Analysis of the Strategies Used by High School Students in Solving Area Problems: A Case Study

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Abstract: *The purpose of this study was to analyze the activities in the Problem-Based Learning (PBL) methodology among high school students when solving problems related to the field of plane figures, both in extra-mathematical and intra-mathematical environments. The research was qualitative and constituted a case study. The technique used to select the research subjects was intentional sampling, which involved the selection of a group of sixth-grade students. The data were collected through tests and interviews. The data analysis techniques employed included data reduction, data presentation, and drawing conclusions. The results showed that the students used perimeter calculation as an erroneous representation of area calculation, and they exhibited a lack of argumentation during problem-solving. Furthermore, they expressed a lack of recognition of certain plane figures and their properties, a characteristic of students at the visualization level, also known as the first level of the Van Hiele levels. Another significant finding in this study was the use of problems involving extra-mathematical contexts, which had a greater impact on the problem-solving process and the understanding of mathematical concepts.*

Keywords: problem-solving, didactic unit, Problem-Based Learning, area of plane figure.

INTRODUCTION

Problem-solving is considered by various authors to be an important part of mathematical activity (Sintema & Mosimege 2023), as it addresses diverse phenomena in the real world and the mathematical realm. Furthermore, "the formulation and resolution of problems enable students to implement strategies for tackling problems and formulating questions, in order to increasingly strengthen their analytical capacity" (Sanabria, 2019, p. 16). In this way, students play an active role, fostering significant learning and promoting the perception of mathematics as a discipline with utilitarian value from a perspective that goes beyond arithmetic. Consequently, it can contribute to improving students' performance (Alifiani 2023, Malvasi & Gil-Quintana, 2022, Maphutha et al., 2023). In the case of geometry, students are more likely to develop their visual perception, intuition, critical analysis, problem-solving skills, and use of arguments and logical evidence (Caviedes et al., 2023, Jupri, 2017, Seah, 2015). Due to its applicability to everyday life issues, the field of plane figures is a fundamental concept that students must master (Herawati et al., 2022, Winarti et al., 2012). Furthermore, the best way for students to achieve good results in the topic of three-dimensional space is to have a solid understanding of the area of plane figures (Battista et al., 1998) and to understand the above, it is important to resort to multiplicative reasoning (Jain et al., 2022).

Given the relationship between problem-solving, geometric reasoning, and understanding of the area of plane figures, an exploration into teaching methodologies and strategies becomes essential. The following research question guides the study: How does the utilization of Problem-Based Learning strategies influence the understanding and application of the concept of area among secondary school students across different Van Hiele geometric reasoning levels?

This question arises from a convergence of observational and theoretical insights. A diagnostic test and a semi-structured interview conducted with a focus group revealed weaknesses in calculating the area of plane figures and their application in problem-solving, prompting an investigation into the depth of these challenges. The student's conceptions did not align with any of the area manifestations outlined by Corberán (1997). They also demonstrated a lack of recognition of certain plane figures and their properties, which is characteristic of students at the visualization level, also known as the first level of the Van Hiele hierarchy (Gutiérrez & Jaime, 1991, Vargas & Gamboa, 2013).

Furthermore, to investigate the different methods, strategies, and techniques used for the development of studies on the learning of mathematical or scientific objects, the categorization taxonomy proposed by Rodríguez and Arias (2020) was used as a basis. This analysis showcases the most commonly used methods, among which Problem-Based Learning (PBL) (Arnal-Bailera & Vera, 2021, Artés et al., 2015, Cruz & Puentes, 2012, Zumbado-Castro, 2019, De Jesus, 2020, Endah et al., 2017, Galviz et al., 2016, Khalid et al., 2020, Salcedo & Ortiz, 2018, Setyaningrum et al., 2018) and Cooperative Learning (Cruz & Puentes, 2012, Fortes & Márquez, 2010, Galviz et al., 2016, Khalid et al., 2020, Villada, 2013) stand out for their extensive utilization. Based on this literature review, the most commonly used strategy is PBL, which will be employed for the development of the mathematical object specific to this study.

Van Hiele Levels

Both Gutierrez & Jaime (1991) and Vargas & Gamboa (2013) agree that the Van Hiele model of geometric reasoning explains how students acquire skills to the point of developing competencies in geometric reasoning. This model divides the process into five levels: visualization, analysis, informal deduction, formal deduction, and rigor. Each level is further subdivided into five phases: information, directed orientation, explication, free orientation, and integration. Upon completing

Phase 5 of a particular level, the student progresses to the next level. The Van Hiele reasoning model demonstrates the interdependence of the different levels, emphasizing that an individual cannot skip any level of reasoning.

Area of Plane Figures

Regarding the concept of area, Freudenthal (1983) states that it is a magnitude used to measure multiple objects, and he identifies three phenomena inherent in the learning of the area concept: (1) area as equal distribution, (2) area as comparison and reproduction of shapes, and (3) area as measurement. In line with this, Corberán (1997) proposes that the concept of area is not limited to a single concept, but is discerned through four manifestations, which are: (1) area as the amount of plane occupied by the surface, (2) area as an autonomous magnitude, (3) area as the number of units that cover the surface, and (4) area as the product of two linear dimensions. According to the author, it is essential that all four manifestations be present in the teaching and learning process. Therefore, this conception of the area was considered for the planning, design, and implementation of this intervention. Each of these manifestations is associated with different actions or procedures developed by the students.

Table 1. Manifestations of area and associated actions (Caviedes et al., 2019).

Manifestations of area (M)	Actions/Procedures (P)
M1. Area as the amount of plane occupied by a surface.	P1. Geometric procedures: P1.1. Comparing surface areas without using numbers. P1.2. Direct comparison of areas by superposition; indirect comparison of areas by cutting and pasting, decomposing the surface.
M2. Area as an autonomous magnitude	P2. Two-dimensional geometric and numerical procedures: P2.1. Decomposing surfaces into equal parts; comparing areas of surfaces and recognizing that different-shaped surfaces can have the same area. P2.2. Measuring the area of the same surface using different units of measurement.
M3. Area as the number of units covering a surface.	P3. Two-dimensional numerical procedures: P3.1. Fractionating the area of a surface and/or counting the number of units covering a surface using a two-dimensional unit of measurement. P3.2. Comparing the area of a surface with the two-dimensional unit that measures that surface.

