Characterization of Primary School Students' Perceptions in Understanding Negative Integer

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***Abstract**: Students' understanding of negative integers is an important aspect of learning mathematics as it is a requirement for learning broader mathematical concepts. The purpose of this research is to explore students' initial perceptions in understanding negative integers on a number line. This descriptive exploratory qualitative research was conducted on fifth grade elementary school students. The subjects were selected based on students' initial perceptions of writing negative integers on a number line before the teacher presented the material. Students' initial perceptions can be seen from the exploration results of their thinking abilities. The data of this research was collected through written tests and interviews. From the exploration results of research subjects, it was found that there were three characteristics of students' thinking perceptions in understanding the concept of negative integers. These three characteristics are: assume there are no negative numbers; unary understanding; and pseudo understanding. From these findings, it is necessary to design learning which provide scaffolding for students to avoid errors in understanding whole numbers.*

Keywords: Characteristics, Negative Integers, Perception, Students

INTRODUCTION

Students' understanding of negative integers is very important to be studied as it is the basis for understanding broader mathematical concepts. This is in line with the results of previous research which states that students' understanding of the concept of numbers is very necessary in solving proportional problems (Parameswari et al., 2023). Furthermore, students' inability to understand integer can be an obstacle in completing tests related to fractional numbers (Susiswo et al., 2021).

The presence of these obstacles resulted in a perception (Ridwan et al., 2022) related to the concept of numbers.

Bishop et al., (2015) revealed several challenges experienced by students in understanding negative integers: First, students develop habit based on initial mathematical experience where integers are used to calculate real objects which made them see numbers only from the perspective of quantity. Second, students assume that the nature of cardinal numbers corresponds to the representation of natural and real numbers which is in contrast to negative integers that cannot be represented by something real or certain objects. Third, during learning, it is impossible for the teacher to give instructions to reduce or delete more numbers than what is available. Furthermore, there are three types of negative integer; unary (structural signifier), binary (operational signifier) and symmetric functions (opposite the positive numbers) (Bofferding, 2014; Vlassis, 2004).

According to Gallardo (2002), the existence of particular problem related to negative integers is very important for the success of students' understanding. He gave the example that it would be easier for students to understand the value of "50 debt" rather than saying "minus 50". The reason for students' difficulties in understanding negative integers is because negative integers cannot be represented concretely or negative integers are "opposed" to real objects (Bishop et al., 2015). In fact, for this reason, negative integers are often considered as fictitious number by students (Vlassis, 2004).

There are several empirical study results from researchers that can be used to convey negative integer material making it easier for students to understand (Fischer, 2005; Beswick, 2011; Bishop et al., 2014; Enzinger, 2015). Beswick (2011) revealed that the use of number lines is still the main tool in giving students experience related to negative integers, however he also believes that there are still many students who represent number lines with objects around them. Meanwhile, Bishop et al., (2014) explained how second grade elementary school students successfully involved the display of number ordering as a reason for solving integer. Enzinger (2015) documented the results of his case study on a student using single and double set drawings (strip drawing) as a medium for solving negative integer problems.

Even though many empirical solutions are offered, there are many students are still facing difficulty in understanding negative integers. Fuadiah & Suryadi (2017) revealed that students are still facing difficulties in dealing with negative integer problems, both at pre-learning and at the formal level. Among many factors, one that can prevent students from understanding negative integers is the minus (negative) sign, which can make it difficult for students to find solutions because they do not understand the meaning of negative integers (Vlassis, 2008). Some students think that subtraction is more difficult than addition (Karantzis, 2010). Therefore, this research aims to explore elementary school students' perceptions in facing negative integer problems before the material is taught by their teacher. The exploration results will be classified in three characters.

LITERATURE REVIEW

Perception and experience are two interrelated things. A new experience from time to time can be built through conceptual embodiment by combining perceptions and actions that develop through the mental world (Sa'adah et al., 2023). In this case, perception is the student's initial understanding of negative integers which can be depicted on a number line. If we look at it from a cognitive development perspective, students of a certain age need concrete examples as a medium for understanding the concept of integers. Likewise, in introducing negative integers to students, teachers must provide relevant examples so that students can easily understand them.

