the value of the centre x coordinate.

Solving those simultaneously helped me find the centre y coordinate and radius.

$(-2-2)^2+(2-y_c)^2=r^2$

$16+(2-y_c)^2=r^2$ — (iii)

$(2-2)^2+(0-y_c)^2=r^2$

$\Rightarrow y_c^2=r^2$ — (iv)

$\therefore$ Putting (iv) into (iii),

$16+(2-y_c)^2=y_c^2$

$16+4-4y_c+y_c^2=y_c^2$

$\Rightarrow y_c=5$

$\therefore C(2,5)$

(iv): $r^2=5^2$

$\Rightarrow r=5$

$\therefore (x-2)^2+(y-5)^2=25.$

By putting in all these values into the general equation of a circle, I found the equation of the graph.

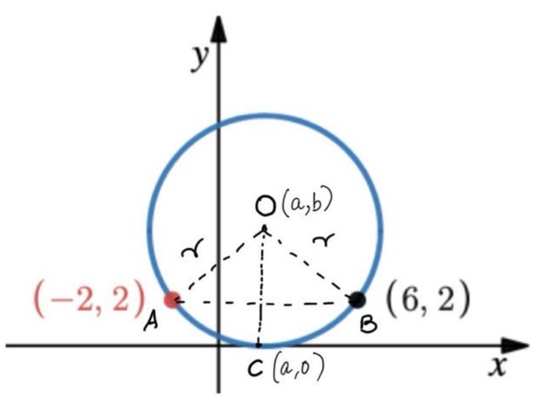
Third solution to problem 20

By Dr. Aradhana Kumari, Borough of Manhattan Community College, USA.

Our alternate solution ingeniously employs the circle equation and the distance formula to first derive the coordinates of the circle's center and its radius. Subsequently, the circle's equation falls into place effortlessly, well done!

Solution: Consider the Circle with center O, radius r as shown below. Let (a, b) be the coordinates of the center.

Hence the equation of the circle is $(x-a)^2+(y-b)^2=r^2$



Since Point A (-2, 2) and point B (6,2) lie on the circle hence we have

$(-2-a)^2 + (2-b)^2 = r^2 = (6-a)^2 + (2-b)^2$

Hence $(-2-a)^2 + (2-b)^2 = (6-a)^2 + (2-b)^2$

$(-2-a)^2 = (6-a)^2$

$4 + a^2 + 4a = 36 + a^2 - 12a$

$4 + 4a = 36 - 12a$

$4a + 12a = 32$

$16a = 32$

$a = \frac{32}{16} = 2$

Hence the x −coordinate of the center is 2.

Draw a line passing from the center $0(2, b)$ of the circle and perpendicular to $x - axis$, this line will meet $x - axis$ at a point, call this point C. The coordinate of point C is (2,0).

Point C (2,0) lies on the circle hence

$(2-2)^2 + (0-b)^2 = r^2 = (-2-a)^2 + (2-b)^2 = (-2-2)^2 + (2-b)^2$

$(2-2)^2 + (0-b)^2 = (-2-2)^2 + (2-b)^2$

$b^2 = 16 + 4 + b^2 - 4b$

$4b = 16 + 4 = 20$

$4b = 20$

$b = 5$

$r = \sqrt{(6-2)^2 + (5-2)^2} = \sqrt{16+9} = 5$

Hence the coordinate of the center of the circle is (2, 5), the radius of the circle is 5 and the equation of the circle is $(x-2)^2 + (y-5)^2 = 5^2$.

Problem 20 was also solved correctly by **Jahidul Islam, Borough of Manhattan Community College, Bangladesh.**

"Tricky" algebraic problem.

Problem 21

Proposed by Ivan Retamoso, BMCC, USA.

Solve the equation below to find all real numbers x that satisfy:

$$\frac{8^x + 27^x}{12^x + 18^x} = \frac{7}{6}$$

First solution to problem 21

By Dr. Aradhana Kumari, Borough of Manhattan Community College, USA.

This well-structured solution first identifies potential solutions through careful inspection. Subsequently, it employs a rigorous methodology, including a clever variable transformation, to systematically derive the final two solutions.

Solution: Consider the below equation

$\frac{8^x+27^x}{12^x+18^x} = \frac{7}{6}$ ....(1)

Just as a trail let's substitute $x = 1$ in the left-hand side of the above equation we get

$$\frac{8^1+27^1}{12^1+18^1} = \frac{35}{30} = \frac{7}{6}$$

Hence $x = 1$ is a solution for the above equation.

Let's substitute $x = -1$ in the left-hand side of the above equation we get

$$\frac{(8)^{-1}+(27)^{-1}}{(12)^{-1}+(18)^{-1}} = \frac{\frac{1}{8}+\frac{1}{27}}{\frac{1}{12}+\frac{1}{18}} = \frac{\frac{(27+8)}{(8\times 27)}}{\frac{18+12)}{(12\times 18)}} = \frac{\frac{35}{216}}{\frac{30}{216}} = \frac{35}{30} = \frac{7}{6}$$

Hence $x = -1$ is a solution for the above equation.