M4. Area as the product of two linear dimensions.	P4. One-dimensional and two-dimensional numerical procedures: P4.1. Calculating the area of polygonal surfaces that can be decomposed into triangles or rectangles. P4.2. Apply formulas for calculating the area of a rectangle or square to determine the areas of triangles and parallelograms.

METHOD

The didactic unit was implemented taking into account the class moments and planning proposed by Diaz-Barriga (2013) and Ártes et al. (2015), considering the action research design presented by Hernández et al. (2010). For this reason, the design of the activities or learning sessions takes as reference those aspects of mathematical knowledge in which students encounter the greatest difficulties, specifically those related to problem-solving in metric thinking. The activities are structured in three stages, namely: the opening stage, which aims to activate students' attention towards the new learning, establish the purpose of the activity, increase interest and motivation for the new learning, provide a preliminary overview of what will be developed, and activate prior knowledge; the development stage, where the focus is on processing new information, directing attention to the new learning, implementing teaching and learning strategies to facilitate the acquisition of new knowledge, and putting the new learning into practice; and finally, the closing stage, where formative assessment takes place. In this stage, it is necessary to review and summarize what has been learned, transfer the new learning, motivate once again, draw conclusions, and conclude the session. Throughout this process, assessment is continuous and occurs in each of the class moments.

After the first activity was implemented, it was shared and evaluated using a checklist, and then the following activities were carried out, repeating the same cycle. Next, the participants and their context are presented, followed by the procedure used for implementing the activities and the sequence of activities in the didactic unit for this project.

Participants

This study was initially conducted in a sixth-grade course at an official educational institution. However, due to situations related to the COVID-19 pandemic, the implementation of the activities in the didactic unit was carried out with the same group of students (37 students aged between 11-14 years) who were part of the diagnostic phase but were in seventh grade (7th) at the time of the activities.

Procedures

This study was conducted over the course of seven class sessions, each lasting 80 minutes. Activity 1 was presented over four sessions, Activity 2 over two sessions, and Activity 3 in one session. These activities were presented in three stages: (1) exploration of prior ideas, where everyday problems were used to activate students' previous knowledge of the concept of polygonal area and to motivate the acquisition of new learning; (2) development, where new ideas about the concept of area were constructed through problem-solving, structuring, and practicing the new learning; and (3) closure, where the aim was to formalize the concept of area.

Didactic Unit

Activity 1. Exploring a Space.

Marcel wishes to replace the entire flooring of his living room and kitchen. He is currently heading to the flooring and tile store. He wants to know how much money he should bring to carry out the renovation. If he has a floor plan of the house (see Figure 1), how would you help him estimate how much money he will spend on the renovation? Justify your answer.

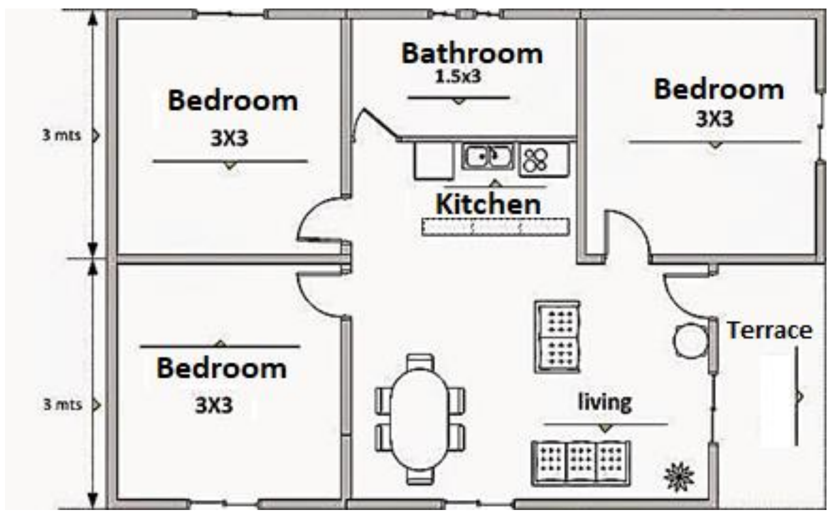


Figure 1. Top plan of Marcel's house. Translated

After the time elapsed, a group discussion was held to identify difficulties in solving the problem, and some previous concepts were activated that could help solve the problem through the following questions: What can you see in the image? What mathematical elements can you find embedded in the image? Why? Do you recognize any flat figures? Which ones? Why do you consider them flat figures? What data do you consider necessary to know the cost of the renovation? Justify your answer. Following this exploratory moment, 20 minutes were given to the students to explore different heuristics to solve the problem on their own, and a tour was made of each of the worktables to investigate how they had approached the problem. After the tour, each worktable was interrupted with thought-provoking questions to help the students reflect. Questions were asked, such as: What mathematical concepts do you think would help you develop the problem? Explain your answer. How would you use the measurements in the plan to help Marcel? Is it possible to calculate the area by adding the measurements? Is it feasible to buy the exact amount you calculated? Why? Does the seller sell the tile in boxes or by square meters? Does Marcel only need to buy the floor to do the renovation? Why?

Activity 2. Building My Home

Considering the information obtained in the previous activity, develop a floor plan of your home, and estimate how much money you would spend to change the flooring in some parts of your house. Four students were asked to share and describe the strategy they used to determine how much money they would spend to change the flooring in some part of their house. Each of the students' interventions was facilitated by the teacher in the same way as Activity 1, using the same guiding questions as a guide.

