Exploring Learning Difficulties in Convergence of Numerical Sequences in Morocco: An Error Analysis Study

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Abstract: *Numerical sequences are one of the mathematical subjects linked to everyday life, and are taught at several levels in Morocco. However, many students still had difficulty teaching this subject, more specifically the limits of numerical sequences and the nature of convergence. The aim of this study was to analyses the learning difficulties of students in the 2nd year of the baccalaureate, experimental sciences series in Morocco. To this end, a written test consisting of 6 questions relating to the notion of convergence of a numerical sequence was administered to 60 students, followed by a questionnaire sent to mathematics teachers at the qualifying secondary level. The results show that the learning difficulties associated with this concept are of several kinds: didactic, pedagogical and epistemological. Using the "Teaching at the Right Level" approach (TARL), teachers can help learners overcome the learning difficulties of numerical sequence convergence by providing support tailored to their individual needs. This promotes progress that is more effective and a better understanding of mathematical concepts. This can help students overcome learning barriers by offering varied teaching aids and promoting a better understanding of the concepts of convergence of numerical sequences.*

Keywords: Learning difficulties, Number sequences, Convergence, Learning obstacles

INTRODUCTION

The concept of convergence of numerical sequences is fundamental in mathematics, particularly in analysis. It allows us to study the behavior of sequences and determine whether they tend towards a specific limit value as the index of the sequence increases.

Learning mathematics is a major challenge for many students around the world, and Morocco is no exception. Among the mathematical concepts that often pose difficulties for science baccalaureate students is the convergence of numerical sequences, which occupies a central place. Understanding this key concept is essential for developing mathematical skills and preparing students for higher education.

There is now a well-documented research literature, testifying to students' difficulties with understanding the concept of limit (Cornu, 1983, Cottrill et al., 1996, Williams, 1991). Here we set out some of the misconceptions reported: - The limit is the last term of an infinite sequence. - A limit is a boundary that cannot be crossed. - A limit is a number that can eventually, theoretically, be reached (Flores & Park, 2016, Swinyard & Larsen, 2012). Mamona-Downs (2001) has proposed three didactic steps for teaching the concept of the limit of a sequence as follows: (i) Initiate and develop intuition by raising problems in class discussion. (ii) Introduce the formal definition and analyze it in conjunction with (i). (iii) Endorse or revoke the opinions expressed in step (i) in relation to the formal definition, in particular via the representation in (ii) above. The difficulties associated with the notion of limit are more conceptual than computational. Current textbooks focus on techniques for calculating limits, and propose either a formal (rigorous) or an informal (intuitive) definition of the limit of a sequence. In reality, the debate between informal and formal teaching of Calculus concepts is "a problem that runs through the history of calculus" (Moreno-Armella, 2014). Students do not seek to understand independently, but are used to being consumers of the teacher's explanations. In this case, students' potential for reflection is not optimal, and their understanding is only partial. Aztekin (2020) aimed to investigate students understanding of the concept of limit in mathematics using the repertory grid technique. The repertory grid technique is a method that involves identifying and analyzing the constructs that individuals use to make sense of a particular subject. In the study, Aztekin applied this technique to a group of mathematics students to explore their conceptualizations and misconceptions related to the limit concept. The findings of the study provided insights into the students' understanding of limits and highlighted common misconceptions that need to be addressed in mathematics education. Arnal-Palacián et al. (2020) desired to find and characterize an infinite limit of sequences that is recognized by specialists in mathematics, and then examine the phenomenology of that limit. They consulted experts to select a suitable definition of the infinite limit of sequences and then explored its phenomenological aspects using intuitive and formal approaches. The study determined and defined phenomena including one-way and return infinite limits, infinite intuitive growth, and endless intuitive reduction that are associated with the infinite limit of sequences. The authors illustrated these phenomena with verbal, graphical, and tabular representation approaches. The study aimed to provide support for pre-university students in understanding and overcoming difficulties associated with the concept of limit in mathematics. Rachma and Rosjanuardi (2021) use an onto-semiotic method to investigate the challenges that students have when learning sequences and series. The study adopts a didactical and interpretive research approach to analyze the epistemological, ontogenic, and didactical obstacles that students encounter. The study findings elucidate that students encounter obstacles in accurately conceptualizing mathematical ideas within the framework of sequences and series. They are able to convert an issue into a mathematical model, but they struggle to apply the right process. The identified obstacles they categorized as didactical, ontogenic, and epistemological obstacles. The onto-semiotic approach used in this study helps better understand the specific difficulties that students face in learning sequences and series. The findings of this study hold practical implications for teachers, allowing them to develop tailored learning environments they better aligned with the needs of students in mastering the topic at hand.