Next, we will prove that $x = 1$ or $x = -1$ are the only solutions for the above equation given by (1).

$$\frac{8^x + 27^x}{12^x + 18^x} = \frac{7}{6}$$

$$\frac{(2^3)^x + (3^3)^x}{6^x[2^x + 3^x]} = \frac{7}{6}$$

$\frac{(2^3)^x+(3^3)^x}{(2^x3^x)[2^x+3^x]} = \frac{7}{6}$ ....(2)

Let $2^x = A$, $3^x = B$

Hence equation given by (2) becomes

$\frac{A^3+B^3}{AB[A+B]} = \frac{7}{6}$

$$\frac{(A+B)^3 - 3AB(A+B)}{AB[A+B]} = \frac{7}{6}$$

$$\frac{(A+B)[(A+B)^2 - 3AB]}{AB[(A+B)]} = \frac{7}{6}$$

$$\frac{[(A+B)^2 - 3AB]}{AB} = \frac{7}{6}$$

$$\frac{A^2 + B^2 + 2AB - 3AB}{AB} = \frac{7}{6}$$

$$\frac{A^2 + B^2 - AB}{AB} = \frac{7}{6}$$

$6(A^2 + B^2 - AB) = 7AB$

$6A^2 + 6B^2 - 6AB = 7AB$

$6A^2 + 6B^2 - 6AB - 7AB = 0$

$6A^2 + 6B^2 - 13AB = 0$

$6A^2 - 13AB + 6B^2 = 0$

$6A^2 - 9AB - 4AB + 6B^2 = 0$

$3A(2A - 3B) - 2B(2A - 3B) = 0$

$(3A - 2B)(2A - 3B) = 0$

$(3A - 2B) = 0$ or $(2A - 3B) = 0$

$3A = 2B$ or $2A = 3B$

$3\ 2^x = 2\ 3^x$ or $2\ 2^x = 3\ 3^x$

$\frac{3}{2} = \left(\frac{3}{2}\right)^x$ or $\left(\frac{2}{3}\right)^x = \frac{3}{2} = \left(\frac{2}{3}\right)^{-1}$

Hence $x = 1$ or $x = -1$

Second solution to problem 21

By Jahidul Islam, Borough of Manhattan Community College, Bangladesh.

The alternative solution avoids finding solutions through inspection and instead, skillfully rewrites the powers of 2 and 3 before proceeding directly to a change of variable. This strategic maneuver ultimately reveals the two possible solutions.

here, $\frac{8^x+27^x}{12^x+18^x} = \frac{7}{6}$

$\Rightarrow \frac{(2^3)^x+(3^3)^x}{(3\cdot 2^2)^x+(2\cdot 3^2)^x} = \frac{7}{6}$

$\Rightarrow \frac{2^{3x}+3^{3x}}{3^x\cdot 2^{2x}+2^x\cdot 3^{2x}} = \frac{7}{6}$

Let, $2^x = m$ and $3^x = n$.

$\therefore \frac{m^3+n^3}{m^2\cdot n+mn^2} = \frac{7}{6}$

$\Rightarrow \frac{(m+n)(m^2-mn+n^2)}{mn(m+n)} = \frac{7}{6}$

$\Rightarrow \frac{m^2-mn+n^2}{mn} = \frac{7}{6}$

$\Rightarrow 6m^2-6mn+6n^2 = 7mn$

$\Rightarrow 6m^2-13mn+6n^2 = 0$

$\Rightarrow 6m^2-9mn-4mn+6n^2 = 0$

$\Rightarrow 3m(2m-3n)-2n(2m-3n) = 0$

$\Rightarrow (2m-3n)(3m-2n) = 0$

$2m-3n = 0$

$\Rightarrow 2m = 3n$

$\Rightarrow \frac{m}{n} = \frac{3}{2}$

$\Rightarrow \frac{3^x}{2^x} = \frac{3}{2}$

$\Rightarrow \left(\frac{3}{2}\right)^x = \frac{3}{2}$

$\Rightarrow x = 1$

$3m-2n = 0$

$\Rightarrow 3m = 2n$

$\Rightarrow \frac{m}{n} = \frac{2}{3}$

$\Rightarrow \frac{3^x}{2^x} = \frac{2}{3}$

$\Rightarrow \left(\frac{3}{2}\right)^x = \left(\frac{3}{2}\right)^{-1}$

$x = -1$

$\therefore x = 1, -1$

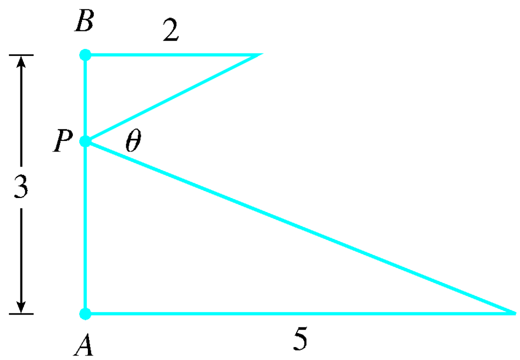
Dear fellow problem solvers,

I'm confident that you had an enjoyable experience solving problems 20 and 21, and you've gained new strategies for your mathematical repertoire. Now, it's time to move on to our next two problems to keep improving.

Problem 22

Proposed by Ivan Retamoso, BMCC, USA.

In the illustration below, at what distance from B should point P be positioned to maximize the angle θ?



Problem 23

Proposed by Ivan Retamoso, BMCC, USA.

Calculate the radius of the circle in which an isosceles triangle, with a base of 24 inches and legs each measuring 15 inches, is inscribed.