Activity 3. Approach to the Formal Concept of Area

Considering the following figure (Figure 2), how could we calculate the area of the rectangle and the triangle without counting the squares? (Justify your answer). Initially, the problem was presented, and students were given 10 minutes to independently identify the relationship between the two surfaces. After this period, we addressed students' errors or difficulties with probing questions: How many square units cover the rectangle? Can you explain your calculation? How many square units cover the central triangle? Can you explain your approach? These questions facilitated a group discussion in which we evaluated the strategies used by the students to calculate the number of square units constituting both figures. After the group discussion and debate about the number of square units covering both the rectangle and the triangle, students were given another 5 minutes to establish a relationship between the areas of the two figures. Subsequently, the students were asked to answer the: How could you determine the area without counting each individual square unit? To conclude the exercise and reinforce the lessons learned, the students' responses were supplemented with the following questions: How would you define the 'area' of a flat figure? Can the concept of area be applied to solve other everyday problems? Could you mention or create a problem where this concept could be utilized? Through this structured approach, students were guided to develop a deep understanding of the concept of area and its practical applications.

RESULTS

Activity 1

It was observed that the students approached the teacher with phrases such as, "Teacher, what do I have to do?" and "I don't understand the problem," as seen in the following classroom situation.

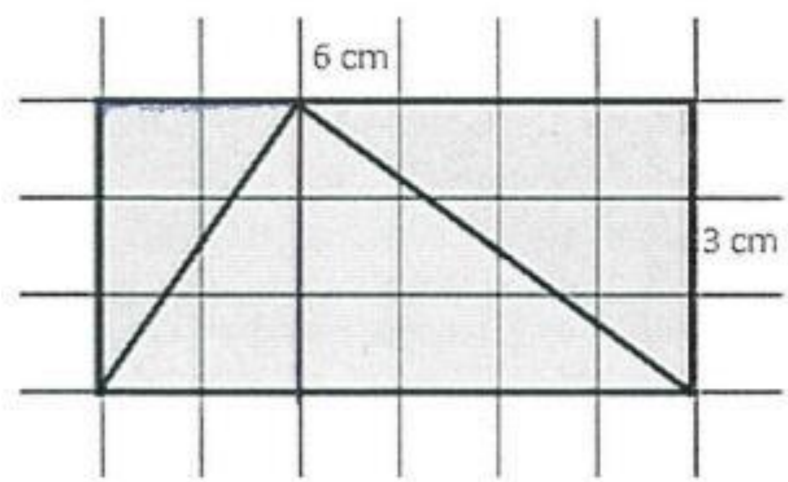


Figure 2. Relationship between areas of two flat figures (Caviedes et al., 2019)

Transcript 1: Initial difficulties in problem development.

Teacher: How are we doing?

Student: we don't understand what else to do.

Teacher: Let's read the problem

Student: Marcel wants to change all the floor...

Teacher: what do we need?

Student: the tile thing, teacher, if we don't know how much we are going to spend.

Teacher: okay, the price of the tile, but what is initially asked in the problem?

Student: how much he is going to spend.

Teacher: and to know how much we are going to spend, what do we need?

Student: the price of the tile.

Student: the perimeter.

Student: but we don't know how many tiles we are going to use.

Teacher: and how do I know how many tiles we are going to use?

Student: Measuring. Teacher: and what do we measure?

Student: the area.

Table 2. Results of the checklist activity 1.

Objectives	Descriptors	Assessment		
		Correct	Acceptable	Incorrect
Propose and develop strategies for estimating, measuring, and calculating areas to solve problems.	1. Recognizes, in different contexts or in problem situations, the calculation of the area of plane figures, and represents them in their different manifestations.	18	19	0
	2. Explain with arguments, verbally or in writing, his/her concept of plane figures.	17	10	10
	3. Explain with arguments, verbally or in writing, his/her concept of area of plane figures.	21	16	0
	4. Calculates and uses the area of plane figures in their different manifestations when solving a problem and concludes.	33	4	0
	5. Solves mathematical problems and tasks involving the calculation of the area of plane figures.	37	0	0
	6. Communicates the processes developed to arrive at the solution to the problem.	21	16	0