Learning difficulties linked to the convergence of a numerical sequence can be of several kinds: pedagogical, didactical and epistemological. To verify or refute the validity of this conjecture, practical investigations will be carried out. This situation raises an interesting question: What are the difficulties encountered in learning about the convergence of numerical sequences? What are the causes of these difficulties? This article focuses on the explicit study of learning difficulties specific to the convergence of a numerical sequence in students in the 2nd year of the baccalaureate, experimental science series. We have used a mixed-methods approach, combining quantitative and qualitative methods. This will enable us, on the one hand, to determine the difficulties encountered, and on the other, to describe the reasons for these difficulties via a test focusing on different axes of the convergence of numerical sequences in the final year. Next, a didactical analysis of the content of the chapter dealing with the convergence of a numerical sequence will be conducted, and the corresponding interpretations and conclusions will constitute the final part of this article.

THEORETICAL FRAMEWORK

In this section, the theoretical framework based on the definition of the limit of a sequence, then the learning obstacles, the typology of errors and the difference between error, obstacle and difficulty.

The notion of limit

The notion of “limits” holds significant importance within the field of analysis due to its interconnectedness with various other fundamental concepts, including derivatives, integrals, and continuity. The comprehension of limits is frequently considered as challenging, primarily due to the involvement of abstract ideas such as infinity, the infinitesimal, and transfinite. Research suggests that students often acquire a superficial understanding of this subject matter, as evidenced by studies conducted by (Arnal et al., 2017, Chorlay, 2019).

The definition of a concept plays a crucial role in its technical application, distinct from its intuitive and colloquial interpretations. In the particular context of characterizing a sequence's infinite limit, as Arnal-Palacián et al. (2020), has already mentioned, this definition is commonly found in textbooks shortly before introducing the concept of the infinite limit of a function, and sequentially following the discussion on the finite limit of a sequence.

In light of the multitude of definitions pertaining to the infinite limit of a sequence, Arnal et al. (2017). Vinner (1991) undertook a consultation process with university professors and secondary education teachers. The objective was to identify a precise definition that would be universally accepted within the mathematical community across various educational levels, the resultant definition that was chosen is as follows:

> "Let F be an ordered field and $\{an\}$ a sequence of elements of F. The sequence $\{an\}$ has a "plus infinite limit", if for each E element of F, there exists a natural number v, so that an > E for all $n \geq v$" (Arnal-Palacián et al., 2020) .

> "Let F be an ordered field and $\{an\}$ a sequence of elements of F. The sequence $\{an\}$ has a "minus infinite limit", if for each E element of F, there exists a natural number v, so that an < E, every time $n \geq v$" (Arnal-Palacián et al., 2020).

The notion of obstacle

Brousseau has provided a definition of obstacles that arise from the interaction between students and a didactic situation during the process of acquiring knowledge. However, this interaction can sometimes result in the formation of misconceptions (Brousseau, 2002). The obstacles encountered during the learning process can be classified into three main categories according to Guy Brousseau. Firstly, epistemological obstacles refer to forms of knowledge that become ineffective in a new context and are inherent to the nature of the targeted concepts, as evidenced by the history of these concepts. Secondly, didactical obstacles are created by the teaching methods themselves, sometimes due to inappropriate approaches or deficiencies in educational resources. Lastly, ontogenical obstacles are related to the specific limitations of the learner at a given moment in their cognitive development, thus reflecting the challenges inherent to their individual progression (Brousseau, 1976).

According to Bachelard (2020), didactical obstacles differ from mathematical errors and can be classified into several categories based on their origins, as follows: linguistic obstacles, notation obstacles, task design obstacles, motivational obstacles, cultural obstacles, obstacles related to negative beliefs, and obstacles related to teachers' mathematical knowledge (Bachelard, 2020).

The typology of errors

Jean-Pierre Astolfi, in his book "L'erreur, un outil pour enseigner", "Mistakes as a teaching tool" (Astolfi, s. d.), analyzes the different natures of errors committed by students, such as comprehension errors, alternative conceptions, cognitive overload and discipline-specific comprehension. He proposes positive listening to what students express, and suggests that teachers should be aware of the clarity of their instructions and understand the difficulties encountered by their students. Constructivist models can be used to integrate errors into the learning process and help students acquire new knowledge. Astolfi has identified 8 types of errors made by students:

- Understanding instructions: The student does not understand the instructions and cannot execute the didactic contract, the problem may stem from the difficulty of the statement.
- Errors resulting from school habits or poor understanding of expectations: The student learns according to a mechanical principle (habitual pedagogy), then finds it difficult to respond to instructions that do not conform to his or her habits.
- Errors due to cognitive overload: memory limitations or inappropriate estimation of the cognitive load of the activity, when the student has to process several pieces of information at the same time.
- Students must systematically reinvest their knowledge in each subject. This requires constant gymnastics on the part of the brain, which sometimes leads to errors.
- Errors reflecting alternative conceptions: “The mind can only be formed by reforming itself”: Bachelard's idea of the obstacle (Bachelard, 2020).
- Errors concerning the approaches adopted: The methods used by students do not necessarily correspond to what the teacher expects. However, teachers, who expect very precise answers, often perceive this approach as a mistake. Pupils already have intellectual representations of the concepts they are studying, so they do not wait for the teacher's lesson to give them explanations.
- Errors caused by the inherent complexity of the content: the teacher has to explain the instructions to the students. If the student pretends not to understand the instruction or the context, an error will result.
- Errors linked to the intellectual operations involved: The student does not have the necessary skills to meet the teacher's expectations.

The difference of difficulty, error and obstacle

When comparing difficulty and obstacle, there are at least two main differences to be noted: the relationship to prior knowledge and the relationship to external factors. According to Brousseau, G (2002), difficulty can be measured as the difference between a correct response and an incorrect response (in relation to the previously treated probability of success), and there is no parasitic knowledge to contend with, meaning that it need not always be the outcome of prior knowledge. According to Bachelard, G (2020), however, difficulties might arise from prior knowledge rather than from a lack of knowledge or external impediments (complexity, fugacity, etc.). Additionally, the history of science and genetic epidemiology emphasize that errors and difficulties are inherent to every envisaged body of knowledge.

These three concepts (error, difficulty, and obstacle) have complementary meanings and connections despite their definitional differences. As an illustration, let us consider Brousseau (2002) definition of an obstacle, which emphasizes that an obstacle in mathematics manifests as a set of shared difficulties among numerous actors (individuals or institutions) who share “an inappropriate conception of a mathematical notion”. Similarly, Alibi and Boilevin (2021) defines difficulty as “something that is difficult, such as an obstacle or a barrier...”. Then it was said that a difficulty might be overcome, circumvented, or redirected, something that is difficult.

It is noted that researchers express one idea by means of another (Alibi & Boilevin, 2021). In fact, Lajoie (2009) uses the word “obstacle” to define the concept of “difficulty”, Vergnaud (1988) and Brousseau (2002) agree. Additionally, Brousseau defines a barrier as a particular group of challenges. Moreover, in addressing the first approach to hurdles, he states that even when difficulties seem to go away, they will reappear and lead to mistakes if the notion is resilient.

METHOD

For this study, we used a mixed methods approach, combining quantitative and qualitative methods. On the one hand, it will enable us to determine the difficulties encountered by students in learning to converge numerical sequences, and on the other, to describe the reasons for these difficulties by means of a test focusing on different aspects of the convergence of numerical sequences in the final year.

Choice of population

The test is designed for students aged between 17 and 18 years old, in the 2nd year of the baccalaureate in the experimental sciences, physical sciences stream, French option, who number 60 and represent two classes at the Lefkih Tetouani secondary school in the town of Salé in the Rabat-Salé-Kenitra regional education and training academy.

Test elaboration

The concept of the convergence of numerical sequences was chosen because of its importance in the mathematics curriculum in qualifying secondary education in Morocco. The test is used to identify the various errors made by students in the convergence of numerical sequences.

Another aspect revealing the relevance of our choice lies in the fact that the convergence of numerical sequences is of significant importance as a transition point between high school and university level in the context of mathematics teaching. This notion occupies a fundamental place in the continuity of students' mathematical learning during their transition from secondary education to higher education.

Numerical Sequences according to the official pedagogical orientations (OPO)

The content of the chapter on numerical sequences according to the school curriculum (MEN, 2007) for classes in the experimental science series. See Table 1.

Table 1. Pedagogical recommendations.

Skills required	Pedagogical recommendations
*Using the limits of reference sequences and convergence criteria is a methodological approach to determining the limits of numerical sequences. * The use of sequences enables us to solve problems from a variety of professional fields, by proposing methods and models for their solution. *Study the convergence of sequences of the form: $U_{n+1} = f(U_n)$ and $V_n = f(U_n)$. * The limit of geometric sequence (a^n).	*Based on the limits of certain reference functions, the limits of reference sequences will be admitted. *Operations on finite and infinite limits are considered valid, and it is essential that students acquire the habit of using them appropriately. * The criteria for convergence of a sequence are accepted, and their approach is based on the compatibility of operations on limits and order in a sequence. $\mathbb{R}$.

The questionnaire is composed of the following questions (Table 2).

Table 2. Test on the convergence of a numerical sequence.