Fischer (2005) suggests that students who are faced with negative integers problems will have varying results. This can happen because students use their cognitive abilities to solve these problems using the number line. Furthermore, Gersten et al., (2009) also revealed that a very powerful tool in supporting students' understanding of various mathematical concepts is the number line. Siegler (2009) also explains that introducing integer through a number line is a very useful way to test students' understanding of numbers because this task requires students to be able to estimate the exact location of numbers on each number line scale. According to Stephan & Akyuz (2012) Students can use their experience with assets (money), debt, and net wealth to find positive and negative integer results. In this concept, number line is used to explore students understanding about negative integers. Earnest (2007) argued that the use of number line is important to help elementary students in identifying numbers.

Students who have obstacles in understanding the concept of negative integers resulted in different perceptions. The existence of these differences can be influenced by each student's thinking ability. According to Faizah et al. (2022a) thinking is a tool for constructing knowledge in mathematics learning. Students can create varied perceptions as they construct knowledge based on their experiences in everyday life. Therefore, the results of their construction are different.

Students' initial perceptions in solving negative integer problems can be categorized into several characteristics based on the location of writing negative symbols on the number line. This categorization can be classified into three main characteristics: assuming that negative sign has no meaning; unary understanding; pseudo understanding. Students assume that negative sign has the same meaning as minus sign in subtraction operation (Vlassis, 2004). Students assume that negative sign is located on the left side on zero (Bofferding, 2014). On the other hand, pseudo understanding is the result of thinking process in which the result is not the real result as the given answer could not represent the thinking result (Subanji, 2016).

The differences in students' perceptions about the location of negative integers on the number line can be overcome by providing scaffolding. Students with little experience of numbers will need more scaffolding from experts than those with more experience. Providing this scaffolding can be done through action from teachers or more expert colleagues (Faizah et al., 2022b). Scaffolding is

given to overcome students' thinking obstacle in understanding negative integers concept. Scaffolding can be given based on students' characteristic results.

METHOD

This research is included in exploratory research with a qualitative approach. This research was conducted on 29 fifth grade elementary school students in Nganjuk district, East Java, Indonesia. Researchers selected students who had never received learning material about negative integers because the researchers wanted to explore students' initial thinking abilities regarding negative integers. Subject selection is also based on students' experience of using the concept of debts and the number line in completing test. Students who do not submit the results of their test work cannot be used as research subjects.

Written tests and interviews were used to collect the data of this research. The test is used to determine students' initial abilities regarding negative integers before students receive the material from the teacher. Students can complete tests based on their knowledge and experience without any interference from researchers or teachers while taking the test. Meanwhile, interviews were used to explore the subject's ability to think about negative integers. Researchers recorded the interview process using a video tape recorder to facilitate the data transcription process. The test instrument used in this research is adopted from Schindler et al. (2016) which is then validated. The written test can be seen in Figure 1.

There are six people talking about their financial problems. They have debt and assets. Randi is known to have 200 in debt while Ayu has 400 in debt. Based on this information, who has more debt?

a. Draw a number line from -500 to +500, then place the number that show Randi's and Ayu's debt in the number line!
b. If the financial situation of these six people are illustrated as follow:

Randi = 200 in debt	Ana = -300	Toni = 300
Ayu = 400 in debt	Tika = -200	Nina = 100

Please write the six numbers above on the number line based on the debts and assets that they have!
c. Please explain the meaning of "minus" sign in front of Ana's and Tika's numbers! Why aren't there any "minus" sign in front of Toni's and Nina's numbers?

Figure 1. Test Instrument

This research uses qualitative data analysis which refers to the results of written tests and task-based interviews. Qualitative data analysis includes: data observation, transcribing interview results, data reduction, data validation, data categorization, data interpretation, and drawing

conclusions (Creswell, 2014). Researchers observed the results of written tests to determine the characteristics of students' thinking perceptions related to negative integers. The researcher transcribed the results of interviews with selected subjects. Data reduction was used to select and focus the data appropriate to the research objectives. Meanwhile, data validity in qualitative research is performed through triangulation. This research uses a triangulation method to determine the suitability of the data obtained from the results of written tests and interviews. Researchers categorized the perceptual characteristics that emerged in subjects when understanding and completing tests about negative integers. Then the researcher interprets the data based on the characterization results for the process of drawing conclusions.