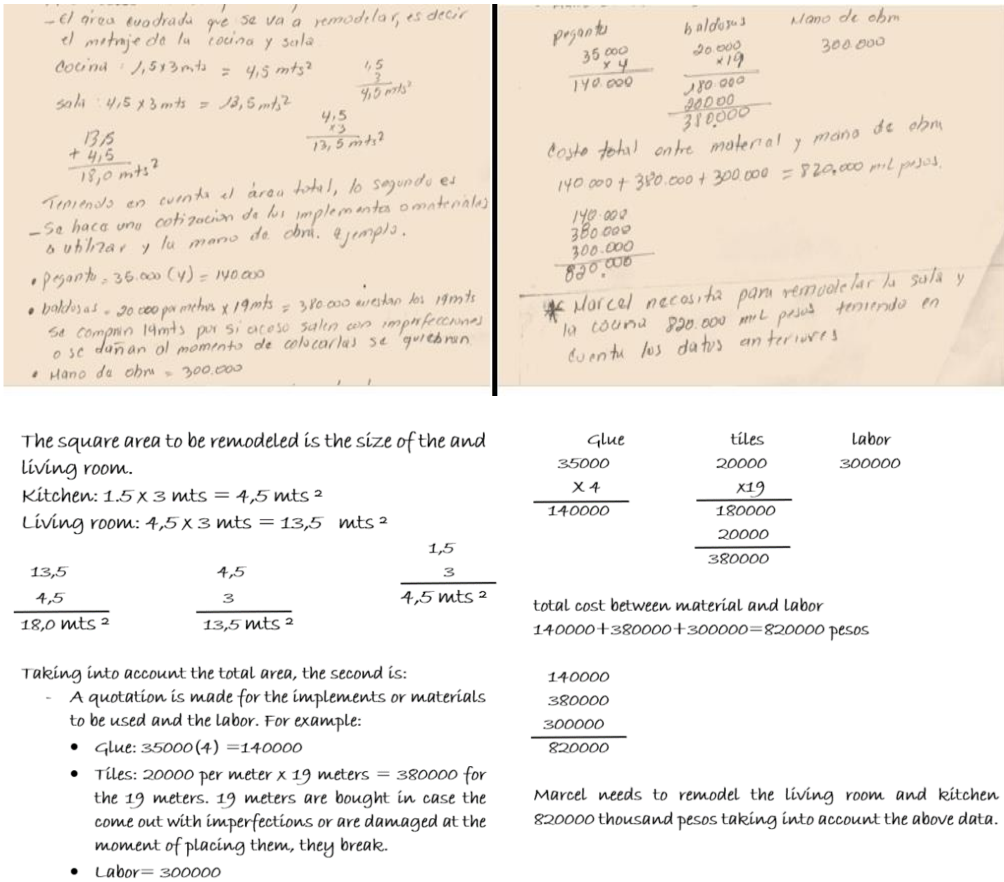


Figure 3. Solution to Problem 1 Example 1

In the previous solution (Figure 3), the student identifies the use of the area of flat figures and represents it as a product of two magnitudes according to Corberan (1997). Additionally, considering the representation, the student identifies the related flat figures (rectangle) and some of their properties, which, according to Vargas & Gamboa (2013), is characteristic of a student at the Van Hiele level of informal deduction. In contrast, in the following solution (Figure 4), the student identifies the importance of the area, but none of the manifestations of the concept of area proposed by Corberan (1997) are visualized. Likewise, although the problem is solved, there is no evidence of the use of area calculation to make decisions, nor does the student communicate all the processes used to solve the problem. This is an example of a solution that, in general terms, is in level of regular compliance with the descriptors.

Activity 2

No initial blocks in the problem approach were observed during its development. This was evidenced by the absence of phrases such as: "I don't understand, teacher", "I don't know how to start". After the development and evaluation of Activity 2, the following results were obtained (Table 3).

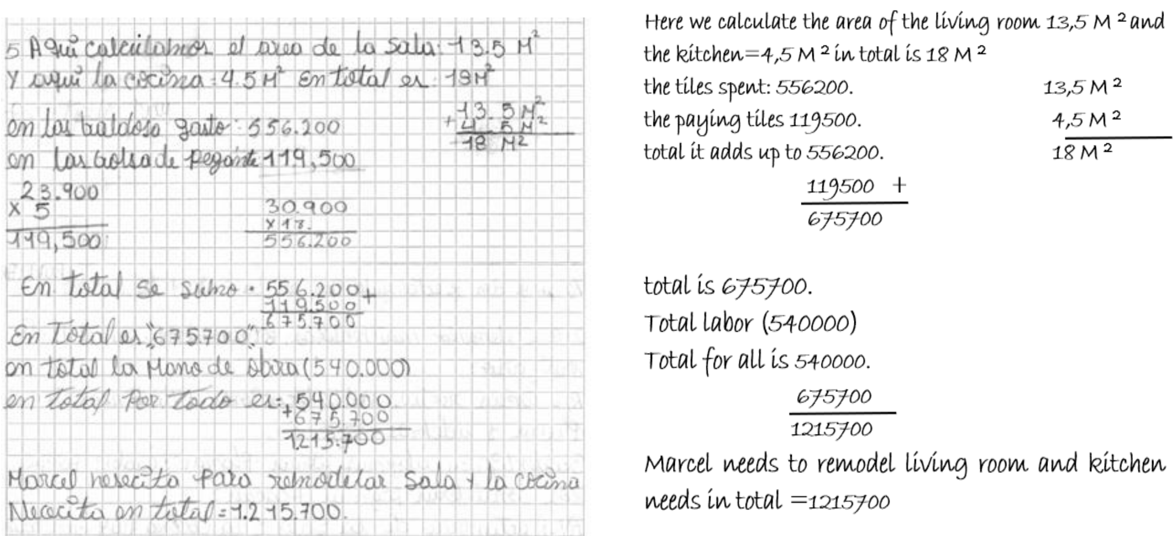


Figure 4. Solution of problem 1 example 2

Table 3. Result of checklist activity 2.

Objectives	Descriptors	Assessment		
		Correct	Acceptable	Incorrect
Propose and develop strategies for estimating, measuring, and calculating areas to solve problems.	1. Recognizes, in different contexts or in problem situations, the calculation of the area of plane figures, and represents them in their different manifestations.	18	9	8
	2. Explain with arguments, verbally or in writing, his/her concept of plane figures.	14	20	3
	3. Explain with arguments, verbally or in writing, his/her concept of area of plane figures.	18	9	10
	4. Calculates and uses the area of plane figures in their different manifestations when solving a problem and concludes.	14	19	4

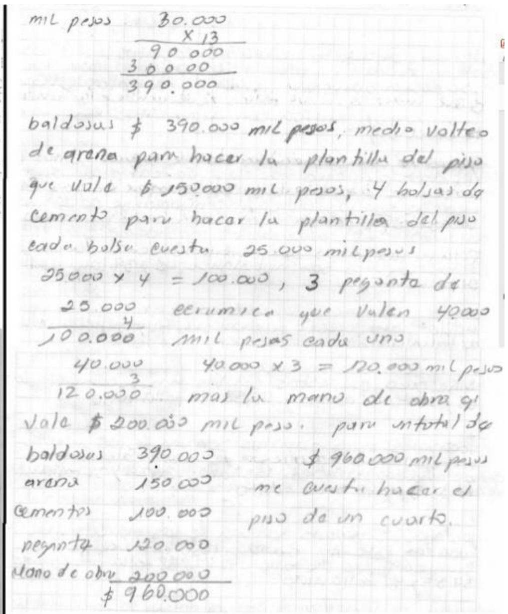
	5. Solves mathematical problems and tasks involving the calculation of the area of plane figures.	21	16	0
	6. Communicates the processes developed to arrive at the solution to the problem.	15	16	6

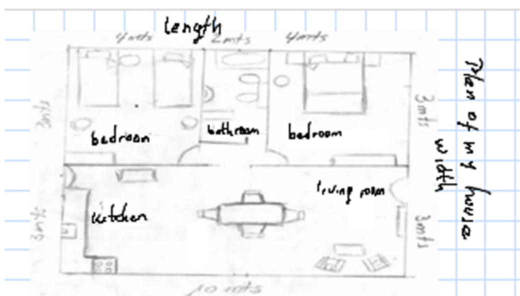
Quantitatively, the results show that Activity 2 constituted a greater cognitive effort for the students, as in most of the descriptors, between 3 and 10 students were placed in the non-compliance (NC) box. The students did not require class management to communicate the decisions and procedures for problem development; therefore, they were more autonomous in the process in general, which is also understood as an indicator of improvement, as seen in the following class examples (Figure 5 and 6).