Questions	Capabilities required
Q1. If you had to explain a convergent sequence to one of your classmates. What would you say?	*Recognize the definition of a convergent sequence.
Q2. Let's consider the sequence (w_n) defined by: $w_n = \ln(u_n)$ such that: $\lim_{n\to+\infty} u_n = 1$ and $(\forall n \in \mathbb{N}): u_n > 0$. Determine $\lim_{n\to+\infty} w_n$.	*Recognize a sequence of type: $v_n = f(u_n)$ and determine its limit.
Q3. Study the convergence of the sequence (u_n) defined by: $(\forall n \in \mathbb{N}): u_n = \frac{2n-3}{3n+2}$.	*Study the convergence of a numerical sequence.
Q4. Let's consider the sequence (w_n) defined by: $(\forall n \in \mathbb{N}^*): w_n = \frac{\sin(n)}{n}$.	*Use convergence criteria to determine the limit of a numerical sequence.

What can we say about the nature of convergence of the sequence (w_n).

Q5. . Let's consider the sequence (v_n) defined by :

$(\forall n \in \mathbb{N}) : v_n = \frac{2^n - 4^n}{4^n + 3^n}$.

Calculate the limit: $\lim_{n \to +\infty} \frac{2^n - 4^n}{4^n + 3^n}$. What can we say about the convergence of the sequence (v_n).

*Limit of geometric sequence (q^n).

Q6. Let's consider the sequence (u_n) defined by: $u_0 \in]0;1]$;

$u_{n+1} = \frac{u_n}{2} + \frac{u_n{}^2}{4}$.

a. Is (u_n) a convergent sequence?

b. Determine the limit of the sequence (u_n).

*Recognize a sequence of type: $u_{n+1} = f(u_n)$ and determine its limit.

RESULTS

There had been a Cronbach's test (Table 3), six questions were used as variables, and a binary approach was adopted for the analysis, where responses were classified as 'yes' or 'no'. According to Cronbach's reliability analysis, the test's alpha coefficient is equal to 0.778. This coefficient is used to assess the internal consistency of the test. An alpha value greater than 0.7 is generally considered acceptable in terms of reliability, although higher values are preferable. With an alpha coefficient of 0.778, our test demonstrates good internal consistency, although additional measures could be taken to further enhance its reliability.

Table 3. Reliability statistics.

Cronbach's Alpha	Number of elements
,778	6

Based on the learning obstacles described in the introduction and following analysis of the students' copies, the following results were revealed:

General results

Depending on the error situation for each question, the distribution of students tested is shown in the figure below (Figure 1):

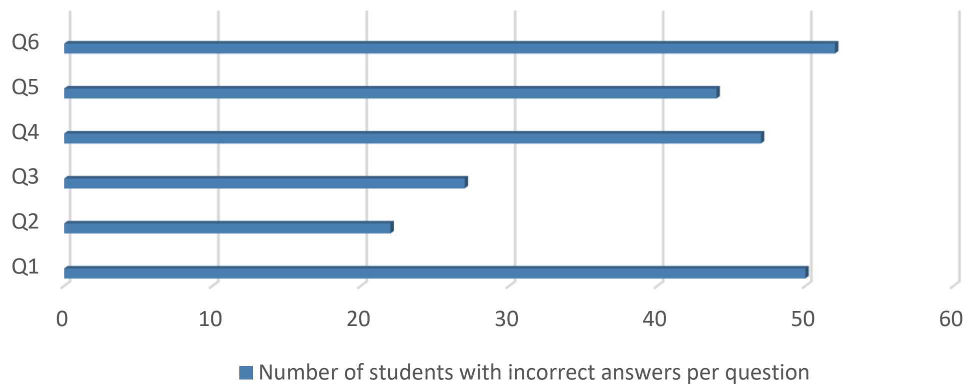


Figure 1. Distribution of students by number of errors made

Analysis of the data collected from 60 students reveals variations in the distribution of errors for the different questions studied in Figure 1. Of the six questions, question 1 shows the highest number of errors, with 50 errors made by the students. This observation suggests that this question posed significant difficulties for a considerable percentage of the students in the sample. Similarly, question 6 also stands out for its high number of errors, with 52 recorded. This finding indicates that this question was particularly difficult for a substantial number of students. Questions 4 and 5 also show relatively high error levels, with 47 and 44 errors recorded respectively. These results suggest that these questions presented significant challenges for students. In contrast, questions 2 and 3 generated a relatively lower number of errors, with 22 and 27 errors respectively. These figures indicate that these questions were relatively more accessible to students than the other questions. Analysis of the distribution of errors reveals significant differences in understanding of the notion of convergence of a numerical sequence for the different questions. Identifying these variations raises questions about the factors that led to these results, leading us to analyze the errors made by the students via an analysis of the obstacles linked to these errors and with the help of a questionnaire addressed to qualifying secondary school teachers.