RESULTS AND DISCUSSION

The results show that only 12 students were able to complete the written test. The twelve students had different perceptual tendencies in understanding the concept of negative integers. These differences can be classified into three types of perception, namely: (1) there are three students who take the test by accumulating debts and assets on the right side of the number line, (2) six students repeat the numbers on the number line symmetrically positive-negative, and (3) three students repeat the numbers on the number line in a positive-positive symmetrical manner. Of the three types, they were chosen randomly to continue with the interview process. Researchers coded the first type of subjects with S1, the second type with S2, and the third type with S3.

From each perception characterization, it can be seen that each subject is able to differentiate between debts and assets. The researcher did not provide any interference when the subject completed the test, but during the interview, the researcher gave the subject simple trigger questions to explore their understanding of negative integers in the context of the difference between debts and assets (money). The subjects said that when they chose debt, the money had to be returned. However, when subjects were asked to mark the location of debt and assets on a number line, they were still confused. From this confusion, students' initial perceptions are obtained when understanding the concept of integers because this material has not been taught by the teacher. The results of the exploration of each subject are as follows:

Subject 1

Subjects with the first type of perception work on questions about negative integers by stacking or placing debt and assets at the same point to the right of the number line. The subject put Tika and Randi at point 200, even though Tika owed Randi 200. The subject also placed Ana and Toni at the same point, even though Ana owed Toni 300. This can be observed from the results of the subject's work in Figure 2.

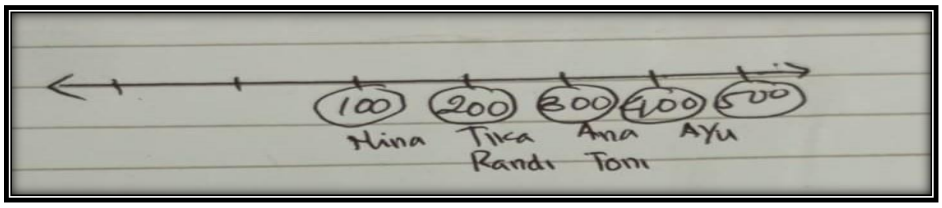

Figure 2. Answer from subjects 1

R: why does Nina occupy that place on the number line?

S1: because Nina has 100 Rupiah

R: Then, why do Tika and Randi occupy the same place?

S1: Because Tika has 200 in cash and Randi has 200 in debt

…

R: try counting backwards starting from 5

S1: Five, four, three, two, one

R: and after that?

S1: zero

R: after zero?

S1: nothing

Perception of Subject 1

Even though the subjects did not give a clear reason why after zero there were no more numbers, the answers they wrote showed that the subjects only knew positive numbers, because they did not pay attention to debts which should be negative numbers. This can be seen in the number line they have drawn, they only use the right side of the number line to place the position of debts and assets. They did not place a negative sign to the left of the number line to indicate debt as an embodiment of a negative integer. To find out whether or not the subject understands negative integers, the researcher provides scaffolding by asking the subject to count backwards starting from 5, when he reaches the number one, the subject stops and does not continue to the zero number.

Initially the subject did not mention zero, but after receiving a question from the researcher, the subject then mentioned zero. But after saying zero, the subject stopped and said that there were no more numbers before zero. One of the factors that causes subjects did not mention zero is because subjects were not used to counting things that are not in that number (Utami et al., 2018). Bofferding (2014) also explains that students will treat negative integer values like positive integers, because students view the minus sign only as a subtraction operation. As a result, the

subject did not differentiate between the location of numbers as assets and debts on the number line, so they accumulated at the same point on one of the line segments.

When the subject said there are no more numbers before zero, this showed that the subject only understands positive integers. Woods et al., (2017) said that indirectly students have developed positive numbers quickly which make the understanding embedded in their mind that there are no more numbers other than positive integers. At that time, students begin to develop abstract mental representations that can generalize numbers from calculation routines (Wynn, 1990).

Subject 2

The second type of subjects completed the test in a different way than the first type of subjects. These differences can be seen in Figure 3.