In the previous example (Figure 5), the student identifies the area to work on and the flat figure that composes it, which according to Vargas & amp; Gamboa (2013) is characteristic of a student at the Van Hiele level of informal deduction. The student also identifies the importance of area calculation and represents it with one of the manifestations proposed by Corberan (1997), which is understood as an empirical argument for the concept of area. Furthermore, the student uses the concept of area in conjunction with strategies based on the heuristics of the Anglo-Saxon school of problem-solving (Rodriguez & amp; Marino, 2009) to respond to the situation and always argues in a written and procedural manner about the decisions made and the results obtained. Therefore, this is an example of compliance with the descriptors on the checklist and thus meets the objective of Activity 2 of the didactic unit.

Contrary to this, in the following example (Figure 6), the absence of representation and application of the calculation of the area of flat figures (Corberan, 1997) for problem development is evident. The student does not respond to the problem question, and although they communicate the area in which the remodeling will be carried out, they do not argue the decisions and procedures that lead to a subsequent response. This is an example of a resolution that, in general terms, is at a level of non-compliance (NC) with the descriptors.

Compared to the results of the previous activity, it was observed that most students did not confuse the concept of area with that of perimeter when solving the problem. On the other hand, based on student perceptions, this problem reinforced the learnings worked on in activity 1, and generated greater motivation since, when asked "how did you find activity 2?", situations such as those observed in Transcript 2 emerged.





Since I am only tiling on room, I need to determine the square footage. The room measures 4 for 3, which equals 12 square meters, and this area I need to tile, my house has a rectangular shape based on its measurements. To calculate the cost of tiling the room, I need to request a quote. I plan to buy 13 meters of ceramic or tiles to account for any potential damage or imperfections during the installation. Each meter of tile costs $30000 pesos

30000
X 13
90000
30000
390000

Tiles $390000 pesos, half truck of sand to make the floor template: $150000 pesos, 4 bags of cement for the floor template, with each bag costing 25000 pesos.

25000 x 4 =100000, 3 ceramic glue containers each priced at 40000 pesos

25000
4
100000

40000 X 3= 120000 pesos

40000
3
120000

labor $200000
for a total of $960000 which is the cost to make the floor of the room.