Distribution of Obstacles by Student Errors: Insights from Teacher Surveys

The teacher survey revealed the following results concerning the most frequent errors linked to the convergence of numerical sequences:

1. Error 1: A significant majority of 95% of the surveyed teachers acknowledged the prevalence of this error in student work.
2. Error 2: The entirety of 100% of the participating teachers identified this error as a commonly occurring mistake.
3. Error 3: The vast majority of 90% of the educators acknowledged the frequent occurrence of this error.

4. Error 4: A consensus of 95% of the teachers concurred with the presence of this error.
5. Error 5: Additionally, 95% of the teachers recognized this error as commonly encountered.
6. Error 6: A substantial proportion of 86% of the surveyed teachers agreed on the recurring nature of this mistake.

These results indicate a high degree of consensus among teachers regarding the most common errors in teaching the convergence of numerical sequences. Errors 2, 4 and 5 were particularly recognized, with agreement from 100% or 95% of teachers. Further analysis of the reasons behind these errors could help develop effective teaching strategies to correct them. See Figure 2.

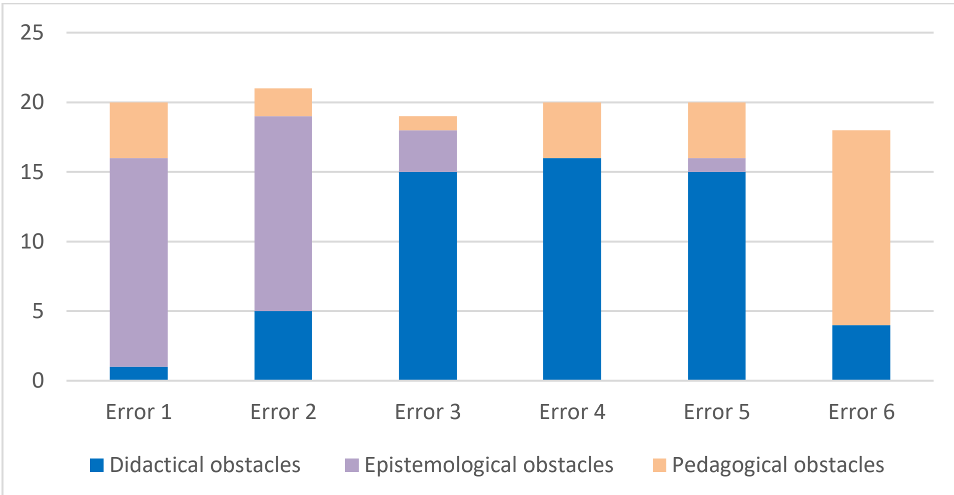


Figure 2. The distribution of obstacles according to student errors

The results reveal that errors 1 and 2 are associated with an epistemological obstacle, while errors 3, 4 and 5 are linked to a didactical obstacle. Error 6, on the other hand, is related to a pedagogical obstacle. Here is a more detailed explanation of these terms in a scientific context:

Epistemological obstacle: Errors 1 and 2, identified as being linked to an epistemological obstacle, indicate a profound conceptual difficulty on the part of learners. This suggests that some students are struggling to build a solid understanding of the fundamental concepts of numerical sequence convergence, perhaps due to their prior misconceptions or lack of understanding of theoretical principles.

Didactical obstacle: Errors 3, 4 and 5, classified as didactic obstacles, refer to difficulties linked to the specific teaching and learning of the convergence of numerical sequences. This may include issues such as inappropriate teaching methods, confusing conceptual presentations or gaps in available teaching resources. These obstacles can hinder the effective transmission of the knowledge and skills needed to correctly understand and apply the concepts of sequence convergence.

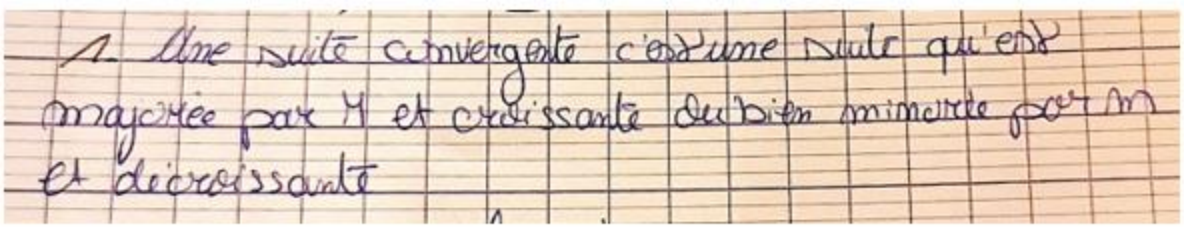
Pedagogical obstacle: Error 6 is associated with a pedagogical obstacle, implying a difficulty in the teaching process itself. This may be due to communication problems between teacher and learners, poor organization of teaching sessions, lack of clarity in explanations or other factors that hinder the effective transmission of knowledge.