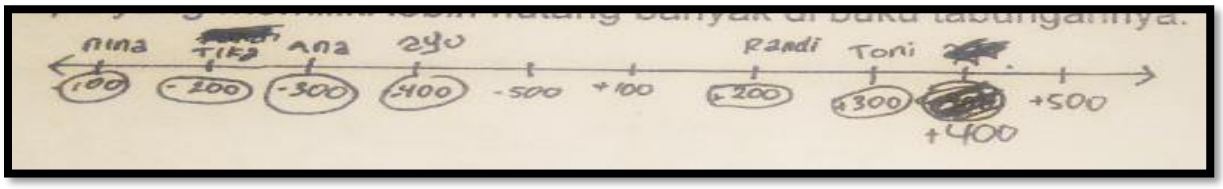

Figure 3. Answer from subjects 2

Based on the subject's answer in Figure 3, it shows that the subject drew a number line and wrote it repeatedly in the form of positive – negative numbers. Then the subject placed the debt and assets numbers correctly, but the position of the numbers was still incorrect. Subjects made errors because they did not write the number zero between the positive and negative number lines. The subject also made errors in writing -100, -200, -300, -400, and -500 starting from the left side, the subject should have started from the middle after zero. Researchers conducted interviews with subjects to determine their initial understanding of negative integers as in the following transcript:

R: try counting from two onwards!

S2: three, four, five, six…

R: try counting backwards now

S2: five, four, three, two, one

R: try continuing again?

S2: min five, min four, min three, min two, min one

R: hmmm...why is that?

S2: yes because this is five hundred plus (while pointing to the right-hand number line) and this is minus five hundred (while pointing to the left-hand number line).

R: So, does that mean before one is minus five?

S2: oh yeah yeah... (starting to feel confused)

Perception of Subject 2

Based on Subject 2's answer, it can be seen that the right side of the number line is the place for positive numbers, and the left side of the number line is the place for negative numbers. However, the subject did not write down number zero to differentiate it, so the subject made mistake when writing the numbers on the number line. According to Fischer (2003), students who understand negative integers on the number line tend to be from an ontogenetic perspective, that is, students understand negative integers by assuming that these numbers tend to be on the left side of the number line. Likewise, the subject's understanding of negative signs. Subject 2 has understood the meaning of negative signs as unary, namely negative signs that follow numbers (Bofferding, 2009; Stephan & Akyuz, 2012).

Tzelgov et al. (2009) stated that students have long-term memory, which states that the numbers 1 to 9 are the only digits that can be represented, while other numbers are produced from nine digits. Therefore, it is very possible that subjects will find and understand negative integers from giving tasks in certain contexts, such as in this study.

Subject 3

Type 3 subjects provided different solutions to type 1 and type 2 subjects. This can be seen in Figure 4.

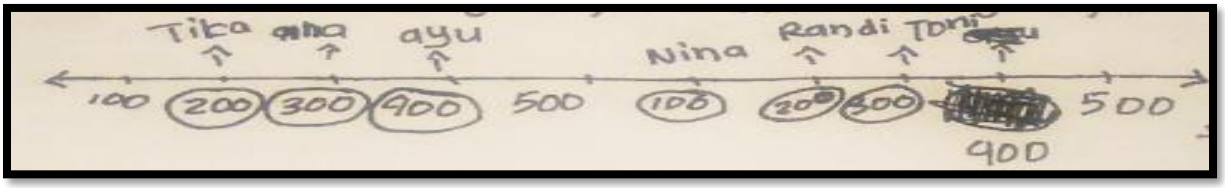

Figure 4. Answer of subjects 3

Figure 4 shows that the subject drew a number line and wrote positive-positive repeatedly. Subjects placed debt on the left and assets on the right of the number line. Even though it was correct in placing debt and assets, the subject still writing negative numbers incorrectly because the subject did not put a negative symbol (-) in front of the number written on the left of the number line, and the subject also did not write the number zero on the number line.

Perception of subjects 3

The subject appeared to have understood the concept of negative integers in its implementation in debts and assets because the subject places the number that indicates debt on the left on the number line. But the subject did not write the number of zero (0) in the middle of the number line, the subject also did not put a negative sign (-) in front of the number written on the left of the number line, and the order of the numbers written was also reversed because the subject should have

written -100 to the left of 0 instead of 500. From the subject's answer, it is known that he has a false understanding of negative integers when depicted on a number line. The subject knew the position of debt as a negative number which is located on the left but did not write a negative sign (-) in front of the number, and the subject also knew that money or assets are positive numbers which are located on the right on the number line. This error occurred because the subject wrote numbers on the number line repeatedly, positive-positive, even though the subject should have written negative sign in front of the number on the left.