Tiles 390000
Sand 150000
Cement 10000
Glue 120000
Labor 200000
$960000

Figure 5. Solution to Problem 2 Example 1

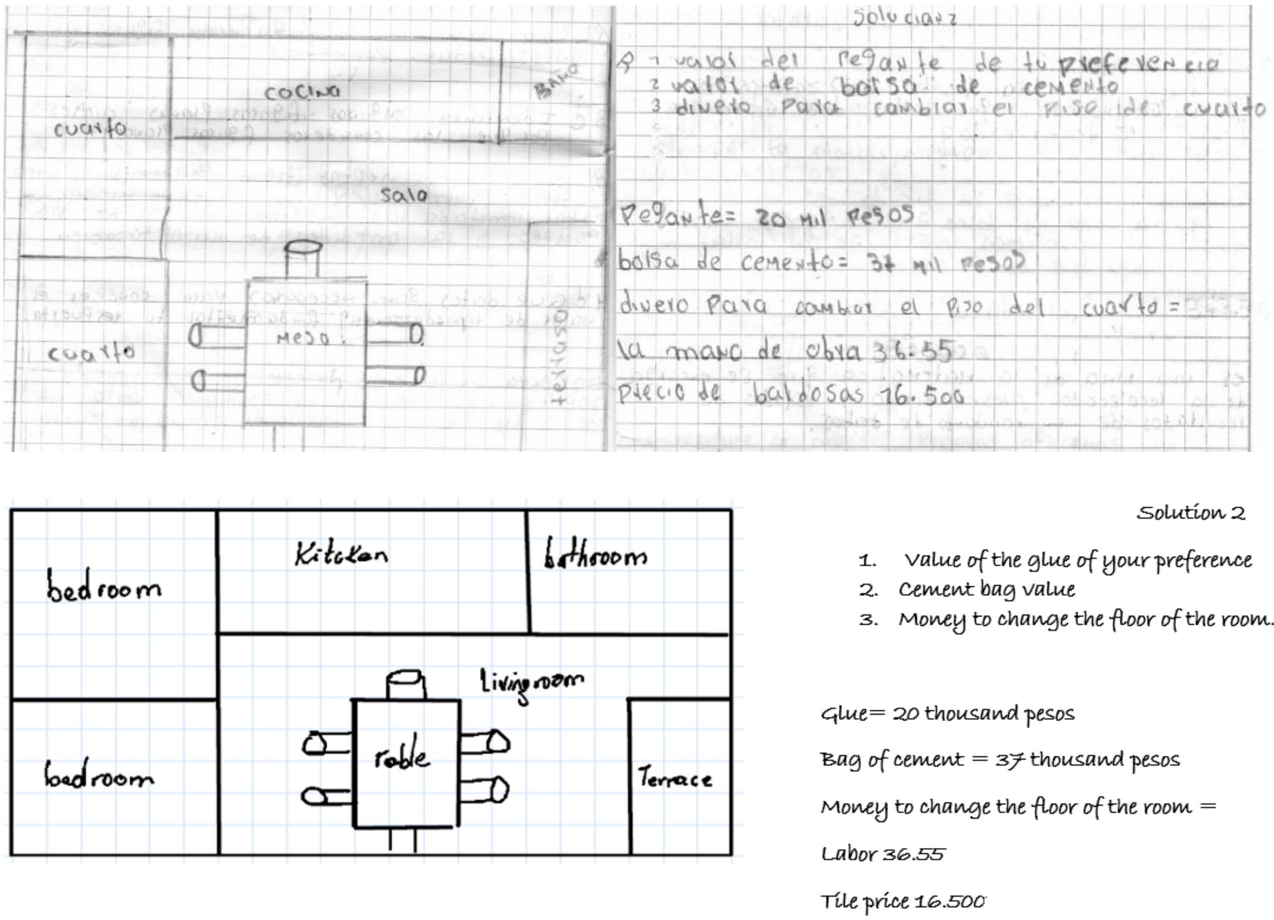


Figure 6. Solution to problem 2 example 2

Transcript 2: Student Perceptions of Activity 2

Teacher: When you were doing this activity, what did you feel?

Students: I felt happy 6 S what the area of plane figures was for.

Teacher: Why?

Students: Teacher because I understood.

Teacher: What did you understand?

Students: what the area of plane figures was for.

Students: that was precisely the intention of this activity, to show you that math is not only on the board, but also outside, for example, the person who laid this floor, went through the same process that you did.

Activity 3

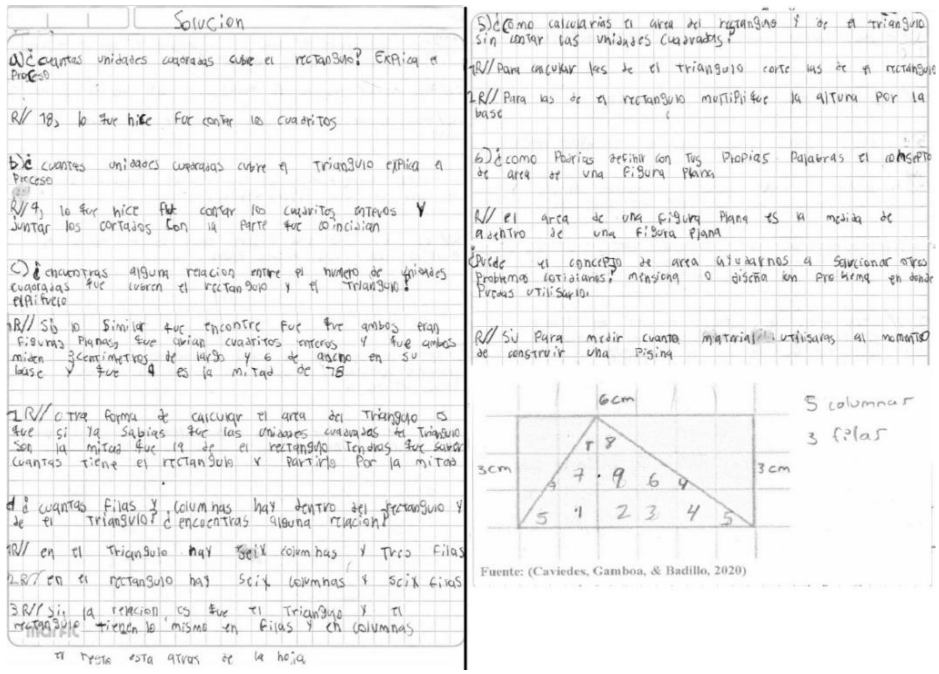
After the development and assessment of Activity 3, the following results were obtained (See Table 4).

Table 4. Result of checklist activity 3.

Objectives	Descriptors	Assessment		
		Correct	Acceptable	Incorrect
Propose and develop strategies for estimating, measuring, and calculating areas to solve problems.	1. Recognizes, in different contexts or in problem situations, the calculation of the area of plane figures, and represents them in their different manifestations.	12	22	3
	2. Explain with arguments, verbally or in writing, his/her concept of plane figures.	4	8	25
	3. Explain with arguments, verbally or in writing, his/her concept of area of plane figures.	9	25	3
	4. Calculates and uses the area of plane figures in their different manifestations when solving a problem and concludes.	8	25	4
	5. Solves mathematical problems and tasks involving the calculation of the area of plane figures.	23	12	2
	6. Communicates the processes developed to arrive at the solution to the problem.	7	12	17

For the students, recognizing, representing in one of its manifestations, and calculating the area of plane figures (Corberan, 1997) for the problem's development was a strength, as most of the students were able to calculate the area of at least one of the two proposed figures. In addition, for the students, communicating the processes developed in this activity did not pose a challenge, as 35 students communicated some or all of the processes developed to answer the problem.

However, describing the concept of the area of a plane figure from a theoretical perspective was a challenge, as only four students were able to fulfill it. Yet, in most cases, the lack of knowledge of the theoretical concept did not have a direct relationship with the development of the concept from a practical point of view, that is, most students did not comply with the explanation of the definition of the area, but they did apply the practical part when calculating the area of the rectangle or the triangle (Figures 7 and 8).



a) how many square units does the rectangle cover? Explain the process.

A// 18, what I did was to count the squares.

b) how many square units does the triangle cover? explain the process

A// 4, what I did was to count the whole squares and join the cut ones with the part that matched.

c) do you find any relation between the number of square units that cover the rectangle and the triangle? explain it

1A// yes, the similarity I found was that both were flat figures that had squares and that both were 3 centimeters long and 6 centimeters wide at their base and that 9 is half of 18.

2A// another way to calculate the area of the triangle is that if you already knew that the square units of the triangle are half that of the rectangle you will have to know how many square units the rectangle has and divide them in half.

d) How many rows and columns are there inside the rectangle and the triangle? do you find any relationship?

1A// in the triangle there are six columns and three rows.

2A// in the rectangle there are six columns and six rows.