The results of the survey confirmed the analysis of student errors in the next section, according to which errors 1 and 2 are linked to an epistemological barrier, errors 3, 4 and 5 are associated with a didactical obstacle and error 6 is related to a pedagogical obstacle. This consistency between the results of the survey and the previous analysis reinforces the validity of the scientific interpretation of the data collected, confirming our research hypotheses.

Individual student errors

From our analysis of the copies and answers to the student questionnaire, we have identified three types of difficulty: epistemological, didactical and pedagogical. We illustrate each type of difficulty with an extract from a copy or an answer to a question.

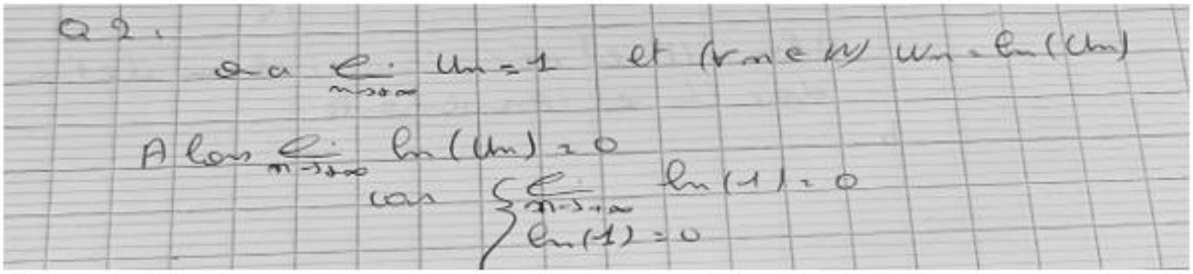
Epistemological obstacles



Translating in English:
A convergent sequence is a sequence that is bounded above by M and increasing, or bounded below by m and decreasing.

Figure 3. Student 1

In question 1, 50 out of 60 students answered in the same way as student 1 (Figure 3), using the characteristic property of a convergent sequence rather than the definition. In this situation, a potential epistemological obstacle is the tendency of students to focus on the characteristic properties of a convergent sequence rather than understanding and applying the formal definition of convergence. This may be the result of previous learning based on specific examples of convergent sequences, where students have associated these characteristic properties with the notion of convergence without being exposed to a deeper understanding of the definition. This can lead to a superficial understanding of the notion of convergence and to conceptual errors. Students may use only properties such as the finite limit or the progressive approach to the terms of the sequence to judge convergence, without fully grasping the idea of convergence to a specific limit value. They may also have difficulty differentiating between the notions of convergence and limit, using these terms interchangeably.



Translating in English:
We have: $\lim_{n\to+\infty} u_n = 1$ and $(\forall n \in \mathbb{N})$: $w_n = \ln(u_n)$. Then: $\lim_{n\to+\infty} \ln(u_n) = 0$ because: $\lim_{n\to+\infty} \ln(1) = 0$ and $\ln(1) = 0$.

Figure 4. Student 2

In question two, it was observed that 48 students gave a correct answer to the question posed but insufficient (Figure 4). However, closer analysis reveals that these students neglected or forgot to take into account the continuity of the function at 1 in their reasoning. Yet, despite the need to consider this continuity, all 48 students omitted or overlooked this key feature when solving the question. This may be due to a lack of understanding or attention to the notion of continuity, or confusion about its importance in the context of the question. This omission of the continuity of the function in 1 may lead to misinterpretation of the results or an incomplete answer. Students may find it difficult to grasp the influence of this continuity on the behavior of the function and to draw accurate conclusions.

Didactical obstacles

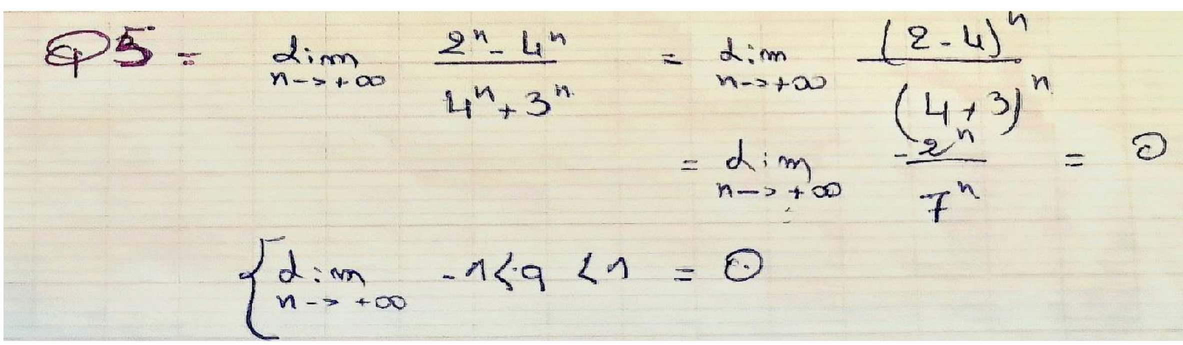


Figure 5. Student 3

In this context, student 3 (Figure 5) made a mistake by incorrectly applying the remarkable identities when calculating this limit. This leads us to a didactic obstacle, as the correct use of remarkable identities is essential for simplifying and solving algebraic expressions. Incorrect application of these identities can lead to errors in the calculations and results obtained. It is therefore important for students to understand and master these remarkable identities correctly, in order to apply them appropriately in their mathematical calculations.