During interviews, subjects gave unclear reasons as to why they wrote numbers repeatedly. Type 3 subjects tend to lead to pseudo or apparent understanding, according to Subanji & Nusantara (2016) who say that pseudo understanding is the condition of students when they understand a concept but do not know the continuation of the concept. This is in line with the result of the research conducted by Young & Booth (2015), which revealed that knowledge of the magnitude of negative integers follows a similar pattern to that of positive numbers, but negative estimation performance is far behind positive. Subject 3's answer is also similar to the number line anticipation proposed by Bofferding (2014) in that students repeat numbers in positive-positive form when understanding the concept of whole numbers. Even though integers contain negative and positive numbers.

Based on the exploration results of each subject, characteristics of students' perceptions in understanding negative integers when depicted on a number line was found. The characterization is presented in Table 1.

Table 1. Characterization of Students' Perceptions of Negative Integers.

Characteristics	Description
Assume there are no negative numbers	• Interpret debt and assets at the same point on the number line. • The negative sign is only present in the subtraction operation.
Unary understanding	• Assuming that negative integers tend to be on the left of the number line. • Assuming that negative signs are signs that only follow positive numbers. • Placing negative numbers on the number line in a positive-negative symmetrical manner.
Pseudo understanding	• Knowledge of negative numbers following the pattern of positive numbers. • Do not give a negative sign because the numbers to the left of the number line are definitely negative. • Place negative numbers on the number line in a positive-positive symmetrical manner.

Table 1 shows that the 1st type of student did not assume that there are no negative numbers. Therefore, the student did not write the "-" symbol before the negative numbers as they assumed that the symbol of "-" is the same as the symbol used in subtraction. On the other hand, unary type of student only understood that the negative numbers are located on the left side of zero on the number line but do not understand the meaning. Students with unary type of thinking face difficulty in implementing the negative integers in debt-asset concept. Last, pseudo type of students understood the existence of negative integers but placed them in the same position with the positive integers. This means that pseudo type of students consider both negative and positive integers are the same, so they did not write negative "-" symbol before the numbers although they write the numbers on the left side of zero on the number line. There are differences from each type of students during the thinking process in understanding the concept of negative integers (Bofferding, 2014, Stephan & Akyuz, 2012, Subanji & Nusantara, 2016, Vlassis, 2004).

Students' inability to understand negative integers can occur due to thinking disorders. This disturbance is caused by one conceptual error which then has an impact on other conceptual errors (Sukoriyanto et al., 2016). Students can experience thinking disorders because of the cognitive conflicts they experience in understanding a certain concept, for example students' errors in understanding the concept of integers which have an impact on conceptual errors regarding fractional numbers. Thinking in mathematics learning can be designed to provide positive encouragement (Faizah & Sudirman, 2022), to help in developing students' confidence in the results of their solutions.

Students can understand the concept of fractional numbers through cognitive that encourage changes in students' way of thinking in defining words or identifying numbers (Pratiwi et al., 2020). To ensure that students do not experience thinking disorders, there needs to be scaffolding from teachers or experts (Anghileri, 2006, Faizah et al., 2022b), so in this case the researcher gives trigger questions to direct the subject to understand negative integers on the number line but the subject cannot accept the scaffolding. Therefore, it is necessary to carry out further research that discusses strategies for providing scaffolding to students who have difficulty in understanding the concept of negative integers.

CONCLUSIONS

The results of the research show that there are three characteristics of students' perceptions in understanding negative integers. The first characteristic is that students assume there are no negative integers, so they put negative integers at the same point with the positive integers on the number line. The second characteristic is that students have a unary understanding because they think that the negative sign in a number is just a meaningless sign that is in a positive integer and is located to the left of the number line. The third characteristic is pseudo understanding where students seem to understand negative integers because they know that they are on the left but when writing on the number line, they do not put a negative sign. The third type of student does not write

the number "0" on the number line which resulted in an error when they write negative numbers on the number line.

This research was conducted when the teacher had not taught negative integer material to students which means that students took the test only based on their initial perception of numbers. Therefore, the results of this research provide an opportunity for other researchers to conduct further research on students' cognitive development in understanding the concept of negative integers when teachers teach this material. If there are students who experiencing difficulties, scaffolding needs to be provided so that different finding can be obtained.

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