3A// Yes, the relationship is that the triangle and the rectangle have the same number of rows and columns.

The rest is at the back of the sheet.

5) How would you calculate the area of the rectangle and the triangle without counting the square units?

1A// to calculate the triangle cut the rectangle.

2A// For those of the rectangle, multiply the height by the base.

6) How could you define in your own words the concept of area of a plane figure?

A// the area of a plane figure is the measurement of the inside of a plane figure.

¿ Can the concept of area help us solve other everyday problems? mention or design a problem where you can use it.

A// yes to measure how much material you will use when building a pool

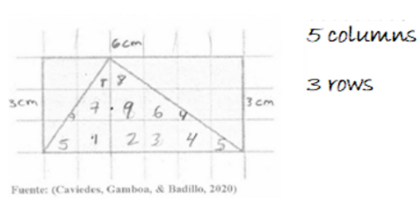
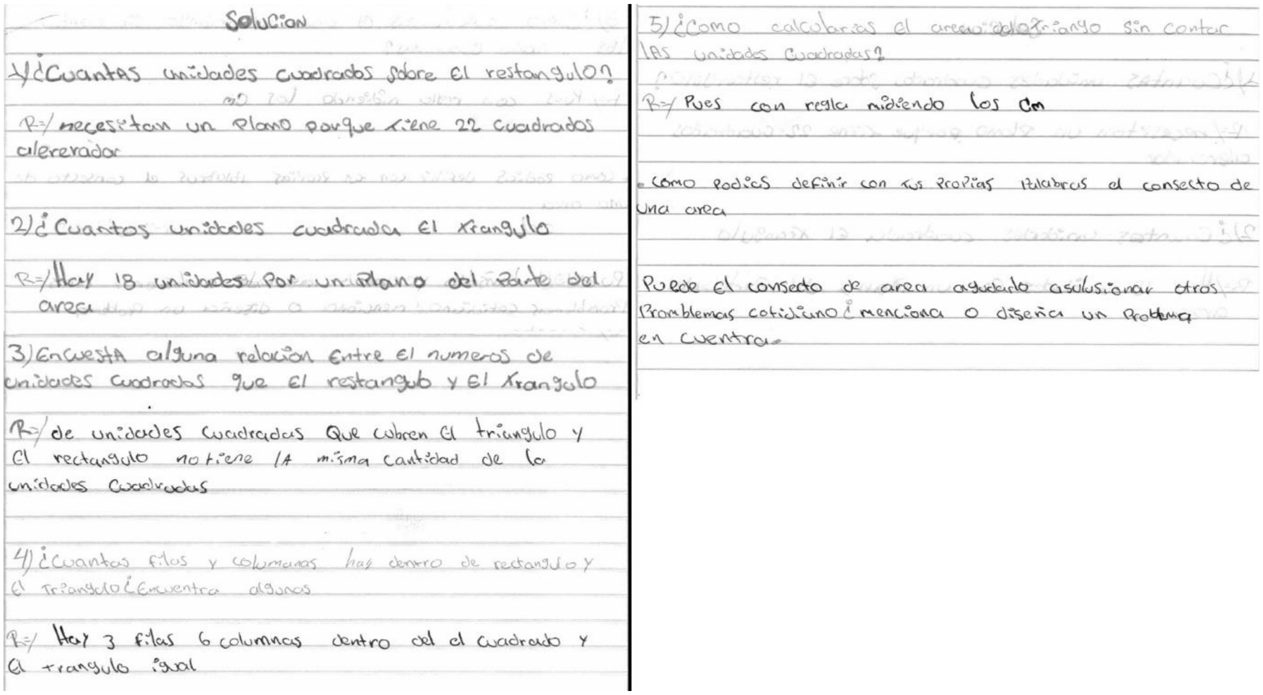


Figure 7. Solution to problem 3 example 1

In the previous image, the student initially identifies the use of the concept of area to respond and represents it in writing. The manifestation of the area as the amount of plane occupied by a surface (Corberan, 1997) is materialized when the student communicates the process carried out to count the square units of both the rectangle and the triangle. Additionally, the student uses the image of the problem to uniquely count the square units, which demonstrates the use of the resource to identify and represent the concept of area. The student also makes this concrete by defining the process and the concept in question considering some mathematical signs and the communicated arguments, which is an example of a resolution that, in general terms, is at a level of compliance (C) with the descriptors inherent to the problem.

In contrast to this, in the example shown in Figure 7, it can be observed that the student attempts to represent the area by means of the manifestation of the area as the number of square units of a surface but does so incorrectly. The student is also not capable of defining the concept of the area of plane figures and does not communicate the procedures developed to arrive at the solution to the problem. This is an example of a resolution that, in general terms, is at a level of non-compliance (NC) with the descriptors.

It was evidenced that the use of mathematical symbols for the generalization of information about the concept and calculation of the area of plane figures represented a weakness for the students, as 17 students did not meet this criterion. This was because when they were asked, "How would you calculate the area of the triangle without counting the square units?" it was expected that they would relate the number of square units used to cover the rectangle with those of the triangle, to conclude that the area of the triangle is half that of the circumscribed rectangle. Taking into account the results of the previous activities, there is an improvement in recognizing the area of plane figures and calculating the area of quadrilaterals, as well as in communicating the processes to solve the problem. However, a weakness in the formal conceptualization of the area of plane figures is observed. See Figure 8.


Solucion

1) ¿Cuantas unidades cuadrados sobre el restangulo?
R=/ necesitan un plano porque tiene 22 cuadrados alrerador

2) ¿Cuantos unidades cuadrada el triangulo
R=/ Hay 18 unidades por un plano del parte del area

3) Encuesta alguna relacion entre el numeros de unidades cuadrados que el restangulo y el triangulo
R=/ de unidades cuadradas que cubren el triangulo y el rectangulo no tiene la misma cantidad de la unidades cuadradas

4) ¿Cuantas filas y columnas hay dentro de rectangulo y el triangulo ¿Encuentra algunos
R=/ Hay 3 filas 6 columnas dentro del el cuadrado y el triangulo igual

5) ¿Como calcularas el area del triangulo sin contar las unidades cuadradas?
R=/ Pues con regla midiendo los cm

¿Como podrías definir con tus propias palabras el consecto de una area

Puede el consecto de area ayudarlo a solucionar otros problemas cotidiano ¿menciona o diseña un problema en cuentra

1) How many square units over the rectangle?
A/ they need a plan because it has 22 squares around it.