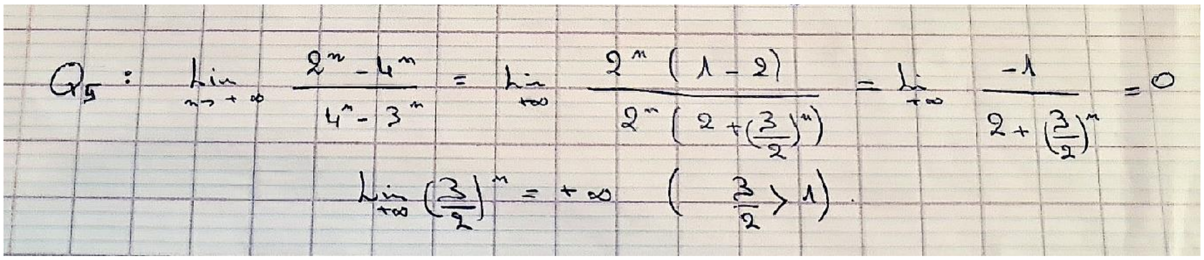


Figure 6. Student 4

Error in factoring can take different forms, as in Student 4 (Figure 6). The student may incorrectly apply these rules or choose an inappropriate method to factor the expression, leading to an incorrect answer. The lack of sufficient examples can be a challenge when it comes to understanding a concept or method. When the examples provided by the teacher are limited in number or variety, this can make it difficult for students to fully grasp the different applications of limit calculus.

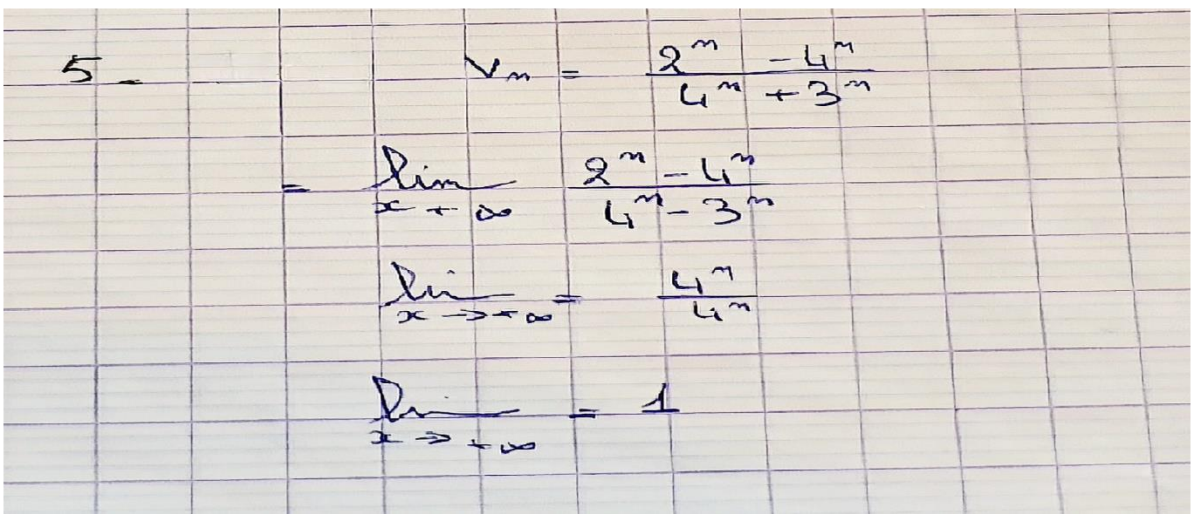


Figure 7. Student 5

Student 5's mistake (Figure 7) was to confuse two different concepts. He probably tried to calculate the limit of a rational function using traditional limit calculation techniques. However, these two concepts are fundamentally different and require distinct approaches. The didactic obstacle here lies in insufficient understanding of the difference between these two notions of limit. This confusion may stem from a misinterpretation of the concepts taught, an incorrect assimilation of limit calculation methods or a lack of practice and understanding of the underlying concepts. To remedy this didactic obstacle, it is important to clarify and clearly distinguish between the concepts of calculating the limit of a geometric sequence and the limit of a rational function. Teachers should provide precise explanations and relevant examples to help students understand the distinct nature of these two notions.