2) How many units square the triangle
A/ there are 18 units for a plan of part of the area.

3) find some relationship between the number of units that the rectangle and the triangle
A/ of square units that cover the triangle and the rectangle do not have the same number of square units.

4) How many rows and columns are there inside the rectangle and triangle? Can you find any?
A/ there are 3 rows 6 columns inside the square and the same triangle.

5) How would you calculate the area of the triangle without counting the square units?
A/ Well, with a ruler measuring the cm.

How could you define the concept of an area in your own words?

Can the concept of area help you solve other everyday problems? Mention or design a problem.

Figure 8. Solution to problem 3 example 2

DISCUSSION

The activities in this study were carefully planned to cover theoretical and practical aspects to unravel the complexity surrounding student involvement and comprehension of the idea of area. The exercises aimed to promote students' holistic, metric thinking by using methods of estimate, measurement, and area computation together with the recognition of flat figures. Looking more closely at the results, a trend emerged showing that students, especially in the early going, mistakenly used the perimeter calculation as a stand-in for the area calculation, demonstrating a concrete misunderstanding. This did not translate to other tasks, suggesting that this misunderstanding may be cleared up with repeated completion of the tasks. But even as practical comprehension appeared to be expanding, there was still a noticeable gap in the area concept's formalization and generalization.

These results are consistent with the research of Winarti et al. (2012), which suggests that students' comprehension of area and perimeter may be significantly enhanced by activities connected to their immediate environments. There is, however, a difference in the application of the contextual activities' focus: whereas Winarti et al. investigated the connection between perimeter and area, the current study directed the contextual activities toward comprehending the notion of area, first from a utilitarian and then from a theoretical perspective. Furthermore, the modification of Activity 3, one of the area activities suggested by Caviedes et al. (2023), was helpful in recognizing various approaches or ways to compute the area. Compared to Caviedes et al., the current study significantly increased the use of geometric processes, which may have been caused by the extra-mathematical context in which the issues were implemented (Activities 1 and 2).

In contrast to the methods suggested by Gutierrez & Jaime (1991), the study proceeded through the first three levels of Van Hiele, guaranteeing a comprehensive investigation of extra- and intra-mathematical activities. Students' development and critical evaluation of the first three Van Hiele levels were greatly aided by this two-pronged approach. Parallel to this, the study showed that the suggested activities strongly emphasized the mathematical process of problem-solving even if they were designed to encourage the learning of the area idea. Thus, as transcript 1 illustrates, the exercises followed Polya's (1945) definition of issues, which calls for investigation and contemplation before coming up with answers. This supports the claims made by Ortiz & Salcedo (2018) about the critical role didactic issues play in mathematical learning, which are further supported by the observable advantages that arise when activities are used in an extra-mathematical setting in this study. However, in order to improve the effectiveness of this approach, a second phase of research can be considered that takes into account the weaknesses of the students. This second phase has been designed in detail to allow teachers to carry it out in different classes.

This guideline integrates theoretical foundations and practical applications to create an instructional manual that addresses student misconceptions, improves plane figure recognition, builds argumentative competencies, and efficiently uses PBL paradigms.

Objective: Address student misconceptions and errors in solving area problems through structured teaching and learning activities.

- Step 1: Directly address misconceptions. Misconceptions about perimeter: Develop activities that clearly differentiate between the concepts of perimeter and area, ensuring that students do not conflate the two.
- Step 2: Facilitate argumentation skills. Encourage verbalization: In problem-solving sessions, encourage students to verbalize their thought processes and reasoning. Promote logical reasoning: Engage students in activities that enhance their logical reasoning and argumentation skills in mathematical problem-solving.
- Step 3: Enhance recognition and understanding of plane figures. Visual Recognition: Use visual aids and physical models to enhance recognition and understanding of different plane figures and their properties. Explore Properties: Engage students in activities that allow them to explore and understand the properties of different plane figures.
- Step 4: Leverage PBL with real-world contexts. Use of real-world problems: Engage students with problems that have tangible, real-world contexts, which, according to the research, enhances understanding. Collaborative problem-solving: Use PBL in a group setting to allow students to share and discuss various strategies for problem-solving.
- Step 5: Integrate continuous assessment. Feedback on misconceptions: Provide continuous feedback, especially targeting misconceptions about area and perimeter, and argumentation during problem-solving. Reflection on Mistakes: Engage students in reflective activities where they analyze and learn from their mistakes and misconceptions.
- Step 6: Explore extra-mathematical and intra-mathematical problems. Diverse problem contexts: Ensure that problems presented to students are diverse, including both extra-mathematical and intra-mathematical contexts, to enhance applicability and theoretical understanding.

CONCLUSIONS

The conclusion drawn from the application of activities in the didactic unit enabled the accurate representation of various manifestations of area and its utilitarian character. Nevertheless, the implementation of these activities did not significantly contribute to the generalization and formalization of the concept of the area of flat figures. Furthermore, these activities facilitated the utilization of various heuristics proposed by the Anglo-Saxon school of problem-solving. The employment of problems involving extra-mathematical contexts can have a pronounced impact on enhancing the process of problem-solving and the appropriation of mathematical objects, as evidenced by the results of activities 1 and 2 (extra-mathematical context), contrasted with the outcomes of activity 3 (intra-mathematical context). Problem-Based Learning, in tandem with the manifestations of the area proposed by Corberan and the levels of Van Hiele, emerges as a viable strategy to promote the problem-solving process related to the area of flat figures.

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