Pedagogical obstacles

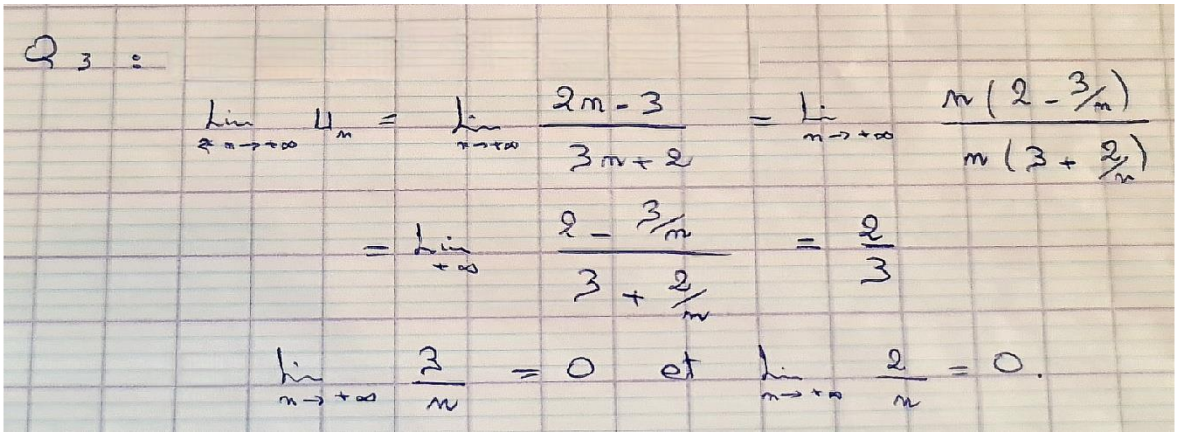


Figure 8. Student 6

Student 6 (Figure 8) provided a correct answer, but his method of arriving at this answer was long and detailed. He demonstrated a precise understanding of the calculation, but might have arrived at the same result by simplifying his method and taking solutions that are more efficient. Although the answer is technically correct, a more concise approach could have been used to solve the problem more efficiently and save time. 99% of the students with a correct answer answered in the same way as student 6 (Figure 8), which led us to orally question a few teachers in this way, and then the majority answered that you have to factor to calculate this limit, and not used the limit of a homographic sequence. After some analysis of the General Pedagogical Guidelines (MEN, 2007), we found that there is no such thing as the limit of a homographic sequence, and that you have to use the usual limits in the form : $\frac{1}{n};\frac{1}{n^2};\frac{1}{n^3};\frac{1}{n^p}$ such that $p \geq 4$. In this context, the general pedagogical guidelines require revision to simplify the task of calculating a homographic sequence.

The pedagogical obstacle identified in question 6 arises from the students' lack of familiarity with the type of question asked. It is possible that they have not encountered this type of question previously, or they may not have sufficiently developed the skills needed to answer it adequately.

CONCLUSIONS

In conclusion, this article confirms the hypotheses put forward previously. The results show that difficulties in learning to converge a numerical sequence in students in the 2nd year of the baccalaureate, experimental science series, are associated with a few obstacles: didactic, epistemological and pedagogical. It is important to recognize these obstacles and implement appropriate pedagogical strategies to overcome them. This can include clear, structured explanations, the use of concrete, relevant examples, hands-on, interactive activities, and pedagogical approaches that help students build links between the convergence of numerical sequences and other previously acquired mathematical concepts. However, this research has certain limitations, such as the complexity of the proofs, the dependence on assumptions, the rigidity of the methods, the limited practical applicability and the sensitivity to round-off errors.

In view of the above, using the TARL (Teaching At the Right Level) approach will be very efficient in addressing the diverse learning needs of students in MOROCCO (Binaoui, A, Moubtassime, M, & Belfakir, L, s. d.). By implementing TARL, teachers can assess the individual competency levels of students and group them accordingly. This allows for tailored instruction that meets each student's specific needs and provides appropriate learning challenges. The TARL approach ensures that students are neither overwhelmed by material beyond their current understanding nor bored by content that is too easy for them. Instead, it promotes an optimal learning environment where students are actively engaged and motivated to learn. By utilizing differentiated teaching strategies, such as targeted resources and interactive activities, teachers can effectively support students at their respective levels and foster continuous progress.

This research will provide crucial information for tailoring future interventions to students' needs in learning to converge numerical sequences. It will enable teachers to design more effective teaching strategies, develop appropriate resources and offer personalized support to help students overcome difficulties and succeed in this specific area of mathematics. Future research should examine the transition from high school to university in the case of the convergence of a numerical sequence.

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